

## ejercicio 02

$$\frac{\partial^2 u}{\partial x^2} = \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{h_x^2}, \quad \frac{\partial u}{\partial x} = \frac{u_{i+1,j} - u_{i-1,j}}{2h_x}$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = w, \text{ reemplazando } \rightarrow \dots$$

$$\frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{h_x^2} + \frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{h_y^2} = w_{i,j}$$

despejando  $u_{i,j}$

$$-4u_{i,j} = -u_{i+1,j} - u_{i-1,j} - u_{i,j+1} - u_{i,j-1} + h^2 w_{i,j}$$

$$u_{i,j} = \frac{1}{4} (u_{i+1,j} + u_{i-1,j} + u_{i,j+1} + u_{i,j-1}) - \frac{h^2}{4} w_{i,j}$$

entonces

$$v \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) = \frac{\partial u}{\partial y} \frac{\partial w}{\partial x} - \frac{\partial u}{\partial x} \frac{\partial w}{\partial y}$$

$$v \left( \frac{w_{i+1,j} - 2w_{i,j} + w_{i-1,j}}{h_x^2} + \frac{w_{i,j+1} - 2w_{i,j} + w_{i,j-1}}{h_y^2} \right) = \dots$$

$$= (u_{i,j+1} - u_{i,j-1}) (w_{i+1,j} - w_{i-1,j}) / 4h^2$$

$$= -(u_{i+1,j} - u_{i-1,j}) (w_{i,j+1} - w_{i,j-1}) / 4h^2$$

$$w_{i,j} = \frac{R}{16} (u_{i+1,j} - u_{i-1,j}) (w_{i,j+1} - w_{i,j-1}) - \frac{R}{16} (u_{i,j+1} - u_{i,j-1}) \dots$$

$$(w_{i+1,j} - w_{i-1,j}) + \frac{1}{4} (w_{i+1,j} + w_{i-1,j} + w_{i,j+1} + w_{i,j-1})$$

demostrado.

## ejercicio 3

$$u(x, y+h) = u(x, y) + \frac{\partial u}{\partial y} h + \frac{h^2}{2} \frac{\partial^2 u}{\partial y^2} + O(h^3)$$

$$w_z = w = \frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} = \frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2}$$

$$w_z = -\frac{\partial v_x}{\partial y}$$

$$w_z = -\frac{\partial^2 u}{\partial y^2}$$



$$u(x, y+h) = u(x, y) - \frac{h^2}{2} w_z$$

↳ despejando  $w_z$

$$w_z = \frac{-2u(x, y+h) - u(x, y)}{h^2} ; w_{i,j}(P_{i,j+h}) = \frac{-2u_{i,j+h} - u_{i,j}}{h^2} \checkmark$$

$$w_z = -\frac{\partial v_y}{\partial x}$$

$$w_z = -\frac{\partial^2 u}{\partial x^2}$$

$$u(x, y+h) = u(x, y) - \frac{h^2}{2} w_z$$

despejando  $w_z$

$$w_z = \frac{-2u(x, y+h) - u(x, y)}{h^2}$$

$$w_{i,j}(P_{i,j+h}) = \frac{-2u_{i,j+h} - u_{i,j}}{h^2} \checkmark \text{ Demostrado}$$