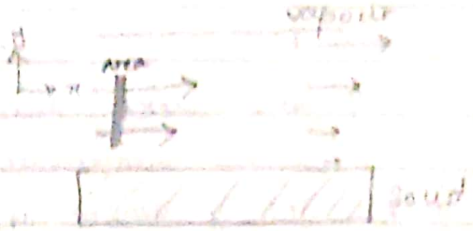
$\tau_{yx}$ ↳ direction of motion
↳ direction of momentum




The stress is caused due to the drag between top and bottom layers (due to diff. in their velocities) and the area is the area prep. to the layers.

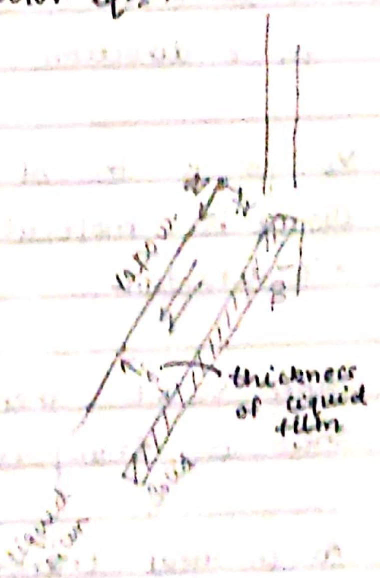
Note :-

* In, most of the cases, the mass transfer is due conc. gradient only.

$N_A$ - amount of mass / moles moving per unit area per second.

→ Shell momentum Balance → Difference ~~Differential~~ eqn.
↓
Diff. eqn. (Gov. eqn.)
←
$\tau, v$ (Boundary cond.)

Flow chart for solving question




The velocity in z-direction is dependent on

$$v_z = f(x, z, \delta, \mu, \beta, g)$$

angle ($\beta$); viscosity ($\mu$); acc. due to gravity ($g$)

So all these have to be taken into consideration while writing the gov. eqn.

$z \to$ length
$y \to$ width
$x \to \delta$

Dimensions of entire film and directions associated with it.

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Momentum transfer is in $x$ and $z$-directions and, $\because$ as there is no flow in $y$ direction, there is no momentum transfer.

In $V_z$ The $y$ doesn't appear as it's very wide in the $y$-direction that the $y$-dependancy can be neglected.

→ Conductive / Molecular Transport of momentum in $x$-direction

→ In $x$-direction there is no net motion, the layers are slipping past each other, the molecules of a faster moving layer would try to drag the molecules of the slower moving layers just below it because of viscosity.

→ $V_x = 0$, $V_y = 0$, $V_z \neq 0$

→ Convection in $z$-direction

→ $V_x = 0$, $V_y = 0$, $V_z \neq 0$

→ Since $V_z \neq 0$, there is actual movement in the $z$-direction, thus there have to be convection in $z$-direction.

→ $V_z$ is a fn. of $x$, then due to viscosity there would be molecular transport of momentum in $+x$ direction.

→ Since $V_x = 0$, there is no transfer of mass, however there is transport of momentum.

→ As in heat transfer, there is no mass transfer but there is molecular transfer of heat and is called conduction, similar is the case with momentum transport in $x$-direction

→ Always choose the dimension in which the velocity is varying as the smaller dimension of your

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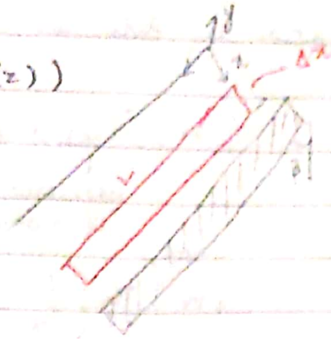
shell. (So we consider our shell in $\delta x$ direction)

$v_z$ isn't a fn of z (i.e $v_z \neq f(z)$)



There are two forces acting on the shell of fluid

→ Gravity which is pulling fluid in downward z-direction

→ And also viscosity acting in -z direction, which is trying to move it.

*Imp assumption :- Steady state; 1D flow — velocity could be a fn of x but not of t

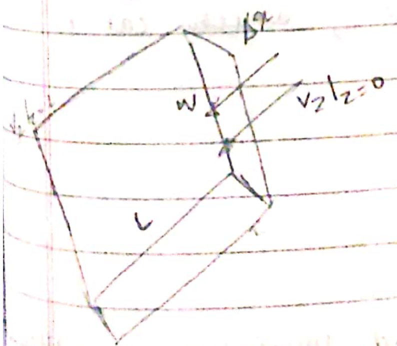
i.e forces are balanced at any location, $\Sigma F = 0$

$\Sigma F = 0$ and $\Delta x \to 0$ we get difference shell eqn.

In convential fluid mech, the predominant surface force is pressure

Rate of $m^2$ in − out + $\Sigma F = 0$

B, S = 0 (pressure, pressure gradient.)



$(w\,\Delta x)\, v_z|_{z=0}$ $\quad \rho v_z|_{z=0}$ = $m^3/s$

$(w\,\delta x)\, v_z|_{z=L}$ $\quad \rho v_z|_{z=L}$

$\Rightarrow v_z|_{z=0} = v_z|_{z=L}$ → So there

and also $v_x = v_y = 0$ } is no convective momentum transport.

Conductive transport of momentum is acting and it acts on L, W

$(LW)\,\tau_{xz}$ → direction of motion
↓
direction of momentum

Net force on the shell

IN OUT

$$\left.\begin{array}{l}(LW)\,\tau_{xz}\big|_{x} - (LW)\,\tau_{xz}\big|_{x+\Delta x}\end{array}\right\}$$ conductive momentum transfer

$$+\ \underbrace{LW\,\Delta x\,\rho g\cos\beta}_{\text{Body force}} = 0$$

$$\lim_{\Delta x \to 0} \tau_{xz}\big|_{x} - \tau_{xz}\big|_{x+\Delta x} = \rho g \cos\beta$$

$$\Rightarrow \boxed{\frac{d}{dx}(\tau_{xz}) = \rho g \cos\beta}$$ – Governing eqn. – ①

Considering the fluid as newtonian fluid, we can substitute

$$\tau_{xz} = -\mu \frac{dv_z}{dx}$$

Boundary Cond.

① → At liquid-air interface, $\tau = 0$ (No shear LV interface)

② → At liquid-solid interface, relative velocity $(v) = 0$ (No slip at L-S cond.)

After solving ①, we get $\tau_{xz} = \rho g \cos\beta + C_1$ (Found using B.Cd ①) and we get another ②, which can be found by using B.Cd ②

$\tau_{xz} = 0,\ x = 0 \rightarrow C_1 = 0$

$V_z = 0,\ x = \delta \Rightarrow C_2$

Exceptions:-

→ Large change in density

→ When the fluid is rarified and continum is challenged