1 It is known that, on average, 2 people in 5 in a certain country are overweight. A random sample of 400 people is chosen. Using a suitable approximation, find the probability that fewer than 165 people in the sample are overweight. [5]

−1

$X \sim B(400, \frac{2}{5})$ $np = 160$ ~~⊞~~ $\sigma^2 = 96$

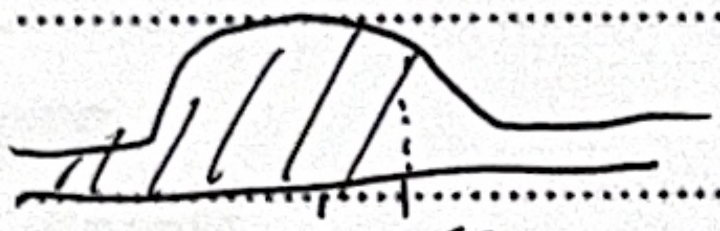
$Y \sim N(160, 96)$ B1 Approx.

~~$P(Y < 165) = P(\ldots <$~~

$P(\cancel{B}X < 165) = P(\cancel{Y} < 165.5) = P(Z < \frac{165.5 - 160}{\sqrt{96}})$ M1 M1

$= P(Z < 0.56134) = \Phi(0.5613)$

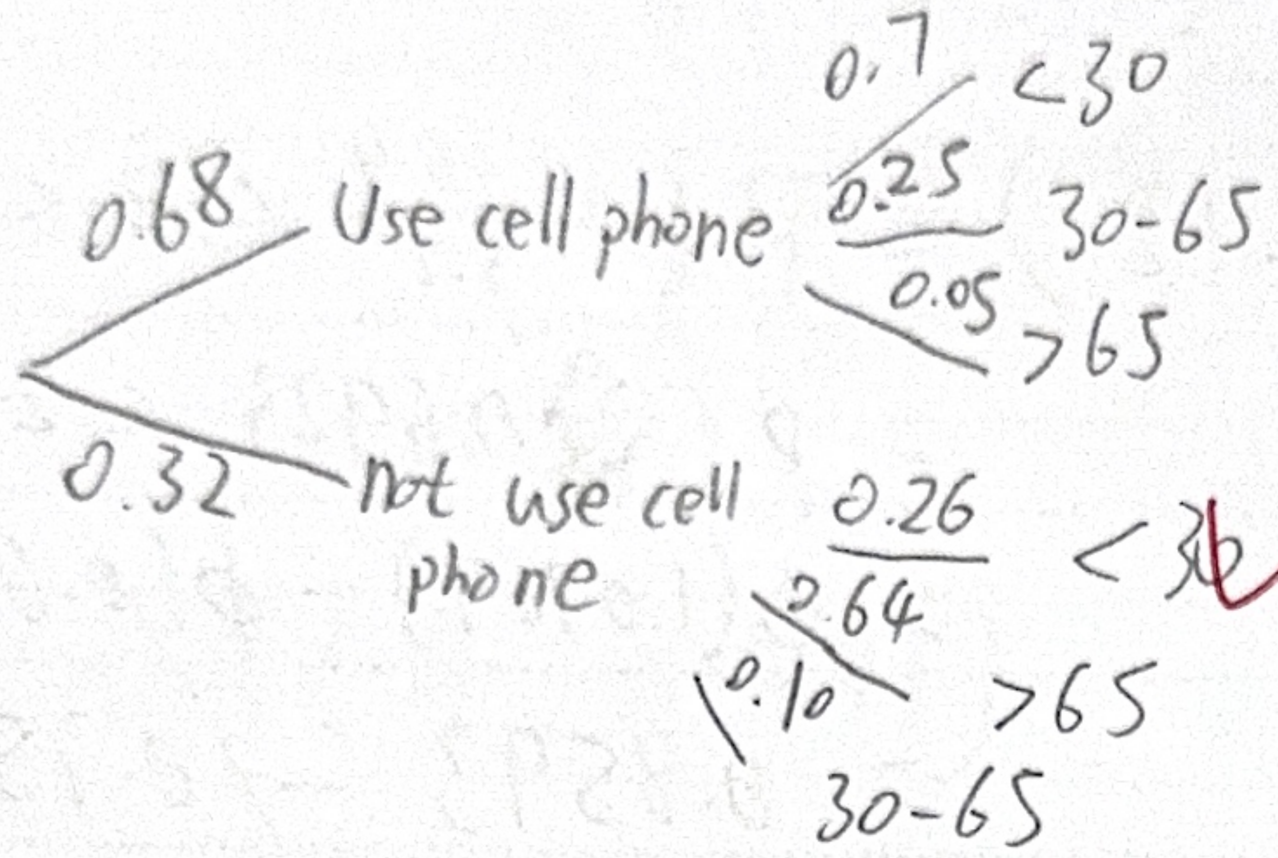
$= 0.7126 \approx 71.3\%$ M1



$P(Z < 0.5613) = \Phi(0.5613) = 0.7126$

2 It was found that 68% of the passengers on a train used a cell phone during their train journey. Of those using a cell phone, 70% were under 30 years old, 25% were between 30 and 65 years old and the rest were over 65 years old. Of those not using a cell phone, 26% were under 30 years old and 64% were over 65 years old.

(i) Draw a tree diagram to represent this information, giving all probabilities as decimals. [2]



(ii) Given that one of the passengers is 45 years old, find the probability of this passenger using a cell phone during the journey. [3]

$P(\text{Use} \mid 30\text{-}65) = \dfrac{P(\text{Use} \cap 30\text{-}65)}{P(30\text{-}65)}$

$P(\text{Use} \cap 30\text{-}65) = 0.68 \times 0.25 = 0.17$

$P(30\text{-}65) = 0.68 \times 0.25 + 0.32 \times 0.10 = 0.202$

$P(\text{Use} \mid 30\text{-}65) = \dfrac{0.17}{0.202} = \cancel{0.8416}\ \dfrac{85}{101}$

3 The random variable X is the daily profit, in thousands of dollars, made by a company. X is normally distributed with mean 6.4 and standard deviation 5.2.

(i) Find the probability that, on a randomly chosen day, the company makes a profit between \$10 000 and \$12 000. [3]

$X \sim N(6.4^{k}, 5.2k^{2})$

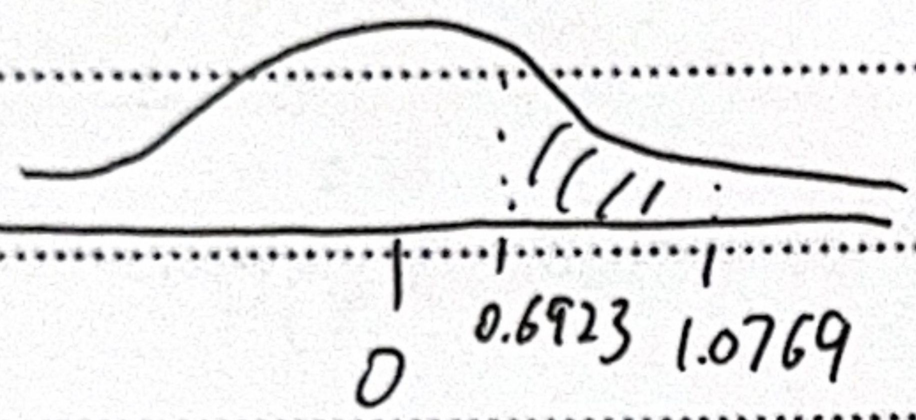
$P(10 < X < 12) = P\left(\frac{10-6.4}{5.2} < Z < \frac{12-6.4}{5.2}\right)$

$= P(0.6923 < Z < 1.0769)$

$= \phi(1.0769) - \phi(0.6923)$

$= 0.8593 - 0.7556$

$= 0.1037$



$\approx$ ~~10.8~~ 10.4% why??

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(ii) Find the probability that the company makes a loss on exactly 1 of the next 4 consecutive days. [4]

$P(X<0) = P\left(Z < \frac{0-6.4}{5.2}\right) = P(Z < -1.231)$

$= \phi(-1.231) = 1 - \phi(1.231)$

$= 1 - 0.8909$

$= 0.1091$

$Y \sim B(4, 0.1091)$

$P(Y=1) = {}^4C_1 \times (0.8909)^3 \times (0.1091)^1$

$= 0.30858$

$\simeq 0.309 \simeq 30.9\%$

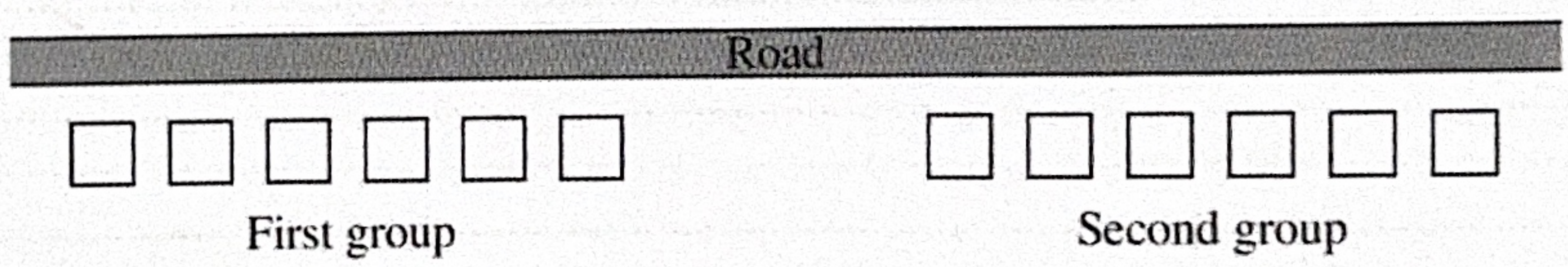
why??

4 A builder is planning to build 12 houses along one side of a road. He will build 2 houses in style A, 2 houses in style B, 3 houses in style C, 4 houses in style D and 1 house in style E.

(i) Find the number of possible arrangements of these 12 houses. [2]

$$\frac{12!}{2! \times 2! \times 3! \times 4!} = 831600$$

(ii)



The 12 houses will be in two groups of 6 (see diagram). Find the number of possible arrangements if all the houses in styles A and D are in the first group and all the houses in styles B, C and E are in the second group. [3]

first group   second group

$\frac{6!}{2! \times 4!} = 15$ $\frac{6!}{2! \times 3!} = 60$

$15 \times 60 = 900$ ways

(iii) Four of the 12 houses will be selected for a survey. Exactly one house must be in style B and exactly one house in style C. Find the number of ways in which these four houses can be selected. [2]

A B C D E
2 2 3 4 1

C B A A = 1

M1

C B ~~A~~D D = 1

−1

C B E ~~A~~A = 1

C B E D = 1

C B A D = 1

5 ways

5 The mean and standard deviation of 20 values of x are 60 and 4 respectively.

(i) Find the values of Σx and Σx^2. [3]

$\cancel{\#}\ \bar{x} = 60 \qquad \sigma = 4 \qquad \sigma^2 = 16$

$\bar{x} = \frac{\Sigma x}{n} \Rightarrow \frac{\Sigma x}{20} = 60$

$\Sigma x = 1200$

$\sigma^2 = \frac{\Sigma x^2}{n} - \bar{x}^2 \Rightarrow 16 = \frac{\Sigma x^2}{20} - 60^2$

$20 \times 3616 = \Sigma x^2$

$\Sigma x^2 = 72320$

Another 10 values of x are such that their sum is 550 and the sum of their squares is 40 500.

(ii) Find the mean and standard deviation of all these 30 values of x. [4]

$\Sigma x = 1200 + 550 = 1750$

$\bar{x} = \cancel{1750}\ \frac{\Sigma x}{\cancel{19}} = \frac{1750}{30} = \cancel{58.}\ \frac{175}{3} = 58.\dot{3}\dot{3}$

$\sigma^2 = \frac{\Sigma x^2}{n} - \bar{x}^2 = \frac{112820}{30} - \frac{30625}{9} = \frac{3221}{9}$

$\Sigma x^2 = 40500 + 72320 = 112820$

$\bar{x}^2 = \left(\frac{175}{3}\right)^2 = \frac{30625}{9}\ \cancel{= 3402.78}$

$\sigma = \sqrt{\sigma^2} = \sqrt{\frac{3221}{9}} \approx 18.918 \approx 18.92$

mean is $\frac{175}{3}$ or 58.33

standard deviation is ~~18.918~~ 18.92

6 The pulse rates, in beats per minute, of a random sample of 15 small animals are shown in the following table.

115	120	158	132	125
104	142	160	145	104
162	117	109	124	134

(i) Draw a stem-and-leaf diagram to represent the data. [3]

stem	leaf
10	4 4 9
11	5 7
12	0 4 5
13	2 4
14	2 5
15	8
16	0 2

10|4 = 104 bpm.

(ii) Find the median and the quartiles. [2]

median: 125

$\frac{15}{2} = 7.5 \Rightarrow$ ~~num~~ 8^{th}

15

7 | 8 | 7

3 | 1 | 3   3 | 1 | 3

4^{th} 12^{th}

11   12

115   145

IQR = 115 ~ 145

= 30

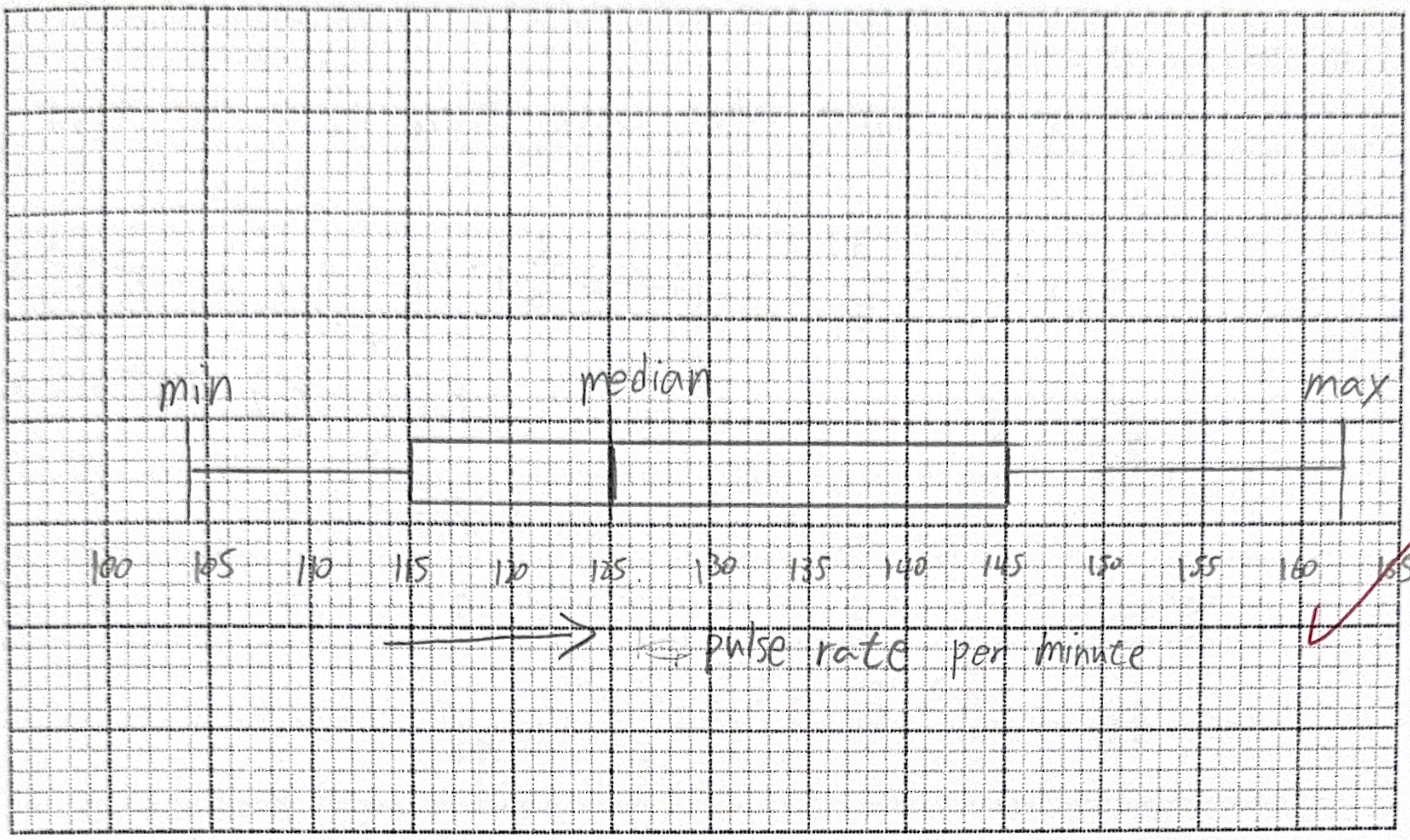
~~first Q~~ 1^{st} Q = 104 − 115

2^{nd} Q = 115 − 125

3^{rd} Q = 125 − 145.

4^{th} Q = 145 − 165

(iii) Draw a box-and-whisker plot to represent this data on the graph paper below. [3]



7 A box contains 2 green apples and 2 red apples. Apples are taken from the box, one at a time, without replacement. When both red apples have been taken, the process stops. The random variable X is the number of apples which have been taken when the process stops.

(i) Show that $P(X = 3) = \frac{1}{3}$. [3]

X : apple taken when there is 2 red taken

$P(X=3) = \cancel{\frac{2}{4} \times \frac{2}{4} \times \frac{1}{3}} \quad P(r,g,r) = \frac{2}{4} \times \frac{2}{3} \times \frac{1}{2} = \frac{1}{6}$

$+ \quad P(g,r,r) = \frac{2}{4} \times \frac{2}{3} \times \frac{1}{2} = \frac{1}{6}$

$= \frac{1}{3}$

(ii) Draw up the probability distribution table for X. [3]

-1

x	2	3	4
$P(X=x)$	$\frac{1}{6}$	$\frac{1}{3}$	~~$\frac{1}{4}$~~ $\frac{1}{2}$

$P(X=2) = P(r,r) = \frac{2}{4} \times \frac{1}{3} = \frac{1}{6}$

$P(X=4) = P(g,g,r,r) = \frac{2}{4} \times \frac{1}{3} \times \frac{1}{2} \times 1 = \frac{1}{12}$ or $1-(P(X=2)+P(X=3))$ =

$+ \; P(r,g,g,r) = \ldots = \frac{1}{12}$

$+ \; P(g,r,g,r) = \ldots = \frac{1}{12}$

$\frac{3}{12} = \frac{1}{4}$

Another box contains 2 yellow peppers and 5 orange peppers. Three peppers are taken at random from the box without replacement.

(iii) Given that at least 2 of the peppers taken from the box are orange, find the probability that all 3 peppers are orange. [5]

3

$$P(3O \mid 2O) = \frac{P(3O \cap 2O)}{P(2O)}$$

$$P(3O \cap 2O) = P(O,O,O) = \frac{5}{7} \times \frac{4}{6} \times \frac{3}{5} = \frac{2}{7}$$

$$P(2O) = \cancel{P(O,O,O)} = \cancel{\frac{5}{7} \times \frac{4}{6} = \frac{10}{21} \times \frac{2}{5} = \frac{4}{21}}$$

$$\cancel{P(O',O,O) \ldots = 7 \ \frac{4}{21} \times \frac{5}{} = \frac{4}{7}}$$

$$P(O,O) = \frac{5}{7} \times \frac{4}{6} = \frac{10}{21}$$

$$P(3O \mid 2O) = \frac{\frac{2}{7}}{\frac{10}{21}} = \frac{3}{5}$$

M1 A1