Formation of Partial D. Eqn

[1] Form a P.D.E. from $z = f(x^2-y) + g(x^2+y)$ where f and F are arbitrary constant.

[2] $y = f(x-at) + F(x+at)$ Eliminate the arbitrary functi-

(*) Convert the equation (..............) into partial Differential Equation by eliminating arbitrary function f and F.

[3] $z = (x-a)^2 + (y-b)^2$

[4] $z = (x^2+a)(y^2+b)$

[5] Expree the equation $z = (x^2+a)(y^2+b)$ into partial differential Eqn where a, b are parameter.

[Midspring 23]

Formation of Partial D.Eqⁿ

solve: $Z=(x-a)^2+(y-b)^2$

Soln: Diff with respect to x & y we get,

$$\frac{\delta z}{\delta x}=2(x-a) \quad ; \quad \frac{\delta^2 z}{\delta x^2}=2$$

$$\frac{\delta z}{\delta y}=2(y-b) \quad ; \quad \frac{\delta^2 z}{\delta y^2}=2$$

$$\therefore \frac{\delta^2 z}{\delta x^2}+\frac{\delta^2 z}{\delta y^2}=2+2=4$$

Ans:

$Z=(x^2+a)(y^2+b)$ [CT Fall23] → 2 section

Soln: Diff with respect to x & y we get,

$$\frac{\delta z}{\delta x}=(y^2+b)\cdot 2x \quad \Big| \quad \frac{\delta z}{\delta y}=(x^2+a)\cdot 2y$$

$$\Rightarrow \frac{\delta^2 z}{\delta x^2}=(y^2+b).2 \quad \Big| \quad \Rightarrow \frac{\delta^2 z}{\delta y^2}=(x^2+a).2$$

Now, $Z=(x^2+a)(y^2+b)$

$$\Rightarrow Z=\frac{1}{2}\frac{\delta^2 z}{\delta y^2}\cdot\frac{1}{2}\frac{\delta^2 z}{\delta x^2}$$

$y = f(x-at) + F(x+at)$

Diff it with respect to x and t we get,

$$\begin{cases} \dfrac{\partial y}{\partial x} = f'(x-at) + F'(x+at) \\[2ex] \dfrac{\partial^2 y}{\partial x^2} = f''(x-at) + F''(x+at) \quad \text{—— ①} \end{cases}$$

$$\begin{cases} \dfrac{\partial y}{\partial t} = f'(x-at)(-a) + F'(x+at)(a) \\[2ex] \dfrac{\partial^2 y}{\partial t^2} = f''(x-at)\, a^2 + F''(x+at)\, a^2 \end{cases}$$

$$\Rightarrow \frac{\partial^2 y}{\partial t^2} = a^2 \left[f''(x-at) + F''(x+at) \right]$$

$$\Rightarrow \frac{1}{a^2} \frac{\partial^2 y}{\partial t^2} = \frac{\partial^2 y}{\partial x^2} \quad \text{[putting value from ①]}$$

which is the Partial Derivative equation.

$z = f(x^2-y) + g(x^2+y)$

Diff it with respect to x, y we get,

$$\frac{\delta z}{\delta x} = f'(x^2-y)\,2x + g'(x^2+y)\,2x$$

$$\frac{\delta^2 z}{\delta x^2} = f''(x^2-y)\,4x^2 + g'(x^2+y)\,4x^2 \quad \text{--- (1)}$$

$$\frac{\delta z}{\delta y} = f'(x^2-y)(-1) + g'(x^2+y)\,1$$

$$\frac{\delta^2 z}{\delta y^2} = + f''(x^2-y) + g''(x^2+y)$$

From (1) $\Rightarrow$

$$\frac{\delta^2 z}{\delta x^2} = 4x^2\left[f''(x^2-y) + g'(x^2+y)\right]$$

$$\Rightarrow \frac{\delta^2 z}{\delta x^2} = 4x^2\,\frac{\delta^2 z}{\delta y^2}$$

$$\Rightarrow \frac{\delta^2 z}{\delta y^2} = \frac{1}{4x^2}\,\frac{\delta^2 z}{\delta x^2}$$

Find the de from

$z = Ae^{-pt} . \cos qx . \sin ry$ — (i) where $p^2 = q^2 + r^2$

differentiating eqn (i) with respect to x, y, t.

$$\frac{\delta z}{\delta x} = Ae^{-pt} . \sin ry . (-\sin qx)\, q$$

$$\Rightarrow \frac{\delta^2 z}{\delta x^2} = -Aqe^{-pt} \sin ry \cos qx \quad \text{— (i)}$$

$$\frac{\delta z}{\delta y} = Ae^{-pt} . \cos qx . \cos ry . r$$

$$\Rightarrow \frac{\delta^2 z}{\delta y^2} = -Ar^2 e^{-pt} . \cos qx . \sin ry \quad \text{— (ii)}$$

$$\frac{\delta z}{\delta t} = A\cos qx \sin ry . e^{-pt} . (-p)$$

$$\Rightarrow \frac{\delta^2 z}{\delta t^2} = +Ap^2 \cos qx \sin ry . e^{-pt} \quad \text{— (iii)}$$

Adding (i), (ii), (iii) we get,

$$\frac{\delta^2 z}{\delta x^2} + \frac{\delta^2 z}{\delta y^2} + \frac{\delta^2 z}{\delta t^2} = Ae^{-pt} \sin ry \cos qx \,(p^2 - q^2 - r^2)$$

$$= Ae^{-pt} \sin ry \cos q \,(p^2 - p^2)$$

$$= 0$$

Ans:

$Z = A.e^{pt}.\sin px$

Soln: Diff with respect to t and x we get,

$$\frac{\delta z}{\delta x} = A.e^{pt} \cos px . p$$

$$\Rightarrow \frac{\delta^2 z}{\delta x^2} = A.e^{pt} \sin px (-p^2)$$

$$\frac{\delta z}{\delta t} = A.\sin px . e^{pt} . p$$

$$\Rightarrow \frac{\delta^2 z}{\delta t^2} = Ap^2 \sin px . e^{pt}$$

Now, Adding we get,

$$\frac{\delta^2 z}{\delta x^2} + \frac{\delta^2 z}{\delta t^2} = -p^2 A e^{pt} \sin px + p^2 A \sin px \, e^{pt}$$

$$= 0$$

Ans: