# Assignment 4.

Q.1.)

(a) $A = \begin{bmatrix} 17 & 12 \\ 8 & -3 \end{bmatrix}$

$$|\lambda I - A| = 0$$

characteristic equation $\left| \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} 17 & 12 \\ 8 & -3 \end{bmatrix} \right| = 0$

$$\begin{vmatrix} \lambda-17 & -12 \\ -8 & \lambda+3 \end{vmatrix} = 0$$

$$(\lambda-17)(\lambda+3) - (8)(12) = \lambda^2 + 3\lambda - 17\lambda - 51 - 96$$

$$= \lambda^2 - 14\lambda - 51 - 96$$

$\cancel{= \lambda^2 + 3\lambda - 17\lambda = 51 + 147}$ $\lambda^2 - 21\lambda + 7\lambda - 147$

$\cancel{= \lambda(\lambda+3) - 17(\lambda+3)}$ $\lambda(\lambda+7) - 21(\lambda+7)$

$\cancel{= (\lambda-17)(\lambda+3)}$ $(\lambda-21)(\lambda+7)$

$\lambda =$ 21, −7 · plug back in $(\lambda I - A)x$ & solve for $x$

$\cancel{\lambda = 17}$

$$\cancel{\left[\begin{bmatrix} 17 & 0 \\ 0 & 17 \end{bmatrix} - \begin{bmatrix} 17 & 12 \\ 8 & -3 \end{bmatrix}\right] x = 0}$$

$$\cancel{\left[\begin{array}{cc|c} 0 & -12 & 0 \\ -8 & 20 & 0 \end{array}\right] = 0} \qquad \cancel{R_1 \to -\frac{5}{3} R_1}$$

$\lambda = 21$

$$\left| \begin{bmatrix} 21 & 0 \\ 0 & 21 \end{bmatrix} - \begin{bmatrix} 17 & 12 \\ 8 & -3 \end{bmatrix} \right| x = 0$$

$$\left[\begin{array}{cc|c} 4 & -12 & 0 \\ -8 & 24 & 0 \end{array}\right] = 0$$ Augmented form.

$$R_2 \rightarrow R_2 \cdot 2,$$

$$\left[\begin{array}{cc|c} 4 & -12 & 0 \\ -4 & 12 & 0 \end{array}\right] = 0 \qquad R_2 \rightarrow R_2 + R_1$$

$$\left[\begin{array}{cc|c} 4 & -12 & 0 \\ 0 & 0 & 0 \end{array}\right].$$

$$y = t.$$

$$4x - 12t = 0$$

$$x = 3t.$$

$$x = \begin{bmatrix} 3 \\ 1 \end{bmatrix} t.$$

$\lambda = -7$

$$\left[\begin{bmatrix} -7 & 0 \\ 0 & -7 \end{bmatrix} - \begin{bmatrix} 17 & 12 \\ 8 & -3 \end{bmatrix}\right] x = 0.$$

$$\left[\begin{array}{cc|c} -24 & -12 & x \\ -8 & -4 & y \end{array}\right] = 0$$

$$\left[\begin{array}{cc|c} -24 & -12 & 0 \\ -8 & -4 & 0 \end{array}\right] = 0 \qquad R_2 \to R_2 * 3$$

$$\left[\begin{array}{cc|c} -24 & -12 & 0 \\ -24 & -12 & 0 \end{array}\right] \qquad R_2 \to R_2 - R_1$$

$$\left[\begin{array}{cc|c} -24 & -12 & 0 \\ 0 & 0 & 0 \end{array}\right].$$

$$y = t$$

$$-24x - 12t = 0$$

$$x = -\frac{12t}{24}$$

$$x = -\frac{1}{2}t.$$

$$x = \begin{bmatrix} -1/2 \\ 1 \end{bmatrix} t. \qquad x = \begin{bmatrix} -1 \\ 2 \end{bmatrix} \frac{t}{2}$$

(b.) $A = \begin{bmatrix} 8 & 0 & 36 \\ 3 & 20 & -9 \\ 6 & 0 & 2 \end{bmatrix}$

$$|\lambda I - A| = 0$$

$$\begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} - \begin{bmatrix} 8 & 0 & 36 \\ 3 & 20 & -9 \\ 6 & 0 & 2 \end{bmatrix} = 0$$

$$\left| \begin{bmatrix} \lambda-8 & 0 & -36 \\ -3 & \lambda-20 & 9 \\ -6 & 0 & \lambda-2 \end{bmatrix} \right| = 0$$

$$(\lambda-8)[(\lambda-20)(\lambda-2)] + 0 - 36[6(\lambda-20)]$$

$$(\lambda-20)[(\lambda-8)(\lambda-2) - 36 \times 6]$$

$$(\lambda-20)[\lambda^2 - 2\lambda - 8\lambda + 16 - 216]$$

$$(\lambda-20)[\lambda^2 - 10\lambda - 200]$$

$$(\lambda-20)[\lambda^2 - 20\lambda + 10\lambda - 200]$$

$$(\lambda-20)[\lambda(\lambda+10) - 20(\lambda+10)]$$

$$(\lambda-20)(\lambda-20)(\lambda+10)$$

$$\lambda = 20, 20, -10$$

$\lambda = -10$

$$\begin{bmatrix} -10 & 0 & 0 \\ 0 & -10 & 0 \\ 0 & 0 & -10 \end{bmatrix} - \begin{bmatrix} 8 & 0 & 36 \\ 3 & 20 & -9 \\ 6 & 0 & 2 \end{bmatrix} = 0$$

$$\begin{bmatrix} -18 & 0 & -36 \\ -3 & -30 & +9 \\ -6 & 0 & -12 \end{bmatrix} x = 0$$

$R_3 \to R_3 * 3$

$$\begin{bmatrix} -18 & 0 & -36 \\ -3 & -30 & +9 \\ -18 & 0 & -36 \end{bmatrix} \Rightarrow R_3 \to R_3 - R_1$$

$$\begin{bmatrix} -18 & 0 & -36 \\ -3 & -30 & 9 \\ 0 & 0 & 0 \end{bmatrix} \quad \cancel{\Rightarrow \begin{bmatrix} -3 & 0 & -6 \\ 3 & -30 & 9 \\ 0 & 0 & 0 \end{bmatrix}}$$

$\cancel{R_1 \to R_1 - R_2}$

$R_2 \to R_2 * 6$

$\cancel{0 \quad 30 \quad 18}$

$\cancel{-3}$

$$\begin{bmatrix} -18 & 0 & -36 \\ -18 & -180 & 54 \\ 0 & 0 & 0 \end{bmatrix} \quad R_2 \to R_2 - R_1$$

$$\begin{bmatrix} -18 & 0 & -36 \\ 0 & -180 & 100 \\ 0 & 0 & 0 \end{bmatrix}$$

$\to$ solving

$$x = \begin{bmatrix} -4 \\ 1 \\ 2 \end{bmatrix} t$$

$\lambda = 20$

$$\begin{bmatrix} 20 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & 20 \end{bmatrix} - \begin{bmatrix} 8 & 0 & 36 \\ 3 & 20 & -9 \\ 6 & 0 & 2 \end{bmatrix} = 0$$

$$\begin{bmatrix} 12 & 0 & -36 \\ -3 & 0 & 9 \\ -6 & 0 & 18 \end{bmatrix} = 0$$

$R_3/2$ ; $R_3 \to R_3 - R_2$

$$\begin{bmatrix} 12 & 0 & -36 \\ -3 & 0 & 9 \\ 0 & 0 & 0 \end{bmatrix}$$

$R_2 * 4$. , $R_2 \to R_2 - R$

$$\begin{bmatrix} 12 & 0 & -36 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0$$

$x_3 = t \qquad x_2 = s$

$12x_1 + x_2(0) + -36x_3 = 0$

$x_1 = \frac{36t}{12} = 3t \qquad x = \begin{bmatrix} 3t \\ s \\ t \end{bmatrix}$

$= s\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$

Q.2. $A = \begin{bmatrix} 1 & 0 & 0 \\ -4 & -7 & 5 \\ -8 & -10 & 8 \end{bmatrix}$

$$A - tI = \begin{bmatrix} 1-t & 0 & 0 \\ -4 & -7-t & 5 \\ -8 & -10 & 8-t \end{bmatrix}$$

$$p(t) = (1-t)\left((-7-t)(8-t) + 50\right)$$
$$= (1-t)\left(-56 + 7t - 8t + t^2 + 50\right)$$
$$= (1-t)\left(t^2 - t - 6\right)$$
$$= (1-t)\left(t^2 - 3t + 2t - 6\right)$$
$$= (1-t)\left(t(t-3) + 2(t-3)\right)$$
$$= (1-t)(t+2)(t-3)$$

$$t = 1, -2, 3.$$

$t = 1.$

$$\begin{bmatrix} 0 & 0 & 0 \\ -4 & -8 & 5 \\ -8 & -10 & 7 \end{bmatrix}$$

Solving we get

Eigen space $E_1$ $\begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}$

$t = -2$

$$\begin{bmatrix} 3 & 0 & 0 \\ -4 & -5 & 5 \\ -8 & -10 & 10 \end{bmatrix}$$

solving we get

$$E_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

$t = 3.$

$$\begin{bmatrix} -2 & 0 & 0 \\ -4 & -10 & 5 \\ -8 & -10 & 5 \end{bmatrix}$$

$$E_3 = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$$

geometric multiplicity equal to algebric multiplicity hence diagonizable

$$V_1 = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix} \quad V_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \quad V_3 = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$$

$$P = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 1 \\ 4 & 1 & 2 \end{bmatrix}$$

$$S^{-1}AS = D$$

$$D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$P^{-1}AP = \text{diag}(t_1, t_2, t_3)$$

$$D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$P|I = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 2 & 1 & 1 & 0 & 1 & 0 \\ 4 & 1 & 2 & 0 & 0 & 1 \end{array}\right]$$

$$R_2 \to R_2 - 2R_1$$

$$R_3 = R_3 - \frac{R_1}{4}$$

solving we get

$$P^{-1} = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 2 & -1 \\ -4 & -2 & 2 \end{bmatrix}$$

to verify $P^{-1}AP = D$

$$\begin{bmatrix} 4 & 0 & 0 \\ 0 & 2 & -1 \\ -4 & -2 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -4 & -7 & 5 \\ -8 & -10 & 8 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 1 \\ 4 & 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 3 \end{bmatrix} = D$$

Hence verified

(Q.4.) 5 directors.

$\underline{D_1}$ $\underline{D_2}$ $\underline{D_3}$ $\underline{D_4}$ $\underline{D_5}$.

(a.) different seating arrangements.

$= 5!$

seats option for $D_1 = 5$.

" " $D_2 = 4$

" " $D_3 = 3$

$D_4 = 2$

$D_5 = 1$

$5 \times 4 \times 3 \times 2 \times 1 = 5! = 120$

(b.) $\underline{D_1}$ $\underline{D_2}$ $\boxed{D_3}$ ← fixed $\underline{D_4}$ $\underline{D_5}$

4 directors, 4 seats.

$4 \times 3 \times 2 \times 1 = 24$.

(c.) $\boxed{D_1}$ ← fixed $\underline{D_2}$ $\underline{D_3}$ $\underline{D_4}$ $\boxed{D_5}$ ← fixed

3 directors ⇒ $3!$

remaining two can sit on opposite ways = 2 ways

$= 3! \times 2 = 12$