Optical Trapping using Doughnut Beam

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Abstract

Under the guidance of Prof. Arijit Kumar De , we modulated the phase of light to create a doughnut shaped beam and used the modulated beam to stably trap neutral dielectric particles. We compared the trap stiffness for doughnut beam of different beam waist with Gaussian beam. We used liquid crystal spatial light modulator to achieve this phase modulation of input plane Gaussian beam and utilise it in the dynamic manipulation of dielectric neutral particle.

1 Introduction to Optical Trapping

The principles of optical trapping were developed by Arthur Ashkin when he was trying to answer the question "Can we observe significant motion of small neutral particles using the forces of radiation pressure from laser light?". In a series of papers, he demonstrated that optical forces could displace and levitate micron-sized dielectric particles in both water and air, and he developed a stable, three-dimensional trap based on counter propagating laser beams. This seminal work eventually led to the development of the single-beam gradient force optical trap, or "optical tweezers".

2 Principle behind Optical Trapping

2.1 Historical Background

The fundamental principle behind optical trapping is the fact that light exerts force on matter. The possibility that light could exert pressure on matter goes back to Johannes Kepler, who in 1619 postulated that the radiation pressure due to the light from the sun is what causes the comet's tail to always point away from the Sun. Here is an interesting sketch by Martin Mc Kenna
First experimental evidence of radiation pressure due to an electromagnetic wave of any wavelength was demonstrated by P. Levedeb in his paper "First experimental evidence for the pressure of light on solid bodies", Annelen De Physik, Vol. 6, 1901.

2.2 Forces due to Radiation Pressure

2.2.1 Momentum carried by light

Consider a beam having a photon flux of N. This translates to N photons crossing an imaginary cross section every second. The momentum flux associated with these photons is given by

$$\frac{nNh}{\lambda}$$

where,

- N indicates the photon flux
- h is the Planck's constant
- λ is the wavelength of light in vacuum
- n is the refractive index of the medium of propagation

This can also be written as

$$\frac{nP}{c}$$

where P is the energy flux which can also be described as the power associated with the beam.

2.2.2 Force exerted on an interface

When light enters from one medium to another medium, it gets reflected and transmitted. Since the directions of reflected and transmitted beams are different from that of the incident beam, there is a change in momentum associated with the electromagnetic wave. This causes a force to act on the material medium.

Consider a light ray incident on a curved interface at an angle θ_i onto a material medium. It is refracted at an angle θ_r. The interface separates two media of refractive indices n_i and n_t. The force exerted on the material can be given by the expression

$$\vec{F} = \frac{n_i P_i}{c}\hat{r_i} - \frac{n_i P_r}{c}\hat{r_r} - \frac{n_t P_t}{c}\hat{r_t}$$

where,

- P indicates the power associated with the respective beam.
- $\hat{r_i}, \hat{r_r}, \hat{r_t}$ indicate the unit vectors along the incident, reflected and transmitted rays respectively
- c is the speed of light

The power carried by the reflected and transmitted rays can be described by the Fresnel equations.

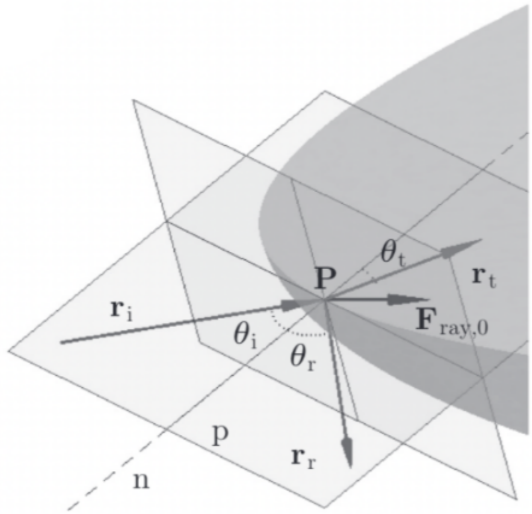


Figure 1: A ray of light incident on a curved surface at an arbitrary angle [1].

2.2.3 Fresnel Equations

Fresnel equations describe the reflection and transmission of light when it travels from one optical medium into another.
The reflection and transmission coefficients depend on the polarization of light.

For s-polarized light, the reflection coefficient is given by,

$$R_s = \left| \frac{n_i cos\theta_i - n_t cos\theta_r}{n_i cos\theta_i + n_t cos\theta_r} \right|$$

For s-polarized light, the transmission coefficient is given by

$$T_s = \frac{4n_i n_t cos\theta_i \theta_r}{|n_i cos\theta_i + n_t cos\theta_r|^2}$$

For p-polarized light, the reflection coefficient is given by

$$R_p = \left| \frac{n_1 cos\theta_2 - n_2 cos\theta_1}{n_1 cos\theta_2 + n_2 cos\theta_1} \right|$$

For p-polarized light, the transmission coefficient is given by

$$T_p = \frac{4n_i n_t cos\theta_i \theta_r}{|n_i cos\theta_r + n_t cos\theta_i|^2}$$

For circularly polarized light,

$$R = \frac{R_s + R_p}{2}$$

$$T = \frac{T_s + T_p}{2}$$

2.3 First observations by Ashkin

The above described theoretical formulation should help us to understand the initial results published by Ashkin.

2.3.1 Result of the experiment

In 1969, Ashkin published the first observation of the acceleration and trapping of micron-sized particles in a stable optical potential well created by forces of radiation pressure from a continuous wave laser. It was observed that the off axis spheres were immediately drawn towards the axis and were accelerated in the direction of propagation of the beam. These latex spheres were dissolved in a relatively transparent media. The bead was accelerated until it hit the other end of the container where it remained trapped. When the laser was blocked, the bead wandered off due to thermal forces resulting in Brownian motion [2].

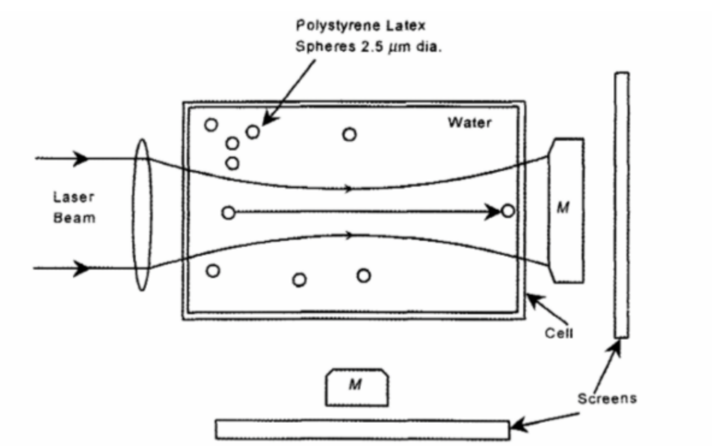


Figure 2: First observation of 0.25 μm transparent latex spheres in water being accelerated using radiation pressure [2].

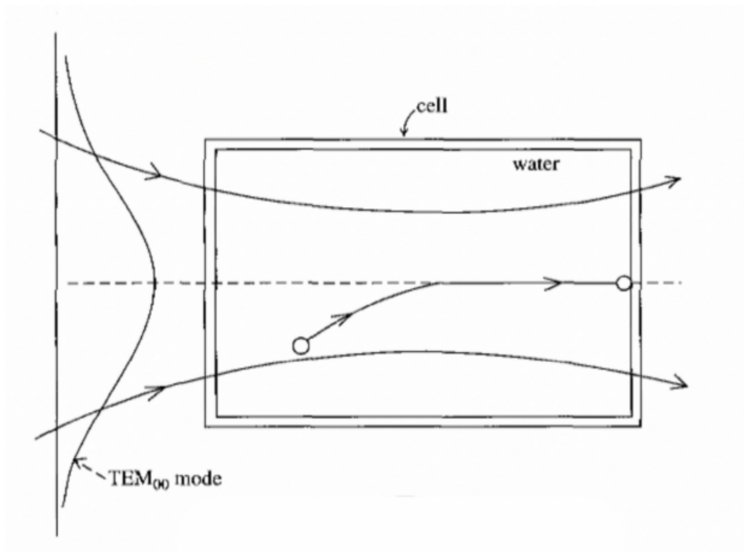


Figure 3: First observation of transverse force pulling the beads towards regions of high intensity created by a Gaussian laser beam [2].

2.3.2 Model proposed to explain the results

The results can be explained using the ray optics model as the size of the bead is much larger compared to the wavelength of laser being used.
Any ray upon interacting with the bead gets deflected due to the processes of reflection and refraction. The bead has a higher refractive index than the medium it is suspended in. The directions of these rays is not in the original direction of the ray thus causing a change in momentum of the ray. This causes an equal and opposite momentum being transferred to the bead.

The intensity profile of the laser is that of a Gaussian thus having more intensity near the axis of beam. Thus there is a net force towards the axis.

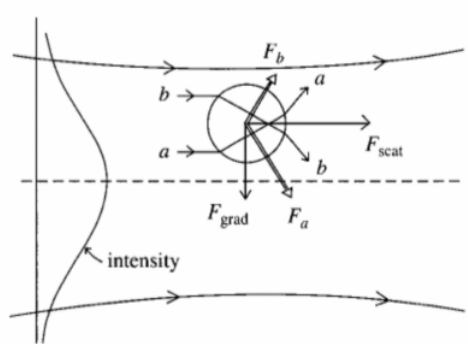


Figure 4: Description of the various forces acting on an off-axis bead [3].

The force acting along the the direction of propagation can be termed as the scattering force and the force in the axial direction to it is the gradient force. The gradient force is directly proportional to the intensity of the gradient. To counter the force acting along the axis of propagation, another counter propagating laser beam has been set up as shown in the figure below. This helped in holding the beads stably in position.

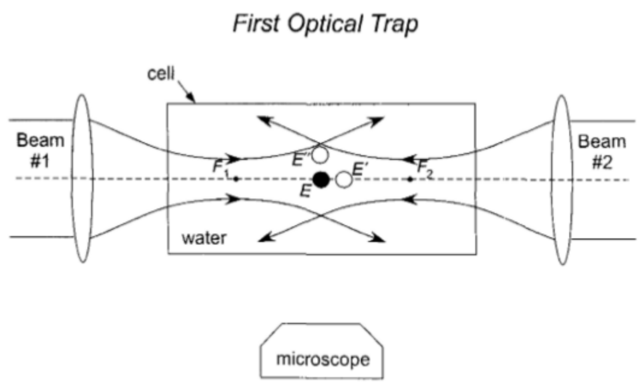


Figure 5: Diagram of the first stable optical trap where E represents the point of equilibrium point generated by two counter propagating laser beams. Displacement of the particle to E' and E'' results in a restoring force [2].

2.4 Single-beam gradient force optical trap for dielectric particles

The above described setup can be replaced by just one laser beam which is tight focused using a lens of high numerical aperture. A single strongly focused laser beam creates an axial gradient force which dominates the scattering force.

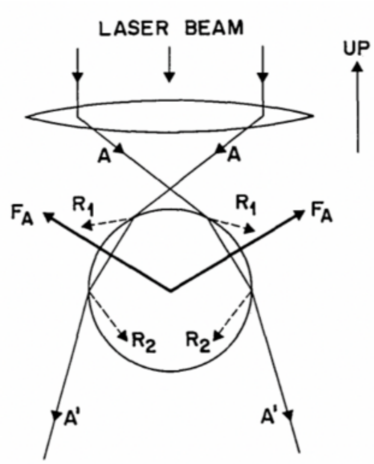


Figure 6: Figure representing a single beam trap [4].

The focused rays when interacting with a bead achieve a forward momentum which is more than the one they possessed before interacting with the bead, thus the bead experiencing a force in the opposite direction to that of the propagation direction of the beam. As the bead is situated on the axis, forces in any other direction cancel out each other due to symmetry. There is a small component of force that pushes the bead in the direction of propagation but it can be reduced by ensuring the beads are sufficiently transparent.

2.5 Dipole Approximation

Now we look at the dipole regime where the particle size is much smaller than the wavelength of the laser being used for trapping, the above developed model based on ray optics does not apply. The conditions of Rayleigh scattering are satisfied and the optical forces are calculated by assuming the particle to be a point dipole.
The scattering force is due to the absorption and re-radiation of light by the dipole. For a sphere of radius r, this force is given by [7]

$$F_{scatt} = \frac{I_0 \sigma n_m}{c}$$

$$\sigma = \frac{128\pi^5 a^6}{3\lambda^4}\left[\frac{m^2-1}{m^2+2}\right]^2$$

where

- I_0 is the intensity of incident light
- σ is the scattering cross section of the sphere
- n_m is the refractive index of the medium
- c is speed of light
- m is the ratio of refractive index of the particle to the refractive index of medium $(\frac{n_p}{n_m})$
- λ is the wavelength of the trapping laser

Scattering force is along the direction of propagation of beam and is proportional to the intensity of laser beam
The gradient force arises due to the potential energy of the dipole in a given inhomogeneous electric field. It is a conservative force and is proportional to the intensity gradient [7].

$$F_{grad} = \frac{2\pi\alpha}{cn_m^2}\nabla I_0$$

α is the polarizability of the sphere and is given by [7]

$$\alpha = n_m^2 a^3\left[\frac{m^2-1}{m^2+2}\right]$$

The gradient force acts along the intensity gradient when $m > 1$, i.e the suspended particles have a higher refractive index than the surrounding medium.
In the next section we look at how one can actually measure the extent to which the above discussed forces contribute to create a stable trap. To do this, one needs to study the trap stiffness.

3 Calibration and Trap Stiffness

The optical tweezers can generate forces in the range of 10^{-13} to 10^{-10} N. The important advantage of using optical tweezers is that we can not only generate these forces but also measure the force being acted on a particle. This can be done by calibrating the setup.

3.1 Equipartition Method

The trapped bead undergoes Brownian motion due to the thermal fluctuations. These can be used to obtain the trap stiffness. For an object in a harmonic potential with stiffness κ [8]:

$$\frac{1}{2}\kappa\langle\Delta x^2\rangle = \frac{1}{2}k_B T$$

where

- k_B is the Boltzmann's constant
- T is the temperature
- $\langle\Delta x^2\rangle$ variance of position of the trapped particle from the mean position

With each degree of freedom associated with the linear system, the potential energy is give by the above relation.
Thus, the shape of the particle, its height above the surface, and the viscosity of the medium need not be known to measure the trap stiffness. The drawback of this method is that any added noise and drift in position measurements serve only to increase the overall variance, thereby decreasing the apparent stiffness estimate [8].
The equipartition method requires accurate distance measurement at high bandwidth along the axis corresponding to the degree of freedom. We can then plot the spectral density of the fluctuations of the trapped bead.

3.2 Mathematical description of a trapped particle

The optical forces acting on a trapped particle are similar to that of a microscopic spring obeying Hooke's Law i.e, the force exerted on the particle when it is displaced along any of the three axes is proportional to its displacement.

The motion of a trapped particle of mass m in a medium of viscosity γ_0 at a temperature T using the famous Einstein–Ornstein–Uhlenbeck theory for Brownian motion and leads to a Langevin equation of the following form [8]:

$$m\frac{\partial^2 x}{\partial t^2} \quad + \gamma\frac{\partial x}{\partial t} + \kappa\Delta x = (2k_B T x)^{1/2}\eta(t)$$

where

- κ denotes the trap stiffness
- k_B is Boltzmann's constant
- $(2k_B T x)^{1/2}$ denotes random Gaussian noise and accounts for all the Brownian forces on the particle at a temperature T
- γ is the Stoke's drag term for a particle of radius r in a medium with density ρ and coefficient of viscosity v is given by $6\pi r\rho v$

3.3 Spectral Density of Fluctuations

A bead in an optical trap is going to fluctuate at a given amplitude for every given frequency. The bead will have a spectrum of fluctuations. The above equation can be solved for the mean quadratic fluctuation $\langle\Delta x^2\rangle$ as a function of frequency and the solution is given by the Lorentzian Power spectrum [8] :

$$\langle\Delta x^2\rangle(\omega) = \frac{4k_B T}{\gamma(\omega^2 + \omega_c^2)}$$

where

- ω denotes the frequency
- ω_c is the corner frequency

This equation tells by how much the bead is displaced on an average from the center of the trap at a particular frequency
Corner frequency is given by the expression.

$$\omega_c = \frac{\kappa}{\gamma}$$

The corner frequency can be obtained from the power spectrum graph. It is that point where the graph takes a turn and starts decaying. Thus, in principle the stiffness of the trap can be obtained by knowing the friction coefficient and corner frequency of the system.
In the next section, we look at how we can improve the trap stiffness by varying the intensity profile of laser beam.

4 Increase in trap stiffness using beam shaping

We can try reasoning out this phenomenon qualitatively. When the particle is on the beam axis, the axial trapping force is produced from the refraction of the off-axis rays within the beam. On-axis rays are detrimental to axial trapping as their back-scattered component results in a force on the particle in the beam propagation direction.
Hence, if a light beam with a doughnut-type mode instead of a Gaussian mode is used, the trapping efficiency is expected to be higher since the on-axis light intensity is zero in a doughnut beam.(Ashkin [1999], Simpson, McGloin, Dholakia, Allen and Padgett [1998], Chang and Lee [1988]).

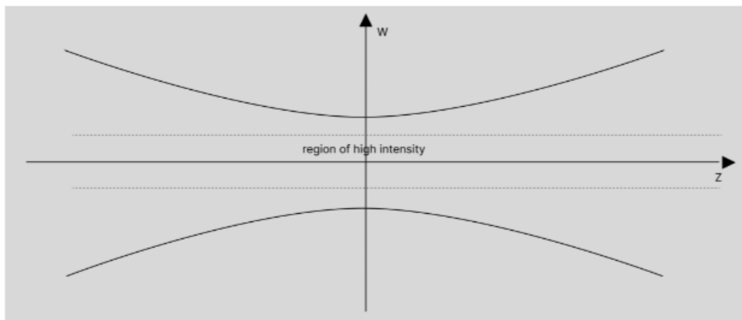


Figure 7: Gaussian beam propagation. Area enclosed by dotted lines represents region of high intensity

The second advantage of using a doughnut beam is that it can trap particles which have a relative refractive index less than unity. The particles in this case are pushed towards the dark centre of the beam and can be stably trapped.

5 The Doughnut beam

A doughnut beam, as the name implies is a ring-shaped light beam with null intensity center on the beam axis in the propagation direction.
The dark spot for a doughnut beam is defined as the full width at half maximum (FWHM) of the radial intensity distribution.

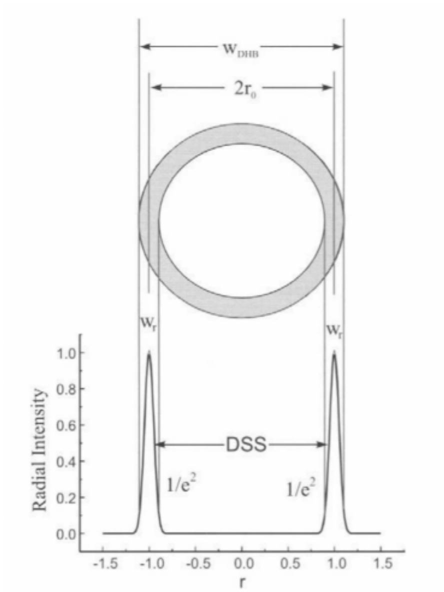
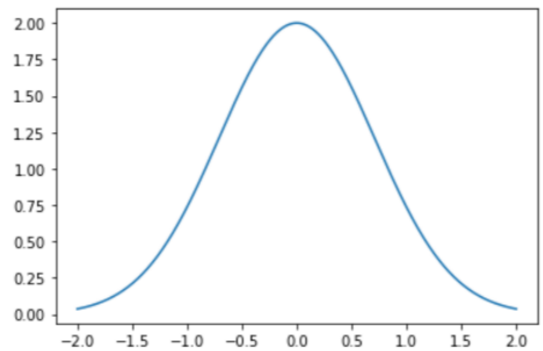


Figure 8: Figure describing intensity profile of Doughnut beam [5].

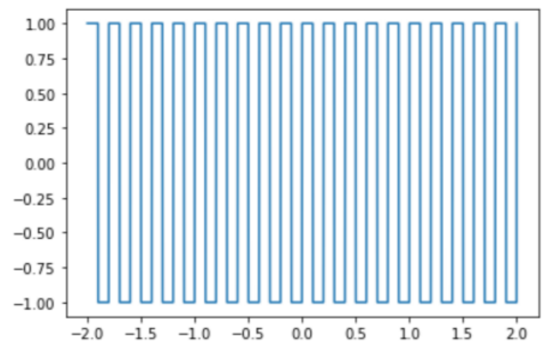
6 Process of beam shaping

6.1 Interaction of Laser beam with Diffraction grating

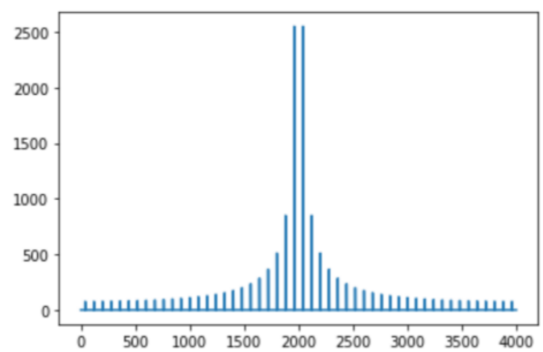
We will be using the principles of Fraunhofer diffraction to arrive at the output beam for an input laser beam projected onto a grating. The Fraunhofer diffraction pattern is the fourier transform of the aperture function. Fraunhofer diffraction can be thought of as the far-field approximation of the Fresnel diffraction pattern.
I have simulated the interaction of laser beam with the diffraction grating. Initially a simple continuous square pulse was used to represent the grating and the resultant diffraction pattern can be easily obtained by taking the fourier transform of the product of incoming signal and grating function.



(a) Gaussian intensity profile of read beam in one dimension



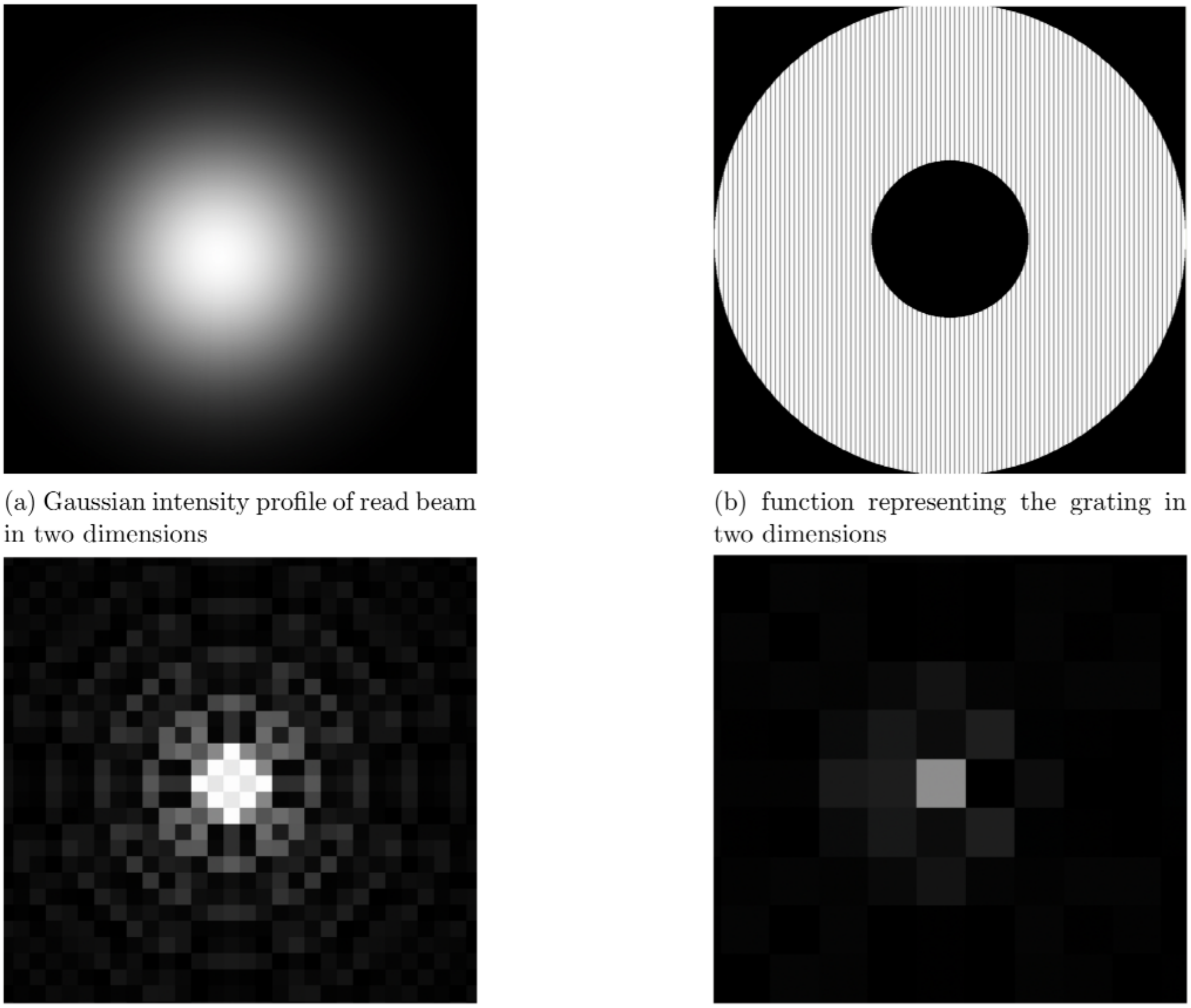
(b) function representing the grating in one dimension



(c) output diffraction orders

Figure 9: Simulating interaction of laser and diffraction grating in one dimension

To simulate the above interaction in 2D, a similar exercise has been carried out by modelling the input laser beam as a two dimensional Gaussian distribution in Grayscale. The grating is now a combination of a binary Grayscale grating onto which our circular aperture mask is superimposed. The results can be seen below.



(a) Gaussian intensity profile of read beam in two dimensions

(b) function representing the grating in two dimensions

(c) zeroth diffraction order

(d) first diffraction order

Figure 10: Simulating interaction of laser and diffraction grating in two dimensions

6.2 Liquid Crystals

In this section we will be studying how the Liquid crystals play a key role in modulating the phase of light. Liquid crystal is a state of matter between solid and liquid. These are much longer than they are wide, thus can be treated as dipoles in the presence of a field resulting in an intermolecular attractive force. Liquid Crystals have an orientation order but no positional order. They all point along the molecular director.

Liquid crystals are birefringent in nature i.e, light travels at different velocities depending on the angle between polarization of light and orientation of liquid crystals. The axis along the length of the crystal is called slow axis and the axis in the direction perpendicular to it is the slow axis. Light experiences a high index of refraction when its polarization is parallel to the slow axis of the liquid crystal.

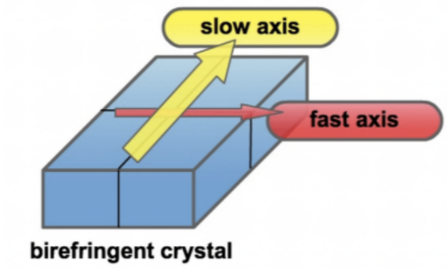


Figure 11: Figure describing the fast and slow axis of the Liquid Crystal source: intro to SLM

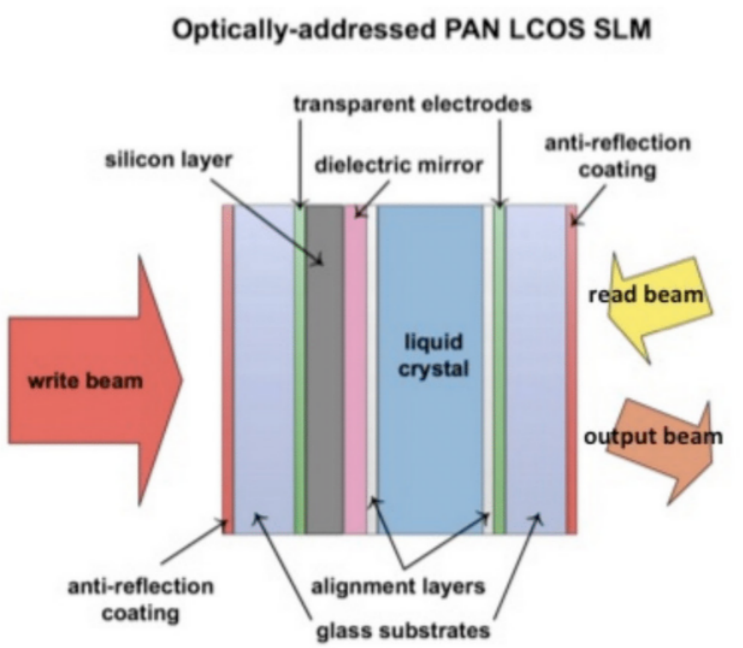


Figure 12: Figure describing the working of SLM source: intro to SLM

The write beam is used to create a computer generated hologram onto a photoconductive material whose resistivity decreases as the intensity of the write beam is increased. With a decrease in resistance, there is a higher voltage that passes to the liquid crystal cell. Areas of intense light causes silicon's resistivity to decrease which causes more voltage to pass through to the LC cell. This increases the strength of Electric field causing a greater tilt in the LC molecules.

When there is no voltage, the liquid crystal molecules are aligned parallel to the surface of the electrode. The voltage now imposed by the write beam tilts the molecular director of the liquid crystals. This changes the relative angle between the polarization of incoming beam (read light) and the molecular director of LC. Thus changing the refractive index that the read beam experiences upon interacting with the SLM. Greater tilt causes more phase modulation.

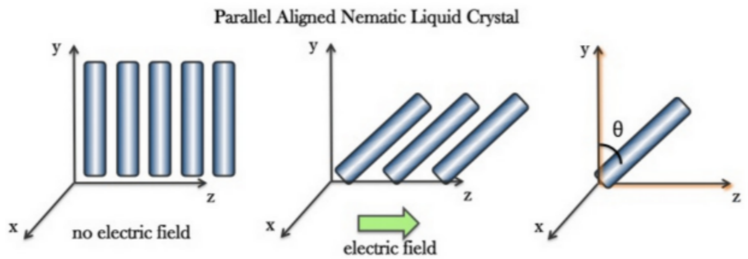


Figure 13: Figure describing phase modulation taking place through the tilt of Liquid Crystal molecules source: intro to SLM

7 Experimental Setup

In this section, we will look at the experimental setup required to perform the calibration on SLM as well as carry out the phase mdoulation. The diagram below gives us a basic idea of how SLM can be used to modulate the phase of light. Thus, it can be used to generate a doughnut beam. The source being used is a continuous wave laser of wavelength 800 nm. The half wave plate is used to rotate the polarization axis of the incoming laser light by 90 degrees.
The aim of our experiment in the lab was to optimize the intensity profile of the output dark hollow beam to obtain a trap with the highest stiffness. This was done by varying the inner radius and also changing the hologram generating the diffraction patterns.

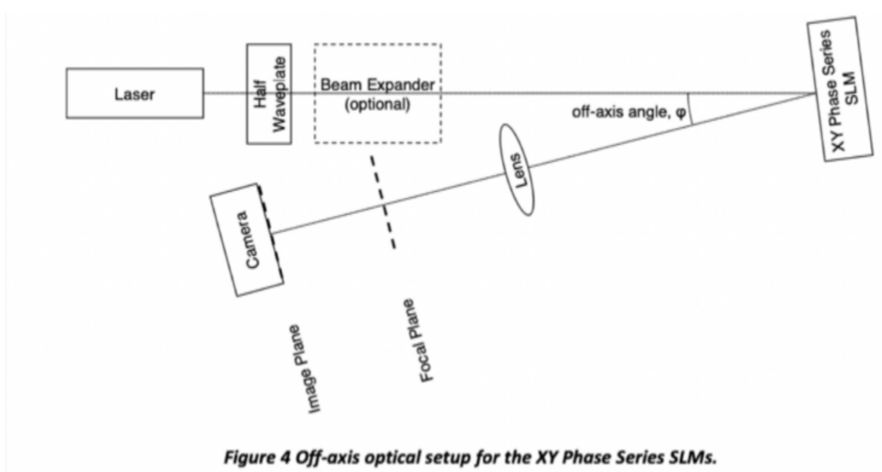


Figure 14: Figure describing our experimental setup to generate dark hollow beam [6]

7.1 Beam expander

A simple Keplerian setup consists of two diverging lenses are separated by a distance equal to the sum of their focal lengths. A Keplerian device focuses the light between its two lenses and then outputs an inverted beam. The beam comes to a focus between the two lenses. This provides an opportunity to spatially filter the beam using a pinhole. This also called mode cleaning. As the beam propagates away from the focus, it starts expanding until it reaches the second lens where it is then collimated.

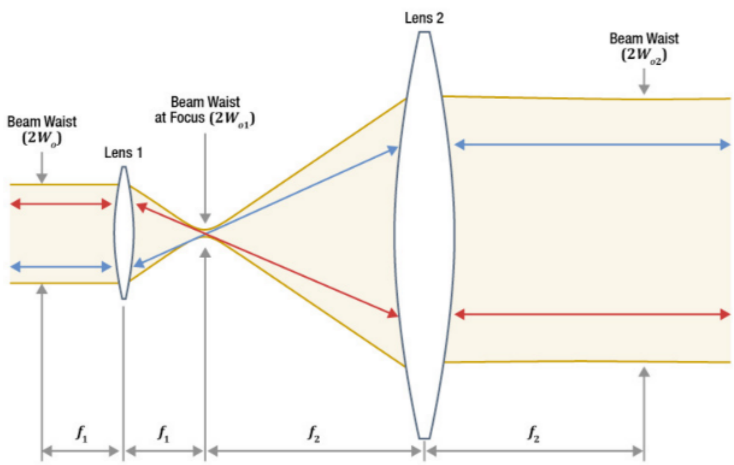


Figure 15: Figure describing Keplerian beam explander source: Thorlabs

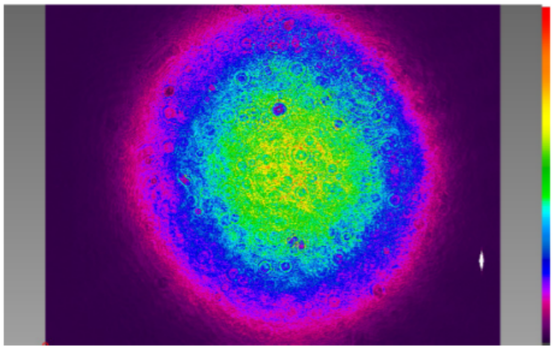
With the increased beam width, the input laser beam also called the read beam is made to fall on the SLM. A computer is used to generate a hologram which will be used to modulate the phase of the light. The output beam now contains diffraction orders. These diffraction orders separate out as the beam propagates. Each diffraction order can be studied separately using a beam profiler. An iris is placed in the path of the output beam which allows only one diffraction order to pass and block the rest of diffraction orders.

7.2 Knife edge technique

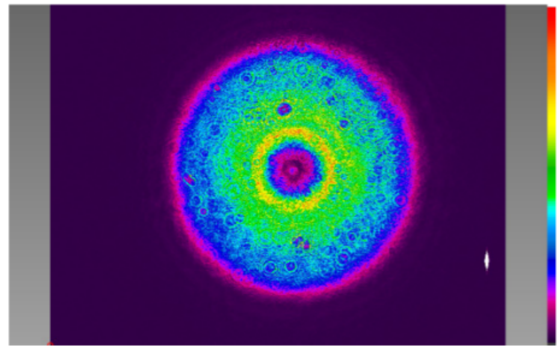
The aim is to study the spot size of the beam at the focus by the objective inside the trapping apparatus.
For this, we focus the beam and use the knife edge technique to reconstruct the intensity profile along one axis. In this technique a knife edge moves perpendicular to the direction of propagation of the laser beam, and the total transmitted power is measured as a function of the knife-edge position.
The width of the linear increasing section in the graph helps us to determine the exact position of the focus along the direction of propagation of beam. When this width is the smallest, it indicates the exact position of the focal spot.

7.3 Intensity distributions as measured by Beam profiler

We finally discuss the preliminary results in this section. The input beam has an intensity profile corresponding to a Gaussian function while the output diffraction orders have different intensity profiles due to the phase modulation carried out by the SLM. The first order of diffraction has a doughnut shaped intensity profile after being phase modulated by the SLM.



(a) Gaussian intensity profile corresponding to first diffraction order without phase modulation



(b) First diffraction order intensity profile of a dark hollow beam

Figure 16: Intensity profiles of first order diffraction corresponding to unmodulated and modulated beams

The first order is now utilized for trapping by allowing it to incident onto the sample by focusing it using an objective of high numerical aperture and magnification.
The backscatter data of the sample is as observed.

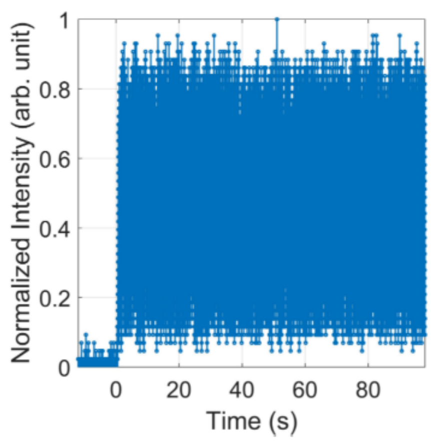


Figure 17: Backscatter data obtained by photomultiplier tube (PMT) from the sample.

8 References

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