Temperature Sensors

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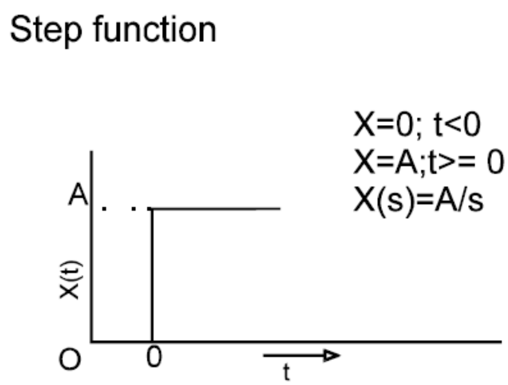
<u>Objective</u> : To study the response of two tank interacting and non-interacting systems.

<u>Theory</u>:

Step response of single capacity system

Step function**:** Mathematically, the step function of magnitude A can be expressed as X (t) = A u (t) where u (t) is a unit step function.
It can be graphically represented as



To study the transient response for step function, consider the system consisting of a tank of uniform cross sectional area $\mathbf{A_1}$ and outlet flow resistance **R** such as a valve. $\mathbf{q_o}$,volumetric flow rate through the resistance, is related to head **h** by a linear relationship

$$q_o = h/R \text{ -------(1)}$$

Writing a transient mass balance around the tank:
Mass flow in - Mass flow out = rate of accumulation of mass in the tank.

$$q(t) - q_o(t) = d(Ah)/dt$$
$$q(t) - q_o(t) = A_1\, dh/dt \text{ ------(2)}$$

Combining equation (1) and (2) to eliminate qo(t) gives the following linear differential equation:

$$q - h/R = A1\, dh/dt \text{ -------(3)}$$

Initially the process is operating at steady state, which means that dh/dt = 0.
Therefore equation (3) becomes as

$$q_s - h_s/R = 0 \text{ -----------(4)}$$

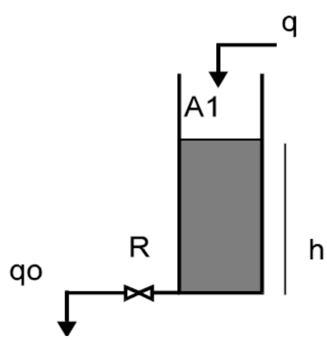
Where, the subscript s indicates the steady state value of the variable.
Subtracting equation (4) from (3)

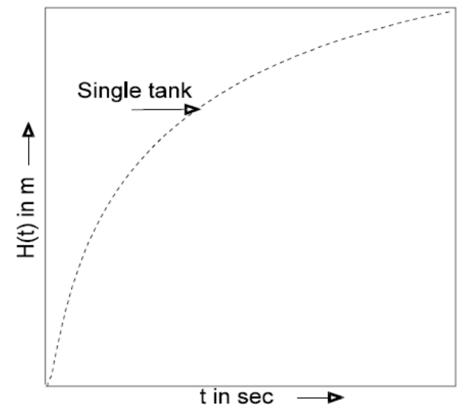
$$(q - q_s) = 1/R\ (h - h_s) + A_1\ d(h - h_s) / dt \text{ ----(5)}$$

Defining deviation variable

$$q - q_s = Q$$
$$h - h_s = H$$

Liquid level system





Transient response of single tank system

Equation (5) can be written as

$$Q = 1/R\ H + A_1\ dH/dt \text{ ---------(6)}$$

Taking a transform of equation (6) gives

$$Q(s) = 1/R\ H(s) + A1s\ H(s) \text{ ----(7)}$$

Equation (7) can be rearranged into standard form of first order system as

$$H(s)/Q(s) = R/(\tau_s +1) \text{ -----------(8)}$$

Where $\tau = A_1R$

For a step change of magnitude A, we can write

$$Q(t) = A\ u(t),\ Q(s) = A/s$$

From equation (8) we can write

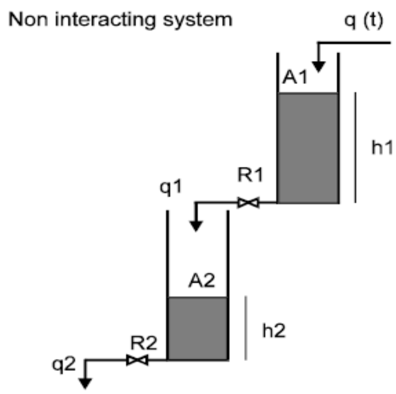
$$H(s) = A/s\ \{R/(\tau_s +1)\} \text{ -------(9)}$$

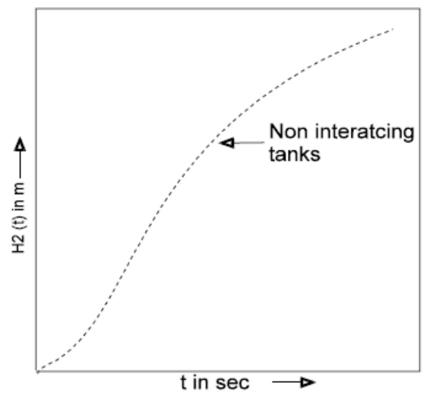
So by taking Laplace transform of equation (9) we get,

$$H(t) = AR \{ (1- e^{-t/\tau}) \} \text{---------}(10)$$

Step response of first order systems arranged in non- interacting mode

In non-interacting system we assume the tanks have uniform cross sectional area and the flow resistance is linear. To find out the transfer function of the system that relates h_2 to q, writing a mass balance around the tanks, we proceed as follows





Transient response of non-interacting system

We can write mass balance at tank1 as

$$q-q_1 = A_1(dh_1 / dt) \ldots\ldots\ldots\ldots (1)$$

A mass balance at tank 2 is given as

$$q_1-q_2 = A_2(dh_2 / dt) \ldots\ldots\ldots\ldots (2)$$

The flow head relationships for the two linear resistances in non-interacting system are given by the expressions

$$q_1=(h_1 / R_1) \ldots\ldots\ldots\ldots (3)$$
$$q_2=(h_2 / R_2) \ldots\ldots\ldots\ldots (4)$$

From (1) and (3)

$$\frac{Q_1(s)}{Q(s)} = \frac{1}{\tau_1 . S + 1} \ldots\ldots\ldots\ldots (5)$$

Where $Q_1 = q_1 - q_1s$, $Q = q - qs$ and $\tau_1 = A_1 R_1$

From (2) and (4)

$$\frac{H_2(s)}{Q_1(s)} = \frac{R_2}{\tau_2 . S + 1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

Where, $H_2 = h_2 - h_2s$ and $\tau_2 = A_2 R_2$

Overall transfer function can be calculated as follows,

$$\frac{H_2(s)}{Q(s)} = \frac{R_2}{(\tau_1 . S + 1)(\tau_2 . S + 1)} \quad \ldots\ldots\ldots\ldots\ldots\ldots (7)$$

For a step change of magnitude A
Q(t) = A u(t)
So, Q(s) = A / s

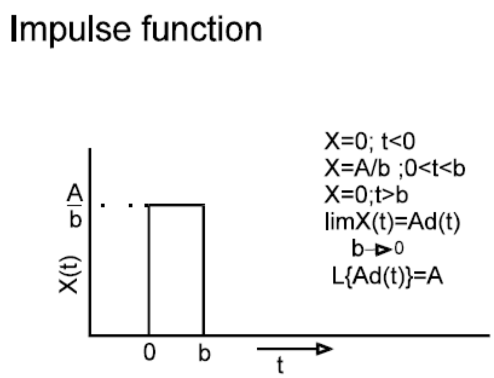
$$H_2(s) = \frac{AxR_2}{s \times (\tau_1 . S + 1)(\tau_2 . S + 1)} \quad \ldots .(8)$$

H_2 at time t is given by,

$$H_2(t) = A R_2 \left[1 - \frac{(\tau_1 \times \tau_2)}{(\tau_1 - \tau_2)} \{ \tau_2^{-1} e^{-t/\tau_1} - \tau_1^{-1} e^{-t/\tau_2} \}\right] \quad \ldots\ldots\ldots\ldots\ldots (9)$$

To study impulse response of first order systems arranged in noninteracting mode

Mathematically, the impulse function of magnitude A is defined as
$X(t) = A^{TM}(t)$
Where $^{TM}(t)$ is the unit impulse function. Graphically it can be described as



Overall transfer function of the system as described in previous experiment

$$\frac{H_2(s)}{Q(s)} = \frac{R_2}{(\tau_1 . S + 1)(\tau_2 . S + 1)} \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(1)}$$

For a impulse change of magnitude V (volume added to the system)

$Q(t) = V^{TM}(t)$
So, $Q(s) = V$

$$H_2(s) = \frac{VxR_2}{(\tau_1 . S + 1)(\tau_2 . S + 1)} \quad \ldots \text{(2)}$$

For impulse change H2 at time t is given by

$$H_2(t) = V R_2 \left[\frac{e^{-t/\tau 1} - e^{-t/\tau 2}}{(\tau 1 - \tau 2)}\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(3)}$$

Considering non-linear resistance at outlet valve of the tank R2 can be calculated as

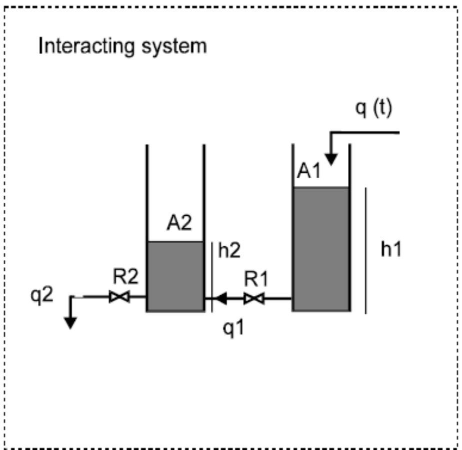
$$R_2 = 2dH_2 / dQ$$

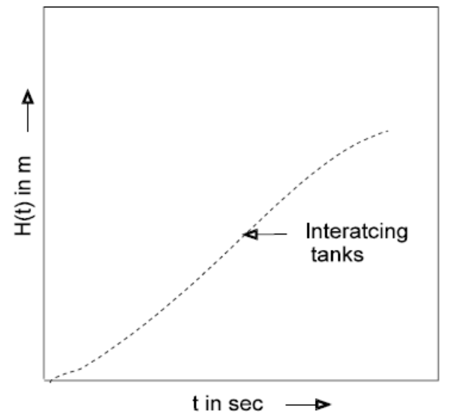
Where dH_2 is changed in the level of tank 2 and dQ is a change of flow from initial to the final state.

Put the values in equation (3) to find out H (t) Predicted and plot the graph of H (t) Predicted and H(t) Observed Vs time.

Step response of first order systems arranged in interacting mode

Assuming the tanks of uniform cross sectional area and valves with linear flow resistance the transfer function of interacting system can be written as:





Transient response of interacting system

$$\frac{H_2\,(s)}{Q\,(s)} = \frac{R_2}{\tau_1\,\tau_2\,s_2 + (\tau_1 + \tau_2 + A_1\,R_2)\,s + 1}$$

Let

$$b = \frac{1}{\tau_1} + \frac{1}{\tau_2} + \frac{A_1\,R_2}{\tau_1\,\tau_2}$$

$$\alpha = (-b/2) + \sqrt{\,[\,(b/2)^2 - (\,1/\,\tau_1\,\tau_2)]}$$
$$\beta = (-b/2) - \sqrt{\,[\,(b/2)^2 - (\,1/\,\tau_1\,\tau_2)]}$$

For a step change of magnitude A

$$H_2\,(t) = AR_2\,\left\{1 - \frac{[(1/\alpha)\,e^{(\alpha t)}] - [(1/\beta)\,e^{(\beta t)}]}{[1/\alpha - 1/\beta]}\right\}$$

In terms of transient response the interacting system is more sluggish than the noninteracting System.

Impulse response of first order systems arranged in interacting mode

As mentioned in theory part of experiment 3, impulse function is described as

$$X\ (t)= A^{TM}(t)$$

Overall transfer function of the system as described in previous experiment

$$\frac{H_2(s)}{Q(s)} = \frac{R_2}{\tau_1\ \tau_2\ s^2 + (\tau_1 + \tau_2 + A_1\ R_2)\ s + 1} \quad \ldots\ldots\ldots(1)$$

For a impulse change of magnitude V (volume added to the system)
$Q(t) = V^{TM}(t)$
So, Q (s) = V

$$H_2(s) = \frac{V\ R_2}{\tau_1\ \tau_2\ s_2 + (\tau_1 + \tau_2 + A_1\ R_2)\ s + 1} \quad \text{---}(2)$$

For impulse change H2 at time t is given by

$$H_2\ (t)= \frac{V\ R_2}{\tau_1\tau_2\ (\alpha-\beta)}\ [e\ (\alpha t) - e\ (\beta t)] \quad \text{-----}(3)$$

(For α, β refer theory part of experiment No. 4)
Considering non- linear valve resistance, the resistance at outlet of both tanks can be calculated as

$$R_1 = 2\ dH_1/dQ \quad \text{------------}(4)$$
$$R_2 = 2\ dH_2/dQ \quad \text{------------}(5)$$

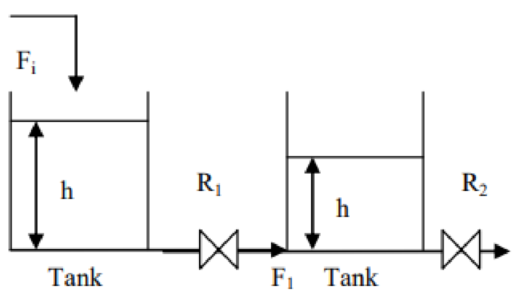
Experimental Setup:

The setup is designed to study dynamic response of single and multi-capacity processes when connected in interacting and noninteracting mode. It is combined to study:

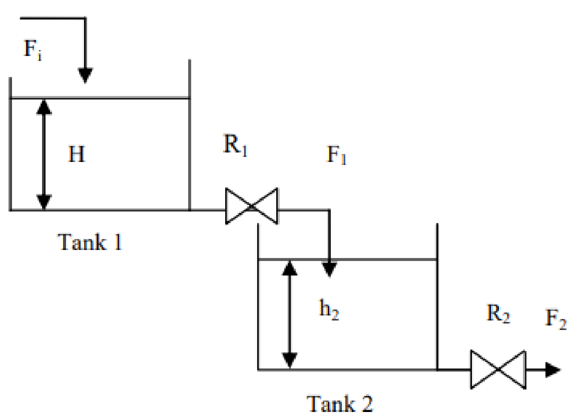
1) Single capacity process,
2) Non-interacting process and
3) Interacting process.

The observed step response of the tank level in different mode can be compared with mathematically predicted response.

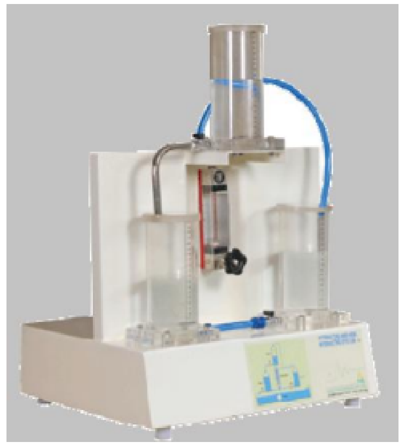
The setup consists of supply tank, pump for water circulation, rotameter for flow measurement, transparent tanks with graduated scales, which can be connected, in interacting and noninteracting mode. The components are assembled on frame to form tabletop mounting.



Schematic diagram of Interacting process



Schematic diagram of Interacting process



Actual experimental setup

Procedure:

1. For step response of the single capacity system,

- The set-up is started. A flexible pipe is provided at the rotameter outlet. The pipe is inserted into the cover of the top Tank 1. We keep the outlet valves (R1 & R2) of Tank 1 & Tank 2 slightly closed.
- We switch on the pump and adjust rotameter flow rates in steps of 10 LPH from 50 to 100 LPH and note steady-state levels for Tank 1 against each flow rate.
- From the data obtained, we select a suitable band for experimentation. (Say 90-100 LPH in which we are getting more readings of tank level).
- We adjust the flow rate at lower value of the band selected (say 90 LPH) and allow the level of the Tank 1 to reach the steady state and record the flow and level at steady state.
- We apply the step change by increasing the rotameter flow by @ 10 LPH.
- We immediately start recording the level of the Tank 1 at the interval of 15 sec, until the level reaches at steady state.
- We then carry out the calculations as mentioned in calculation part and compare the
- predicted and observed values of the tank level.
- We repeat the experiment by throttling outlet valve (R_1) to change resistance.

2. For Step response of first order systems arranged in non-interacting Mode

- The setup is started. A flexible pipe is provided at the rotameter outlet. The pipe is inserted in to the cover of the top Tank 1. We keep the outlet valves (R_1 & R_2) of both Tank 1 & Tank 2 slightly closed. Ensure that the valve (R_3) between Tank 2 and Tank 3 is fully closed.
- We switch on the pump and adjust the flow to @90 LPH, allowing the level of both the tanks (Tank 1 & tank 2) to reach at steady state and record the initial flow and steady state levels of both tanks.
- We apply the step change with increasing the rotameter flow by @ 10 LPH.

- We record the level of Tank 2 at the interval of 30 sec, until the level reaches at steady state.
- We record final flow and steady state level of Tank1
- We carry out the calculations as mentioned in calculation part and compare the predicted and observed values of the tank level.
- The experiment is repeated by throttling outlet valve (R1) to change resistance.

3. <u>For Impulse response of first order systems arranged in non-interacting Mode</u>

- The setup is started.A flexible pipe is provided at the rotameter outlet. The pipe is inserted in to the cover of the top tank (T_1) keeping the outlet valves (R_1 & R_2) of both Tank1 & Tank2 slightly closed. We ensure that the valve (R_3) between two bottom tanks T_2 and T_3 is fully closed.
- We switch on the pump and adjust the flow to @90 LPH and allow the level of both Tank1 and Tank 2, to reach the steady state and record the initial flow and steady state levels of both tanks.
- We apply impulse input by adding 0.5 lit of water in Tank 1.
- We record the level of the Tank 2 at the interval of 30 sec, until the level reaches to steady state.
- We record final steady state level of Tank1
- We carry out the calculations as mentioned in calculation part and compare the predicted and observed values of the tank level.
- The experiment is repeated by throttling outlet valve (R_1) to change resistance.

4. <u>For Step response of first order systems arranged in interacting Mode</u>

- The setup is started. A flexible pipe is provided at the rotameter outlet. The pipe is inserted in to the cover of the Tank 3 keeping the outlet valve (R2) of Tank 2 slightly closed. We ensure that the valve (R3) between Tank 2 and Tank 3 is also slightly closed.
- We switch on the pump and adjust the flow to @90 LPH. Allow the level of both Tank 2 and Tank 3, to reach the steady state and record the initial flow and steady state levels of both tanks.
- We apply the step change with increasing the rotameter flow by @ 10 LPH.
- We record the level of the Tank 2 at the interval of 30 sec, until the level reaches at steady state.
- We record final steady state flow and level of Tank 3
- We carry out the calculations as mentioned in calculation part and compare the predicted and observed values of the tank level.
- We repeat the experiment by throttling outlet valve (R1) to change resistance.

5. For Impulse response of first-order systems arranged in interacting mode,

- The setup is started.A flexible pipe is provided at the rotameter outlet. The pipe is inserted in to the cover of Tank 3 keeping the outlet valve (R2) of Tank 2 slightly closed. We ensure that the
- valve (R3) between both Tank 2 and Tank 3 is slightly closed.
- We switch on the pump and adjust the flow to @90 LPH allowing the level of both the
- tanks to reach at steady state and record the initial flow and steady state levels.
- We apply impulse input by adding 0.5 lit of water in Tank 3.
- We record the level of the Tank 2 at the interval of 30 sec, until the level reaches to
- steady state.
- We record final steady state level of Tank3.
- We carry out the calculations as mentioned in calculation part and compare the
- predicted and observed values of the tank level.
- We repeat the experiment by throttling outlet valve (R_1) to change resistance.

Observations and Calculations

Step response of single capacity system

Time(sec)	Level (mm)	$h_{observed}$ (mm)	$h_{predicted}$ (mm)
0	41.5	0	0
15	47	5.5	5.0178
30	50	8.5	8.1721
45	52	10.5	10.1548
60	53	11.5	11.4012
75	54	12.5	12.1847
90	54.5	13	12.677
105	55	13.5	12.9868

Conclusion:

Observed response fairly tallies with theoretically calculated response. Deviations observed may be due to the following factors:

- Non-linearity of valve resistance.
- A step change is not instantaneous.
- Visual errors in recording observations.
- Accuracy of rotameters

Diameter of the tank = 92 mm

Initial Flow Rate = 90 lph

Final Flow Rate = 100 lph

Initial steady state tank level = 41.5 mm

Final steady state tank level = 55 mm

$$A = 10 \text{ lph} = \frac{10 \times 10^{-3}}{3600} m^3/s = 2.78 \times 10^{-6} m^3/s$$

$$R = \frac{dH}{dQ} \approx \frac{\Delta H}{\Delta Q} = \frac{55 - 41.5}{100 - 90} \times 3600 = 4860 \text{ s}/m^2$$

$$\text{Time constant, } \tau = A_1 R = \frac{\pi d^2}{4} R = \frac{\pi}{4} (0.092)^2 \times 4860$$

$$= 32.31 s$$

$$\text{Now, } h_{pred} = AR\left(1 - e^{-t/\tau}\right)$$

$$\text{For } t = 15 s, \ h_{pred} = 2.78 \times 10^{-6} \times 4860 \left(1 - e^{-15/32.31}\right)$$

$$= 5.018 \times 10^{-3} m \sim 5.02 \text{ mm}$$

$$h_{obs} = (\text{Level at } 10s - \text{Level at } 0s)$$

$$= 47 - 41.5 = 5.5 \text{mm}$$

Step response of first-order systems arranged in non-interacting mode

Time(sec)	Level tank 2 (mm)	$h_{observed}$ (mm)	$h_{predicted}$ (mm)
0	80	0	0.0115
15	80.5	0.5	1.12
30	83	3	3.5
45	86	6	6.21
60	89	9	8.88
75	91	11	11.06
90	93	13	12.95
105	94.5	14.5	14.47
120	96	16	15.68
135	97	17	16.61
150	97.5	17.5	17.32
165	98	18	17.86
180	98	18	18.27

Conclusion:

Observed response fairly tallies with theoretically calculated response. Deviations observed may be due to following factors:

- Non-linearity of valve resistance.
- Step change is not instantaneous.
- Visual errors in recording observations.
- Accuracy of rotameters

Diameter of tank : 92 mm (for both tank 1 and tank 2)

Initial Flow rate : 90 lph

Final Flow rate : 100 lph

Initial steady state water level (Tank 2) = 80 mm

Final steady state water level (Tank 2) : 99.5 mm

$$A_1 = A_2 = \frac{\pi d^2}{4} = \frac{\pi}{4}(0.092)^2 = 6.65 \times 10^{-3}\ m^2$$

$$A = 10\ lph = 2.77 \times 10^{-6}\ m^3/s$$

$$R_1 = \left(\frac{dH}{d\theta}\right)_1 = \frac{55 - 41.5}{100 - 90} \times 3600 = 4860\ s/m^2$$

$$R_2 = \left(\frac{dH}{d\theta}\right)_2 = \frac{99.5 - 80}{100 - 90} \times 3600 = 7020\ s/m^2$$

$$\tau_1 = A_1 R_1 = 32.32\ s$$

$$\tau_2 = A_2 R_2 = 46.68\ s$$

Taking data for $t = 15s$,

h_{obs} = level at $t = 15$ - Initial level

$= 80.5 - 80 = 0.5\ mm$

$$h_{pred} : AR_2 \left(1 - \frac{\tau_1 \tau_2}{\tau_1 - \tau_2}\left(\frac{e^{-t/\tau_1}}{\tau_2} - \frac{e^{-t/\tau_2}}{\tau_1}\right)\right)$$

$$= 0.0194454 \left(1 + 105.06 \left(\frac{e^{-15/32.32}}{46.68} - \frac{e^{-15/46.68}}{32.32}\right)\right)$$

$$= 1.122\ mm$$

Impulse response of first order systems arranged in non-interacting Mode

Time(sec)	Level tank 2 (mm)	$h_{observed}$ (mm)	$h_{predicted}$ (mm)
0	75	0	0
15	110	35	23.54
30	113	38	31.87
45	111	36	32.41
60	107	32	29.36
75	103	28	24.96
90	98	23	20.42
105	92	17	16.26
120	87	12	12.7
135	85	10	9.78
150	82	7	7.46
165	80	5	5.63
180	78	2	4.23
195	78	2	3.16
210	78	2	2.346

Conclusion:

Observed response fairly tallies with theoretically calculated response. Deviations observed may be due to following factors:

- Non-linearity of valve resistance.
- Step change is not instantaneous.
- Visual errors in recording observations.
- Accuracy of rotameters

Initial Volumetric flow rate = 90 lph

Volume Added = 0.5 L

Diameter of tanks = 92 mm

Initial steady state level (tank 2) = 75 mm

Final steady state level (tank 2) = 78 mm

τ_1, τ_2 and R_2 were already calculated in last part as written below :

$\tau_1 = 32.32s$, $\tau_2 = 46.68s$

$R_2 = 7020$ s/m^2 $v = 0.5\,L = 5 \times 10^{-4} m^3$

Let us take, $t = 15\,s$

h_{obs} = Level at 15s - Initial level = 110 - 75
= 35 mm

$$h_{pred} = vR_2 \left(\frac{e^{-t/\tau_1} - e^{-t/\tau_2}}{\tau_1 - \tau_2} \right)$$

$$= -\frac{3.51}{14.36} \left(e^{-t/32.32} - e^{-t/46.68} \right) = 0.0235\,m$$

$\therefore$ h_{pred} = 23.5 mm

Step response of first order systems arranged in interacting mode

Time(sec)	Level tank 2 (mm)	$h_{observed}$ (mm)	$h_{predicted}$ (mm)
0	70.5	0	0
15	73	2.5	0.733
30	75	4.5	2.418
45	78	7.5	4.55
60	79	8.5	6.88
75	81	10.5	9.26
90	83	12.5	11.63
105	84	13.5	13.94
120	86	15.5	16.18
135	87	16.5	18.34
150	88	17.5	20.425
165	89	18.5	22.42
180	90	19.5	24.34
195	90.5	20	26.18
210	91	20.5	27.95
225	92	21.5	29.65
240	92	21.5	31.27

Conclusion:

Observed response fairly tallies with theoretically calculated response. Deviations observed may be due to following factors:

- Non-linearity of valve resistance.
- Step change is not instantaneous.
- Visual errors in recording observations.
- Accuracy of rotameters.

Initial flow rate = 40 lph

Final flow rate = 100 lph

Tank 2 : Initial steady state level = 70.5 mm

Final steady state level = 92mm

Tank 3 : Initial steady state level = 131.5 mm

Final steady state level = 166.5 mm

As calculated previously,

$$A = 2.77 \times 10^{-6} \ m^3/s$$

$$A_1 = A_2 = 6.65 \times 10^{-3} \ m^2$$

Now, $R_2 = 2 \frac{dH_2}{dB} = 2 \times \frac{166.5 - 131.5}{10} \times 3600$

$$= 25200 \ s/m^2$$

$$R_1 = \frac{dH_1}{dB} = \frac{92 - 70.5}{10} \times 3600 = 7740 \ s/m^2$$

$\tau_1 = A_1 R_1 = 51.471 s$, $\tau_2 = A_2 R_2 = 167.58 s$

$$b = \tau_1^{-1} + \tau_2^{-1} + A_1 R_2 \tau_1^{-1} \tau_2^{-1} = 0.0488 \ s^{-1}$$

$$\alpha = -b/2 + \sqrt{\left(\frac{b}{2}\right)^2 - \frac{1}{\tau_1 \tau_2}} = -0.00276$$

$$\beta = -b/2 - \sqrt{(b/2)^2 - \frac{1}{\tau_1 \tau_2}} = -0.042$$

hobs = (level at time t - initial level) (At t = 15s (say))

$= 73 - 70.5 = 2.5$ mm

$$h_{pred} = AR_2 \left[1 - \frac{\frac{e^{\alpha t}}{\alpha} - \frac{e^{\beta t}}{\beta}}{\frac{1}{\alpha} - \frac{1}{\beta}} \right] = 7.33 \times 10^{-4} \ m$$

$= 0.733$ mm

Impulse response of first order systems arranged in interacting mode

Time(sec)	Level tank 2 (mm)	$h_{observed}$ (mm)	$h_{predicted}$ (mm)
0	74	0	0
15	91	17	9.813973644
30	97	23	14.56523634
45	96	22	16.49054624
60	95	21	16.86620488
75	93	19	16.41807311
90	91	17	15.55676615
105	90	16	14.51254798
120	88	14	13.41268171
135	86	12	12.32576742
150	85	11	11.28714258
165	83	9	10.3134204
180	82	8	9.410799054
195	81	7	8.57979973
210	80	6	7.817958447
225	79	5	7.121346083
240	78	4	6.485418328

Conclusion:

Observed response fairly tallies with theoretically calculated response. Deviations observed may be due to following factors:

- Non-linearity of valve resistance.
- Step change is not instantaneous.
- Visual errors in recording observations.
- Accuracy of rotameters.

Initial Flow Rate = 90 lph

Final Flow Rate = 100 lph

Tank 2 : Initial steady state Level = 74 mm
Final steady state level = 78 mm

Tank 3 : Initial Steady State level = 136 mm
Final steady state level = 134 mm

Volume Added = 0.5 L = $5\times10^{-4} m^3$

we have, $A_1 = A_2 = 6.65\times10^{-3} m^2$

$A = 2.77\times10^{-6} m^3/s$

$$R_1 = 2\frac{dH_1}{dQ} = 2\times\frac{78-74}{10}\times 3600 = 2880\ s/m^2$$

$$R_2 = 2\frac{dH_2}{dQ} = 2\times\frac{|134-136|}{10}\times 3600 = 1440\ s/m^2$$

$\tau_1 = A_1 R_1 = 19.152 s$

$\tau_2 = A_2 R_2 = 9.576 s$

$b = \tau_1^{-1} + \tau_2^{-1} + A_1 R_1 \tau_1^{-1} \tau_2^{-1} = 0.2088$

$\alpha = -b/2 + \sqrt{(b/2)^2 - (\tau_1\tau_2)^{-1}} = -0.0306$

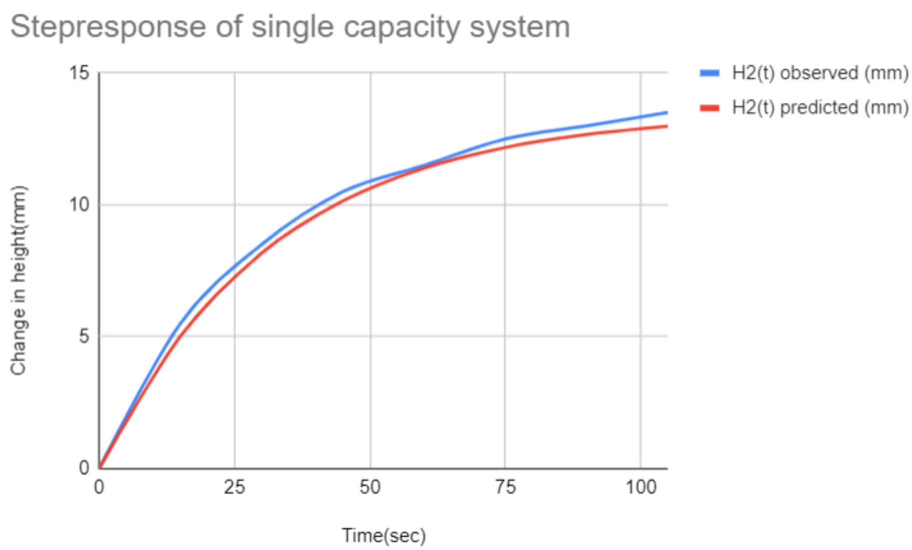
$\beta = -b/2 - \sqrt{(b/2)^2 - (\tau_1\tau_2)^{-1}} = -0.178$

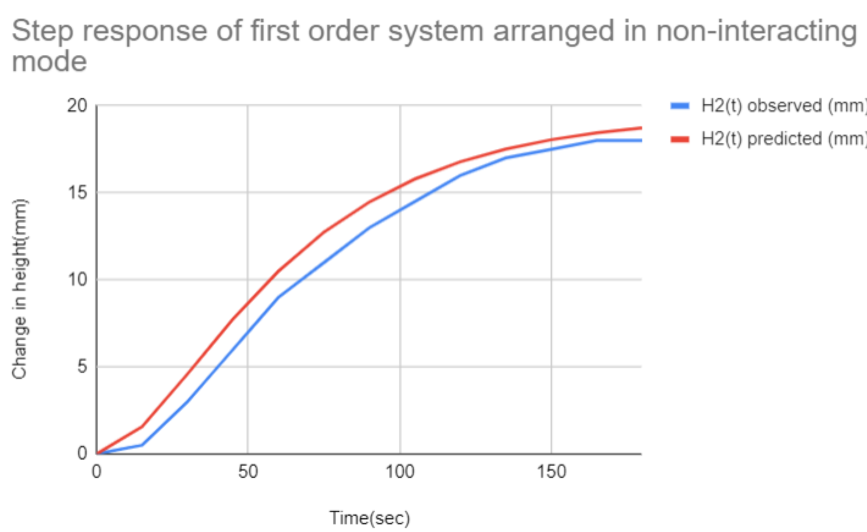
For $t = 15 s$, $h_{obs} = \begin{pmatrix}\text{Level at} \\ t = 15s\end{pmatrix} - \begin{pmatrix}\text{level at} \\ t = 0s\end{pmatrix} = 91 - 74$
$= 17$ mm

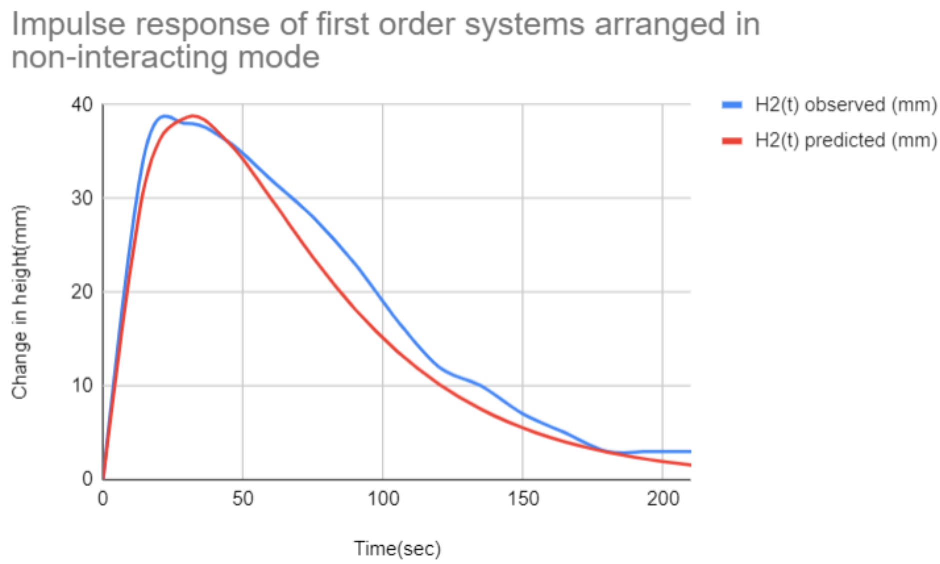
$$h_{pred} = \frac{VR_2}{\tau_1\tau_2}\left(e^{\alpha t} - e^{\beta t}\right) = 0.0266\left(e^{-0.0306\times15} - e^{-0.178\times15}\right)$$

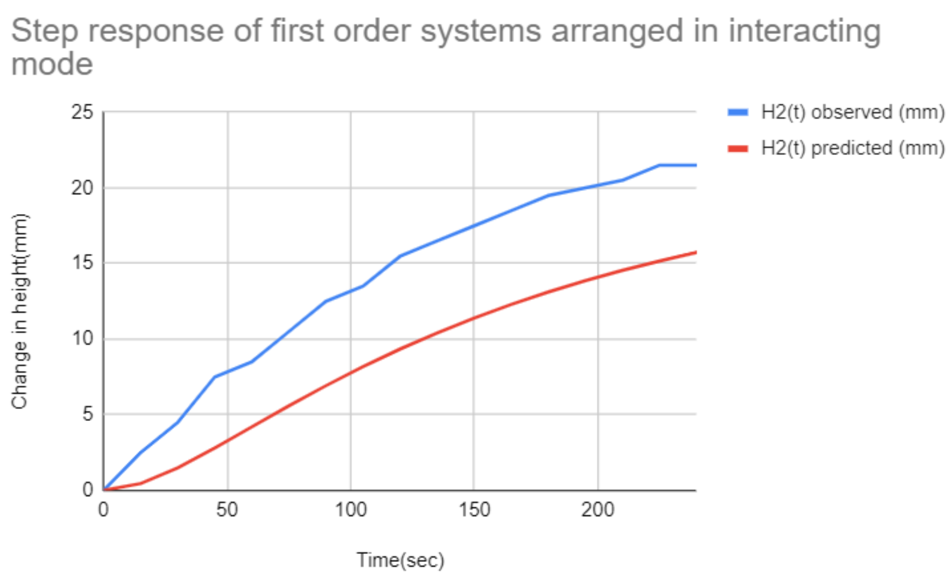
$= 0.01498$ m $= 14.98$ mm

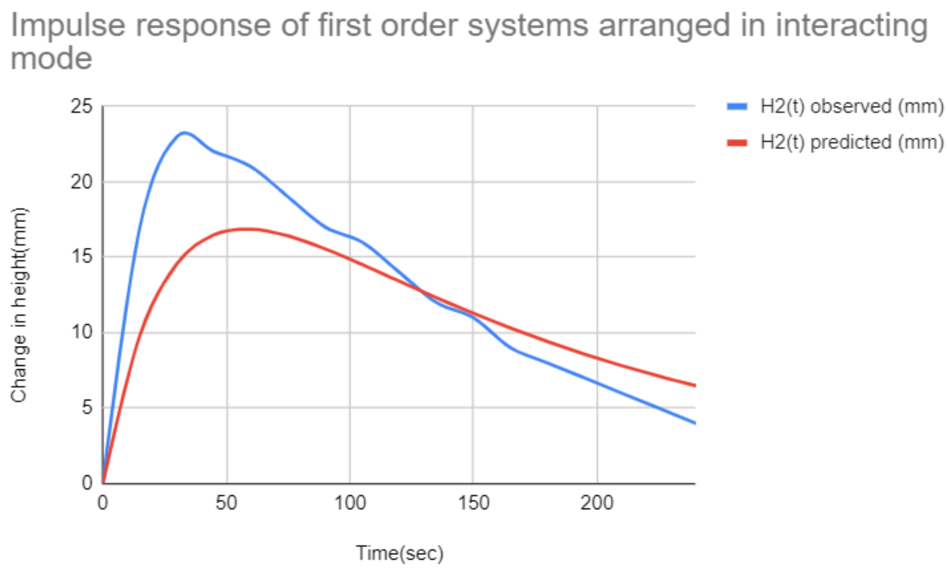
Plots


Stepresponse of single capacity system
H2(t) observed (mm)
H2(t) predicted (mm)
Change in height(mm)
Time(sec)
15
10
5
0
0
25
50
75
100


Step response of first order system arranged in non-interacting mode
H2(t) observed (mm)
H2(t) predicted (mm)
Change in height(mm)
Time(sec)
20
15
10
5
0
0
50
100
150


Impulse response of first order systems arranged in non-interacting mode
H2(t) observed (mm)
H2(t) predicted (mm)
Change in height(mm)
Time(sec)
0
10
20
30
40
0
50
100
150
200


Step response of first order systems arranged in interacting mode
H2(t) observed (mm)
H2(t) predicted (mm)
Change in height(mm)
Time(sec)
0
5
10
15
20
25
0
50
100
150
200


Impulse response of first order systems arranged in interacting mode
H2(t) observed (mm)
H2(t) predicted (mm)
Change in height(mm)
Time(sec)
0
5
10
15
20
25
0
50
100
150
200

Additional Questions:

1) $H(t) = AR\,(1 - e^{-t/\tau})$

$$\frac{dH(t)}{dt} = \frac{AR}{\tau}\,e^{-t/\tau}$$

$$\left.\frac{dH(t)}{dt}\right|_{t=0} = AR/\tau$$

Slope of response is inversely proportional to the time constant. For the system to respond fast to the change, larger slope for response curve is expected and vice-versa

2) Non-Interacting System

$$G(s) = \frac{R_2}{\tau_1\tau_2 s^2 + (\tau_1+\tau_2)s + 1} \quad \text{--- ①}$$

$$G(s) = \frac{K_p}{\tau^2 s^2 + 2\psi\tau s + 1} \quad \text{--- ②}$$

Comparing ① and ②

$$\tau^2 = \tau_1\tau_2 \Rightarrow \tau = \sqrt{\tau_1\tau_2}$$

$$2\psi\tau = \tau_1 + \tau_2$$

$$\psi = \frac{\tau_1+\tau_2}{2\sqrt{\tau_1\tau_2}} \quad \text{(Damping factor)}$$

$\tau_1 = 21.58\ s^{-1}$, $\tau_2 = 46.77\ s^{-1}$

$$\therefore \psi = 1.075\ s^{-1}$$

3) Interacting systems

$$G(s) = \frac{R_2}{\tau_1\tau_2 s^2 + (\tau_1+\tau_2+A_1R_2)s+1} \quad - (1)$$

$$= \frac{K_p}{\tau^2 s^2 + 2\zeta\tau s + 1} \quad - (2)$$

$\zeta \rightarrow$ Damping Factor

Comparing (1) and (2)

$$\tau_1+\tau_2+A_1R_2 = 2\zeta\tau \quad , \quad \tau = \sqrt{\tau_1\tau_2}$$

$$\therefore \zeta = \frac{\tau_1+\tau_2+A_1R_2}{2\sqrt{\tau_1\tau_2}}$$

[$\tau_1 = 13.79\, s^{-1}$
$\tau_2 = 51.47\, s^{-1}$
$A_1 = 0.0665$
$R_2 = 7740$]

$$\therefore \zeta = 1.42\, s^{-1}$$

4) Two-tank systems (both interacting and non-interacting) show second-order response and thus may be described by a second-order ordinary differential equation.

$$C'_{A2}(s) = \frac{\dfrac{1}{(\tau_1 s+1)(\tau_2 s+1)}}{\left(1+\dfrac{C_{Ac}/F}{(\tau_1 s+1)(\tau_2 s+1)}\dfrac{K_v}{\tau_v s+1}K_cK_s\right)}C'_{Ai}(s) + \frac{\dfrac{C_{Ac}/F}{(\tau_1 s+1)(\tau_2 s+1)}\dfrac{K_v}{\tau_v s+1}K_cK_s}{\left(1+\dfrac{C_{Ac}/F}{(\tau_1 s+1)(\tau_2 s+1)}\dfrac{K_v}{\tau_v s+1}K_cK_s\right)}C'_{A2,sp}(s)$$