MTL122 - Real and complex analysis

Assignment-6

Department of Mathematics
Indian Institute of Technology Delhi

Question 1

Find the argument for each of the following complex numbers.

(a) $-3 + i3$

(b) $(1 - i)(-\sqrt{3} + i)$

(c) $\dfrac{-1 + i\sqrt{3}}{2 + i2}$

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- Write $(1-i)(-\sqrt{3}+i) = r_1 e^{i\theta_1} r_2 e^{i\theta_2} = r_1 r_2 e^{i(\theta_1+\theta_2)}$.

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- Hence $(1-i)(-\sqrt{3}+i) = 2\sqrt{2}e^{i\frac{5\pi}{12}}$.

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If z is a complex number, written in polar form as $z = r(\cos x + i \sin x)$. then the n-th roots of z are given by $r^{\frac{1}{n}} \left(\cos \frac{x+2\pi k}{n} + i \sin \frac{x+2\pi k}{n}\right)$ where k varies over the integer values from 0 to $n-1$.

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- We know $z^5 = 1 = \cos(0) + i \sin(0)$.
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- So $z = 1, e^{\frac{2\pi i}{5}}, e^{\frac{4\pi i}{5}}, e^{\frac{6\pi i}{5}}, e^{\frac{8\pi i}{5}}$.

Question 3

Show that for each of the following functions, Cauchy-Riemann equations are satisfied at the origin. Also determine whether these functions are diferentiable at $z = 0$. Are these functions analytic at $z = 0$?

(i) $f(z) = \sqrt{|Re(z)Im(z)|}$,

(ii) $f(z) = xy^2 + iyx^2$.

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- So f is not differentiable at $z = 0$, and hence f is not analytic at $z = 0$.

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- Let $f(z)$ be a real valued function of complex variable.
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- Since f is real valued , thus $f(x, y) = u(x, y) + iv(x, y)$ where $v(x, y) \equiv 0$.

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is continuous at $z = 0$, first order partial derivatives of its real and imaginary part exist at $z = 0$, but $f(z)$ is not differentiable at $z = 0$.

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Question 5

Show that the function

$$f(z) = \begin{cases} \dfrac{z^5}{|z|^4} \text{ if } z \neq 0 \\ 0 \text{ if } z = 0 \end{cases}$$

is continuous at $z = 0$, first order partial derivatives of its real and imaginary part exist at $z = 0$, but $f(z)$ is not differentiable at $z = 0$.

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Question 6

Is there an analytic function $f(z) = u(x, y) + iv(x, y)$, where $z = x + iy$, defined on some open subset of $\mathbb{C}$ with $u = x^3 - 3xy^2 - 2x^2 + 2y^2 + 1$? If so, find all such $f(z)$.

Solution

- We have $u = x^3 - 3xy^2 - 2x^2 + 2y^2 + 1$, this gives $u_{xx} = 6x - 4$ and $u_{yy} = -6x + 4$.

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- $u_{xx} + u_{yy} = 0$, this implies u is harmonic.
- Now we will find out $v(x,y)$ of an analytic function whose real part u.
- Since f is analytic, it must satisfy CR-equations, i.e $u_x = v_y$ and $u_y = -v_x$.
- Consider $v_y = u_x = 3x^2 - 3y^2 - 4x$, on integrating both sides w.r.t y we get.

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- Now either $\cos(x) = 0$ or $e^{y} = e^{-y} = 1$. If the second case is true then $sin(x) = 2$ which is impossible. Thus $cos(x) = 0$. Thus $x = (2n+1)\frac{\pi}{2}$

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- Since $sin(x)\left(\frac{e^{-y}+e^{y}}{2}\right) = 2$,we have sin(x) cannot be -1. Thus $x = (4n+1)\frac{\pi}{2}$

Question 8 Contd...

- Now since $e^y + e^{-y} = 4$, solving quadratic equation we have $y = \pm ln(2 + \sqrt{3})$.
- Thus $z = (4n+1)\frac{\pi}{2} \pm i \ln(2 + \sqrt{3})$

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- Now if the Cauchy Riemann equations are to hold then we can see that it will hold only on isolated points namely $z = (2n + 1)\frac{\pi}{2}$
- Thus f(z) is not analytic anywhere since cauchy riemann equations do not hold in any open set.