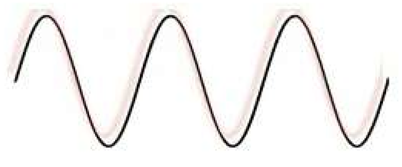
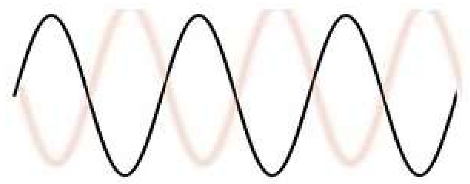
Wave on String

Power Transmission in Travelling Wave on String

$P = -T\left(\frac{\partial y}{\partial t}\right)\left(\frac{\partial y}{\partial x}\right)$ valid for all waves on string

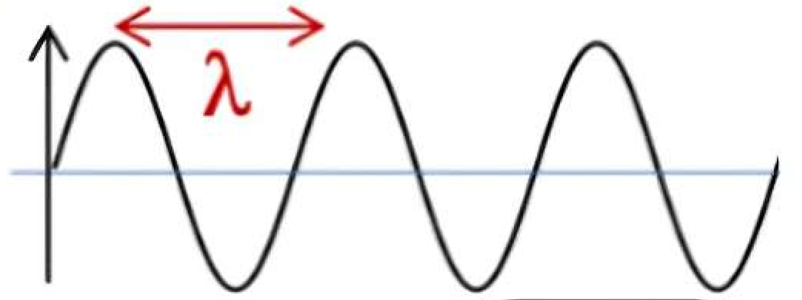
$\langle P \rangle = \underbrace{\frac{1}{2}\sqrt{T\mu}}_{\text{Property of medium}}\ \overbrace{A^2\omega^2}^{\text{Property of Source}}$





$y = A\sin(kx - \omega t)$	$y = A\sin(kx + \omega t)$
Wave travelling in + dir.	Wave travelling in − dir.

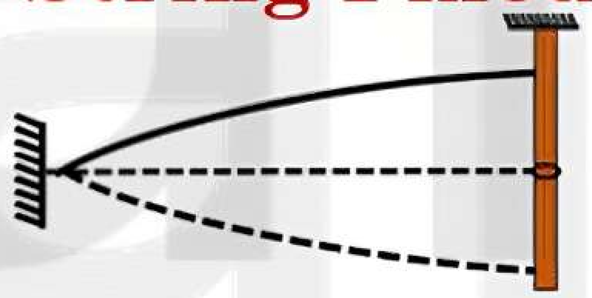
Sinusoidal Wave Equation



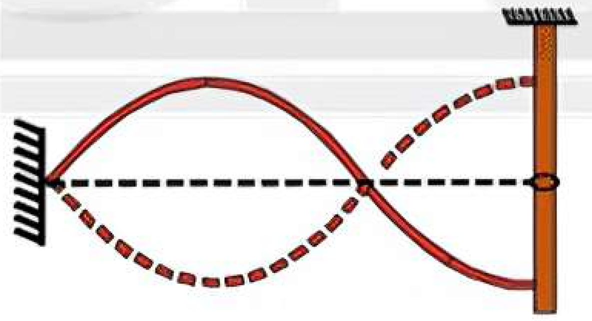
$y = A\sin(\omega t - kx + \phi)$

$\boxed{\lambda = \frac{2\pi}{k}}$ $\boxed{T = \frac{2\pi}{\omega}}$ Angular wave number, $\boxed{k = \frac{2\pi}{\lambda}}$

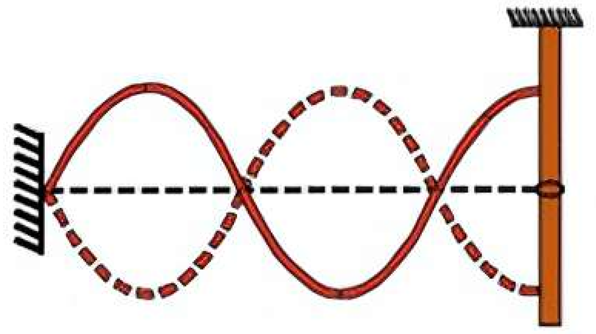
String Fixed at One End & Free at Other



$\frac{\lambda}{4} = \ell$ $f = \frac{v}{4\ell}$ Fundamental Frequency



$\frac{3\lambda}{4} = \ell$ $f = \frac{3v}{4\ell}$ 3rd Harmonic, 1st Overtone



$\frac{5\lambda}{4} = \ell$ $f = \frac{5v}{4\ell}$ 5th Harmonic, 2nd Overtone

$\frac{(2n+1)\lambda}{4} = \ell$ $f = \frac{(2n+1)v}{4\ell}$ (2n +1)Harmonic, nth Overtone

General Equation of Standing Wave

$y = 2A\sin(kx + \phi_1)\sin(\omega t + \phi_2)$

Amplitude of component wave = A

Coefficient of x = k

Wavelength of component waves = $\boxed{\frac{2\pi}{k}}$

Loop length = $\frac{\lambda}{2} = \frac{\pi}{k}$ $\quad k = \frac{2\pi}{\lambda}$

Velocity & Acceleration of Particle

$v_P = \frac{\partial y}{\partial t} = \omega\sqrt{A^2 - y^2}$

$a_P = \frac{\partial v_P}{\partial t} = -\omega^2 y$

$y = A\sin(\omega t - kx)$ → Displacement

Velocity of Wave on String

$v_w = \sqrt{\frac{T}{\mu}}$

μ = mass per unit length

$\mu = \frac{m}{L}$ $\quad v_w = \sqrt{\frac{\text{Stress}}{\rho}}$

Standing Waves

$y_{res} = 2A\sin(kx)\cos(\omega t)$

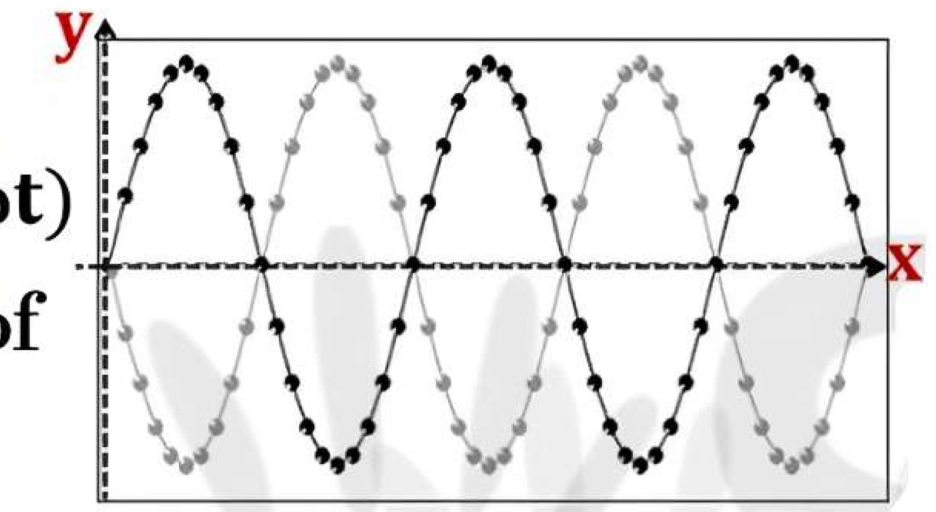
For $x = x_0$ Equation of Motion

$y = \underbrace{2A\sin(kx_0)}_{A'}\cos(\omega t)$

$y = A'\cos(\omega t)$

↓

SHM

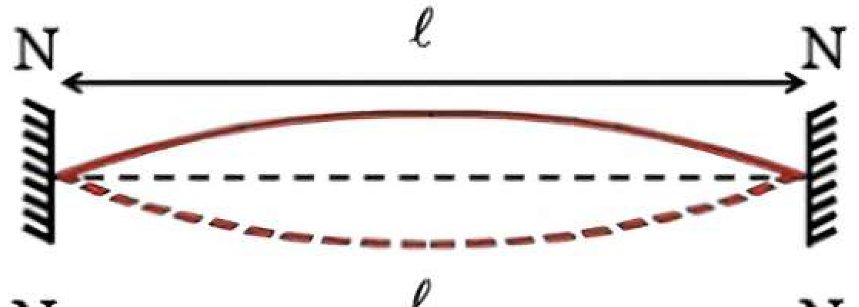


$y_{res} = 2A\sin(kx)\cos(\omega t)$

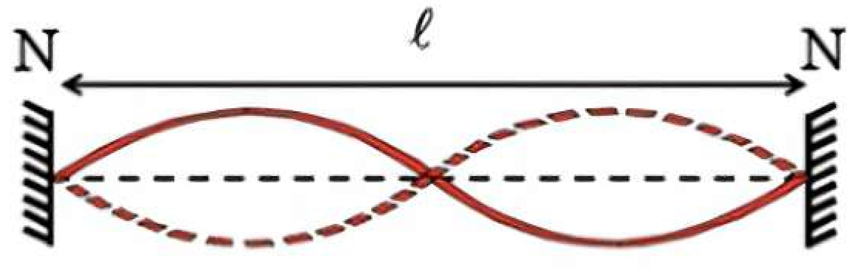
$v_P = \frac{\partial y}{\partial t} = \omega\sqrt{A^2 - y^2}$

$a_p = \frac{\partial^2 y}{\partial t^2} = -\omega^2 y$

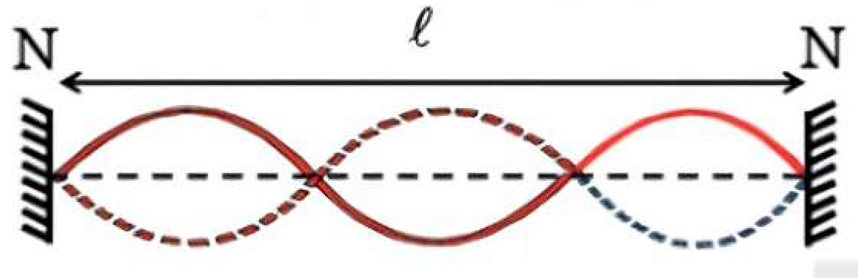
Normal Modes String Fixed at Both Ends



$\frac{\lambda}{2} = \ell$ $f_0 = \frac{v}{2\ell}$ Fundamental Frequency



$\frac{2\lambda}{2} = \ell$ $f = \frac{2v}{2\ell}$ 2nd Harmonic, 1st Overtone



$\frac{3\lambda}{2} = \ell$ $f = \frac{3v}{2\ell}$ 3rd Harmonic, 2nd Overtone

$\frac{n\lambda}{2} = \ell$ $f = \frac{nv}{2\ell}$ n th Harmonic, (n–1)th Overtone

For fundamental frequency, n = 1

(1) $\frac{n\lambda}{2} = \ell$

(2) $\boxed{f = \frac{nv}{2\ell}}$

(3) $\boxed{f = \frac{n}{2\ell}\sqrt{\frac{T}{\mu}}}$

(4) $f = n f_0$