chapter-2

The role of Time value in Finance:

Financial managers and investors can make money by investing in projects or interest bearing options. Time value of money is important because when you get or spend money matters. It's based on the idea that a dollar today is worth more than the same dollar in the future. In Finance, we look at two things:

Future value which about how much money will be worth in the future.

Present value which is about how much money is worth right now.

$$\text{Future value, } FV = CF(1+r)^n$$

$$\text{Present value, } PV = \frac{CF}{(1+r)^n}$$

Here,

CF = Cash flow

r = interest rate

n = time

In present value,

r = discount rate

Basic patterns of cash flows:

Single amount:

A lump sum is a big amount of money that you either have right now or will get in the future.

Annuity:

A level periodic stream of cash flow. for our purposes, we'll work primarily with annual cash flows.

Mixed stream:

A stream of unequal periodic cash flows that reflect no particular pattern.

Interest rate:

An interest rate is a rate of return that reflects the relationship between differently dated cash flows.

Interest rates have three perspectives:-

1. They are the least amount of profit an investor requires to choose an investment.

2. They work as discount rates.

3. They show the value that investors give up when they choose one option over another.

Real Risk-free interest rate:

The real risk-free interest rate is the interest on a super-safe investment if there's no expectation of prices going-up. It shows how people decide between enjoying things now or later based on time preferences.

Difference between Liquidity premium and Maturity premium.

Liquidity	Maturity
The liquidity premium is like a payment to investors for the risk of losing money if they need to quickly	Maturity pre. is like an extra reward for investors who take on the risk of longer-term investment.
Reflects difficulty in quickly buying or selling an asset.	Reflects the time value of money and uncertainty about future interest rate.
Deals with risk of losing value during fast buying or selling an asset.	Deals with risk related to changes in interest rates over time.

Compound interest:

Compound interest is when you earn interest not only on your starting money but also on the interest you've already earned. It's like your money growing faster by adding interest on top of interest.

Annually Present value, $PV = \left(\frac{CF}{1+r/m}\right)^{mn}$

বা বললে Future value, $FV = PV\left(1+r/m\right)$

where,

r = Annual interest year

n = number of years

m = number of periods based on compounding frequency.

যদি বলে,

Quaterly, $m = 4$

Semi-annually, $m = 2$

monthly, $m = 12$

$\rightarrow PV = \frac{CF}{(1+r/m)^{mn}}$

$FV = CF(1+r/m)^{mn}$

Annually বললে, $PV = \frac{CF}{(1+r)^n}$

_ Practices _

(1) Given,

$PV = \$100,000$

interest rate, $r = 6\% = 0.06$

$m = 12$

$n = 1$

$FV = ?$

$FV = PV \left(1 + \frac{r}{m}\right)^{mn}$

$= 10,00,000 \left(1 + \frac{0.06}{12}\right)^{12 \times 1}$

$= 10,61,677.812$

(2) Given,

$PV = 11,576$

discounted rate, $r = 5\% = 0.05$

$n = 3$

$m = 4$

$\boxed{\begin{array}{ll} \cancel{PV = pv / (1+r/m)^{mn}} & \cancel{FV = CF \left(1 + \frac{r}{m}\right)^{mn}} \\ \cancel{FV = PV \left(1 + \frac{r}{m}\right)^{mn}} & \cancel{= 10,000 \left(1 + \frac{0.05}{2}\right)^{2 \times 3}} \\ \cancel{= 11,576 \left(1 + \frac{0.05}{4}\right)^{12}} & \cancel{= 11,596.93} \end{array}}$

$PV = \frac{CF}{(1 + r/m)^{mn}}$

$= \frac{11,576}{\left(1 + \frac{0.05}{4}\right)^{4 \times 3}}$

$= 9972.82$

(3) Given,

$CF = \$10,000$

$r = 5\% = 0.05$

$m = 2$

$n = 3$

$FV = CF(1+r/m)^{mn}$

$= 10,000 \left(1+\frac{0.05}{2}\right)^{2\times3}$

$= 11,596.93$

(4) Given,

$CF = 11,576$

$r = 5\% = 0.05$

$n = 5$

$m = 4$

$FV = CF(1+r/m)^{mn}$

$= 11,576 \left(1+\frac{0.05}{4}\right)^{4\times5}$

$= 14,840.86$

(5) Given,

$CF = 78,000$

$r = 7.5\% = 0.075$

$m = 12$

$n = 5$

$PV = \frac{CF}{(1+r/m)^{mn}}$

$= \frac{78,000}{\left(1+\frac{0.075}{12}\right)^{12\times5}}$

$= 53671.16$

(6) Given,

$CF = 65,000$

$r = 1.1\% = 0.011$

$n = 5$

$PV = \frac{CF}{(1+r)^n}$

$= \frac{65,000}{(1+0.011)^5}$

$= 61540.012$

Given,

$n = 2$

$r = 8\% = 0.08$

$m = 4$

$CF = 10,000$

$FV = CF(1+r/m)^{mn}$

$= 10,000(1 + \frac{0.08}{4})^{4\times2}$

$= 11716.59$

Continuous compounding :

continuously compounded interest is the mathematical limit of the general compound interest formula.

present value, $PV = \frac{FV}{e^{in}}$

Future value, $FV = PV \times e^{in}$

Given,

$CF = 10,0000$

$r = 12\% = 0.012$

$n = 10$

For annually,

$$FV = CF(1+r)^n$$

$$= 100000(1+0.12)^{10}$$

$$= 310584.82$$

For semi annually, $m = 2$

$$FV = CF\left(1+\frac{r}{m}\right)^{mn}$$

$$= 1,00000\left(1+\frac{0.12}{2}\right)^{2\times10}$$

$$= 320713.54$$

For continuously,

$$FV = CF \times e^{in}$$

$$= 100000 \times (e)^{0.12\times10}$$

$$= 332011.69$$

Annuities:

An annuity is a long-term insurance product that provides guaranteed income.

Ordinary annuity:

An annuity for which the cashflow occurs at the end of each period.

Annuity due:

An annuity for which the cash flow occurs at the begining of each period.

for ordinary annuity -

$$FV = P \times \frac{(1+r)^n - 1}{r}$$

$$PV = P \times \frac{1-(1+r)^{-n}}{r}$$

for annuity due -

$$FV_A = P \times \frac{(1+r)^n - 1}{r} \times (1+r)$$

$$PV_A = P \times \frac{1-(1+r)^{-n}}{r} \times (1+r)$$

(1) Annuity A,

$P = 1,000$

$r = 7\% = 0.07$

$n = 5$

$$PV = P \times \frac{(1+r)^n - 1}{r}$$

$$= 1,000 \times \frac{(1+0.07)^5 - 1}{0.07}$$

$= 5750.73$

Annuity B,

$P = 1,000$

$r = 7\% = 0.07$

$n = 5$

$$FV_{\text{Annuity due}} = P \times \frac{(1+r)^n - 1}{r} \times (1+r)$$

$$= 1,000 \times \frac{(1+0.07)^5 - 1}{0.07} \times (1+0.07)$$

$$= 6153.29$$

2 Given,

$P = 700$

$n = 5$

$r = 8\% = 0.08$

$$PV_0 = P \times \frac{1-(1+r)^{-n}}{r}$$

$$= 700 \times \frac{1-(1+0.08)^{-5}}{0.08}$$

$$= 2794.89$$

③ Project A,

$P = 1,77,000$

$r = 11.75\% = 0.1175$

$n = 5$

$$PV_{Annuity} = P \times \frac{1-(1+r)^{-n}}{r}$$

$$= 1,77,000 \; \frac{1-(1+0.1175)^{-5}}{0.1175}$$

$$= 642016.84$$

Project B,

$P = 1,43,000$

$r = 14.25\% = 0.1425$

$n = 5$

$$PV_{annuity\ due} = P \times \frac{1-(1+r)^{-n}}{r} \times (1+r)$$

$$= 1,43,000 \; \frac{1-(1+0.1425)^{-5}}{0.1425} \times (1+0.1425)$$

$$= 560653.55.$$