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Assignment:

Ans to the que no-1

we know,

Time dependent Schrodinger Eqn,

$$i\hbar \frac{\partial}{\partial x}\left(\Psi(x,t)\right) = \frac{-\hbar^2}{2m} \cdot \frac{\partial^2}{\partial x^2}\Psi(x,t) + V(x)\Psi(x,t)$$

assuming a seperate variable to be,

$$\Psi(x,t) = \psi(x) \cdot \phi(t)$$

substituting, the eqn,

$$i\hbar \frac{\partial}{\partial t}\left(\psi(x) \cdot \phi(t)\right) = \frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\left(\psi(x) \cdot \phi(t)\right) + V(x) \cdot \psi(x)\phi(t)$$

$$\Rightarrow i\hbar\, \psi(x)\frac{\partial}{\partial t}\phi(t) = \left(\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x) + V(x)\psi(x)\right)\phi(t)$$

$$\Rightarrow i\hbar \frac{1}{\phi(t)}\frac{\partial}{\partial t}\phi(t) = \frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x) + V(x)$$

$$\Rightarrow i\hbar \frac{1}{\phi(t)}\frac{d}{dt}\phi(t) = \frac{-\hbar^2}{2m}\frac{d^2}{dx^2}\psi(x) + V(x)$$ [both side depends on one variable from differential]

$$\Rightarrow i\hbar \frac{1}{\phi(t)}\frac{d}{dt}\phi(t) = \frac{-\hbar^2}{2m}\frac{d^2}{dx}\psi(x) + V(x) = C$$

[LHS and RHS can be equal if both are equal to a constant. Here C is the constant]

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Therefore we can write,

$$i\hbar \frac{1}{\phi(t)} \cdot \frac{d}{dt} \phi(t) = c$$

And,

$$\frac{\hbar^2}{2m} \frac{1}{\psi(x)} \frac{d^2}{dx^2} \psi(x) + v(x) = c$$ [Time independent Schrodinger Eqn]

Now,

$$i\hbar \frac{1}{\phi(t)} \cdot \frac{d}{dt} \phi(t) = c$$

$$\Rightarrow i\hbar \frac{1}{\phi(t)} \, d\phi(t) = c \, dt$$

$$\Rightarrow i\hbar \int \frac{1}{\phi(t)} \, d\phi(t) = c \int dt$$

$$\Rightarrow \ln \phi(t) = \frac{c}{i\hbar} t$$

$$\therefore \phi(t) = e^{\frac{c}{i\hbar} \cdot t}$$

So,

$$\Psi(x,t) = \psi(x) \cdot e^{\frac{c}{i\hbar} t}$$

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Ans to the que no-2

The wave for a free particle is,

$$\psi(x,t) = Ae^{i(kx-\omega t)}$$

differentiating with respect to x,

$$\frac{\partial}{\partial x}\psi(x,t) = \frac{\partial}{\partial x}Ae^{i(kx-\omega t)}$$

$$\Rightarrow \frac{\partial \psi}{\partial x}(x,t) = Ae^{i(kx-\omega t)}.(ik - i\omega t)$$

$$\Rightarrow \frac{\partial^2 \psi(x,t)}{\partial x^2} = A i^2 k^2 e^{i(kx-\omega t)}$$

$$= -A\frac{p^2}{\hbar^2}e^{i(kx-\omega t)}$$

$$= -\frac{p^2}{\hbar^2}\psi(x,t) \quad \text{—(i)}$$

$$\Rightarrow -\hbar^2\frac{\partial^2}{\partial x^2}\psi(x,t) = p^2\psi(x,t)$$

We know, $\lambda = \frac{h}{p}$

$$\Rightarrow \frac{1}{\lambda} = p/h$$

$$\Rightarrow \frac{2\pi}{\lambda} = \frac{p \times 2\pi}{h}$$

$$\therefore k = p/\hbar$$

Additionally differentiating with respect to t,

$$\frac{\partial}{\partial t}\psi(x,t) = \frac{\partial}{\partial t}Ae^{i/\hbar(px-Et)}$$

$$\Rightarrow \frac{\partial}{\partial t}\psi(x,t) = A\frac{i}{\hbar}E\, e^{i/\hbar(px-Et)}$$

$$= \frac{iE}{\hbar}\psi(x,t)$$

$$\Rightarrow E\psi(x,t) = -\frac{\hbar}{i}\frac{\partial}{\partial t}\psi(x,t) \quad \text{—(ii)}$$

$E = h\nu$

$$\Rightarrow \frac{E}{2\pi} = \frac{h\nu}{2\pi}$$

$$\Rightarrow E = \frac{h\nu}{2\pi}2\pi$$

$$\Rightarrow E = \hbar\omega$$

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Energy conservation,

$$E = E_k + E_p$$

$$\Rightarrow E = \frac{p^2}{2m} + V(x,t)$$

$$\Rightarrow E\psi(x,t) = \frac{p^2}{2m}\psi(x,t) + V(x,t)\psi(x,t)$$

$$\Rightarrow -\frac{\hbar}{i}\frac{\partial}{\partial t}\psi(x,t) = \frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x,t) + V(x,t)\psi(x,t)$$

[from eqn (i) and (ii)]

This equation is Time Dependent Schrodinger Equation.

Ans to the que no-3

We know,

TDSE,

$$i\hbar\frac{\partial}{\partial t}\psi(x,t) = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x,t) + V(x,t)\psi(x,t)$$

→ $\psi(x,t)$ is the wave function that gives us the probability of a particle residing in a position x at time t,

→ $i\hbar\frac{\partial}{\partial t}\psi(x,t)$ is a operation form of the total energy.

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→ $\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x,t)$ is the kinetic Energy operator. It is similar to $\frac{p^2}{2m}$ from classical Physics.

→ $V(x,t)\psi(x,t)$ is the potential Energy operator

<u>Ans to the que no-4</u>

~~[illegible]~~

We know,

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x)+V(x).\psi(x)=E\psi(x)$$

here,

$\psi(x)$ is a spatial wave function

$V(x)$ is the independent potential

E is the total energy

The equation describes particles with stationary state. It also gives a probability of finding a particle at position x with respect to time

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Ans to the que no-5

We know,

$$i\hbar \frac{\partial}{\partial t} \psi(x,t) = \frac{-\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \psi(x,t) + V(x,t)\,\psi(x,t) \quad [TDSE]$$

Here given, $V = 0$. So,

$$i\hbar \frac{\partial}{\partial t} \psi(x,t) = \frac{-\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \psi(x,t)$$

This equation shows that the time for a free particles depends only on its kinetic energy.

Ans to the que no-6

We know,

TISE,

$$\frac{-\hbar^2}{2m} \frac{1}{\psi(x)} \cdot \frac{d^2}{dx^2} \psi(x) + V(x)\,\psi(x) = c \quad \left[\begin{matrix}\text{from}\\ \text{Ans-1}\end{matrix}\right]$$

When $V=0$,

$$-\frac{\hbar^2}{2m} \frac{1}{\psi(x)} \frac{d^2}{dx^2} \psi(x) = c$$