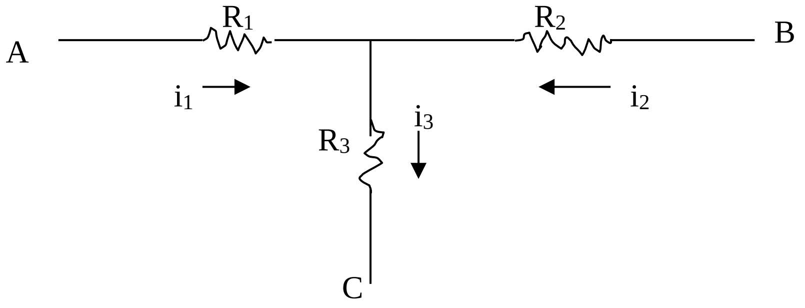
ESO 208A: Computational Methods in Engineering
Problem Set 3

1. For the circuit shown in the figure, find the currents through the elements using (a) Gauss elimination (b) Gauss Jordan and (c) Gauss Seidel methods. (Use R_1=10 Ohm; R_2=20 Ohm; R_3=40 Ohm; V_A-V_C = 200 Volt; V_B-V_C = 100 Volt)



2. In the above problem, how much will be the change in current in R_3 due to unit change in voltage difference (V_A-V_C)? Which of the currents is the most sensitive to a change in voltage at B?

3. Solve the following set of equations using (a) Doolittle and Crout decomposition (b) Thomas algorithm and (c) Cholesky decomposition

$$4x_1 + x_2 = 6$$
$$x_1 + 4x_2 + x_3 = 12$$
$$x_2 + 4x_3 = 14$$

4. Solve the following system of equations by (a) Gauss Elimination, (b) Gauss-Jordon, (c) Doolittle's method, (d) Crout's method, and (e) Cholesky decomposition:

$$\begin{bmatrix} 9.3746 & 3.0416 & -2.4371 \\ 3.0416 & 6.1832 & 1.2163 \\ -2.4371 & 1.2163 & 8.4429 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 9.2333 \\ 8.2049 \\ 3.9339 \end{bmatrix}$$

5. Consider the following matrix:

$$\boldsymbol{A} = \begin{bmatrix} 9 & 3 & -2 \\ 3 & 6 & 1 \\ -2 & 1 & 9 \end{bmatrix}$$

a) Reduce it to an upper triangular matrix using Gauss Elimination Procedure.

b) Synthesize a lower triangular matrix ***L*** and an upper triangular matrix ***U*** from the steps of (a) above such that ***A*** = ***LU***.

c) Compute $\boldsymbol{A}^{-1}$ using the ***LU*** decomposition obtained in (b).

d) Using the above results, compute the determinant and condition number of ***A.*** Use row sum norms for the matrices.

6. Solve the following system of equation using Thomas algorithm:

$$\begin{bmatrix} -2 & 1 & 0 & 0 \\ 1 & -4 & 1 & 0 \\ 0 & 1 & -4 & 1 \\ 0 & 0 & 1 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 2 \\ -2 \end{bmatrix}$$

7. Solve the following system of equations using Jacobi and Gauss Seidel methods using initial guess as zero for all the variables and compare the number iterations required for solution using two methods:

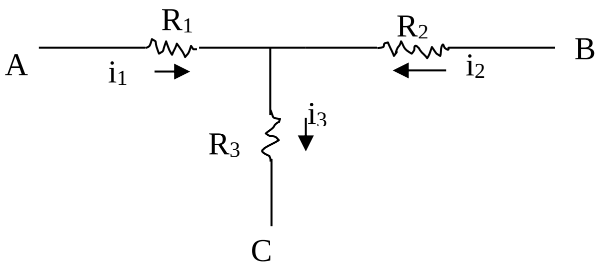
$$\begin{aligned} & x_1 + 2x_2 - x_4 = 1 \\ & x_2 + 2x_3 = 1.5 \\ & -x_3 + 2x_4 = 1.5 \\ & x_1 + 2x_3 - x_4 = 2 \end{aligned}$$

8. Two approximate solutions to the following set of equations are given as described below:

$$\begin{bmatrix} 10 & 7 & 8 & 7 \\ 7 & 5 & 6 & 5 \\ 8 & 6 & 10 & 9 \\ 7 & 5 & 9 & 10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 32 \\ 23 \\ 33 \\ 31 \end{bmatrix}$$

An approximation to the *x*-values as [-7.2, 14.6, -2.5, 3.1] yields the right hand side vector as [31.9, 23.1, 32.9, 31.1]. A very different set of *x*-values [0.18, 2.36, 0.65, 1.21] also yields a very close right hand side vector as [31.99, 23.01, 32.99, 31.01]. It is not clear whether any of the *x*-values are close to the true solution. Use Crout's decomposition and improve the solution starting from each of the above approximations of *x*-values.

9. For the circuit shown below, compute the currents and inverse of the coefficient matrix together using Gauss-Jordon. What is the upper bound for the relative change in the currents in this circuit due to 5% variation in each of the resistances R_1 and R_2? Given: $R_1 = 10\ \Omega$, $R_2 = 30\ \Omega$, $R_3 = 50\ \Omega$, $V_A - V_C = 24$ V and $V_A - V_B = 12$ V.



Use *maximum norm* for vectors and *column-sum norm* for the matrices.

10. Consider the following matrix:

$$A = \begin{bmatrix} 10 & 7 & 8 & 7 \\ 7 & 5 & 6 & 5 \\ 8 & 6 & 10 & 9 \\ 7 & 5 & 9 & 10 \end{bmatrix}$$

a) Compute A^{-1} using Cholesky decomposition.

b) Find the solution of $Ax=b$ where, $b = [4\ \ 3\ \ 3\ \ 1]^T$.

c) If b is perturbed by δb such that $\| \delta b \|_\infty = 0.01$, find an upper bound for the corresponding perturbation vector $\| \delta x \|_\infty$.

d) Compute the condition number $\ell(A)$ and check the inequality $\dfrac{\|\delta x\|_\infty}{\|x\|_\infty} \Big/ \dfrac{\|\delta b\|_\infty}{\|b\|_\infty} \leq \ell(A)$.

11. Consider the following set of equations:

$$\begin{bmatrix} 10^{-5} & 10^{-5} & 1 \\ 10^{-5} & -10^{-5} & 1 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \times 10^{-5} \\ -2 \times 10^{-5} \\ 1 \end{bmatrix}$$

a) Solve the system using Gaussian elimination, without pivoting, using 3-digit floating-point arithmetic with round-off.

b) Perform complete pivoting and carry out Gaussian elimination steps once again using 3-digit floating-point arithmetic with round-off. Explain the results.

c) Rewrite the set of equations after scaling according to $x'_3 = 10^5 \times x_3$ and equilibration. Solve the system with the same precision for floating point operations.

12. Let $\boldsymbol{A}$ be a given nonsingular $n \times n$ matrix, and $\boldsymbol{X_0}$ an arbitrary $n \times n$ matrix. We define a sequence of matrices by

$$\boldsymbol{X}_{k+1} = \boldsymbol{X}_k + \boldsymbol{X}_k(\boldsymbol{I} - \boldsymbol{A}\boldsymbol{X}_k), \quad k = 0, 1, 2, \ldots.$$

Prove that $\lim_{k \to \infty} X_k = A^{-1}$ if and only if $\rho(\boldsymbol{I} - \boldsymbol{A}\boldsymbol{X_0}) < 1$.

13. Consider the following set of linear equations:

$$\begin{aligned} d_1 x + by + cz &= f_1 \\ bx + d_2 y + az &= f_2 \\ cx + ay + d_3 z &= f_3 \end{aligned}$$

where, the mean ± standard deviation of the values of the constants are, $d_1 = 4.34 \pm 0.05$, $d_2 = 7.8 \pm 0.10$, $d_3 = 4.2 \pm 0.07$, $b = 2.1 \pm 0.02$, $a = 1.8 \pm 0.01$, $c = -2.4 \pm 0.11$, $f_1 = 87.65 \pm 0.56$, $f_2 = 121.76 \pm 1.80$ and $f_3 = -2.0 \pm 0.03$.

(a) Obtain a *LU* decomposition of the coefficient matrix using *Cholesky's* method by considering the mean values of the constants.

(b) Obtain the solution vector using the *LU* decomposition of (a) and the mean values for the constants in the right hand side vector.

(c) Compute the inverse of the coefficient matrix using the *LU* decomposition in (a).

(d) Derive an analytical expression for the maximum norm of relative error in the solution vector for small perturbations in both, coefficient matrix and the right hand side vectors.

(e) Using the results of (c) and (d), obtain the maximum norm of the relative error in the solution vector for one standard deviation perturbations in all the constants (in both coefficient matrix and right hand side vector) of the set of equation. Use *column sum norm* for the matrices and L_∞ *-norm* for the vectors.