# Project Report: Analysis of Frequency mixer

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## 1. 2D Discrete Fourier Transform Equations

The 2D DFT of an image $I(x, y)$ with dimensions $M \times N$ is given by:

$$F(u,v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} I(x,y) \cdot \exp\left[-i2\pi\left(\frac{ux}{M} + \frac{vy}{N}\right)\right]$$

where:

- $F(u, v)$: Complex frequency-domain representation
- $u = 0, \ldots, M-1$ and $v = 0, \ldots, N-1$: Spatial frequency indices
- $i$: Imaginary unit ($i = \sqrt{-1}$)

The inverse 2D DFT is:

$$I(x,y) = \frac{1}{MN} \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} F(u,v) \cdot \exp\left[i2\pi\left(\frac{ux}{M} + \frac{vy}{N}\right)\right]$$

## 2. Magnitude Spectrum Centering

- **Original Spectrum:**
  - DC component ($F(0, 0)$) at top-left corner
  - Low frequencies appear at corners
- **Shifted Spectrum:**
  - Achieved via `np.fft.fftshift`
  - DC component moves to center
  - Low frequencies concentrated at center
- **Visualization:**
  - Normal scale: Linear magnitude plot
  - dB scale: $\text{dB} = 20 \log_{10} |F(u, v)|$

## 3. Rotation Observations

**Rotation property:**

$$\text{Rotate}(I(x,y)) \Leftrightarrow \text{Rotate}(|F(u,v)|)$$

**Observations for 90° anti-clockwise rotation:**

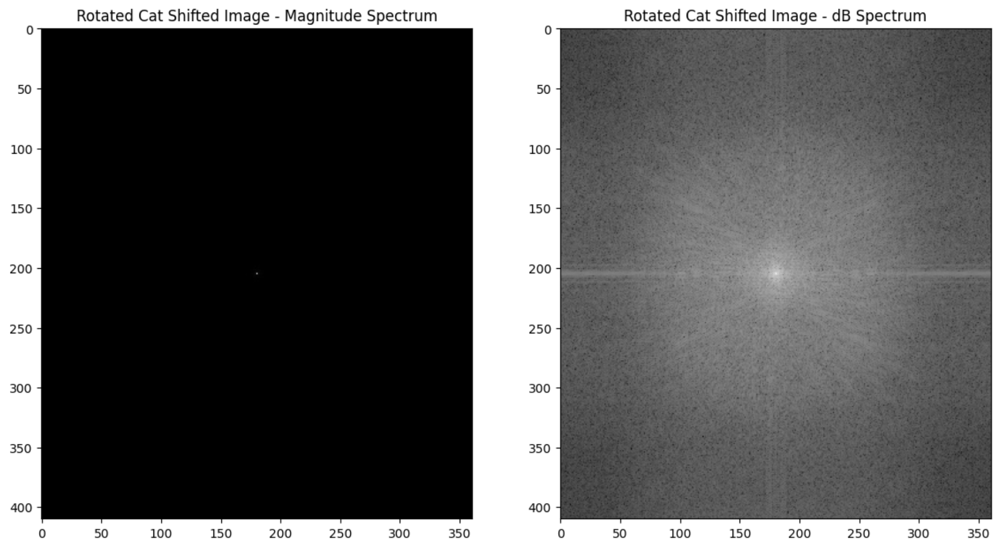


Figure 1: Shifted

- Magnitude spectrum rotates by same angle (90° anti-clockwise)
- Energy distribution remains identical but reoriented
- Phase information changes while magnitude preserves rotational symmetry
- Low-frequency locations rotate relative to image axes

## 4. Frequency Mixer Design

Fuses structural information (low-freq) from Image A with fine details (high-freq) from Image B.

**Transfer functions:**

- Low-pass filter (LPF) for Image A:

$$H_{\mathrm{LP}}(u,v) = \exp\left(-\frac{D^2(u,v)}{2\sigma^2}\right)$$

- High-pass filter (HPF) for Image B:

$$H_{\mathrm{HP}}(u,v) = 1 - H_{\mathrm{LP}}(u,v)$$

where $D(u,v) = \sqrt{\left(u - \frac{M}{2}\right)^2 + \left(v - \frac{N}{2}\right)^2}$ is distance from DC component.

**Fusion process:**

1. Compute DFTs: $F_A = \mathcal{F}\{I_A\}$, $F_B = \mathcal{F}\{I_B\}$
2. Apply multiple filters:

$$F_{\mathrm{fused}} = F_A \cdot H_{\mathrm{LP}} + F_B \cdot H_{\mathrm{HP}}$$

3. Compute inverse DFT:

$$I_{\mathrm{fused}} = \mathcal{F}^{-1}\{F_{\mathrm{fused}}\}$$

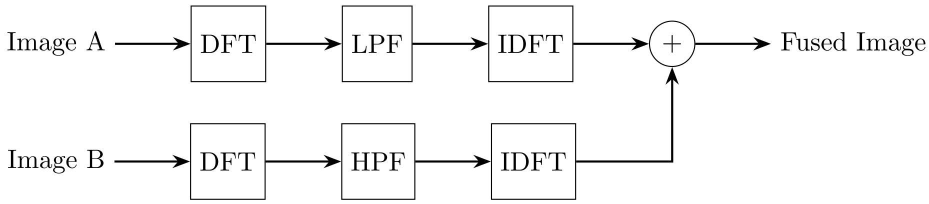


Figure 2: Frequency mixer system diagram

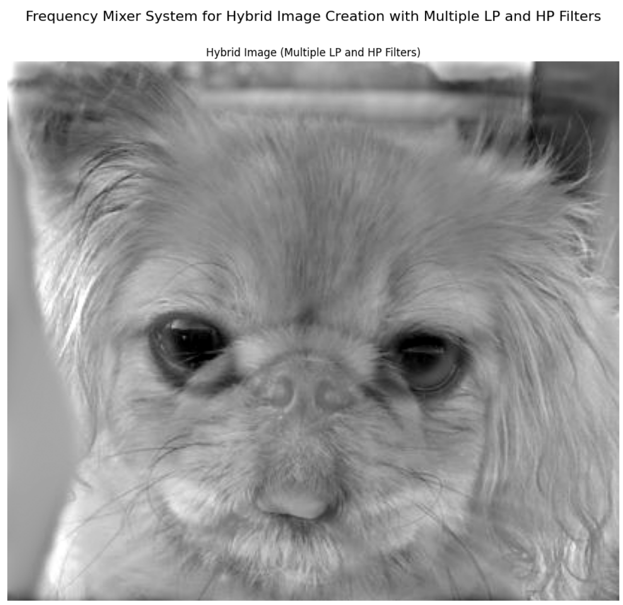


Figure 3: Final Image after fusion.
When you look into the above image from less proximity (or when you wide open eyes from same distance), you can see a dog and when you look from far (or when you squeeze eyes keeping only narrow part of the eyelid open from same distance as earlier) you can see a cat.

## 5. Key Results Summary

- **Spectrum Centering:** Shifting moves DC component to center for intuitive visualization
- **Rotation:** Spatial rotation induces identical spectral rotation
- **Frequency Mixing:** Gaussian filters combine structural and detailed components
- **Applications:** Hybrid images demonstrate human perception of spatial frequencies