Relation:-

$$A \times B = \{ (a,b) \mid a \in A, b \in B \}$$

$$R \subseteq A \times B$$

aRb or $(a,b) \in R$

- Reflexive : A & R $\forall a \in A$ aRa
- Symmetric : $aRb \Rightarrow bRa$ $\forall a,b \in A$
- Transitive : aRb & $bRc \Rightarrow aRc$ $\forall a,b,c \in A$

Ex:- $A = \{1, 2, .., 5\}$

$R = \{ (a,b) \mid a \leq b \}$

Transitive + Ref

Ex:- S = Set of st. line in a plane

$R = \{ (a,b) \mid a \perp b \}$

symm

- Equivalence :- symmetric, transitive and reflexive.

Ex:- congruence modulo n

$a \equiv b$, $a \equiv b \pmod{n}$

$n \mid a-b$

equivalence class

$$[a] = \{ b \mid a \equiv b \}$$

$$= \{ a + kn \mid k \in \mathbb{Z} \}$$

$[a] = \{ b \in S : aRb \}$

Partition :-

A partition of a set is a collection of non-empty disjoint subsets of S whose union is S.

Statement :- The equivalence classes of an equivalence rel^n on a set S constitute a partition of S.

Conversely, if there is a partition P of S, there is an equivalence rel^n on S whose equivalence classes are elements of P.

Proof :- Define an equivalence relation R on S.

for $a \in S$

$aRa \Rightarrow a \in [a]$

$$\bigcup_{a \in S} [a] = S$$

Claim :- $[a], [b]$ distinct equivalence classes then $[a] \cap [b] = \phi$

let $c \in [a] \cap [b]$

cRa, cRb

$[a] \subseteq [b]$ $x \in [a]$ xRa aRc xRc xRb $\Rightarrow x \in [b]$

$[b] \supseteq [b]$

$\Rightarrow [a] = [b]$

$\rightarrow\leftarrow$

Converse Proof:- $\{A_i\}$ of set S

$A_i \cap A_j = \phi \quad , i \neq j$

$\bigcup_i A_i = S$

Define rel^n R on S

aRb if a and b are in same set A

Function :-

$$\phi : \underset{\text{Domain}}{A} \longrightarrow \underset{\text{Co-domain}}{B}$$

a function is a mapping from set A to B such that every element of A maps to exactly one element of set B.

$\phi(A)$ — Range

$\phi(a) = b \quad a \in A \; , \; b \in B$

One-one function:-

$\phi : A \longrightarrow B$ is one one if $\phi(a_1) = \phi(a_2) \Rightarrow a_1 = a_2$

contrapositive :— $a_1 \neq a_2 \Rightarrow \phi(a_1) \neq \phi(a_2)$

Onto function:-

$\phi : A \longrightarrow B$ is onto, if each element of B has atleast one pre-image in A.

Ex:-

	Domain	Codomain	
1	$\mathbb{Z}$	$\mathbb{Z}$	one-one
	$x \longmapsto x^3$		
	$\mathbb{R}$	$\mathbb{R}$	bijective (both)

Composition of function:-

$\phi : A \longrightarrow B$, $\psi : B \longrightarrow C$, $\gamma : C \longrightarrow D$

$\psi \times \phi : A \longrightarrow C$

$\psi . \phi (a) = \psi(\phi(a))$

$\gamma . (\psi . \phi) = (\gamma . \psi) . \phi$

Inverse :-

if $\phi : A \longrightarrow B$ is 1-1 onto

then $\exists$ $\psi B \rightarrow A$

$\phi . \psi = I_B$

$\psi . \phi = I_A$

$\phi^{-1} = \psi$

Theorem:- if $f : A \longrightarrow B$ and $g : B \rightarrow A$

$g o f : A \rightarrow A$

if $gof = I_A$, then f is one-one & g is onto

if $fog = I_B$, then g is one-one & f is onto

Theorems :-

① $f(A \cup B) = f(A) \cup f(B)$

② $f(A \cap B) \subseteq f(A) \cap f(B)$ equality holds if f is one-one

③ $f^{-1}(P \cup Q) = f^{-1}(P) \cup f^{-1}(Q)$

④ $f^{-1}(P \cap Q) = f^{-1}(P) \cap f^{-1}(Q)$

Proof :- ① let $y \in f(A \cup B)$

$\exists\ x \in A \cup B$

$f(x) = y$

$x \in A$ or $x \in B$

$f(x) \in f(A)$ or $f(x) \in f(B)$

$\therefore\ y \in f(A) \cup f(B)$

$\therefore\ f(A \cup B) \subset f(A) \cup f(B)$

Similarily others can be proved

Ex for ② :-

$R \longrightarrow R$

$x \longrightarrow x^2$

$A = [-1, 0]$, $B = [0, 1]$

$f(A \cap B) = f\{0\} = \{0\}$

$f(A) \cap f(B) = [0, 1]$

Principle of Mathematical Induction :-

S be a set of integers containing 'a', Suppose S has a property that whenever some integer $n \geq a$ belongs to S then integer $n+1$ also belongs to S.
Then S contains every Integer greater than or equal to a.

De Morgan's Law:-

$$(A \cup B)' = A' \cap B'$$
$$(A \cap B)' = A' \cup B'$$
$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$
$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

$$A' \cup (B \cup C)' = (A \cap B)' \cap (A \cap C)'$$