# MATHS 7027 Mathematical Foundations of Data Science
# Assignment 2
## Due: 4:59pm Tuesday 27 August 2019 via Canvas (PDF only)

1. In lectures we proved the result

$$\sum_{i=1}^{n} i^2 = \frac{n}{6}(2n+1)(n+1)$$

via a somewhat circuitous route. Prove this result instead by using the principle of mathematical induction.

2. In lectures we derived an expression for the Maclaurin polynomial for $\cos(x)$.

   (a) Using this expression, find the Maclaurin polynomial of degree $n = 2k$ for $f(x) = \cos(2x)$.

   (b) Use Taylor's theorem to estimate how many terms need to be used to approximate $\cos(2)$ to within 0.001. (Hint: For $f^{(n+1}(z)$, think about what $y$-values $\cos(x)$ and $\sin(x)$ are both bounded by. You'll need to use some trial and error to find $n$ once you have a bound for the remainder.)

3. Find the Taylor series for $f(x) = \ln(x)$, centred at $a = 3$, along with its interval of convergence.

4. Consider the series $\sum_{n=0}^{\infty} a_n$, with terms $a_n$ defined *recursively* by the equations:

$$a_0 = 2, \quad a_{n+1} = \frac{4n+1}{kn+3} a_n$$

for some given value of $k \in \mathbb{N}$.

   (a) Write out the first 6 terms of the series (i.e., up to $n = 5$).

   (b) Use the ratio test to show that the series converges for $k \geq 5$, and diverges for $k \leq 3$.

5. Use Maclaurin series to compute the limit

$$\lim_{x \to 0} \frac{1 - \cos(x)}{1 + x + -e^x}.$$