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$$T_s = \text{constant}, \quad Nu = \frac{hD}{k} = 3.66$$

$$Nu = 3.66 + \frac{0.065 (D/L) Re Pr}{1 + 0.04 (D/L) Pr Re}^{1/3}$$

↑ Entrance region

$$Nu = 1.86 \left(\frac{Re Pr D}{L} \right)^{1/3} \left(\frac{\mu_b}{\mu_s} \right)^{0.14}$$

If $Re > 10,000$ (Turbulent flow)

$$f = (0.79 \ln Re - 1.64)^{-2}$$

$$Nu = 0.125 f Re Pr^{1/2}$$

In fully developed turbulent smooth pipe,
 $f = 0.184 Re^{-0.2}$

$$Nu = 0.023 Re^{0.8} Pr^{1/3} \rightarrow 0.7 < Pr < 160$$

$$Re > 1000$$

↳ Colburn Eqn:

So as to correct the error in the above,

$$\left. \begin{array}{l} Nu = 0.023 Re^{0.8} Pr^n \\ n = 0.4 \text{ for heating} \\ n = 0.3 \text{ for cooling} \end{array} \right\} \text{Dittus-Boelter Eqn.}$$

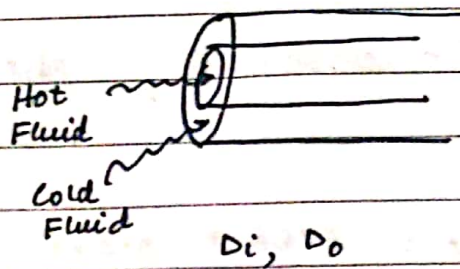
Liquid Metals ($0.004 < Pr < 0.01$)

$$T_s = \text{constant} \rightarrow Nu = 4.8 + 0.0156 Re^{0.085} Pr^{0.93}$$

$$q_s'' = \text{constant} \rightarrow Nu = 6.3 + 0.0167 Re^{0.85} Pr^{0.93}$$

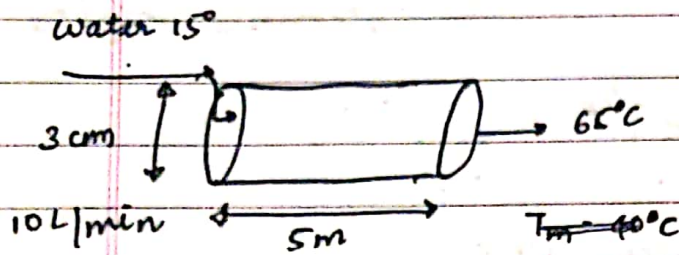
$$D_h = \frac{4A_c}{P}$$

$$D_h = D_o - D_i$$



$$Nu_i = \frac{h_i D_h}{k}$$

$$Nu_o = \frac{h_o D_h}{k}$$



② heater power

③ linear T_s

$$\rho = 992 \cdot 1 \text{ kg/m}^3$$

$$k = 0.631 \text{ W/m}^\circ\text{C}$$

$$v = 0.658 \times 10^{-6} \text{ m}^2/\text{s}$$

$$C_p = 4179 \text{ J/kg}^\circ\text{C}$$

$$Pr = 4.32$$

$$\dot{m} = \rho \dot{V} = 0.165 \text{ kg/s}$$

$$\dot{Q} = \dot{m} C_p \Delta T$$

$$= 34.6 \text{ kW}$$

$$\dot{q}_c = \frac{\dot{Q}}{A_c} = 13.5 \text{ kW/m}^2$$

$$\dot{q}_c = h(T_s - T_m)$$

$$Re = 10900 \left(\frac{v_m \rho}{v} \right)$$

$$V = \frac{\dot{V}}{A_c} = 0.24 \text{ m/s}$$

$$Nu = 0.023 Re^{0.8} Pr^{0.4}$$

$$= \frac{h D}{k} \approx 70$$

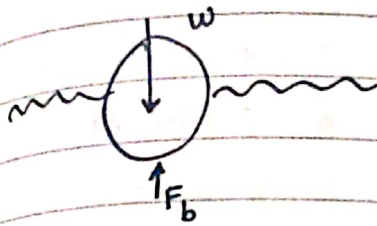
$$T_s = T_m + \frac{\dot{q}_s}{h}$$

$$h = 1479 \text{ W/m}^2\text{C}$$

$$= 65 + \frac{13500}{1479}$$

$$= 115^\circ\text{C}$$

Free convection (Natural convection)



$$F_{net} = (\rho_b - \rho_f) g V_{body}$$

Vol. expansion

$$\text{coeff } (\beta) = -\frac{1}{\rho} \left(\frac{\partial \rho}{\partial T} \right)_P \left[\frac{1}{K} \right]$$

For ideal gas

$$\rho = \frac{P}{RT} \Rightarrow \beta = \frac{1}{T}$$

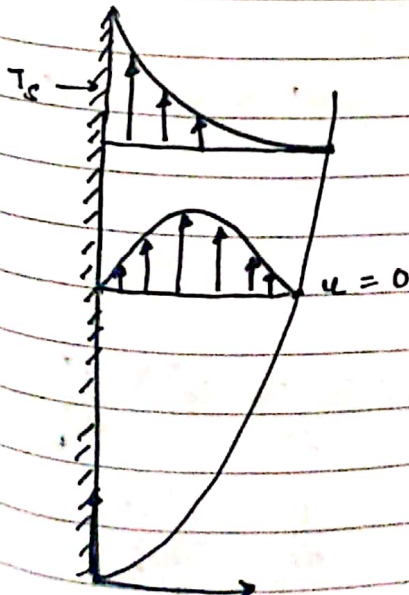
$$\beta = -\frac{1}{\rho} \frac{\rho_\infty - \rho}{T_\infty - T} \quad (\text{At const. } P)$$

$$\rho_\infty - \rho = \rho \beta (T - T_\infty)$$

Buoyancy force $\propto \Delta \rho \propto \Delta T$ (At const. P)

larger $\Delta T \Rightarrow$ larger $F_b \Rightarrow$ Stronger Free conv

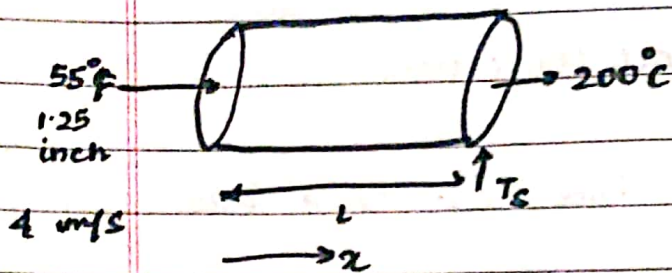
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Higher HT Rate



$$\rho \left(u \frac{\partial v}{\partial x} + v \frac{\partial u}{\partial y} \right)$$

$$= \mu \frac{\partial^2 u}{\partial y^2} - \frac{\partial P}{\partial x} - \rho g$$

$$\frac{\partial P}{\partial x} = -\rho_0 g$$



$$Pr = 3.368$$

$$C_p = 1 \text{ Btu} / ^\circ\text{F}$$

$$\nu = 0.57 \times 10^{-5} \text{ ft}^2/\text{s}$$

$$k = 0.374 \text{ Btu} / \text{ft}^\circ\text{F}$$

$$Re = 11000$$

$$Nu = 475$$

$$h = 1666 \text{ Btu} / \text{hft}^2^\circ\text{F}$$

$$\dot{q}_s = h(T_s - T_e)$$

$$P \left(u \frac{du}{dx} + v \frac{du}{dy} \right) = \mu \frac{\partial^2 u}{\partial y^2} + (P_\infty - P)g$$

$$\beta = -\frac{1}{P} \left(\frac{\partial P}{\partial T} \right)_P$$

$$P_\infty - P = P\beta(T - T_\infty) \quad (\text{at const. } P)$$

$$u \frac{du}{dx} + v \frac{du}{dy} = \nu \frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty)$$

$$\nu = \frac{Re \nu}{L_c}$$

$L_c \leftarrow \text{char. length}$

On non-dimensionalising,

$$x^* = \frac{x}{L_c}, \quad y^* = \frac{y}{L_c}, \quad u^* = \frac{u}{V}, \quad v^* = \frac{v}{V}$$

$$T^* = \frac{T - T_\infty}{T_s - T_\infty}$$

$$u^* \frac{du^*}{dx^*} + v^* \frac{du^*}{dy^*} = \left(\frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2} \right) \frac{T^*}{Re^2} + \frac{1}{Re} \frac{\partial^2 u^*}{\partial y^{*2}}$$

$$Gr_L \text{ (Grashof No.)} = \frac{g \beta (T_s - T_o) L^3}{\nu^2}$$

kinematic viscosity

For vertical plate, $Gr_{critical} = 10^9$ of order

Turbulent if $> 10^9$, in vertical Hot plane.



If $Gr/Re^2 \ll 1$, free convection dominates

If $Gr/Re^2 \gg 1$, natural convection

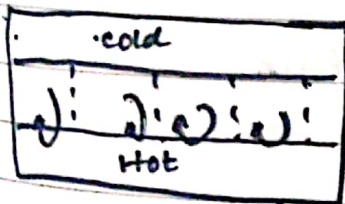
If $Gr/Re^2 \approx 1$, can't neglect any

$$Nu_L = \frac{h L_c}{k} = c (Gr_L Pr)^n = c Ra_L^n$$

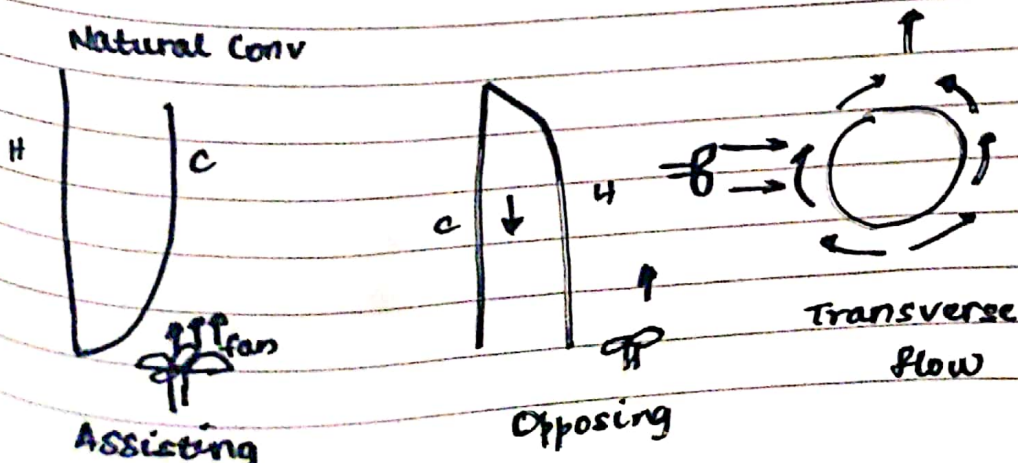
\downarrow
 $c < 1$

$\hookrightarrow n = \frac{1}{4}$ laminar
 $n = \frac{1}{3}$ turbulent

$$\dot{Q} = h A_s (T_s - T_o)$$



Bénard Cells (Only when $Ra > 1700$)
 When $Ra > 3 \times 10^5$, then
 cells are broken with turbulent



The combined Nu number,

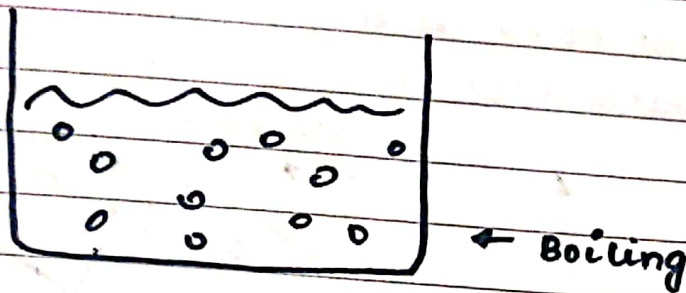
$$Nu_c = (Nu_f^n + \overset{\text{Assisting / Transverse}}{Nu_n^n})^{1/n}$$

← Opposing

$$\dot{q}_{conv} = h A_s (T_s - T_{\infty})$$

Inv. prop

Boiled convection



Formation of the bubbles is dependent on latent heat of vap, surface tension.

Evaporation liq. vapour Interface

Boiling

60% RH, 20°C, water evaporates

2.3 kPa = p_{sat}

v.p = 14 kPa

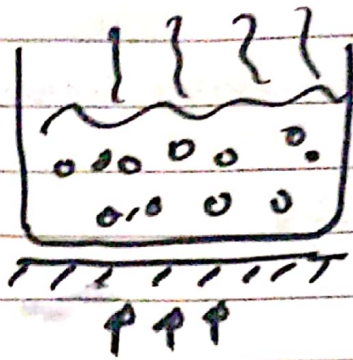
Solid liq. interface

$T_s \gg T_{sat}$

$$q_b = h (T_s - T_{sat}) = h \Delta T_{excess} \quad (W/m^2)$$

Pool boiling - No bulk motion of flow

Flow boiling - Forced convection induced boiling.



Pool boiling

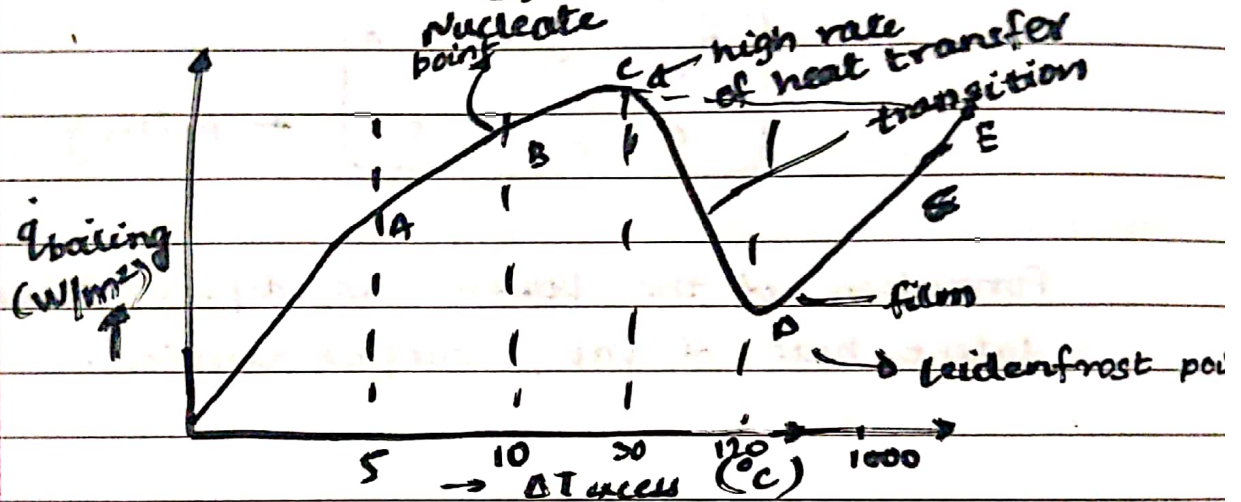


Flow boiling

Subcooled boiling
Saturated boiling

Subcooled boiling, $T_b < T_{sat}$

Saturated boiling, $T_b = T_{sat}$



Nucleate point - bubbles are moving up

At C - critical / max heat flux
then CD is transition B-P

From pt. D - film boiling

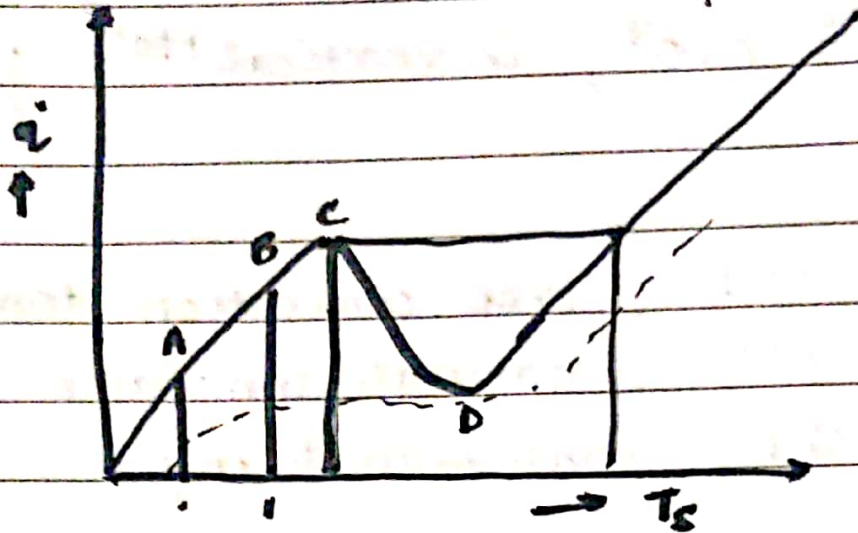
$$Nu = f \left(Pr, \frac{g(\rho_l - \rho_v)L^3}{\mu^2}, Ja, Bo \right)$$

Jacob no. \rightarrow
Bondi \rightarrow

$$Ja = \frac{c_p \Delta T}{h_{fg}}$$

$$Bo = \frac{g(\rho_l - \rho_v)L}{\mu}$$

Boiled convection



A \rightarrow natural convection boiling

(A \rightarrow B) \rightarrow Change from natural to nucleid boiling

BC \rightarrow Bubbles detach from the q surface and comes up to the surface

C \rightarrow Critical heat flux

D \rightarrow level

$$\dot{q}_{nuc} = \eta h_{fg} \left[\frac{g(\rho_l - \rho_g)}{\sigma} \right]^{1/2} \left[\frac{C_{sf} h_{fg} Pr_l^n}{\rho_l (T_s - T_{sat})} \right]^3$$

↑
const. depends
on fluid and
surface const.

$$\dot{q}_{max} = C_{cr} h_{fg} \left[\frac{g \rho_v^2 (\rho_l - \rho_v)}{\sigma} \right]^{1/4}$$

$$\dot{q}_{rad} = \epsilon \sigma (T_s^4 - T_{sat}^4)$$

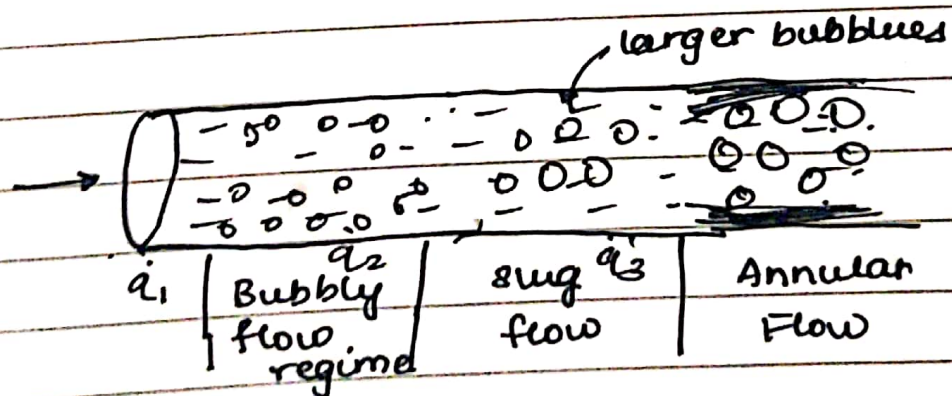
$$T_w = \left(\frac{T_s + T_{sat}}{2} \right)$$

$$\dot{q}_{total} = \dot{q}_{film} + \frac{3}{4} \dot{q}_{rad}$$

$C_{cr} \rightarrow$ depends on shape / material of the

Flow boiling \rightarrow Internal
 \downarrow
External

Internal



$$\dot{q}_{total} = \dot{q}_1 + \dot{q}_2 + \dot{q}_3$$