

1. (2 marks) Today is your 30th birthday. Your company announces that you will receive a bonus of Rs 10000 on your next birthday, one year from now. This bonus amount will grow by 2% each year, paid on your birthday till your 40th birthday (inclusive). Assume that the risk-free interest rate for ten bonus payments remains 5% per annum. Calculate the present value of payments you will be receiving.

**Ans:**  $83881.056 = C(1 - \xi^{10})/(1 + r)(1 - \xi)$ ,  $\xi = (1 + g)/(1 + r)$ ,  $r = 0.05$ ,  $g = 0.02$ ,  $C = 10000$ .

2. (2 marks) Suppose you brought a futures contract of size 100 on a stock currently trading at Rs 720. An initial balance of Rs 10000 is needed to enter into this trade. The balance in the trading account is earning interest at 0.05% per day. Let the minimum balance maintained in the trading account be 10% of the value of the contract on the day when the contract was initiated. The stock falls sharply to 672.45 on the next day and rises again to 721.84 on day two. What is your balance in the trading account at the end of day two?

**Ans:** Value of the contract 72000 and 10% of it is 7200.

At  $t = 0$ , margin account has Rs 10000. At  $t = 1$  margin is  $10000(1 + 0.0005) - 4755 = 5245$  Call raised. Balance deposited to make it 7200. At  $t = 2$ , the margin account has  $7200(1 + 0.0005) + 4939 = 12142.6$

3. (1 mark) What is the effective interest rate per annum compounded semiannually when the interest rate is 5.75% per annum compounded weekly? (An error of  $\pm 0.1$  is allowed in numerical value).

**Ans:**  $(1 + r/2)^2 = (1 + (5.75/52 \times 100))^{52}$ . So,  $r = 5.8302\%$

4. (1 marks) Let  $A$  be the risk-free asset and  $S$  be the risky asset. Let  $A(0) = 90$ ,  $A(1) = 100$ ,  $S(0) = 25$  and

$$S(1) = \begin{cases} 30 & \text{with } p = 0.7 \\ 20 & \text{with } p = 0.3 \end{cases}$$

Let you owe  $x$  shares of risk-free asset  $A$  and  $y$  shares of risky asset  $S$  and value of your portfolio  $(x, y)$  at  $t = 1$  be

$$V(1) = \begin{cases} 1160 & \text{with } p = 0.7 \\ 1040 & \text{with } p = 0.3 \end{cases}$$

Under the no-arbitrage principle, determine the portfolio's value at  $t = 0$ .

**Ans:**  $100x + 30y = 1160$ ,  $120x + 20y = 1040$ . So,  $x = 8$ ,  $y = 12$ .  $V(0) = 1020$ .

5. (1 mark) A watch is purchased by paying Rs 2000 cash and quarterly installments of Rs 300 each for a period of two years, paid at the end of the quarter. Let the interest on the loan be 9.2% per annum. How much does the watch cost you?

**Ans:** Two answers are allowed both when cost is calculated at  $t = 0$  and at  $t = 2$  years.

Here  $r = 9.2/4\% = 0.023$ .

At  $t=0$ , it is present value = 4179.5 and at  $t=2$  years, it is future value 5001.374.

6. (1 mark) A stock is trading at Rs 50 at present. The stock is expected to pay dividends of Rs 2 on the 30th day and Rs 3 on the 90th day from now. What is the price of the 70-day forward on this stock if the risk-free interest is 4% per annum (1 year = 365 days)?

**Ans:**  $F = 50(1.04)^{70/365} - 2(1.04)^{40/365} = 48.3688$

7. (1 mark) The underlying stock is priced at Rs 620. The risk-free interest rate is 5% per annum continuously compounded, and the dividend yield is 1.23% continuously compounded. Determine the value of the 91-day forward on the 31st day when the underlying stock price is Rs 648.

**Ans:**  $S_0 e^{(r-\delta)T} = 625.85496$ , where  $T = 91/365$  and  $r = 0.05$ ,  $\delta = 0.0123$

$$V_{31} = 648 - 625.85496 e^{-0.05 \times 60/365} = 27.268$$

8. (1 mark) Consider two dealers offering currency exchange as follows:

Dealer A	USD/INR	82.67 (bid)	82.72 (ask)
Dealer B	USD/INR	82.59 (bid)	82.64 (ask)

Determine the arbitrage value of 10000 INR. (An error of  $\pm 0.1$  is allowed in numerical value)

**Ans:** Go to B to convert Rs to USD at 82.64 RS = 1 USD. Then go to A to convert USD in Rs.  
The arbitrage =  $Rs(82.67/82.64) \times 10000 = Rs\ 3.63$