Assignment 6 April 2021

Real and Complex Analysis

MTL122/MTL503/MTL506

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(1) Find the argument for each of the following complex numbers.
   (a) $-3 + i3$
   (b) $(1 - i)(-\sqrt{3} + i)$
   (c) $\dfrac{-1 + i\sqrt{3}}{2 + i2}$.

(2) Solve the equation $z^5 = 1$ for all complex numbers z.

(3) Show that for each of the following functions Cauchy-Riemann equations are satisfied at the origin. Also determine whether these functions are differentiable at $z = 0$. Are these functions analytic at $z = 0$?
   (a) $f(z) = \sqrt{|\operatorname{Re}(z)\operatorname{Im}(z)|}$
   (b) $f(z) = xy^2 + iyx^2$, where $z = x + iy$.

(4) Show that the derivative of a real valued function $f(z)$ of a complex variable z, at any point, is either zero or it does not exist.

(5) Show that the function

$$f(z) = \begin{cases} \dfrac{z^5}{|z|^4} \text{ if } z \neq 0 \\ 0 \text{ if } z = 0 \end{cases}$$

is continuous at $z = 0$, first order partial derivatives of its real and imaginary part exist at $z = 0$, but $f(z)$ is not differentiable at $z = 0$.

(6) Is there an analytic function $f(z) = u(x, y) + iv(x, y)$, where $z = x + iy$, defined on some open subset of $\mathbb{C}$ with $u = x^3 - 3xy^2 - 2x^2 + 2y^2 + 1$? If so, find all such $f(z)$.

(7) Show that the function $\log(z - i)$ is analytic everywhere except on the half line $y = 1, x \leq 0$.

(8) Solve the equation $\sin z = 2$ for z by equating real and imaginary parts in that equation.

(9) Show that neither $\sin \bar{z}$ nor $\cos \bar{z}$ is an analytic function of z anywhere.