STATS 2103: Probability and Statistics II: Assignment 2

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CHECKLIST

- ☐: Have you shown all of your working, including probability notation where necessary?
- ☐: Have you given all numbers to 3 decimal places unless otherwise stated?
- ☐: Have you included all R output and plots to support your answers where necessary?
- ☐: Have you included all of your R code?
- ☐: Have you made sure that all plots and tables each have a caption?
- ☐: If before the deadline, have you submitted your assignment via the online submission on Canvas?
- ☐: Is your submission a single pdf file - correctly orientated, easy to read? If not, penalties apply.
- ☐: Penalties for more than one document - 10% of final mark for each extra document. Note that you may resubmit and your final version is marked, but the final document should be a single file.
- ☐: Penalties for late submission - within 24 hours 40% of final mark. After 24 hours, assignment is not marked and you get zero.
- ☐: Assignments emailed instead of submitted by the online submission on Canvas will not be marked and will receive zero.
- ☐: Have you checked that the assignment submitted is the correct one, as we cannot accept other submissions after the due date?

Due date: Friday 3rd April 2020 (Week 5), 5pm.

$$\left(1 - \frac{50}{52}\times\frac{49}{51}\times\frac{48}{50}\times\frac{47}{49}\times\frac{46}{48}\right)$$

Q1: Discrete random variables

a. A standard deck of cards is shuffled and 5 cards are randomly selected. What is the probability of at least one red queen if

1 - No Red queen

- the 5 cards are selected without replacement?
- the 5 cards are selected with replacement? $1-\left(\frac{50}{52}\right)^5$

[4 marks]

b. Customers arrive at a checkout according to a Poisson distribution with an average of 7 per hour. During a given hour, what are the probabilities that

b.) 1 - (0,1)

$1 - e^{-7}(1+7)$

0.9927

$\lambda = 7$ $\quad P(Y) = \frac{e^{-\lambda}\lambda^y}{y!}$

- no more than three customers arrive? $P(0)+P(1)+P(2)+P(3)$
- at least two customers arrive?
- exactly five customers arrive?

$e^{-7}\left(\frac{7^0}{0}+\frac{7^1}{1}+\frac{7^2}{2}+\frac{7^3}{6}\right)$ $\quad \frac{6+42+147+343}{6}$ $= 0.0817$ (b.1)

$y=5$ $\quad e^{-7}\left(\frac{7^5}{5!}\right) = 0.1277$

[6 marks]

[Question total: 10]

Q2: Recursive binomial Let Y be a binomial random variable with number of trials n and probability of success p, i.e.,

$$Y \sim Bin(n, p).$$

a. Show that

$$\frac{\cancel{(y-1)!}(n-y+1)!}{y!\cancel{(n-y)!}} \quad p\,(1-p)^{-1}$$

$n = 10$
$p = 2x$
$p' = x$

$P(Y=y) = {}^{n}C_{y}(p)^{y}(1-p)^{n-y}$
$P(Y=y-1) = {}^{n}C_{y-1}(p)^{(y-1)}(1-p)^{n-y+1}$ =

$\dfrac{\dfrac{n!}{y!(n-y)!}}{\dfrac{n!}{(y-1)!(n-y+1)!}}$ — $p^{y-y+1)}$

$(1-p)^{\cancel{n-y-n}+y-1}$

$$P(Y = y) = \frac{n - y + 1}{y} \times \frac{p}{1 - p} \times P(Y = y - 1).$$

[3 marks]

b. If it is known that for a particular binomial random variable with 10 trials the probability of getting one success is exactly twice the probability of getting no successes, what is the value of p?

$P(1) = {}^{10}C_{1}(p)^{1}(1-p)^{9} = 2({}^{10}C_{0}(p)^{0}(1-p)^{10}$
$= 5p\,\cancel{(1-p)^{9}} = (1-p)^{10}$ $\quad 1 = 6p \quad (p = \frac{1}{6})$

[4 marks]

[Question total: 7]

Q3: Binomial MGF If Y has a binomial distribution with n trials and probability of success p show that the moment generating function for Y is

$E[Y] = np$

$$m(t) = (pe^t + q)^n,$$

$m(t) := E[e^{tY}]$

where

$$q = 1 - p.$$

$m(t) = E[e^{t}$

Using this result, find

- $E[Y]$,
- $E[Y^2]$ and hence,
- $\text{var}(Y)$.

[7 marks]

[Question total: 7]

Q4: Poisson question In a class with N students, where N is Poisson distributed with parameter λ, each student attends the Tuesday morning lecture with probability γ. (We assume that the students decide whether or not to attend the lecture independently of each other.)

a. Show that the number of students attending the Tuesday morning lecture is Poisson distributed with parameter $\lambda\gamma$.

$x \sim$ Pois(lambda * gamma)
$= P(x$ intersect $N)$
$P(x) =$ sum $P(x|N)P(N)$
$\Sigma P(x \cap N)$

[6 marks]

[Question total: 6]

[[Assignment total: 30]]

$P(x|N)$ binomial
$= \binom{n}{x} \gamma^{p}(1-\gamma)^{n-x}$

Let N = the no of students enrolled in course

N, No of students enrolled
x, no of student attending tuesday

$\sim \quad pe^{t}$