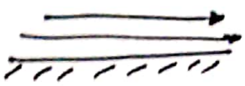
① Film Theory,

→ steady state

→ $N_A + N_B = 0$

At $x = 0$, $C_A = C_{As}$

$x = \delta$, $C_A = C_{Af}$



From Fick's second law,

$$\frac{d^2 C_A}{dx^2} = 0$$

On integration, $\frac{dC_A}{dx} = k_1 \Rightarrow C_A = k_1 x + k_2$

$x = 0 \Rightarrow k_2 = C_{As}$

$x = \delta \rightarrow k_1 = \frac{C_{Af} - C_{As}}{\delta}$

$$\therefore C_A = \left(\frac{C_{Af} - C_{As}}{\delta}\right) x + C_{As}$$

$$\frac{dC_A}{dx} = \frac{C_{Af} - C_{As}}{\delta} \Rightarrow N_A = -D_{AB}\frac{dC_A}{dx}$$

$$\therefore N_A = \frac{D_{AB}}{\delta}(C_{As} - C_{Af}) \Rightarrow K_L = \frac{D_{AB}}{\delta}$$

Penetration Theory

$t = 0$, $z \geq 0$, $C_A = C_{Af}$ $\quad t_c \rightarrow$ contact time

$t > 0$, $z = 0$, $C_A = C_{Ai}$

$t > 0$, $z = \infty$, $C_A = C_{Af}$

$$\left(\frac{C_A - C_{Ab}}{C_{Ac} - C_{Ab}}\right) = 1 - \text{erf}(\eta) \quad \left[\eta = \frac{z}{2\sqrt{D_{AB}t}}\right]$$

$$N_A(t) = -D_{AB}\left(\frac{\partial C_A}{\partial z}\right)_{z=0} = \sqrt{\frac{D_{AB}}{\pi t}}(C_{Ai} - C_{Ab})$$

$$N_{A,avg} = \left[2\sqrt{\frac{D_{AB}}{\pi t}}(C_{Ai} - C_{Ab})\right]$$

$$K_L(t) = \sqrt{\frac{D_{AB}}{\pi t}} \Rightarrow K_{L,avg} = \left(2\sqrt{\frac{D_{AB}}{\pi t_c}}\right)$$

$$\therefore K_L \propto D_{AB}^{0.5}$$

Surface Renewal Theory,

$$K_c = \sqrt{D_A s} \longrightarrow \text{surface renewal state}$$

② Boundary layer Theory

$$Sh_x = \frac{K_{Lx} \cdot x}{D_{AB}} = 0.332 \, (Re_x)^{1/2} (Sc)^{1/3}$$

$$K_{Lx} = \left[\frac{0.332 \, Re_x^{1/2} \, Sc^{1/3}}{x} \right] D_{AB}$$

$$\frac{1}{L}\int_0^L K_{Lx} \, dx = \left(\frac{Sc^{1/3} D_{AB}}{L} \right) \int_0^L \frac{0.332 \left(\frac{\rho u_\infty x}{\mu} \right)^{1/2} dx}{x}$$

$$= \left(\frac{Sc^{1/3} D_{AB}}{L} \right) 0.332 \left(\frac{\rho u_\infty}{\mu} \right)^{1/2} \times 2 \times L^{1/2}$$

$$= 0.664 \, Re^{1/2} \, Sc^{1/3} \, \frac{D_{AB}}{L}$$

$$\therefore K_{L,avg} = 0.664 \, Re^{1/2} \, Sc^{1/3} \, \frac{D_{AB}}{L}$$

$$\frac{K_{L,avg} \, L}{D_{AB}} = 0.6643 \, Re^{1/2} \, Sc^{1/3}$$

$$\therefore Sh_{avg} = 0.664 \, Re^{1/2} \, Sc^{1/3}$$

③ $Sh = Sh_0 + c \, Re^m \, Sc^n$

Using analogous correlation from heat transfer,

$$Nu = 2 + 0.3 \, Re^{0.5} \, Pr^{0.33}$$

Similarly, $Sh = 2 + 0.3 \, Re^{0.5} \, Sc^{0.33}$

Now, $Sh_0 = 2$ when $Re \ll 1$ (when surrounding atmosphere is stagnant)

$$Sh = 2 + 0.3 \, Re^{0.5} \, Sc^{0.33}$$

As $Re \ll 1 \rightarrow 0$, $Sh = 2 + (\cancel{}) \rightarrow 0$.

$\therefore Sh = 2$ at low Reynold's no.