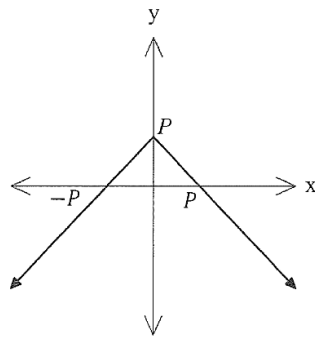


**Maths Test**  
**Question 1**

The graph shows the function  $y = P - |x|$ .



Find the value of  $P$ , given:

$$\int_{-P}^P (P - |x|) dx = 16$$

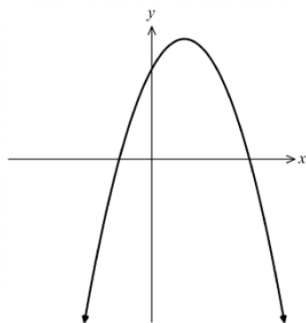
- A.  $P = 4$
- B.  $P = 8$
- C.  $P = -4$
- D.  $P = -8$

---

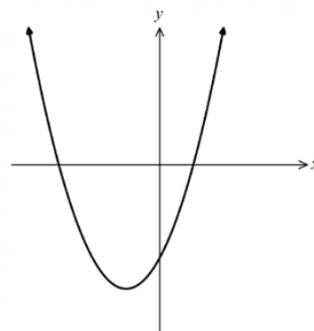
**Question 2**

Which of the following graphs best represents  $y = x^2 + 2x - 3$ ?

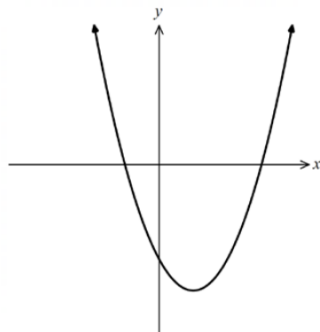
A.



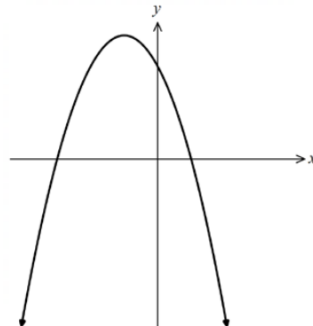
B.



C.

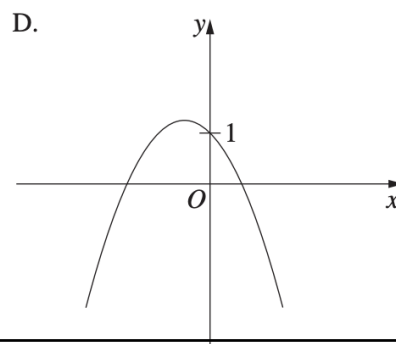
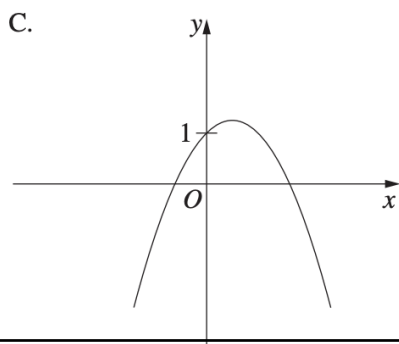
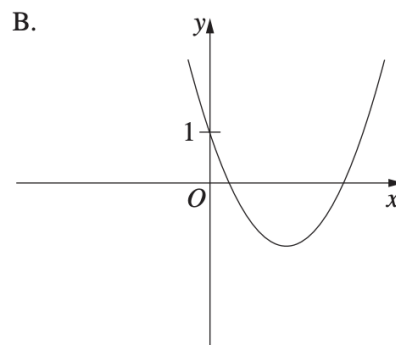
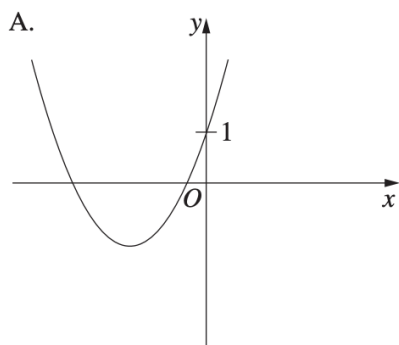


D.



**Question 3**

Which of the following could represent the graph of  $y = -x^2 + bx + 1$ , where  $b > 0$ ?

**Question 4**

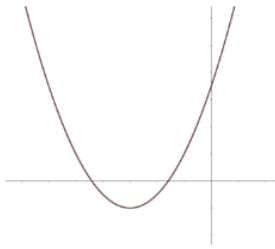
Which of the following correctly describes the gradient and concavity of the function  $f(x) = 2x^2 + 3x - 1$  at the origin?

- (A) The gradient is positive and the function is concave up
- (B) The gradient is positive and the function is concave down
- (C) The gradient is negative and the function is concave up
- (D) The gradient is negative and the function is concave down

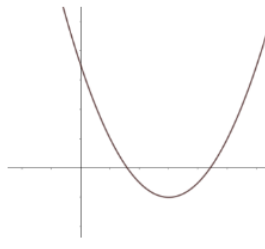
**Question 5**

Which diagram below best shows the graph  $y = 2 - (x + 3)^2$ ?

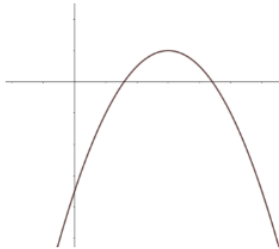
A.



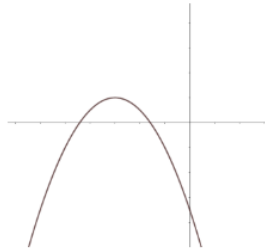
B.



C.



D.



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**Question 6**

Let  $h(x) = \frac{f(x)}{g(x)}$ , where

$$\begin{array}{ll} f(1) = 2 & f'(1) = 4 \\ g(1) = 8 & g'(1) = 12. \end{array}$$

What is the gradient of the tangent to the graph of  $y = h(x)$  at  $x = 1$ ?

A.  $-8$ B.  $8$ C.  $-\frac{1}{8}$ D.  $\frac{1}{8}$ 

---

**Question 7**

Which of the following is the angle of inclination of the line  $y = \frac{9}{8}x - 5$ ?

(A)  $42^\circ$ (B)  $48^\circ$ (C)  $132^\circ$ (D)  $138^\circ$ 

---

**Question 8**

What is the solution to the inequation  $x^2 + 4x + 3 \geq 0$ ?

- A.  $x \leq -1$  or  $x \leq -3$
  - B.  $x \leq -1$  or  $x \geq -3$
  - C.  $x \geq -1$  or  $x \geq -3$
  - D.  $x > -1$  or  $x < -3$
- 

**Question 9**

What is the domain of the function  $y = \frac{1}{\sqrt{x-9}}$ ?

- (A) Domain:  $[9, \infty)$
  - (B) Domain:  $(9, \infty)$
  - (C) Domain:  $(-\infty, \infty)$
  - (D) Domain:  $[-3, 3]$
- 

**Question 10**

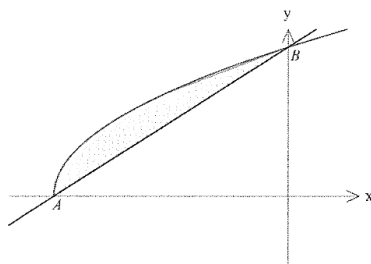
Which of the following is an odd function?

- (A)  $y = x^2 + x$
  - (B)  $y = x^2 + 1$
  - (C)  $y = x^3 + x$
  - (D)  $y = x^3 + 1$
-

**Maths Test**  
**Question 11**

In the following diagram, the curves  $y = \sqrt{x+9}$  and the line  $y = \frac{1}{3}x + 3$  intersect at  $A$  and  $B$ .

4



Calculate the area bounded between the two curves.

**Question 12**

The graph of the function  $f(x) = x^2$  is translated  $m$  units to the right, dilated vertically by a scale factor of  $k$  and then translated 5 units down. The equation of the transformed function is  $g(x) = 3x^2 - 12x + 7$ .

3

Find the values of  $m$  and  $k$ .

**Question 13**

Solve:

2

$$\frac{243}{6\sqrt{3} - \sqrt{27}} = 3^x$$

**Question 14**

Find the exact gradient of the tangent to the curve  $y = x \tan x$  at the point where  $x = \frac{\pi}{3}$ .

3

**Question 15**

The equation of a circle is given by

$$x^2 + y^2 + 4x - 2y - 20 = 0$$

a) Find the centre and radius of the circle.

2

b) Hence evaluate

2

$$\int_{-7}^3 1 + \sqrt{21 - 4x - x^2} \, dx$$

**Question 16**

Without using calculus, sketch the graph of  $y = 2 + \frac{1}{x+4}$ , showing the asymptotes and the  $x$  and  $y$  intercepts.

3

**Question 17**

Find the equation of the line that passes through the point  $(0, -3)$  and has an angle of inclination of  $30^\circ$ . Leave your answer in gradient-intercept form.

2

**Question 18**

Consider the curve  $y = 3x^4 - 16x^3 + 24x^2 - 9$ .

- a) Show that the first and second derivatives are respectively 2

$$y' = 12x(x - 2)^2$$

$$y'' = 12(x - 2)(3x - 2)$$

- b) Find and classify all stationary points and points of inflexion. 4

- c) Over which interval(s) is the curve decreasing? 1

- d) Sketch the curve, ensuring you demonstrate all features found, including the intercept with the  $y$ -axis (you may ignore calculating any intercepts with the  $x$ -axis). 3

**Question 19**

The circle  $x^2 - 6x + y^2 + 4y - 3 = 0$  is reflected in the  $x$ -axis. 3

Sketch the reflected circle, showing the coordinates of the centre and the radius.

**Question 20**

A shoe store buys shoes for \$50 a pair and sells them for \$80. If their weekly rent and salary costs are \$2100 a week, how many pairs must they sell each week to break even? 2

## Multiple Choice Answers

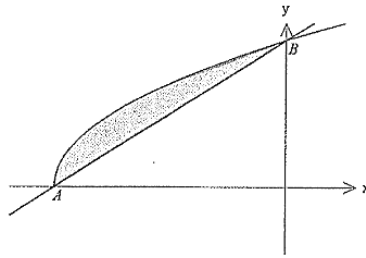
- 1: A
- 2: B
- 3: C
- 4: A
- 5: D
- 6: D
- 7: B
- 8: D
- 9: B
- 10: C

# Short Answer Solutions

## Question 11

In the following diagram, the curves  $y = \sqrt{x+9}$  and the line  $y = \frac{1}{3}x + 3$  intersect at A and B.

4



Calculate the area bounded between the two curves.

$$A: (-9, 0) \quad B(0, 3)$$

$$\text{Area} = \int_{-9}^0 \sqrt{x+9} - \left(\frac{1}{3}x + 3\right) dx$$

$$= \int_{-9}^0 (x+9)^{\frac{1}{2}} - \frac{1}{3}x - 3 dx$$

$$= \left[ \frac{(x+9)^{\frac{3}{2}}}{\frac{3}{2}} - \frac{1}{6}x^2 - 3x \right]_{-9}^0$$

$$= \frac{9^{\frac{3}{2}}}{\frac{3}{2}} - 0 - 0 - \left(0 - \frac{1}{6} \times 9^2 - 3 \times -9\right)$$

$$= 18 - \frac{27}{2}$$

$$= 4.5 \text{ u}^2$$



### Question 12

Criteria	Marks
• Provides correct solution	3
• Writes $g(x)$ in terms of $m$ and $k$ , and finds value of $k$ , or equivalent merit	2
• Writes $g(x)$ in terms of $m$ and $k$ , or equivalent merit	1

**Sample answer:**

$$\begin{aligned}
 g(x) &= k(x - m)^2 - 5 \\
 &= k(x^2 - 2xm + m^2) - 5 \\
 &= kx^2 - 2mkx + m^2k - 5
 \end{aligned}$$

By inspection  $k = 3$

By equating coefficient of  $x$

$$2mk = 12$$

$$6m = 12$$

$$m = 2$$

ALTERNATE SOLUTION

$$\begin{aligned}
 g(x) &= k(x - m)^2 - 5 \\
 g(x) &= 3x^2 - 12x + 7 \\
 &= 3(x^2 - 4x) + 7 \\
 &= 3(x^2 - 4x + 4) + 7 - 12 \\
 &= 3(x - 2)^2 - 5
 \end{aligned}$$

So  $k = 3$  and  $m = 2$

### Question 13

Solve:

2

$$\frac{243}{6\sqrt{3} - \sqrt{27}} = 3^x$$

$$\begin{aligned}
 &\frac{243}{6\sqrt{3} - 3\sqrt{3}} \\
 &= \frac{3^5}{3\sqrt{3}} \\
 &= \frac{3^4}{\sqrt{3}} \\
 &= 3^3\sqrt{3} \\
 &= 3^{3\frac{1}{2}} \\
 &\therefore x = 3\frac{1}{2}
 \end{aligned}$$

### Question 14

Criteria	Marks
• Provides correct solution	3
• Finds the derivative and attempts to calculate gradient	2
• Applies the product rule, or equivalent merit	1

**Sample answer:**

$$y = x \tan x$$

$$\frac{dy}{dx} = x \sec^2 x + \tan x$$

$$\text{Gradient of tangent} = \frac{\pi}{3} \sec^2\left(\frac{\pi}{3}\right) + \tan\left(\frac{\pi}{3}\right)$$

$$= \frac{\pi}{3} \frac{1}{\left(\frac{1}{2}\right)^2} + \sqrt{3}$$

$$= \frac{4\pi}{3} + \sqrt{3}$$

### Question 15

The equation of a circle is given by

$$x^2 + y^2 + 4x - 2y - 20 = 0$$

- a) Find the centre and radius of the circle.

2

$$x^2 + 4x + 4 + y^2 - 2y + 1 = 25$$

$$(x+2)^2 + (y-1)^2 = 25$$

$$\text{centre: } (-2, 1)$$

$$\text{radius} = 5 \text{ units}$$

b) Hence evaluate

2

$$\int_{-7}^3 1 + \sqrt{21 - 4x - x^2} \, dx$$

$$21 - (x^2 + 4x) = 21 - ((x+2)^2 - 4) \\ = 25 - (x+2)^2$$

OR  $(x+2)^2 + (y-1)^2 = 25$

$$(y-1)^2 = 25 - (x+2)^2$$

$$y-1 = \sqrt{25 - (x+2)^2}$$

$$y = 1 + \sqrt{21 - 4x - x^2}$$

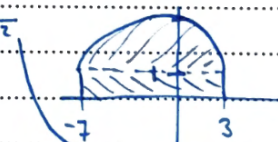
Hence  $\int_{-7}^3 1 + \sqrt{21 - 4x - x^2} \, dx$

= Area of rectangle + Area of semicircle — ①

$$= (3 - (-7)) \times 1 + \frac{1}{2} \pi (5)^2$$

$$= 10 + \frac{25\pi}{2} \text{ u}^2 \text{ — ①}$$

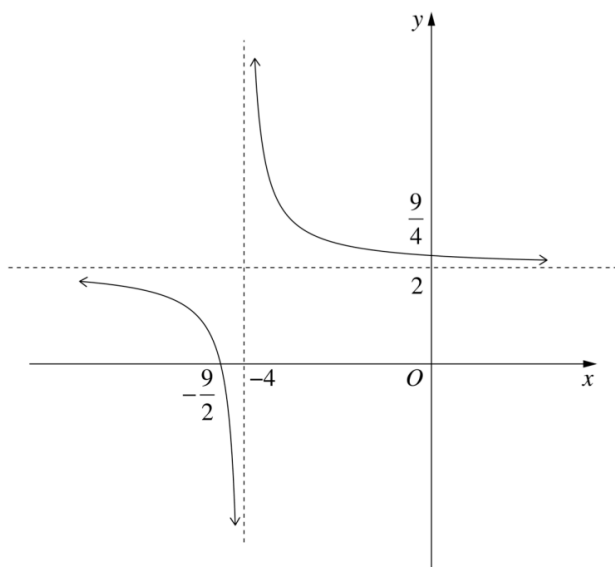
OR  $= 49.27 \text{ u}^2 \text{ (2dp)}$



#### Question 16

Criteria	Marks
• Provides correct solution	3
• Provides correct graph without x and y intercepts, or equivalent merit	2
• Draws a hyperbola, or equivalent merit	1

Sample answer:



y-intercept:  $y = 2 + \frac{1}{4}$

$$= \frac{9}{4}$$

x-intercept:  $0 = 2 + \frac{1}{x+4}$

$$x = -\frac{9}{2}$$

### Question 17

Find the equation of the line that passes through the point  $(0, -3)$  and has an angle of inclination of  $30^\circ$ . Leave your answer in gradient-intercept form.

2

$$\begin{aligned} m &= \tan \theta \\ &= \tan 30^\circ \\ &= \frac{1}{\sqrt{3}} \end{aligned}$$

$$y = \frac{1}{\sqrt{3}}x - 3$$

### Question 18

Consider the curve  $y = 3x^4 - 16x^3 + 24x^2 - 9$ .

- a) Show that the first and second derivatives are respectively

2

$$y' = 12x(x-2)^2$$

$$y'' = 12(x-2)(3x-2)$$

$$y = 3x^4 - 16x^3 + 24x^2 - 9$$

$$y' = 12x^3 - 48x^2 + 48x$$

$$= 12x(x^2 - 4x + 4)$$

$$= 12x(x-2)^2$$

Note: ans given

$$y'' = 36x^2 - 96x + 48$$

$$= 12(3x^2 - 8x + 4)$$

$$= 12(3x-2)(x-2)$$

$$= 12(x-2)(3x-2)$$

Note: ans. given.

- b) Find and classify all stationary points and points of inflexion.

Stationary points occur when  $y' = 0$  ie  $12x(x-2)^2 = 0$

When  $x=0$   $y=-9$

ie  $x=0, 2$

$x=2$   $y=7$

Stationary points are found at  $(0, -9)$  &  $(2, 7)$

Testing their nature at  $(0, -9)$ ,  $y'' = 12(-2)(-2)$

$= 48 > 0$  ie concave up

ie min at  $(0, -9)$

Testing  $(2, 7)$ ,  $y'' = 12(0)(4) = 0$

ie possible point of inflexion (horizontal)

Test point either side

$x$	1	2	3
$y''$	-12	0	84

Changes sign from concave down to concave up. ie  $(2, 7)$  is a horizontal point of inflexion.

Test to see if other points of inflexion when  $y'' = 0$

ie  $12(x-2)(3x-2) = 0$

$x=2$   $x=\frac{2}{3}$

ie Possible P.O.I. at  $(\frac{2}{3}, -\frac{67}{27})$

$x$	0	$\frac{2}{3}$	1
$y''$	-12	0	-12

changes concavity ie  $(\frac{2}{3}, -\frac{67}{27})$  P.O.I.

ax  
2 mks for  
points only  
with no  
reasons

Note: you must  
write values in  
table, not just  
say +ve or -ve

1mk

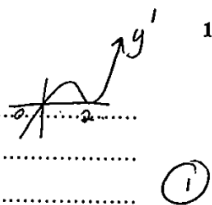
1mk

1mk

1mk

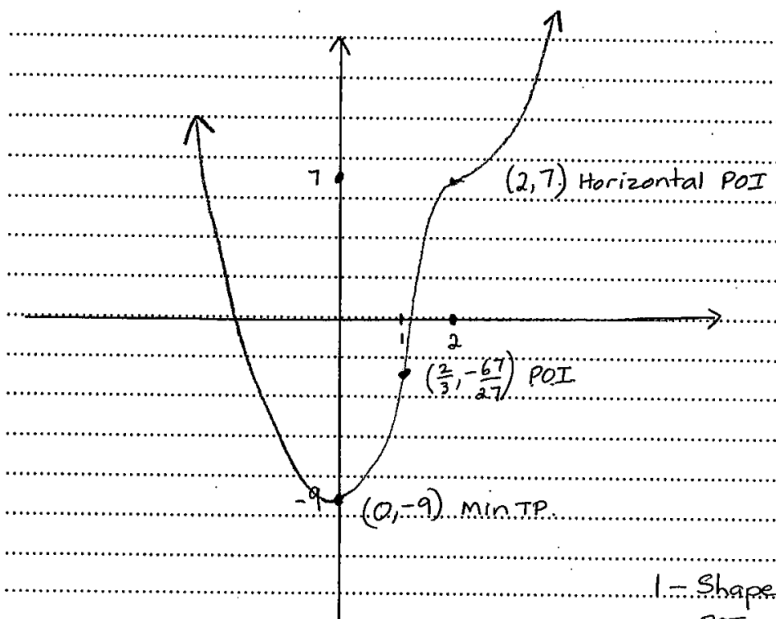
- c) Over which interval(s) is the curve decreasing?

Curve is decreasing when  $y' < 0$   
 ie  $12x(x-2) < 0$   
 Decreasing when  $x < 0$



- d) Sketch the curve, ensuring you demonstrate all features found, including the intercept with the y-axis (you may ignore calculating any intercepts with the x-axis).

3



1 - Shape + label  
 POI at  $(\frac{2}{3}, -\frac{67}{27})$   
 1 - Min. at  $(0, -9)$   
 1 - Horizontal POI  
 labelled at  $(2, 7)$

End Section II

### Question 19

Criteria	Marks
• Provides the correct solution	3
• Finds the centre and the radius of the original circle or of the reflected circle, or equivalent merit	2
• Completes the square for the equation, or equivalent merit	1

**Sample answer:**

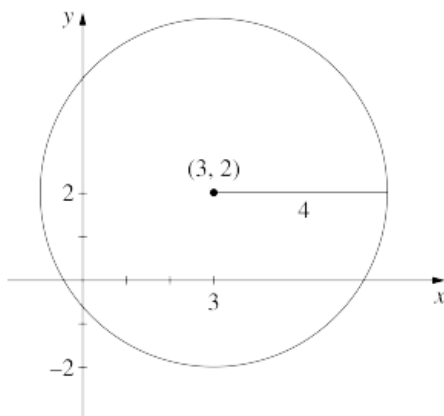
$$x^2 - 6x + y^2 + 4y - 3 = 0$$

$$x^2 - 6x + 9 + y^2 + 4y + 4 = 3 + 9 + 4$$

$$(x - 3)^2 + (y + 2)^2 = 16$$

The original circle has centre  $(3, -2)$  and radius 4.

The reflected circle has centre  $(3, 2)$  and radius 4.



### Question 20

A shoe store buys shoes for \$50 a pair and sells them for \$80. If their weekly rent and salary costs are \$2100 a week, how many pairs must they sell each week to break even? 2

let  $x$  be the number of shoes sold

$$R = 80x, \quad C = 50x + 2100$$

for break even  $R = C$

$$80x = 50x + 2100$$

$$30x = 2100$$

$$x = 70 \text{ pairs}$$

Almost all students were successful in obtaining the correct answer and were generously awarded full credit, however there is much scope for improvement in setting out. It was not uncommon to see the following, apropos of nothing:

$$80 - 50 = 30 \text{ (a true fact, but what is this calculating?)}$$

$$\frac{2100}{30} = 70 \text{ (again a correct numerical fact unexplained)}$$

To improve, students should:

- Introduce a pronumeral before using it.
- Start with the definition of “breakeven” - costs equal revenues
- Answer the question