Zero Knowledge Proofs and its applications in Blockchain

Shubham Sahai Srivastava

CS-731: Lecture 16

Department of Computer Science and Engineering, IIT Kanpur.

Blockchain and Privacy?

- Blockchain: immutability, censorship-resistance, and open and permissionless
- Bitcoin (Blockchain 1.0): pseudo-anonymity, public ledger, patterns, de-anonymization … transactions
- Ethereum (Blockchain 2.0): smart contracts, DApp … data
- Fabric (Blockchain 3.0): Privacy and channels, … trust?

Do we need to continue transacting in the open to enjoy the benefits?

Applications?

- Cryptocurrencies, Wallets … manage transactions
- CryptoKitties, Identity management, Microblogging … manage data
- Supply chain management, Traceability (IBM FoodTrust, Diamond mining etc.) … manage business logic

What else can we do?

Can we integrate **non-blockchain** solutions with blockchain to achieve something more?

Mathematics (or Crypto) so far…

- One way functions … cryptographic hashes
- Digital signature … signing transactions
- Authentication mechanisms

- Zero Knowledge Proofs
- zkSNARKS

Zero Knowledge Proofs

The general idea of proof systems and zero knowledge

- First proposed in 1985 by MIT researchers: Shafi Goldwasser, Silvio Micali, and Charles Rackoff
- Decades of research, but generating excitement lately!
- Potential to redefine the concept of online privacy.

Let’s Begin …

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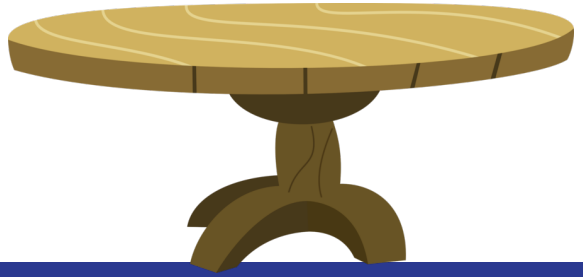

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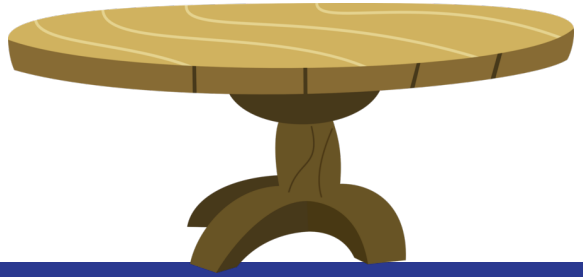

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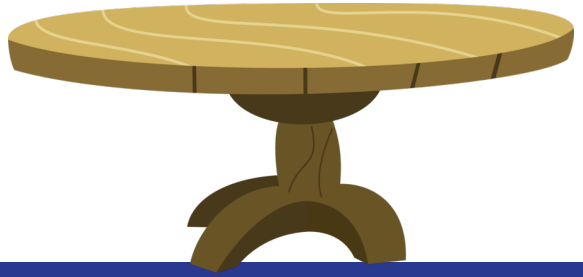

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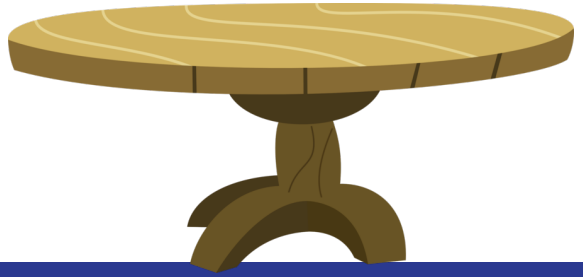

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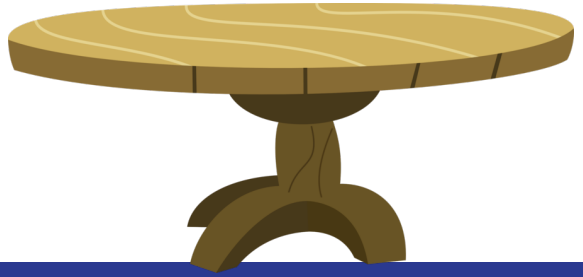

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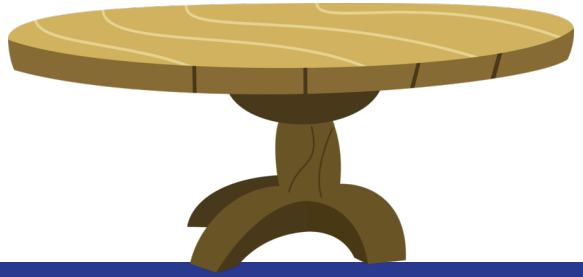

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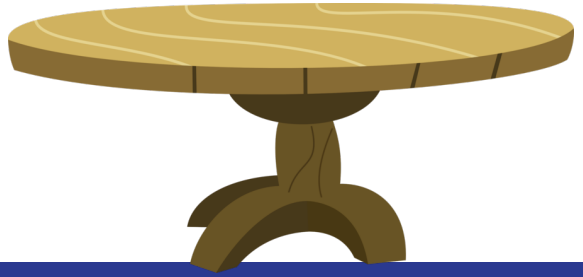

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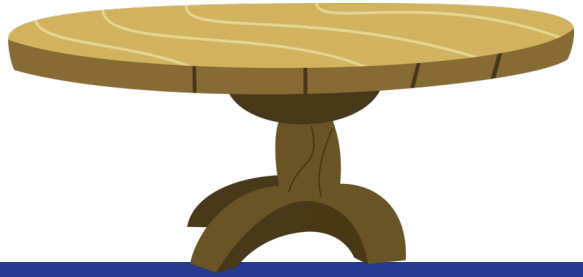

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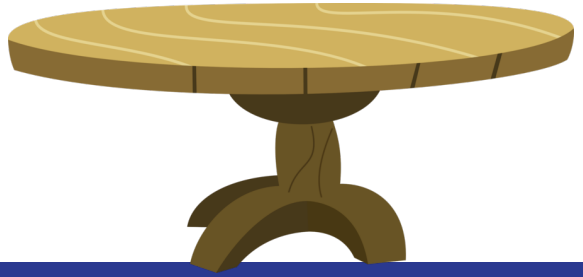

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$$\frac{1}{2}$$

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If Alice is cheating, then probability of fooling Bob :

$$\left(\frac{1}{2}\right)^n$$

Let's Begin …

If Alice is cheating, then probability of fooling Bob :

$$\left(\frac{1}{2}\right)^n$$

For n = 15 :

0.0000305

In the end …

In the end …

Alice Convinces Bob, that with very high probability :

“She has **different** coloured hats.”

In the end …

Alice Convinces Bob, that with very high probability :
“She has **different** coloured hats.”

However, Bob has no idea :
“What are the **colors** of the hats?”

Zero Knowledge Proofs

- A method by which one party: **“Prover”**, can convince another party: **“Verifier”**; about the **“truthfulness”** of a statement.
- The protocol conveys nothing apart from the veracity of the statement.

(Zero Knowledge)

- A zero knowledge proof must satisfy :
    - **Completeness** : Honest Prover can convince an honest Verifier.
    - **Soundness** : Cheating Prover can convince an honest Verifier with negligible probability.
    - **Zero Knowledge** : Verifier learns nothing apart from the veracity of the statement.

Interactive Zero Knowledge Proofs

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All languages in NP have a Zero Knowledge Proof !

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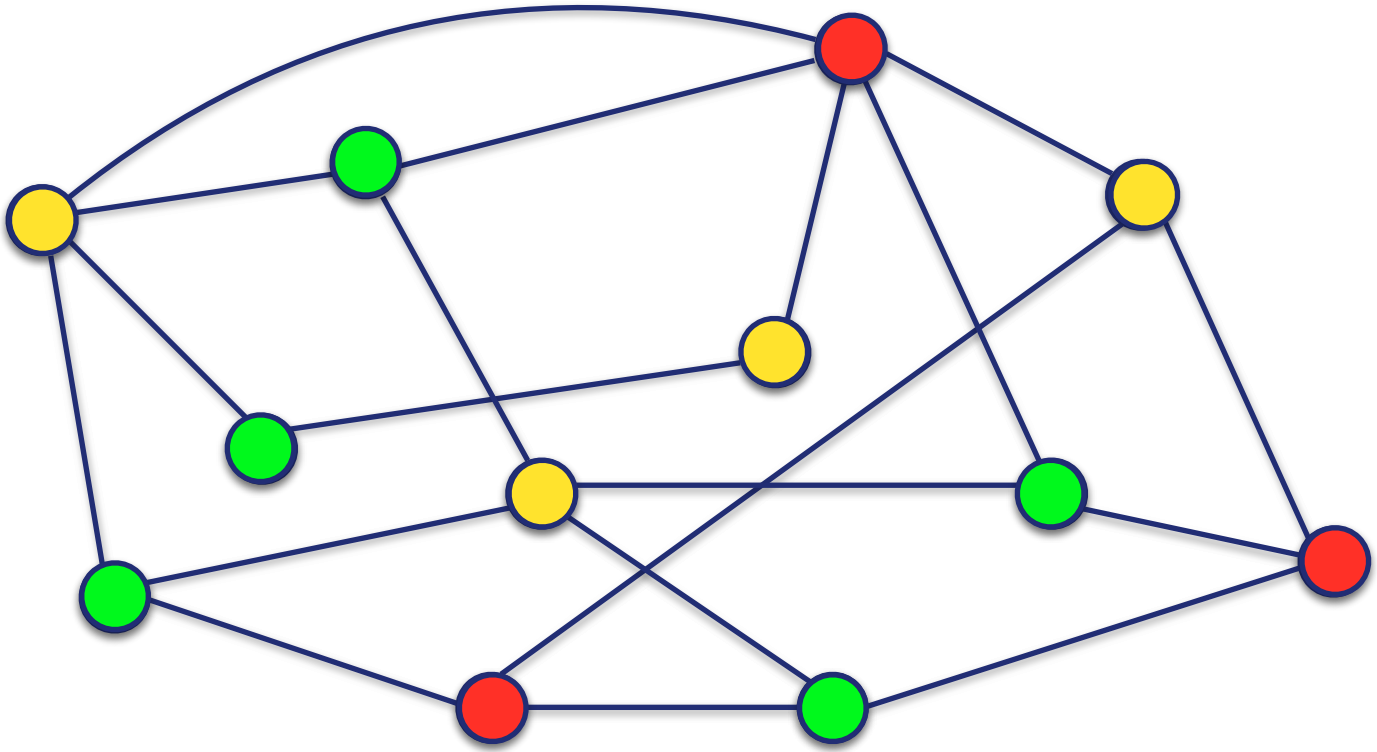
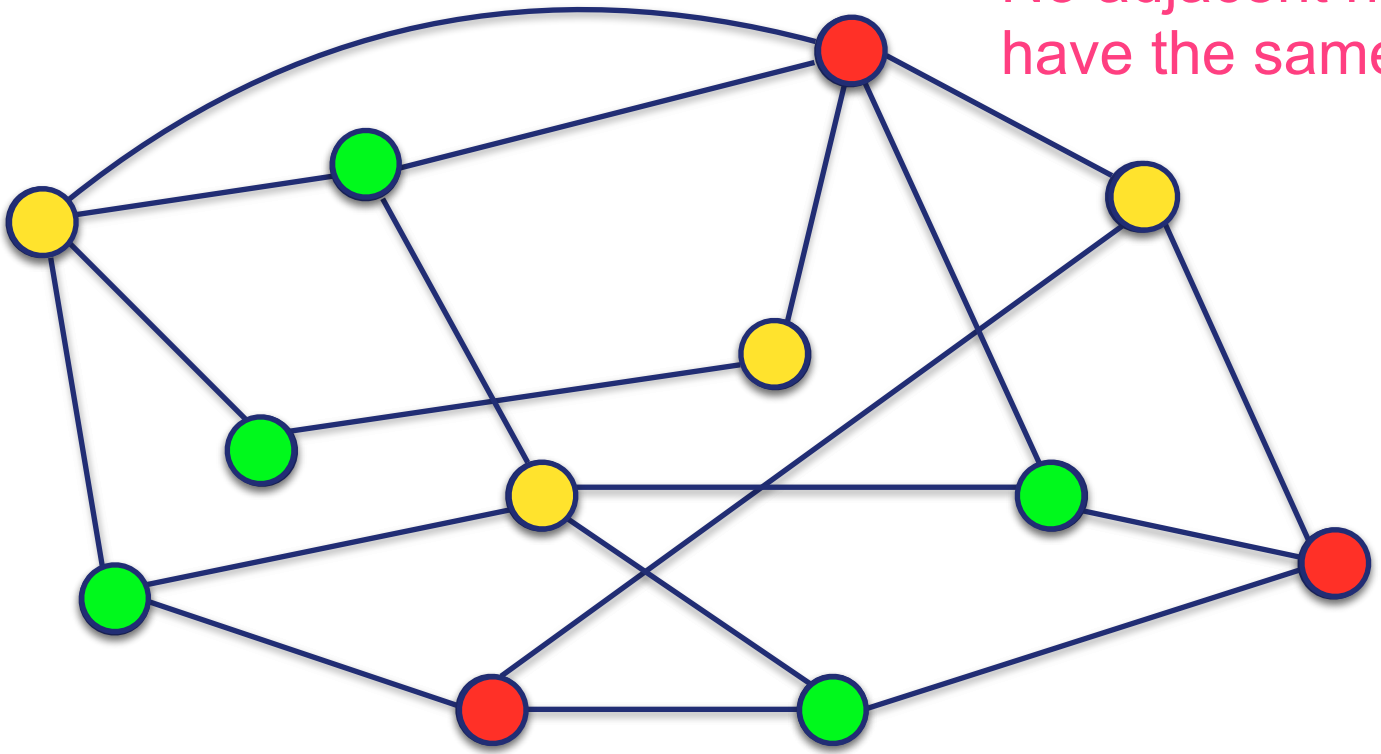
NP is the set of decision problems for which the problem instances, where the answer is **"yes"**, have proofs **verifiable in polynomial time.**

Graph 3-Coloring (G3C)



Graph 3-Coloring (G3C)

No adjacent nodes have the same color.



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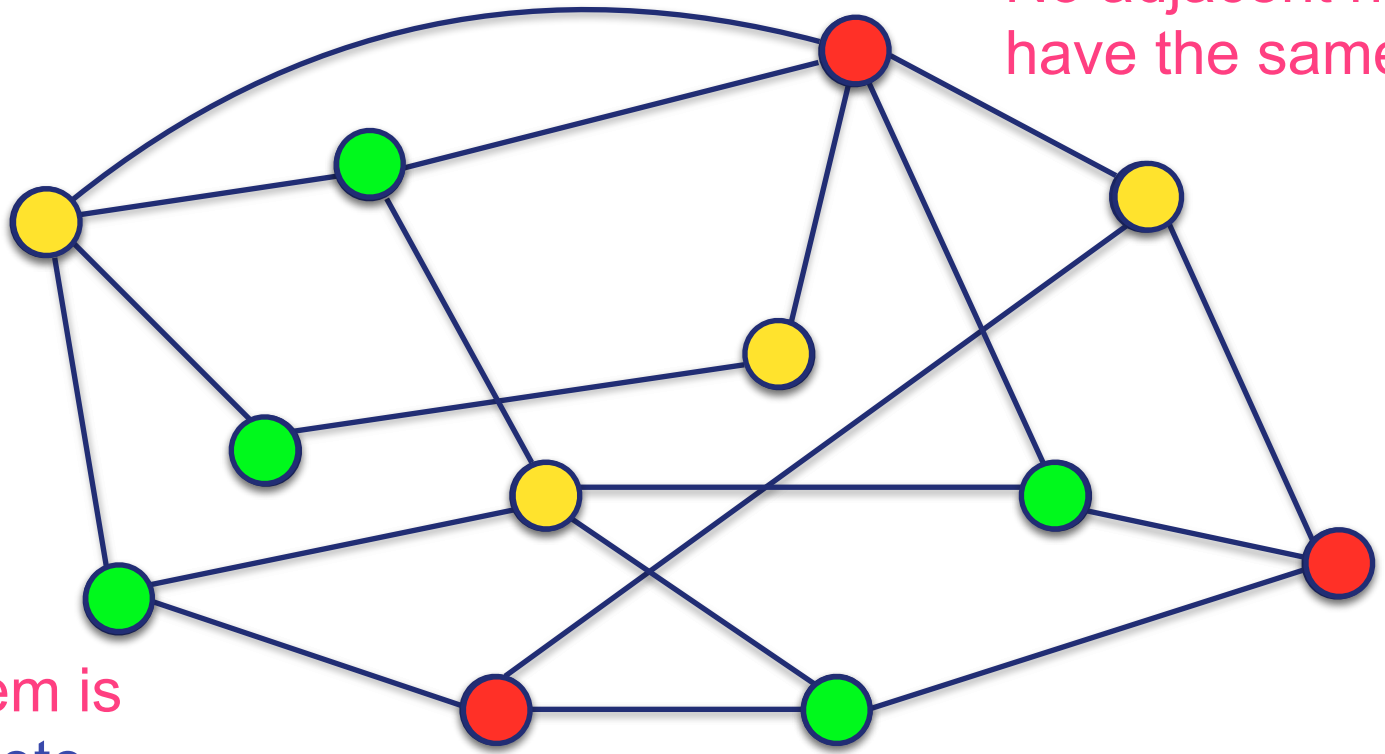

The problem is NP-Complete

G3C : Problem …

Given a Graph G with **n** vertices and **m** edges

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G is 3-colorable!

I know the colouring

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Really?
Prove it?

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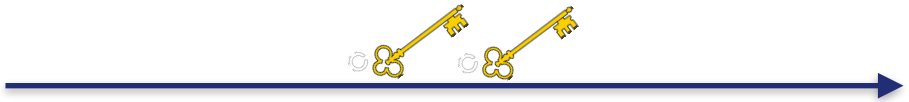

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"The edge chosen was correctly coloured"

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Repeat the same experiment again!

G3C: Repeat! How many times ?

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For m = 7 :

0.000524

G3C: Is it a Zero Knowledge Proof?

It can be shown that the protocol satisfies:

- **Completeness:** Honest Prover can convince an honest Verifier.
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ZKP: Complexity Class Standpoint

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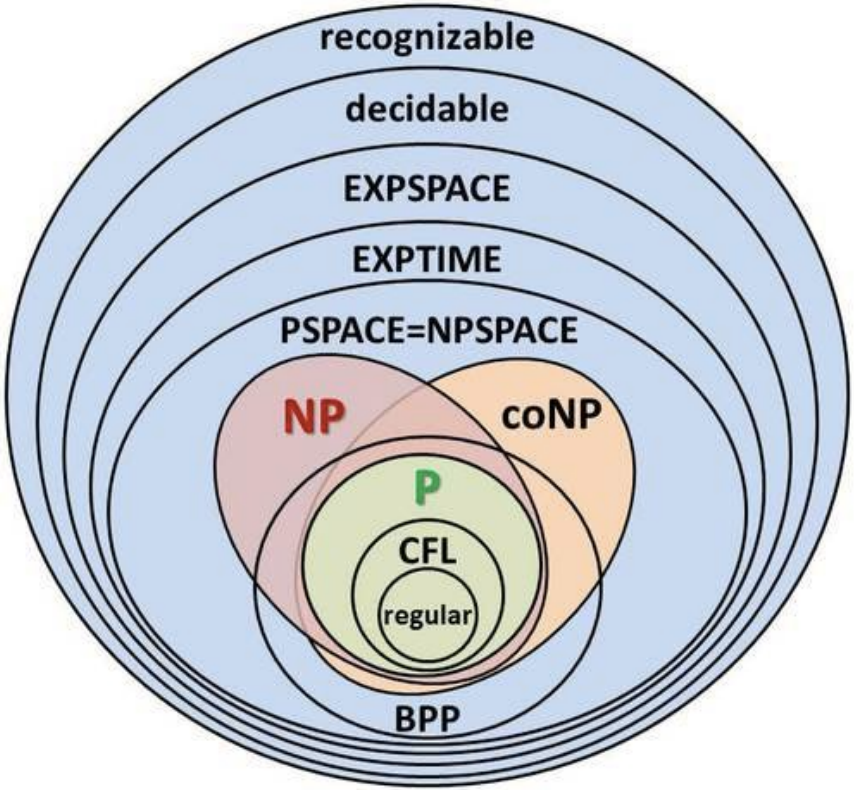

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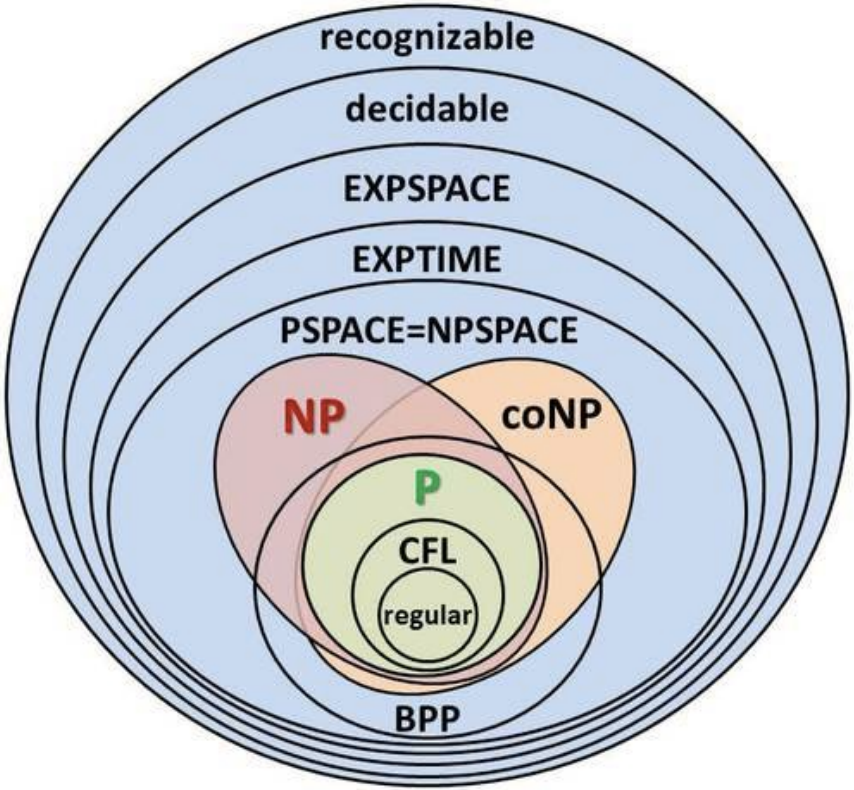

Computational Zero Knowledge Proofs

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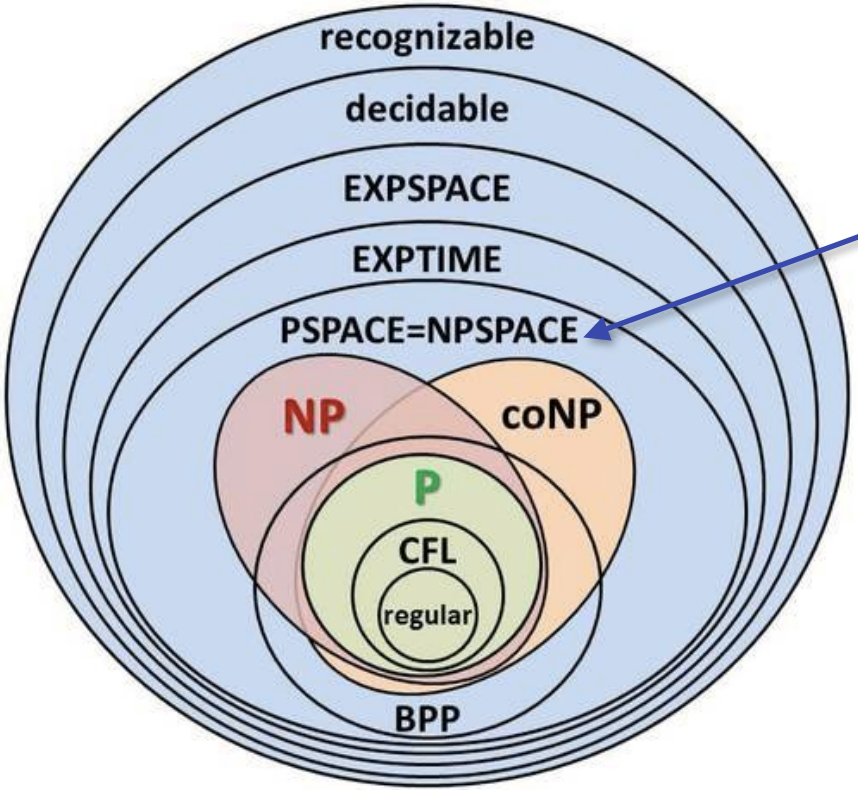

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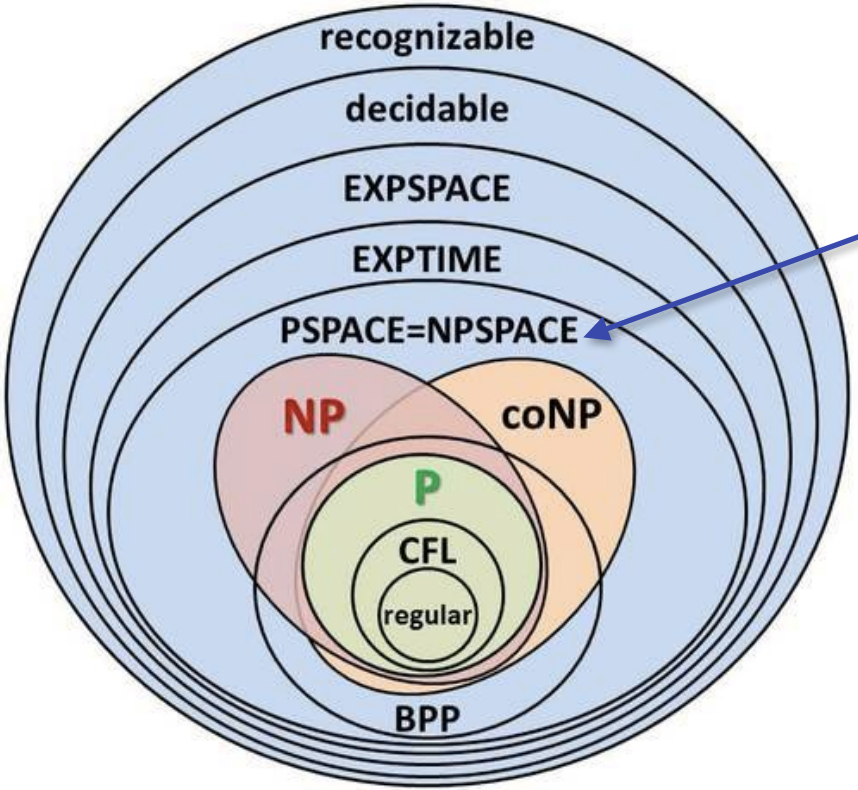

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CZKP = PSPACE = NPSPACE = IP

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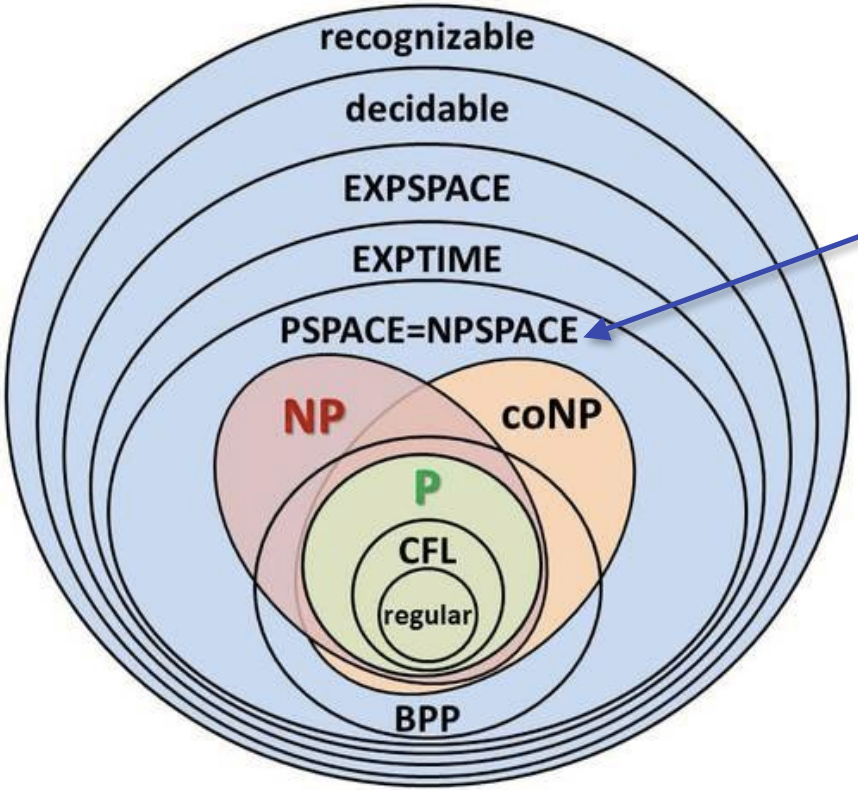

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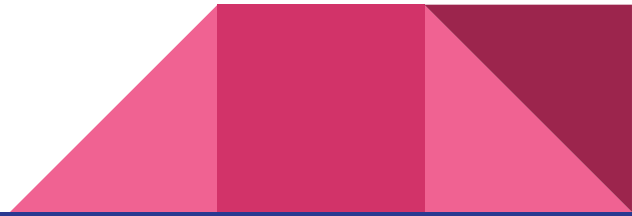
All languages in NP have a Zero Knowledge Proof !

Application of ZKP in Blockchains?



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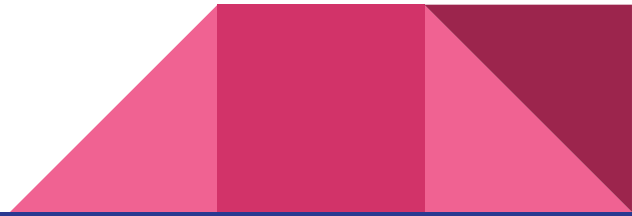
Application of ZKP in Blockchains?

- The proofs seen so far are **interactive**
- Not suitable for blockchain applications. **Why?**
- Desirable properties from a ZKP variant to be usable in blockchain:
    - Non-interactiveness
    - Small in size
    - Fast verification



zk-SNARK

“it’s really mind boggling”
- Sergey Brin
(Blockchain Summit-2018)

“

Zero **K**nowledge

- **S**uccinct
- **N**on-interactive
- **AR**guments of
- **K**nowledge

Non Interactive Zero Knowledge Proofs

- No interaction required between the Prover and the Verifier
- Impossible in standard cryptographic model
  [Goldreich and Oren; 1993]
- Possible in **common reference string** and **random oracle model.**
- Common reference string can yield computational zero knowledge.

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- To establish this, we need to establish some shared "reference string"
- This is done during Setup Phase…generates toxic waste
- For every function (circuit) we need to run the Setup Phase

What can we do with it?

What can we do with it?

I know a ‘x’, such that :

$x^3 + x + 5 == 35$

What can we do with it?

I know a ‘x’, such that :

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Really?
Why don’t you
prove it?

Lets work out this example…

$$x^3 + x + 5 == 35$$

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Function:

```
def qeval(x):
    y = x**3
    return x + y + 5
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Flattened Code:

$$\begin{aligned} sym_1 &= x * x \\ y &= sym_1 * x \\ sym_2 &= y + x \\ \sim out &= sym_2 + 5 \end{aligned}$$

Arithmetic Circuit from code…

Flattened Code:

$sym_1 = x * x$

$y = sym_1 * x$

$sym_2 = y + x$

$\sim out = sym_2 + 5$

Arithmetic Circuit from code…

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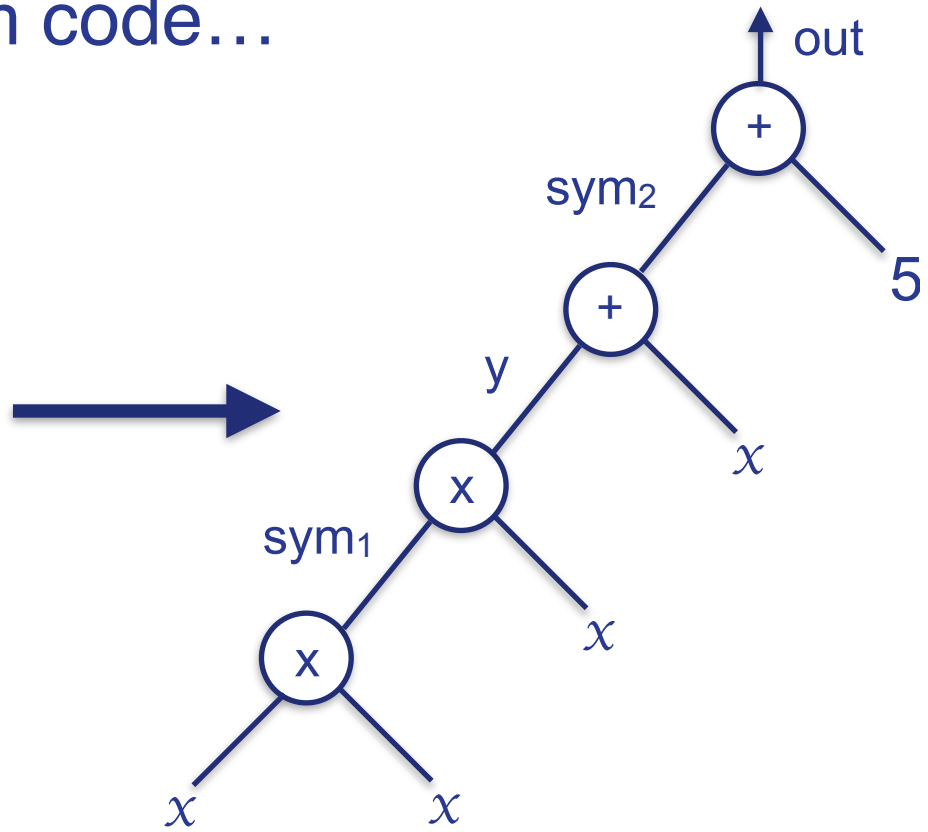
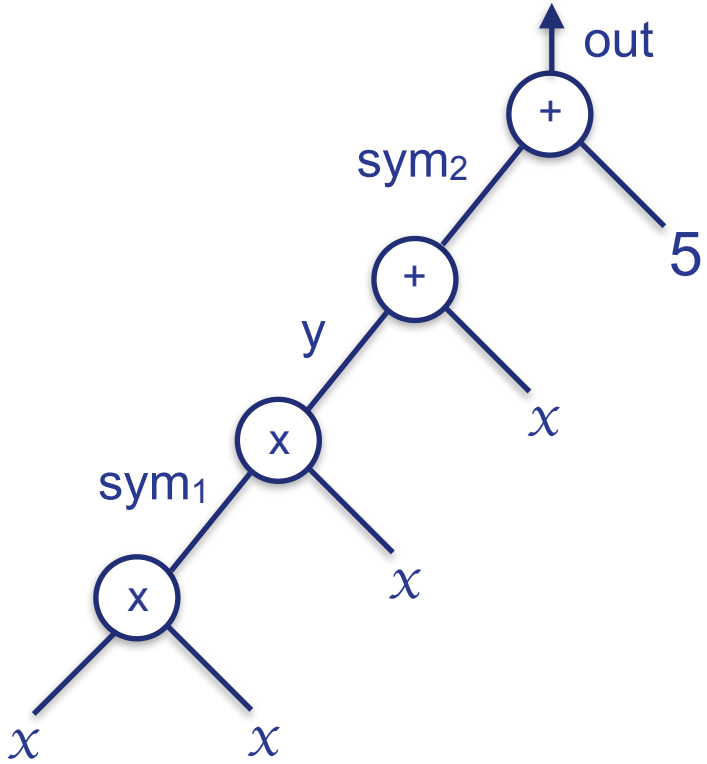


Constraints on variables …



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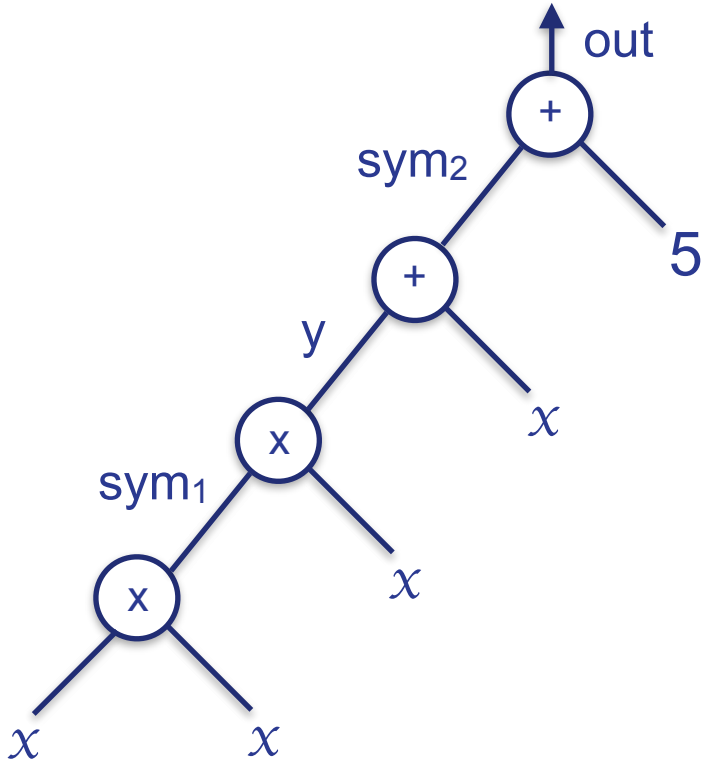

Every gate in the circuit, imposes a constraint on its **inputs** and **outputs**.

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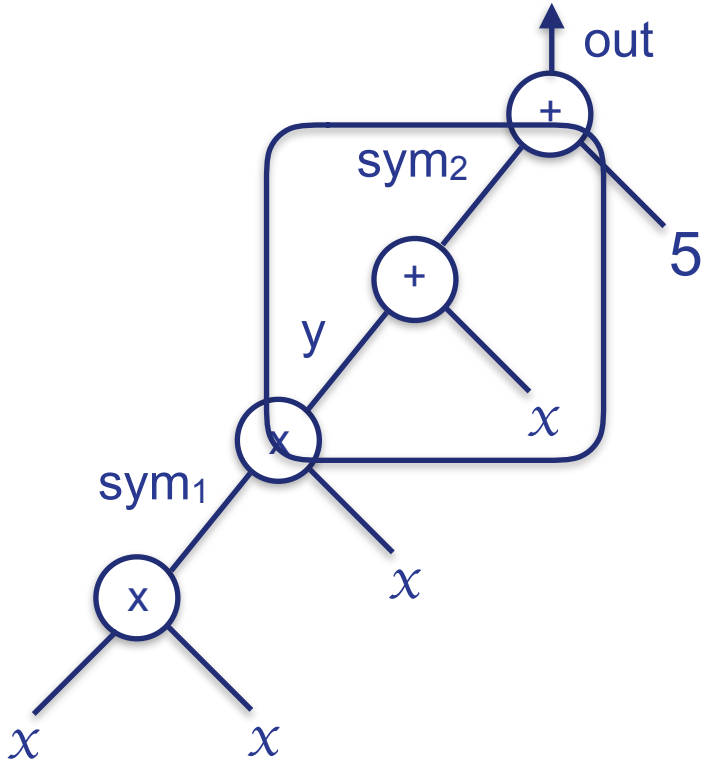

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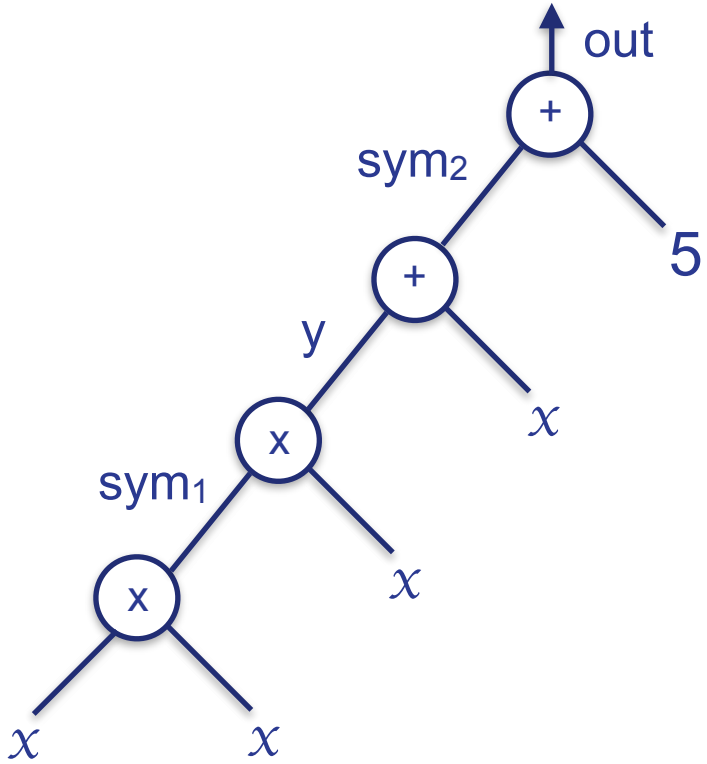

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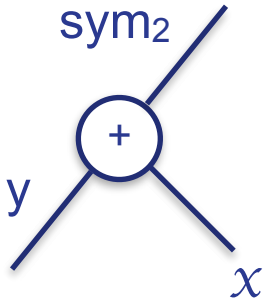

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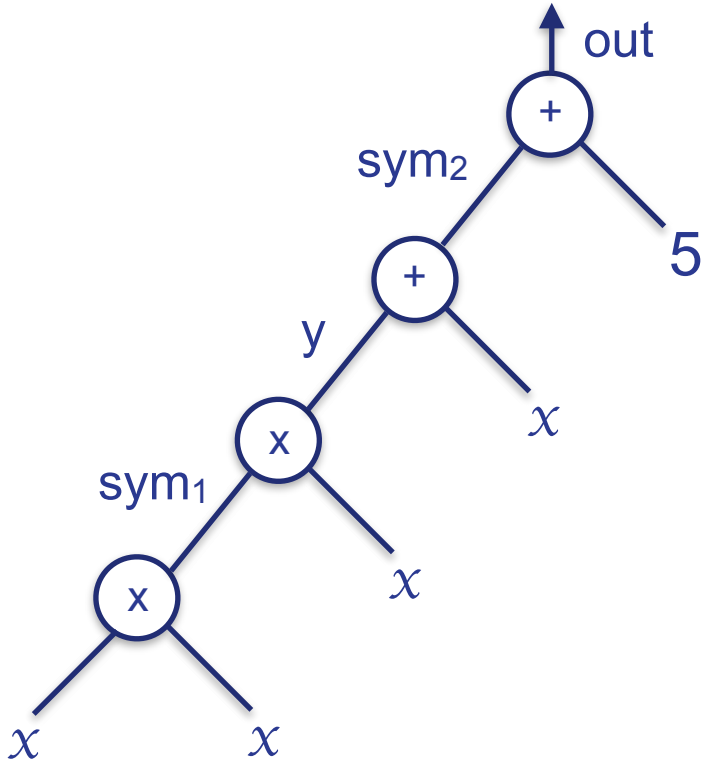

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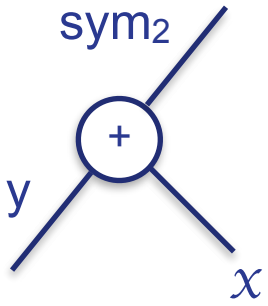
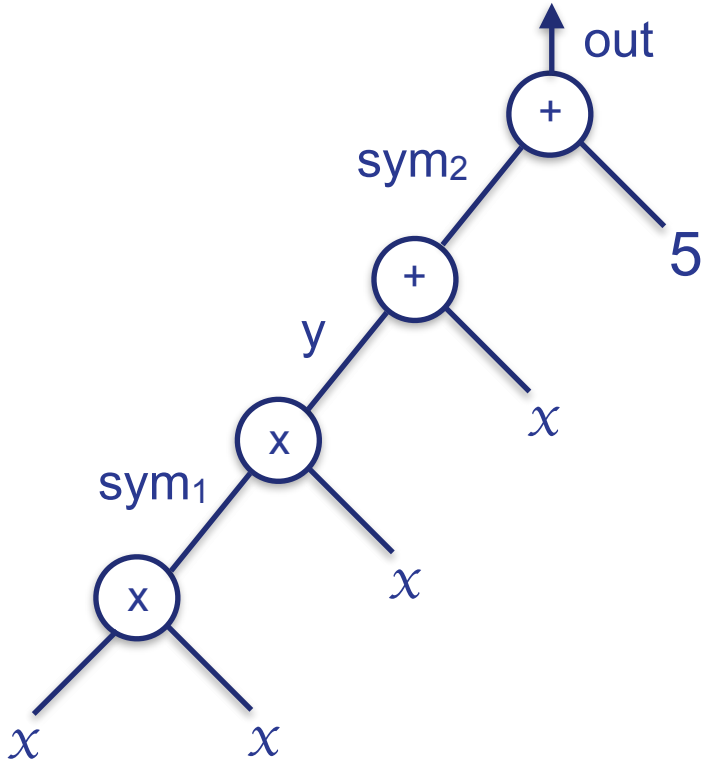


$x + y = sym_2$

Constraints on variables …



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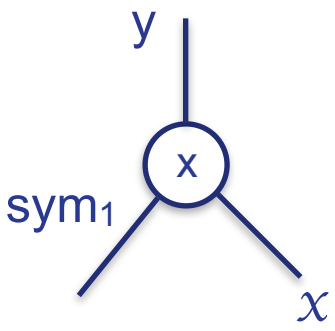

$x * sym_1 = y$

Rank 1 Constraint System (R1CS)

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$$s.a * s.b - s.c = 0$$

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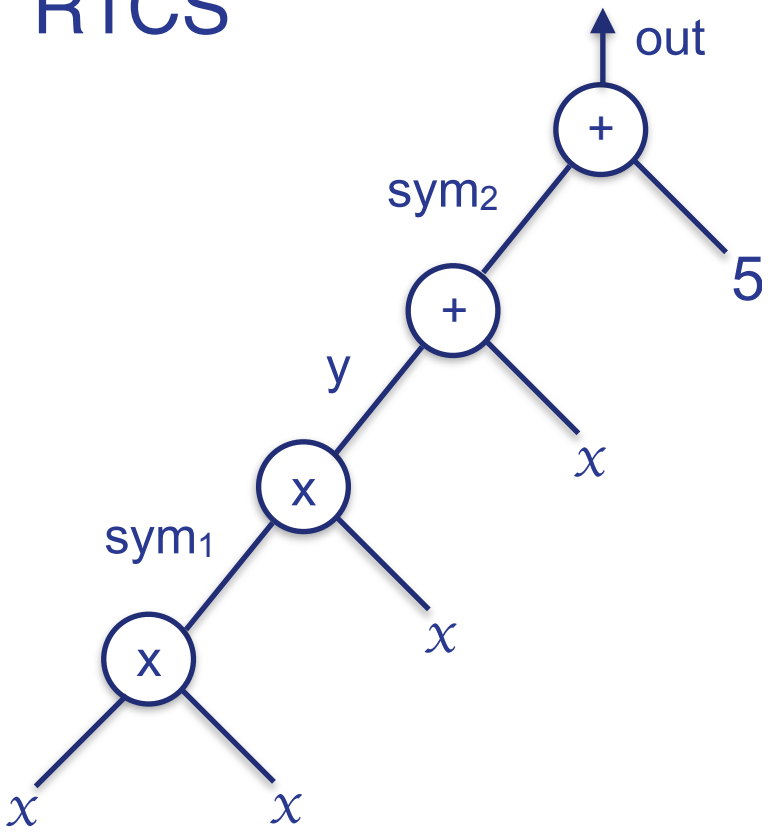
- Sequence of 3 vectors (a,b,c) and a solution vector s, which satisfy:

$$s.a * s.b - s.c = 0$$

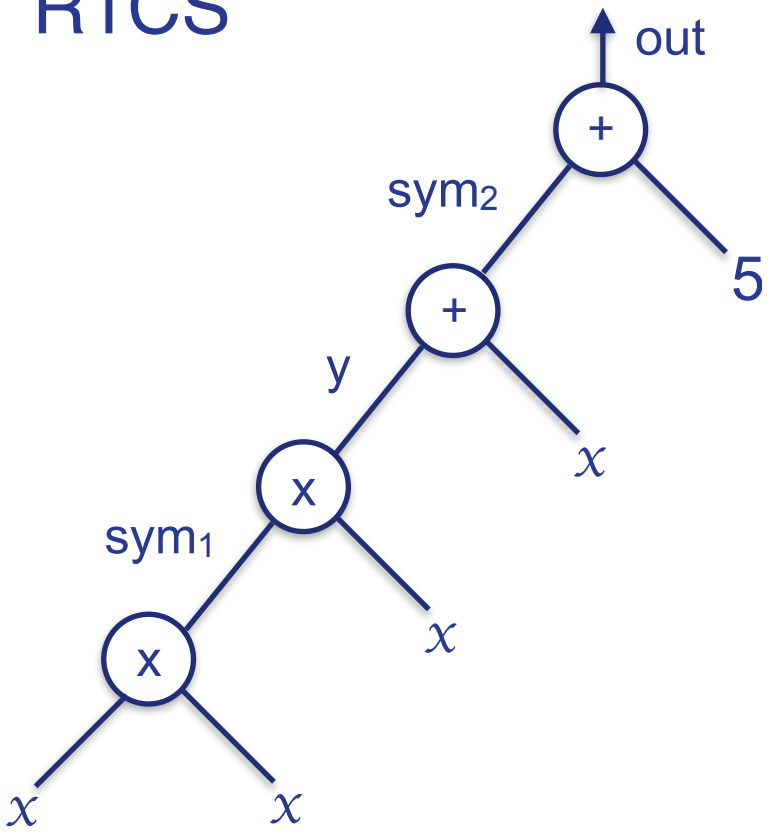
- Represent constraint corresponding to each gate as a R1CS
- The vectors will be:
    - **a** : Left input to the gate
    - **b** : Right input to the gate
    - **c** : Output of the gate
    - **s** : Solution vector

R1CS

R1CS

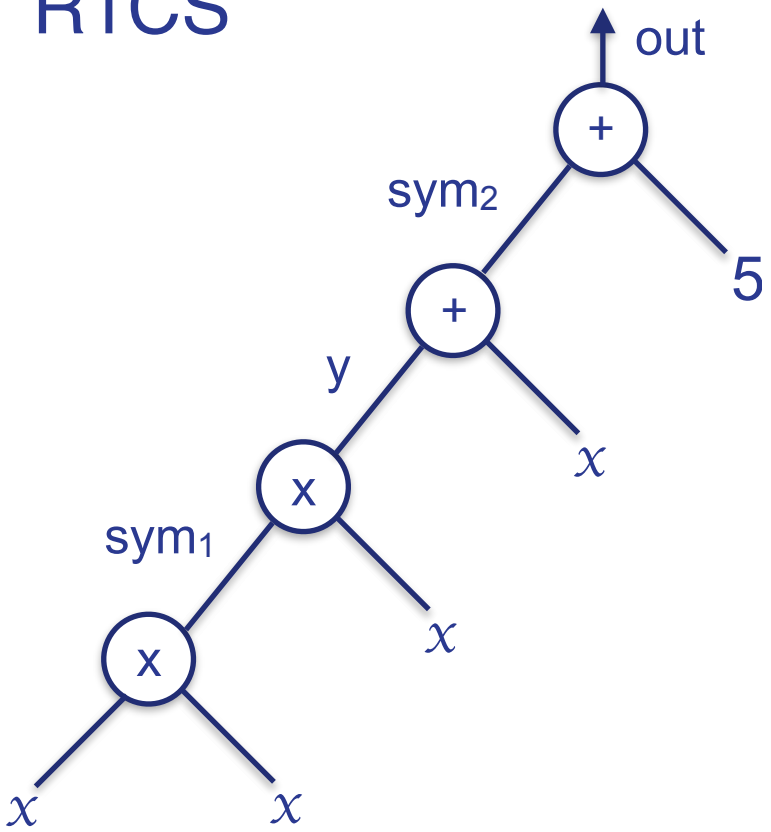


R1CS



Fix a variable ordering amongst all the variables of the circuit

R1CS



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[ONE, x, sym_1, y, sym_2, out]

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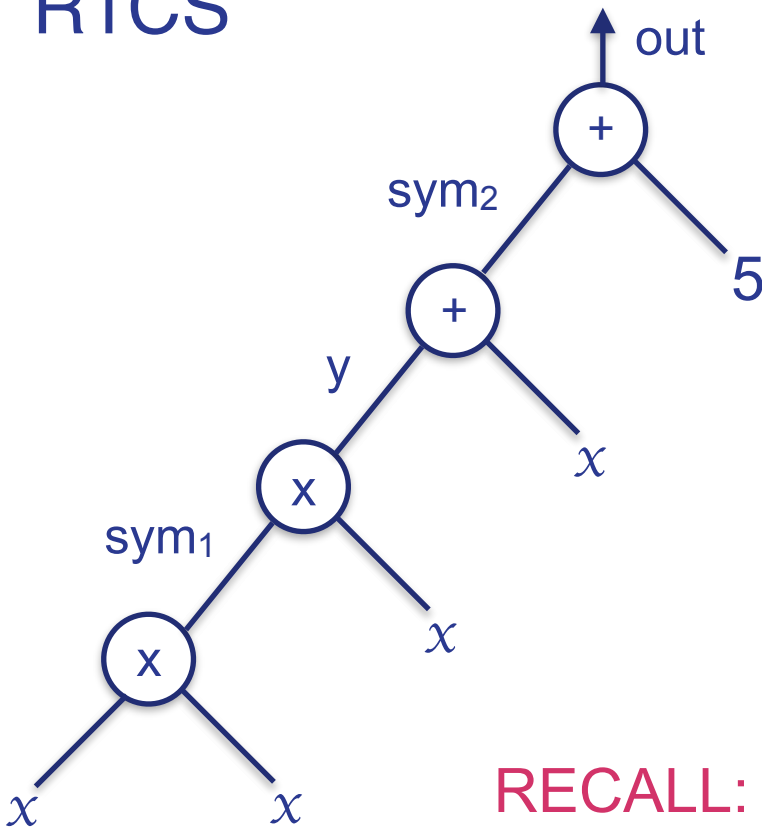

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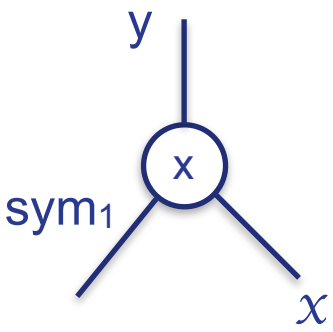
RECALL: Every gate corresponds to a constraint

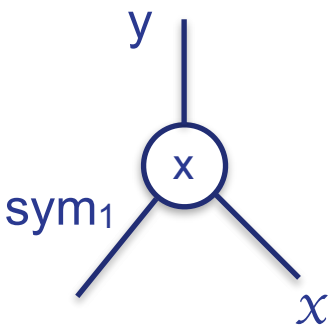
R1CS …



[ONE, x, OUT, sym_1, y, sym_2]

R1CS …

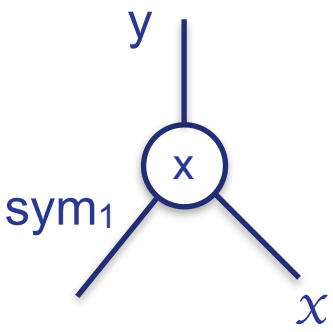
$[\ \text{ONE}, x, \text{OUT}, \text{sym}_1, \text{y}, \text{sym}_2\ ]$



$$a = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

R1CS …

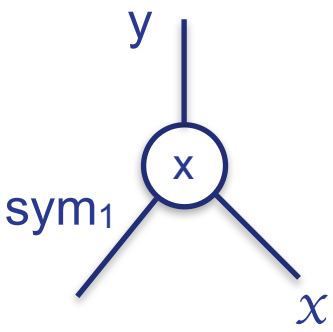
[ONE, x, OUT, sym_1, y, sym_2]



$$a = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \quad b = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad c = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

R1CS …

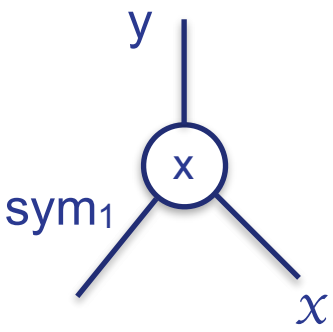
[ONE, x, OUT, sym_1, y, sym_2]



$$a = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \quad b = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad c = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \quad s = \begin{bmatrix} 0 \\ 3 \\ 0 \\ 5 \\ 15 \\ 0 \end{bmatrix}$$

R1CS …

[ONE, x, OUT, sym_1, y, sym_2]



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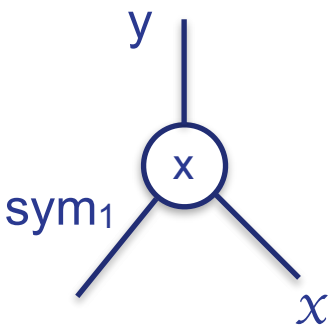
Easy to see s.a * s.b - s.c = 0 holds

R1CS …

[ONE, x, OUT, sym_1, y, sym_2]



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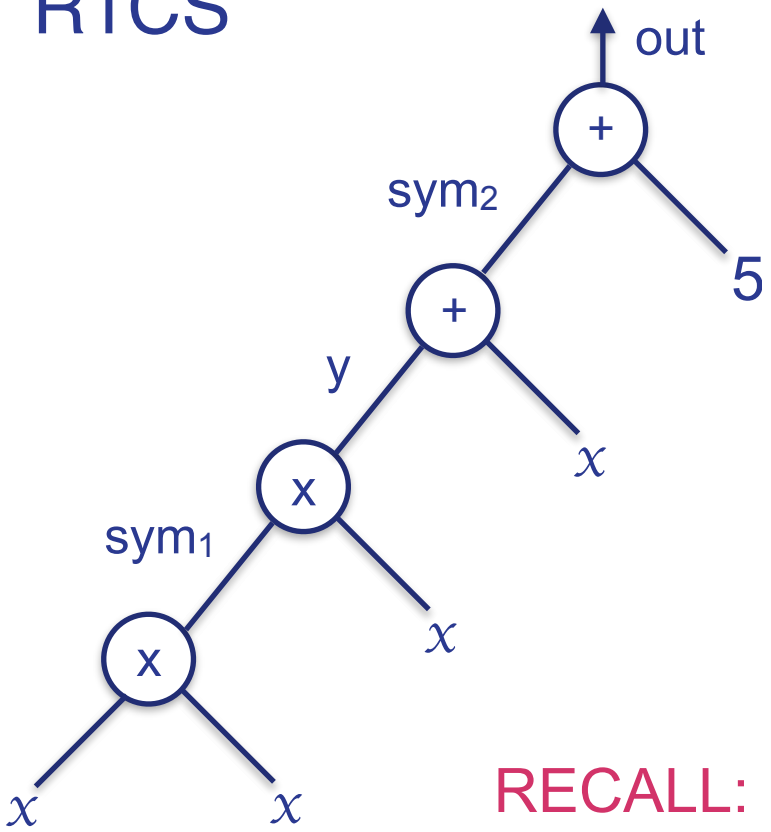
Represent all the gates as R1CS

Easy to see s.a * s.b - s.c = 0 holds

R1CS



Fix a variable ordering amongst all the variables of the circuit

[ONE, x, sym_1, y, sym_2, out]

RECALL: Every gate corresponds to a constraint

R1CS Constraints …

[ONE, x, OUT, sym_1, y, sym_2]

A	B	C
[0, 1, 0, 0, 0, 0]	[0, 1, 0, 0, 0, 0]	[0, 0, 0, 1, 0, 0]
[0, 0, 0, 1, 0, 0]	[0, 1, 0, 0, 0, 0]	[0, 0, 0, 0, 1, 0]
[0, 1, 0, 0, 1, 0]	[1, 0, 0, 0, 0, 0]	[0, 0, 0, 0, 0, 1]
[5, 0, 0, 0, 0, 1]	[1, 0, 0, 0, 0, 0]	[0, 0, 1, 0, 0, 0]

R1CS Constraints …

[ONE, x, OUT, sym_1, y, sym_2]

A	B	C
[0, 1, 0, 0, 0, 0]	[0, 1, 0, 0, 0, 0]	[0, 0, 0, 1, 0, 0]
[0, 0, 0, 1, 0, 0]	[0, 1, 0, 0, 0, 0]	[0, 0, 0, 0, 1, 0]
[0, 1, 0, 0, 1, 0]	[1, 0, 0, 0, 0, 0]	[0, 0, 0, 0, 0, 1]
[5, 0, 0, 0, 0, 1]	[1, 0, 0, 0, 0, 0]	[0, 0, 1, 0, 0, 0]

S = [1, 3, 35, 9, 27, 30]

R1CS

R1CS

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R1CS

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- To verify: Solve each R1CS equation corresponding to a solution vector.
- Can we do better?

Quadratic Arithmetic Program (QAP)

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- Implement the same logic as R1CS, but using polynomials instead of dot products.

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- To verify: Solve QAP for the system at once.

$$A(x) * B(x) = C(x)$$

R1CS to QAP

A	B	C
[0, 1, 0, 0, 0, 0]	[0, 1, 0, 0, 0, 0]	[0, 0, 0, 1, 0, 0]
[0, 0, 0, 1, 0, 0]	[0, 1, 0, 0, 0, 0]	[0, 0, 0, 0, 1, 0]
[0, 1, 0, 0, 1, 0]	[1, 0, 0, 0, 0, 0]	[0, 0, 0, 0, 0, 1]
[5, 0, 0, 0, 0, 1]	[1, 0, 0, 0, 0, 0]	[0, 0, 1, 0, 0, 0]

R1CS to QAP

A	B	C
[0, 1, 0, 0, 0, 0]	[0, 1, 0, 0, 0, 0]	[0, 0, 0, 1, 0, 0]
[0, 0, 0, 1, 0, 0]	[0, 1, 0, 0, 0, 0]	[0, 0, 0, 0, 1, 0]
[0, 1, 0, 0, 1, 0]	[1, 0, 0, 0, 0, 0]	[0, 0, 0, 0, 0, 1]
[5, 0, 0, 0, 0, 1]	[1, 0, 0, 0, 0, 0]	[0, 0, 1, 0, 0, 0]

R1CS to QAP

A	B	C
[0, 1, 0, 0, 0, 0]	[0, 1, 0, 0, 0, 0]	[0, 0, 0, 1, 0, 0]
[0, 0, 0, 1, 0, 0]	[0, 1, 0, 0, 0, 0]	[0, 0, 0, 0, 1, 0]
[0, 1, 0, 0, 1, 0]	[1, 0, 0, 0, 0, 0]	[0, 0, 0, 0, 0, 1]
[5, 0, 0, 0, 0, 1]	[1, 0, 0, 0, 0, 0]	[0, 0, 1, 0, 0, 0]

$A_1(x)$

R1CS to QAP

A	B	C
[0, 1, 0, 0, 0, 0]	[0, 1, 0, 0, 0, 0]	[0, 0, 0, 1, 0, 0]
[0, 0, 0, 1, 0, 0]	[0, 1, 0, 0, 0, 0]	[0, 0, 0, 0, 1, 0]
[0, 1, 0, 0, 1, 0]	[1, 0, 0, 0, 0, 0]	[0, 0, 0, 0, 0, 1]
[5, 0, 0, 0, 0, 1]	[1, 0, 0, 0, 0, 0]	[0, 0, 1, 0, 0, 0]

$A_2(x)$

R1CS to QAP

A

[0, 1, 0, 0, 0, 0]
[0, 0, 0, 1, 0, 0]
[0, 1, 0, 0, 1, 0]
[5, 0, 0, 0, 0, 1]

$A_2(x)$

B

[0, 1, 0, 0, 0, 0]
[0, 1, 0, 0, 0, 0]
[1, 0, 0, 0, 0, 0]
[1, 0, 0, 0, 0, 0]

$B_2(x)$

C

[0, 0, 0, 1, 0, 0]
[0, 0, 0, 0, 1, 0]
[0, 0, 0, 0, 0, 1]
[0, 0, 1, 0, 0, 0]

R1CS to QAP

A

[0, 1, 0, 0, 0, 0]
[0, 0, 0, 1, 0, 0]
[0, 1, 0, 0, 1, 0]
[5, 0, 0, 0, 0, 1]

$A_2(x)$

B

[0, 1, 0, 0, 0, 0]
[0, 1, 0, 0, 0, 0]
[1, 0, 0, 0, 0, 0]
[1, 0, 0, 0, 0, 0]

$B_2(x)$

C

[0, 0, 0, 1, 0, 0]
[0, 0, 0, 0, 1, 0]
[0, 0, 0, 0, 0, 1]
[0, 0, 1, 0, 0, 0]

$C_4(x)$

Quadratic Arithmetic Programs (QAPs)

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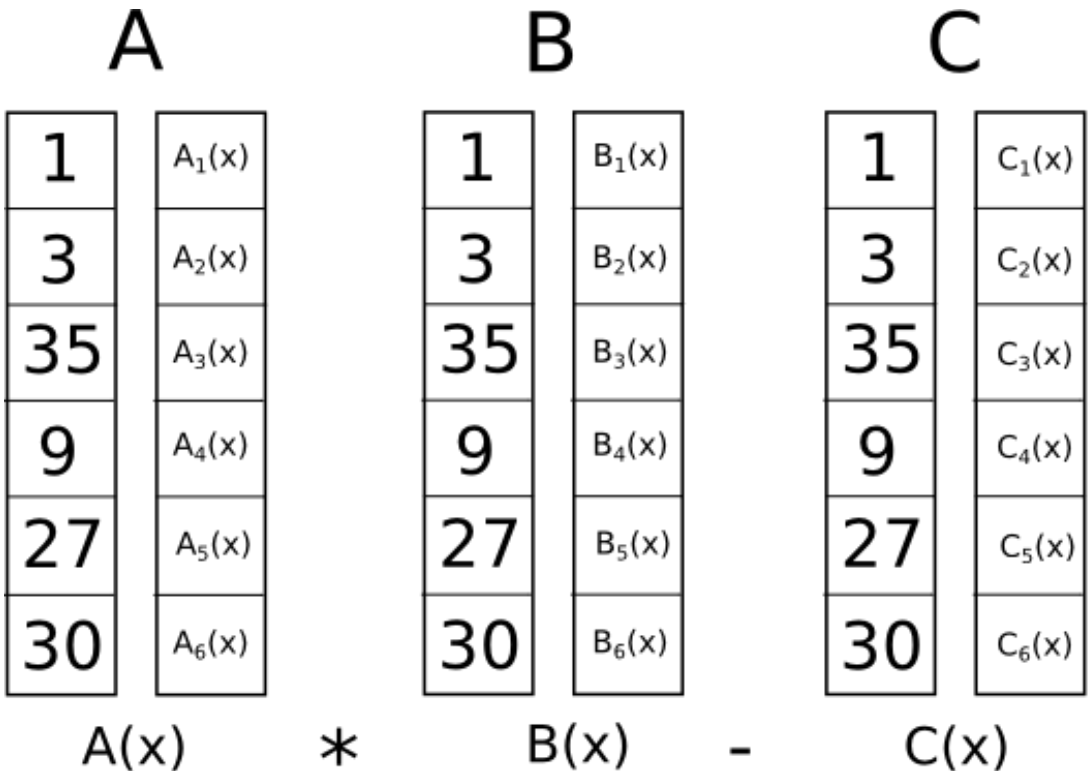
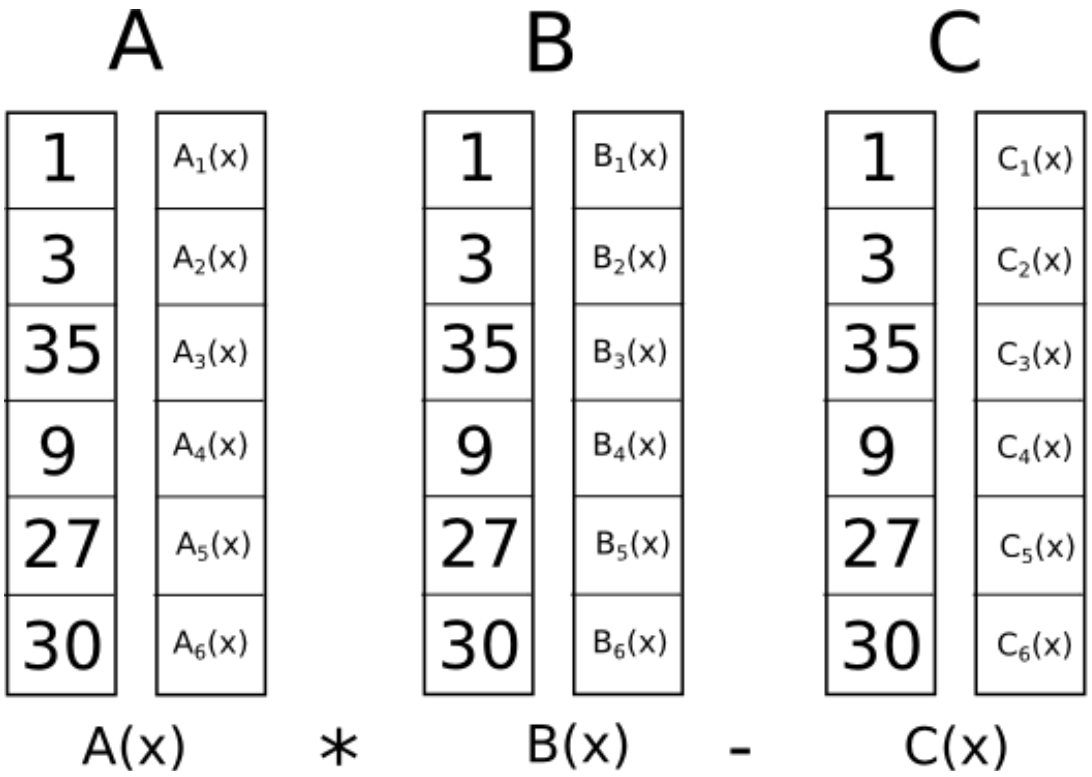


Quadratic Arithmetic Programs (QAPs)



$$A(x) = \Sigma\, s_i\, A_i(x)$$

$$B(x) = \Sigma\, s_i\, B_i(x)$$

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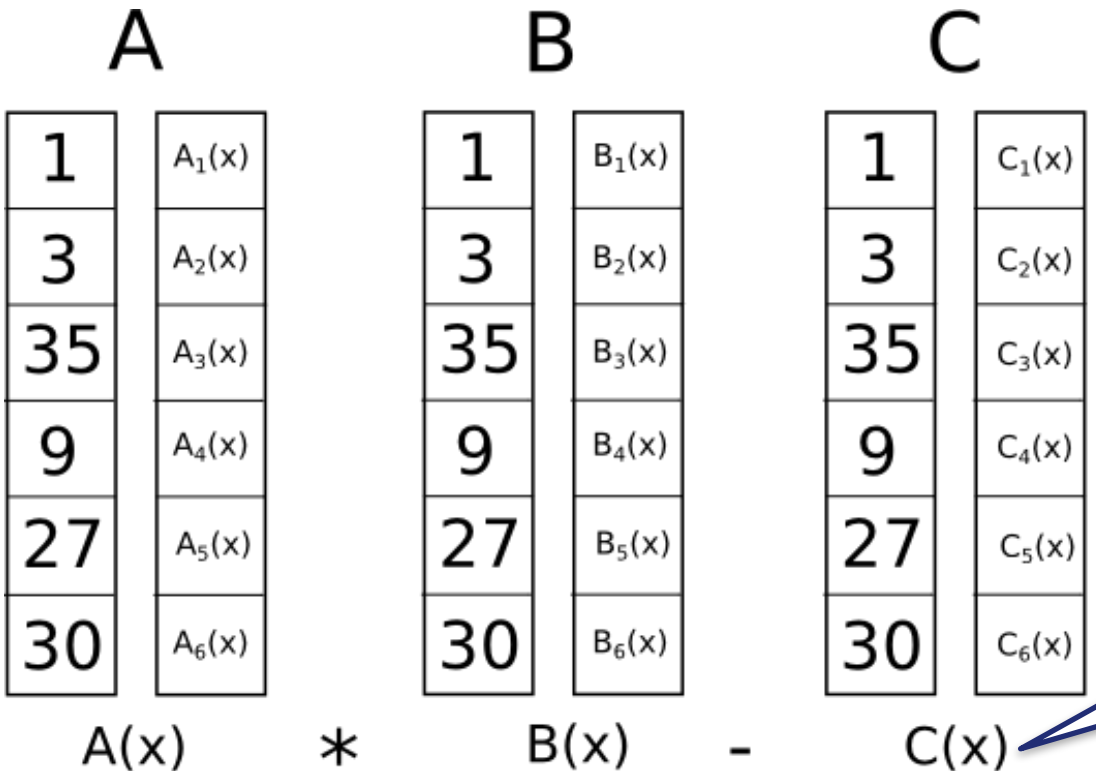
Quadratic Arithmetic Programs (QAPs)



$$A(x) = \Sigma\, s_i\, A_i(x)$$

$$B(x) = \Sigma\, s_i\, B_i(x)$$

$$C(x) = \Sigma\, s_i\, C_i(x)$$

Should be 0 for all x corresponding to circuit gates.

QAP …

QAP …

$$A(x) * B(x) - C(x) = 0, \quad \forall x \in [1, |Gates|]$$

QAP …

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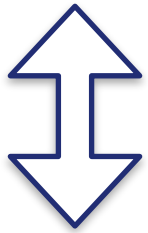

$$A(x) * B(x) - C(x) \text{ is divisible by } \prod_{i=1}^{|Gates|} (x-i)$$

QAP …

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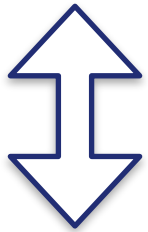

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Target Polynomial (T(x))

QAP …

$$A(x) * B(x) - C(x) = H(x) * T(x)$$

QAP …

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QAP …

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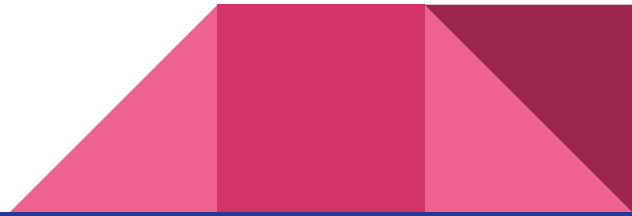
If we know the solution vector (**s**), then:

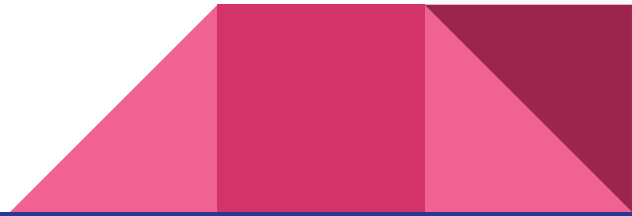
- We know: s_i, $\forall i \in [1, |Vars|]$
- For a given **e**, can compute : A(e), B(e), C(e)
- T(x) is public, so we can compute H(e)

Proof



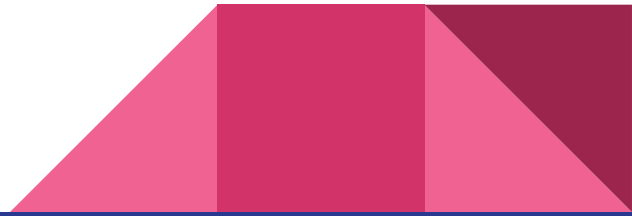
Proof

- The evaluation point e is chosen randomly by the *Verifier* and sent to the *Prover*.



Proof

- The evaluation point e is chosen randomly by the *Verifier* and sent to the *Prover*.
- A(e), B(e), C(e), H(e) is the desired proof.



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- The point e is send in an “hidden” form.

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- Polynomials have to be evaluated “blindly”.

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- The point e is send in an “hidden” form.
- Polynomials have to be evaluated “blindly”.
- Elliptic Curve pairings are used as a underlying “hiding scheme”.

Questions?

Thank You!