DEPARTMENT OF MATHEMATICS
INDIAN INSTITUTE OF TECHNOLOGY DELHI
RE-MINOR TEST DUE TO MEDICAL REASONS 2022-2023 SECOND SEMESTER
MTL107 (NUMERICAL METHODS AND COMPUTATION)

Time: 1 hour **Max. Marks: 25**

** Answer to each question should begin on a new page **

** All notations are standard. Exhibit clearly all the steps to deserve full credit. **

1. Assume $g(x)$ and $g'(x)$ are continuous for $a \le x \le b$, and assume g satisfies the property $a \le x \le b \Longrightarrow a \le g(x) \le b$. Further assume that $\lambda \equiv$ Maximum $(|\, g'(x)\,|) < 1$ for all x in $[a, b]$. Then prove or disprove the following:
(i) There is a unique solution α of $x = g(x)$ in the interval $[a, b]$.
(ii) For any initial approximation x_0 in $[a, b]$, the iterates x_n generated by $x_{n+1} = g(x_n)$ satisfy

$$|\, \alpha - x_n \,| \le \frac{\lambda^n}{1-\lambda} \,|\, x_0 - x_1 \,| \ , \quad n \ge 0.$$

(4)

2a. Find the Hermite interpolating polynomial that fits the data

x	$f(x)$	$f'(x)$
0.5	4	-16
1	1	-12

(4)

2b. Approximate $f(x) = \sqrt[3]{x}$ by a straight line in the interval [0,1], in the least square sense with the weight function $w(x) = 1$. Also find the norm of the error function for the best approximation.

(4)

3. For the method

$$f'(x_0) = \frac{-3f(x_0) + 4f(x_1) - f(x_2)}{2h} + \frac{h^2}{3} f'''(\xi), \quad x_0 < \xi < x_2$$

determine the optimum value of h, using the criteria $|\, RE \,| = |\, TE \,|$. Using this method and the value of h obtained from the criteria $|\, RE \,| = |\, TE \,|$, determine an approximate value of $f'(2.0)$ from the following tabulated values of $f(x) = \log x$

x	2.0	2.01	2.02	2.06	2.12
f(x)	0.69315	0.69813	0.70310	0.72271	0.75142

given that the maximum roundoff error in function evaluation is 5×10^{-6}.

(4)

4. Compute

$$I = \int_0^1 e^{2x}\, dx$$

by Romberg integration method correct to three decimal places, using Trapezoidal rule. (4)

5. Using Gauss-Jordan method, find the inverse of

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

and hence solve the system $Ax = \begin{bmatrix} 0 \\ -2 \\ 3 \end{bmatrix}$. (3)

6. Using Euler's method, find the solution of the initil vlue problem $y' = 3x + \frac{1}{2}y$, $y(0) = 1$ at $x = 0.1$ and $x = 0.2$ (2)

LU L $A^{-1} = (LL^T)^{-1}$

$A^{-1} = (L^T)^{-1}(L)$

MTL107: NUMERICAL METHODS AND COMPUTATION
MINOR: 2

Total Marks: 20 **Time: One Hour**

1 (5 Marks) Find $PA = LU$ factorization of the following matrix using partial pivoting:

$$A = \begin{bmatrix} 2 & 1 & 5 \\ 3 & 3 & -3 \\ 2 & 6 & 2 \end{bmatrix}$$

where P is permutation matrix, L is lower triangular matrix with diagonal entries 1 and U is upper triangular matrix.

2. (5 Marks) Prove that the following matrix is symmetric positive definite.

$$A = \begin{bmatrix} 9 & 6 & 3 \\ 6 & 8 & 6 \\ 3 & 6 & 14 \end{bmatrix}$$

Then determine its Cholesky Factorization.

3. (5 Marks) Consider the linear system of equations,

$$\begin{bmatrix} 3 & 1 & -1 \\ 2 & 4 & 1 \\ -1 & 2 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 1 \end{bmatrix}$$

Use zero vector as initial guess and calculate two iterations of Jacobi and two iterations with Gauss-Seidel Method.

4. (5 Marks) Consider a function f witch take values, $(0, 3)$, $(1, -2)$, and $(2, 1)$. Use newton divided difference to find interpolating polynomial for the function. Furthermore, if f is smooth and $|f^{(3)}(x)| < K \ \forall x$, then estimate the maximum error.

DEPARTMENT OF MATHEMATICS

INDIAN INSTITUTE OF TECHNOLOGY DELHI

MINOR TEST II 2015-2016 FIRST SEMESTER

MTL 107/MAL 230 (NUMERICAL METHODS AND COMPUTATION)

Time: 1 hour

Max. Marks: 25

**** Answer to each question should begin on a new page ****

1a. The interpolating polynomial for the function $f(x)$ on the set of distinct points $x_0, x_1, \ldots\ldots, x_n$ is given as $P_n(x) = \sum_{k=0}^{n} l_k(x) f(x_k)$. Find an explicit expression for $\sum_{k=0}^{n} l_k(0) x_k^{n+1}$. (2)

1b. A function $f(x)$ is defined on [0,1] and $| f^{(m)}(x) | \leq m!$ for $m = 1, 2, \ldots\ldots$. Let $P_n(x)$ be the interpolating polynomial of $f(x)$ at the points $1, q, q^2, \ldots\ldots, q^n$ where $0 < q < 1$. Then prove or disprove that $\lim_{n \to \infty} P_n(0) = f(0)$. (3)

2a. Suppose $f^* = \sum_{j=0}^{j=n} c_j^* \Phi_j$ be the least squares approximation to a given function f. Then prove or disprove $\|f - f^*\|_2^2 = \|f\|_2^2 - \|f^*\|_2^2$. (2)

2b. Approximate $f(x) = \sqrt[3]{x}$ by a straight line in the interval [0,1], in the least square sense with the weight function $w(x) = 1$. Also find the norm of the error function for the best approximation. (4)

2c. Find a polynomial of second degree which is the best approximation in maximum norm to $\sqrt{x}$ on the point set $\{0, \frac{1}{9}, \frac{4}{9}, 1\}$. (3)

3. For the method

$$f'(x_0) = \frac{-3f(x_0) + 4f(x_1) - f(x_2)}{2h} + \frac{h^2}{3} f'''(\xi), \quad x_0 < \xi < x_2$$

determine the optimum value of h, using the criteria $| RE | = | TE |$. Using this method and the value of h obtained from the criteria $| RE | = | TE |$, determine an approximate value of $f'(2.0)$ from the following tabulated values of $f(x) = \log x$

x	2.0	2.01	2.02	2.06	2.12
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4a. Find the number of subintervals n and the step size h so that the error for the composite Simpson's $\frac{1}{3}$ rd rule is less than 5×10^{-9} for the approximation $\int_2^7 \frac{dx}{x}$. (3)

4b. Compute

$$I = \int_0^1 e^{2x}\, dx$$

by Romberg integration method correct to three decimal places, using Trapezoidal rule. (4)

0.33 0.66 1

$e^{.5}$ $e^{1.5}$ e^2