MTP290 Spring 2023
Assignment 1

1   Instructions

1. The assignment can be attempted in teams of 2. Only one team member has to submit the folder on Moodle. Discussions should only happen within the team. Your code will be checked for plagiarism.

2. Any descriptive answer should be written at the top of the code. Use '%' to comment inside the code.

3. Make a separate MATLAB file for each problem. Name them $A1Qn.m$, where n is the question number.

4. Submit a zip file with folder name <EntryNo1_EntryNo2>_A1 (e.g: 2019MT10648_2019MT10649_A1.zip containing the folder 2019MT10648_2019MT10649_A1) or <EntryNo1>_A1 (e.g: 2019MT10648_A1.zip containing the folder 2019MT10648_A1) according to your team composition. The folder should contain your MATLAB code files.

5. The problems are worth 2, 3, 3 and 2 marks respectively.

6. **Assignment Deadline: 11.59pm January 27, 2023**

2   Problems

1. Use the false point method to find the root of $f(x) = (x-4)^2(x+2) = 0$, using the initial guesses of $x_L = -2.5$ and $x_U = -1.0$, and a pre-specified tolerance of 0.1%

2. Use Newton's Method for the following:
   (a) Approximate $x = \frac{1}{a}$ for any given $a(\neq 0)$ **without using division**.
   (b) The sum of two real numbers is 20. If each number is added to its square root, the product of the two sums is 155.55. Determine the two numbers to within 10^{-8}.

3. **Recall** : Newton's method converges quadratically.
   If a sequence x_k converges to x_∞, denote $e_k = |x_\infty - x_k|$,
   The sequence converges quadratically if $\lim_{k \to \infty} \frac{e_{k+1}}{e_k^2} = c, c > 0$

   Let $f(x) = 27x^4 + 162x^3 - 180x^2 + 62x - 7$
   (a) Show that $f(x)$ has a zero of multiplicity 3 at $x = 1/3$.
   (b) Use Newton's Method to solve this equation with $x_0 = 0$ within 10^{-8}. Show your iterations.
   Observe and comment on the order of convergence in this case.
   (c) The modified Newton's Method uses the update rule $x_{n+1} = x_n$ - $m[f(x_n)/f'(x_n)]$ where m is the multiplicity of the repeated root.
   Use modified Newton's Method to solve this equation with $x_0 = 0$ within 10^{-8}, show your iterations and comment on the convergence relative to part (b).

4. Write a function that combines the Bisection and Newton's method, call it the Hybrid Method.
   This method uses the following idea:
   Start with the bisection method with an initial interval $[a, b]$ and switch to Newton's method when the length of the current interval in the bisection method becomes less than $s(b - a)$.
   Try the Hybrid Method on $\tanh(x) = 0$ with an initial interval $[-10, 15]$ and $s = 0.1$.
   How does it compare it to using just the regular (a) Newton's Method and (b) Bisection Method?