Assignment 3

(1.) $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ $B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Conditions on a, b, c, d such that A & B commute

ie $AB = BA$

$\therefore$ $$\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} (*) \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

Multiplying the above Matrices we get.

$$\cancel{a} - c = \cancel{a} - b \Rightarrow b - c = 0 \quad (1)$$
$$\cancel{b} - d = -a + \cancel{b} \Rightarrow a - d = 0 \quad (2)$$
$$-a + \cancel{c} = \cancel{c} - d \quad\quad a - d = 0 \quad (3)$$
$$-b + \cancel{d} = -c + \cancel{d} \quad\quad b - c = 0 \quad (4)$$

Simplifying (1) and (4) are same

(2) and (3) are same

$a - d = 0$

$b - c = 0$

Gauss Jordan E. Method

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 & 0 \end{array}\right]$$

This is already in reduced echlon form

$a = d.$

$b = c$

$$B = \begin{bmatrix} d & c \\ c & d \end{bmatrix}$$

$$\boxed{B. = d\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + c\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}}$$

(2.)

$$x_1 + 4x_2 - 6x_3 - 3x_4 = 3$$
$$x_1 - x_2 + 2x_4 = -5.$$
$$x_1 + x_3 + x_4 = 1$$
$$x_2 + x_3 + x_4 = 0$$

$$\left[\begin{array}{cccc|c} \boxed{1} & 4 & -6 & -3 & 3 \\ 1 & -1 & 0 & +2 & -5 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{array}\right] \quad \begin{array}{l} R_2 = R_2 - R_1 \\ R_3 = R_3 - R_1. \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 4 & -6 & -3 & 3 \\ 0 & -5 & 6 & 5 & -8 \\ 0 & -4 & 7 & 4 & -2 \\ 0 & 1 & 1 & 1 & 0 \end{array}\right] \quad R_2 \leftrightarrow R_4.$$

$$\left[\begin{array}{cccc|c} 1 & 4 & -6 & -3 & 3 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & -4 & 7 & 4 & -2 \\ 0 & -5 & 6 & 5 & -8 \end{array}\right] \quad \begin{array}{l} R_3 \to R_3 + 4R_2 \\ R_4 \to R_4 + 5R_2. \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 4 & -6 & -3 & 3 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 11 & 8 & -2 \\ 0 & 0 & 11 & 10 & -8 \end{array}\right] \quad \begin{array}{l} R_4 \to R_4 - R_3. \\ R_3 \to R_3/11 \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 4 & -6 & -3 & 3 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & (8/11) & (-2/11) \\ 0 & 0 & 0 & 2 & -6 \end{array}\right] \quad R_1 \to R_1 - 4R_2.$$

Row echlon form. —

$$\left[\begin{array}{cccc|c} 1 & 0 & -10 & -7 & 3 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 8/11 & -2/11 \\ 0 & 0 & 0 & 2 & -6 \end{array}\right] \quad \begin{array}{l} R_1 \to R_1 + 10R_3. \\ R_2 \to R_2 - R_3. \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & \left(-7+\frac{80}{11}\right) & 3-\frac{20}{11} \\ 0 & 1 & 0 & 3/11 & 2/11 \\ 0 & 0 & 1 & 8/11 & -2/11 \\ 0 & 0 & 0 & 2 & -6 \end{array}\right] \quad R_4 = \frac{R_4}{2}.$$

$$\boxed{\left[\begin{array}{cccc|c} 1 & 0 & 0 & \left(-7+\frac{80}{11}\right) & 3-\frac{20}{11} \\ 0 & 1 & 0 & 3/11 & 2/11 \\ 0 & 0 & 1 & 8/11 & -2/11 \\ 0 & 0 & 0 & 1 & -3 \end{array}\right]}$$

$\boxed{x_4 = -3}$

$x_3 + \frac{8}{11}(-3) = \frac{-2}{11}$

$x_3 = -\frac{2}{11} + \frac{24}{11} = +2$

$\boxed{x_3 = +2}$

$x_2 + \frac{3}{11}(-3) = \frac{2}{11}$

$x_2 = \frac{2}{11} + \frac{9}{11} = 1$

$\boxed{x_2 = 1}$

$x_1 + \left(-7+\frac{80}{11}\right)(-3) = 3 - \frac{20}{11}$

$x_1 + \left(\frac{3}{11}\right)(-3) = \frac{13}{11}$

$x_1 = \frac{13}{11} + \frac{9}{11} = \frac{22}{11} = 2$

$\boxed{x_1 = 2}$

$$\begin{bmatrix} 1 & 4 & -6 & -3 \\ 1 & -1 & 0 & 2 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 2 \\ -3 \end{bmatrix} = \begin{bmatrix} 3 \\ -5 \\ 1 \\ 0 \end{bmatrix}$$

$$\begin{matrix} 2+4-12+9 \Rightarrow 3 \\ 2-1+0-6 \Rightarrow -5 \\ 2+0+2-3 \Rightarrow 1 \\ 0+1+2-3 \Rightarrow 0 \end{matrix} = \begin{bmatrix} 3 \\ -5 \\ 1 \\ 0 \end{bmatrix}$$

Hence verified

Q.3

$$x + y + z = 0$$
$$2x - 6y - 2z = 0$$
$$2x + z = 0$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 2 & -6 & -2 & 0 \\ 2 & 0 & 1 & 0 \end{array}\right] \begin{matrix} R_2 \rightarrow R_2 - 2R_1 \\ R_3 \rightarrow R_3 - 2R_1 \end{matrix}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & -8 & -4 & 0 \\ 0 & -2 & -1 & 0 \end{array}\right] R_3 \leftrightarrow R_4$$

$$\cancel{\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & -8 & -4 & 0 \\ 0 & -2 & -1 & 0 \end{array}\right]}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & -2 & -1 & 0 \\ 0 & -8 & -4 & 0 \end{array}\right] \begin{array}{l} R_4 \to R_4 - 4R_2 \\ R_2 \to R_2/-2 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & 1/2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \underset{R_1 \to R_1 - R_2}{\Rightarrow} \underline{\underline{0 = 0}}$$

system might have infinitely many solution

$$\left[\begin{array}{ccc|c} 1 & 0 & 1/2 & 0 \\ 0 & 1 & 1/2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$x + \frac{z}{2} = 0$$

$$y + \frac{z}{2} = 0$$

$$x = -y$$

$\boxed{x + y = 0}$ infinitely many solutions

4.)

$$A = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 2 & 0 \end{bmatrix}_{(3\times 2)}, \quad B = \overset{(2\times 3)}{\begin{bmatrix} 1 & -2 & -1 \\ 2 & 3 & 0 \end{bmatrix}}, \quad C = \overset{(2\times 2)}{\begin{bmatrix} 4 & 1 \\ 2 & 1 \end{bmatrix}}$$

$$D = \begin{bmatrix} 8 & x \\ -4 & 3 \end{bmatrix}_{(2\times 2)}$$

(a) $|AB|$.

$$AB = (3\times \underline{2)(2}\times 3) = (3\times 3)$$

$$\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 1 & -2 & -1 \\ 2 & 3 & 0 \end{bmatrix} = \begin{bmatrix} 1-4 & -2-6 & -1+0 \\ 1+2 & -2+3 & -1+0 \\ 2+0 & -4+0 & -2+0 \end{bmatrix}$$

$$A*B = \begin{bmatrix} -3 & -8 & -1 \\ 3 & +1 & -1 \\ 2 & -4 & -2 \end{bmatrix}$$

$$\Rightarrow |AB| = \left[-3(-2\pm 4) + 8(-6+2) - 1(-12-2)\right]$$

$$\cancel{= \left[+18 \; -32 \; +14 \right]}$$

$=$

$+18 - 32 + 14$

$= \underline{\underline{0}}$

(b.) $C = \begin{bmatrix} 4 & 1 \\ 2 & 1 \end{bmatrix}$

$$\underset{A}{\begin{bmatrix} 4 & 1 \\ 2 & 1 \end{bmatrix}} \left| \underset{I}{\begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}} \right] \qquad CX = I, \quad X = C^{-1}$$

$$R_2 \leftrightarrow R_1 \Rightarrow \left[\begin{array}{cc|cc} 2 & 1 & 0 & 1 \\ 4 & 1 & 1 & 0 \end{array}\right] \quad \begin{matrix} R_2 \to R_2 - 2R_1 \\ R_1 = R_1/2 \end{matrix}$$

$$\left[\begin{array}{cc|cc} 1 & 1/2 & 0 & 1/2 \\ 0 & -1 & 1 & -2 \end{array}\right] \quad \begin{matrix} R_1 \to R_1 + \frac{1}{2}R_2 \\ R_2 \leftrightarrow -R_2 \end{matrix}$$

$$\left[\begin{array}{cc|cc} 1 & 0 & 1/2 & -1/2 \\ 0 & 1 & -1 & +2 \end{array}\right] \xRightarrow{C^{-1}} \begin{bmatrix} 1/2 & -1/2 \\ -1 & 2 \end{bmatrix}$$

$\quad I \qquad\qquad C^{-1}$

$$|C^{-1}| = 2\left(\tfrac{1}{2}\right) - \left(-1 \times -1/2\right) = 1 - \frac{1}{2} = \boxed{\frac{1}{2} = |C^{-1}|}$$

(C.)

$$D = \begin{bmatrix} 8 & x \\ -4 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad R_1 \Leftrightarrow R_2$$

$$D \qquad\qquad I.$$

$$\left[\begin{array}{cc|cc} -4 & 3 & 0 & 1 \\ 8 & x & 1 & 0 \end{array}\right] \quad \begin{array}{l} R_2 \to R_2 + 2R_1 \\ R_1 \to \frac{R_1}{-4} \end{array}$$

$$\left[\begin{array}{cc|cc} 1 & -3/4 & 0 & 1/4 \\ 0 & x+6 & 1 & 2 \end{array}\right]$$

from this it is clear that $x \neq -6$.

or

$$D = \begin{bmatrix} 8 & x \\ -4 & 3 \end{bmatrix}$$

for inverse to exist $|D| \neq 0$

$$8 \times 3 - (-4 * x) \neq 0$$

$$24 + 4x \neq 0$$

$$\boxed{x \neq -6}$$

$\therefore$ ~~[illegible]~~

$\Rightarrow \forall x \in R - \{-6\}$.

<u>OR</u> $x \in (-\infty, -6) \cup (-6, \infty)$.

for D to be invertible

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