

# STATS 2103: Probability and Statistics II: Assignment 4

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## CHECKLIST

- ☐: Have you shown all of your working, including probability notation where necessary?
- ☐: Have you given all numbers to 3 decimal places unless otherwise stated?
- ☐: Have you included all R output and plots to support your answers where necessary?
- ☐: Have you included all of your R code?
- ☐: Have you made sure that all plots and tables each have a caption?
- ☐: If before the deadline, have you submitted your assignment via the online submission on Canvas?
- ☐: Is your submission a single pdf file - correctly orientated, easy to read? If not, penalties apply.
- ☐: Penalties for more than one document - 10% of final mark for each extra document. Note that you may resubmit and your final version is marked, but the final document should be a single file.
- ☐: Penalties for late submission - within 24 hours 40% of final mark. After 24 hours, assignment is not marked and you get zero.
- ☐: Assignments emailed instead of submitted by the online submission on Canvas will not be marked and will receive zero.
- ☐: Have you checked that the assignment submitted is the correct one, as we cannot accept other submissions after the due date?

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**Due date: Friday 15th May 2020 (Week 9), 5pm.**

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**Q1: Transformation of random variables.** Let  $Y$  be a random variable with probability density function given by

$$f(y) = \begin{cases} \frac{3}{2}y^2, & -1 \leq y \leq 1, \\ 0, & \text{otherwise.} \end{cases}$$

- a. Find the density of  $X = 2Y - 1$ . *Hint: if you use the CDF method, ensure that you write the complete CDF.*

[5 marks]

- b. Find the density of  $Z = Y^2$ .

[5 marks]

[Question total: 10]

**Q2 Multinomial distribution.** A version of the game “two-up” is played.

Two fair coins are tossed and the possible outcomes are two heads, two tails, or a head and a tail. If the game is played 5 times what is the probability of seeing at least one of each outcome.

HH, TH, HT, TT  
 $\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}$

~~0~~ ~~x~~ ~~Y~~ ~~x~~ ~~Y~~ ~~x~~ ~~Y~~ ~~x~~ ~~Y~~

[6 marks]

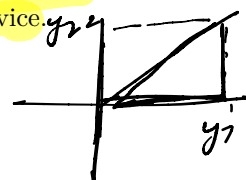
[Question total: 6]

$$\int_0^1 \int_{\frac{y_2}{2}}^1 1 \, dy_1 \, dy_2$$

**Q3 Continuous bivariate random variables.** An environmental engineer measures the amount (by weight) of particulate pollution in air samples of a certain volume collected over two smokestacks at a coal-operated power plant. One of the stacks is equipped with a cleaning device. Let  $Y_1$  denote the amount of pollutant per sample collected above the stack that has no cleaning device, and let  $Y_2$  denote the amount of pollutant per sample collected above the stack that is equipped with the cleaning device.

Suppose that the joint behaviour of  $Y_1$  and  $Y_2$  can be modelled by

$$f(y_1, y_2) = c \quad \text{for } 0 \leq y_1 \leq 2, \quad 0 \leq y_2 \leq 1, \quad 2y_2 \leq y_1.$$



That is,  $Y_1$  and  $Y_2$  are uniformly distributed over the region inside the triangle bounded by  $y_1 = 2$ ,  $y_2 = 0$  and  $2y_2 = y_1$ .

- a. Find the value of  $c$  that makes this function a valid probability density function.

[3 marks]

- b. Find  $P(Y_1 \geq 3Y_2)$ . That is, find the probability that the cleaning device reduced the amount of pollutant by one-third or more.

$$\int_0^1 \int_{2y_2}^1 c \, dy_1 \, dy_2$$

$$\int_0^1 \int_{2y_2}^1 c \, dy_1 \, dy_2 = \int_0^1 c(1 - 2y_2) \, dy_2$$

[3 marks]

**Q4 Continuous bivariate random variables.** Consider two random variables  $Y_1$  and  $Y_2$  with the joint probability density function

$$f(y_1, y_2) = \begin{cases} k(1 - y_2) & 0 \leq y_1 \leq y_2 \leq 1, \\ 0 & \text{elsewhere.} \end{cases}$$

- a. Draw a sketch of the area where the probability density function is nonzero.

[2 marks]

- b. Show that for this to be a valid probability density function, we need  $k = 6$ .

[3 marks]

- c. Draw the appropriate integration region and find  $P(Y_1 \leq 1/4, Y_2 \leq 3/4)$ .

[3 marks]

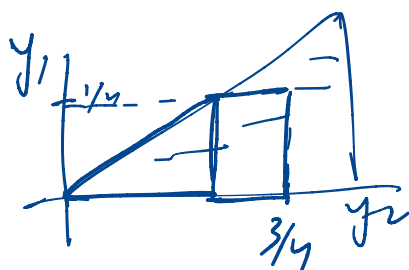
- d. Find the marginal density function of  $Y_1$ .

$$\int_0^1 \int_{y_1}^1 k(1 - y_2) \, dy_2 \, dy_1$$

[2 marks]

- e. Find the marginal density function of  $Y_2$ .

[2 marks]



$$k y_2 - \frac{k y_2^2}{2}$$

$$\int_0^{1/4} \int_{y_1}^{3/4} 6(1 - y_2) \, dy_2 \, dy_1$$

[Question total: 12]

[[Assignment total: 34]]