Tutorial 8

1. Solve the differential equation $dy/dt = -100\,y + 99\,e^{-t}$ with the initial condition $y(0)=2$ using, (a) Euler's forward (explicit) method, and (b) Euler backward (implicit) method, to obtain the value of y at $t=0.1$. Use time steps of 0.01, 0.02 and 0.025. Find the analytical solution and compare the errors for these time steps.

2. The amount of lowering of water level, s, in a well at a time t, due to pumping from groundwater is governed by an equation of the form $s=C\ W(u)$, where C is a constant (proportional to the discharge), W is called the Well Function, and u is inversely proportional to t. The well function is given by the equation $dW(u)/du = -Exp(-u)/u$. If the value of $W(1)$ is 0.21938393, find the value of $W(0.5)$ using (a) Romberg integration algorithm with accuracy $O(h^6)$ (b) Modified Euler with $h= -0.5$ (c) Heun's method with $h= -0.25$ and (d) Fourth-order Runge-Kutta method with $h= -0.25$. Perform an error analysis using the true value of $W(0.5)$ as 0.55977359.