Distributional Similarity

Problems with thesaurus-based meaning

- We don’t have a thesaurus for every language
- Even if we do, they have problems with **recall**
  - Many words are missing
  - Most (if not all) phrases are missing
  - Some connections between senses are missing
  - Thesauri work less well for verbs, adjectives
    - Adjectives and verbs have less structured hyponymy relations

Distributional models of meaning

- Also called vector-space models of meaning
- Offer **much higher recall than hand-built thesauri**
  - Although they tend to have **lower precision**
- Zellig Harris (1954): "**oculist** and **eye-doctor** … occur in almost the same environments…. **If A and B have almost identical environments we say that they are synonyms**.
- Firth (1957): "You shall know a word by the company it keeps!"

Intuition of distributional word similarity

- Nida example:

```
A bottle of tesgüino is on the table
Everybody likes tesgüino
Tesgüino makes you drunk
We make tesgüino out of corn.
```

- From context words humans can guess ***tesgüino*** means
  - an alcoholic beverage like **beer**
- Intuition for algorithm:
  - Two words are similar if they have similar word contexts.

Reminder: Term-document matrix

- Each cell: count of term t in a document d: $\mathrm{tf}_{t,d}$:
  - Each document is a count vector in $\mathbb{N}^v$: a column below

	As You Like It	Twelfth Night	Julius Caesar	Henry V
battle	1	1	8	15
soldier	2	2	12	36
fool	37	58	1	5
clown	6	117	0	0

Reminder: Term-document matrix

- Two documents are similar if their vectors are similar

	As You Like It	Twelfth Night	Julius Caesar	Henry V
battle	1	1	8	15
soldier	2	2	12	36
fool	37	58	1	5
clown	6	117	0	0

The words in a term-document matrix

- Each word is a $\boxed{\text{count vector}}$ in $\mathbb{N}^D$: a row below

	As You Like It	Twelfth Night	Julius Caesar	Henry V
battle	1	1	8	15
soldier	2	2	12	36
fool	37	58	1	5
clown	6	117	0	0

The words in a term-document matrix

- Two **words** are similar if their vectors are similar

	As You Like It	Twelfth Night	Julius Caesar	Henry V
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The Term-Context matrix

- Instead of using entire documents, use smaller contexts
  - Paragraph
  - Window of 10 words
- A word is now defined by a vector over counts of context words

Sample contexts: 20 words (Brown corpus)

- equal amount of sugar, a sliced lemon, a tablespoonful of **apricot** preserve or jam, a pinch each of clove and nutmeg,
- on board for their enjoyment. Cautiously she sampled her first **pineapple** and another fruit whose taste she likened to that of
- of a recursive type well suited to programming on the **digital** computer. In finding the optimal R-stage policy from that of
- substantially affect commerce, for the purpose of gathering data and **information** necessary for the study authorized in the first section of this

Term-context matrix for word similarity

- Two **words** are similar in meaning if their context vectors are similar

	aardvark	computer	data	pinch	result	sugar	…
apricot	0	0	0	1	0	1	
pineapple	0	0	0	1	0	1	
digital	0	2	1	0	1	0	
information	0	1	6	0	4	0	

Should we use raw counts?

- Raw word frequency is not a great measure of association between words
  - It’s very skewed
    - “the” and “of” are very frequent, but maybe not the most discriminative
- We’d rather have a measure that asks whether a context word is **particularly informative** about the target word.

Should we use raw counts?

- For the term-document matrix
  - We used tf-idf instead of raw term counts
- For the term-context matrix
  - Positive Pointwise Mutual Information (PPMI) is common

Term frequency (tf)

$\mathrm{tf}_{t,d} = \mathrm{count}(t,d)$

Instead of using raw count, we squash a bit:

$\mathrm{tf}_{t,d} = \log_{10}(\mathrm{count}(t,d)+1)$

Document frequency (df)

df_t *is* the number of documents *t* occurs in.

(note this is not collection frequency: total count across all documents)

"*Romeo*" is very distinctive for one Shakespeare play:

	Collection Frequency	Document Frequency
Romeo	113	1
action	113	31

Inverse document frequency (idf)

$$\mathrm{idf}_t = \log_{10}\left(\frac{N}{\mathrm{df}_t}\right)$$

N is the total number of documents in the collection

Word	df	idf
Romeo	1	1.57
salad	2	1.27
Falstaff	4	0.967
forest	12	0.489
battle	21	0.246
wit	34	0.037
fool	36	0.012
good	37	0
sweet	37	0

What is a document?

Could be a play or a Wikipedia article

But for the purposes of tf-idf, documents can be **anything**; we often call each paragraph a document!

Final tf-idf weighted value for a word

$$w_{t,d} = \text{tf}_{t,d} \times \text{idf}_t$$

Raw counts:

	As You Like It	Twelfth Night	Julius Caesar	Henry V
battle	1	0	7	13
good	114	80	62	89
fool	36	58	1	4
wit	20	15	2	3

tf-idf:

tf = log(count + 1)

	As You Like It	Twelfth Night	Julius Caesar	Henry V
battle	0.074	0	0.22	0.28
good	0	0	0	0
fool	0.019	0.021	0.0036	0.0083
wit	0.049	0.044	0.018	0.022

Pointwise Mutual Information

- **Pointwise mutual information**:
  - Do events x and y co-occur more than if they were independent?

$$\text{PMI}(X,Y) = \log_2 \frac{P(x,y)}{P(x)P(y)}$$

- **PMI between two words**: (Church & Hanks 1989)
  - Do words x and y co-occur more than if they were independent?

$$\text{PMI}(word_1, word_2) = \log_2 \frac{P(word_1, word_2)}{P(word_1)P(word_2)}$$

- **Positive PMI between two words** (Niwa & Nitta 1994)
  - Replace all PMI values less than 0 with zero

Computing PPMI on a term-context matrix

- Matrix F with W rows (words) and C columns (contexts)
- f_{ij} is # of times w_i occurs in context c_j

	aardvark	computer	data	pinch	result	sugar
apricot	0	0	0	1	0	1
pineapple	0	0	0	1	0	1
digital	0	2	1	0	1	0
information	0	1	6	0	4	0

$$p_{ij} = \frac{f_{ij}}{\sum_{i=1}^{W}\sum_{j=1}^{C} f_{ij}} \qquad p_{i*} = \frac{\sum_{j=1}^{C} f_{ij}}{\sum_{i=1}^{W}\sum_{j=1}^{C} f_{ij}} \qquad p_{*j} = \frac{\sum_{i=1}^{W} f_{ij}}{\sum_{i=1}^{W}\sum_{j=1}^{C} f_{ij}}$$

$$pmi_{ij} = \log_2 \frac{p_{ij}}{p_{i*}p_{*j}} \qquad ppmi_{ij} = \begin{cases} pmi_{ij} & \text{if } pmi_{ij} > 0 \\ 0 & \text{otherwise} \end{cases}$$

$$p_{ij} = \frac{f_{ij}}{\sum_{i=1}^{W}\sum_{j=1}^{C} f_{ij}}$$

Count(w,context)

	computer	data	pinch	result	sugar
apricot	0	0	1	0	1
pineapple	0	0	1	0	1
digital	2	1	0	1	0
information	1	6	0	4	0

$$p(w_i) = \frac{\sum_{j=1}^{C} f_{ij}}{N} \qquad p(c_j) = \frac{\sum_{i=1}^{W} f_{ij}}{N}$$

p(w=information,c=data) = 6/19 = .32

p(w=information) = 11/19 = .58

p(c=data) = 7/19 = .37

PPMI(w,context)

	computer	data	pinch	result	sugar
apricot	-	-	2.25	-	2.25
pineapple	-	-	2.25	-	2.25
digital	1.66	0.00	-	0.00	-
information	0.00	0.57	-	0.47	-

$$pmi_{ij} = \log_2 \frac{p_{ij}}{p_{i*}p_{*j}}$$

	p(w,context)					p(w)
	computer	data	pinch	result	sugar	
apricot	0.00	0.00	0.05	0.00	0.05	0.11
pineapple	0.00	0.00	0.05	0.00	0.05	0.11
digital	0.11	0.05	0.00	0.05	0.00	0.21
information	0.05	0.32	0.00	0.21	0.00	0.58
p(context)	0.16	0.37	0.11	0.26	0.11	

- pmi(information,data) = $\log_2$ (.32 / (.37*.58)) = .58

(.57 using full precision)

PPMI(w,context)

	computer	data	pinch	result	sugar
apricot	-	-	2.25	-	2.25
pineapple	-	-	2.25	-	2.25
digital	1.66	0.00	-	0.00	-
information	0.00	0.57	-	0.47	-

Weighing PMI

- PMI is biased toward infrequent events
- Various weighting schemes help alleviate this
  - See Turney and Pantel (2010)
- Add-one smoothing can also help

Add-2 Smoothed Count(w,context

	computer	data	pinch	result	sugar
apricot	2	2	3	2	3
pineapple	2	2	3	2	3
digital	4	3	2	3	2
information	3	8	2	6	2

	p(w,context) [add-2]					**p(w)**
	computer	data	pinch	result	sugar	
apricot	0.03	0.03	0.05	0.03	0.05	0.20
pineapple	0.03	0.03	0.05	0.03	0.05	0.20
digital	0.07	0.05	0.03	0.05	0.03	0.24
information	0.05	0.14	0.03	0.10	0.03	0.36
p(context)	0.19	0.25	0.17	0.22	0.17	

PPMI(w,context)

	computer	data	pinch	result	sugar
apricot	-	-	2.25	-	2.25
pineapple	-	-	2.25	-	2.25
digital	1.66	0.00	-	0.00	-
information	0.00	0.57	-	0.47	-

PPMI(w,context) [add-2]

	computer	data	pinch	result	sugar
apricot	0.00	0.00	0.56	0.00	0.56
pineapple	0.00	0.00	0.56	0.00	0.56
digital	0.62	0.00	0.00	0.00	0.00
information	0.00	0.58	0.00	0.37	0.00

More on MI and PMI

- Board work!

Using syntax to define a word's context

- Zellig Harris (1968)
  - "The meaning of entities, and the meaning of grammatical relations among them, is related to the restriction of combinations of these entities relative to other entities"
- Two words are similar if they have similar parse contexts
- **Duty** and **responsibility** (Chris Callison-Burch's example)

Modified by adjectives	additional, administrative, assumed, collective, congressional, constitutional …
Objects of verbs	assert, assign, assume, attend to, avoid, become, breach …

Co-occurrence vectors based on syntactic dependencies

Dekang Lin, 1998 “Automatic Retrieval and Clustering of Similar Words”

- The contexts C are different dependency relations
  - Subject-of- “absorb”
  - Prepositional-object of “inside”
- Counts for the word cell:

	subj-of, absorb	subj-of, adapt	subj-of, behave	…	pobj-of, inside	pobj-of, into	…	nmod-of, abnormality	nmod-of, anemia	nmod-of, architecture	…	obj-of, attack	obj-of, call	obj-of, come from	obj-of, decorate	…	nmod, bacteria	nmod, body	nmod, bone marrow
cell	1	1	1		16	30		3	8	1		6	11	3	2		3	2	2

PMI applied to dependency relations

Hindle, Don. 1990. Noun Classification from Predicate-Argument Structure. ACL

Object of "drink"	Count	PMI
tea	2	11.8
liquid	2	10.5
wine	2	9.3
anything	3	5.2
it	3	1.3

- "`Drink it`" more common than "`drink wine`"
- But "`wine`" is a better "drinkable" thing than "`it`"

Measuring similarity

- Given 2 target words *v* and *w*
- We'll need a way to measure their similarity.
- Most measure of vectors similarity are based on the:
- **Dot product** or **inner product** from linear algebra

$$\text{dot-product}(\vec{v},\vec{w}) = \vec{v}\cdot\vec{w} = \sum_{i=1}^{N} v_i w_i = v_1 w_1 + v_2 w_2 + \ldots + v_N w_N$$

  - High when two vectors have large values in same dimensions.
  - Low (in fact 0) for **orthogonal vectors** with zeros in complementary distribution

Problem with dot product

$$\text{dot-product}(\vec{v},\vec{w}) = \vec{v}\cdot\vec{w} = \sum_{i=1}^{N} v_i w_i = v_1 w_1 + v_2 w_2 + \ldots + v_N w_N$$

- Dot product is longer if the vector is longer. Vector length:

$$|\vec{v}| = \sqrt{\sum_{i=1}^{N} v_i^2}$$

- Vectors are longer if they have higher values in each dimension
- That means more frequent words will have higher dot products
- That's bad: we don't want a similarity metric to be sensitive to word frequency

Reminder: cosine for computing similarity

Dot product

Unit vectors

$$\cos(\vec{v},\vec{w}) = \frac{\vec{v}\cdot\vec{w}}{|\vec{v}||\vec{w}|} = \frac{\vec{v}}{|\vec{v}|}\cdot\frac{\vec{w}}{|\vec{w}|} = \frac{\sum_{i=1}^{N} v_i w_i}{\sqrt{\sum_{i=1}^{N} v_i^2}\sqrt{\sum_{i=1}^{N} w_i^2}}$$

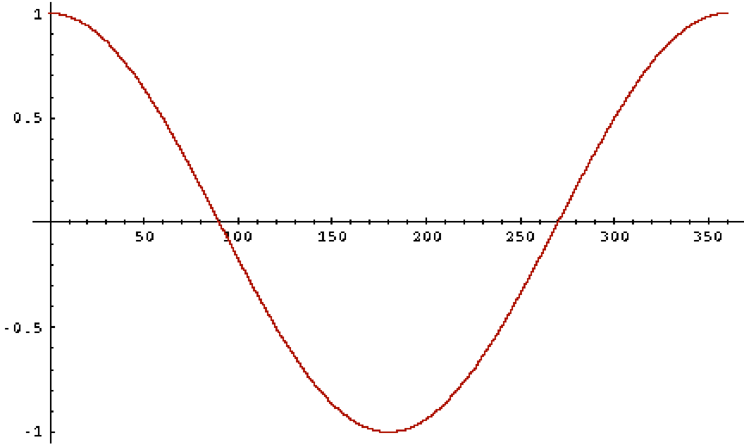
v_i is the PPMI value for word v in context i

w_i is the PPMI value for word w in context i.

$\text{Cos}(\vec{v},\vec{w})$ is the cosine similarity of $\vec{v}$ and $\vec{w}$

Cosine as a similarity metric

- -1: vectors point in opposite directions
- +1: vectors point in same directions
- 0: vectors are orthogonal



- Raw frequency or PPMI are non-negative, so cosine range 0-1

$$\cos(\vec{v},\vec{w}) = \frac{\vec{v}\cdot\vec{w}}{|\vec{v}||\vec{w}|} = \frac{\vec{v}}{|\vec{v}|}\cdot\frac{\vec{w}}{|\vec{w}|} = \frac{\sum_{i=1}^{N} v_i w_i}{\sqrt{\sum_{i=1}^{N} v_i^2}\sqrt{\sum_{i=1}^{N} w_i^2}}$$

	large	data	computer
apricot	1	0	0
digital	0	1	2
information	1	6	1

Which pair of words is more similar?

cosine(apricot,information) = $\frac{1+0+0}{\sqrt{1+0+0}\ \sqrt{1+36+1}} = \frac{1}{\sqrt{38}} = .16$

cosine(digital,information) = $\frac{0+6+2}{\sqrt{0+1+4}\ \sqrt{1+36+1}} = \frac{8}{\sqrt{38}\sqrt{5}} = .58$

cosine(apricot,digital) = $\frac{0+0+0}{\sqrt{1+0+0}\ \sqrt{0+1+4}} = 0$

Other possible similarity measures

$$\mathrm{sim}_{\mathrm{cosine}}(\vec{v},\vec{w}) = \frac{\vec{v}\cdot\vec{w}}{|\vec{v}||\vec{w}|} = \frac{\sum_{i=1}^{N} v_i \times w_i}{\sqrt{\sum_{i=1}^{N} v_i^2}\sqrt{\sum_{i=1}^{N} w_i^2}}$$

$$\mathrm{sim}_{\mathrm{Jaccard}}(\vec{v},\vec{w}) = \frac{\sum_{i=1}^{N} \min(v_i,w_i)}{\sum_{i=1}^{N} \max(v_i,w_i)}$$

$$\mathrm{sim}_{\mathrm{Dice}}(\vec{v},\vec{w}) = \frac{2\times\sum_{i=1}^{N} \min(v_i,w_i)}{\sum_{i=1}^{N} (v_i+w_i)}$$

$$\mathrm{sim}_{\mathrm{JS}}(\vec{v}||\vec{w}) = D(\vec{v}|\frac{\vec{v}+\vec{w}}{2}) + D(\vec{w}|\frac{\vec{v}+\vec{w}}{2})$$

Evaluating similarity (the same as for thesaurus-based)

- Intrinsic Evaluation:
  - Correlation between algorithm and human word similarity ratings
- Extrinsic (task-based, end-to-end) Evaluation:
  - Spelling error detection, WSD, essay grading
  - Taking TOEFL multiple-choice vocabulary tests

Levied is closest in meaning to which of these:

imposed, believed, requested, correlated