

**MATHS 7027 Mathematical Foundations of  
Data Science  
Assignment 3**  
**Due: 4:59pm Tuesday 10 September 2019 via Canvas  
(PDF only)**

1. Consider the matrix

$$A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix},$$

and the general  $2 \times 2$  matrix

$$B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}.$$

Find the conditions on  $a$ ,  $b$ ,  $c$ , and  $d$ , such that  $A$  and  $B$  commute (i.e.,  $AB = BA$ ). Therefore, write out the most general form of the matrix  $B$  that commutes with  $A$ .

2. Consider the system of equations

$$\begin{aligned} x_1 + 4x_2 - 6x_3 - 3x_4 &= 3 \\ x_1 - x_2 + 2x_4 &= -5 \\ x_1 + x_3 + x_4 &= 1 \\ x_2 + x_3 + x_4 &= 0. \end{aligned}$$

- (a) Convert this system to augmented matrix form and solve using Gauss-Jordan elimination (or explain why no solution exists). Make sure you show all of your steps, by writing out the row operations performed on the augmented matrix.
- (b) Check your answer either by hand by performing an appropriate matrix multiplication on your solution  $(x_1, x_2, x_3, x_4)$ .
3. Consider the homogeneous system of equations

$$\begin{aligned} x + y + z &= 0 \\ 2x - 6y - 2z &= 0 \\ 2x + z &= 0. \end{aligned}$$

Convert to augmented matrix form and use Gauss-Jordan elimination to find **all** solutions of the system of equations (i.e., not just the trivial solution  $x = y = z = 0$ ).

4. Consider the matrices

$$A = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 2 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 & -2 & -1 \\ 2 & 3 & 0 \end{bmatrix}, C = \begin{bmatrix} 4 & 1 \\ 2 & 1 \end{bmatrix}, D = \begin{bmatrix} 8 & x \\ -4 & 3 \end{bmatrix}.$$

Find:

- (a)  $|AB|$ ,
- (b)  $|C^{-1}|$ ,
- (c) The set of all values of  $x$  such that the matrix  $D$  is invertible. (Note: make sure that you use appropriate set notation to write your answer!)