①(i) N_A is const. , $A\uparrow$, $B \neq 0$, $N_B = 0$

$$N_A = N_A \frac{P_A}{P} - \frac{D_{AB}}{RT}\frac{dP_A}{dx}$$

$$N_A \, dx = -\frac{D_{AB}\, P}{RT\,(P-P_A)} \, dP_A$$

$$N_A \int_0^L dx = -\frac{D_{AB}\, P}{RT} \int_{P_{A1}}^{P_{A2}} \frac{dP_A}{P-P_A}$$

$$N_A = \frac{D_{AB}\, P}{RT\, L} \ln \frac{P-P_{A2}}{P-P_{A1}}$$

At $x = 0$, $P_A = P_{A1}$

$x = L$, $P_A = P_{A2}$

Total pressure is uniform

$$P = P_{A1} + P_{B1} = P_{A2} + P_{B2}$$

$$P - P_{A1} = P_{B1}$$

$$P - P_{A2} = P_{B2}$$

$$\Rightarrow P_{A1} - P_{A2} = P_{B1} - P_{B2}$$

①

(ii) WKT,

$$N_A = \frac{D_{AB}\ c}{L\ (C_B)_{lm}} (C_{A_1} - C_{A_2})$$

where, $(C_B)_{lm} = \dfrac{C_{A_1} - C_{A_2}}{\ln(C_{A_1}/C_{A_2})}$

WKT, $x_{A_1} = \dfrac{C_{A_1}}{c}$, $x_{A_2} = \dfrac{C_{A_2}}{c}$

$$C_{Blm} = \frac{c\ (x_{A_1} - x_{A_2})}{\ln\left(\frac{x_{A_1}}{x_{A_2}}\right)}$$

$$\frac{C_{Blm}}{c} = \frac{(x_{A_1} - x_{A_2})}{\ln(x_{A_1}/x_{A_2})} \Rightarrow x_{Blm} = \frac{C_{Blm}}{c} = \left(\frac{x_{A_1} - x_{A_2}}{\ln(x_{A_1}/x_{A_2})}\right)$$

so,

$$N_A = \frac{D_{AB}\ c}{L\ x_{Blm}} (x_{A_1} - x_{A_2})$$

$$\Rightarrow c = P/RT$$

$$\Rightarrow N_A = \frac{D_{AB}\ P}{RT\ L\ x_{Blm}} (x_{A_1} - x_{A_2})$$

Also, $\dfrac{P_{A_1}}{RT} = C_{A_1}$ & $\dfrac{P_{A_2}}{RT} = C_{A_2}$

Scanned with CamScanner

(2) (i) From Fick's second law,

$$\frac{d^2 C_A}{dx^2} = 0$$

On integrating

$$\frac{dC_A}{dx} = k \quad -(1)$$

On integrating further,

$$C_A = kx + m \quad -(2)$$

For binary, non-reactive, steady state, single phase, const. geometry

$$N_A = -D_{AB} \frac{dC_A}{dx}$$

$$\therefore \quad k = -\frac{N_A}{D_{AB}}$$

Also, $N_A = \frac{D_{AB}}{L} (C_{A_1} - C_{A_2})$

At $x = 0$

$$C_A = C_{A_1} = m$$

If $C_A = C$ at x

Then $$C = -\frac{N_A}{D_{AB}} x + C_{A_1}$$

$$\therefore \quad \frac{C - C_{A_1}}{C_{A_1} - C_{A_2}} = -x/L$$

③ (ii) show that $\frac{C - C_{A_2}}{C - C_{A_1}} = \left(\frac{C - C_{A_2}}{C - C_{A_1}}\right)^{x/L}$

From Fick's law,

$$N_A = -D_{AB}\frac{dC_A}{dx} + \frac{C_A}{C}N_A$$

$$\frac{N_A\,dx}{D_{AB}\,C} = -\frac{dC_A}{C_A - C} \Rightarrow N_A = \frac{D_A\,C}{C - C_A}\frac{dC_A}{dx}$$

$$\therefore \frac{dN_A}{dx} = 0$$

$$\Rightarrow \frac{d}{dx}\left(\frac{D_A\,C}{C_A - C_A}\cdot\frac{dC_A}{dx}\right) = 0$$

$$C_A|_0 = C_{A_1}, \qquad C_A|_L = C_{A_2}$$

$$\frac{dC_A}{dx\,(C - C_A)} = C_1 \Rightarrow \int_{C_{A_1}}^{C_{A_2}}\frac{dC_A}{C - C_A} = \int C_1\,dx$$

$$\therefore \ln(C - C_A) = C_1 x + C_2$$

At $x = 0$, $\ln(C - C_{A_1}) = C_2$

At $x = L$, $\ln(C - C_{A_2}) = \ln(C - C_{A_1}) + C_1 L$

$$\therefore \frac{1}{L}\ln\left(\frac{C - C_{A_2}}{C - C_{A_1}}\right) = C_1$$

$$\Rightarrow \frac{C - C_A}{C - C_{A_1}} = \left(\frac{C - C_{A_2}}{C - C_{A_1}}\right)^{x/L}$$

③ $$D_{AB}\frac{dC_A}{dr} - \frac{C_A}{c}(N_A + N_B) = -N_A$$

$(N_B = 0)$

$$D_{AB}\frac{dC_A}{dr} = N_A\left(\frac{C_A - C}{C}\right)$$

$$\int\frac{dC_A}{\left(\frac{C_A - C}{C}\right)} = \int\frac{\pi r^2}{\pi r^2}\times\frac{N_A}{D_{AB}}\,dr$$

$$\int_{C_{A\infty}}^{C_{As}}\frac{dC_A}{\left(\frac{C_A - C}{C}\right)} = \frac{W}{4\pi D_{AB}}\int_{r=\infty}^{r=r_s}\frac{dr}{r^2}$$

$$c\ln\left(\frac{C - C_{As}}{C - C_{A\infty}}\right) = \frac{W}{4\pi D_{AB}}\times -\frac{1}{r}$$

$$\therefore W = 4\pi D_{AB}\, c\cdot r\cdot\ln\left(\frac{C - C_{A\infty}}{C - C_{As}}\right)$$

④ Sublimation / Release time

$$w = -\frac{d}{dt}\left(\frac{4}{3}\pi r^3 \frac{\rho_A}{M_A}\right)$$

$$= -4\pi r^2 \frac{dr}{dt} \cdot \frac{\rho_A}{M_A}$$

$$= r c D_{AB} \ln\left(\frac{c - c_\infty}{c - c_{AS}}\right)$$

$$-r\frac{dr}{dt} = \frac{M_A}{\rho_A} c D_{AB} \ln\left(\frac{c - c_\infty}{c - c_{AS}}\right)$$

$$-\int_{r_0}^{0} r\,dr = \frac{M_A c D_{AB}}{\rho_A} \ln\left(\frac{c - c_\infty}{c - c_{AS}}\right)\int_0^t dt$$

$$\frac{r_0^2}{2} = \frac{M_A c D_{AB}}{\rho_A} \ln\left(\frac{c - c_\infty}{c - c_{AS}}\right) t$$

$$\therefore\ t = \frac{r_0^2}{2}\frac{\rho}{M_A c D_{AB} \ln(c - c_\infty / c - c_{AS})}$$

⑤ Equimolar counter diffusion (flux at any poistion of L in tapered tube)

$$N_A = -\frac{D_{AB}}{RT}\left(\frac{dP_A}{dx}\right) \quad - ①$$

$$\pi r^2 N_A = w_1^2 - \pi r^2 \frac{D_{AB}}{RT}\frac{dP_A}{dx}$$

$$r = r_1 + \left(\frac{r_2 - r_1}{L}\right)x \quad - ②$$

Substituting N_A and r, $x = 0,\ P = P_{A_1}$, $x = L,\ P = P_{A_2}$

$$-\int_{P_{A_1}}^{P_{A_2}} dP_A = \frac{w_1 RT}{\pi D_{AB}}\int_0^L \frac{dx}{r_1 + \left(\frac{r_2 - r_1}{L}\right)x}$$

$$w_1 = \frac{\pi D_{AB}}{RT}\frac{r_1 r_2}{L}(P_{A_1} - A_2)$$

Now using $c = P/RT \Rightarrow P = c/RT$, $C_{A_1} = P_{A_1}/RT$, $C_{A_2} = P_{A_2}/RT$

$$W = \frac{\pi D_{AB} r_1 r_2}{l}(C_{A_1} - C_{A_2})$$