

Given the following grammar:

$$\begin{aligned} A &\rightarrow \text{id} = E \\ E &\rightarrow E+T \mid E \rightarrow E-T \mid T \\ T &\rightarrow T * F \mid T / F \mid F \\ F &\rightarrow (E) \mid \text{id} \\ \text{id} &\rightarrow a \mid b \mid c \dots z \text{ etc} \end{aligned}$$

Use bottom up parsing algorithm for the following input strings:

- a) $a=b+c-d$
- b) $a=b*c+d*e$
- c) $a=b*c*d$
- d) $a=a/b-c*d$
- e) $a=b+c-d-f$

Answer:

- a) $a=b+c-d$

Right Most Derivation

$$\begin{aligned} A &\rightarrow \text{id} = E \\ &\rightarrow \text{id} = \mathbf{E} - \mathbf{T} \\ &\rightarrow \text{id} = E - \mathbf{F} \\ &\rightarrow \text{id} = E - \mathbf{id} \\ &\rightarrow \text{id} = E - \mathbf{d} \\ &\rightarrow \text{id} = \mathbf{E} + \mathbf{T} - d \\ &\rightarrow \text{id} = E + \mathbf{F} - d \\ &\rightarrow \text{id} = E + \mathbf{id} - d \\ &\rightarrow \text{id} = E + \mathbf{c} - d \\ &\rightarrow \text{id} = \mathbf{T} + c - d \\ &\rightarrow \text{id} = \mathbf{F} + c - d \\ &\rightarrow \text{id} = \mathbf{id} + c - d \\ &\rightarrow \text{id} = \mathbf{b} + c - d \\ &\rightarrow \mathbf{a} = b + c - d \end{aligned}$$

Handle sequence:

$$\mathbf{a} \rightarrow \mathbf{b} \rightarrow \mathbf{id} \rightarrow \mathbf{F} \rightarrow \mathbf{T} \rightarrow \mathbf{c} \rightarrow \mathbf{id} \rightarrow \mathbf{F} \rightarrow \mathbf{E+T} \rightarrow \mathbf{d} \rightarrow \mathbf{id} \rightarrow \mathbf{F} \rightarrow \mathbf{E-T} \rightarrow \mathbf{id=E}$$

Stack	Input	Action
\$	a=b+c-d\$	Shift
\$a	=b+c-d\$	Reduce by id \rightarrow a
\$id	=b+c-d\$	Shift
\$id=	b+c-d\$	Shift
\$id=b	+c-d\$	Reduce by id \rightarrow b
\$id=id	+c-d\$	Reduce by F \rightarrow id
\$id=F	+c-d\$	Reduce by T \rightarrow F
\$id=T	+c-d\$	Reduce by E \rightarrow T
\$id=E	+c-d\$	Shift
\$id=E+	c-d\$	Shift
\$id=E+c	-d\$	Reduce by id \rightarrow c
\$id=E+id	-d\$	Reduce by F \rightarrow id
\$id=E+F	-d\$	Reduce by T \rightarrow F
\$id=E+T	-d\$	Reduce by E \rightarrow E+T
\$id=E	-d\$	Shift
\$id=E-	d\$	Shift
\$id=E-d	\$	Reduce by id \rightarrow d
\$id=E-id	\$	Reduce by F \rightarrow id
\$id=E-F	\$	Reduce by T \rightarrow F
\$id=E-T	\$	Reduce by E \rightarrow E-T
\$id=E	\$	Reduce by A \rightarrow id=E
\$A	\$	Accept

b) $a=b*c+d*e$

Right Most Derivation

$A \rightarrow id = E$
 $\rightarrow id = E + T$
 $\rightarrow id = E + T * F$
 $\rightarrow id = E + T * id$
 $\rightarrow id = E + T * e$
 $\rightarrow id = E + F * e$
 $\rightarrow id = E + id * e$
 $\rightarrow id = E + d * e$
 $\rightarrow id = T + d * e$
 $\rightarrow id = T * F + d * e$
 $\rightarrow id = T * id + d * e$
 $\rightarrow id = T * c + d * e$

$\rightarrow id = F * c + d * e$
 $\rightarrow id = id * c + d * e$
 $\rightarrow id = b * c + d * e$
 $\rightarrow a = b * c + d * e$

Handle Sequence:

$a \rightarrow b \rightarrow id \rightarrow F \rightarrow c \rightarrow id \rightarrow T * F \rightarrow T \rightarrow d \rightarrow id \rightarrow F \rightarrow e \rightarrow id \rightarrow T * F \rightarrow E + T$
 $\rightarrow id = E$

Stack	Input	Action
\$	a=b*c+d*e\$	Shift
\$a	=b*c+d*e\$	Reduce by $id \rightarrow a$
\$id	=b*c+d*e\$	Shift
\$id=	b*c+d*e\$	Shift
\$id=b	*c+d*e\$	Reduce by $id \rightarrow b$
\$id=id	*c+d*e\$	Reduce by $F \rightarrow id$
\$id=F	*c+d*e\$	Reduce by $T \rightarrow F$
\$id=T	*c+d*e\$	Shift
\$id=T*	c+d*e\$	Shift
\$id=T*c	+d*e\$	Reduce by $id \rightarrow c$
\$id=T*id	+d*e\$	Reduce by $F \rightarrow id$
\$id=T*F	+d*e\$	Reduce by $T \rightarrow T*F$
\$id=T	+d*e\$	Reduce by $E \rightarrow T$
\$id=E	+d*e\$	Shift
\$id=E+	d*e\$	Shift
\$id=E+d	*e\$	Reduce by $id \rightarrow d$
\$id=E+id	*e\$	Reduce by $F \rightarrow id$
\$id=E+F	*e\$	Reduce by $T \rightarrow F$
\$id=E+T	*e\$	Shift
\$id=E+T*	e\$	Shift
\$id=E+T*e	\$	Reduce by $id \rightarrow e$
\$id=E+T*id	\$	Reduce by $F \rightarrow id$
\$id=E+T*F	\$	Reduce by $T \rightarrow T*F$
\$id=E+T	\$	Reduce by $E \rightarrow E+T$
\$id=E	\$	Reduce by $A \rightarrow id=E$
\$A	\$	Accept

c) $a = b * c * d$

Right Most Derivation

$A \rightarrow id = E$
 $\rightarrow id = T * F$
 $\rightarrow id = T * id$
 $\rightarrow id = T * d$
 $\rightarrow id = T * F * d$
 $\rightarrow id = T * id * d$
 $\rightarrow id = T * c * d$
 $\rightarrow id = F * c * d$
 $\rightarrow id = id * c * d$
 $\rightarrow id = b * c * d$
 $\rightarrow a = b * c * d$

Handle Sequence:

$a \rightarrow b \rightarrow id \rightarrow F \rightarrow c \rightarrow id \rightarrow F \rightarrow T * F \rightarrow d \rightarrow id \rightarrow T * F \rightarrow id = E$

$ab\#Fc\#FT*Fd\#T*F\#=E$

Stack	Input	Action
\$	$a = b * c * d \$$	Shift
\$a	$= b * c * d \$$	Reduce by $id \rightarrow a$
\$id	$= b * c * d \$$	Shift
\$id =	$b * c * d \$$	Shift
\$id = b	$* c * d \$$	Reduce by $id \rightarrow b$
\$id = id	$* c * d \$$	Reduce by $F \rightarrow id$
\$id = F	$* c * d \$$	Reduce by $T \rightarrow F$
\$id = T	$* c * d \$$	Shift
\$id = T *	$c * d \$$	Shift
\$id = T * c	$* d \$$	Reduce by $id \rightarrow c$
\$id = T * id	$* d \$$	Reduce by $F \rightarrow id$
\$id = T * F	$* d \$$	Reduce by $T \rightarrow T * F$
\$id = T	$* d \$$	Shift
\$id = T *	$d \$$	Shift
\$id = T * d	$\$$	Reduce by $id \rightarrow d$
\$id = T * id	$\$$	Reduce by $F \rightarrow id$
\$id = T * F	$\$$	Reduce by $E \rightarrow T * F$
\$id = E	$\$$	Reduce by $A \rightarrow id = E$
\$A	$\$$	Accept

d) $a = a/b - c * d$

Right Most Derivation

$A \rightarrow id = E$
 $\rightarrow id = E - T$
 $\rightarrow id = E - T * F$
 $\rightarrow id = E - T * id$
 $\rightarrow id = E - T * d$
 $\rightarrow id = E - F * d$
 $\rightarrow id = E - id * d$
 $\rightarrow id = E - c * d$
 $\rightarrow id = T - c * d$
 $\rightarrow id = T/F - c * d$
 $\rightarrow id = T/id - c * d$
 $\rightarrow id = T/b - c * d$
 $\rightarrow id = F/b - c * d$
 $\rightarrow id = id/b - c * d$
 $\rightarrow id = a/b - c * d$
 $\rightarrow a = a/b - c * d$

Handle sequence:

$a \rightarrow a \rightarrow id \rightarrow F \rightarrow b \rightarrow id \rightarrow T/F \rightarrow T \rightarrow c \rightarrow id \rightarrow F \rightarrow d \rightarrow id \rightarrow T * F \rightarrow$
 $E - T \rightarrow id = E$

aa#Fb#T/FTc#Fd#T*FE-T#=E

Stack	Input	Action
\$	$a = a/b - c * d \$$	Shift
\$a	$= a/b - c * d \$$	Reduce by $id \rightarrow a$
\$id	$= a/b - c * d \$$	Shift
\$id=	$= a/b - c * d \$$	Shift
\$id=a	$/b - c * d \$$	Reduce by $id \rightarrow a$
\$id=id	$/b - c * d \$$	Reduce by $F \rightarrow id$
\$id=F	$/b - c * d \$$	Reduce by $T \rightarrow F$
\$id=T	$/b - c * d \$$	Shift
\$id=T/	$b - c * d \$$	Shift
\$id=T/b	$-c * d \$$	Reduce by $id \rightarrow b$
\$id=T/id	$-c * d \$$	Reduce by $F \rightarrow id$
\$id=T/F	$-c * d \$$	Reduce by $T \rightarrow T/F$
\$id=T	$-c * d \$$	Reduce by $E \rightarrow T$
\$id=E	$-c * d \$$	Shift

\$id=E-	c*d\$	Shift
\$id=E-c	*d\$	Reduce by id \rightarrow c
\$id=E-id	*d\$	Reduce by F \rightarrow id
\$id=E-F	*d\$	Reduce by T \rightarrow F
\$id=E-T	*d\$	Shift
\$id=E-T*	d\$	Shift
\$id=E-T*d	\$	Reduce by id \rightarrow d
\$id=E-T*id	\$	Reduce by F \rightarrow id
\$id=E-T*F	\$	Reduce by T \rightarrow T*F
\$id=E-T	\$	Reduce by E \rightarrow E-T
\$id=E	\$	Reduce by A \rightarrow id=E
\$A	\$	Accept

e) $a = b + c - d - f$

Right most derivation

$A \rightarrow \mathbf{id = E}$

$\rightarrow \mathbf{id = E - T}$

$\rightarrow \mathbf{id = E - F}$

$\rightarrow \mathbf{id = E - id}$

$\rightarrow \mathbf{id = E - f}$

$\rightarrow \mathbf{id = E - T - f}$

$\rightarrow \mathbf{id = E - F - f}$

$\rightarrow \mathbf{id = E - id - f}$

$\rightarrow \mathbf{id = E - d - f}$

$\rightarrow \mathbf{id = E + T - d - f}$

$\rightarrow \mathbf{id = E + F - d - f}$

$\rightarrow \mathbf{id = E + id - d - f}$

$\rightarrow \mathbf{id = E + c - d - f}$

$\rightarrow \mathbf{id = T + c - d - f}$

$\rightarrow \mathbf{id = F + c - d - f}$

$\rightarrow \mathbf{id = id + c - d - f}$

$\rightarrow \mathbf{id = b + c - d - f}$

$\rightarrow \mathbf{a = b + c - d - f}$

Handle Sequence:

$\mathbf{a \rightarrow b \rightarrow id \rightarrow F \rightarrow T \rightarrow c \rightarrow id \rightarrow F \rightarrow E+T \rightarrow d \rightarrow id \rightarrow F \rightarrow E-T \rightarrow f \rightarrow id}$
 $\rightarrow \mathbf{F \rightarrow E-T \rightarrow id=E}$

Stack	Input	Action
\$	a=b+c-d-f\$	Shift
\$a	=b+c-d-f\$	Reduce by id \rightarrow a
\$id	=b+c-d-f\$	Shift
\$id=	b+c-d-f\$	Shift
\$id=b	+c-d-f\$	Reduce by id \rightarrow b
\$id=id	+c-d-f\$	Reduce by F \rightarrow id
\$id=F	+c-d-f\$	Reduce by T \rightarrow F
\$id=T	+c-d-f\$	Reduce by E \rightarrow T
\$id=E	+c-d-f\$	Shift
\$id=E+	c-d-f\$	Shift
\$id=E+c	-d-f\$	Reduce by id \rightarrow c
\$id=E+id	-d-f\$	Reduce by F \rightarrow id
\$id=E+F	-d-f\$	Reduce by T \rightarrow F
\$id=E+T	-d-f\$	Reduce by E \rightarrow E+T
\$id=E	-d-f\$	Shift
\$id=E-	d-f\$	Shift
\$id=E-d	-f\$	Reduce by id \rightarrow d
\$id=E-id	-f\$	Reduce by F \rightarrow id
\$id=E-F	-f\$	Reduce by T \rightarrow F
\$id=E-T	-f\$	Reduce by E \rightarrow E-T
\$id=E	-f\$	Shift
\$id=E-	f\$	Shift
\$id=E-f	\$	Reduce by id \rightarrow f
\$id=E-id	\$	Reduce by F \rightarrow id
\$id=E-F	\$	Reduce by T \rightarrow F
\$id=E-T	\$	Reduce by E \rightarrow E-T
\$id=E	\$	Reduce by A \rightarrow id=E
\$A	\$	Accept