Course Name: MATHEMATICS – II

Webpage: https://sites.google.com/view/lecma10002

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Syllabus:

- **Linear Algebra (11 Lectures)**
- **Integral Calculus (11 Lectures)**
- **Numerical Analysis (9 Lectures)**
- **Vector Calculus (9 Lectures)**

Linear Algebra (11 Lectures)

Algebra of matrices – **Solution of system of linear equations** – Gauss Elimination

Vector spaces, basis and dimension, linear dependence and independence of vectors, rank of a matrix and its properties, Solution of system of equations using rank concept

Linear Transformations – Matrix representation of linear transformations

Hermitian, Skew Hermitian and Unitary matrices, **eigenvalues, eigenvectors** and eigenvalues of Hermitian, Skew Hermitian and Unitary matrices

Similarity of matrices & Diagonalization - Applications

Numerical Analysis (9 Lectures)

Iterative method for solution of system of linear equations: Jacobi and Gauss Seidel method

Solution of transcendental equations: Bisection, Fixed point Iteration, Newton-Raphson methods

Interpolation: Finite differences, interpolation, error in interpolation polynomial, Newton's forward and backward interpolation formulae, Lagrange's interpolation and error estimates

Numerical integration: Trapezoidal rule and Simpson's 1/3rd rule and their geometrical interpretation.

Integral Calculus (11 Lectures)

Convergence of **improper integrals**, test of convergence. **Beta and Gamma functions** with their elementary properties

Differentiation of integrals with variable limits - **Leibnitz rule**

Double integrals, Change in order of integration, Change of variables in double integrals - Jacobians of transformations

Triple integrals, change of order, change of variables

Applications of Multiple Integrals - Computations of surfaces, area and volumes

Vector Calculus (9 Lectures)

Scalar and vector fields, level surfaces; limit, continuity and differentiability of vector functions, Curves and Arc-Length

Directional derivative, Gradient, Curl and Divergence and geometrical Interpretation

Line and surface integrals, theorems of **Green, Gauss and Stokes**

Literature

- E. **Kreyszig**: Advanced Engineering Mathematics

- S. **Narayan** and R. K. **Mittal**: Integral Calculus

- N. **Piskunov**: Differential and Integral Calculus, Volume I & II

- **Lecture Notes**

Linear Algebra

- ❑ System of Linear Equation - Introduction
- ❑ Solution – Geometrical Interpretation
- ❑ Solution of the System - Gauss Elimination
- ❑ Consistency of Solution

System of Linear Equations

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1$$
$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2$$
$$\vdots$$
$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m$$

m equations and n unknowns

Matrix form:

$$A\,x = b \qquad A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \qquad b = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$$

A system of equation is **consistent** if it has at least one solution, and **inconsistent** if it has no solution.

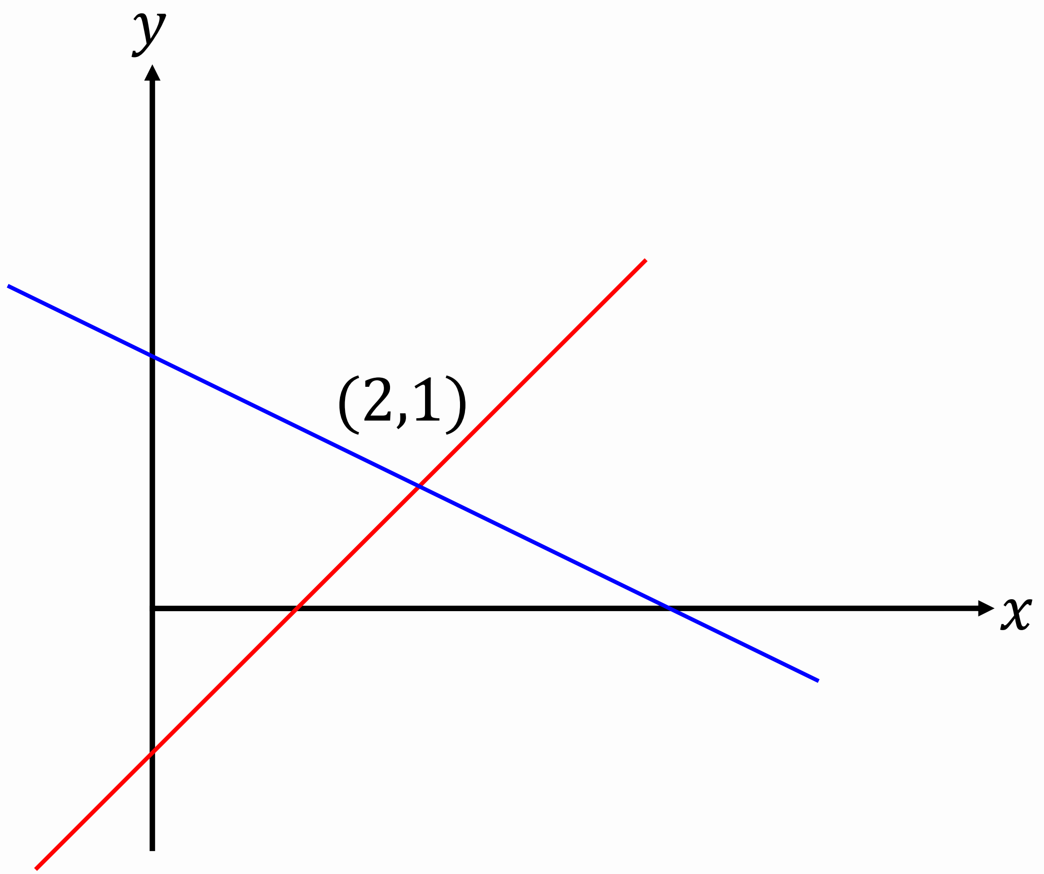
System of Linear Equations

Consider $\quad x + 2y = 4 \qquad (L_1)$

$\qquad\qquad x - y = 1 \qquad (L_2)$

OR

$$\begin{bmatrix} 1 & 2 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$$



Case of Unique Solution

System of Linear Equations (vectors interpretation)

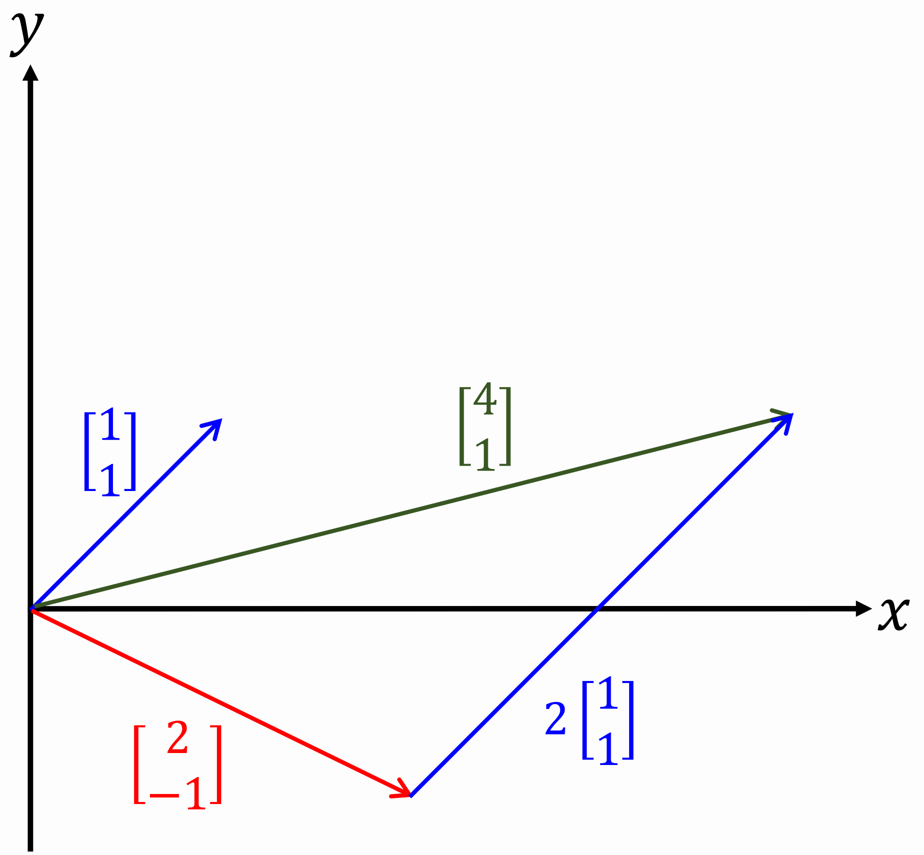
$x + 2y = 4 \qquad x - y = 1$

OR

$$\begin{bmatrix} 1 & 2 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$$

OR

$$x \begin{bmatrix} 1 \\ 1 \end{bmatrix} + y \begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$$

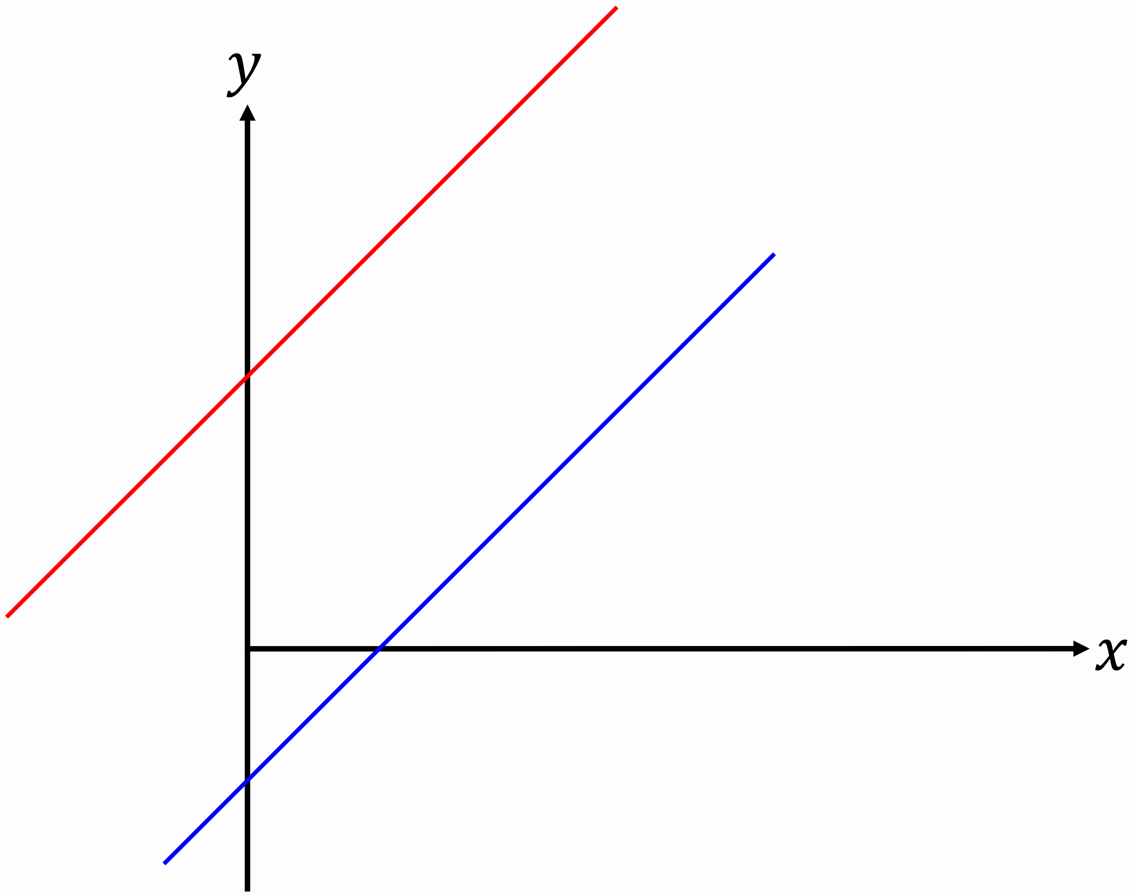


The vector $\begin{bmatrix} 4 \\ 1 \end{bmatrix}$ can only be produced by ONE linear combination of col-1 & col-2. Hence this is the case of unique solution.

System of Linear Equations

Consider $\quad x - y = 1 \qquad (L_1)$

$$-x + y = 2 \qquad (L_2)$$



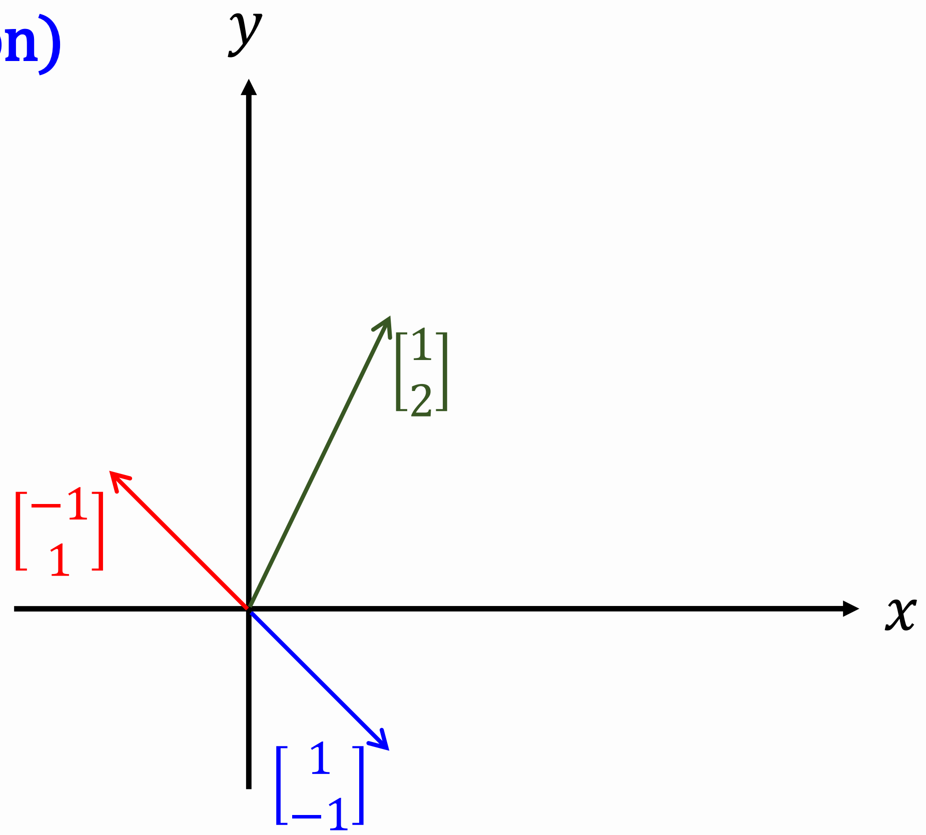
Lines do not intersect.

$\boxed{\text{Case of No Solution}}$

System of Linear Equations (vectors interpretation)

Consider $x - y = 1 \qquad -x + y = 2$

$$x \begin{bmatrix} 1 \\ -1 \end{bmatrix} + y \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$



It is not possible to produce $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ by any linear combination of col-1 & col-2. Hence this is the case of no solution.

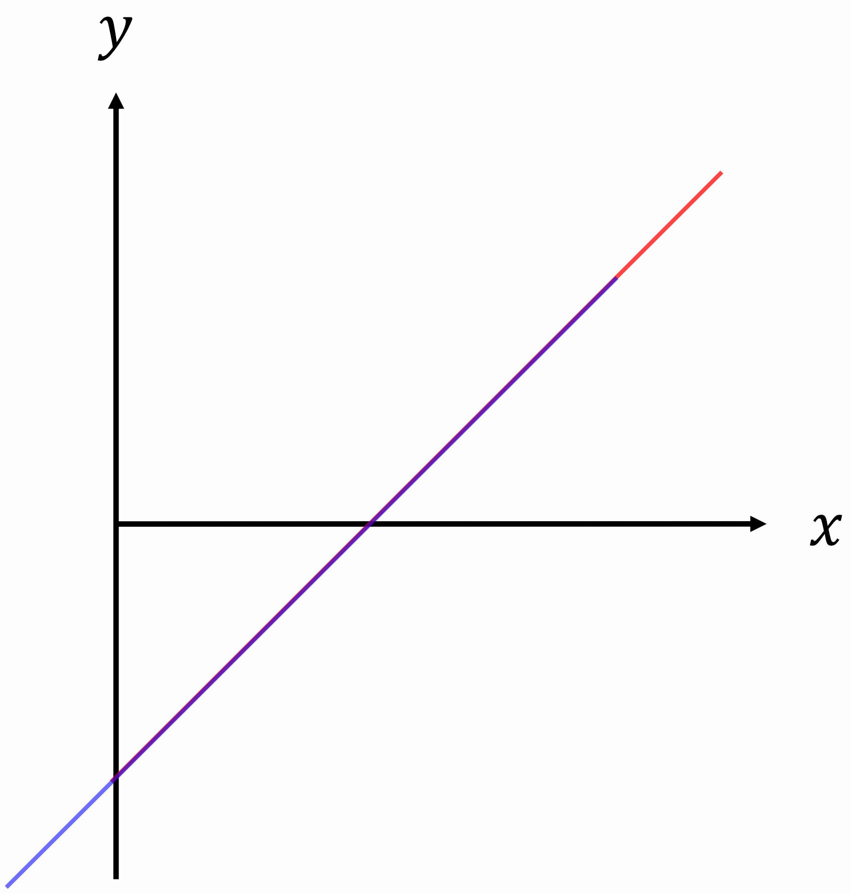
System of Linear Equations

Consider $\quad x - y = 2 \qquad (L_1)$

$\qquad\qquad -x + y = -2 \qquad (L_2)$

Both are the same equation. Any point on the line is a solution of the given system

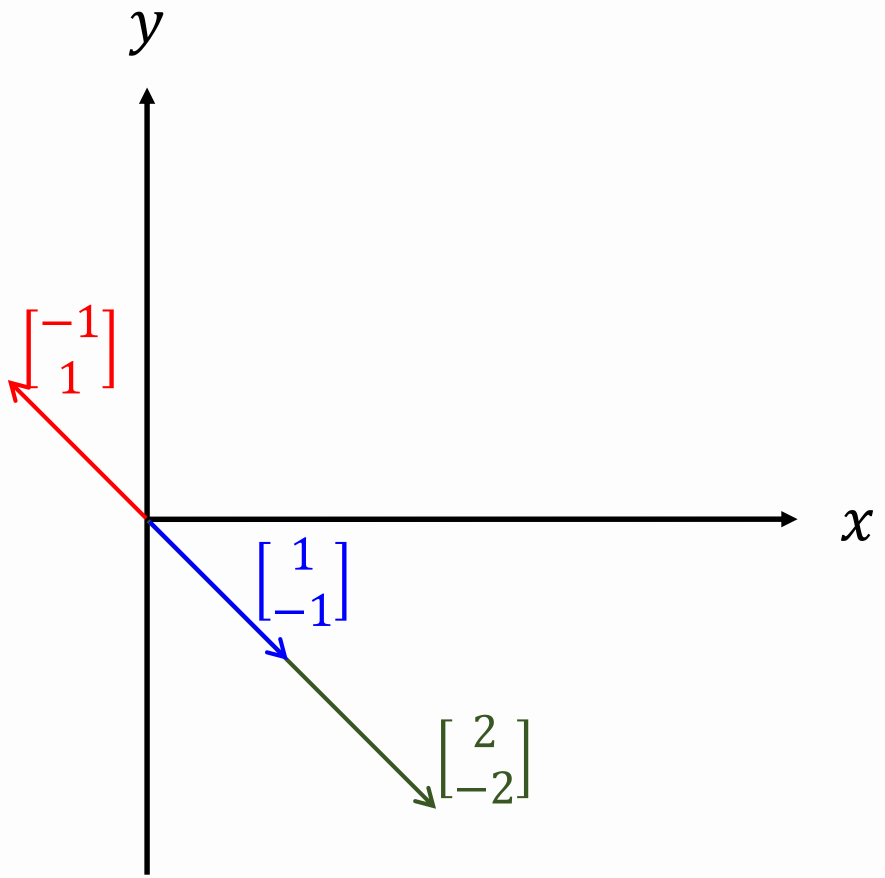
Case of Infinitely Many Solutions



System of Linear Equations (vectors interpretation)

$x - y \ = 2 \qquad -x + y \ = -2$

$$x\begin{bmatrix}1\\-1\end{bmatrix} + y\begin{bmatrix}-1\\1\end{bmatrix} = \begin{bmatrix}2\\-2\end{bmatrix}$$



The vector $\begin{bmatrix}2\\-2\end{bmatrix}$ can be produced by many linear combinations of col-1 & col-2. Hence this is the case of infinitely many solution.

Summary:

System of Linear Equations

- Unique Solution
- Infinitely Many Solutions
- No Solution

System of Linear Equations: Solution Methods

- Method of Determinants: Cramer's rule
- Matrix Inversion Method: $Ax = b \Rightarrow x = A^{-1}b$
- Gauss Elimination Method

direct method
(exact solution)

- Iterative Method – Jacobi & Gauss-Seidel method

approximate
solution

System of Linear Equations: Gauss Elimination Method

Elementary Row Operations

- Interchange of i-th and j-th rows $\left(R_i \leftrightarrow R_j\right)$
- Multiplication of the i-th row by a nonzero number λ $(R_i \leftarrow \lambda R_i)$
- Addition of λ times the j-th row to the i-th row $\left(R_i \leftarrow R_i + \lambda R_j\right)$

Gauss Elimination: Example - 1

$$x_1 + x_2 + x_3 = 4$$
$$2x_1 + 3x_2 + x_3 = 7$$
$$x_1 + 2x_2 + 3x_3 = 9$$

Augmented Matrix

$$[A|b] = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 2 & 3 & 1 & 7 \\ 1 & 2 & 3 & 9 \end{array}\right]$$

$$R_2 \leftarrow R_2 - 2R_1$$
$$R_3 \leftarrow R_3 - R_1$$

$$x_1 + x_2 + x_3 = 4$$
$$x_2 - x_3 = -1$$
$$x_2 + 2x_3 = 5$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 0 & 1 & -1 & -1 \\ 0 & 1 & 2 & 5 \end{array}\right]$$

$$R_3 \leftarrow R_3 - R_2$$

$$x_1 + x_2 + x_3 = 4$$
$$x_2 - x_3 = -1$$
$$3x_3 = 6$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 3 & 6 \end{array}\right]$$

Gauss Elimination: Example - 1

$$
\begin{aligned}
x_1 + x_2 + x_3 &= 4 \\
x_2 - x_3 &= -1 \\
3x_3 &= 6
\end{aligned}
$$

$$[A|b] \sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 3 & 6 \end{array}\right]$$

Echelon form

Solution:

$$x_3 = 2$$

$$x_2 = -1 + 2 = 1$$

$$x_1 = 4 - 1 - 2 = 1$$

Back substitution:

$$x_3 = 2$$

$$x_2 = -1 + x_3 = 1$$

$$x_1 = 4 - x_2 - x_3 = 1$$

Number of Pivots = Number of Unknowns $\Longrightarrow$ Unique Solution

OR every column has a pivot