SOLVING SYSTEM OF LINEAR EQUATIONS :-

→ matrix notation
→ Basic operations on the matrix / elements of matrix (addition / subtraction / multiplying by a scalar)
→ matrix multiplication → code

$$C = AB$$

$$\Rightarrow c_{ij} = \sum_{x=1}^{n} a_{ix} b_{xj}$$

columns in A = # columns in B = n

$$\therefore A_{n \times m} B_{m \times l} \longrightarrow C_{n \times l}$$

→ Inverse of a matrix (later classes)
→ Representing linear equations in matrix form.

$$\left.\begin{cases} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2 \\ \quad\vdots \qquad\qquad \vdots \qquad\qquad\qquad \vdots \qquad\quad \vdots \\ a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = b_n \end{cases}\right\} \text{ --- } (*)$$

$$\Rightarrow Ax = B$$

where, $A \longrightarrow n \times n$ matrix (coefficient matrix)

$x \longrightarrow n \times 1 \longrightarrow$ unknowns

$b \longrightarrow n \times 1 \longrightarrow$ constants

$$\therefore A^{-1}(Ax) = A^{-1}b \quad \Longrightarrow \quad \boxed{x = A^{-1}b}$$

In ⊛, multiply eq^n (i) by $\frac{a_{21}}{a_{11}}$, i.e.,

$$a_{21}x_1 + \frac{a_{21}a_{12}}{a_{11}}x_2 + \dots + \frac{a_{21}a_{1n}}{a_{11}}x_n = \frac{a_{21}b_1}{a_{11}} \quad -(a)$$

∴ Doing (a) − eq^n (ii) of ⊛, gives,

$$\underbrace{\left(a_{22} - \frac{a_{21}a_{12}}{a_{11}}\right)}_{a_{22}'}x_2 + \dots + \underbrace{\left(a_{2n} - \frac{a_{21}a_{1n}}{a_{11}}\right)}_{a_{2n}'}x_n = \underbrace{b_2 - \frac{a_{21}b_1}{a_{11}}}_{b_2'}$$

∴ Follow similarily ∀ eq^n & get,

$$\begin{aligned}
a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n &= b_1 \\
a_{22}'x_2 + \dots + a_{2n}x_n &= b_2' \\
\vdots \qquad\qquad \vdots \qquad &\quad \vdots \\
a_{n2}'x_2 + \dots + a_{nn}x_n &= b_n'
\end{aligned}$$

∴ Finally, we get,

$$\begin{aligned}
a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \dots + a_{1n}x_n &= b_1 \\
a_{22}'x_2 + a_{23}'x_3 + \dots + a_{2n}'x_n &= b_2' \\
a_{33}''x_3 + \dots + a_{3n}''x_n &= b_3'' \\
\ddots \qquad \vdots \qquad &\quad \vdots \\
a_{nn}^{(n-1)}x_n &= b_n^{(n-1)}
\end{aligned}$$

$$\therefore \boxed{x_n = \frac{b_n^{(n-1)}}{a_{nn}^{(n-1)}}}$$

using x_n, find x_{n-1} from second last eq^n & so on till the first eq^n.

$\therefore$ In general, $\boxed{x_i = \dfrac{b_i^{(i-1)} - \sum_{j=i+1}^{n} a_{ij}^{(i-1)} x_j}{a_{ii}^{(i-1)}}}$

for $i = n-1, \ldots, 2, 1$

PSEUDOCODE :-

a) Forward elimination :-

```
DO FOR  K=1 to n-1                          ] k → pivot eqn
    DO FOR  i= K+1 to n
        factor = aik / akk
        DO FOR  j = K+1 to n
For           aij = aij - factor . akj      ] j → current column
Row     END FOR
        bi = bi - factor . bk
    ENDFOR
END FOR
```

(b) Backward substitution :- (more efficient)

```
xn = bn / ann
DOFOR  i = n-1 to 1
    sum = bi
    DOFOR  j = i+1 to n
        sum = sum - aij xj
    ENDFOR
    xi = sum / aii
```

END FOR

FORWARD

$$\frac{n^3}{3} + O(n) \Rightarrow \text{addition / substraction}$$

$$\frac{n^3}{3} + O(n^2) \Rightarrow \text{multiplication}$$

$$\frac{2n^3}{3} + O(n^2) \Rightarrow \text{operations}$$

BACKWARD

$$n^2 + O(n) \Rightarrow \text{operations}$$

Total: $\frac{2n^3}{3} + O(n^2) \Rightarrow$ operations

→ Elimination dominates the running time.