Measuring Stellar Elemental Abundance

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March 2025

Motivation

Research Questions:

1. Estimate the ratio of Na atoms in the ground state to Na atoms in excited states.
2. Estimate the ratio of neutral Na atoms to ionized Na atoms.
3. Compute the total column density of Na atoms in the sun's photosphere.
4. What is the abundance of Na relative to H?

Goals

- **Understand the physical properties of orbiting planets**
  By determining the elemental abundance of their stars
- **Can infer the compositions of exoplanetary systems**
  Through their sun's chemical abundances

Approach

- **Analyze the atomic and ionization states of Na**
  (strong absorption features in stellar spectra)
- **Absorption lines provide valuable insights**
  (temperature, electron pressure, ionization balance)

Process

- **Apply Growth Method**
  Measure the elemental abundance of Na in a Sun-like star
- **Compute number densities of Na atoms in various states**
  To determine total Na abundance

Methods

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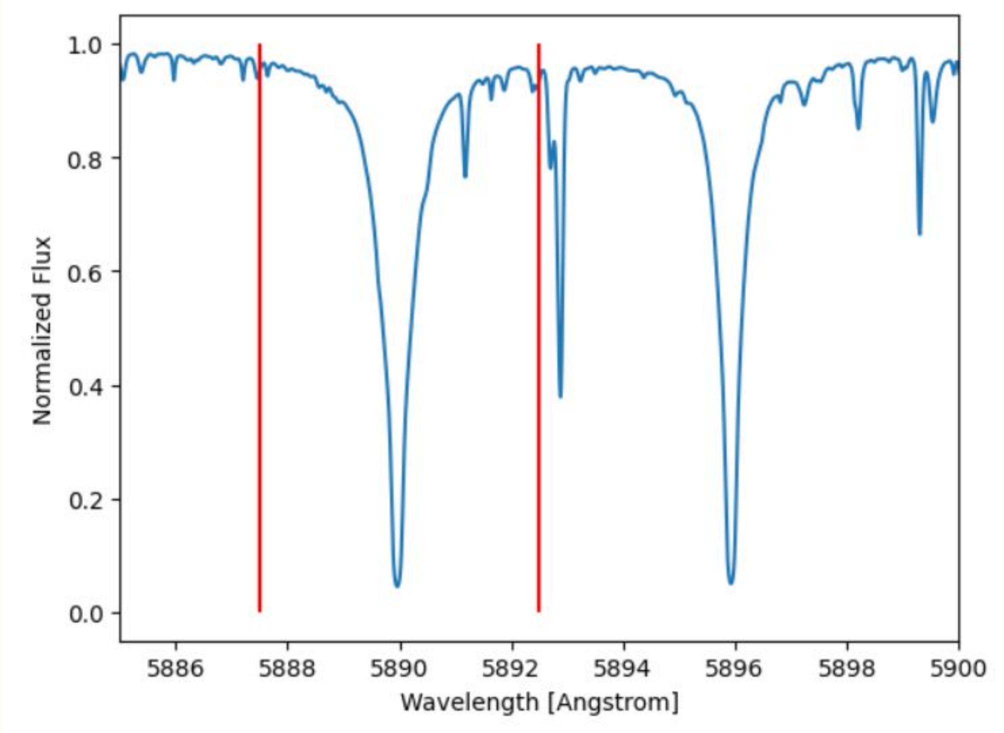
Sodium Spectrum



Calculating equivalent width

Equation:

$$ew = \sum_{\lambda_1}^{\lambda_2} (1 - F_\lambda)\, d\lambda$$

Defined as the width of a rectangle with a height equal to that of continuum emission such that the area of the rectangle is equal to the area in the spectral line.

Ratios of Excited and Ionized Na States

2.1

Use wavelength at first peak of Na to calculate energy

(5890 ˚A)

Equation to find energy at peak wavelength:

$$E = \frac{\hbar c}{\lambda}$$

Find a ratio of excited Na to ground state Na atoms

Using degenergy factors of g=2 and g=6, and the Boltzmann Equation:

$$\frac{N_2}{N_1} = \frac{g_2}{g_1} e^{\frac{-E_1}{k_B T}}$$

Find the ratio of ionized to neutral Na atoms

Saha Equation:

$$\frac{Na_{II}}{Na_I} = \frac{2k_BT}{P_e}\left(\frac{2\pi m_e k_B T}{\hbar^2}\right)^{3/2} e^{\frac{-\chi}{k_B T}}$$

Parameters:

$$T = 5772 \text{ K}$$
$$P_e = 1 \text{ Pa}$$
$$Z_I = 1$$
$$Z_{II} = 2.4$$
$$\chi = 5.6 \text{ eV}$$

Column Density of Na in the Sun's Photosphere 2.2

Equation:
(Given an assumed total of Na atoms)

$$N_{Na} = N_1 \left(1 + \frac{N_2}{N_1}\right)\left(1 + \frac{Na_{II}}{Na_I}\right)$$

Relative Abundance of Na to H 2.3

Equation:
(Given a column density of H being 6.6×10^{23})

$$\frac{H}{Na} = 12 + \log \frac{N_{Na}}{N_H}$$

Number Densities of Various Na States

2.4

Total Na number density

$$N_{\text{Na tot}} = \frac{N_{Na}}{1 + \frac{Na_{II}}{Na_{I}}}$$

Ground state Na number density

$$N_{\text{Na ground}} = N_{\text{Na tot}} \frac{g_{\text{ground}}}{Z_{Na}} e^{k_B T}$$

Ionized Na number density

$$N_{\text{Na ionized}} = N_{\text{Na ground}} \frac{Na_{II}}{Na_{I}} \times ew$$

Neutral Na number density

$$N_{\text{Na neutral}} = N_{\text{Na ground}} \times ew$$

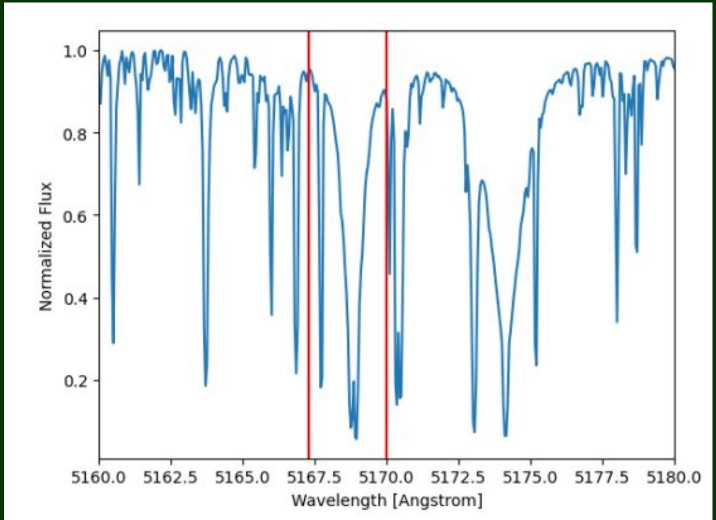
Curve of Growth Method for Mg 2.5

Followed same process/ equations as Na

Changes to parameters:

- $\lambda = 5167$
- $g_1 = 2$
- $g_2 = 3$
- $\chi = 7.6462$ eV

Magnesium Spectrum



Results

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Na	
Ground state to Excited state (N_2/N_1)	0.0436
Neutral to Ionized (Na_{II}/Na_I)	5222
Total Column Density (N_{Na})	4.49×10^{18} cm^{-2}
H to Na	6.83

Mg	
Ground state to Excited state (N_2/N_1)	0.0218
Neutral to Ionized (Mg_{II}/Mg_I)	85.36
Total Column Density (N_{Mg})	6.49×10^{16} cm^{-2}
H to Mg, Na to Mg	4.99, 1.84

Interpreting Results

Both Na and Mg ratios indicate

That nearly all Na (and significantly less Mg atoms) atoms in stellar atmosphere are ionized and are thermally excited due to high temperatures

Large neutral to ionized ratio

Nearly all Na atoms in stellar atmosphere are ionized due to high temperatures

Expectations from Saha equation

Predicts increased ionization at high temperatures and low electron pressures

Small ground to excited state ratio

Most Na atoms are not in their ground state, but are thermally excited

Mg follows a similar pattern

But has a significantly lower ionization ratio, which can be attributed to its higher ionization potential compared to Na

Conclusion

Analyzed the ionization and excitation states of Na and Mg in a Sun-like stellar atmosphere

Mg is more resistant to ionization
(neutral-to-ionized ratio of 85.36)

The vast majority of Na atoms exist in an ionized state
(neutral-to-ionized ratio of 5222)

Total column densities provide insight into the elemental composition of the stellar atmosphere

Most neutral sodium atoms are in excited states rather than the ground state
(ground-to-excited ratio of 0.0436)

Findings are consistent with expectations given extreme temperatures of stellar atmospheres
Helps understanding of chemical abundances in Sun-like stars, and can infer compositions of exoplanetary systems

Thank You