ESO 208A: Computational Methods in Engineering
Problem Set 2

1. For $x, a \in \Re$ but $\neq 0$, show that the sequence $x_{k+1} = x_k + x_k(1 - ax_k)$ converges to $1/a$ if and only if $|1 - ax_0| < 1$.

*2. Find a simple root (other than $x = 0$) of the equation: $f(x) = \sin x - (x/2)^2$ using Bisection method, Regula-Falsi method, Fixed Point method, Newton-Raphson's method and Secant method. In each case, calculate true relative error and approximate relative error at each iteration. Plot both of these errors as Log (%error) vs. iteration number for each of the methods. Terminate the iterations when the approximate relative error is less than 0.01 %. For the error calculations the true root may be taken as 1.93375496. Use starting points for Bisection, Regula-Falsi and Secant methods as $x = 1$ and $x = 2$.

*3.Find a root of $f(x) = -1+0.5x+0.75x^2-0.25x^3$ using the Newton-Raphson method starting with $x_0=0$. Comment on the results. Repeat with the starting guess of 0.5.

4. Find a root of the following equation using Secant and Mueller's method to an approximate error of $\varepsilon_a \leq 0.1\%$:

$$x^4 \sin x - e^x = 0$$

Take starting values of 1 and 2 in the Secant method and three starting values as 1, 2 and 3 in the Mueller's method. Do they converge to the same root? Compare the number of iterations required in two methods.

*5. Find the root of the polynomial by (a) Mueller's method and (b) Bairstow's method using $\varepsilon = 0.01\%$.

$$x^4 - 2x^3 - 53x^2 + 54x + 504 = 0$$

6. If α is a zero of $f(x)$ of multiplicity $m > 1$, show the following:

a) Newton Raphson method given by $x_{k+1} = x_k - \dfrac{f(x_k)}{f'(x_k)}$ is first order.

b) If we modify the Newton Raphson method as $x_{k+1} = x_k - \dfrac{mf(x_k)}{f'(x_k)}$, the method becomes at least 2nd order.

7. Let the function $f(x)$ be four times continuously differentiable and have a simple zero at ξ. Successive approximations x_n, for the root ξ are computed from

$$x_{n+1} = \frac{(x'_{n+1} + x''_{n+1})}{2} \quad \text{where,}$$

$$x'_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x''_{n+1} = x_n - \frac{u(x_n)}{u'(x_n)}, \quad u(x) = \frac{f(x)}{f'(x)}$$

Prove that if the sequence $\{x_n\}$ converges to ξ, then the rate of convergence is cubic.