ESO 208A: Computational Methods in Engineering
Problem Set 1

1. Let

$$F(x) = x + \frac{a^3}{x^2}\text{, where } a = \text{constant parameter} \tag{1}$$

and let x_1 and x_2 be <u>two positive values</u> of x that satisfy the following relation:

$$a^3\left(\frac{1}{x_1} - \frac{1}{x_2}\right) = \frac{1}{4}\left(x_2^2 - x_1^2\right),\ x_2 > x_1 \tag{2}$$

It is desired to compute the difference $\Delta F = F(x_1) - F(x_2)$. with low precision arithmetic, by simulating a computer that performs floating point operations rounding *mantissa* to 6 decimals.

For the case in which $a = 6.870429497$ and $x_1 = 8.583454139$, compute ΔF directly from the definition of F in (1) and using (2) to compute x_2. True value of ΔF computed using double precision algorithm is 0.103622333×10^{-4}. What is the true relative error in ΔF using this algorithm?

Using (1) and (2), express ΔF as:

$$\Delta F = \frac{x_1(r-1)^3}{4r}\text{, where } r = x_2/x_1 \tag{3}$$

Estimate r using (2) and ΔF using (3) with the same low precision arithmetic. What is the true relative error in ΔF using this algorithm? Comment on your result.

2. Consider computation of the following expression:

$$G(\varepsilon) = \frac{\sqrt{(1+8\varepsilon^2)} - 1}{2}$$

Compute the value of G for $\varepsilon = 0.001$ and estimate the corresponding relative error, performing operations by rounding all mantissas to six decimals. Improve your computation of G by employing a Taylor series expansion, and estimate the corresponding relative error. Use double precision calculation to obtain the true solution that is needed to calculate the relative errors.

3. One has measured the two sides and the included angle of a triangle as: $a = 100.0 \pm 0.1$, $b = 101.0 \pm 0.1$, and the angle $C = 1.00^\circ \pm 0.01^\circ$. How accurately is it possible to give the third side c?

4. The following set of equations is to be solved to get the value of x for a given δ. For what values of δ will this problem be well-conditioned?

$$x + y = 2$$
$$x + (1-\delta)y = 1$$