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Bernoulli's Experiment

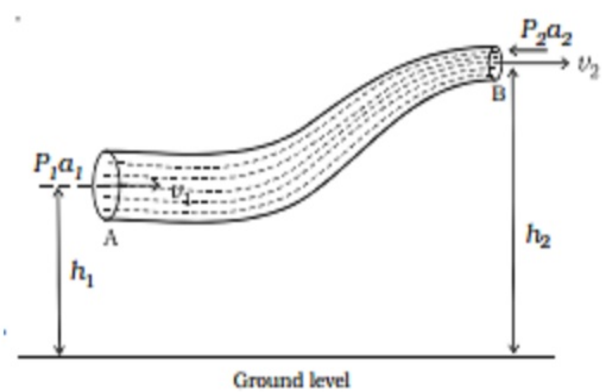
Objective: To verify Bernoulli's theorem

Theory:

Bernoulli's Principle states that the sum of pressure energy per unit volume, kinetic energy per unit volume and potential energy per unit volume of an incompressible, non-viscous fluid in a streamlined irrotational flow remains constant along a streamline.

$$p + \frac{1}{2}\rho v^2 + \rho g h = \text{constant}$$

P is known as the pressure head, $\rho v^2/2$ is known as the kinetic head, ρgh is known as the potential head



Consider two points as shown in the figure,

ρ = fluid density
g = acceleration due to gravity
P_1= pressure at elevation 1
v_1= velocity at elevation 1
h_1= height of elevation 1
P_2= pressure at elevation 2
v_2= velocity at elevation 2
h_2= height at elevation 2

Using Bernoulli's Theorem we get,

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

However, the velocity we consider is the average, because of which we get a difference while performing the experiment. Thus we consider a kinetic energy coefficient (α).

$$\alpha = \frac{1}{V^2}\frac{\int u^3\,dA}{\int u\,dA} = \frac{1}{AV^3}\int u^3\,dA$$

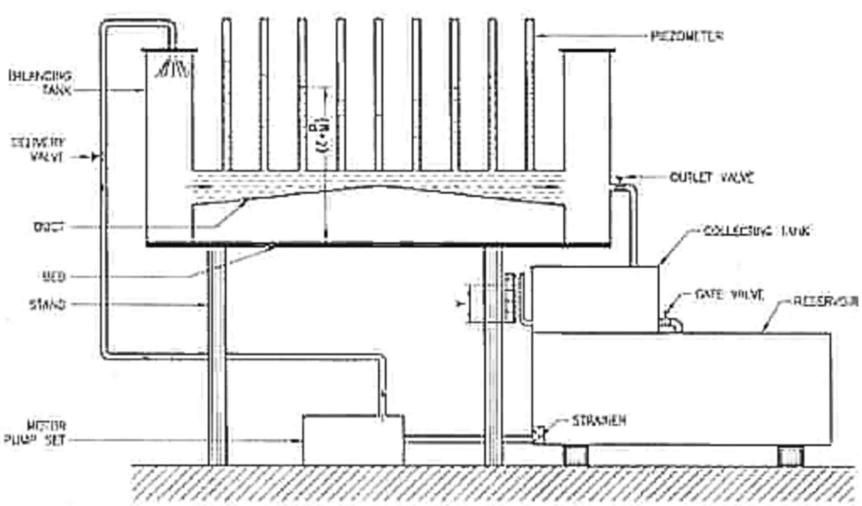
Considering two points at different locations, as shown in the above figure, we get,

$$\frac{P_1}{\rho} + \frac{\alpha_1 v_1^2}{2} + gh_1 = \frac{P_2}{\rho} + \frac{\alpha_2 v_2^2}{2} + gh_2$$

Where,
α_1 is the kinetic coefficient factor at location 1
α_2 is the kinetic coefficient factor at location 2

For laminar flow $\alpha = 2.0$
For turbulent flow, having a large Reynold's number $\alpha \approx 1.0$

Schematic -



Observation Table

S. No	Water Flow Rate	Water Height in piezometric tubes		Cross-Sectional Area	Velocity	Velocity head	Level of water flow	Total Head
	Q	Pt		A	V	$v^2/2g$	z	$Pt + v^2/2g + z$
	LPM	cm		cm^2	cm/s	cm	cm	cm
1	15	P-1	15.9	20.88	11.97	0.073	22.2	38.173
		P-2	16.0	19.15	13.05	0.087		38.287
		P-3	16.1	17.376	14.39	0.106		38.406
		P-4	16.1	15.60	16.03	0.131		38.431
2	20	P-1	17	20.88	15.96	0.13	23.4	40.53
		P-2	17.1	19.15	17.41	0.155		40.655
		P-3	17	17.376	19.18	0.188		40.588
		P-4	17	15.6	21.37	0.233		40.633
3	25	P-1	9.1	20.88	19.96	0.203	15.5	24.803
		P-2	9.2	19.15	21.76	0.242		24.942
		P-3	9.0	17.376	23.98	0.293		24.793
		P-4	9.0	15.6	26.71	0.364		24.864
4.	30	P-1	27.6	20.88	23.95	0.293	34.2	62.093
		P-2	27.7	19.15	26.11	0.348		62.248
		P-3	27.6	17.376	28.78	0.422		62.222
		P-4	27.5	15.80	31.65	0.511		62.211

Calculations

Consider the following cases,

$Q = 15\,LPM$

$Pt_1 = 15.9$ cm $\quad A_1 = 20.88\ cm^2 \quad Z = 22.2$ cm

$Pt_2 = 16$ cm $\quad A_2 = 19.15\ cm^2$

Thus, $Q = 250\ cm^3/s$

$g = 980\ cm/s^2$

$$V_1 = \frac{Q}{A_1} = \frac{250}{20.88} = 11.97\ cm/s$$

$$V_2 = \frac{Q}{A_2} = \frac{250}{19.15} = 13.05\ cm/s$$

The velocity head,

$$\frac{V_1^2}{2g} = \frac{11.97^2}{2 \times 980} = 0.073\ cm$$

$$\frac{V_2^2}{2g} = \frac{13.05^2}{2 \times 980} = 0.087\ cm$$

The velocities found above are average velocities, thus to avoid error we introduce a correction factor α

$$L_1 = D_1 = 2\sqrt{\frac{A_1}{\pi}} = 0.052\ m$$

$$L_2 = D_2 = 2\sqrt{\frac{A_2}{\pi}} = 0.049\ m$$

$$Re_1 = \frac{\rho V_1 L_1}{\mu} = \frac{1000 \times 11.97 \times 10^{-2} \times 0.052}{1 \times 10^{-3} \times 10^{-3}} = 6934.58$$

$$Re_2 = \frac{\rho V_2 L_2}{\mu} = \frac{1000 \times 13.05 \times 10^{-2} \times 0.049}{1 \times 10^{-3} \times 10^{-3}} = 7235.793$$

Clearly Re_1, Re_2 are very large and are in the flow range of turbulence

WKT, for very high Re in a turbulent flow,

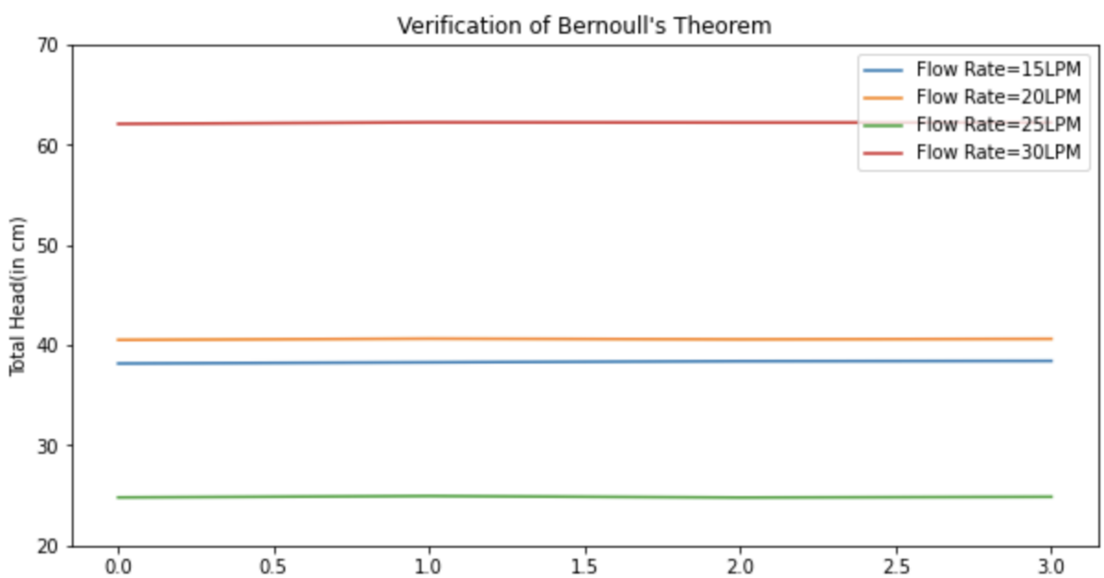
$$\alpha_1 = \alpha_2 = 1$$

$$(\text{Total Head})_1 = Pt_1 + \frac{\alpha_1 v_1^2}{2g} + z = 38.173 \text{ cm}$$

$$(\text{Total Head})_2 = Pt_2 + \frac{\alpha_2 v_2^2}{2g} + z = 38.287 \text{ cm}$$

$$|Th_1 - Th_2| < 1$$

Plot:



Conclusion :

The objective of this experiment was to verify Bernoulli's theorem and from the calculations and the graph it’s clear that the total head is almost constant. The difference between the values is because we didn’t take the minor and major losses into account. From the calculation it is very clear that with decrease in area of the flow velocity increases and pressure decreases. From the difference in the total head can be said that water is not an ideal fluid. This is because of the friction losses in the real fluid; ideal fluid does not have friction losses. These errors can be attributed to the assumptions like frictionless flow, streamline flow, incompressible flow. To account for these losses we introduce two variables, minor losses (sudden change in area, bend in the pipe) and major losses (pressure drop due to friction). Thus, the modified Bernoulli’s equations for two points as shown in Figure 1 is,

$$\frac{P_1}{\rho} + \frac{\alpha_1 v_1^2}{2} + gz_1 = \frac{P_2}{\rho} + \frac{\alpha_2 v_2^2}{2} + gz_2 + h_{LT}$$

$$h_{LT} = h_L + h_{LM}$$

Where, h_{LT} = Total losses
h_{LM} = Minor losses
h_L= Major Losses

Thus, Bernoulli’s Theorem is verified.