INDIAN INSTITUTE OF TECHNOLOGY DELHI
DEPARTMENT OF MATHEMATICS
SEMESTER II, 2020−21
MTL 122 (REAL AND COMPLEX ANALYSIS)
Minor Examination

DATE: 19/03/2021   Total Marks: 20   Time: 3.00 − 4.30 pm

MARKS WILL BE AWARDED ONLY FOR THOSE ANSWERS WITH PROPER JUSTIFICATION

Question 1: Let (X, e) be a metric space. Suppose $f : (\mathbb{R}, d_{\text{euc}}) \to (X, e)$ be a continuous, bijective function such that f^{-1} is uniformly continuous. Is (X, e) a complete metric space? Prove or disprove your claim. [5]

Question 2: Let S be an open subset of $\mathbb{R}^3$, where $(\mathbb{R}^3, d_{\text{euc}})$ is a metric space. Then every point of S is a limit point of S. Is this true/false? Justify. Also is this true for general metric space? Give justifications.
What happens when S is closed? Prove or disprove your claim. [3+2+2]

Question 3: Suppose $A \subset \mathbb{R}$. For every non empty bounded subset $B \subset A$, $\sup B$ and $\inf B$ are in A. Show that A is closed with respect to the Euclidean metric. [4]

Question 4: Let
$$f(x) = \begin{cases} \sin(x) & \text{if } x \in [0, \pi] \cap \mathbb{Q} \\ 0 & \text{if } x \in [0, \pi] \setminus \mathbb{Q}. \end{cases}$$

Discuss the continuity of the function f on $[0, \pi]$. [4]

——ALL THE BEST——