Tutorial 5

1. Approximate the function, $f(t) = e^t$ in the interval $[-1, 3]$.
   (a) Use a second-degree polynomial using both the conventional form of polynomials and the Legendre polynomials. Then, use a third-degree polynomial and comment on the additional computational effort required in both the methods.
   (b) Obtain the second-degree Tchebycheff fit and compare the error with second-degree Legendre fit.

   The Legendre Polynomials are: $P_0(x)=1$; $P_1(x) = x$; $P_2(x) = (-1+3x^2)/2$; $P_3(x) = (-3x+5x^3)/2$

   The Tchebycheff Polynomials are: $T_0(x)=1$; $T_1(x) = x$; $T_2(x) = -1+2x^2$

   Following integrals are useful:

$$\int xe^{2x+1}dx = e^{2x+1}(2x-1)/4; \int x^2e^{2x+1}dx = e^{2x+1}(2x^2-2x+1)/4;$$

$$\int x^3e^{x}dx = e^{2x+1}(4x^3-6x^2+6x-3)/8$$

$$\int_{-1}^{1} \frac{e^{2x+1}}{\sqrt{1-x^2}}dx = 19.4671; \int_{-1}^{1} \frac{xe^{2x+1}}{\sqrt{1-x^2}}dx = 13.5836; \int_{-1}^{1} \frac{x^2e^{2x+1}}{\sqrt{1-x^2}}dx = 12.6752$$

2. Estimate the value of the function at $x = 4$ from the table of data given below, using, (a) Lagrange interpolating polynomial of 2nd degree using the points x=2,3,5; (b) Newton's interpolating polynomial of 4th degree.

x	***f(x)***
1	1
2	12
3	54
5	375
6	756