FIITJEE

JEE (Advanced)-2019

ALL INDIA TEST SERIES

PART TEST – I

Paper 1

Time Allotted: 3 Hours **Maximum Marks: 180**

- Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.
- You are not allowed to leave the Examination Hall before the end of the test.

INSTRUCTIONS

A. General Instructions

1. Attempt ALL the questions. Answers have to be marked on the OMR sheets.
2. This question paper contains Three Parts.
3. **Part-I** is Physics, **Part-II** is Chemistry and **Part-III** is Mathematics.
4. Each part is further divided into Two sections: **Section-A & Section-D**
5. Rough spaces are provided for rough work inside the question paper. No additional sheets will be provided for rough work.
6. Blank Papers, clip boards, log tables, slide rule, calculator, cellular phones, pagers and electronic devices, in any form, are not allowed.

B. Filling of OMR Sheet

1. Ensure matching of OMR sheet with the Question paper before you start marking your answers on OMR sheet.
2. On the OMR sheet, darken the appropriate bubble with black pen for each character of your Enrolment No. and write your Name, Test Centre and other details at the designated places.
3. OMR sheet contains alphabets, numerals & special characters for marking answers.

C. Marking Scheme For All Three Parts.

1. ***Section-A (01– 06, 19 – 24, 37 - 42)*** *contains 18 multiple choice questions which have* ***one or more than one correct*** *answer. Each question carries* ***+4 marks*** *for all correct answer.*
   *Partial Marks : **+3** If all the four options are correct but ONLY three options are chosen.*
   *Partial Marks : **+2** If three or more options are correct but ONLY two options are chosen, both of which are correct options.*
   *Partial Marks : **+1** If two or more options are correct but ONLY one option is chosen and it is a correct option.*
   *Zero Marks : **0** If none of the options is chosen (i.e. the question is unanswered).*
   *Negative Marks : **–2** In all other cases.*

   Section-A (07 – 10, 25 – 28, 43 - 46) *contains 12 questions. Based on* this section contains **TWO (02)** paragraphs. Based on each paragraph, there are **TWO (02)** questions.
   Each question has ***only one correct*** *answer and carries* ***+3 marks*** *for correct answer and* ***–1 mark*** *for wrong answer.*

2. ***Section-D (11 – 18, 29 – 36, 47 – 54)*** *contains 24 Numerical answer type questions with answer XXXXX.XX and each question carries* ***+3 marks*** *for correct answer. There is no negative marking.*

Name of the Candidate	
Enrolment No.	

Useful Data

PHYSICS

Acceleration due to gravity	$g = 10\ m/s^2$
Planck constant	$h = 6.6 \times 10^{-34}$ J-s
Charge of electron	$e = 1.6 \times 10^{-19}$ C
Mass of electron	$m_e = 9.1 \times 10^{-31}$ kg
Permittivity of free space	$\varepsilon_0 = 8.85 \times 10^{-12}\ C^2/N\text{-}m^2$
Density of water	$\rho_{water} = 10^3\ kg/m^3$
Atmospheric pressure	$P_a = 10^5\ N/m^2$
Gas constant	$R = 8.314\ J\ K^{-1}\ mol^{-1}$

CHEMISTRY

Gas Constant	R	=	$8.314\ J\ K^{-1}\ mol^{-1}$
		=	0.0821 Lit atm $K^{-1}\ mol^{-1}$
		=	$1.987 \approx 2$ Cal $K^{-1}\ mol^{-1}$
Avogadro's Number	N_a	=	6.023×10^{23}
Planck's constant	h	=	6.625×10^{-34} J·s
		=	6.625×10^{-27} erg·s
1 Faraday		=	96500 coulomb
1 calorie		=	4.2 joule
1 amu		=	1.66×10^{-27} kg
1 eV		=	1.6×10^{-19} J

Atomic No: H=1, He = 2, Li=3, Be=4, B=5, C=6, N=7, O=8, N=9, Na=11, Mg=12, Si=14, Al=13, P=15, S=16, Cl=17, Ar=18, K =19, Ca=20, Cr=24, Mn=25, Fe=26, Co=27, Ni=28, Cu = 29, Zn=30, As=33, Br=35, Ag=47, Sn=50, I=53, Xe=54, Ba=56, Pb=82, U=92.

Atomic masses: H=1, He=4, Li=7, Be=9, B=11, C=12, N=14, O=16, F=19, Na=23, Mg=24, Al = 27, Si=28, P=31, S=32, Cl=35.5, K=39, Ca=40, Cr=52, Mn=55, Fe=56, Co=59, Ni=58.7, Cu=63.5, Zn=65.4, As=75, Br=80, Ag=108, Sn=118.7, I=127, Xe=131, Ba=137, Pb=207, U=238.

PART – I (Physics), PART – II (Chemistry), PART – III (Mathematics):

(SECTION – D)

For questions **11 to 18, 29 to 36, 47 to 54.**
Numerical answer type questions with answer XXXXX. XX

If answer is 348.4 / 251.37 / 213

Correct Method :

0	0	3	4	8	.	4	0
0	0	2	5	1	.	3	7
0	0	2	1	3	.	0	0

Wrong Method :

	3	4	8		.	4	
3	4	8			.		4
		3	4	8	.		4
	3		4	8	.	4	
	2		5	1	.	3	7
		2	1	3	.		
		2	1	3	.	0	
		2	1	3	.		0
		3	4	8	.	4	0
		2	5	1	.	3	7
		2	1	3	.	0	0

Physics

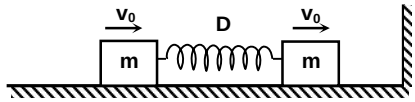
PART – I

SECTION – A

(One or More than one correct type)

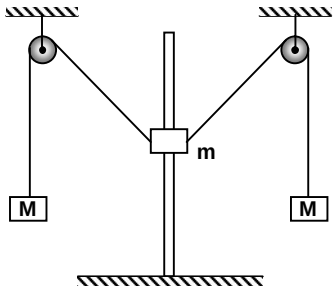
This section contains **6** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four options is(are) correct.

1. Bodies of mass m=0.5 kg, resting on a horizontal frictionless tabletop, are connected with an unstretched spring of length L=20 cm, and of spring constant k =16 N/m. The mass of the spring is negligible. At a certain moment the bodies are given an initial speed of v_0=0.36 m/s, towards the wall on the right. The body at the right collides with the wall totally elastically.



(A) The greatest compression of the spring during the motion is 9cm .
(B) The right body hit the wall again, when the spring is unstretched
(C) The change in the linear momentum of the system after all collisions will be 0.72kgm/s.
(D) Finally both the bodies will have same velocity .

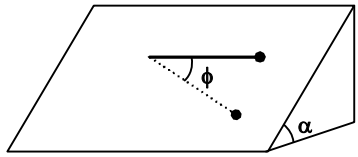
2. A point like object of mass m is able to move up and down on a vertical fixed rod. There is a vertical hole at the centre of the object, and the rod fits into this hole, such that the object can slide along the rod frictionlessly. Two pieces of thin light threads are attached to the object, and initially the angle between both threads and the vertical is $\alpha = 45^o$. Each thread goes through a pulley, which are at the same height, and at the other end of each thread an object of mass M =1 kg is tied. The system is released from rest. After releasing the system it stops at the position where the threads attached to it are horizontal.



(A) $m = 2M(\sqrt{2}-1)$
(B) The acceleration of the object of mass m when it starts to move back is zero.
(C) The acceleration of the object of mass m when it starts to move back is g.
(D) The acceleration of the other two objects of mass M, when they start to move back is g/2.

Space for Rough work

3. One end of a thread of length l = 1 m is attached to an inclined plane of angle of elevation of $\alpha = 30^{o}$. A point-like body of mass m = 1 kg is attached to the other end of the thread as shown in the figure. The body is released without initial speed, such that the thread is tight and horizontal. The coefficient of friction between the slope and the body is $\mu = 0.2$.



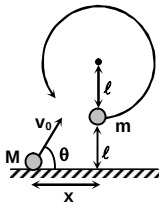
(A) The tension in the thread when the angle between the thread and the horizontal is φ is $T = (3 \sin\varphi \sin\alpha - 2\mu\varphi \cos\alpha)mg$

(B) The tension in the string will be maximum when $\varphi = 90^{0}$.

(C) The tension in the string will be maximum when $\phi = \cos^{-1}\left(\frac{2\mu}{3\tan\alpha}\right)$

(D) The maximum speed of the particle will be when $\varphi = \cos^{-1}(\mu\cot\alpha)$

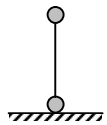
4. A small ball of mass m=0.1 kg is attached to a ℓ =1 m-long thread and is hung to a horizontal peg. The small ball is at rest and another small ball of mass M=0.2 kg, is projected from the ground and collides with it, such that the collision is totally elastic and head on, and after the collision the ball at the end of the thread completes a whole circle around the peg. The distance between the peg and the ground is 2ℓ. The value of x can be :



(A) 0.5m (B) 1m
(C) 2m (D) 2.5m

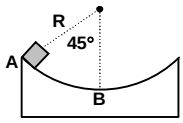
Space for Rough work

5. Two tennis balls of mass 60 g are attached with a massless rubber thread, and held in the vertical position as shown in the figure. In this position the unstretched length of the rubber thread is 40 cm. The upper ball is slowly raised vertically upward, until the lower ball just becomes unsupported by the ground. At this time the length of the thread is 1 m. The rubber thread exerts a force which is proportional to its extension.



(A) Work done by external agent while the upper ball was raised is 0.53J.
(B) Work done by external agent while the upper ball was raised is 0.23J.
(C) Releasing the upper ball, the speed which it hits the lower one is 5.1m/s.
(D) The time that elapses between the release of the upper ball and the collision is 0.34 sec.

6. A small rubber eraser is placed at one edge of a quarter-circle-shaped track of radius *R* that lies in a vertical plane and has its axis of symmetry vertical (*see* figure); it is then released. The coefficient of friction between the eraser and the surface of the track is $\mu = 0.6$. Will the eraser reach the lowest point of the track?



(A) If the particle slides from A to B work done by frictional force will be $\frac{0.6mgR}{\sqrt{2}}$.

(B) If the particle slides from A to B work done by frictional force will be greater than $\frac{0.6mg\pi R}{4\sqrt{2}}$.

(C) The particle will never be able to go from A to B.
(D) If μ is $\sqrt{2}$ the particle will not begin to slide.

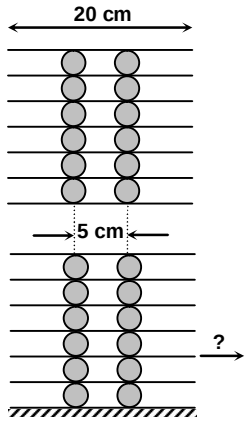
Space for Rough work

(Paragraph Type)

This section contains **TWO** paragraphs. Based on the paragraph, there are **TWO** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is correct.

Paragraph for Question Nos. 7 and 8

A tall tower is built from alike tubes, which have thin walls, and from rectangular sheets, which have negligible mass with respect to that of the tubes, as shown *in the figure.* rectangular sheets are numbered from bottom as S_1 , S_2 , S_3 …..(Static friction is big enough, such that the tubes does not slip.) The radius of each tube is 1 cm.



7. The minimum distance by which S_2 should be pulled towards right so that the tower collapses is :
   (A) 2cm (B) 3cm
   (C) 4cm (D) 5cm

8. The minimum distance by which S_6 should be pulled towards right so that the tower collapses is :
   (A) 2cm (B) 3cm
   (C) 4cm (D) 5cm

Space for Rough work

Paragraph for Question Nos. 9 and 10

A particle is suspended by three equal strings, of length a, from three points forming an equilateral triangle of side 2b, in the horizontal plane.

9. The tension in the string will be

(A) $\frac{3mga}{\sqrt{3(3a^2-4b^2)}}$ (B) $\frac{mga}{\sqrt{3(3a^2-4b^2)}}$

(C) $\frac{3mga}{\sqrt{(4a^2-3b^2)}}$ (D) $\frac{mga}{\sqrt{(4a^2-3b^2)}}$

10. If one string be cut, the tension of each of the other two strings will be changed in the ratio (i.e. the ratio of final tension after cutting the string to tension before cutting the string):

(A) $\frac{3a^2-4b^2}{2(a^2-b^2)}$ (B) $\frac{4a^2-3b^2}{2(a^2-b^2)}$

(C) $\frac{3a^2-4b^2}{(a^2-b^2)}$ (D) $\frac{4a^2-3b^2}{(a^2-b^2)}$

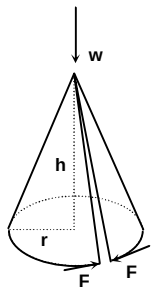
Space for Rough work

SECTION – D

(Numerical Answer Type)

This section contains **EIGHT** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**; e.g. xxxxx.xx).

11. A cone with height h= 1 m and a base circle of radius r = 1 m is formed from a sector- shaped sheet of paper. The sheet is of such a size and shape that its two straight edges almost touch on the sloping surface of the cone. In this state the cone is stress-free. The cone is placed on a horizontal, slippery table-top, and loaded at its apex with a vertical force of magnitude w = 2π, without collapsing. The splaying of the cone is opposed by a pair of forces of magnitude F acting tangentially at the join in the base circle (see figure). Ignoring any frictional or bending effects in the paper, find the value of F.



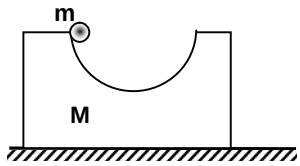
12. The two ends of a 40 cm long chain are fixed at the same height, as shown in the figure. Find the radius of curvature of the chain at its lowest point in cm.

13. A student of height h jumps vertically up from the "squat" position. At the top point of the jump, the student's center of mass is at a height 3h/4 from the ground. Find the average force F acting on the floor prior to the moment when the student loses contact with the floor. It is known that when the student stands on the floor, the center of mass is at a height h/2 from the floor; in the "squat" position, the center of mass is at a height h/4 from the floor. The mass of the student is m= 0.1kg.(take g=10m/s^2)

14. Two points, A and B, are located on the ground a certain distance d = $10\sqrt{2}$m apart. Two rocks are launched simultaneously from points A and B with equal speeds but at different angles. Each rock lands at the launch point of the other. Knowing that one of the rocks is launched at an angle $\theta = 37°$ with the horizontal, what is the minimum distance between the rocks during the flight? (given cos 37° = 3/5)

Space for Rough work

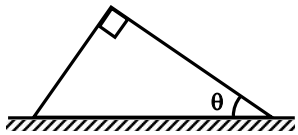
15. A semi-cylinder is cut out of a block and the block is kept on the horizontal surface. A small body of mass m is released at the top of the cylinder shaped hole. (Friction is negligible.)
If the block is able to move on the horizontal surface without friction, then the force exerted on the block by the small body at the lowest point is 7mg/2 . Find the ratio of M/m?



16. A projectile launched from the ground explodes into three fragments of equal mass at the top point of the trajectory. One of the fragments lands t seconds after the explosion two other fragments land simultaneously 2t seconds after the explosion. How high above the ground does the projectile explode? (take t= 2/5 seconds and g= 10m/s^2)

17. A vertical rod of mass 4 kg is hanging on a rope and a 3-kg cat is grasping the bottom end of the rod. The rope is cut off and the frightened cat begins to run up on the rod. While the rod is falling vertically, the cat remains at the same height, with respect to the ground. The acceleration of the rod is kg/4 m/s^2 where g is the acceleration due to gravity. Find k.

18. One stick leans on another as shown in Figure . A right angle is formed where they meet, and the right stick makes an angle θ with the horizontal. The left stick extends infinitesimally beyond the end of the right stick. The coefficient of friction between the two sticks is μ. The sticks have the same mass density per unit length and are both hinged at the ground. The minimum angle θ for which the sticks don't fall is 30^0. Find μ .



Space for Rough work

Chemistry

PART – II

SECTION – A

(One or More than one correct type)

This section contains **6** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four options is(are) correct.

19. A mixture of 2 moles of Argon and 4 moles of PCl_5 (g) were introduced in a 80 litre evacuated vessel at 527°C. The following equilibrium was established

$$PCl_5(g) \rightleftharpoons PCl_3(g) + Cl_2(g)$$

The total pressure of the gaseous mixture in the vessel at equilibrium was found to be 6.568 atm. Which of the following statement(s) regarding the above equilibrium is/are correct?

(A) K_c for the equilibrium is 0.025
(B) K_P for the equilibrium is 1.642 atm
(C) Total number of moles of gaseous species at equilibrium is 6
(D) Number of moles of PCl_5 at equilibrium is 2

20. The correct statement(s) among the following is/are

(A) Bond angle in NH_3 and NF_3 are same
(B) Bond angle in CH_4 and CF_4 are same
(C) Bond angle in PH_3 is greater than AsH_3
(D) Bond angle in BH_4^- and BF_4^- are same

21. The correct reaction(s) among the following is/are

(A) $3B_2O_3 + P_2O_5 \longrightarrow 2P(BO_2)_3 + O_2$
(B) $B_2O_3 + CoO \longrightarrow Co(BO_2)_2$
(C) $H_3BO_3 + 3HF \longrightarrow 2BF_3 + 3H_2O$
(D) $BCl_3 + 3H_2O \longrightarrow H_3BO_3 + 3HCl$

22. Which of the following statement(s) is/are correct?

(A) In the solid $NaHCO_3$ the HCO_3^- ions are linked by hydrogen bond forming an infinite chain.
(B) In the solid H_3BO_3, the $B(OH)_3$ units are linked by hydrogen bond.
(C) In chloral hydrate there is intramolecular hydrogen bond.
(D) In solid $KHCO_3$ the HCO_3^- ions are linked by hydrogen bond to form dimeric anion.

23. $KMnO_4$ reacts with $K_2C_2O_4$ and H_2SO_4 to form $MnSO_4$, CO_2, K_2SO_4 and H_2O. Which of the following statement(s) is/are correct for the reaction?

(A) One mole of $KMnO_4$ reacts completely to produce 112 litre of CO_2(g) at STP.
(B) One mole of $KMnO_4$ requires 5 mole of $K_2C_2O_4$ in presence of H_2SO_4 to react completely.
(C) One mole of $K_2C_2O_4$ reacts completely to produce 2 moles of CO_2.
(D) The ratio of stoichiometric coefficient of $KMnO_4$ and $K_2C_2O_4$ in this reaction is 1 : 2.

24. Which of the following hydroxide(s) is/are soluble in excess of NaOH solution?

(A) $Zn(OH)_2$
(B) $Al(OH)_3$
(C) $Fe(OH)_3$
(D) $Ni(OH)_2$

Space for Rough work

(Paragraph Type)

This section contains **TWO** paragraphs. Based on the paragraph, there are **TWO** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is correct.

Paragraph for Question Nos. 25 to 26

The process of selectively precipitating an ion from a solution of more than one ion is called selective precipitation. The selective precipitation of ions from a solution in the form of salt can be done by adding precipitating agent slowly. $AgNO_3$ solid is slowly added to one litre of a solution (without changing the volume) containing 0.1 mole of Cl^- and 0.1 mole of PO_4^{3-}. $K_{sp}AgCl = 1.2 \times 10^{-10}$ and $K_{sp}Ag_3PO_4 = 2.7 \times 10^{-18}$..

Answer the following questions based on the above paragraph.

25. Minimum concentration of Ag^+ required for the precipitation of Ag_3PO_4 is
(A) 1.6×10^{-9} M (B) 3×10^{-6} M
(C) 9×10^{-6} M (D) 3×10^{-9} M

26. The concentration of Cl^- in the solution when Ag_3PO_4 starts precipitating is
(A) 1.2×10^{-9} M (B) 4×10^{-11} M
(C) 4×10^{-6} M (D) 4×10^{-5} M

Paragraph for Question Nos. 27 to 28

Solid NH_4HS was heated at 100^oC in a closed container. The following equilibrium was established

$NH_4HS(s) \rightleftharpoons NH_3(g) + H_2S(g)$

K_P for the equilibrium was found to be 0.36 atm^2. Answer the following question based on the above equilibrium.

27. The total pressure of the mixture of a gases at equilibrium is
(A) 0.6 atm (B) 1.2 atm
(C) 0.18 atm (D) 0.9 atm

28. Which of the following statement is correct for the above equilibrium
(A) Addition of NH_4HS (s) at equilibrium increases the concentration of NH_3(g)
(B) Addition of inert gas at constant volume increases the number of moles of H_2S
(C) Addition of inert gas at constant pressure increases the number of moles of NH_3(g)
(D) Addition of NH_3 at equilibrium increases K_p for the reaction

Space for Rough work

SECTION – D

(Numerical Answer Type)

This section contains **EIGHT** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**; e.g. xxxxx.xx).

29. At a certain temperature, the equilibrium constant (K_c) is 0.36 for the reaction

$$SO_2(g) + NO_2(g) \rightleftharpoons SO_3(g) + NO(g)$$

If 1.8 mole each of all the four gases are added in 1 litre container, the number of moles of $SO_3(g)$ at equilibrium is

30. 20 ml of $\frac{M}{10}$ H_3PO_4 solution is treated with 40 ml of $\frac{M}{10}$ NaOH solution pK_{a_1}, pK_{a_2} and pK_{a_3} of H_3PO_4 are 2.15, 7.2 and 12.3 respectively. The pH of the resulting solution is

31. Total energy of electron in the 1st orbit of hydrogen atom is 13.6 eV/atom. Kinetic energy of electron in the 1st excited state of Li^{2+} ion in eV is

32. The half-life of first order decomposition of NH_2NO_2 is 6.93 hrs. at 12°C. If 6.2 g of NH_2NO_2 is allowed to decompose, then the time taken in hrs for 99% decomposition of NH_2NO_2 is

33. A catalyst decreases activation energy of a reaction from 42 kJ mol^{-1} to x kJ $mole^{-1}$. The rate of reaction in the absence of the catalyst at 500 K is equal to the rate of the reaction in the presence of catalyst at 450 K. The value of x is:

34. The half-life period of a 1st order reaction is 60 min. What percent of the reactant will be left after 180 min?

35. The weight in gram of $K_2Cr_2O_7$ required to produce 5.6 L of CO_2 at STP from excess of oxalic acid in H_2SO_4 is : (MW of $K_2Cr_2O_7$ = 294)

36. The wavelength of radiation in $\overset{o}{A}$ required to excite an electron in the ground state of He^{2+} to the 2nd energy level is: $\left(\frac{1}{R} = 911.7\,\overset{o}{A}\right)$

Space for Rough work

Mathematics

PART – III

SECTION – A

(One or More than one correct type)

This section contains **6** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four options is(are) correct.

37. If $\lim\limits_{n\to\infty} \dfrac{\sqrt[n^2]{1!\,2!\,.....\,n!}}{n^{\alpha}} = \beta$, where β is a non-zero real number, then

(A) $\alpha = \dfrac{3}{4}$ (B) $\alpha = \dfrac{1}{2}$

(C) $\beta = e^{-3/4}$ (D) $\beta = e^{-1/2}$

38. Ordered triple(s) (a, b, c) of positive reals that satisfy [a]bc = 3, a[b]c = 4, ab[c] = 5 (where [.] denotes the greatest integer function) is/are

(A) $\left(\dfrac{\sqrt{30}}{3}, \dfrac{\sqrt{30}}{4}, \dfrac{2\sqrt{30}}{5}\right)$ (B) $\left(\dfrac{\sqrt{30}}{3}, \dfrac{\sqrt{30}}{2}, \dfrac{\sqrt{30}}{5}\right)$

(C) $\left(\dfrac{\sqrt{20}}{3}, \dfrac{\sqrt{20}}{3}, \dfrac{2\sqrt{20}}{5}\right)$ (D) $\left(\dfrac{\sqrt{20}}{3}, \dfrac{\sqrt{20}}{3}, \dfrac{\sqrt{20}}{5}\right)$

39. The function f: R $\to$ R satisfies $f(x^2)f''(x) = f'(x)f'(x^2)\ \forall\ x \in R$, given that f(1) = 1 and f‴(1) = 8, then

(A) f′(1) = 2 (B) f″(1) = 4

(C) f′(1) = 4 (D) f″(1) = 2

40. For $n \geq 0$, we have $I_n = \int\limits_{-\pi}^{\pi} \dfrac{\sin nx}{\left(1+2^x\right)\sin x}dx$, then

(A) if n is even, then $I_n = 0$ (B) if n is odd, then $I_n = \pi$

(C) if n is even, then $I_n = \pi$ (D) if n is odd, then $I_n = 0$

Space for rough work

41. Suppose a and b are real numbers such that $\lim_{x\to 0}\frac{\sin^2 x}{e^{ax}-bx-1}=\frac{1}{2}$, then

(A) a = 2, b = 2 (B) a = –2, b = –2
(C) a = 2, b = –2 (D) a = –2, b = 2

42. Let a, b be real numbers such that $\lim_{x\to 1}\frac{\left(\ln(2-x)\right)^2}{x^2+ax+b}=1$, then

(A) a = 2 (B) b = 1
(C) a = –2 (D) b = –1

(Paragraph Type)

This section contains **TWO** paragraphs. Based on the paragraph, there are **TWO** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is correct.

Paragraph for Question Nos. 43 to 44

Read the following write up carefully and answer the following questions:

An equation is called variable separable if it is of the form $\frac{dy}{dx}=f(x)g(y)$. In this case we formally separate the variable and write $\int\frac{dy}{g(y)}=\int f(x)dx$, which after the integration gives the solution in implicit form.

43. Differentiable function f: R $\to$ R satisfying the equation $f(x)=(1+x^2)\left[1+\int_0^x\frac{f(t)}{1+t^2}dt\right]$ is

(A) $f(x) = ce^x(1 - x^2)$ (B) $f(x) = ce^x(1 + x^3)$
(C) $f(x) = ce^x(1 + x^2)$ (D) $f(x) = ce^x(1 - x^3)$

44. Suppose f(x) and g(x) are differentiable functions such that $xg(f(x))f'(g(x))g'(x) = f(g(x))g'(f(x))f'(x)$ $\forall\ x \in R$. Moreover, f(x) is non-negative, g(x) is positive and $\int_0^x f(g(t))dt = 1-\frac{e^{-2x}}{2}\ \forall\ x \in R$.

If g(f(0)) = 1, then g(f(4)) is

(A) e^{-16} (B) e^{-8}
(C) e^{-4} (D) e^{-2}

Space for rough work

Paragraph for Question Nos. 45 to 46

Read the following write up carefully and answer the following questions:

If $g'(x) = f(x)$, then $\int_a^b f(x)\,dx = g(b) - g(a)$.

45. Evaluate $\int_1^{\infty}\left(\frac{\ln x}{x}\right)^{2018} dx$

(A) $\frac{2016!}{(2017)^{2019}}$ (B) $\frac{2017!}{(2018)^{2019}}$

(C) $\frac{2017!}{(2019)^{2018}}$ (D) $\frac{2018!}{(2017)^{2019}}$

46. The value of integral $\int_{-1}^{1}\frac{2x^{332}+x^{998}+4x^{1664}\sin x^{691}}{1+x^{666}}dx$

(A) $\frac{2}{333}\left(1+\frac{\pi}{3}\right)$ (B) $\frac{2}{333}\left(1+\frac{\pi}{5}\right)$

(C) $\frac{2}{333}\left(1-\frac{\pi}{4}\right)$ (D) $\frac{2}{333}\left(1+\frac{\pi}{4}\right)$

SECTION – D

(Numerical Answer Type)

This section contains **EIGHT** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**; e.g. xxxxx.xx).

47. Let f: R $\to$ R be a continuous function with $\int_0^1 f(x)f'(x) = 0$ and $\int_0^1 (f(x))^2 f'(x)dx = 18$, then the value of $\int_0^1 (f(x))^4 f'(x)dx$ is _____

Space for rough work

48. Let f: R $\to$ R be a differentiable function such that f(0) = 0, f(1) = 1 and $|f'(x)| < 2\ \forall\ x \in R$, if a and b are real numbers such that the set of possible values of $\int_0^1 f(x)dx$ is the open interval (a, b), then (b – a) is _____

49. Let f: R $\to$ R be a continuous function which satisfies the identity $f(2x) = 3f(x)\ \forall\ x \in R$.
If $\int_0^1 f(x)dx = 1$, then $\frac{1}{2}\int_1^2 f(x)dx$ is _____

50. Let $f^1(x) = f(x)$ and for $n \geq 1$ $f^{n+1}(x) = f^n(f(x))$. If $f(x) = x^3 - \frac{3}{2}x^2 + x + \frac{1}{4}$, then $\int_0^1 f^{2004}(x)dx$ is _____

51. Let p(x) be a fourth degree polynomial, with derivative p′(x),
such that p(1) = p(3) = p(5) = p′(7) = 0. If x is a real number such that p(x) = 0 and x is not equal to 1, 3 or 5, then 11x is _____

52. If $L = \lim_{x\to\infty} x^p\left(\sqrt[3]{x+1} + \sqrt[3]{x-1} - 2\sqrt[3]{x}\right)$, where L is some non-zero real number, then $\frac{p^2}{L}$ is _____

53. If $\int_{-\infty}^{\infty} e^{tx}f(x)dx = \sin^{-1}\left(t - \sqrt{\frac{1}{2}}\right)$, then $\frac{1}{2\sqrt{2}}\int_{-\infty}^{\infty} x\,f(x)dx$ is _____

54. Let f: $R^+ \to R$ be a differentiable function such that the tangent to curve y = f(x) always meets the y-axis at point whose ordinate is one less than the ordinate of the point of contact. If f(1) = 0, then $\frac{1}{5}e^{f(2)}$ is _____

Space for rough work