EARTHSC 5205 Project 4 - Build-a-Planet

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1 Motivation

Understanding terrestrial planet formation can give us key insights as to how life is created in the universe. Various parameters such as core and mantle composition, surface density, pressure, gravity, temperature, and its host star can significantly impact the chances of life being found in the universe. Through the use of modern-day simulations, we can model the formation of a range of exoplanets. Through altering the parameters mentioned above, we can simulate properties of planets, such as mass and radius, to give us further insight into finding habitable worlds.

In this project, we explore the formation of terrestrial planets. in the form of a multitude of simulations using ExoPlex.The planet we have selected to explore is TRAPPIST-1e from the TRAPPIST-1 system. Through previous papers we have read this semester (Fulton et al., 2017; Seager et al., 2007; Chen and Kipping, 2016), we can confirm that this planet is likely rocky, although smaller than Earth but with similar density. Our goals are to be able to :

1. Effectively use ExoPlex to calculate a planet's possible density and structure.
2. Calculate planet structures from its mass-proportions compared to stellar.
3. Place the structure and composition in the context of mineral proportions and orbital parameters.

2 Methods

2.1 Warm Ups

To begin this planet-creation journey, we first did a few warm ups to become familiar with ExoPlex. The goal of these warm ups were to test the accuracy of ExoPlex under various conditions. We first modeled the radii of Mercury, Earth, and Mars using the default solar compositions and known masses of the planets. We then explored what effects adjusting the molar ratio of Fe/Mg has on the size and structure of a planet. From there we explored the consequences of moving Fe from the core to the mantle as FeO, pushing ExoPlex to the limit as we did so. After that we added light elements to the core and analyzed that effect, again pushing ExoPlex to it's limit. Finally, we analyzed the mineral proportions through the mantel and how they changed with Si/Mg molar ratios. By the end of the warm ups, we had a pretty good grasp on how to properly use ExoPlex for our own planet simulations.

2.2 Molar Ratios

The planet we have selected is TRAPPIST-1e from the TRAPPIST-1 system. We can confirm this planet is likely rocky after consult some of the papers we previously read this semester (Fulton et al., 2017; Seager et al., 2007; Chen and Kipping, 2016). The planet is is smaller than Earth, but has around the same density, which indicates a rocky exterior.

We found the radius and mass of this planet using the NASA Exoplanet Archive. The $\left[\frac{\mathrm{Fe}}{\mathrm{H}}\right]$ value was found via IOPscience. By referencing Figure 3 in Griffith et al. (2020), we estimated: $\left[\frac{\mathrm{Mg}}{\mathrm{Fe}}\right] \approx 0.08$ and $\left[\frac{\mathrm{Si}}{\mathrm{Fe}}\right] \approx 0.07$. We can find our $\left[\frac{\mathrm{X}}{\mathrm{H}}\right]$ values using the following general formula:

$$\frac{X}{H} = \frac{X}{Fe} + \frac{Fe}{H} \tag{1}$$

Knowing $\left[\frac{\mathrm{Fe}}{\mathrm{H}}\right] \approx 0.04$ for our TRAPPIST-1 star, we can further estimate: $\left[\frac{\mathrm{Mg}}{\mathrm{H}}\right] \approx 0.12$ and $\left[\frac{\mathrm{Si}}{\mathrm{H}}\right] \approx 0.11$.

To find $[\frac{Fe}{Mg}]$ and $[\frac{Si}{Mg}]$, we use this formula:

$$\frac{X}{Mg} = 10^{\frac{X}{H} - \frac{Mg}{H}} \tag{2}$$

where X is either Fe or Mg.

2.3 Range of Planet Structure

We created a function to try and determine the values for $[\frac{Fe}{Mg}]$ and $[\frac{Si}{Mg}]$. By holding the mass of the planet constant and adjusting the molar ratios, mantle, and core compositions, we were able to get a range of possible planet structure by computing various radii and densities.

3 Results

3.1 Warm Up Results

The goal of the warm up was to get comfortable with Exoplex. By using it and changing one parameter at a time, we were able to get a feel for how altering each parameter impacts the formation of the planet. We found that when it comes to strictly reproducing the sizes of the planets that the $[\frac{Fe}{Mg}]$ and $[\frac{Si}{Mg}]$ have the greatest impact on the predicted size of the planet. This is due to the must large relative abundance of Fe, Si, and Mg compared to the other elements in the simulated planets.

3.2 Molar Ratios

Using equations 1 and 2, we found molar ratios of :
$[\frac{Fe}{Mg}] = 0.832$
$[\frac{Si}{Mg}] = 0.977$

3.3 Size and Distance in Context

This planet's radius is about 0.7 times that of Earth, which puts it in the range of likely rocky planets. In general, planets smaller than around 1.6 Earth radii are expected to have solid, rocky compositions without thick gas envelopes.

3.4 Refractory Composition

The luminosity of TRAPPIST-1 is about 0.000522 times the Sun's. Thus, the planet ends up receiving over four times the solar flux Earth gets, which indicates that it has a high surface temperature and atmosphere retention.

The likely refractory composition of the star indicates that TRAPPIST-1e likely has a solid rocky makeup. Using our estimated molar ratios of $[\frac{Fe}{Mg}]$ and $[\frac{Si}{Mg}]$ and known mass of 0.772 $M_{\oplus}$ for our planet, we have a known radius of 0.910 $R_{\oplus}$ and density of 1.024 $\rho_{\oplus}$. It is important to note that we had varied results given the uncertainties in the measured mass of (+0.026, -0.027) Earth masses. Small variations in this value create large discrepancies when trying to replicate this planet using ExoPlex.

3.5 Range of Planet Structure

There is no combination of parameters that would decrease our estimated radius to 0.91 Earth radii, our known radius, without increasing our $[\frac{Fe}{Mg}]$ value, which indicates that TRAPPIST-1e has a very iron-abundant core.

We can also show a few iterations using our estimated $[\frac{Fe}{Mg}]$ value of 0.832 that would give us radii close to 0.91 Earth radii; these estimated radii will fall within the known uncertainty bounds of our exoplanet: (+0.026, -0.027). As we can see from the rows 2-4, we cannot estimate a radius of less than 0.935 Earth radii and a $[\frac{Fe}{Mg}]$ ratio of 0.832, no matter the combination of parameters.

Radius ($R_\oplus$)	Mass ($M_\oplus$)	Density ($\rho/\rho_\oplus$)	[Fe/Mg]	[Si/Mg]	FeO (mantle)	Si (core)	O (core)	S (core)	Mantle Comp
0.910	0.772	1.024	1.300	0.977	0	0	0	0	SiO=54.6 MgO=37.5 CaO=3.7 AlO=4.3
0.935	0.772	0.944	0.832	0.977	0.15	0.03	0	0.01	SiO=50.3 MgO=36.2 CaO=3.5 AlO=4.1
0.934	0.772	0.947	0.832	0.977	0	0	0	0	SiO=54.6 MgO=37.5 CaO=3.7 AlO=4.3
0.935	0.772	0.944	0.832	0.977	0.15	0.05	0	0.03	SiO=47.8 MgO=35.5 CaO=3.5 AlO=4.1
0.923	0.772	0.982	1.023	0.977	0.12	0.005	0	0.035	SiO=53.6 MgO=37.2 CaO=3.6 AlO=4.2
0.910	0.772	1.024	1.160	0.670	0.2	0.03	0	0.05	SiO=53.6 MgO=37.2 CaO=3.6 AlO=4.2
0.910	0.772	1.024	1.361	0.977	0.15	0.015	0.01	0.025	SiO=50.8 MgO=36.4 CaO=3.5 AlO=4.1

Table 1: Planetary model parameters, normalized to Earth values. Mantle composition is given in oxide percentages.

We can do a combination of parameter adjustments and increase our $[\frac{Fe}{Mg}]$ ratio within a reasonable amount (lower end of the spread of probable $[\frac{X}{Fe}]$ ratios from Figure 3 in Griffiths et al. 2020) and then adjust our parameters as indicated in the chart and obtain an estimated radius of 0.923 Earth radii and density of 0.982 Earth densities.

Many studies (like this one) indicate that TRAPPIST-1e cannot accommodate a Hydrogen-dominated atmosphere, so the issue is not due to a lack of atmosphere (and if there was an atmosphere, it would likely be a thick, Earth-like atmosphere). Thus, our issue is the fact that our core is too light. To fix this, we can increase our $[\frac{Fe}{Mg}]$ ratio, decrease our $[\frac{Si}{Mg}]$ ratio, increase FeO in the mantle, increase Si and S in the core and decrease O, or some combination of all of the above. Thus, we have a dense planet, similar to Earths, with a lot of heavy materials within its core. We tried to increase the abundance of Fe within the core and mantle as much as possible.

The parameters of our most realistic simulated planet is as follows:

Parameter	Value
Mass ($M_\oplus$)	0.772
[Fe/Mg]	1.361
[Si/Mg]	0.977
Molar Fraction of Fe (mantle)	0.15
Weight Fraction of Si (core)	0.015
Weight Fraction of O (core)	0.005
Weight Fraction of S (core)	0.025

Table 2: Selected compositional parameters for the planet model.

which gave us an estimated radius of 0.910 Earth radii and a density of 1.024 Earth densities, which is equal to our known values.

Mantle Mineralogy	TRAPPIST-1e (%)	Earth (%)
Fe	0.07	0
FeO	5.04	0
SiO_2	50.8	52.5
MgO	36.4	39.2
CaO	3.5	3.8
Al_2O_3	4.1	4.5

Table 3: Comparison of mantle mineralogy between TRAPPIST-1e and Earth.

3.6 Mantle Mineralogy

Here, we can see the differences between the mantle mineralogy of TRAPPIST-1e and Earth. For the most part, these values are quite similar. This is because they have nearly identical densities. The main difference is iron, which can be found in the mantle of TRAPPIST-1e, but is solely within the core of Earth, according to ExoPlex. That being said, there are hundreds of combinations of parameters that would lead to similar results, and there is a lot of uncertainty when it comes to the 'known' mass and radius estimates. We can, however, conclude that TRAPPIST-1e is similar in terms of composition to the Earth, and has a very heavy core (high abundance of iron and other heavy elements).

4 Conclusion

Through a range of simulations using ExoPlex, we were able to model TRAPPIST-1e with a realistic internal structure and composition. By adjusting molar ratios of key refractory elements like Fe and Si, and fine-tuning the distribution of core and mantle materials, we found that a planet with a mass of 0.772 $M_{\oplus}$, radius of 0.910 $R_{\oplus}$, and density of 1.024 $\rho_{\oplus}$ aligns well with observations. Our results suggest that TRAPPIST-1e is iron-rich, requiring a relatively heavy core and the presence of FeO in the mantle to match the measured radius. We determined that a $[\frac{Fe}{Mg}]$ ratio of 1.361 and $[\frac{Si}{Mg}]$ ratio of 0.977 were the most effective in achieving a model consistent with known planetary values.

The final mantle composition of TRAPPIST-1e is remarkably close to that of Earth, with notable differences primarily in iron content. This supports the idea that TRAPPIST-1e is a dense, rocky planet with a similar overall structure to Earth, albeit with heavier core components. While uncertainties in observational data leave room for variation, our model demonstrates how planetary interiors can be reconstructed through stellar abundance data and bulk planet properties. With continued improvements in exoplanet detection and modeling, studies like this can provide further insight into the likelihood of habitability on worlds beyond our solar system.