MODULE 2

Fuzzy Set:-
If X is a collection of objects (universe of disclosure) and x be any particular element of X.
Then fuzzy set A defined on X is collection of Ordered pairs,

$$A = \{(x, \mu_A(x)\} / x \in X \}$$

where $\mu_A(x) : X \longrightarrow [0, 1]$ is called membership function.

Types of Universe of Disclosure:
i) discrete non ordered universe
ii) discrete ordered universe.
iii) continuous universe.

Crisp Set: -

- A set defined using a characteristic function that assigns a value of either 0 or 1 to each element of the universe with strict boundaries.
- U= All students
- A= In 10th: Elements in boundary of set A belong 100% to it and out of boundary belongs 0%
- B= In 12th: Elements in boundary of set B belong 100% to it and out of boundary belongs 0%

Difference between Crisp Set and Fuzzy Set: -

1	Crisp set defines the value is either 0 or 1.	Fuzzy set defines the value between 0 and 1 including both 0 and 1.
2	It is also called a classical set.	It specifies the degree to which something is true.
3	It shows full membership	It shows partial membership.
4	Eg1. She is 18 years old. Eg2. Rahul is 1.6m tall	Eg1. She is about 18 years old. Eg2. Rahul is about 1.6m tall.
5	Crisp set application used for digital design.	Fuzzy set used in the fuzzy controller.
6	It is bi-valued function logic.	It is infinite valued function logic
7	Full membership means totally true/false, yes/no, 0/1.	Partial membership means true to false, yes to no, 0 to 1.

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FUZZY LOGIC:-

- Fuzzy Logic (FL) is a method of reasoning that resembles human reasoning.
- RELATIONAL LOGIC + BOOLEAN LOGIC + PREDICATE LOGIC = FUZZY LOGIC

Fuzzy rules are used within fuzzy logic systems to infer an output based on input variables.

If x in A then y in B

x in A $\longrightarrow$ antecendent or premise

y in B $\longrightarrow$ consequence or conclusion

Fuzzy rule: 'R' denoted as

$$R : A \longrightarrow B$$

Example :- If temp is high, then pressure is low.

$T_{HIGH} = \{(25, 0.1), (30, 0.2), (35\ 0.5), (40, 0.6)\}$

$P_{LOW} = \{(2, 0.3), (5, 0.5), (6, 0.4)\}$

$$R : T_{HIGH} \longrightarrow P_{LOW}$$

$$R = \begin{array}{c c} & \begin{array}{ccc} 2 & 5 & 6 \end{array} \\ \begin{array}{c} 25 \\ 30 \\ 35 \\ 40 \end{array} & \begin{bmatrix} 0.1 & 0.1 & 0.1 \\ 0.2 & 0.2 & 0.2 \\ 0.3 & 0.5 & 0.4 \\ 0.3 & 0.5 & 0.4 \end{bmatrix} \end{array} \qquad (R = A \times B)$$

FUZZY INFERENCE SYSTEM :-

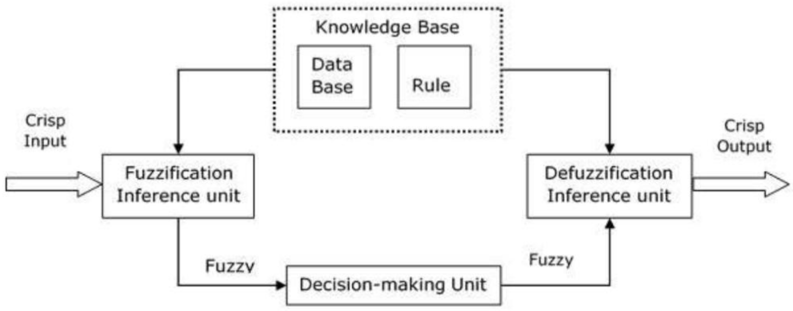
- Fuzzy inference system is key component of any fuzzy logic system.
- It uses fuzzy set theory, IF-THEN rules and fuzzy reasoning process to find the output corresponding to crisp inputs.
- Predicates in IF-THEN rules are connected using and or or logical connectives.

Functional Blocks of FIS: -

The following five functional blocks will help us to understand the construction of FIS −

- Rule Base − It contains fuzzy IF-THEN rules.
- Database − It defines the membership functions of fuzzy sets used in fuzzy rules.
- Decision-making Unit − It performs operation on rules.
- Fuzzification Interface Unit − It converts the crisp quantities into fuzzy quantities.
- Defuzzification Interface Unit − It converts the fuzzy quantities into crisp quantities.

Following is a block diagram of fuzzy interference system.



Working of FIS

- The working of the FIS consists of the following steps −
- A fuzzification unit supports the application of numerous fuzzification methods and converts the crisp input into fuzzy input.
- A knowledge base - collection of rule base and database is formed upon the conversion of crisp input into fuzzy input.
- The defuzzification unit fuzzy input is finally converted into crisp output.

- This fuzzy inference system is used in microwaves, air conditioner, pattern recognizition, vaccum cleaners, heaters etc.

Two types of Fuzzy Inference System are: -
1. Mamdani System
2. Sugeno System

Crisp Relation:-

Crisp relation is defined over the cartesign product on two crisp sets.

Suppose A and B are two crisp sets. Then cartesign product denoted as $A \times B$ is a collection of order pairs, such that

$$A \times B = \{(a,b) \mid a \in A \text{ and } b \in B\}$$

Note:-

$A \times B \neq B \times A$

$|A \times B| = |A| \times |B|$

$A \times B$ provides a mapping from $a \in A$ to $b \in B$

Relation is very useful concept in many fields. It is useful in logic, pattern recognition, control system, classification etc.

Example:-

Consider the elements A and B,

$A = \{2, 4, 6, 8\}$
$B = \{3, 7, 8, 9\}$

Cartesign product of two sets:

$$A \times B = \{(2,3)\ (2,7)\ (2,8)\ (2,9)\ (4,3)\ (4,7)\ (4,8)\ (4,9)\ (6,3)\ (6,7)\ (6,8)\ (6,9)\ (8,3)\ (8,7)\ (8,8)\ (8,9)\}$$

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8.00 A particular mapping is done from $a \in A$, to $b \in B$ which is denoted by R. (relation)

Let us define a relation,

$R = \{(a,b) \mid a = b-1 \; ; \; (a,b) \in A \times B\}$

then $R = \{(2,3)(8,9)\}$

Crisp Relations :-

We can represent R in a matrix form

$$R = \begin{array}{c|cccc} & 3 & 7 & 8 & 9 \\ \hline 2 & 1 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 \\ 6 & 0 & 0 & 0 & 0 \\ 8 & 0 & 0 & 0 & 1 \end{array}$$

Operations on crisp relations :-

Let R and S be two separate relations on cartesign $A \times B$, defined over two crisp sets $a \in A$ and $b \in B$.

$R = \{2, 4, 6, 8\}$ $S = \{3, 7, 8, 9\}$

$R = \{(a,b) \mid a = b-1\}$ $S = \{(a,b) \mid b = a\}$

$R = \{(2,3)(8,9)\}$ $S = \{(8,8)\}$

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$$\begin{array}{c|cccc} & 3 & 7 & 8 & 9 \\ \hline 2 & 1 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 \\ 6 & 0 & 0 & 0 & 0 \\ 8 & 0 & 0 & 0 & 1 \end{array} \quad R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{array}{c|cccc} & 3 & 7 & 8 & 9 \\ \hline 2 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 \\ 6 & 0 & 0 & 0 & 0 \\ 8 & 0 & 0 & 1 & 0 \end{array} \quad S = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Find the following:-

1] $R \cup S = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

Union :- $R(a,b) \cup S(a,b) = \max(R(a,b), S(a,b))$

2] $R \cap S = \cancel{\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix}} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Intersection :- $R(a,b) \cap S(a,b) = \min(R(a,b), S(a,b))$

3] $\bar{R} = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix}$

Complement :- $\bar{R}(a,b) = 1 - R(a,b)$

4] Containment :-
$R \subseteq S \rightarrow R(a,b) \leq S(a,b)$

8.00 Cardinality of Crisp Set:-
i) Cardinality defines the number of
9.00 elements in the given set.
ii) The Let A and B be the crisp sets with
10.00 cardinality n and m respectively.
iii) The cardinality of crisp relation defined
11.00 over Carteign product A×B will be n×m
Ex:-
12.00 Let $A = \{1, 2\}$, $B = \{3, 4, 5\}$
Here $n = |A| = 2$ and $m = |B| = 3$, $(n \times m) = 6$
1.00 The Carteign product of both sets
will be
2.00 $A \times B = \{(1,3)\ (1,4)\ (1,5)\ (2,3)\ (2,4)\ (2,5)\}$
Cardinality of carteign product is,
3.00 $|A \times B| = 6 = n \times m$.

4.00 Composition between two crisp sets:-
→ The composition between two relations is
5.00 defined as $R \cdot S$
$R \cdot S = \{(x,z) \mid (x,y) \in R \text{ and } (y,z) \in S\ \forall y \in Y\}$
6.00 Consider the universal sets:-
$A = \{a_1, a_2, a_3\}$
7.00 $B = \{b_1, b_2, b_3\}$
$C = \{c_1, c_2, c_3\}$
8.00

Let relations R and S can be formed as
$R = A \times B = \{(a_1, \underline{b_1}), (a_1, b_3)\ (a_2, \underline{b_2})\}$
Notes $S = B \times C = \{(\underline{b_1}, c_2), (b_1, c_3)\ (\underline{b_2}, c_3)\}$

$R \cdot S = \{(a_1, c_2), (a_2, c_3)\}$

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$$R \cdot S = \begin{array}{c|ccc} & c_1 & c_2 & c_3 \\ \hline a_1 & 0 & 1 & 0 \\ a_2 & 0 & 0 & 1 \\ a_3 & 0 & 0 & 0 \end{array}$$

Composition of Relation is computed in two different ways.

i) Max-Min Composition:-

Given the two relation matrices R and S, the max-min composition is defined as

$$T = R \circ S$$

$$T(a,c) = \max\{\min\{R(a,b), S(b,c)\}\}$$

$$\chi_T(a,c) = \bigvee_{b \in B} \left[\chi_R(a,b) \wedge \chi_S(b,c) \right]$$

ii) Max-product composition

$$T = R \cdot S$$

$$\chi_T(a,c) = T(a,c) = \bigvee_{b \in B} \left[\chi_R(a,b) \cdot \chi_S(b,c) \right]$$

Few properties:-

i) Associative $(R \cdot S) \cdot M = R \cdot (S \cdot M)$

ii) Cummulative $R \cdot S \neq S \cdot R$

iii) Inverse $(R \cdot S)^{-1} = S^{-1} \cdot R^{-1}$

Ex:-

Given $A = \{2, 4, 6\}$, $B = \{3, 5, 7\}$

$R = \{(a,b) \mid b = a+1\}$
$S = \{(a,b) \mid b < a\}$

$A \times B = \{(2,3)\ (2,5)\ (2,7)\ (4,3)\ (4,5)\ (4,7),\ (6,3)\ (6,5)\ (6,7)\}$

$R = \{(2,3)\ (4,5)\ (6,7)\}$
$S = \{(4,3),\ (6,3)\ (6,5)\}$

$$R = \begin{array}{c|c} & \begin{matrix} 3 & 5 & 7 \end{matrix} \\ \begin{matrix} 2 \\ 4 \\ 6 \end{matrix} & \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array} \qquad S = \begin{array}{c|c} & \begin{matrix} 3 & 5 & 7 \end{matrix} \\ \begin{matrix} 2 \\ 4 \\ 6 \end{matrix} & \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix} \end{array}$$

Max-Min Composition:-

$$R \circ S = \begin{array}{c|c} & \begin{matrix} 3 & 5 & 7 \end{matrix} \\ \begin{matrix} 2 \\ 4 \\ 6 \end{matrix} & \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array} \quad \begin{array}{c|c} & \begin{matrix} 3 & 5 & 7 \end{matrix} \\ \begin{matrix} 2 \\ 4 \\ 6 \end{matrix} & \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix} \end{array}$$

$$= \max\left[\min\left(M_R(a,b),\ M_S(a,b)\right)\right]$$

$$R \cdot S = \begin{bmatrix} \max(0.0\,0) & \max(0,0,0) & \max(0,0,0) \\ \max(0\,1\,0) & \max(0,0\,0) & \max(0\,0\,0) \\ \max(0\,0\,1) & \max(0\,0\,1) & \max(0\,0\,0) \end{bmatrix}$$

Notes

$$R \cdot S = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

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Max - Product Composition :-

$$R = \begin{bmatrix} 0.6 & 0.3 \\ 0.2 & 0.9 \end{bmatrix} \qquad S = \begin{bmatrix} 1 & 0.5 & 0.3 \\ 0.8 & 0.4 & 0.7 \end{bmatrix}$$

$$T = R \cdot S$$
$$= \max\left[\min\left(M_R(a,b) \cdot M_S(a,b) \right) \right]$$

$$R \cdot S = \begin{bmatrix} \max(0.6, 0.24) & \max(0.3, 0.12) \\ \max(0.18, 0.21) & \max(0.2, 0.72) \\ \max(0.10, 0.36) & \max(0.06, 0.63) \end{bmatrix}$$

$$= \begin{bmatrix} 0.6 & 0.3 \\ 0.21 & 0.72 \\ 0.36 & 0.63 \end{bmatrix}$$

Fuzzy Relations:-

Fuzzy relations defines the mapping of variables from one fuzzy set to another.
fuzzy relations relates the element of one universal set (X) to another universal set (Y) through the cartesian product of two universal set.

$$A \in X,\ B \in Y,\ R = A \times B \subset X \times Y$$

if $A = \{(a, 0\cdot2), (b, 0\cdot7), (c, 0\cdot4)\}$ and
$B = \{(a, 0\cdot5), (b, 0\cdot6)\}$

$$\mu_R(X, Y) = \mu_{A \times B}(X, Y)$$
$$= \min(\mu_A(X), \mu_B(Y))$$

$$\mu_R(X, Y) = \begin{array}{c c} & \begin{array}{cc} a & b \end{array} \\ \begin{array}{c} a \\ b \\ c \end{array} & \begin{bmatrix} 0\cdot2 & 0\cdot2 \\ 0\cdot5 & 0\cdot6 \\ 0\cdot4 & 0\cdot4 \end{bmatrix} \end{array}$$

The matrix representing a fuzzy relation is called fuzzy matrix.

Operations on fuzzy Relations:-

a) Union:-

$$\mu_{R \cup S}(X, Y) = \max\{\mu_R(X, Y), \mu_S(X, Y)\}$$

let $\mu_R(X,Y) = \begin{array}{c|cc} & a & b \\ \hline a & 0.2 & 0.2 \\ b & 0.5 & 0.6 \\ c & 0.4 & 0.4 \end{array}$, $\mu_S = \begin{array}{c|cc} & a & b \\ \hline a & 0.9 & 0.5 \\ b & 0.1 & 0.4 \\ c & 0.3 & 0.8 \end{array}$

$\mu_{(R \cup S)}(X,Y) = \begin{array}{c|cc} & a & b \\ \hline a & 0.3 & 0.5 \\ b & 0.5 & 0.6 \\ c & 0.4 & 0.6 \end{array}$

ii) Intersection :-

$\mu_{R \cap S}(X,Y) = \begin{array}{c|cc} & a & b \\ \hline a & 0.2 & 0.2 \\ b & 0.1 & 0.4 \\ c & 0.3 & 0.4 \end{array}$

iii) Complement :-

$\overline{\mu_R(X,Y)} = \begin{array}{c|cc} & a & b \\ \hline a & 0.8 & 0.8 \\ b & 0.5 & 0.4 \\ c & 0.6 & 0.6 \end{array}$

iv) Projection :-

The projection of $R(X,Y)$ on X denoted by R_1 is given by,

$$R_1 = \{ (X, \max_y \mu_R(X,Y) \mid (X,Y) \in X \times Y \}$$

The projection of $R(X,Y)$ on Y denoted by R_2 is given by

$$R_2 = \{ Y, \max_x \mu(R(X,Y) \mid (X,Y) \in X \times Y \}$$

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Example :-

Let R be a fuzzy relation defined by the following relational matrix,

$$R = \begin{bmatrix} 0.1 & 0.2 & 0.4 & 0.8 & 1 & 0.8 \\ 0.2 & 0.4 & 0.8 & 1 & 0.8 & 0.6 \\ 0.4 & 0.8 & 1 & 0.8 & 0.4 & 0.2 \end{bmatrix}$$

The projection of $R(X,Y)$ on X is,

$$\mu_{R_1}(X_1) = \max\{0.1, 0.2, 0.4, 0.8, 1, 0.3\}$$
$$= 1$$

$$\mu_{R_1}(X_2) = \max\{0.2, 0.4, 0.8, 1, 0.8, 0.6\}$$
$$= 1$$

$$\mu_{R_1}(X_3) = \max\{0.4, 0.8, 1, 0.8, 0.4, 0.2\}$$
$$= 1$$

So, the projection X is,

$$R_1 = \{(X_1, 1), (X_2, 1), (X_3, 1)\}$$

The projection of $R(X,Y)$ on Y is calculated as,

$$\mu_{R_2}(Y_1) = \max\{0.1, 0.2, 0.4\} = 0.4$$
$$\mu_{R_2}(Y_2) = \max\{0.2, 0.4, 0.8\} = 0.8$$
$$\mu_{R_2}(Y_3) = \max\{0.4, 0.8, 0.1\} = 0.8$$
$$\mu_{R_2}(Y_4) = \max\{1, 0.8, 0.4\} = 1$$
$$\mu_{R_2}(Y_5) = \max\{0.8, 0.6, 0.2\} = 0.8$$

The projection of $R(X,Y)$ on Y is

$$R_2 = \{(Y_1, 0.4), (Y_2, 0.8), (Y_3, 0.8), (Y_4, 1), (Y_5, 0.8)\}$$

Cylindrical Extension of fuzzy relation:-
Filling all the columns of the related matrix by X-projection and vice-versa.

$$R_2 = \begin{bmatrix} 0.4 & 0.8 & 1 & 1 & 1 & 0.8 \\ 0.9 & 0.8 & 1 & 1 & 1 & 0.8 \\ 0.4 & 0.8 & 1 & 1 & 1 & 0.8 \end{bmatrix}$$

$$R_1 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Compositions on Fuzzy Relations:-

Fuzzy Composition:-
The operations which are executed on two compatible fuzzy relations to get a single fuzzy relation is called Fuzzy Composition.

Let $R = X \times Y$ and $S = Y \times Z$ be the two fuzzy relations.
i) Fuzzy max-min composition.
ii) Fuzzy max-product composition.

Fuzzy max-min composition : $T_{x\cdot z}$

$\mu_{R\cdot S} = \max\{\min(\mu_R(x,y), \mu_S(y,z))\}$

Fuzzy max-product composition

$\mu_{R\cdot S} = \max\{\mu_R(x,y)\cdot\mu_S(y,z)\}$

i] Fuzzy max-min composition:-

$$R_{x,y} = \begin{array}{c} \\ x_1 \\ x_2 \end{array}\begin{array}{c} \begin{array}{cc} y_1 & y_2 \end{array} \\ \begin{bmatrix} 0.6 & 0.3 \\ 0.2 & 0.9 \end{bmatrix} \end{array} \qquad S_{y,z} = \begin{array}{c} \\ y_1 \\ y_2 \end{array}\begin{array}{c} \begin{array}{ccc} z_1 & z_2 & z_3 \end{array} \\ \begin{bmatrix} 1 & 0.5 & 0.3 \\ 0.8 & 0.4 & 0.7 \end{bmatrix} \end{array}$$

$$\mu_{R\cdot S} = T_{x\cdot z} = \begin{array}{c} \\ x_1 \\ x_2 \end{array}\begin{array}{c} \begin{array}{ccc} z_1 & z_2 & z_3 \end{array} \\ \begin{bmatrix} 0.6 & 0.5 & 0.3 \\ 0.8 & 0.4 & 0.7 \end{bmatrix} \end{array}$$

$(x_1, z_1) = \max[\min(x_1, y_1)(y_1, z_1), \min(x_1, y_2)(y_2, z_1)]$
$= 0.6$

$(x_1, z_2) = \max[\min(0.6, 0.5), \min(0.3, 0.4)] = 0.5$

$(x_1, z_3) = \max[\min(0.6, 0.3), \min(0.3, 0.7)] = 0.3$

$(x_2, z_1) = \max[\min(0.2, 0.1), \min(0.9, 0.8)] = 0.8$

$(x_2, z_2) = \max[\min(0.2, 0.5), \min(0.9, 0.4)] = 0.4$

$(x_2, z_3) = \max[\min(0.2, 0.3), \min(0.4, 0.7)] = 0.7$

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Max-Product Classification:-

$R(X,Y)$ and $S(Y,Z)$ are two relations, then fuzzy max-product composition.

$$\mu_{R \cdot S} = \max(\mu_R(X,Y), \mu_S(Y,Z))$$

Let
$$R = \begin{array}{c|cc} & Y_1 & Y_2 \\ \hline X_1 & 0.6 & 0.3 \\ X_2 & 0.2 & 0.9 \end{array} \qquad S = \begin{array}{c|ccc} & Z_1 & Z_2 & Z_3 \\ \hline Y_1 & 1 & 0.5 & 0.3 \\ Y_2 & 0.8 & 0.4 & 0.7 \end{array}$$

$$\mu_{R \cdot S} = \begin{array}{c|ccc} & Z_1 & Z_2 & Z_3 \\ \hline X_1 & 0.6 & 0.30 & 0.21 \\ X_2 & 0.72 & 0.36 & 0.63 \end{array}$$

$$\mu(X_1, Z_1) = \max[\mu_R(X_1,Y_1) \cdot \mu_S(Y_1,Z_1), \mu_R(X_1,Y_2) \cdot \mu_S(Y_2 Z_1)]$$
$$= \max[0.6 \times 1, 0.3 \times 0.8]$$
$$= 0.6$$

$$\mu(X_1, Z_2) = \max[0.6 \times 0.5, 0.3 \times 0.4]$$
$$= 0.30$$

$$\mu(X_1, Z_3) = \max[0.6 \times 0.3, 0.3 \times 0.7] = 0.21$$
$$\mu(X_2, Z_1) = \max[0.2 \times 1, 0.9 \times 0.8] = 0.72$$
$$\mu(X_2, Z_2) = \max[0.2 \times 0.5, 0.9 \times 0.4] = 0.36$$
$$\mu(X_2, Z_3) = \max[0.2 \times 0.3, 0.9 \times 0.7] = 0.63$$

Membership Function:-

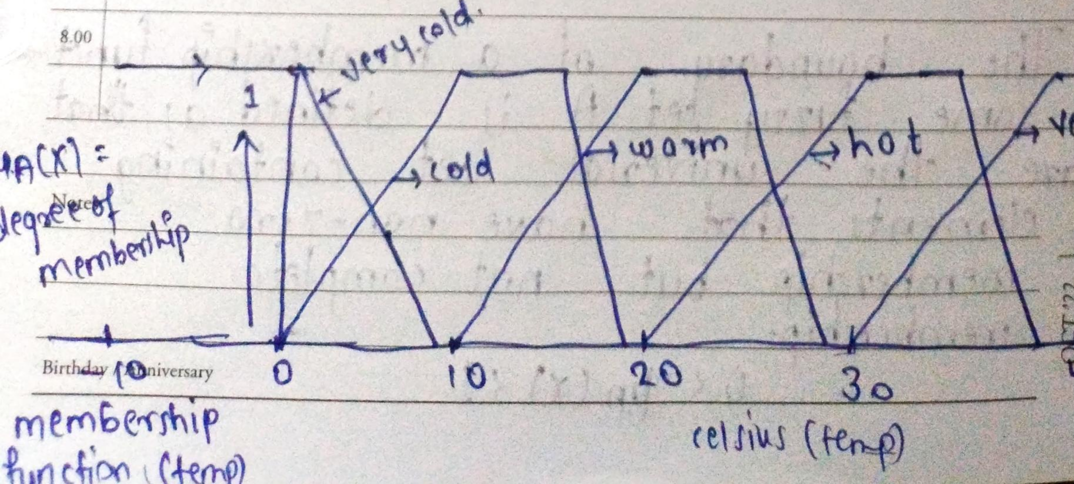
Fuzzy set A of universal set X is defined by function $\mu_A(x)$ called the membership function of set A.

$\mu_A(x) : X \rightarrow [0,1]$, where $\mu_A(x) = 1$, if x is totally in A.
$\mu_A(x) = 0$, if x is not in A.
$0 < \mu_A(x) < 1$, if x is partially in A.

Membership function: It is a function that specifies the degree to which a given input belongs to a set.

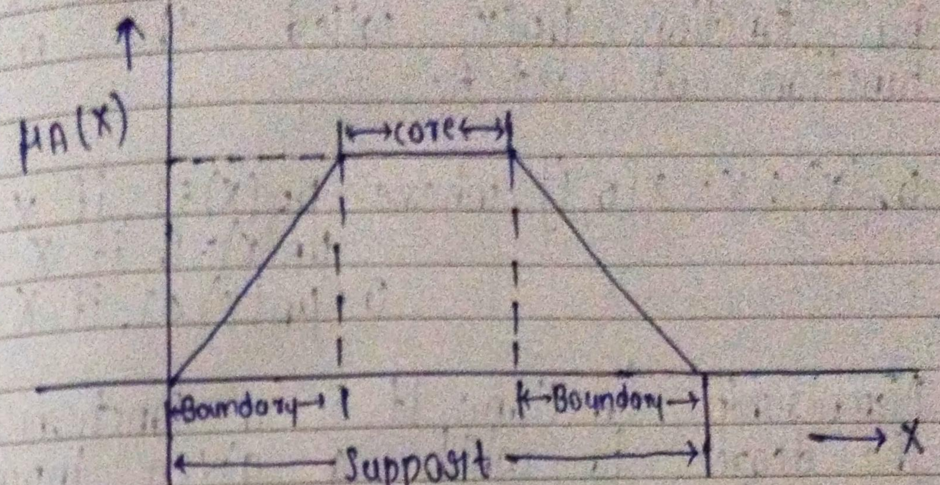
Degree of membership:-

A value between 0 and 1, represents the degree of membership, also called membership value of element X in set A. It is the output of membership function. Membership function can be defined as the technique to solve practical problem by experience rather of knowledge (degree of truth).
Membership function are used in fuzzification and defuzzification of a fuzzy logic system.



{very cold, cold, warm, hot, very hot}

Features of membership function :-



1) Core: The core of a membership function for some fuzzy set A is defined as that region of the universe that is characterized by complete and full membership in the set.

$\mu_A(x) = 1$

2) Support: The support of a membership function for some fuzzy set A is defined as that region of the universe that is characterized by nonzero membership in the set.

$\mu_A(x) > 0$

3) Boundary: The boundary of a membership function for some fuzzy set A is defined as that region are the universal set containing elements that have non-zero membership but not complete membership.

$0 < \mu_A(x) < 1$