STATS 2103: Probability and Statistics II: Assignment 3

Jono Tuke

Semester 1 2020

CHECKLIST

- ☐: Have you shown all of your working, including probability notation where necessary?
- ☐: Have you given all numbers to 3 decimal places unless otherwise stated?
- ☐: Have you included all R output and plots to support your answers where necessary?
- ☐: Have you included all of your R code?
- ☐: Have you made sure that all plots and tables each have a caption?
- ☐: If before the deadline, have you submitted your assignment via the online submission on Canvas?
- ☐: Is your submission a single pdf file - correctly orientated, easy to read? If not, penalties apply.
- ☐: Penalties for more than one document - 10% of final mark for each extra document. Note that you may resubmit and your final version is marked, but the final document should be a single file.
- ☐: Penalties for late submission - within 24 hours 40% of final mark. After 24 hours, assignment is not marked and you get zero.
- ☐: Assignments emailed instead of submitted by the online submission on Canvas will not be marked and will receive zero.
- ☐: Have you checked that the assignment submitted is the correct one, as we cannot accept other submissions after the due date?

Due date: Friday 1st May 2020 (Week 7), 5pm.

Q1: PDF

Suppose we make parts on a production line and measure the amount Y by which the weight of the part differs from the target weight. From experience, we know that the distribution of Y is given by the probability density function

$$f(y) = c(1 - y^3), \quad -1 \leq y \leq 1,$$

where $f(y)$ takes value 0 elsewhere.

a. Find the constant c. [3 marks]

b. Find the cumulative distribution function $F(y)$. [5 marks]

c. Find $P(Y < 0.5)$. [2 marks]

d. Find the expected value of Y, μ_Y.

e. Find the standard deviation of Y, σ_Y. [2 marks]

[3 marks]

[Question total: 15]

Q2: Expected reciprocal

Let Y be uniformly distributed on the interval $(1, 2)$.

a. Find $E(1/Y)$. [2 marks]

b. Is $E(1/Y) = 1/E(Y)$? Justify your answer. [2 marks]

[Question total: 4]

Q3: Transformation of random variables

The length of time Y necessary to complete a key operation in the construction of houses has an exponential distribution with mean 10 hours. The formula

$$C = 100 + 40Y + 3Y^2$$

relates the cost C of completing this operation to the square of the time to completion.

a. Find the mean of C. [6 marks]

b. Find the variance of C. [6 marks]

[Question total: 12]

Q4: Poisson process

Buses arrive at a city according to a Poisson process with a rate of 5 per hour.

a. What is the probability that no buses arrive between 9 a.m. and 11 a.m. on a randomly selected morning. [4 marks]

b. Given three buses have arrived in the first hour, what is the probability that the next bus arrives in the next 1/2 an hour? [4 marks]

c. What is the probability that the fifth bus of the day arrives after midday given they start arriving at 9 a.m. [4 marks]

d. What is the probability that at most 2 buses arrive in the first hour? [2 marks]

[Question total: 14]

[[Assignment total: 45]]