

STATS 2103: Probability and Statistics II: Assignment 4

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Semester 1 2020

CHECKLIST

- Have you shown all of your working, including probability notation where necessary?
- Have you given all numbers to 3 decimal places unless otherwise stated?
- Have you included all R output and plots to support your answers where necessary?
- Have you included all of your R code?
- Have you made sure that all plots and tables each have a caption?
- If before the deadline, have you submitted your assignment via the online submission on Canvas?
- Is your submission a single pdf file - correctly orientated, easy to read? If not, penalties apply.
- Penalties for more than one document - 10% of final mark for each extra document. Note that you may resubmit and your final version is marked, but the final document should be a single file.
- Penalties for late submission - within 24 hours 40% of final mark. After 24 hours, assignment is not marked and you get zero.
- Assignments emailed instead of submitted by the online submission on Canvas will not be marked and will receive zero.
- Have you checked that the assignment submitted is the correct one, as we cannot accept other submissions after the due date?

Due date: Friday 15th May 2020 (Week 9), 5pm.

Q1: Transformation of random variables. Let Y be a random variable with probability density function given by

$$f(y) = \begin{cases} \frac{3}{2}y^2, & -1 \leq y \leq 1, \\ 0, & \text{otherwise.} \end{cases}$$

- a. Find the density of $X = 2Y - 1$. Hint: if you use the CDF method, ensure that you write the complete CDF.

[5 marks]

- b. Find the density of $Z = Y^2$.

[5 marks]

[Question total: 10]

Q2 Multinomial distribution. A version of the game “two-up” is played.

Two fair coins are tossed and the possible outcomes are two heads, two tails, or a head and a tail. If the game is played 5 times what is the probability of seeing at least one of each outcome.

HH, TH, HT, TT
 $\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{4}$

$\underline{0} \times \underline{1} \times \underline{1} \times \underline{1} \times \underline{1}$

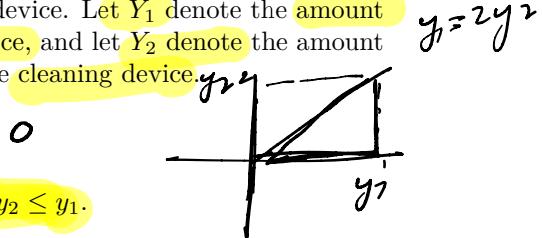
[6 marks]

[Question total: 6]

$$\int_0^2 \int_{0}^{y_1} 1 dy_2 dy_1$$

Q3 Continuous bivariate random variables. An environmental engineer measures the amount (by weight) of particulate pollution in air samples of a certain volume collected over two smokestacks at a coal-operated power plant. One of the stacks is equipped with a cleaning device. Let Y_1 denote the amount of pollutant per sample collected above the stack that has no cleaning device, and let Y_2 denote the amount of pollutant per sample collected above the stack that is equipped with the cleaning device.

Suppose that the joint behaviour of Y_1 and Y_2 can be modelled by



$$f(y_1, y_2) = c \quad \text{for } 0 \leq y_1 \leq 2, \quad 0 \leq y_2 \leq 1, \quad 2y_2 \leq y_1.$$

That is, Y_1 and Y_2 are uniformly distributed over the region inside the triangle bounded by $y_1 = 2$, $y_2 = 0$ and $2y_2 = y_1$.

- a. Find the value of c that makes this function a valid probability density function. [3 marks]

$$\int_0^2 \int_{0}^{y_1} c dy_2 dy_1$$

- b. Find $P(Y_1 \geq 3Y_2)$. That is, find the probability that the cleaning device reduced the amount of pollutant by one-third or more.

$$\int_0^1 \int_{0}^{y_1} c dy_2 dy_1 = c \left(\frac{y_1^2}{2} \right) = c \left(\frac{2y_2^2}{2} \right) = c y_2^2 \quad [3 \text{ marks}]$$

$$= c - c y_2 \quad [Question total: 6]$$

$$c y_2 - \frac{c y_2^2}{2} \quad \frac{c}{2} = 3/1$$

$$\frac{c}{2} = 1 \quad \underline{\underline{c=2}}$$

Q4 Continuous bivariate random variables. Consider two random variables Y_1 and Y_2 with the joint probability density function

$$f(y_1, y_2) = \begin{cases} k(1-y_2) & 0 \leq y_1 \leq y_2 \leq 1, \\ 0 & \text{elsewhere.} \end{cases}$$

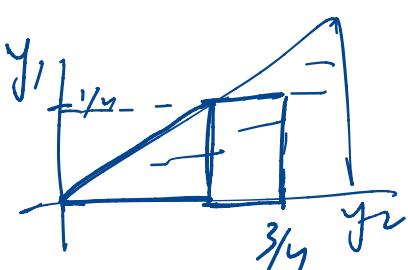
- a. Draw a sketch of the area where the probability density function is nonzero. [2 marks]

- b. Show that for this to be a valid probability density function, we need $k = 6$. [3 marks]

- c. Draw the appropriate integration region and find $P(Y_1 \leq 1/4, Y_2 \leq 3/4)$. [3 marks]

- d. Find the marginal density function of Y_1 . $\int_0^1 \int_{y_1}^1 k(1-y_2) dy_2 dy_1$ [2 marks]

- e. Find the marginal density function of Y_2 .



$$k y_2 - \frac{k y_2^2}{2}$$

$$6(1-y_2)$$

$$\int_0^{1/4} \int_{y_1}^{3/4} 6(1-y_2) dy_2 dy_1$$

[2 marks]

[Question total: 12]

[[Assignment total: 34]]