Punto 1

Sabemos y comprobamos que la función es continua, definida y acotada en el intervalo $[-T/2, T/2]$, comprobando si su sumatoria converge planteamos su sumatoria como serie de Fourier

$$f(t) = \sum_{n=-\infty}^{0} C \cdot n \cdot e^{inW_0 t} + \sum_{n=1}^{\infty} C \cdot n \cdot e^{inW_0 t}$$

lo cual tanto con signo positivo como negativo tendrá sentido, por lo que:

$$f(t) = C_0 + \sum_{n=1}^{\infty} C(-n) e^{-inW_0 t} + \sum_{n=1}^{\infty} C_n e^{inW_0 t}$$

Sabemos que: $g_n(t) = C_n e^{inW_0 t}$ tenderá a

$$|g_n(t)| = |C_n|$$

acotando de manera correcta la integral:

$$C_n = \frac{1}{T} \int_{-T/2}^{T/2} f(t) e^{-inW_0 t} \, dt$$

que converge.

Utilizando teorema de Parseval en la integral

$$C_n = \frac{1}{T} \int_{-T/2}^{T/2} f(t) e^{-inW_0 t} \, dt \leq \left| \frac{1}{T} \int_{-T/2}^{T/2} f(t) e^{-inW_0 t} \, dt \right| \leq \ldots$$

b)

$$\dots \frac{1}{T}\int_{-T/2}^{T/2} |f(t)|dt \leq \frac{1}{T}\int_{-T/2}^{T/2} |f(t)|^2 dt$$

Dando que:

$\sum_{n=-\infty}^{\infty} |c_n|^2$ por lo que la sumatoria converge y converge uniformemente.

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + b_n \sin(\cancel{m}\omega_0 t)$$

Ya que si es uniforme:

$$\frac{d}{dt} f(t) = \sum_{n=1}^{\infty} \frac{d}{dt}\left(a_n(\cos(n\omega_0 t)) + b_n \sin(n\omega_0 t)\right)$$

$$= \sum_{n=1}^{\infty} n\omega_0\left(-a_n \sin(n\omega_0 t) + b_n \cos(n\omega_0 t)\right)$$

Integrando

$$\int_{t_1}^{t_2} f(t)dt = \int_{t_1}^{t_2} \sum_{n=1}^{\infty}\left(a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t) + \frac{a_0}{2}\right) dt$$

$$= \frac{a_0}{2}(t_2 - t_1) + \sum_{n=1}^{\infty} \frac{1}{n\omega_0}\big((-b_n(\cos(n\omega_0 t_2) -$$

$$\cos(n\omega_0 t_1)) + a_n(\sin(n\omega_0 t_2) -$$

$\sin(n\omega_0 t_1))$

Usando la serie en el intervalo $-\pi \leq t \leq \pi$ y $f(t+2\pi)$

$$t^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}\cos(nt)$$

$$\int t^2 \rightarrow \frac{1}{12}t(t^2-\pi^2) = \sum_{n=1}^{\infty} \frac{(-1)^n}{n^3}\sin(nt)$$

De nuevo con Parseval:

$$\frac{1}{T}\int_{-T/2}^{T/2} f(t)\,dt = \frac{1}{4}a_0^2 + \frac{1}{2}\sum_{n=1}^{\infty}(a_n^2+b_n^2)$$

Ya que es impar:

$$\frac{1}{2\pi}\int_{-\pi}^{\pi}\left(\frac{1}{12}t(t^2-\pi^2)\right)^2 dt = \frac{1}{2}\sum_{n=1}^{\infty}\frac{(-1)^{2n}}{n^6}$$

Suma tiene valor:

$$\sum_{n=1}^{\infty}\frac{1}{n^6} = \frac{\pi^6}{945}$$

Cálculos matemáticos.

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## Presentación de Funciones.

$f(t)=t$, donde $-\pi\le t\le\pi$, $f(t+2\pi)=f(t)$

$$a_0=\frac{1}{\pi}\int_{-\pi}^{\pi}t\,dt=\frac{1}{\pi}\frac{t^2}{2}\Big|_{-\pi}^{\pi}=\frac{1}{\pi}\left(\frac{\pi^2}{2}-\frac{\pi^2}{2}\right)=0$$

$$a_n=\frac{1}{\pi}\int_{-\pi}^{\pi}t\cos\left(\frac{n\pi t}{\pi}\right)=\frac{1}{\pi}\int_{-\pi}^{\pi}t\cos(nt)dt=\frac{1}{\pi}\left(\frac{1}{n}t\,sen(nt)\right)\Big|_{-\pi}^{\pi}-\frac{1}{n}\int_{-\pi}^{\pi}sen(nt)dt$$

$$=\frac{1}{\pi n^2}\cos(n\pi)-\cos(n\pi)=0$$

$$b_n=\frac{1}{\pi}\left(\frac{-t\cos(nt)}{n}\Big|_{-\pi}^{\pi}+\int_{-\pi}^{\pi}\frac{\cos(nt)}{n}dt\right)=-\frac{1}{n\pi}\left(t\cos(nt)\Big|_{-\pi}^{\pi}\right)$$

$$=\frac{-1}{n\pi}\left(\pi\cos(n\pi)+\pi\cos(n-\pi)\right)=\frac{-1}{n\pi}2\pi\cos(n\pi)=\frac{2}{n}\left(-\cos(n\pi)\right)=\frac{2}{n}(-1)^{n-1}$$

$$f(t)=\sum_{n=1}^{\infty}\frac{2}{n}(-1)^{n+1}sen(nt)$$

## Función $\zeta(s)$ de Riemann.

$f(t)=t^2$; donde $-\pi\le t\le\pi$; $f(t+2\pi)=f(t)$

$$a_0=\frac{1}{\pi}\int_{-\pi}^{\pi}t^2dt=\frac{1}{\pi}\left(\frac{t^3}{3}\right)_{-\pi}^{\pi}=\frac{2}{3}\pi^2$$

$$a_n=\frac{1}{\pi}\int_{-\pi}^{\pi}t^2\cos(n\pi t)dt=\frac{1}{\pi}\left(t^2\frac{1}{n}sen(nt)-\frac{2}{n}\int_{-\pi}^{\pi}sen(nt)t\,dt\right)$$

$$=\frac{1}{n\pi}\left(t^2sen(nt)+\frac{2}{n}t\cos(nt)-\frac{2}{n^2}sen(nt)\right)_{-\pi}^{\pi}=\frac{4}{n^2}(-1)^n$$

$$b_n=\frac{1}{\pi}\int_{-\pi}^{\pi}t^2sen(nt)dt=0$$

$$f(t)=\frac{1}{3}\pi^2+4\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\cos(nt)$$

$$\int_{-\pi}^{\pi}f(t)=\int_{-\pi}^{\pi}\frac{1}{3}\pi^2+4\sum_{n=1}^{\infty}\frac{-1^n}{n^2}\int_{-\pi}^{\pi}\cos(nt)=\frac{1}{12}t(t^2-\pi^2)=\sum_{n=1}^{\infty}\frac{(-1)^n}{n^3}sen(nt)$$

Usamos la identidad de Parseval, tenemos que:

$$\frac{1}{2\pi}\int_{-\pi}^{\pi}\left(\frac{1}{12}t(t^2-\pi^2)\right)^2 dt = \frac{1}{2}\sum_{n=1}^{\infty}\frac{(-1)^{2n}}{n^6}$$

$$\zeta(6)=\sum_{n=1}^{\infty}\frac{1}{n^6} = \frac{1}{\pi}\int_{-\pi}^{\pi}\left(\frac{t}{12}(t^2-\pi^2)\right)^2 dt$$