ESO 208A: Computational Methods in Engineering
Problem Set 4

1. Solve the following set of non-linear equations using fixed-point iteration and Newton-Raphson method with relative approximate error of 0.01%, starting with initial guesses of x=1 and y=1:

$$x^2 - x + y - 0.5 = 0$$
$$x^2 - 5xy - y = 0$$

2. Determine to five decimal places of accuracy, the real solutions of the following set of equations using Newton Raphson method with initial guess as (1,1),

$$x = \sin(x + y) \qquad\qquad y = \cos(x - y)$$

3. Consider the following non-linear system of equations:

$$x^2 + y^2 - 2 = 0$$
$$x^2 - y - 0.5x + 0.1 = 0$$

(a) Express y in terms of x using the second equation and substitute in the first equation to obtain a 4th order polynomial. Compute all the *real* roots using Bairstow's method with 3 complete iterations or with a maximum norm of approximate relative error (in r and s) of 0.01%, whichever is earlier. Choose initial values of r=0.3 and s=0.9.

(b) Using the roots of the polynomial, compute all the *real* solutions (x-y pairs) of the non-linear system of equation.

4. Use multi-equation Newton-Raphson method to determine the roots of

$$x_1^2 + x_1 x_2 = 10$$
$$x_2 + 3x_1 x_2^2 = 57$$

Use initial guesses x_1=1.5 and x_2=3.5 and perform only one iteration. Formulate the multi-equation form of the Newton-Raphson method from the single equation formula (formula sheet).

5. Equations of two curves in 2-dimension are given as follows:

$$(x^2 + y^2)^2 = (x^2 - y^2)$$
$$(x^2 + y^2)^2 = xy^2$$

a) Compute one point of intersection between the two curves using two-variable Newton Raphson method. Use the initial guesses as $x = y = 0.25$ and the approximate relative error criterion of 0.1%.

b) From the geometry and the computed point of intersection, identify the other point of intersection? (*Hint: you may use polar co-ordinates*)

6. Consider the following matrix:

$$\begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 4 & -1 & 0 \\ 0 & -1 & 4 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}$$

a) Find the eigenvalue of maximum absolute magnitude and the corresponding eigenvector using the Power method with an accuracy of 0.001% of relative approximate error on the eigenvalue.

b) Find the eigenvalue of minimum absolute magnitude and the corresponding eigenvector using the Inverse Power method with an accuracy of 0.001% of relative approximate error on the eigenvalue.

c) Formulate the characteristic polynomial using Fadeev-Leverrier method. Solve the polynomial equation using the Bairstow's method (with relative approximate error of < 0.01%) for all the eigenvalues of the matrix.

d) Obtain all the eigenvalues using *QR* algorithm and compare with those obtained in (a), (b) and (c) above.

7. Consider the following Matrix:

$$\begin{bmatrix} 7 & -2 & 1 \\ -2 & 10 & -2 \\ 1 & -2 & 7 \end{bmatrix}$$

a) Obtain the Equation of the Characteristic polynomial using Fadeev-Leverrier Method.

b) Perform one complete iteration of the Bairstow's method with the starting values as $r = 18$ and $s = -10$. Compute the approximate eigenvalues of the matrix after this iteration.

8. Consider the following matrix:

$$\begin{bmatrix} 3 & 4 & 1 \\ 3 & 5 & 1 \\ 2 & 2 & 1 \end{bmatrix}$$

a) Find the eigenvalue of maximum absolute magnitude and the corresponding eigenvector using the Power method.

b) Find the eigenvalue of minimum absolute magnitude and the corresponding eigenvector using the Inverse Power method.

c) Formulate the characteristic polynomial using Fadeev-Leverrier method.

d) Obtain the eigenvalues using *QR* algorithm. Compare with the results of (a) and (b). Check whether the eigenvalues obtained are the roots of the polynomial obtained in (c).