

DP is used to solve MDP's and similar problems.
DP aim to find an optimal policy

Dynamic Programming

- DP is a method of solving complex prob. by breaking them down into sub problems. The solutions to the sub-problems are combined to solve overall problem.
- The basic assumption is that the model of the env is known.
- The two req. properties of DP are -

1) Optimal substructure -
optimal solⁿ of the sub-problem can be used to solve the overall problem.

2) Overlapping sub-problems -
sub-problems occur many times. solⁿ can be cached & reused.

- Two popular DP algos are -
 - a) Policy iteration
 - b) Value iteration.
- } to find optimal policy of MDP

POLICY ITERATION:

- Involves iteratively improving a policy until it converges to the optimal policy.
- After initil^y initializing a random policy, this method iterates betⁿ 2 steps
 - Policy evaluation
 - Policy improvement

$$V_{\pi}(s) = E[R_{\pi}(s)]$$

~~$$Q_{\pi}(s, a) = r + \gamma E[V_{\pi}(s') | s, a]$$~~

$$Q_{\pi}(s, a) = r + \gamma V_{\pi}(s')$$

Policy Evaluation:

i) Evaluate the current policy by ^{computing} evaluating the expected value of reward obtained by following the policy.

$$E[R_{\pi}(s)]$$

ii) Measures how good the policy is by calculating all the state value function $V_{\pi}(s)$ for all states until the state value function is converged.

iii) Also known as the prediction problem.

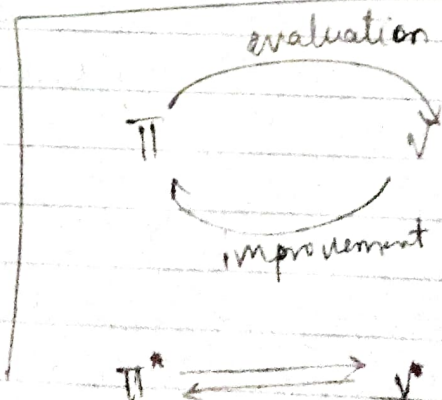
Policy Improvement:

i) Update the policy to be greedy w.r.t. current q-value function

ii) This means the policy chooses the action ~~with max~~ that maximises the expected reward from the current state $q_{\pi}(s, a)$ for each state

$$\pi'(a|s) = \begin{cases} 1, & a = \underset{a}{\operatorname{argmax}} [\sum_{s'} \sum_r P(s'|sa) [r + \gamma V_{\pi}(s')]] \\ 0, & \text{otherwise} \end{cases}$$

iii) Control Problem



VALUE ITERATION:

- Simpler than policy iteration.
- only performs one step of policy evaluation in each iteration.
- the optimal value function is defined.
- It is an iterative algo that starts with an arbitrary value function and updates it until it converges to optimal value function.
- optimal value function - max. expected reward obtained from each state in the given policy.

$$V(s) = \max_a [r_s + \gamma \sum_{s'} P(s'|s,a) V(s')]]$$

s.t.,

$V(s)$ - current estimate of the optimal value function for state s

$P(s'|s,a)$ - model of the environment.

γ - discount factor.

$V(s+1) =$

(PI)

Policy Iteration

Strength i) Converges to optimal policy faster than VI in same cases

ii) Can handle larger class of MDP's ~~than~~ including those with stochastic policies

iii) More stable & less sensitive to discount factor

Weak-
ness

i) Can be computationally expensive

ii) Can get stuck in local optima

iii) May not converge if the MDP has infinite horizon

(VI)

Value Iteration

i) Converges to optimal value faster than PI

ii) Can handle MDP's with infinite horizon

iii) Simpler & easy to implement

i) sensitive to the choice of discount factor

ii) cannot handle stochastic values directly

iii) May require a large no. of iterations to converge