

## MTL732: Financial Mathematics, Practice Sheet 1

1. Guptaaji is planning ahead for his sons education. He is eight now and will start college in 10 years. How much will Guptaaji have to set aside at the end of each year to have Rs 65000 in 10 years if the annual interest rate is 7%?
2. Raj's uncle is going to give him Rs 250 a month for the next two years starting today. If Raj deposits every payment in an account paying an annual interest rate of 6% compounded monthly, how much will he have at the end of three years?
3. A company is considering its options for a machine to use in production. At a cost of 470 the company can make some small repairs on current machine which will make it last for 2 more years. At a higher cost of 900, the company can make more extensive repairs on current machine making it last for 4 more years. A new machine costs 3000 and will last for 8 years. The company is facing an interest rate of 10%. Advice the best action.
4. You plan to buy a mobile that cost Rs 25700. You made a down payment Rs 4000. The remaining cost is financed over a period of 3 years. You will repay the loan by making equal monthly installments. The interest quoted is 8% per annum with monthly compounding. What is your monthly payment plan when the payments are to be made at the end of the month.
5. You are valuing an investment that will pay you Rs 12000 the first year, Rs 14000 the second year, Rs 17000 the third year, Rs 19000 the fourth year, Rs 23000 the fifth year, and Rs 29000 the sixth year (all payments are at the end of each year). What is the value of the investment to you now if the appropriate annual discount rate is 10%?
6. Spot an arbitrage opportunity in the following situation:  
Suppose a dealer A offers to buy British pounds in an year from now at a rate Rs 79 to a pound, while dealer B would sell British pounds immediately at a rate Rs 80 to a pound. Suppose that a rupee can be borrowed at an annual rate of 4% and a British pound can be invested in a bank at 6% annual interest.
7. You are given these exchange rates in a bank (buying(bid)-selling(ask)) data:  
JPY/EUR    139.3449    139.4782  
JPY/AUD    92.5606    92.6883  
AUD/EUR    1.4854    1.4966  
You start with AUD 1000. Is there an arbitrage opportunity? If so, what trades do you make, and what profit will you earn? (Hint: trade AUD to EURO, EURO to JPY, JPY to AUD)
8. Suppose that you are given the following information about the CAD (Canadian dollar) and the GBP:  
The spot rate is CAD/GBP 1.7791 (i.e., CAD 1.7791 = GBP 1.00) The 1-year risk-free GBP rate is 2% The 1-year risk-free CAD rate is 2.4% Find if this situation has an arbitrage opportunity.
9. Find the arbitrage opportunity: at a time, a stock is trading both in New York stock exchange and London stock exchange at prices USD 154 and BP 100, respectively, and the exchange rate is USD 1.55 per BP.
10. The price of gold currently is Rs 25000 per 10 gram. The forward price of delivery in one year is Rs 25500. If you can borrow money at 10% compounded annually, then can you get an arbitrage profit? If yes, then explain how?
11. Let  $A(0) = \text{Rs}100$ ;  $A(1) = \text{Rs}115$  and  $S(0) = 35$ . Can the forward price of the stock be Rs 41 at the delivery time 1 unit, assuming no arbitrage principle?
12. The 2 months interest rate in Switzerland and USA are 3% and 8% per annum respectively with continuous compounding. The spot price of a Swiss franc is USD 0.65. The forward price for a contract deliverable in 2 months is USD 0.66. Can an arbitrage opportunity be created? If so, how?
13. Suppose that the price of a stock is Rs 45 at the beginning of the year, the risk-free annual rate is 6% and Rs 2 dividend will be paid after half a year. Then for a long forward position with delivery in one year, find the value of the contract after nine months if the stock price at that time turns out to be Rs 50.
14. Stock XYZ has a current price of 100. The forward price for delivery of this stock in 1 year is 110. Unless otherwise indicated, the stock pays no dividends and the annual effective risk-free interest rate is 10%. Determine which of the following statements is FALSE.  
(A) The time-1 profit diagram and the time-1 payoff diagram for long positions in this forward contract are identical.

- (B) The time-1 profit for a long position in this forward contract is exactly opposite to the time-1 profit for the corresponding short forward position.
- (C) There is no comparative advantage to investing in the stock versus investing in the forward contract.
- (D) If the 10% interest rate was continuously compounded instead of annual effective, then it would be more beneficial to invest in the stock, rather than the forward contract.
- (E) If there was a dividend of 3.00 paid 6 months from now, then it would be more beneficial to invest in the stock, rather than the forward contract.
15. The current price of a stock is 200, and the continuously compounded annual risk-free interest rate is 4%. A dividend will be paid every quarter for the next 3 years, with the first dividend occurring 3 months from now. The amount of the first dividend is 1.50, but each subsequent dividend will be 1% higher than the one previously paid. Calculate the fair price of a 3-year forward contract on this stock.
16. Suppose the XYZ index is currently Rs 950 and initial margin 10%. You wish to enter into 10 XYZ futures contracts. If continuous compounding rate is 5% on margin balance, and marking is done weekly, maintenance margin is 80% of initial margin. What is the maximum futures price 1 week from today at which you will receive a margin call?
17. A stock is expected to pay a dividend of Rs 1 per share in 2 months and 5 months. The stock price at present is Rs 50 and risk free continuous compounding interest is 8% per annum. An investor has just taken a short position in a 6-month forward contract on stock. What are the forward price and initial value of forward contract. Three months later, the stock is at Rs 48 and no change in risk free interest rate. What are the forward price and value of the short position in contract? (Hint: Convince yourself that the value of a forward contract at  $t$  to the holder of the contract, is  $f_{0,t,T} = S_t - Fe^{-r(T-t)}$ )