Exoplanet Detection Methods

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We present the results using data from NASA's exoplanet and comparing their radii, masses, period, and orbital distances from their star. Furthermore, in this report it includes the radial, transit, and imaging sensitivities. Moreover, these plots show the different detection methods used to find these exoplanets. The results of these plots dictate that these detecting methods are quite effective to find gas giants; however, these detecting methods are less effective with finding terrestrial planets. These results are a consequence of how limiting these detection methods can be for detecting terrestrial planets. Moreover, these detection methods work more effectively depending on how close to their parent star these planets are, whether using radii or mass, and using periods or days.

Initially, we started out by creating the sensitivity lines for the different types of direct imaging. For finding the sensitivity lines for the radial velocity it is imperative to examine the formula for detection rate.

$$K = \frac{m_p}{m_*} \cdot \sqrt{\frac{G\,m_*}{a}} \cdot sin\, i$$

This formula is used for detection rate of radial velocity where m_p represents the mass of the planet, m_* represents the mass of the star that the exoplanet is orbiting, G is the gravitational constant, a is the semi major axis, and i is angle that the planet is detected from. However, when creating the sensitivity lines we determined that the angle that we observed these orbits from was 90 degrees, thereby assuming these planets are edge-on orbits. Afterwards, we used the value of K to be 0.5 m/s, and our minimum star to be 0.5 solar masses. This allowed us to create a graph depicting a wide array of minimum radii that can be detected. The rearranged formula is shown below.

$$m_p = K \cdot m_* \cdot \sqrt{\frac{a}{Gm_*}}$$

To determine the transit sensitivity, we used the following formula. Where the limiting case would be around 1 -1.5 $R_\oplus$. The current formula for finding the transit sensitivity is as follows.

$$R_\oplus = \frac{m_p}{\sqrt{3 \cdot \sqrt{\frac{period}{time}}}}$$

Using an edge on orbit and rewriting the equation we can find the minimum radius to be the following.

$$r_p = \sqrt{3 \cdot \sqrt{\frac{period}{time}}} \cdot R_\oplus$$

In our data plots we assume that the earth Radii is around 1 $R_\oplus$. By using this equation, this gives a range of periods that give radii sensitivity. This thereby gives a relationship between the radius and mass to find the minimum mass that can be feasibly detected.

The final sensitivity that was considered was direct imaging. This can be done by observing the planet-to-star contrast determined by both the reflected and emitted parts. The reflected planet- to- star contrast is based on the light reflected by the planet, whereas the emitted is determined by the planet's own luminosity. These two parts can be written as $f = f_{reflected} + f_{emitted}$. If we maximize our reflection from the star we can simply the formula to get f to be the following.

$$f = \left(\frac{R_p}{R_*}\right)^2 \left[\frac{1}{4}\left(\frac{1}{aR_*}\right)^2 + \frac{(exp(\frac{hc}{\lambda k_B T_*}) - 1)}{(exp(\frac{hc}{\lambda k_B T_p}) - 1)}\right]$$

Since $f_{reflected} \ll f_{emitted}$ then we can rearrange the formula to solve for R_p give the following.

$$R_p = R_* \sqrt{\frac{f}{\frac{(exp(\frac{hc}{\lambda k_B T_*})-1)}{(exp(\frac{hc}{\lambda k_B T_p})-1)}}}$$

We assume that f is at minimum 10^{-7}, temperature of the star is roughly equivalent to the sun, and the temperature of the planet is T_p =130K, h is planck's constant, c is the speed of light, and k_b is the boltzmann constant. Using λ to be roughly 10^{-5}nanometers, which is the wavelength of infrared. Using these values and correlating them to find the minimum mass of these planets. Doing this gives the minimum radii to be roughly $16.41R_{\oplus}$ and the minimum mass to be 114.7 $M_{\oplus}$.

After finding these values we can find the smallest semi-major axis using the equation $\theta = 1.22\frac{\lambda}{D}$, where D is the telescopes diameter which is 8 meters, and λ being the wavelength which is roughly 22.3 nanometers. Then by using a small angle approximation, finding the semi-major axes is simply a $= d \cdot \theta$. Where d is the distance to the star from earth, which we set to 8 . Applying this method gives the result of 11239.5 days with a minimum separation of about 8 au.

By plotting several graphs, these graphs Mass [$M_{\oplus}$] v.s Period (days), Mass v.s Semi-Major Axis, and Radius v.s Period. These plots depict where different detection methods find these exoplanets. These plots clearly demonstrate that certain detection methods are more effective depending on the certain conditions. Also, overlaying all these plots with information about our solar system it is evident that the planets that are detected were mostly gas giants, and barely any of these planets come close to terrestrial planets in our solar system. The data points of planets detected by different detection methods; for radial velocity it is 99.3% accurate, for transits it is 98.9% accurate, and 86.6% accuracy for direct imaging accurate.

Looking at the Mass v.s Period graph it is evident that the points have a mixture of radial velocity and transit detection method with very few direct imaging. Furthermore, the sensitivities are all above the terrestrial planets in our solar system, so most of these techniques mostly do not detect these types of planets. These techniques mostly get gas giants. Moreover, transit method works effectively around a period of 1 to 100 days; whereas, radial velocity works generally pretty well around periods of 10 to 10^5days. However, direct imaging did not give a lot of information on this plot, only points in the range of $10^2$to $10^5 M_\oplus$ and around $10^4$to 10^6 $R_\oplus$.

Examining the Mass v.s Semi - Major Axis graph, there are several aspects that carry over from the Mass v.s Period graphs. These mainly being that radial velocity detection gets a wider spread of planets compared to transit and direct imaging. However, on this graph there appears to be more direct imaging planets detected at distances furthest away from their star. Furthermore, transits are able to detect planets that are closer to their star more effectively. Moreover, the radial and transit sensitivities are above the terrestrial planets in our solar system.

Finally, for both the Radius v.s Period graph and Radius and Semi-Major Axis graphs both plots give mostly transit detection with large group around 1 to 10 $R_\oplus$. In the Radius v.s Period graph, most of the points are around 0.55 days and roughly over 1,000 days. With the Radius v.s Semi-Major Axis, the points were around 0.055 au and 0.55 au.

These results show that these detection methods are most effective for certain conditions. Based on these plots it is clear that transit is most useful to find the radii of planets with a large span of orbital periods, but only able to detect planets near their star. However, radial velocities are useful to detect a wide range of planetary masses from distances from their stars. Transits can be used to find masses of planets but it can most effectively be done Finally, direct imaging is useful for distances that are extremely far away from their star.

Finally, using these factors to help detect a Jupiter sized planet around a sun like star, gave some very interesting results. Using the same assumptions and equations used in radial velocity, we found that the radial velocity of this type of planet is around $17.6\ \mathrm{m}s^{-1}$. This speed would be able to be detected my our current state of the art systems. Furthermore, the time of the transit is roughly 2.47 days. This would be detectable with our current technology, which has a limit of a year. This calculation was done by rearranging the equation for transits and solving for time. As previously discussed the minimum distance and radius that our system can detect is around $16.4\ R\oplus$, and a semi major axis of 8 au. With Jupiter being roughly $11R\oplus$, and a distance of 5.2 au, our Jupiter would not be detected by direct imaging.

In conclusion, these different types of detection methods work effectively depending on how close you detect them from their parent star or their orbital period. Transits work effectively with short semi-major axes, short periods, and measuring planet's radii. However, radial velocities work better with a wider spread of semi-major axes, and periods, and determining planets masses. These plots do not effectively show any terrestrial planets that fit under our solar system due to the limitations of these detection methods. As a result there are possibly more planets in these start systems that are not accounted for. Moreover, these detection methods are capable of capturing more gas giants than terrestrial planets.