DEPARTMENT OF MATHEMATICS
INDIAN INSTITUTE OF TECHNOLOGY DELHI
MINOR-I TEST 2022-2023 SECOND SEMESTER
MTL107 (NUMERICAL METHODS AND COMPUTATION)

Time: 1 hour **Max. Marks: 25**

** Answer to each question should begin on a new page **
** All notations are standard. Exhibit clearly all the steps to deserve full credit. **

1a. How many normalized numbers are there in the floating point system defined by

$$y = \pm\ .d_1 d_2 \ldots\ldots d_t \times \beta^e,$$

where each digit d_i satisfies $0 \le d_i \le \beta - 1$, $d_1 \neq 0$ and $e_{\min} \le e \le e_{\max}$? (3)

1b. If $\beta = 10, t = 4$ floating point arithmetic is used, what will be the unit roundoff for rounded arithmetic on machine. Also, find $fl(fl(1 + 0.0004))$. (2)

2a. Write whether the following statements are TRUE or FALSE. No justification is required. Each question carries $\frac{1}{2}$ mark.

In **IEEE** arithmetic, assume that a and b are normalized floating point numbers. Then
(i) $fl(a - b) = -fl(b - a)$
(ii) $fl(a + a) = fl(2 * a)$
(iii) $fl(0.5a) = fl(a/2)$
(iv) $fl((a + b) + c) = fl(a + (b + c))$ (2)

2b. If x, y and z are numbers in F_s (IEEE single precision floating point system) expressed in their binary form, what upper bound can be given for the relative roundoff error in computing $fl(z \times fl(x + y))$, with rounding to the closest. (3)

3a. If f is such that $|f''(x)| \le 3$ for all x and $|f'(x)| \ge 1$ for all x, and if the initial error in the Newton-Raphson method is less than $\frac{1}{2}$, what is the upper bound on the error at each of the first three steps? (4)

3b. For the following nonlinear system $4x_1^2 + 9x_2^2 - 36 = 0,\quad 16x_1^2 - 9x_2^2 - 36 = 0$ consider the fixed point method

$$x_1 = \phi_1(x) = \frac{1}{4}\sqrt{36 + 9x_2^2},$$

$$x_2 = \phi_2(x) = \frac{1}{3}\sqrt{36 - 4x_1^2}.$$

Does it converge to the root $(1.8974, 1.5492)^T$ in some region around $(1.8974, 1.5492)$ with initial approximation $(1, 1)^T$? Justify your answer. (3)

P.T.O.

4. Assume $g(x)$ and $g'(x)$ are continuous for $a \leq x \leq b$, and assume g satisfies the property $a \leq x \leq b \Longrightarrow a \leq g(x) \leq b$. Further assume that $\lambda \equiv$ Maximum $(| g'(x) |) < 1$ for all x in $[a, b]$. Then prove or disprove the following:

(i) There is a unique solution α of $x = g(x)$ in the interval $[a, b]$.

(ii) For any initial approximation x_0 in $[a, b]$, the iterates x_n generated by $x_{n+1} = g(x_n)$ satisfy

$$| \alpha - x_n | \leq \frac{\lambda^n}{1-\lambda} | x_0 - x_1 | \ , \quad n \geq 0.$$

(4)

5. The number of positive real roots of polynomial equation $P_n(x) = 0$ can not exceed the number of sign changes in $P_n(x)$ and the number of negative real roots of $P_n(x) = 0$ can not exceed the number of sign changes in $P_n(-x)$. Find the interval of unit length in which the smallest root of the equation $x^5 - x + 1 = 0$ lies. Taking midpoint of that interval as initial approximation perform one iteration of the second order Birge-Vieta method (Do calculations with four decimal places). (4)

DEPARTMENT OF MATHEMATICS

INDIAN INSTITUTE OF TECHNOLOGY DELHI

MINOR TEST 2020-2021 FIRST SEMESTER

MTL107 (NUMERICAL METHODS AND COMPUTATION)

Time: 1 hour 10 minutes **Max. Marks: 30**

** **Answer to each question should begin on a new page** **

1a. How many normalized numbers are there in the floating point system defined by

$$y = \pm\ .d_1 d_2 \ldots\ldots d_t \ \times\ \beta^e,$$

where each digit d_i satisfies $0 \le d_i \le \beta - 1$, $d_1 \ne 0$ and $e_{\min} \le e \le e_{\max}$? (2)

1b. If $\beta = 10, t = 4$ floating point arithmetic is used, what will be the unit roundoff for rounded arithmetic on machine. Also, find $fl(fl(1 + 0.0004))$. (2)

2a. If f is such that $\mid f''(x) \mid \le 3$ for all x and $\mid f'(x) \mid \ge 1$ for all x, and if the initial error in the Newton-Raphson method is less than $\frac{1}{2}$, what is the upper bound on the error at each of the first three steps? (3)

2b. For the following nonlinear system $4x_1^2 + 9x_2^2 - 36 = 0, \quad 16x_1^2 - 9x_2^2 - 36 = 0$ consider the fixed point method

$$x_1 = \phi_1(x) = \frac{1}{4}\sqrt{36 + 9x_2^2},$$
$$x_2 = \phi_2(x) = \frac{1}{3}\sqrt{36 - 4x_1^2}.$$

Does it converge to the root $(1.8974, 1.5492)^T$ in some region around $(1.8974, 1.5492)$ with initial approximation $(1, 1)^T$?. Justify your answer. (3)

2c. The number of positive real roots of polynomial equation $P_n(x) = 0$ can not exceed the number of sign changes in $P_n(x)$ and the number of negative real roots of $P_n(x) = 0$ can not exceed the number of sign changes in $P_n(-x)$. Find the interval of unit length in which the smallest root of the equation $x^5 - x + 1 = 0$ lies. Taking midpoint of that interval as initial approximation perform one iteration of the second order Birge-Vieta method (Do calculations with four decimal places). (4)

3a. Consider the problem of finding a quadratic polynomial $p(x)$ for which $p(x_0) = y_0, \ \ p'(x_1) = y_1', \ \ p(x_2) = y_2$ with $x_0 \ne x_2$ and $\{y_0, y_1', y_2\}$ the given data. Assuming that the nodes x_0, x_1, x_2 are real, what conditions must be satisfied for such a $p(x)$ to exist and be unique?. (3)

3b. Let $x_0, x_1, \ldots\ldots x_n$ be $n + 1$ distinct points in $[a, b]$. Let $f_0, f_1 \ldots\ldots f_n$ be the values of $f(x)$ at these points. Then prove or disprove that for $k \ge 0$

$$f[x_n, \ldots\ldots, x_k] = \frac{1}{(n-k)!h^{n-k}} \nabla^{n-k} f_n.$$

(3)

3c. Let $f \in C^4[a,b]$. Let $x = a$ and $x = b$ be the nodes and $H(x)$ be Hermite interpolating polynomial of f . Then prove or disprove

$$\|f - H\|_\infty \leq \frac{(b-a)^4}{384}\|f^{(4)}\|_\infty.$$

(3)

4. Define

$$s(x) = \begin{cases} -\frac{11}{2}x^3 + 26x^2 - \frac{75}{2}x + 18, & 1 \leq x \leq 2 \\ \\ \frac{11}{2}x^3 - 40x^2 + \frac{189}{2}x - 70, & 2 \leq x \leq 3 \end{cases}$$

Examine whether $s(x)$ is a cubic spline function or not on [1,3]?. Is $s(x)$ a natural cubic spline function on [1,3]? Justify your answer. (3)

5a. Suppose $f^* = \sum_{j=0}^{j=n} c_j^* \phi_j$ be the least squares approximation to a given function f. Then TRUE or FALSE, justify the statement

$$\|f - f^*\|_2^2 = \|f\|_2^2 - \|f^*\|_2^2.$$

(2)

5b. Find the least squares straight line fit for $y = x\sqrt{1-x^2}$ on $[-1,1]$ using Chebyshev polynomials. (2)

Department of Mathematics

MTL107:Numerical Methods and Computations

Minor-1,

Max Marks: 25 Max Time: 1 Hour

Answer ALL questions

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1. State and prove the theorems for convergence of Bisection and Fixed Point Iteration algorithms. [3+3 Marks]

2. Using False position method compute two iterations $x^{(2)}$ and $x^{(3)}$ of root of $\cos(x-1)+\ln(x-1)$ on $[1.3, 2]$ given $x^{(0)} = 1.3$ and $x^{(1)} = 2$. [3 Marks]

3. Using Newton's method compute first iteration $x^{(1)}$ of solution of the nonlinear system

$$x_1(1-x_1)+4x_2 = 12$$

$$(x_1-2)^2+(2x_2-3)^2 = 25$$

given $x^{(0)} = (0 \quad 0)^t$. [4 Marks]

4. Using Jacobi and Gauss-Seidel methods compute first iteration $x^{(1)}$ of solution of the linear system

$$4x_1 + x_2 - x_3 = 5$$

$$-x_1 + 3x_2 + x_3 = -4$$

$$2x_1 + 2x_2 + 5x_3 = 1$$

given $x^{(0)} = (1 \quad 2 \quad -1)^t$. [4 Marks]

5. Using Conjugate-Gradient method compute first two iterations $x^{(1)}$ and $x^{(2)}$ of solution of the linear system

$$2x_1 - x_2 = 3$$

$$-x_1 + 2x_2 - x_3 = -4$$

$$-x_2 + 2x_3 = 3$$

given $x^{(0)} = (0 \quad 0 \quad 0)^t$. [8 Marks]

DEPARTMENT OF MATHEMATICS

INDIAN INSTITUTE OF TECHNOLOGY DELHI
MINOR TEST I 2015-2016 FIRST SEMESTER
MTL 107/MAL 230 (NUMERICAL METHODS AND COMPUTATION)

Time: 1 hour Max. Marks: 25

**** Answer to each question should begin on a new page ****

1a. Let $a = 1 \times 10^{308}$, $b = 1.01 \times 10^{308}$ and $c = -1.001 \times 10^{308}$ be three floating point numbers in F_D (IEEE double precision floating point system) expressed in their decimal form. Then find the values of $fl(a + fl(b + c))$ and $fl(fl(a + b) + c)$. (2)

1b. If x, y and z are numbers in F_s (IEEE single precision floating point system) expressed in their binary form, what upper bound can be given for the relative roundoff error in computing $fl(z \times fl(x + y))$, with rounding to the closest. (3)

2a. Determine the number of iterations required by **bisection method** to find the zero of $f(x) = x^3 - x^2 - 1$ on $[1, 2]$ with an absolute error of no more than 10^{-6}. (2)

2b. Consider a function f which satisfies the properties:
(i) There exists a unique root $\xi \in [0, 1]$;
(ii) For all real x we have $f'(x) \geq 2$ and $0 \leq f''(x) \leq 3$.
With initial approximation $x_0 = \frac{1}{2}$, how many iterations are required to get 10^{-6} accuracy by Newton-Raphson method? (4)

3. Using Sturm sequence, find the exact number of real roots of the equation

$$x^3 - 11x^2 + 32x - 22 = 0$$

lying in the interval $(3, 7)$. Perform one iteration of Newton-Raphson method to find the largest root of the above equation. (4)

4a. Assume $g(x)$ and $g'(x)$ are continuous for $a \leq x \leq b$, and assume g satisfies the property $a \leq x \leq b \Longrightarrow a \leq g(x) \leq b$. Further assume that $\lambda \equiv$ Maximum $(|\ g'(x)\ |) < 1$ for all x in $[a, b]$. Then prove or disprove the following:
(i) There is a unique solution α of $x = g(x)$ in the interval $[a, b]$.
(ii) For any initial approximation x_0 in $[a, b]$, the iterates x_n generated by $x_{n+1} = g(x_n)$ satisfy

$$|\ \alpha - x_n\ | \leq \frac{\lambda^n}{1 - \lambda}\ |\ x_0 - x_1\ |\ , \quad n \geq 0.$$

(4)

4b. For the following nonlinear system $4x_1^2 + 9x_2^2 - 36 = 0$, $\quad 16x_1^2 - 9x_2^2 - 36 = 0$ consider the fixed point method

$$x_1 = \phi_1(x) = \frac{1}{4}\sqrt{36 + 9x_2^2},$$

$$x_2 = \phi_2(x) = \frac{1}{3}\sqrt{36 - 4x_1^2}.$$

P.T.O.

Does it converge to the root $(1.8974, 1.5492)^T$ in some region around $(1.8974, 1.5492)$ with initial approximation $(1,1)^T$?. Justify your answer.

(3)

4c. Let $x_0, x_1, \ldots\ldots x_n$ be $n+1$ distinct points in $[a,b]$. Let $f_0, f_1 \ldots\ldots f_n$ be the values of $f(x)$ at these points. Then prove or disprove that for $k \geq 0$

$$f[x_n, \ldots\ldots, x_k] = \frac{1}{(n-k)!h^{n-k}} \nabla^{n-k} f_n.$$

(3)

DEPARTMENT OF MATHEMATICS

INDIAN INSTITUTE OF TECHNOLOGY DELHI

RE-MINOR TEST DUE TO MEDICAL REASONS 2022-2023 SECOND SEMESTER

MTL107 (NUMERICAL METHODS AND COMPUTATION)

Time: 1 hour **Max. Marks: 25**

** Answer to each question should begin on a new page **

** All notations are standard. Exhibit clearly all the steps to deserve full credit. **

1. Assume $g(x)$ and $g'(x)$ are continuous for $a \le x \le b$, and assume g satisfies the property $a \le x \le b \Longrightarrow a \le g(x) \le b$. Further assume that $\lambda \equiv$ Maximum $(\mid g'(x) \mid) < 1$ for all x in $[a, b]$. Then prove or disprove the following:

(i) There is a unique solution α of $x = g(x)$ in the interval $[a, b]$.

(ii) For any initial approximation x_0 in $[a, b]$, the iterates x_n generated by $x_{n+1} = g(x_n)$ satisfy

$$\mid \alpha - x_n \mid \le \frac{\lambda^n}{1-\lambda} \mid x_0 - x_1 \mid , \quad n \ge 0.$$

(4)

2a. Find the Hermite interpolating polynomial that fits the data

x	$f(x)$	$f'(x)$
0.5	4	−16
1	1	−12

(4)

2b. Approximate $f(x) = \sqrt[3]{x}$ by a straight line in the interval [0,1], in the least square sense with the weight function $w(x) = 1$. Also find the norm of the error function for the best approximation.

(4)

3. For the method

$$f'(x_0) = \frac{-3f(x_0) + 4f(x_1) - f(x_2)}{2h} + \frac{h^2}{3} f'''(\xi), \quad x_0 < \xi < x_2$$

determine the optimum value of h, using the criteria $\mid RE \mid = \mid TE \mid$. Using this method and the value of h obtained from the criteria $\mid RE \mid = \mid TE \mid$, determine an approximate value of $f'(2.0)$ from the following tabulated values of $f(x) = \log x$

x	2.0	2.01	2.02	2.06	2.12
f(x)	0.69315	0.69813	0.70310	0.72271	0.75142

given that the maximum roundoff error in function evaluation is 5×10^{-6}.

(4)

4. Compute

$$I = \int_0^1 e^{2x}\, dx$$

by Romberg integration method correct to three decimal places, using Trapezoidal rule. (4)

5. Using Gauss-Jordan method, find the inverse of

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

and hence solve the system $Ax = \begin{bmatrix} 0 \\ -2 \\ 3 \end{bmatrix}$. (3)

6. Using Euler's method, find the solution of the initil vlue problem $y' = 3x + \frac{1}{2}y,\ y(0) = 1$ at $x = 0.1$ and $x = 0.2$ (2)

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$A^{-1} = (L^T)^{-1}(L)$