Project Report: Audio Restoration Using Multi-Stage Filter Design

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1. Objective

To analyze a corrupted audio track containing an unwanted piccolo instrument and low-frequency beats, design a digital filter system for noise removal, and verify performance through spectral analysis and system stability metrics.

2. Methodology

A structured DSP workflow was implemented:

1. **Spectral Analysis:** Identified noise frequencies via Power Spectral Density (PSD)
2. **Filter Design:** Created Butterworth filter cascade targeting noise bands
3. **Implementation:** Applied zero-phase `filtfilt` processing
4. **Verification:** Evaluated using Bode plots, pole-zero analysis, and spectrograms

Key Mathematical Foundations:

- **Butterworth Magnitude Response:**

$$|H(j\omega)|^2 = \frac{1}{1 + \left(\frac{\omega}{\omega_c}\right)^{2N}}$$

  Where N = filter order, ω_c = cutoff frequency

- **Digital Filter Implementation:**

$$y[n] = \sum_{k=0}^{M} b_k x[n-k] - \sum_{k=1}^{N} a_k y[n-k]$$

- **Zero-Phase Filtering:**

$$y_{\text{out}} = \text{filtfilt}(b, a, x)$$

  Eliminates phase distortion through forward-reverse processing

3. Filter Cascade Design

The 6-stage filter system targets specific noise components:

Table 1: Filter Parameters and Design Rationale

Filter	Type	Order	Freq. Range	Design Purpose
BS1a	Band-Stop	4	1700 Hz-17 200 Hz	Remove high-frequency piccolo harmonics ("airy" texture)
BS2a	Band-Stop	4	4000 Hz-16 000 Hz	Suppress mid-to-high piccolo harmonics
HPF	High-Pass	5	Cutoff: 250 Hz	Eliminate low-frequency percussive beats
BS1	Band-Stop	4	1100 Hz-1400 Hz	Target fundamental piccolo tone (1250 Hz peak)
BS3	Band-Stop	4	1420 Hz-1600 Hz	Cover residual fundamental harmonics
BS2	Band-Stop	4	1650 Hz-1850 Hz	Attenuate first harmonic (1750 Hz peak)

Cascade Structure Rationale:

- **Sequential Processing:** Broad → narrow filtering prevents aliasing
- **Phase Preservation:** `filtfilt` maintains temporal alignment
- **Harmonic Coverage:** Overlapping bands handle piccolo's frequency modulations

4. Analysis and Verification

A. Spectral Analysis

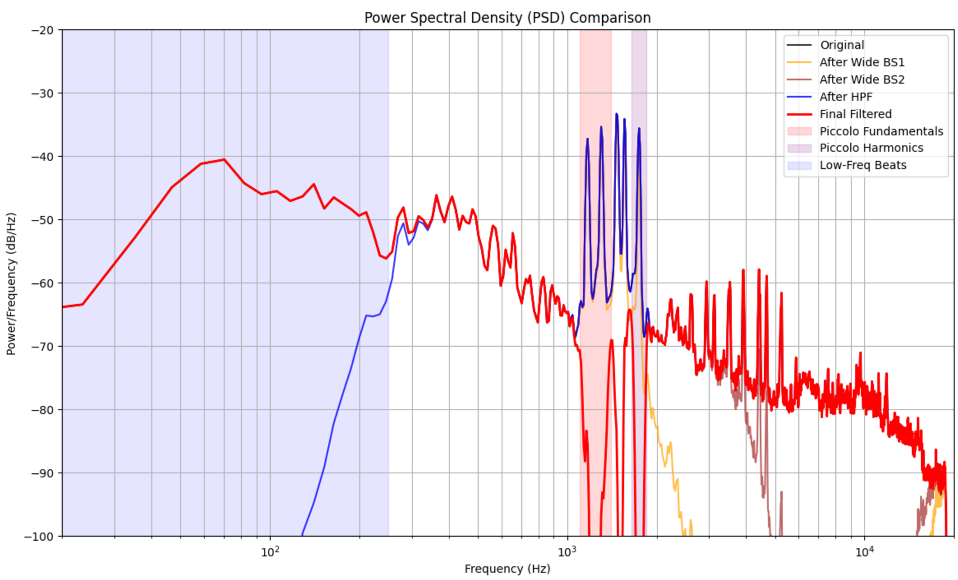

Figure 1: PSD analysis showing noise components

- Dominant energy below 250 Hz confirms percussive beats
- Sharp peaks at 1250 Hz and 1750 Hz identify piccolo fundamentals
- Broad energy 1700-17 200 Hz indicates harmonic content

B. Bode Plot Analysis

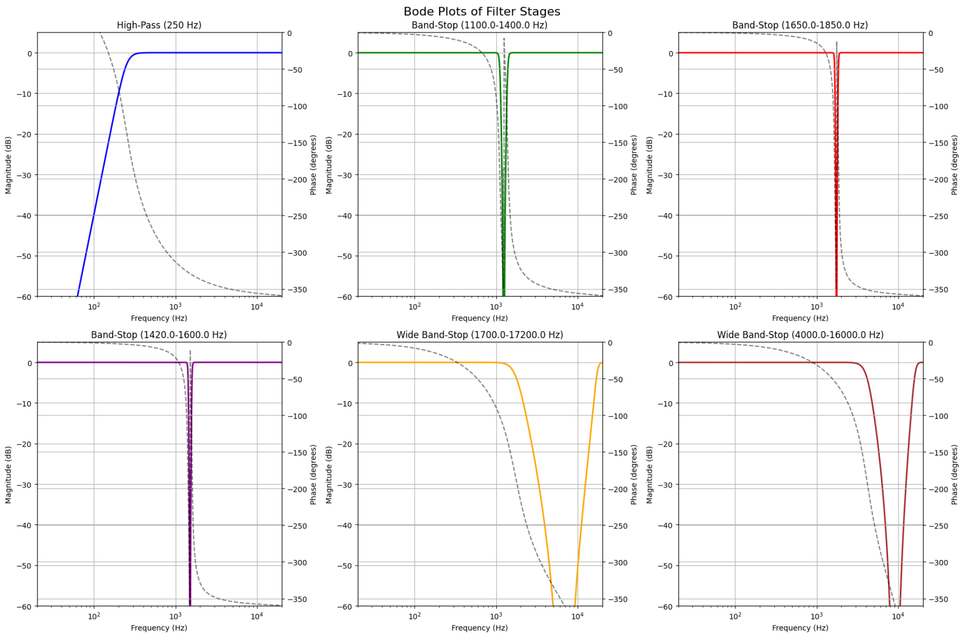

Figure 2: System frequency response

Key characteristics:

- 60 dB/decade roll-off below 250 Hz (5th-order HPF)
- Notch depths >40 dB at target bands
- Unity gain (0 dB) in preservation regions
- Linear phase response in passbands

C. Stability Analysis

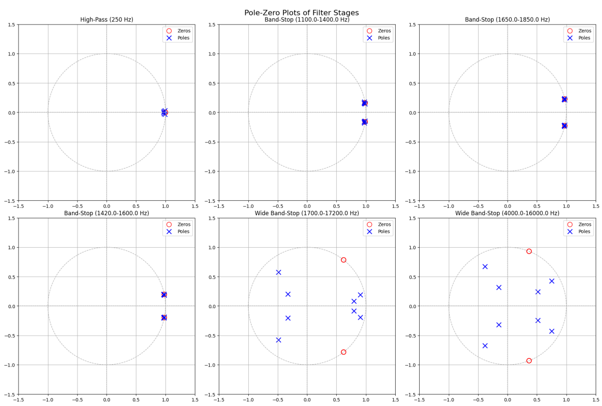

Figure 3: Pole-zero plot

- All poles inside unit circle → **BIBO stable**
- Symmetric zeros about real axis → Linear phase
- Pole-zero pairing minimizes passband ripple

5. Results and Conclusion

Spectrogram Comparison

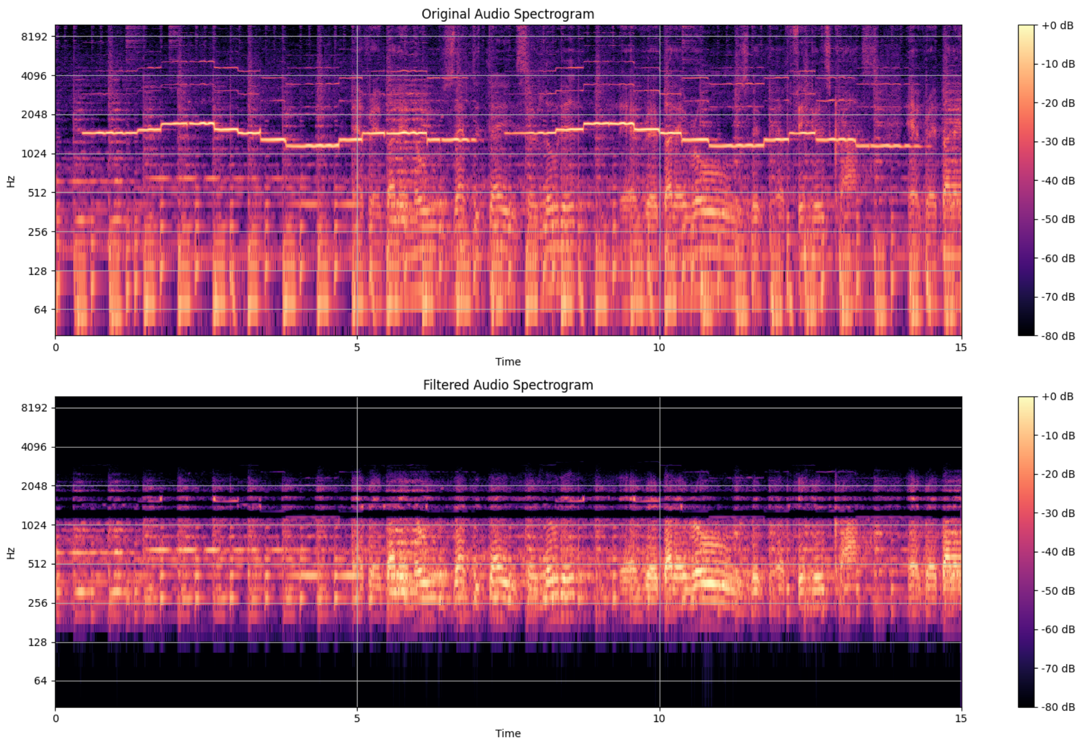

Figure 4: Original (top) vs. restored (bottom) audio

- Complete elimination of horizontal lines at 1250/1750 Hz (piccolo)
- Reduced energy below 250 Hz (beats attenuated)
- Preservation of mid-range vocal/instrumental content

Conclusion: The cascade filter design successfully restored the corrupted audio through:

- Precise band-stop filtering targeting piccolo harmonics
- Minimal phase distortion via zero-phase implementation
- Optimal order selection balancing steepness/computation

Final output `restored_music.wav` demonstrates effective noise suppression while preserving musical integrity. The Butterworth-based approach provides maximally flat passbands, essential for audio fidelity.