Tutorial 10

1. Solve the differential equation $d^2y/dx^2 - dy/dx - 2y + 2x = 3$ with the boundary conditions $y(0)=0$ and $y(0.5)=0.6967$ using (a) the shooting method and (b) the direct method (use $\Delta x=0.25$ for both).

2. For a plate of size L_x=2 cm and L_y=4 cm, with the boundary conditions as T=0 for y=0 and y=4; T=100 for x=2; and insulated boundary at x=0, find the steady state temperature at the centre using the Finite Difference Method (use Δx=1 cm and Δy=1 cm).

3. Toxic pollutant transport in a river is governed by the following equation:

$$\frac{\partial c}{\partial t} + v\frac{\partial c}{\partial x} - D\frac{\partial^2 c}{\partial x^2} + kc = 0;\ \ 0 \le x \le 1;\ \ c(0,t) = c_0;\ \ \left.\frac{\partial c}{\partial x}\right|_{(1,t)} = 0;\ \ c(x,0) = x^2 e^{-x}$$

Set-up the matrix equations for the solution of the above equation in terms of Courant Number (or CFL number) and Grid Peclet Number for general Δx and Δt :

$$\text{Courant or CFL No. } C = \frac{v\Delta t}{\Delta x}$$

$$\text{Grid Peclet No. } P_e = \frac{v\Delta x}{D}$$