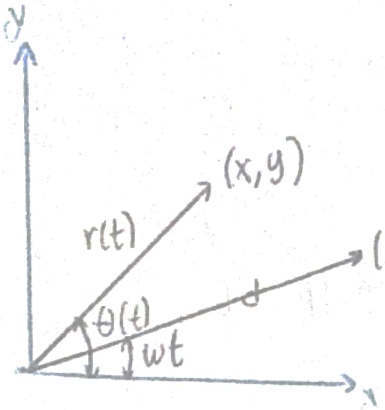
Calculos "Lunar Rocket", metodos computacionales 2.

c) muestre que la distancia

$$r_2(r,\theta,t) = \sqrt{r(t)^2+d^2-2r(t)\cdot d\cdot\cos(\theta-\omega t)}$$



$x = r\cos(\theta)$ | $x_L = d\cos(\omega t)$

$y = r\sin(\theta)$ | $y_L = d\sin(\omega t)$

$$\Rightarrow r_L^2 = [r\cos(\theta)-d\cos(\omega t)]^2 + [r\sin(\theta)-d\sin(\omega t)]^2$$

$$r_L^2 = r^2\cos^2(\theta)+r^2\sin^2\theta - 2rd\cos\theta\cos(\omega t) - 2rd\sin(\theta)\sin(\omega t) + d^2\cos^2(\omega t)+d^2\sin^2(\omega t)$$

$$= r^2[\cos^2\theta+\sin^2\theta] - 2rd[\cos\theta\cos(\omega t)+\sin\theta\sin(\omega t)] + d^2[\cos^2(\omega t)+\sin^2(\omega t)]$$

$$= r^2+d^2-2rd\cos(\theta-\omega t)$$

$$r_L(r,\theta,t) = \sqrt{r^2+d^2-2rd\cos(\theta-\omega t)}$$ ✓ Comprobado

d) Hallando lagrangiano ($\mathcal{L}$)

$\mathcal{L} = K - U$; K = E. cinetica, U = E. Potencial

$$H = P_r\cdot\dot r + P_f\cdot\phi - \mathcal{L}$$

$$P_r = (m\dot r)\dot r = m\dot r^2 = \frac{P_r^2}{m} \; ; \; P_f = (mr^2\dot\phi)\dot\phi = \frac{m^2 r\phi^2}{mr^2} = \frac{P_f^2}{mr^2}$$

$$K = \frac{1}{2}m(\dot x^2+\dot y^2) \; ; \; x = r\cos(\phi(t)),\; y = r\sin(\theta(t))$$

$$\dot x = \dot r\cos(\phi(t)) - r\sin(\phi(t))\cdot\phi(t) \; ; \; \dot y = \dot r\sin(\phi(t)) + r\cos(\phi(t))\dot\phi(t)$$

$$\dot x^2 = \dot r^2\cos^2(\phi(t)) - 2\dot r r\sin(\phi(t))\cdot\cos(\phi(t))\phi(t) + r^2\sin^2(\phi(t))\cdot\dot\phi(t)^2$$

$$\dot y^2 = \dot r^2\sin^2(\theta t) + 2\dot r r\sin(\phi(t))\cdot\cos(\phi(t))\,\phi(t) + r^2\cos^2(\phi(t))\cdot\dot\phi(t)^2$$

$$\dot{x}^2+\dot{y}^2 = \dot{r}^2+r^2\dot{\phi}(t)^2$$

$$k = \frac{1}{2}m\dot{r}^2 + \frac{1}{2}r^2\dot{\phi}(t)^2 = \frac{P_r^2}{2m} + \frac{P_\phi^2}{2mr^2}$$

$$U = U_{Ter} + U_{Lun} \;;\; U = -\frac{GmM}{r} = -G\frac{m \cdot M_L}{r(r,\phi,t)}$$

$$H = \frac{P_r^2}{2m} + \frac{P_\phi^2}{2mr^2} - G\frac{mM_T}{r} - G\frac{mM_L}{r(r,\phi,t)} \quad -\mathcal{L} : \mathcal{L} = H - \frac{P_r^2}{2m} + \frac{P_\phi^2}{2mr^2} - \frac{GmM_T}{r} - \dots$$

e) Deduzca las ecuaciones del movimiento a partir del Hamiltoniano

$$\dot{r} = \frac{\partial H}{\partial P_r} \;;\; \dot{r} = \frac{P_r}{m} \quad \text{como } \dot{\phi} = \frac{\partial H}{\partial P_\phi} \;,\; \dot{\phi} = \frac{P_\phi}{mr}$$

$$\frac{\partial}{\partial r}(r_L^{-1}) = -\frac{(r - d\cos(\phi - \omega t))}{r_L^3}$$

$$\frac{\partial}{\partial \phi}(r_L^{-1}) = -\frac{1}{r_L^2} \cdot \frac{[2rd\sin(\phi-\omega t)]}{2r_L} = -\frac{rd\sin(\phi-\omega t)}{r_L^3}$$

$\dot{P}_r = -\frac{\partial H}{\partial r}$ (y ya lo habíamos hallado antes); $P_\theta = P_\phi$; $\dot{P}_\phi = -\frac{\partial H}{\partial \phi}$

$$\boxed{\dot{P}_r = \frac{P_\theta^2}{mr^3} - \frac{GmM_T}{r^2} - \frac{GmM_L}{r_L^3}[r - d\cos(\theta-\omega t)]}$$

$$\boxed{\dot{P}_\theta = -\frac{GmM}{r_L^3}[rd\sin(\theta-\omega t)]}$$

f) Deduzca las ecuaciones de movimiento con el siguiente cambio de variable

$$\vec{r} = \frac{r}{d} \;,\; \theta = \phi \;,\; \tilde{P}_r = \frac{P_r}{md} \;,\; \tilde{P}_\phi = \frac{P_\phi}{md^2}$$

si $\tilde{r} = \frac{r}{d}$; $\dot{r} = d\dot{\tilde{r}}$ y $P_r = md\dot{\tilde{r}}$

$$md\tilde{P}_r = md\dot{\tilde{r}}$$

$$\boxed{\tilde{P}_r = \dot{\tilde{r}}}$$

$$\tilde{P}_\phi = mr^2\dot{\phi} = \frac{r^2\dot{\phi}}{d^2}$$

$$\boxed{\dot{\phi} = \frac{\tilde{P}_\phi}{\tilde{r}^2}} \ \star$$

$$md\dot{\tilde{P}}_r = \frac{\cancel{m^2 d^2}\tilde{P}_\phi^2}{\cancel{m\tilde{r}^3 d^3}} - \frac{\Delta m}{\tilde{r}d^2} - \frac{(m_L m[\tilde{r}d - d\cos(\phi-\omega t)]}{(\tilde{r}d^2 + d^2 - 2\tilde{r}d^2\cos(\theta\omega t))^{3/2}}$$

$$\tilde{P}_r = \frac{\tilde{P}_\phi^2}{\tilde{r}^3} - \frac{\Delta}{\tilde{r}^2} - \frac{6m_L}{d^3} \cdot \frac{[\tilde{r}\cdot\cos(\phi-\omega t)]}{[\tilde{r}+1-2\tilde{r}\cos(\theta-\omega t)]^{3/2}}$$

$$\tilde{r}^2 = \tilde{r}^2 - 2\tilde{r}\cos(\phi-\omega t)$$

$$\Delta N = \frac{6m_L}{d3}$$

$$\boxed{\begin{aligned}\dot{\tilde{P}}_r &= \frac{\tilde{P}_\phi^2}{\tilde{r}^3} - \Delta\left[\frac{1}{\tilde{r}^2} + \frac{N}{\tilde{r}^3}\cdot[\tilde{r}-\cos(\phi-\omega t)]\right] \\ \dot{\tilde{P}}_\phi &= -\frac{\Delta N\tilde{r}}{r^3}\sin(\phi-\omega t)\end{aligned}} \ \star$$

g) Encuentre expresiones para los momentos

$$\tilde{P}_r^{\circ} = \frac{P_r^{\circ}}{md} = \frac{1}{d}\cdot\frac{dr}{dt}$$

$$= \frac{1}{d}\cdot\frac{d}{dt}\left[\sqrt{x^2+y^2}\right] = \frac{\cancel{2}x\cdot\dot{x} + \cancel{2}y\cdot\dot{y}}{2rd} = \frac{x\cdot\dot{x}+y\cdot\dot{y}}{rd}$$

$$= \frac{x\cdot V_0\cos\theta + y\cdot V_0\sin\theta}{rd} = \frac{V_0 r}{rd}[\cos\phi\cdot\cos\theta + \sin\phi\sin\theta]$$

$$\boxed{= \frac{V_0}{d}[\cos(\theta-\phi)] = \tilde{V}\cdot\cos(\theta-\phi)}$$

$$\tilde{P}_\phi^{\circ} = \frac{P_\phi}{md^2} = \frac{\cancel{m}r^2\dot{\phi}}{\cancel{m}d^2} = \tilde{r}\cdot\frac{\partial}{\partial t}\arctan\left(\frac{y}{x}\right) = \frac{\tilde{r}^2}{1+(y^2/x^2)}\frac{d}{dt}\left(\frac{y}{x}\right)$$

$$= \left(\frac{\dot{y}x - y\dot{x}}{\cancel{x^2}}\right)\frac{\cancel{x^2}\cdot\tilde{r}^2}{x^2+y^2} = \frac{\tilde{r}^2}{r^2}(\dot{y}x - y\dot{x})$$

$$= \frac{\bar{r}^2}{r}\left(r\cos(\phi)V_0\sin(\theta) - r\sin(\phi)\,V_0\cos\theta\right) = \frac{\bar{r}^2}{r}\sin(\theta-\phi)\,V_0$$

$$= \frac{\bar{r}^2\,V_0}{d\cdot r}\sin(\theta-\phi) = \bar{r}_0\cdot\bar{V}_0\cdot\sin(\theta-\phi)$$