EARTHSC 5205 Project 3 - Measuring Stellar Elemental Abundance

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1 Motivation

Determining the elemental abundance of stars is essential for understanding their physical properties and the characteristics of orbiting planets. One key approach involves analyzing the atomic and ionization states of elements in stellar atmospheres. Na serves as a valuable tracer due to its strong absorption features in stellar spectra. These absorption lines provide insights into temperature, electron pressure, and ionization balance, all of which are crucial for studying stars and their planetary systems. By applying the Boltzmann and Saha equations, we can determine the excitation and ionization states of Na under different conditions, allowing us to infer its abundance and distribution relative to hydrogen.

In this paper, we apply the curve of growth method to measure the elemental abundance of Na in a Sun-like star. We compute the number densities of Na atoms in various states—ground-state, neutral, and ionized—to determine the total Na abundance. This approach provides a detailed characterization of Na in stellar atmospheres, improving our understanding of its role in astrophysical processes.

2 Methods

To begin, we firstly imported a data file containing an Na spectrum and plot it (Fig. 1). We then calculate equivalent width using Equation (1),

$$ew = \sum_{\lambda_1}^{\lambda_2} (1 - F_\lambda)\, d\lambda \tag{1}$$

where F_λ is our observed wavelength.

2.1 Ratios of Excited and Ionized Na States

To start, we are given equivalent width (defined as the width of a rectangle with a height equal to that of continuum emission such that the area of the rectangle is equal to the area in the spectral line.), as well as the atomic flux.

We begin these calculation for number densities by finding the energy of various excited states of Na. To do so, we first find wavelength at the first peak of Na (which we observe to be at 5890 Å), and use Equation (2) to find energy.

$$E = \frac{\hbar c}{\lambda} \tag{2}$$

From this, we use degeneracy factors of $g = 2$ and $g = 6$ coupled with the Boltzmann Equation (3) to find a ratio of excited Na atoms to ground state Na.

$$\frac{N_2}{N_1} = \frac{g_2}{g_1} e^{\frac{-E_1}{k_B T}} \tag{3}$$

Next we find the ratio of ionized to neutral Na atoms. To do this, we will use the Saha equation. We are given parameters for unknowns that we are unable to solve for, and directly plug in to the Saha equation. Both the parameters and Saha Equation (4) are shown below.

$T = 5772$ K
$P_e = 1$ Pa
$Z_I = 1$
$Z_{II} = 2.4$
$\chi = 5.6$ eV

$$\frac{Na_{II}}{Na_I} = \frac{2k_B T}{P_e} \left(\frac{2\pi m_e k_B T}{\hbar^2} \right)^{3/2} e^{\frac{-\chi}{k_B T}} \tag{4}$$

2.2 Column Density of Na in the Sun's Photosphere

We then computed the total column density of Na atoms in the Sun's photosphere with an assumed total number of Na atoms. To compute the total column density, we used Equation (5) below.

$$N_{Na} = N_1 \left(1 + \frac{N_2}{N_1}\right) \left(1 + \frac{Na_{II}}{Na_I}\right) \tag{5}$$

2.3 Relative Abundance of Na to H

We were given the column density of H being 6.6×10^{23}. With it, we can compute the abundance of H to Na as a ratio, which is given below by Equation (6).

$$\frac{H}{Na} = 12 + \log \frac{N_{Na}}{N_H} \tag{6}$$

2.4 Number Densities of Various Na States

From the many ratios we have calculated, we are able to take these a step further to solve for the number densities of ground state, ionized, and Na. This can be done simply with a few equations, all of which are shown below. But first we define a few more parameters.

$g_{ground} = 2$
$Z_{Na} = 2$

For total Na number density -

$$N_{\text{Na tot}} = \frac{N_{Na}}{1 + \frac{Na_{II}}{Na_I}} \tag{7}$$

For ground state Na number density -

$$N_{\text{Na ground}} = N_{\text{Na tot}} \frac{g_{\text{ground}}}{Z_{Na}} e^{k_B T} \tag{8}$$

For ionized Na number density -

$$N_{\text{Na ionized}} = N_{\text{Na ground}} \frac{Na_{II}}{Na_I} \times ew \tag{9}$$

For neutral Na number density -

$$N_{\text{Na neutral}} = N_{\text{Na ground}} \times ew \tag{10}$$

2.5 Curve of Growth Method for Mg

Finally, we were given the opportunity to use the Curve of Growth method for a different element, and here we chose Mg. Using mostly the same code, we changed our data file from a Na spectrum to a Mg spectrum, and following the same process as the above steps (the only changes being $\lambda = 5167$, $g_1 = 2$ and $g_2 = 3$, as well as $\chi = 7.6462$ eV), we were able to find the number densities for the various states of Mg. See Fig 2. for the spectra plot, and the Results section for the number densities.

3 Results

Using the process described in the Methods section, we found ratios for Na as follows -

$$\text{Ground state to Excited state}: \frac{N_2}{N_1} = 0.0436$$

$$\text{Neutral to Ionized}: \frac{Na_{II}}{Na_I} = 5222$$

$$\text{Total Column Density}: N_{Na} = 4.49 \times 10^{18}\text{cm}^{-2}$$

$$\text{H to Na}: \frac{H}{Na} = 6.83$$

The number densities for Na are as follows -

$$\text{Ground state}: 4.47 \times 10^{24} \frac{\text{atoms}}{\text{cm}^2}$$

$$\text{Ionized state}: 1.95 \times 10^{28} \frac{\text{atoms}}{\text{cm}^2}$$

$$\text{Neutral state}: 3.73 \times 10^{24} \frac{\text{atoms}}{\text{cm}^2}$$

$$\text{Total Na}: 2.34 \times 10^{28} \frac{\text{atoms}}{\text{cm}^2}$$

Using the process described in the latter part of the Methods section, we found ratios for Mg as follows -

$$\text{Ground state to Excited state}: \frac{N_2}{N_1} = 0.0218$$

$$\text{Neutral to Ionized}: \frac{Mg_{II}}{Mg_I} = 85.36$$

$$\text{Total Column Density}: N_{Mg} = 6.49 \times 10^{16} \text{cm}^{-2}$$

$$\text{H to Na}: \frac{H}{Mg} = 4.99$$

$$\text{Na to Mg}: \frac{Na}{Mg} = 1.84$$

The number densities for Mg are as follows -

$$\text{Ground state}: 3.91 \times 10^{24} \frac{\text{atoms}}{\text{cm}^2}$$

$$\text{Ionized state}: 3.34 \times 10^{26} \frac{\text{atoms}}{\text{cm}^2}$$

$$\text{Neutral state}: 3.80 \times 10^{24} \frac{\text{atoms}}{\text{cm}^2}$$

$$\text{Total Na}: 3.38 \times 10^{2} \frac{\text{atoms}}{\text{cm}^2}$$

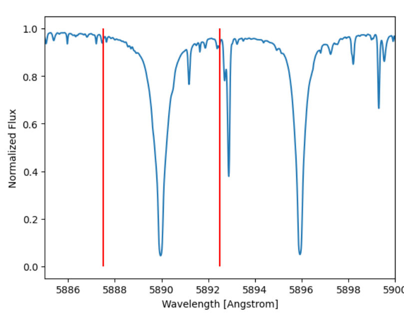
The large neutral to ionized ratio (and number density) for Na indicates that nearly all Na atoms in the stellar atmosphere are ionized due to the high temperatures present. This aligns with expectations from the Saha equation, which predicts increased ionization at high temperatures and low electron pressures. Similarly, the ground state to excited state ratio suggests that most Na atoms are not in their ground state but are thermally excited. Mg follows a similar pattern, but with a significantly lower ionization ratio, which can be attributed to its higher ionization potential compared to Na.

4 Conclusion

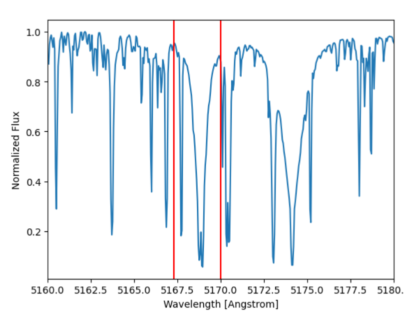
In this project, we analyzed the ionization and excitation states of Na and Mg in a Sun-like stellar atmosphere using the Boltzmann and Saha equations as a guide. Our results indicated that the vast majority of Na atoms exist in an ionized state with a neutral-to-ionized ratio of $\frac{Na_{II}}{Na_{I}} = 5222$.Similarly, the ground state to excited state ratio of $\frac{N_2}{N_1} = 0.0436$ suggests that most neutral sodium atoms are in excited states rather tahn the ground state. These findings are consistent with what we expect to see given the extreme temperatures we find in stellar atmospheres. Mg followed a similar trend to Na, but its significantly lower ionization ratio of $\frac{Mg_{II}}{Mg_{I}} = 85.36$ suggests that it is more resistant to ionization.

The total column densities of sodium and magnesium provide insight into the elemental composition of the stellar atmosphere, with a ratio of $\frac{H}{Na}$ ratio of 6.83 and $\frac{H}{Mg}$ ratio of 4.99. These values contribute to our broader understanding of chemical abundances in Sun-like stars, which can be used to infer the compositions of exoplanetary systems. Our results highlight the importance of using atomic spectroscopy to study stellar properties and demonstrate the effectiveness of the curve of growth method in determining elemental abundances.

5 Appendix



(a) Na Spectrum



(b) Mg Spectrum

Figure 1: Na and Mg Spectra