Assignment 7		April 2021
	Real and Complex Analysis	
	MTL122/ MTL503/ MTL506	
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(1) Verify that the following functions u are harmonic, and in each case give a conjugate harmonic function v.
  (a) $u(x,y) = 3x^2y + 2x^2 - y^3 - 2y^2$,
  (b) $u(x,y) = \ln(x^2+y^2)$.

(2) Find the contour integral $\int_\gamma \bar{z} dz$ for
  (a) γ is the triangle ABC oriented counterclockwise, where $A = 0, B = 1+i$ and $C = -2$;
  (b) γ is the circle $|z-i| = 2$ oriented clockwise.

(3) Evaluate the following integrals.
  (a) $\int_C \frac{2dz}{z^2-1}$, where C is the circle with radius $1/2$ centre 1 , positively oriented.
  (b) $\int_C \left(e^z + \frac{1}{z}\right) dz$, where C is the lower half of the circle with radius 1 . centre 0 clockwise oriented.
  (c) $\int_C ze^z dz$, where C is any contour.
  (d) $\int_C \cosh z dz$, where C is any contour.

(4) Let C_R be the circle with radius R, centre 0 , counterclockwise. Show that

$$\lim_{R\to\infty} \int_{C_R} \frac{z^2+4z+7}{(z^2+4)(z^2+2z+2)} dz = 0$$

Use this fact to prove that

$$\int_C \frac{z^2+4z+7}{(z^2+4)(z^2+2z+2)} dz = 0$$

where C is a circle with radius 5 , centre 2, positively oriented.

(5) Find the value of the integral $g(z)$ around the circle $|z-i| = 2$ oriented counterclockwise when
  (a) $g(z) = \frac{1}{z^2+4}$
  (b) $g(z) = \frac{1}{z(z^2+4)}$.

(6) Let C_R be the circle $|z| = R(R > 1)$ oriented counterclockwise. Show that

$$\left| \int_{C_R} \frac{\log(z^2)}{z^2} dz \right| < 4\pi \left(\frac{\pi + \ln R}{R} \right)$$

and then

$$\lim_{R\to\infty} \int_{C_R} \frac{\log(z^2)}{z^2} dz = 0$$

(7) Without evaluating the integral, show that

$$\left| \int_C \frac{dz}{\bar{z}^2 + \bar{z} + 1} \right| \le \frac{9\pi}{16}$$

where C is the arc of the circle $|z| = 3$ from $z = 3$ to $z = 3i$ lying in the first quadrant.

(8) Find where

$$\arctan(z) = \frac{i}{2} \log \frac{i+z}{i-z}$$

is analytic?