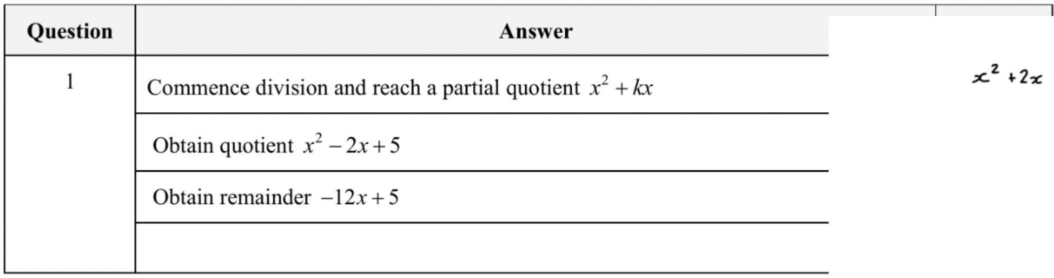
Q1



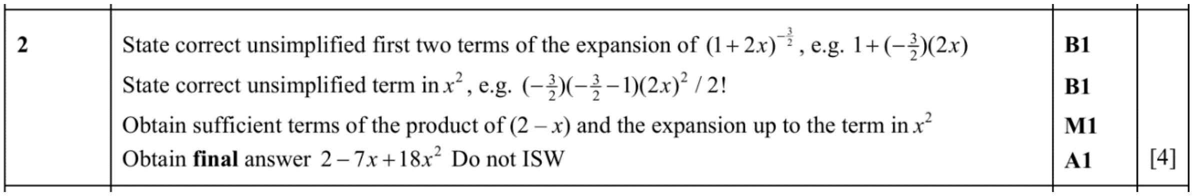
| Question | Answer |
|---|---|
| 1 | Commence division and reach a partial quotient $x^2 + kx$ |
| | Obtain quotient $x^2 - 2x + 5$ |
| | Obtain remainder $-12x + 5$ |
| | |

$$\begin{array}{r|l} & x^2 - 2x + 5 \\ \hline x^2 + 2x - 1 & x^4 \\ & x^4 + 2x^3 - x^2 \\ \hline & -2x^3 + x^2 \\ & -2x^3 - 4x^2 + 2x \\ \hline & 5x^2 - 2x \\ & 5x^2 + 10x - 5 \\ \hline & -12x + 5 \end{array}$$

Quotient : $x^2 - 2x + 5$

Remainder : $-12x + 5$

Q2

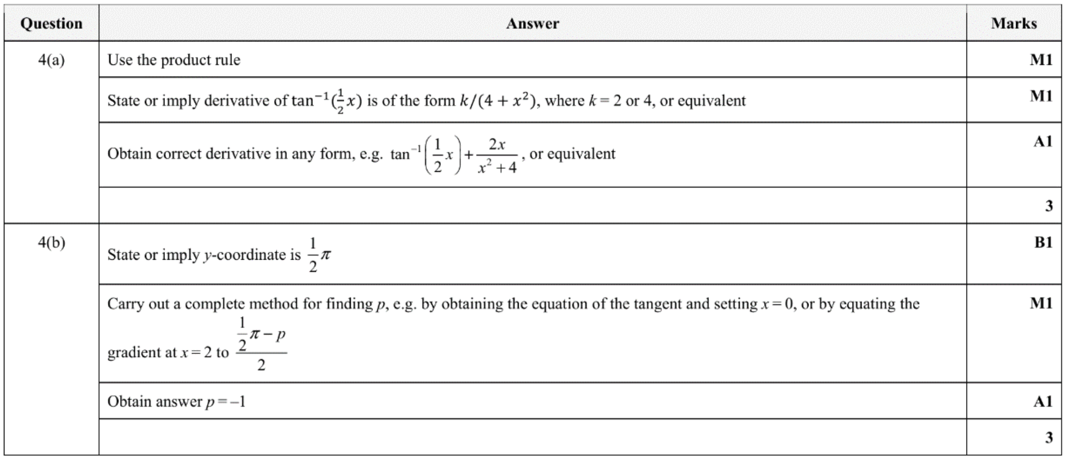


| | | | |
|---|---|---|---|
| **2** | State correct unsimplified first two terms of the expansion of $(1+2x)^{-\frac{3}{2}}$, e.g. $1+(-\frac{3}{2})(2x)$ | **B1** | |
| | State correct unsimplified term in $x^2$, e.g. $(-\frac{3}{2})(-\frac{3}{2}-1)(2x)^2 / 2!$ | **B1** | |
| | Obtain sufficient terms of the product of $(2-x)$ and the expansion up to the term in $x^2$ | **M1** | |
| | Obtain **final** answer $2-7x+18x^2$ Do not ISW | **A1** | [4] |

$$(1+2x)^{-\frac{3}{2}} = 1 + (-\tfrac{3}{2}(2x)) + \frac{-\frac{3}{2}(-\frac{5}{2})}{2}(2x)^2 + \ldots$$
$$= 1 - 3x + \tfrac{15}{2}x^2 + \ldots$$

$$(2-x)(1 - 3x + \tfrac{15}{2}x^2 + \ldots) = 2 - 6x + 15x^2 - x + 3x^2$$
$$= 2 - 7x + 18x^2 + \ldots$$

Q3



| Question | Answer | Marks |
|---|---|---|
| 4(a) | Use the product rule | M1 |
| | State or imply derivative of $\tan^{-1}(\frac{1}{2}x)$ is of the form $k/(4+x^2)$, where $k = 2$ or $4$, or equivalent | M1 |
| | Obtain correct derivative in any form, e.g. $\tan^{-1}\left(\frac{1}{2}x\right)+\frac{2x}{x^2+4}$, or equivalent | A1 |
| | | 3 |
| 4(b) | State or imply $y$-coordinate is $\frac{1}{2}\pi$ | B1 |
| | Carry out a complete method for finding $p$, e.g. by obtaining the equation of the tangent and setting $x = 0$, or by equating the gradient at $x = 2$ to $\frac{\frac{1}{2}\pi - p}{2}$ | M1 |
| | Obtain answer $p = -1$ | A1 |
| | | 3 |

a)

$$y = x\tan^{-1}(\tfrac{1}{2}x)$$
$$\frac{dy}{dx} = x\left(\frac{1}{2+\frac{1}{2}x^2}\right) + \tan^{-1}(\tfrac{1}{2}x)$$
$$= \frac{2x}{4+x^2} + \tan^{-1}(\tfrac{1}{2}x)$$

let $u = \frac{1}{2}x$

$$\frac{du}{dx} = \frac{1}{2}$$
$$\frac{dy}{du} = \frac{1}{1+u^2}$$
$$\frac{dy}{dx} = \frac{1}{2}\left(\frac{1}{1+\frac{1}{4}x^2}\right)$$

b)

$$f'(2) = \frac{4}{8} + \frac{\pi}{4}$$

$x = 2$

$$y = 2\left(\frac{\pi}{4}\right) = \frac{\pi}{2}$$

$$\Rightarrow \quad y - \frac{\pi}{2} = \left(\frac{1}{2} + \frac{\pi}{4}\right)(x-2)$$

$x = 0$

$$y = \frac{\pi}{2} - 1 - \frac{\pi}{2}$$
$$p = -1$$

Q4

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4 | State that $\frac{du}{dx} = \frac{1}{2\sqrt{x}}$ or $du = \frac{1}{2\sqrt{x}}\, dx$ | **B1** | |
| | Substitute throughout for $x$ and d$x$ | **M1** | |
| | Obtain a correct integral with integrand $\frac{2}{u^2+1}$ | **A1** | |
| | Integrate and obtain term of the form $k\tan^{-1}u$ | **M1** | $(2\tan^{-1}u)$ |
| | Use limits $\sqrt{3}$ and $\infty$ for $u$ or equivalent and evaluate trig. | **A1** | e.g. $2\left(\frac{\pi}{2}-\frac{\pi}{3}\right)$ Must be working in radians. |
| | Obtain answer $\frac{1}{3}\pi$ | **A1** | Or equivalent single term. |
| | | | |

$$\int_3^\infty \frac{1}{(x+1)\sqrt{x}}\, dx$$

$$= \int_3^\infty \frac{1}{(u^2+1)\sqrt{x}}\; 2\sqrt{x}\; du$$

$$= \int_{\sqrt{3}}^\infty \frac{2}{u^2+1}\, du$$

$$= 2\left[\tan^{-1}(u)\right]_{\sqrt{3}}^{\infty}$$

$$= 2\left[\frac{\pi}{2} - \frac{\pi}{3}\right]$$

$$= \frac{\pi}{3}$$

let $u = x^{\frac{1}{2}}$

$\frac{du}{dx} = \frac{1}{2\sqrt{x}}$

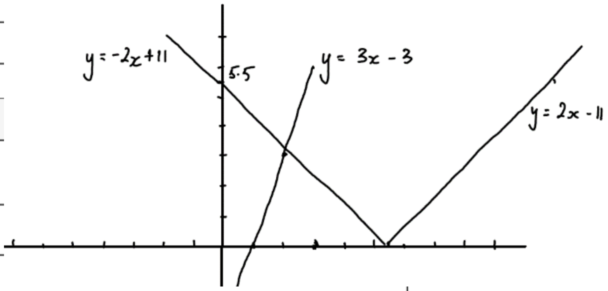
$dx = 2\sqrt{x}\, du$

$x = u^2$

$x+1 = u^2+1$

Q5

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(a) | Draw V-shaped graph with vertex on positive $x$-axis | **B1** | |
| | Draw approximately correct graph of $y = 3x - 3$ with greater gradient | **B1** | Crossing $x$-axis between origin and vertex of first graph. |
| | | 2 | |



| Question | Answer | Marks | |
|---|---|---|---|
| 4(b) | Attempt solution of linear equation where signs of $2x$ and $3x$ are different | **M1** | |
| | Solve $-2x+11=3x-3$ to obtain $x = \frac{14}{5}$ | **A1** | OE |
| | Conclude $x > \frac{14}{5}$ | **A1** | OE |
| | **Alternative method for Question 4(b)** | | |
| | Attempt solution of 3-term equation $(2x-11)^2 = (3x-3)^2$ to obtain at least one value of $x$ | **M1** | Or equivalent inequality. |
| | Obtain at least $x = \frac{14}{5}$ | **A1** | OE |
| | Conclude $x > \frac{14}{5}$ | **A1** | OE |
| | | 3 | |
| 4(c) | Attempt value of $N$ (maybe non-integer at this stage) using logarithms and *their* answer to part **(b)**. | **M1** | |
| | Conclude with single integer 17 | **A1** | |
| | | 2 | |

$\ln N = \frac{14}{5}$

$N = 16.44$

$\therefore N = 17$ smallest value of N

Q6

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(a) | Show sufficient working to justify the given answer | **B1** | |
| | | **1** | |
| 7(b) | Correct separation of variables | **B1** | e.g. $-\int \frac{1}{t}\mathrm{d}t = \int \frac{1}{x\ln x}\mathrm{d}x$ |
| | Obtain term $\ln(\ln x)$ | **B1** | |
| | Obtain term $-\ln t$ | **B1** | |
| | Evaluate a constant or use $x = \mathrm{e}$ and $t = 2$ as limits in an expression involving $\ln(\ln x)$ | **M1** | |
| | Obtain correct solution in any form, e.g. $\ln(\ln x) = -\ln t + \ln 2$ | **A1** | |
| | Use log laws to enable removal of logarithms | **M1** | |
| | Obtain answer $x = \mathrm{e}^{\frac{2}{t}}$, or simplified equivalent | **A1** | |
| | | **7** | |
| 7(c) | State that $x$ tends to 1 coming from $x = \mathrm{e}^{\frac{k}{t}}$ | **B1** | |
| | | **1** | |

$$y = \ln(\ln x)$$

$$y = \ln(u)$$

$$\frac{dy}{du} = \frac{1}{u}$$

$$\frac{dy}{dx} = \frac{1}{x\ln x}$$

let $u = \ln x$

$$\frac{du}{dx} = \frac{1}{x}$$

b)

$$t\frac{1}{dt} = -x\ln x \frac{1}{dx}$$

$$\int \frac{1}{t}\, dt = -\int \frac{1}{x\ln x}\, dx$$

$$\ln t = -\ln(\ln x) + c$$

$x = e$ $t = 2$ $\ln(2) = c$

$$\ln(\ln x) = \ln(2) - \ln(t)$$

$$\ln(\ln x) = \ln\left(\frac{2}{t}\right)$$

$$x = e^{\frac{2}{t}}$$

c) $t \to \infty$ $x \to e^{0}$

$\to 1$

Q7

| Question | Answer | Marks |
|---|---|---|
| 7(a) | Square $x + \mathrm{i}y$ and equate real and imaginary parts to 8 and –15 | M1 |
| | Obtain $x^2 - y^2 = 8$ and $2xy = -15$ | A1 |
| | Eliminate one unknown and find a horizontal equation in the other | M1 |
| | Obtain $4x^4 - 32x^2 - 225 = 0$ or $4y^4 + 32y^2 - 225 = 0$, or three term equivalent | A1 |
| | Obtain answers $\pm\frac{1}{\sqrt{2}}(5 - 3\mathrm{i})$ or equivalent | A1 |
| | | 5 |
| 7(b) | Show a circle with centre $2 + \mathrm{i}$ in a relatively correct position | B1 |
| | Show a circle with radius 2 and centre not at the origin | B1 |
| | Show line through i at an angle of $\frac{1}{4}\pi$ to the real axis | B1 |
| | Shade the correct region | B1 |
| | | 4 |

a) $u = 8 - 15i$

$\sqrt{8-15i} = a + ib$

$8 - 15i = a^2 - b^2 + 2iab.$

$a^2 - \frac{225}{4a^2} = 8$

$2ab = -5$

$b = -\frac{15}{2a}$

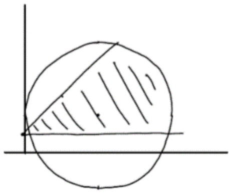
$4a^4 - 32a^2 - 225 = 0$

$a^2 = \frac{32 \pm \sqrt{32^2 - 4(4)(-225)}}{8}$

$= \frac{25}{2}, -\frac{9}{2}$

$\Rightarrow a = \pm\sqrt{\frac{25}{2}}$

$\sqrt{u} = \frac{5}{\sqrt{2}} - i\frac{3}{\sqrt{2}}$ and $\sqrt{u} = -\frac{5}{\sqrt{2}} + i\frac{3}{\sqrt{2}}$



Q8

| Question | Answer | Marks |
|---|---|---|
| 8(i) | Integrate by parts and reach $lx\mathrm{e}^{-\frac{1}{2}x} + m\int \mathrm{e}^{-\frac{1}{2}x}\,\mathrm{d}x$ | M1* |
| | Obtain $-2x\mathrm{e}^{-\frac{1}{2}x} + 2\int \mathrm{e}^{-\frac{1}{2}x}\,\mathrm{d}x$ | A1 |
| | Complete the integration and obtain $-2x\mathrm{e}^{-\frac{1}{2}x} - 4\mathrm{e}^{-\frac{1}{2}x}$, or equivalent | A1 |
| | Having integrated twice, use limits and equate result to 2 | M1(dep*) |
| | Obtain the given equation correctly | A1 |
| | | 5 |
| 8(ii) | Calculate values of a relevant expression or pair of expressions at $a = 3$ and $a = 3.5$ | M1 |
| | Complete the argument correctly with correct calculated values | A1 |
| | | 2 |
| 8(iii) | Use the iterative formula $a_{n+1} = 2\ln(a_n + 2)$ correctly at least once | M1 |
| | Obtain final answer 3.36 | A1 |
| | Show sufficient iterations to 4 d.p. to justify 3.36 to 2 d.p., or show there is a sign change in the interval (3.355, 3.365) | A1 |
| | | 3 |

i) $\int_0^a xe^{-\frac{1}{2}x}\,dx = 2$

let $u = x$ $\quad \frac{dv}{dx} = e^{-\frac{1}{2}x}$

$\frac{du}{dx} = 1$ $\quad v = -2e^{-\frac{1}{2}x}$

$\int_0^a xe^{-\frac{1}{2}x}\,dx = -2xe^{-\frac{1}{2}x} + 2\int_0^a e^{-\frac{1}{2}x}\,dx$

$= \left[-2xe^{-\frac{1}{2}x} - 4e^{-\frac{1}{2}x}\right]_0^a$

$\Rightarrow -2ae^{-\frac{1}{2}a} - 4e^{-\frac{1}{2}a} + 4 = 2$

$-2e^{-\frac{1}{2}a}(a+2) = -2$

$(a+2) = e^{\frac{1}{2}a}$

$\ln(a+2) = \frac{1}{2}a$

$a = 2\ln(a+2)$

ii) $a = 3$ $\quad 3 - 2\ln(5) = -0.218\ldots$

$a = 3.5$ $\quad 3.5 - 2\ln(5.5) = 0.0905\ldots$

iii) $u_1 = 3$ $\quad u_2 = 2\ln(3+2)$

$= 3.2188..$

$u_3 = 3.3045$

$u_4 = 3.3371$

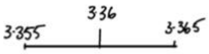
$u_5 = 3.3493$

$u_6 = 3.3539$

$u_7 = 3.3556$

$u_8 = 3.3563$

$u_9 = 3.3565$



$3.355 - 2\ln(3.355) = -1.06 \times 10^{-3}$

$3.365 - 2\ln(3.365) = 5.2 \times 10^{-3}$

$\therefore a = 3.36$ to 3 s.f.

## Q9

| | | | |
|---|---|---|---|
| **(i)** | State or imply the form $A+\dfrac{B}{2x+1}+\dfrac{C}{x+2}$ | **B1** | |
| | State or obtain $A = 2$ | **B1** | |
| | Use a correct method for finding a constant | **M1** | |
| | Obtain one of $B = 1$, $C = -2$ | **A1** | |
| | Obtain the other value | **A1** | [5] |
| **(ii)** | Integrate and obtain terms $2x+\frac{1}{2}\ln(2x+1)-2\ln(x+2)$ | **B3**√ | |
| | Substitute correct limits correctly in an integral with terms $a\ln(2x+1)$ and $b\ln(x+2)$, where $ab \neq 0$ | **M1** | |
| | Obtain the given answer after full and correct working | **A1** | [5] |

i)

$$f(x) = 2 - \frac{3x}{(2x+1)(x+2)}$$

$$\begin{array}{r} 2 \\ 2x^2+5x+2\,\overline{)\,4x^2+7x+4} \\ \underline{4x^2+10x+4} \\ -3x \end{array}$$

$$-3x = A(x+2) + B(2x+1)$$

$$x = -2 \qquad 6 = -3B \qquad x = -\tfrac{1}{2} \qquad \tfrac{3}{2} = \tfrac{3}{2}A$$

$$B = -2 \qquad A = 1$$

$$\therefore \quad f(x) = 2 + \frac{1}{2x+1} - \frac{2}{x+2}$$

ii)

$$\int_0^4 2 + \frac{1}{2x+1} - \frac{2}{x+2}\,dx = \Big[2x + \tfrac{1}{2}\ln|2x+1| - 2\ln|x+2|\Big]_0^4$$

$$= \Big[8 + \tfrac{1}{2}\ln(9) - 2\ln(6) - (-2\ln(2))\Big]$$

$$= 8 + \ln 3 - 2\ln(6) + 2\ln(2)$$

$$= 8 + \ln 3 + \ln\left(\tfrac{1}{9}\right)$$

$$= 8 + \ln\left(\tfrac{1}{3}\right)$$

$$= 8 - \ln(3)$$

## Q10

| | | | | |
|---|---|---|---|---|
| **8** | **(i)** | State $2\sin x\cos x\,.\,\dfrac{\cos x}{\sin x}$ | **B1** | |
| | | Simplify to confirm $2\cos^2 x$ | **B1** | [2] |
| | **(ii) (a)** | Use $\cos 2x = 2\cos^2 x - 1$ | **B1** | |
| | | Express in terms of $\cos x$ | **M1** | |
| | | Obtain $16\cos^2 x + 3$ or equivalent | **A1** | |
| | | State 3, following their expression of form $a\cos^2 x + b$ | **A1** | [4] |
| | **(b)** | Obtain integrand as $\dfrac{1}{2}\sec^2 2x$ | **B1** | |
| | | Integrate to obtain form $k\tan 2x$ | **M1*** | |
| | | Obtain correct $\dfrac{1}{4}\tan 2x$ | **A1** | |
| | | Apply limits correctly | **dep M1*** | |
| | | Obtain $\dfrac{1}{4}\sqrt{3}-\dfrac{1}{4}$ or exact equivalent | **A1** | [5] |

i)

$$\text{LHS} = 2\sin x\cos x \cdot \frac{\cos x}{\sin x}$$

$$= 2\cos^2 x$$

ii) a)

$$6\cos^2 x + 5\cos 2x + 8 = 6\cos^2 x + 5(2\cos^2 x - 1) + 8$$

$$= 16\cos^2 x + 3$$

$$y = 16\cos^2 x + 3$$

$\therefore$ the least value $= 3$

b)

$$\int_{\frac{\pi}{8}}^{\frac{\pi}{6}} \operatorname{cosec} 4x \tan 2x \,dx = \int_{\frac{\pi}{8}}^{\frac{\pi}{6}} \frac{\tan 2x}{\sin 4x}\,dx$$

$$= \int_{\frac{\pi}{8}}^{\frac{\pi}{6}} \frac{\frac{\sin 2x}{\cos 2x}}{2\sin 2x\cos 2x}\,dx$$

$$= \int_{\frac{\pi}{8}}^{\frac{\pi}{6}} \frac{1}{2\cos^2 2x}\,dx$$

$$= \int_{\frac{\pi}{8}}^{\frac{\pi}{6}} \frac{1}{2}\sec^2 2x\,dx$$

$$= \frac{1}{4}\Big[\tan 2x\Big]_{\frac{\pi}{8}}^{\frac{\pi}{6}}$$

$$= \frac{1}{4}\Big[\sqrt{3} - 1\Big]$$