# Type 1

**Ex:** $\left(\frac{y^2 z}{x}\right)p + xzq = y^2$ [CT Fall 2023]

**Soln:** Given eqn is of the formula $Pp+Qq=R$

Lagrange's Auxiliary Equation is,

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$$

$$\Rightarrow \frac{dx}{\left(\frac{y^2 z}{x}\right)} = \frac{dy}{xz} = \frac{dz}{y^2}$$

$$\Rightarrow \frac{x\,dx}{y^2 z} = \frac{dy}{xz} = \frac{dz}{y^2}$$

Taking 1st two fraction,

$$\frac{x\,dx}{y^2 z} = \frac{dy}{xz}$$

$$\Rightarrow x^2 dx = y^2 dy$$

$\Rightarrow$ Now integrating it we get,

$$\int x^2 dx = \int y^2 dy$$

$$\Rightarrow \frac{x^3}{3} = \frac{y^3}{3} + \frac{c_1}{3}$$

$$\Rightarrow x^3 - y^3 = c_1$$

$$\therefore u(x,y,z) = x^3 - y^3 - c_1 = 0$$

Again choosing 1st and third fraction,

$$\frac{x\,dx}{y^2 z} = \frac{dz}{y^2}$$

$$\Rightarrow \frac{x\,dx}{z} = \frac{dz}{1}$$

$$\Rightarrow \int x\,dx = \int z\,dz$$

$$\Rightarrow \frac{x^2}{2} = \frac{z^2}{2} + \frac{c_2}{2}$$

$$\Rightarrow x^2 - z^2 = c_2$$

$$\therefore v(x,y,z) = x^2 - z^2 - c_2 = 0$$

General Solution is,

$$\phi(x^3 - y^3, x^2 - z^2) = 0$$

or, $\phi(x^3 - y^3) = x^2 - z^2$

or, $\phi(x^2 - z^2) = x^3 - y^3$ Ans!

**Ex:** $xyp + yzq = zx$ [CT : Fall 2023] (2sec)

**Soln:** Given eqn is of the formula $Pp + Qq = R$

Lagrange's Auxiliary Equation is,

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$$

$$\Rightarrow \frac{dx}{xy} = \frac{dy}{yz} = \frac{dz}{zx}$$

Taking 1st two fraction,

$$\frac{dx}{xy} = \frac{dy}{yz}$$

$$\Rightarrow \frac{dx}{x} = \frac{dy}{z}$$

$$\cancel{\Rightarrow \int \frac{1}{x} dx = \int \frac{1}{z} dy}$$

$$\cancel{\Rightarrow \ln x = \ln y + \ln c_1}$$

$$\cancel{\Rightarrow \frac{x}{y} = c_1}$$

$$\Rightarrow \int z\, dx = \int x\, dy$$

$$\Rightarrow zx = xy + c_1$$

$$\Rightarrow zx - xy = c_1$$

$$u(x, y, z) = zx - xy - c_1 = 0$$

Again choosing 1st and third fraction,

$$\frac{dx}{xy} = \frac{dz}{zx}$$

$$\Rightarrow \frac{dx}{y} = \frac{dz}{z}$$

$$\Rightarrow \int z\, dx = \int y\, dz$$

$$\Rightarrow zx - yz = c_2$$

$\therefore$ G.S. $\Rightarrow$

$$\phi(zx - xy, zx - yz) = 0$$

**Ans:**

Ex: $p\tan x + q\tan y = \tan z$ [CT Fall 2023] 2sec

Soln: Given eqn is of the formula $Pp + Qq = R$

Lagrange's Auxiliary equation is

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$$

$$\Rightarrow \frac{dx}{\tan x} = \frac{dy}{\tan y} = \frac{dz}{\tan z}$$

taking 1st two fraction,

$$\frac{dx}{\tan x} = \frac{dy}{\tan y}$$

$$\Rightarrow \int \cot x\, dx = \int \cot y\, dy$$

$$\Rightarrow \ln|\sin x| = \ln|\sin y| + \ln c_1$$

$$\Rightarrow \ln|\sin x| - \ln|\sin y| = \ln c_1$$

$$\Rightarrow \frac{\sin x}{\sin y} = c_1$$

$$\therefore u(x,y,z) = \frac{\sin x}{\sin y} - c_1 = 0$$

taking 1st and 3rd,

$$\frac{dx}{\tan x} = \frac{dz}{\tan z}$$

$$\Rightarrow \int \cot x\, dx = \int \cot z\, dz$$

$$\Rightarrow \ln|\sin x| = \ln|\sin z| + \ln c_2$$

$$\Rightarrow \frac{\sin x}{\sin z} = c_2$$

$$\therefore v(x,y,z) = \frac{\sin x}{\sin z} - c_2 = 0$$

$$\therefore \text{G.S. is} \Rightarrow \phi\left(\frac{\sin x}{\sin y}, \frac{\sin x}{\sin z}\right) = 0$$

Ans:

Ex: $y^2p + xyq = x$ [CT: Fall 2023]

Soln: Given equn is of the Formula $Pp + Qq = R$

Lagrange's Auxiliary Equation is,

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$$

$$\Rightarrow \frac{dx}{y^2} = \frac{dy}{-xy} = \frac{dz}{x}$$

taking first two fraction,

$$\frac{dx}{y^2} = \frac{dy}{-xy}$$

$$\Rightarrow \frac{dx}{y} = \frac{dy}{-x}$$

$$\Rightarrow \int x\,dx = \int -y\,dy$$

$$\Rightarrow \frac{x^2}{2} = -\frac{y^2}{2} + \frac{c_1}{2}$$

$$\Rightarrow x^2 + y^2 = c_1$$

$\therefore u(x,y,z) = x^2 + y^2 - c_1 = 0$

taking ~~2nd~~ and 3rd fraction,

$$\frac{\cancel{dx}}{\cancel{y^2}} = \frac{\cancel{dz}}{\cancel{x}}$$

$$\frac{dy}{-xy} = \frac{dz}{x}$$

$$\Rightarrow \frac{dy}{-y} = \frac{dz}{1}$$

$$\Rightarrow \int dy = \int -y\,dz$$

$$\Rightarrow y = -yz + c_2$$

$$\Rightarrow y + yz = c_2$$

$\therefore v(x,y,z) = y + yz - c_2 = 0$

G.S. is

$$\phi(x^2+y^2,\ y+yz) = 0$$

Ans:

Ex: $a(p+q) = z \Rightarrow ap + aq = z$

Soln: Given eqn is of the formula $Pp + Qq = R$

Lagrange's Auxiliary equation is,

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$$

$$\Rightarrow \frac{dx}{a} = \frac{dy}{a} = \frac{dz}{z}$$

taking first two fraction,

$$\frac{dx}{a} = \frac{dy}{a} \quad (ii)$$

$$\Rightarrow \int dx = \int dy$$

$$\Rightarrow x - y = c_1$$

$$\therefore u(x,y,z) = x - y - c_1 = 0$$

taking 1st and 3rd fraction,

$$\frac{dx}{a} = \frac{dz}{z}$$

$$\Rightarrow \int \frac{1}{a}\,dx = \int \frac{1}{z}\,dz$$

$$\Rightarrow \frac{1}{a} \cdot x = \ln z + c_2$$

$$\Rightarrow \frac{x}{a} = \ln z + c_2$$

$$\Rightarrow \frac{x}{a} - \ln z = c_2$$

$$\therefore v(x,y,z) = \frac{x}{a} - \ln z - c_2 = 0$$

G.S. is,

$$\varphi\left(x - y,\ \frac{x}{a} - \ln z\right) = 0$$

Ans: