Maths Test
Question 1
The integral

$$\int_0^6 |x-2|\, dx$$

evaluates to which of the following?

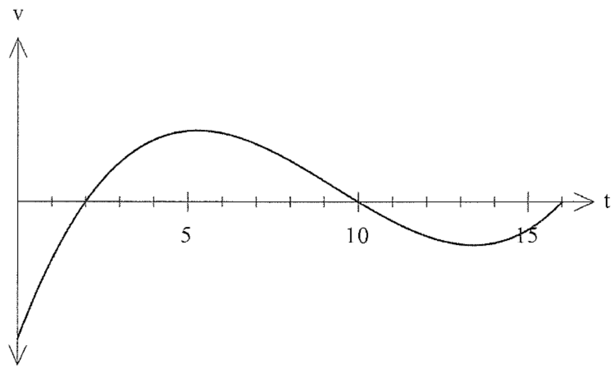
A. 10

B. 20

C. 30

D. None of the above.

Question 2
The graph below shows the velocity function of a particle during its travel for the first 16 seconds.



Given that the particle begins at the origin, which of the following statements is true?

A. At $t = 2$, the particle is to the left of the origin, and travelling towards the origin.

B. At $t = 5$, the particle is at the origin, and at rest.

C. At $t = 10$, the particle is to the right of the origin, and at rest.

D. At $t = 15$, the particle is at the origin, and travelling from left to right.

Question 3
What is the domain of $y = \ln(x-1)$?

A. $x > 0$

B. $x \geq 1$

C. $x \geq -1$

D. $x > 1$

Question 4

What is the domain of the function $y = \frac{1}{\sqrt{x-9}}$?

(A) Domain: $[9, \infty)$

(B) Domain: $(9, \infty)$

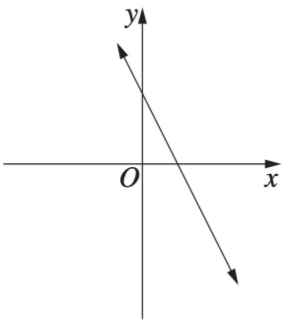
(C) Domain: $(-\infty, \infty)$

(D) Domain: $[-3, 3]$

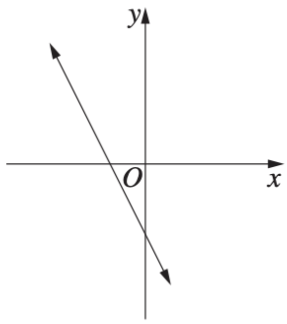
Question 5

Which of the following could be the graph of $y = -2x + 2$?

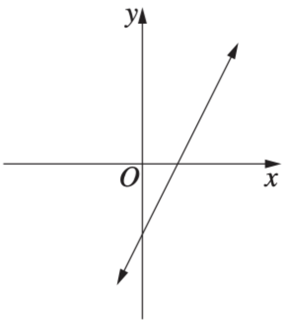
A.



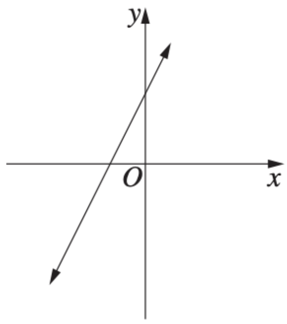
B.



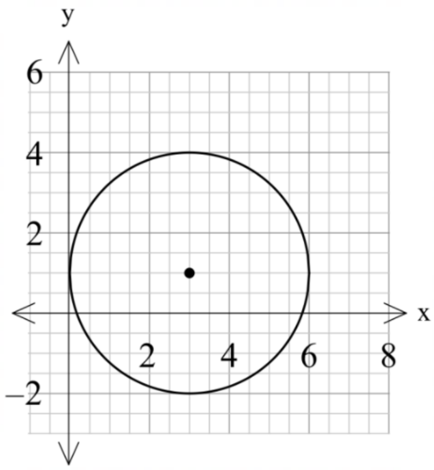
C.



D.



Question 6



Which of the following is the equation for the circle shown in the diagram above?

A. $x^2 + 6x + y^2 + 2y + 1 = 0$

B. $x^2 - 6x + y^2 + 2y - 1 = 0$

C. $x^2 - 6x + y^2 - 2y + 1 = 0$

D. $x^2 - 6x + y^2 - 2y - 1 = 0$

Question 7

For what values of x is the curve $f(x) = 2x^3 + x^2$ concave down?

A. $x < -\frac{1}{6}$

B. $x > -\frac{1}{6}$

C. $x < -6$

D. $x > 6$

Question 8

What is the x coordinate of the point on the curve $y = e^{2x}$ where the tangent is parallel to the line $y = 4x - 1$?

A. $x = \frac{1}{2}\ln 2$

B. $x = \ln 2$

C. $x = -\frac{1}{2}\ln 2$

D. $x = 2$

Question 9

What is the domain and range of $f(x) = \sqrt{x-4}$?

A. Domain: $(-\infty, \infty)$ and Range: $(0, \infty)$

B. Domain: $[4, \infty)$ and Range: $[0, \infty)$

C. Domain: $[0, \infty)$ and Range: $[4, \infty)$

D. Domain: $[2, \infty)$and Range: $[0, \infty)$

Question 10

Which of the following is an odd function?

(A) $y = x^2 + x$

(B) $y = x^2 + 1$

(C) $y = x^3 + x$

(D) $y = x^3 + 1$

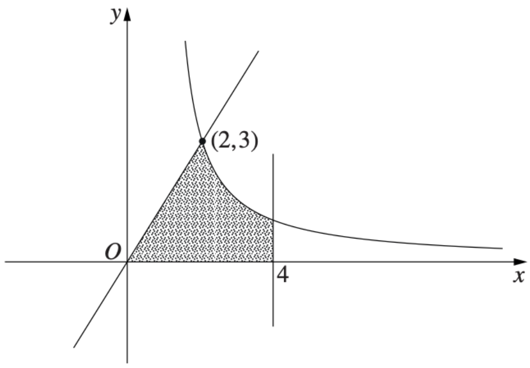
Maths Test
Question 11

Solve $|3x - 8| = 2$. **2**

Question 12

The curve $y = \dfrac{3}{x-1}$ intersects the line $y = \dfrac{3}{2}x$ at the point $(2, 3)$. **3**

The region bounded by the curve $y = \dfrac{3}{x-1}$, the line $y = \dfrac{3}{2}x$, the x-axis and the line $x = 4$ is shaded in the diagram.



Find the exact area of the shaded region.

Question 13

Consider the curve $y = 3x^4 - 4x^3 + 2$.

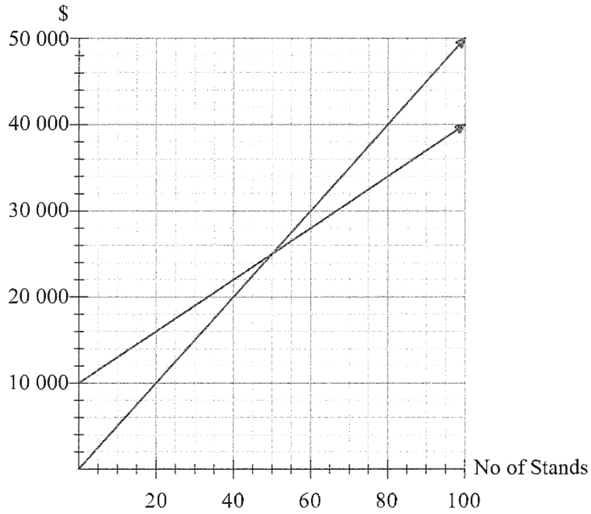
(a) Find the coordinates of any stationary points on the curve and determine their nature. **3**

(b) Find the coordinates of any points of inflection on the curve. **2**

(c) Hence, or otherwise, sketch the graph of $y = 3(x-1)^4 - 4(x-1)^3 + 2$, showing the coordinates of the stationary points and points of inflection. **2**

Question 14

The company, Upstanding Televisions, makes a television stand that sells for \$500. It costs \$300 to manufacture each unit and the machinery and warehouse costs \$10 000 per year.
This is shown in the graph below:



(a) How many stands must they sell to break even? **1**

(b) How much profit do they make if they sell 80 stands? **1**

Question 15

Let $f(x) = xe^{-2x}$.

It is given that $f'(x) = e^{-2x} - 2xe^{-2x}$.

(a) Show that $f''(x) = 4(x-1)e^{-2x}$. **2**

(b) Find any stationary points of $f(x)$ and determine their nature. **2**

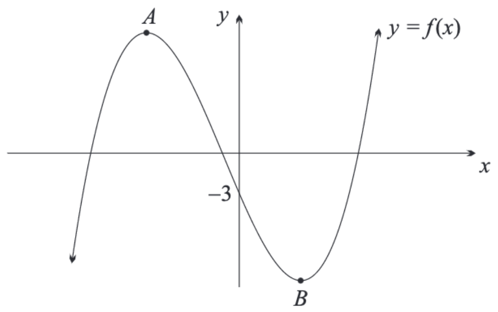
(c) Sketch the curve $y = xe^{-2x}$, showing any stationary points, points of inflection and intercepts with the axes. **3**

Question 16

Find the domain of the function $f(x) = \sqrt{3x-2}$. $\boxed{1}$

Question 17

The diagram below shows the graph of a function $f(x)$ with a y-intercept of -3, a maximum turning point at A, and a minimum turning point at B. The derivative of the function is $f'(x) = 3x^2 + 2x - 8$.



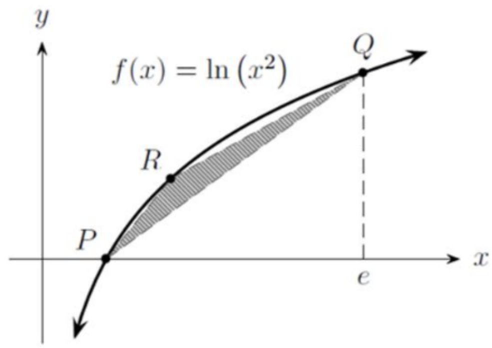
(a) Find the equation of $f(x)$. $\boxed{2}$

(b) Find the coordinates of A. $\boxed{3}$

(c) The graph of $y = g(x)$ is obtained by reflecting the graph of $y = f(x)$ in the x-axis, shifting 5 units up, and then dilating horizontally by a factor of $\frac{1}{2}$. $\boxed{2}$
Find the coordinates of the point A', the image of the point A, after these successive transformations have been applied.

Question 18

The diagram shows the graph of the function $y = \ln(x^2)$, where $x > 0$.
The points $P(1, 0)$, $Q(e, 2)$ and $R(t, \ln t^2)$ all lie on the curve.
The area of ΔPQR is maximum when the tangent at R is parallel to the line through P and Q.



a) Show that R has coordinates $(e - 1, \ln[(e-1)^2])$ for ΔPQR to have maximum area. **2**

b) Hence find the size of $\angle RPQ$ correct to the nearest degree. **2**

c) Hence or otherwise, find the maximum area of ΔPQR, correct to 2 decimal places. **2**

Question 19

The temperature T inside an oven, measured in degrees Celsius, m minutes after it is switched on, can be modelled by the equation $T = 200 - 175e^{-km}$, where k is a constant.

(a) Write down the initial temperature of the oven. **1**

(b) If the oven reached a temperature of 150°C after 16 minutes and 15 seconds, find the value of k correct to three significant figures. **2**

(c) What will be the temperature in the oven after 10 minutes? **1**

Question 20

Given that $f(x)=x^2+3$ and $g(x)=x+4$:

(a) Find simplified expressions for $f\left(g(x)\right)$ and $g\left(f(x)\right)$. **2**

(b) Show that $f\left(g(x)\right)+g\left(f(x)\right)=0$ has no real roots. **2**

Multiple Choice Answers

1: A
2: C
3: S
4: B
5: A
6: C
7: A
8: A
9: B
10: C

Short Answer Solutions

Question 11

Solve $|3x - 8| = 2$. **2**

$3x - 8 = 2$ or $3x - 8 = -2$

$3x = 10$ $3x = 6$

$x = 10/3$ $x = 2$

$\therefore x = \frac{10}{3}, 2$

Question 12

Criteria	Marks
• Provides the correct solution	3
• Evaluates $\int_2^4 \frac{3}{x-1}dx$, or equivalent merit	2
• Calculates the area of the triangle, or equivalent merit	1

Sample answer:

$$\text{Area} = \frac{2\times 3}{2} + \int_2^4 \frac{3}{x-1}dx$$

$$= 3 + 3\left[\ln(x-1)\right]_2^4$$

$$= (3 + 3\ln 3)\text{ units}^2$$

Question 13

Consider the curve $y = 3x^4 - 4x^3 + 2$.

(a) Find the coordinates of any stationary points on the curve and determine their nature. **3**

$y' = 12x^3 - 12x^2$

$= 12x^2(x-1)$

$y'' = 36x^2 - 24x$

for stat points $y' = 0$

$\therefore x = 0, 1$

$y = 2, 1$

$y''(0) = 0$

$\therefore$ horizontal inflection point at $(0,2)$

$y''(1) = 12 > 0$

$\therefore$ minimum at $(1,1)$

In part (a) Generally well done. Students should know the difference between point of inflection and stationary point of inflection.

(0, 2) is a stationary point of inflection because both $f'(0) = 0$ and $f''(0) = 0$.

Or a gradient table could be used.

Most students were successful in obtaining the correct stationary points and their nature and were generously awarded full credit.

(b) Find the coordinates of any points of inflection on the curve. **2**

for inflection points $y''=0$

$36x^2-24x=0$

$12x(3x-2)=0$

$\therefore x=0,\ \frac{2}{3}$

Horizontal point of inflection at (0,2) shown in (a)

x	1/3	2/3	1
y''	– 4	0	12
	$\cap$	–	$\cup$

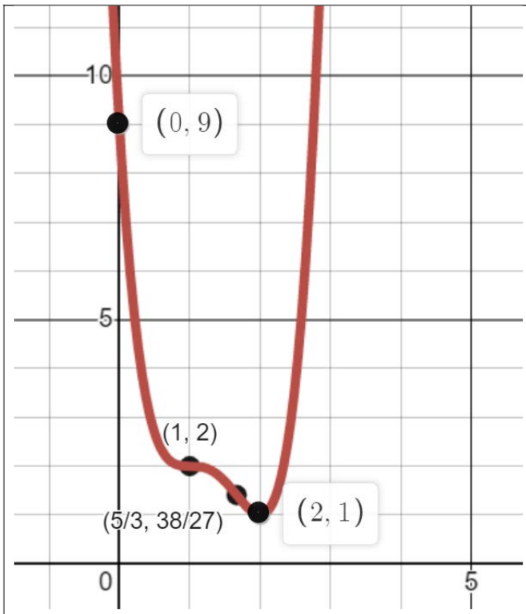
Therefore points of inflection at (0, 2) and (2/3, 38/27)

Part (b) many students did not construct a concavity table.

To improve, students should:

1. Find $f''(x)$
2. At $f''(x)=0$, solve and find the point
3. Construct a concavity table

(c) Hence, or otherwise, sketch the graph of $y=3(x-1)^4-4(x-1)^3+2$, showing the coordinates of the stationary points and points of inflection. **2**

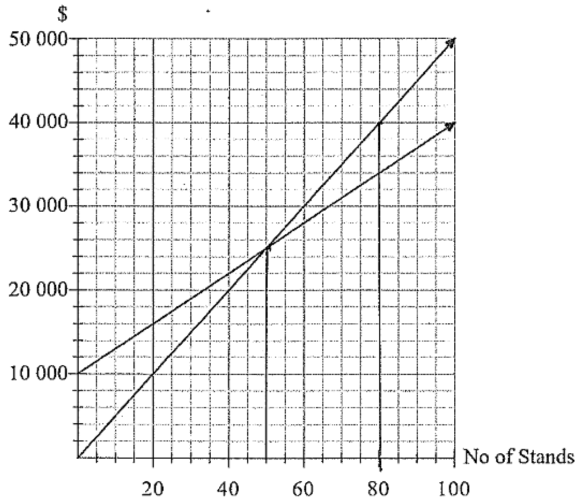


Part (c) Students should shift the curve $y=3x^4-4x^3+2$ to the right 1 unit.

Question 14

The company, Upstanding Televisions, makes a television stand that sells for \$500. It costs \$300 to manufacture each unit and the machinery and warehouse costs \$10 000 per year.

This is shown in the graph below:



(a) How many stands must they sell to break even? **1**

50 stands

(b) How much profit do they make if they sell 80 stands? **1**

40 000 − 34 000 = 6000

\$6000 profit

Question 15

Criteria	Marks
• Provides correct solution	2
• Attempts to use the product rule	1

Sample answer:

$$f'(x) = e^{-2x} - 2xe^{-2x}$$

$$f''(x) = -2e^{-2x} - 2\left(x \times -2e^{-2x} + e^{-2x}\right)$$

$$= -2e^{-2x} + 4xe^{-2x} - 2e^{-2x}$$

$$= (4x-4)e^{-2x}$$

$$= 4(x-1)e^{-2x}$$

Criteria	Marks
• Provides correct solution	2
• Finds the *x* value of the stationary point	1

Sample answer:

Stationary points where $f'(x) = 0$

$$(1-2x)e^{-2x} = 0$$

$$x = \frac{1}{2}, \qquad \text{since } e^{-2x} \neq 0, \quad f\left(\frac{1}{2}\right) = \frac{1}{2}e^{-1} = \frac{1}{2e}$$

$$f''\left(\frac{1}{2}\right) = 4 \times -\frac{1}{2} \times e^{-1} = -2e^{-1} < 0$$

$\therefore \left(\frac{1}{2}, \frac{1}{2e}\right)$ is a local maximum.

Criteria	Marks
• Provides correct graph	3
• Sketches the curve showing most of the main features	2
• Provides a sketch showing the stationary point and 1 other feature	1

Sample answer:

Possible point of inflection where $f''(x) = 0$

$4(x-1)e^{-2x} = 0$

$\therefore x = 1, \quad$ since $e^{-2x} \neq 0$

x	0	1	2
$f''(x)$	-4	0	$4e^{-4}$

Changes concavity at $x = 1$

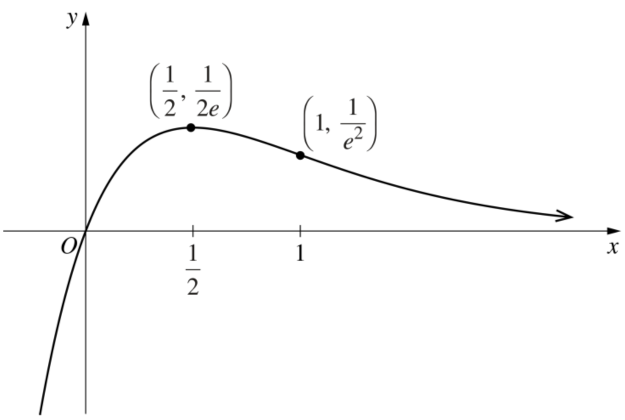
$\therefore$ Inflection point at $x = 1$

$$f(1) = 1e^{-2\times 1}$$
$$= e^{-2}$$

$\left(1, \frac{1}{e^2}\right)$ — Inflection point

$f(0) = 0 \quad \therefore$ graph passes through $(0, 0)$

As $x \to \infty, \quad f(x) \to 0$



Question 16

Find the domain of the function $f(x) = \sqrt{3x - 2}$. $\boxed{1}$

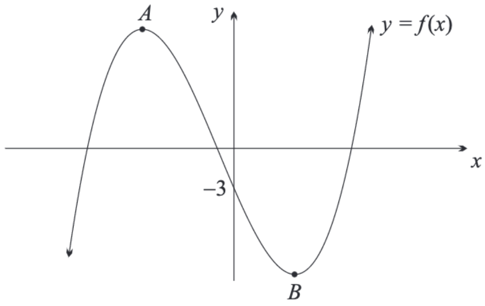
$3x - 2 \geq 0$ Common error: $x > \frac{2}{3}$

$3x \geq 2$

$x \geq \frac{2}{3}$ ✓

Question 17

The diagram below shows the graph of a function $f(x)$ with a y-intercept of -3, a maximum turning point at A, and a minimum turning point at B. The derivative of the function is $f'(x) = 3x^2 + 2x - 8$.



(a) Find the equation of $f(x)$. $\boxed{2}$

$f'(x) = 3x^2 + 2x - 8$

$f(x) = x^3 + x^2 - 8x + c$ ✓

when $x = 0$, $y = -3$

$-3 = 0 + 0 - 0 + c$

$c = -3$

$f(x) = x^3 + x^2 - 8x - 3$ ✓

(b) Find the coordinates of A. $\boxed{3}$

stationary points when $f'(x) = 0$

$0 = 3x^2 + 2x - 8$

$0 = (3x - 4)(x + 2)$

$x = \frac{4}{3}, -2$ ✓

$f''(x) = 6x + 2$

$f''(-2) = 6 \times (-2) + 2 = -10$, so max at $x = -2$

as A is to the left of the y-axis, x-coordinate of A is -2 ✓

$f(-2) = (-2)^3 + (-2)^2 - 8(-2) - 3$

$= 9$

$A(-2, 9)$ ✓

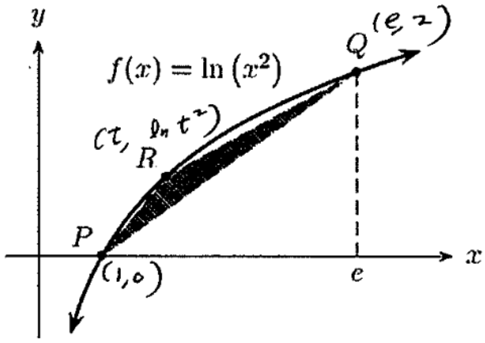
(c) The graph of $y = g(x)$ is obtained by reflecting the graph of $y = f(x)$ in the x-axis, shifting 5 units up, and then dilating horizontally by a factor of $\frac{1}{2}$. **2**
Find the coordinates of the point A', the image of the point A, after these successive transformations have been applied.

$A'\left(-2 \times \frac{1}{2}, -9+5\right)$

$A'(-1, -4)$

Question 18

The diagram shows the graph of the function $y = \ln(x^2)$, where $x > 0$.
The points $P(1, 0)$, $Q(e, 2)$ and $R(t, \ln t^2)$ all lie on the curve.
The area of ΔPQR is maximum when the tangent at R is parallel to the line through P and Q.



a) Show that R has coordinates $(e-1, \ln[(e-1)^2])$ for ΔPQR to have maximum area. **2**

$f(x) = \ln(x^2)$

$f'(x) = \frac{2x}{x^2} = \frac{2}{x}$ 1m

$m(PQ) = \frac{2-0}{e-1} = \frac{2}{e-1}$

$\therefore \frac{2}{x} = \frac{2}{e-1}$ $\therefore x = e-1$

$y = f(x) = \ln(e-1)^2$

$\therefore R = \left((e-1), \ln(e-1)^2\right)$ 1m

b) Hence find the size of $\angle RPQ$ correct to the nearest degree. 2

Let $m(PQ) = \frac{2}{e-1} = \tan\alpha$ 1m for $m(PQ)$, $m(RP)$

1m for correct angle $7°$ with working

$\alpha = 49°20'$

$m(RP) = \frac{\ln(e-1)^2 - 0}{e-1-1} = \frac{\ln(e-1)^2}{e-2}$

Let $m(RP) = \tan\beta$ $\beta = 56°26'$

$\angle RPQ = 7°6' \doteqdot \underline{7°}$ (nearest degree)

See below for alternative ☆

c) Hence or otherwise, find the maximum area of ΔPQR, correct to 2 decimal places. 2

$\max|PQR| = \frac{1}{2}ab\sin\theta$ 1m

$= \frac{1}{2}|PQ| \cdot |PR| \sin 7°$ ← plus correct value of at least $|PQ|$ or $|PR|$ correct

$|PQ| = \sqrt{(e-1)^2 + 2^2} = \sqrt{e^2 - 2e + 5} \doteqdot \sqrt{6.9527} = 2.6368$

$|PR| = \sqrt{(e-2)^2 + (\ln(e-1)^2)^2} \doteqdot \sqrt{1.688} = 1.2992$

$\max|PQR| = \frac{1}{2} \times 2.6368 \times 1.299 \sin 7°$

$= 0.2088$

$\doteqdot \underline{0.21 \text{ u}^2}$ 1m

☆ b) $\cos\theta = \frac{PR^2 + PQ^2 - RQ^2}{2\, PR \cdot PQ} = \frac{1.688 + 6.9527 - 1.8415}{2\sqrt{1.688 \times 6.9527}}$

Question 19

The temperature T inside an oven, measured in degrees Celsius, m minutes after it is switched on, can be modelled by the equation $T = 200 - 175e^{-km}$, where k is a constant.

(a) Write down the initial temperature of the oven. 1

When $m = 0$, $T = 200 - 175e^{-0}$ $T = 25°$
Pretty well done. In part (a) a few students could not successfully evaluate the initial temperature as they incorrectly used $e^0 = 0$. Be careful with this, which also arises when evaluating definite integrals involving exponentials.

(b) If the oven reached a temperature of 150°C after 16 minutes and 15 seconds, find the value of k correct to three significant figures. **2**

When $m = 16.25,\ T = 150$, $150 = 200 - 175e^{-16.25k}$ $\frac{-50}{-175} = e^{-16.25k}$ $-16.25k = \ln\frac{2}{7}$ $k = \frac{-\ln\left(\frac{2}{7}\right)}{16.25} = \frac{4}{65}\ln\left(\frac{7}{2}\right) \approx 0.0771$ to 3 s.f.
In part (b) most students could manipulate the resulting exponential equation correctly. Common errors tended to be in using the calculator incorrectly or in rounding correctly to three significant figures. Revisit significant figures if required.

(c) What will be the temperature in the oven after 10 minutes? **1**

When $m = 10$, $T = 200 - 175e^{-10\times\frac{4}{65}\ln\left(\frac{7}{2}\right)}$ $= 119.05°\text{C}$
Part (c) was generally well done barring a few calculator errors. Exercise care to enter expressions correctly into the calculator. Using the ANS key or a pronumeral using the STO function on your calculator can make it easier to enter complex expressions.

Question 20

Given that $f(x) = x^2 + 3$ and $g(x) = x + 4$:

(a) Find simplified expressions for $f(g(x))$ and $g(f(x))$. **2**

$f(g(x)) = (x+4)^2 + 3$ $= x^2 + 8x + 19$ $g(f(x)) = (x^2 + 3) + 4$ $= x^2 + 7$
Well done

(b) Show that $f(g(x)) + g(f(x)) = 0$ has no real roots. **2**

$f(g(x)) + g(f(x)) = x^2 + 8x + 19 + x^2 + 7$ $= 2x^2 + 8x + 26$ $\Delta = 8^2 - 4\times 2\times 26$ $= -144$ < 0 Therefore no real roots.
Reasonably well done. Students should use the discriminant. They don't need to solve the quadratic equation