CS 731: Blockchain Technology And Applications

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  - ….

Today is all about Crypto

- We will discuss the crypto basics that are essential for blockchain technology
- Hash functions and their properties
- Public Key Cryptosystems
- Digital Signatures
- Hash Puzzles
- Hash Pointers
- Merkle Data Structures

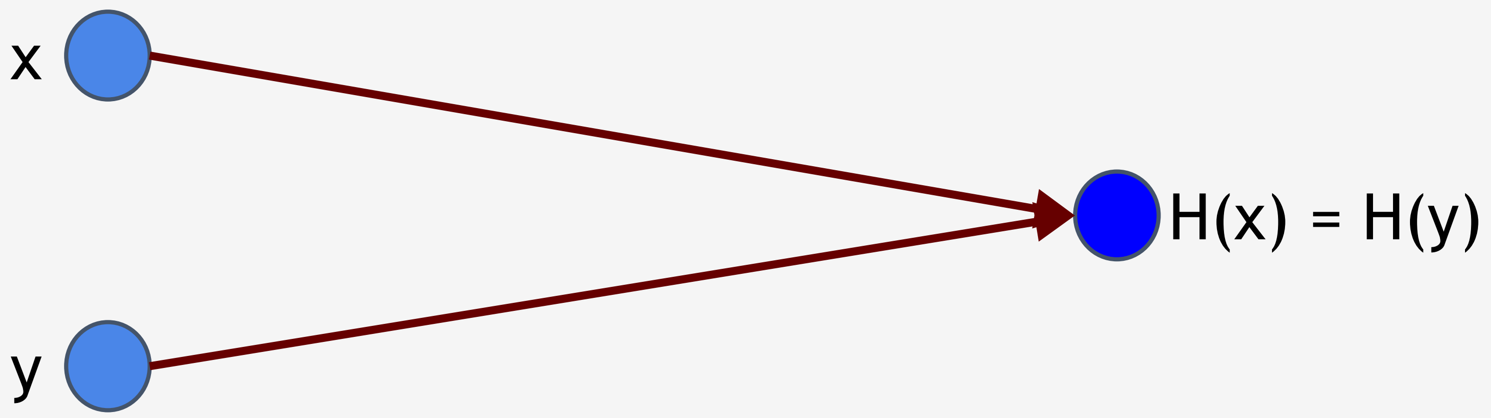
Cryptographic Hash Functions

- Hash function:
  - takes any string as input
  - fixed-size output (we’ll use 256 bits)
    - efficiently computable
  - Security properties:
    - collision-free
    - hiding
    - puzzle-friendly

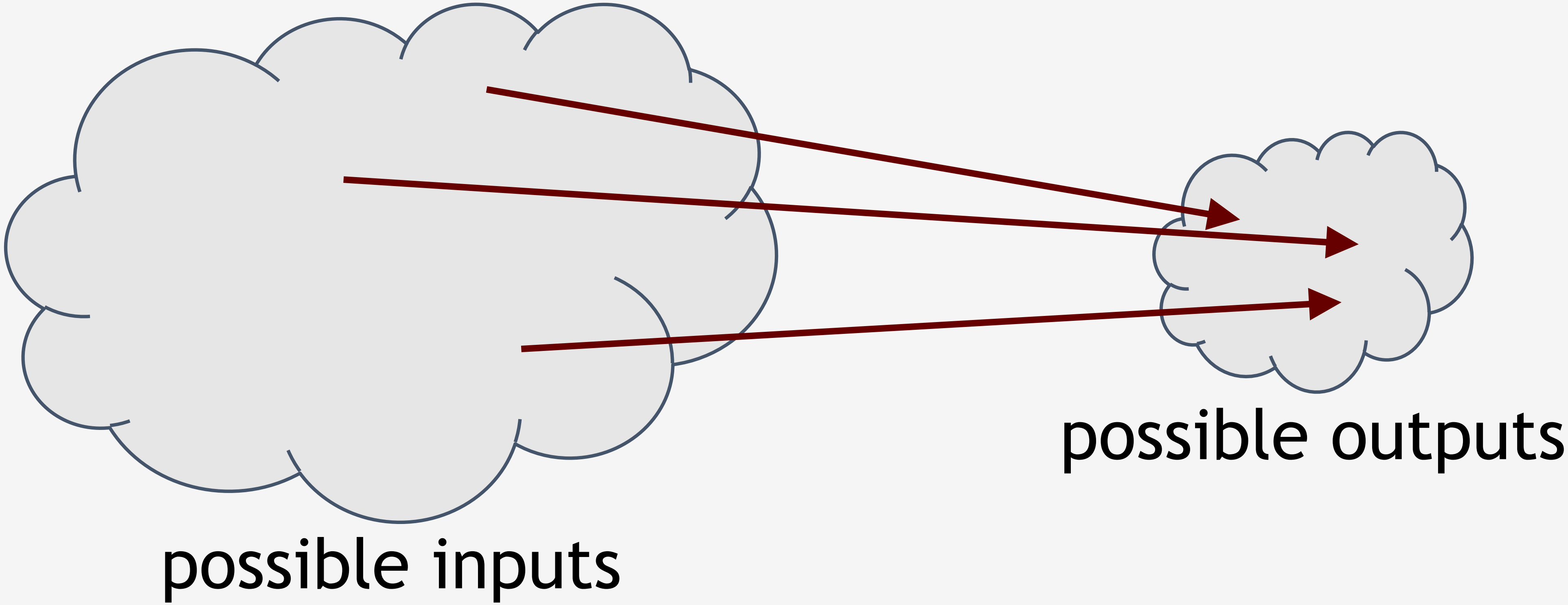
Hash property 1: Collision-free

Nobody can find x and y such that

x != y and H(x)=H(y)



Collisions do exist ...



... but can anyone find them?

How to find a collision

try 2^{130} randomly chosen inputs
99.8% chance that two of them will collide

This works no matter what H is …
… but it takes too long to matter

Is there a faster way to find collisions?

- For some possible H’s, yes.
- For others, we don’t know of one.

No H has been <u>proven</u> collision-free.

Application: Hash as message digest

- If we know H(x) = H(y),
  - it’s safe to assume that x = y.

- To recognize a file that we saw before,
  - just remember its hash.

- Useful because the hash is small.

Hash property 2: Hiding

We want something like this:

Given H(x), it is infeasible to find x.

H(“heads”)

H(“tails”)

easy to find x!

Hash property 2: Hiding

- <u>Hiding property</u>:
  - If r is chosen from a probability distribution that has *high min-entropy*, then given H(r | x), it is infeasible to find x.

- High min-entropy means
  - the distribution is “very spread out”, so that no particular value is chosen with more than negligible probability.

Application: Commitment

Want to “seal a value in an envelope”, and
“open the envelope” later.

Commit to a value, reveal it later.

Commitment API

(*com, key*) := commit(*msg*)
match := verify(*com, key, msg*)

To seal *msg* in envelope:
  (*com, key*) := commit(*msg*) -- then publish *com*
To open envelope:
  publish *key, msg*
  anyone can use verify() to check validity

Commitment API

(*com, key*) := commit(*msg*)
match := verify(*com, key, msg*)

Security properties:
Hiding: Given *com*, infeasible to find *msg*.
Binding: Infeasible to find *msg != msg'* such that
verify(commit(*msg*), *msg'*) == true

Commitment API

commit(*msg)* := (H(*key | msg*), H(*key)*)
where *key* is a random 256-bit value
verify(*com, key, msg*) := (H(*key | msg) == com*)

Security properties:
Hiding: Given H(*key | msg)*, infeasible to find *msg*.
Binding: Infeasible to find *msg != msg'* such that
H(*key | msg) ==* H(*key | msg')*

Hash property 3: Puzzle-friendly

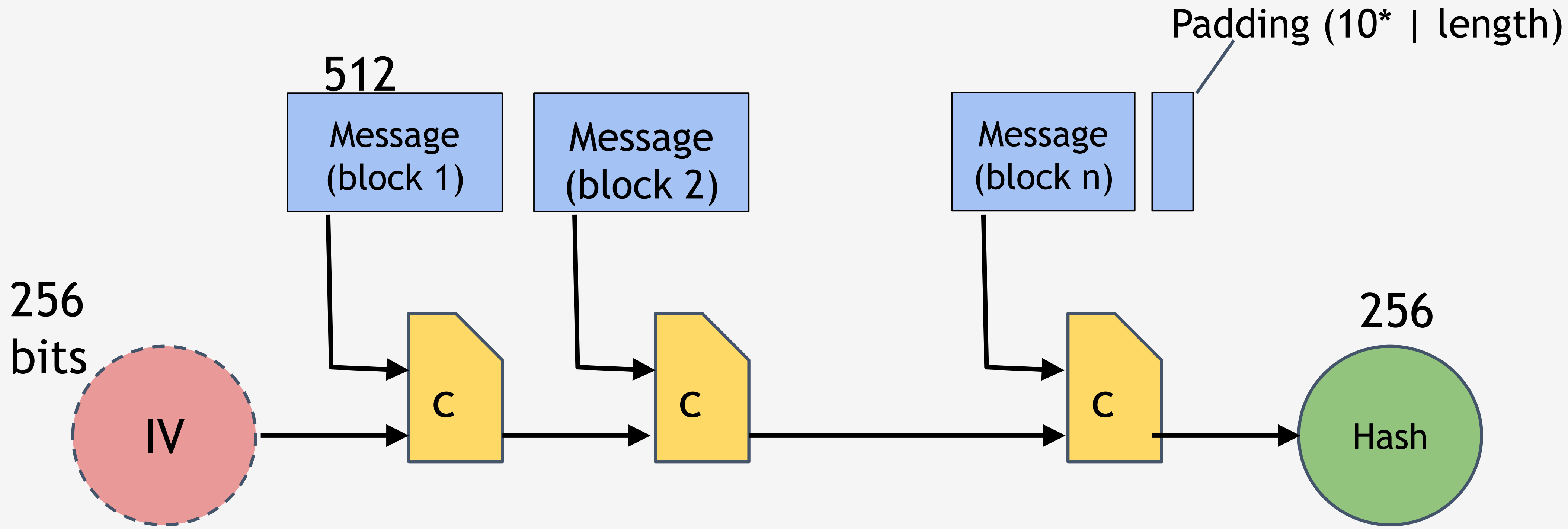
Puzzle-friendly:
For every possible output value y,
if k is chosen from a distribution with high min-entropy,
then it is infeasible to find x such that H(k | x) = y.

Application: Search puzzle

Given a "puzzle ID" *id* (from high min-entropy distrib.),
and a target set *Y*:
Try to find a "solution" *x* such that
$$H(id \mid x) \in Y.$$

Puzzle-friendly property implies that no solving strategy is much better than trying random values of *x*.

SHA-256 hash function

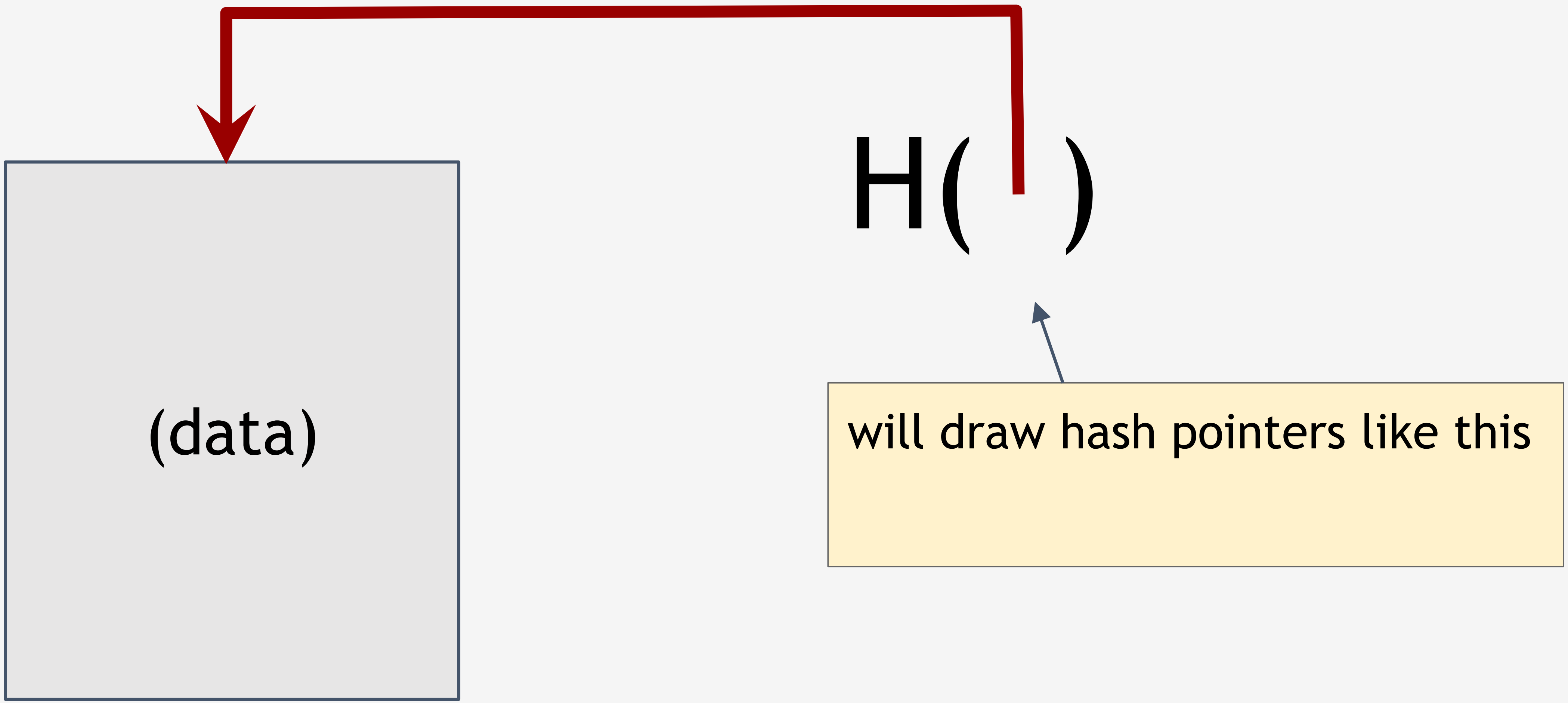


Theorem: If c is collision-free, then SHA-256 is collision-free.

Hash Pointers and Data Structures

- hash pointer is:
  - * pointer to where some info is stored, and
  - * (cryptographic) hash of the info

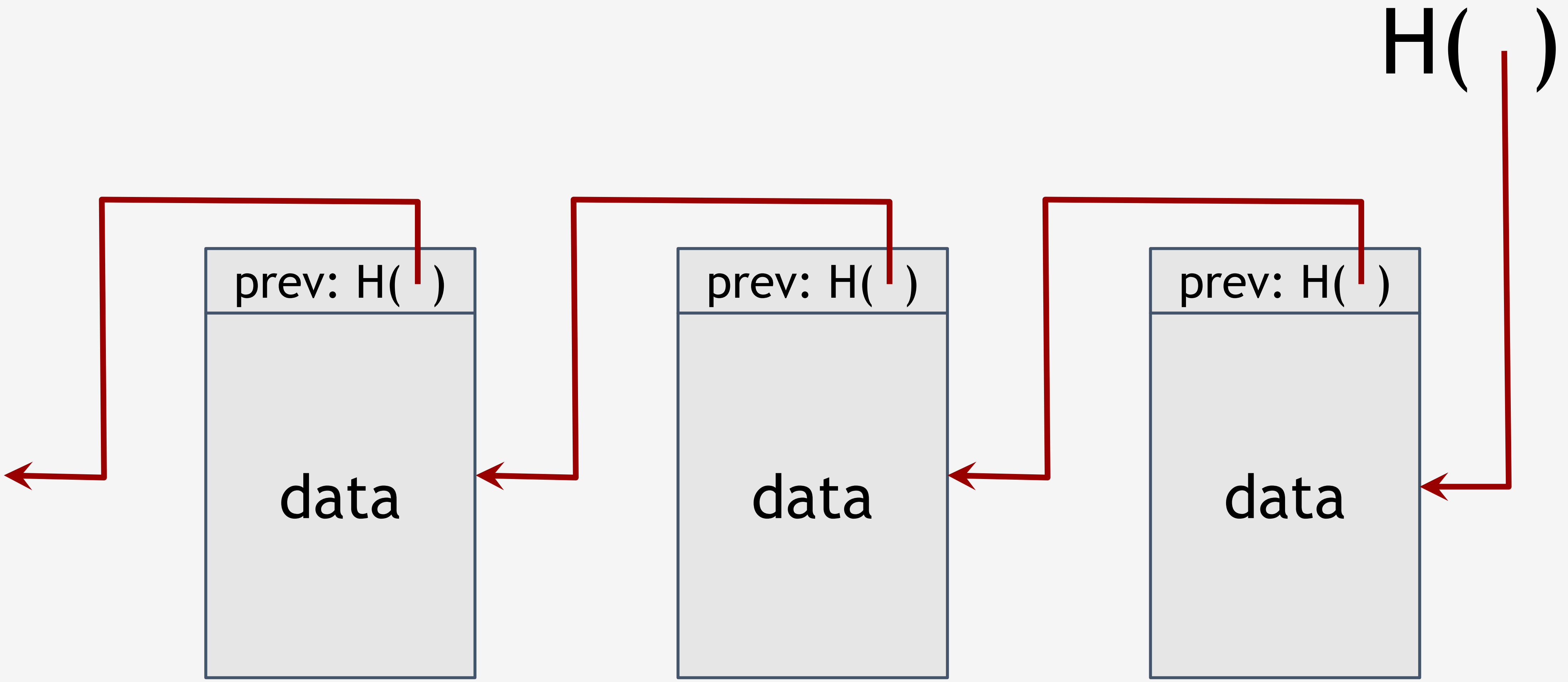
- if we have a hash pointer, we can
  - * ask to get the info back, and
  - * verify that it hasn't changed


(data)
H( )
will draw hash pointers like this

key idea:

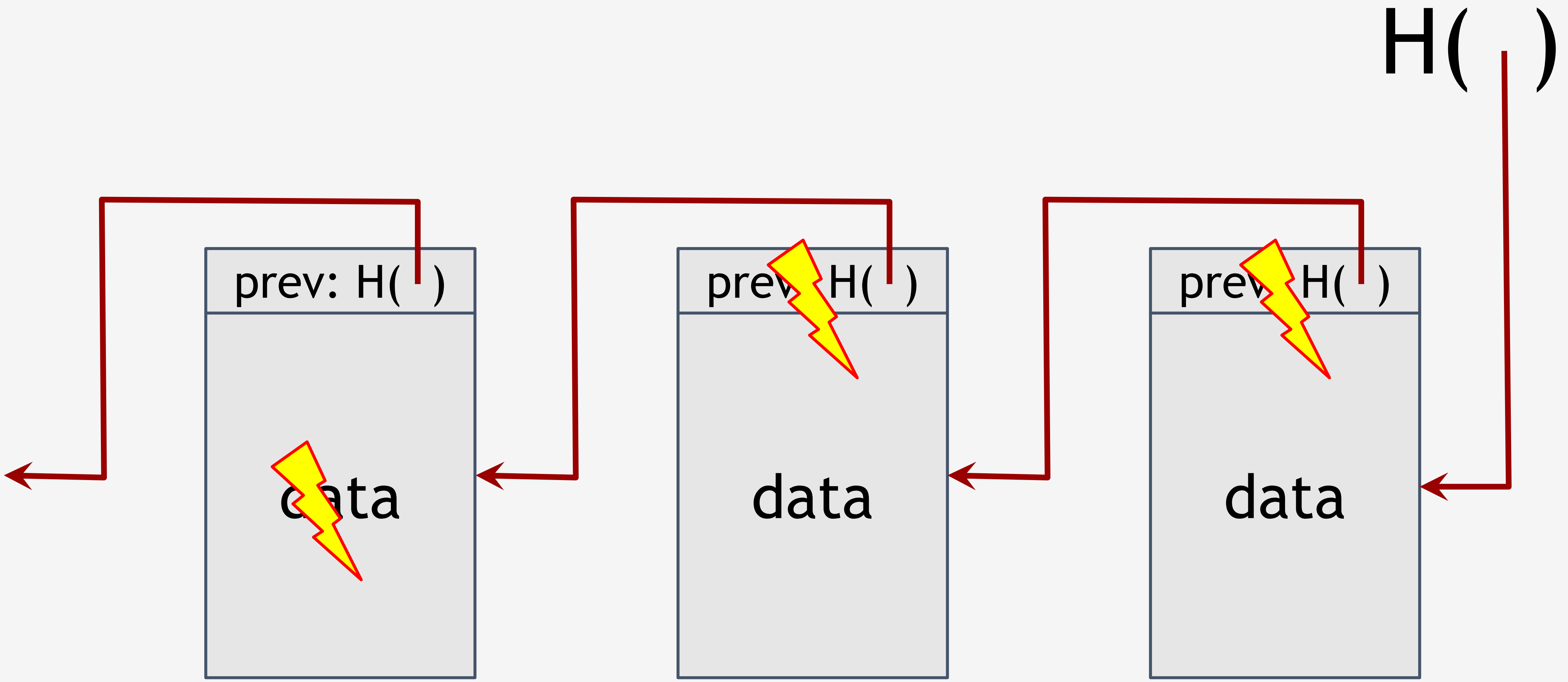
build data structures with hash pointers

linked list with hash pointers = "block chain"



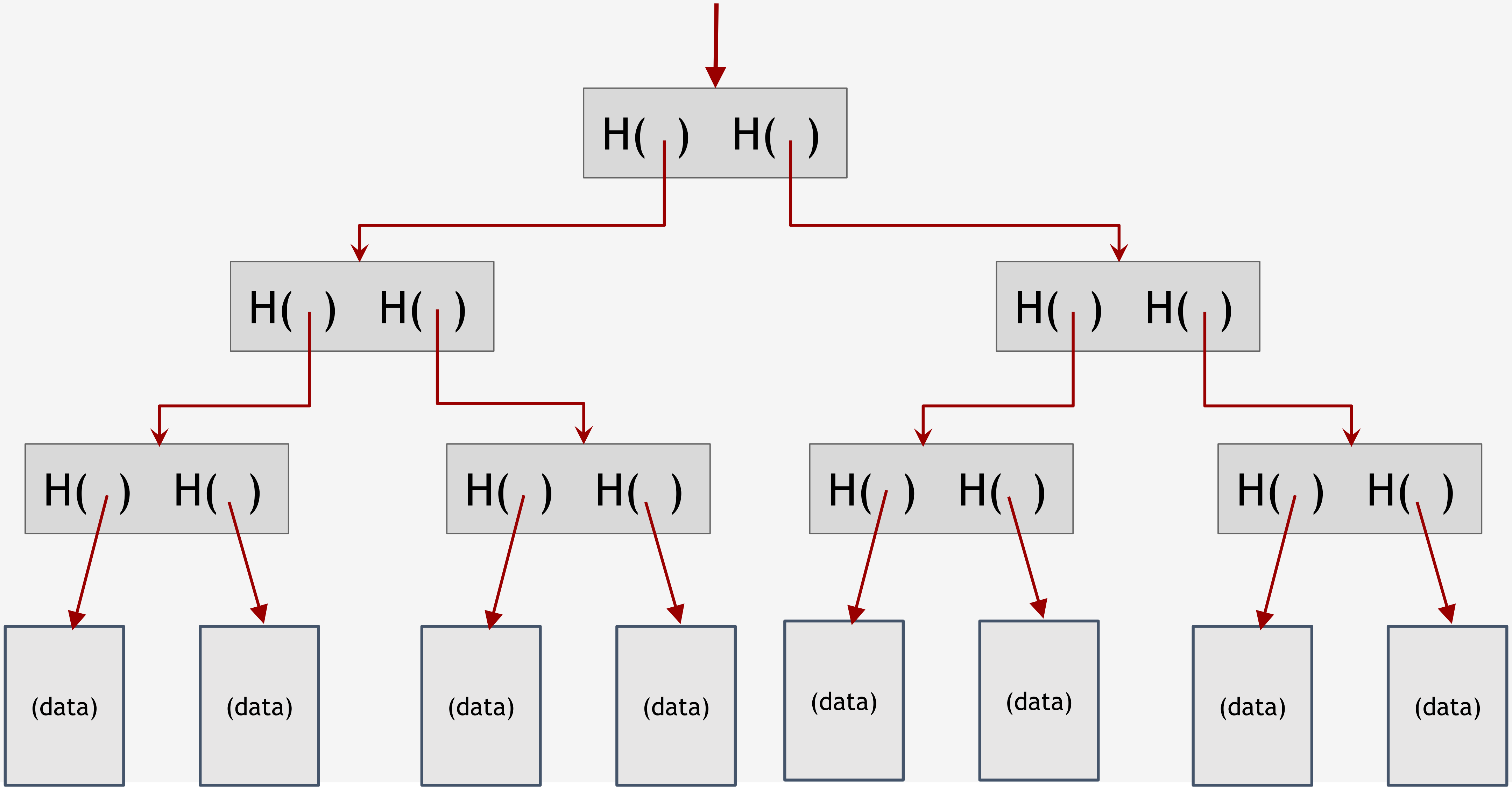
use case: tamper-evident log

detecting tampering



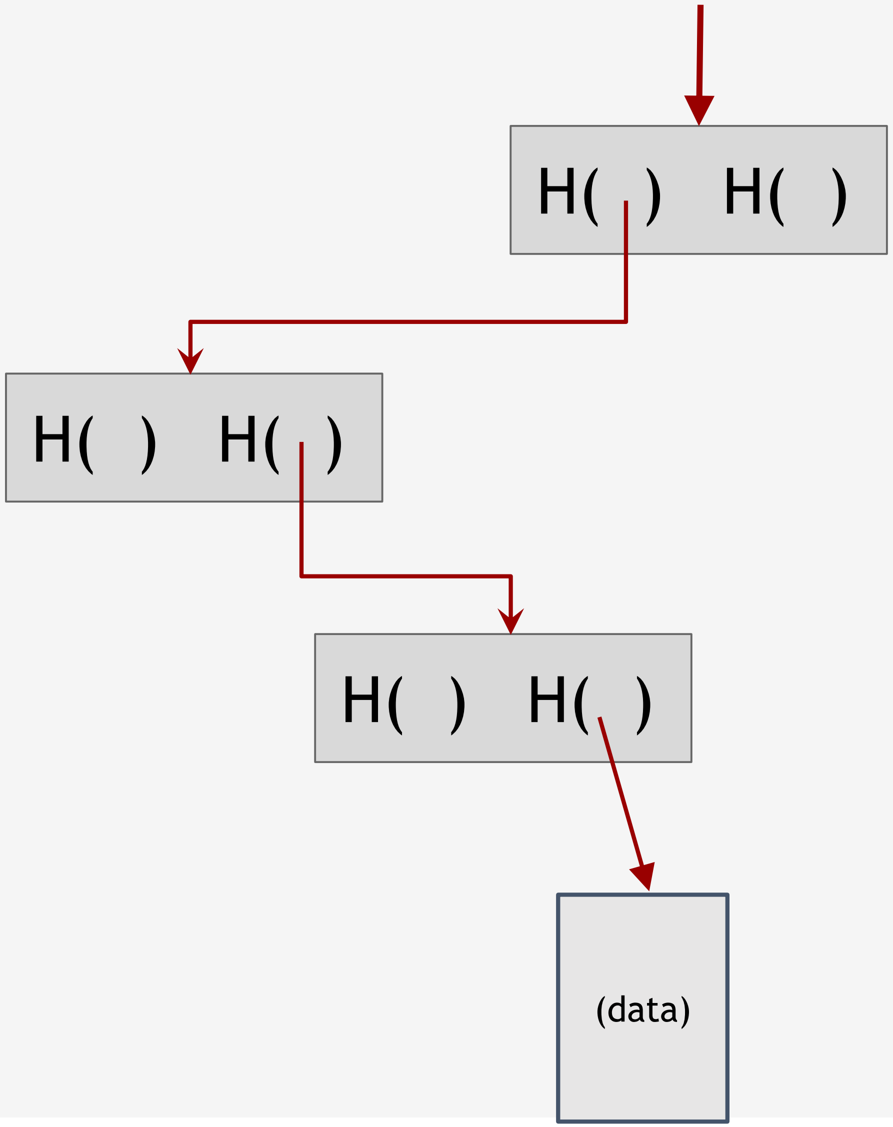
use case: tamper-evident log

binary tree with hash pointers = “Merkle tree”



proving membership in a Merkle tree

show O(log n) items



Advantages of Merkle trees

- Tree holds many items
    - but just need to remember the root hash
- Can verify membership in O(log n) time/space

- Variant: sorted Merkle tree
    - can verify non-membership in O(log n)
- (show items before, after the missing one)

More generally ...

can use hash pointers in any pointer-based
data structure that has no cycles

Digital Signatures

What we want from signatures

Only you can sign, but anyone can verify

Signature is tied to a particular document

can’t be cut-and-pasted to another doc

API for digital signatures

(sk, pk) := generateKeys(keysize)

sk: secret signing key

pk: public verification key

sig := sign(sk, message)

isValid := verify(pk, message, sig)

can be randomized algorithms

Requirements for signatures

"valid signatures verify"

verify(pk, message, sign(sk, message)) == true

"can't forge signatures"

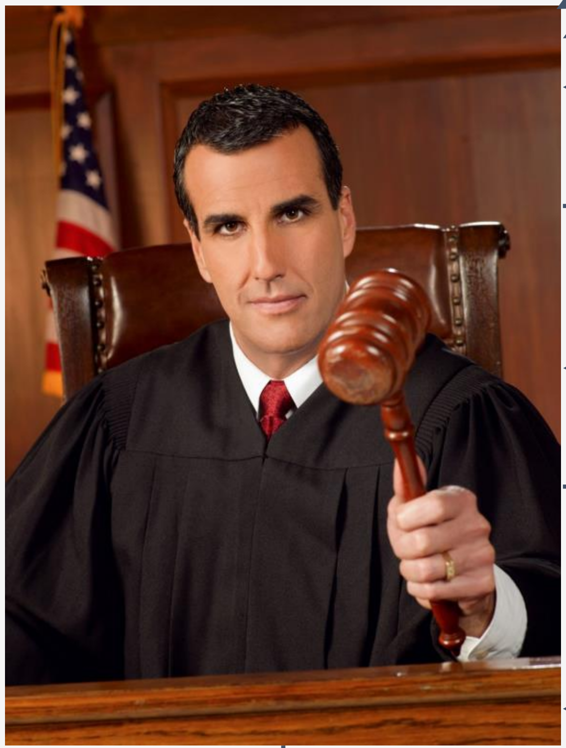
adversary who:

knows pk

gets to see signatures on messages of his choice

can't produce a verifiable signature on another message

(sk, pk)



m$_0$

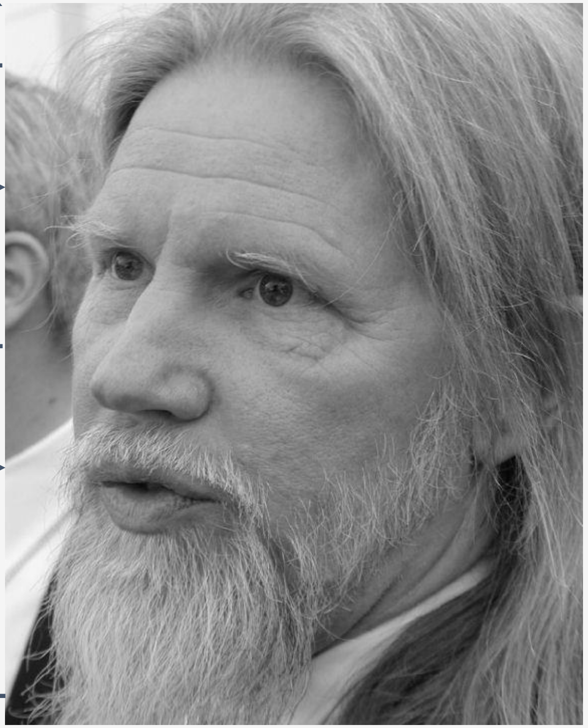
sign(sk, m$_0$)

m$_1$

sign(sk, m$_1$)

• • •

M, sig



challenger

attacker

M not in { m$_0$, m$_1$, … }

verify(pk, M, sig)

if true, attacker wins

Practical stuff...

algorithms are randomized

need good source of randomness

limit on message size

fix: use Hash(message) rather than message

fun trick: sign a hash pointer

signature “covers” the whole structure

Bitcoin uses ECDSA standard

Elliptic Curve Digital Signature Algorithm

relies on hairy math

will skip the details here --- look it up if you care

good randomness is essential

foul this up in generateKeys() or sign() ?

probably leaked your private key

Public Keys as Identities

Useful trick: public key == an identity

if you see *sig* such that *verify(pk, msg, sig)==true*, think of it as
pk says, "*[msg]*".

to "speak for" *pk*, you must know matching secret key *sk*

How to make a new identity

create a new, random key-pair *(sk, pk)*
pk is the public "name" you can use
[usually better to use Hash(pk)]
sk lets you "speak for" the identity

you control the identity, because only you know *sk*
if *pk* "looks random", nobody needs to know who you are

Decentralized identity management

anybody can make a new identity at any time
  make as many as you want!

no central point of coordination

<u>These identities are called “addresses” in Bitcoin.</u>

Privacy

Addresses not directly connected to real-world identity.

But observer can link together an address's activity over time, make inferences.