STATS 2103: Probability and Statistics II: Assignment 1

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CHECKLIST

- ☐: Have you shown all of your working, including probability notation where necessary?
- ☐: Have you given all numbers to 3 decimal places unless otherwise stated?
- ☐: Have you included all R output and plots to support your answers where necessary?
- ☐: Have you included all of your R code?
- ☐: Have you made sure that all plots and tables each have a caption?
- ☐: If before the deadline, have you submitted your assignment via the online submission on Canvas?
- ☐: Is your submission a single pdf file - correctly orientated, easy to read? If not, penalties apply.
- ☐: Penalties for more than one document - 10% of final mark for each extra document. Note that you may resubmit and your final version is marked, but the final document should be a single file.
- ☐: Penalties for late submission - within 24 hours 40% of final mark. After 24 hours, assignment is not marked and you get zero.
- ☐: Assignments emailed instead of submitted by the online submission on Canvas will not be marked and will receive zero.
- ☐: Have you checked that the assignment submitted is the correct one, as we cannot accept other submissions after the due date?

Due date: Friday 20th March 2020 (Week 3), 5pm.

$A|B = \frac{A \cap B}{P(B)} = \frac{\frac{1}{4}}{\frac{2}{4}}$

Q1: Set probability question $S = (g,g)\ (g,b)\ (b,g)\ (b,b)$

A couple have two children.

a. What is the probability that both are girls given that the oldest is a girl? $B = (b,g)\ (g,g)$ $A = (g,g)$ $\left(\frac{1}{2}\right)$

[3 marks]

b. What is the probability that both are girls given that one of them is a girl?

You may assume that the probability that a child is female is 1/2, and that the gender of one child does not affect the gender of the other child.

$A = (g,g)$ $B = (g,b)\ (b,g)$ $\frac{\frac{1}{4}}{\frac{2}{4}} = \frac{1}{2}$

[4 marks]

[Question total: 7]

Q2: Axiom question

a. If A and B are independent events, then prove that A^c and B^c are independent. $P(A|B) = P(A)$

[5 marks]

b. Suppose that $A \subset B$, and that $P(A) > 0$ and $P(B) > 0$. When are A and B independent? Prove your answer.

$P(A) + P(B) = 1$

$1 - A^c + 1 - P(B^c) = 1$

$A^c + B^c = 1$

[2 marks]

[Question total: 7]

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Q3: Counting question

Nine taxis are to be dispatched to three airports in such a way that three go to airport A, five go to airport B, and one goes to airport C.

a. In how many ways can this be accomplished?

[2 marks]

b. If exactly one taxi is in need of repair, what is the probability that it is dispatched to airport C?

[3 marks]

c. If exactly three of the taxis are in need of repair, what is the probability that every airport receives one of the taxis needing repair?

[3 marks]

[Question total: 8]

Q4: Bayes question

A rare disease affects only 1 percent of the population. A blood test has been developed. It has a sensitivity of 99%, and a specificity of 95%.

Let D be the event that a randomly selected patient has the disease, and let T be the event that a randomly select patient has a positive blood test.

- **Sensitivity**: is the probability of a positive blood test given a patient has the disease.
- **Specificity**: is the probability of a negative blood test given a patient does not have the disease.

a. Write the given information in probability notation.

[3 marks]

b. Calculate the probability of a positive blood test in a randomly selected person.

[3 marks]

c. Hence, or otherwise, calculate the probability of a patient having the disease given they have a positive blood test.

[2 marks]

[Question total: 8]

Q5: Probability mass function question

You are playing a game of two-up (https://en.wikipedia.org/wiki/Two-up). Basically the rules are as follows:

1. Toss two fair coins into the air.
2. For two heads you win \$1.
3. For two tails you win \$2.
4. For a head and a tail you lose \$1.

a. Write down the PMF as a table.

[6 marks]

b. What are your expected winnings from a single game.

[2 marks]

[Question total: 8]

[[Assignment total: 38]]