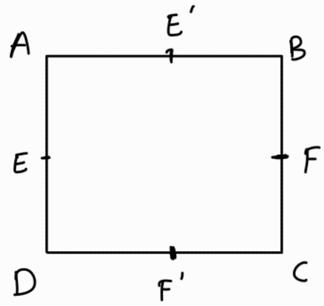
Symmetries

Rotation about O (the centre) by an angle an integer multiple of $90^\circ = \frac{\pi}{2}$

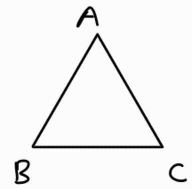
Reflection about diagonals.

Reflection about EF or E'F'.



In checking the identity there are eight symmetries for a square.

For an equilateral triangle rotation by an integer multiple of $\frac{2\pi}{3} = 120^\circ$, through 3 distinct symmetries.



Reflection about medians, there are another 3 symmetries.

Isoceles triangle - 2 symmetries

Regular pentagon:

Rotation about the centre by an integer multiple of 72°

There are 5 lines of symmetries, reflection about them are symmetric.

There are $5 \times 2 = 10$ lines of symmetries.

For a regular n-gon, there are 2n symmetries.

1. Rotation by an integer multiple of $2\pi/n$.
2. There are n-lines of symmetry

If n is odd, for every vertex there is a unique opposite side, a line of symmetry is obtained by joining the vertex to the mid-point of the opposite side.

If n is even, for every vertex there is an opposite vertex, the line joining them is a line of symmetry and for every side there is an opposite side, the line joining their mid-points are lines of symmetry.

Observe that combination of any two symmetries is again a symmetry. For any regular n-gon the symmetries form a group, under composition of symmetries.

We denote this group by D_n. Then the number of elements of D_n is $2n$.

Suppose we are given a letter say 'a' then our alphabet is $\{a, a^{-1}\}$

Then words are $\{a^n : n \in \mathbb{Z}\}$

Here $aa^{-1} = 1$ empty word

$a^m a^{-n} = a^{m-n}$

Suppose two letters are given say a, b then alphabet is $\{a, b, a^{-1}, b^{-1}\}$

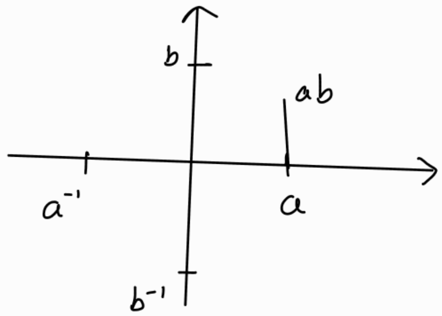
These words form a group under concatenation

If W_1 and W_2 are two words then $w_1 w_2$ and $W_2 W_1$ are also words.

Eg: $W_1 = ab^{-1}$ and $W_2 = ba^2$. Then $W_1 W_2 = a^3$, $W_2 W_1 = ba^3 b^{-1}$

Two words W_1 and W_2 are same if we get the same word W by cancellation on W_1 & W_2.

More generally words obtained from $\{a_1, a_1^{-1}, a_2, \dots, a_n, a_n^{-1}\}$ form a group. Remember the operation and when two are same.



The group of words obtained using n letters (and their inverses) is called the free group on n letters. Denote it by $F(S)$ where $S = \{a_1, a_2, \dots, a_n\}$ & $\{a^{-1}, a_2^{-1}, \dots, a_n^{-1}\}$ is the alphabet.

What is the inverse of ab?

$(ab)^{-1} = b^{-1}a^{-1}$

Because $(ab)(b^{-1}a^{-1}) = abb^{-1}a^{-1} = a1a^{-1} = aa^{-1} = 1$

& $||^{r}ly$ $(b^{-1}a^{-1})(ab) = 1$

Order of an element in a group

D_3 if x = rotation by angle $120°$ about the centre of the equilateral triangle

Then, $xxx = 1$

$\Rightarrow x^3 = 1$

y = rotation by angle $240°$ $y^3 = 1$

The order of any re