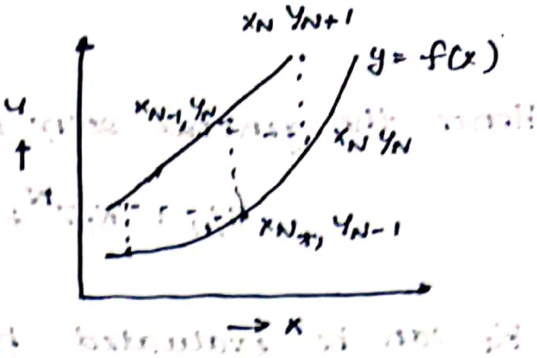
① $y = f(x) = \alpha x$



$$G_s(y_{n+1} - y_1) = L_s(x_N - x_0)$$

$$y_{N+1} = y_1 + \frac{L_s}{G_s}(x_N - x_0)$$

$$y_{N+1} = y_1 + \frac{L_s}{G_s}\frac{y_N}{\alpha} - \frac{L_s}{G_s}x_0$$

Let $\frac{L_s}{G_s \alpha} = \bar{A} \Rightarrow \frac{L_s}{G_s} = \alpha \bar{A}$

$$\therefore y_{N+1} = y_1 + \bar{A} y_N - \alpha \bar{A} x_0$$

$$\Rightarrow y_{N+1} - \bar{A} y_N = y_1 - \bar{A} \alpha x_0$$

② Considering the eqn:,

$$y_{N+1} - \bar{A} y_N = y_1 - \bar{A}\alpha x_0 \quad -Ⓐ$$

where, $\bar{A} = \frac{L_s}{G_s \alpha} \rightarrow$ Absorption factor

Considering the homogeneous eqn:,

$$y_{N+1} - \bar{A} y_N = 0 \quad -①$$

Assuming a soln: of the form, $y_N = k_1 z^N$ and substituting in eqn ① we get,

$$k_1 z^{N+1} - \bar{A} k_1 z^N = 0 \Rightarrow z = \bar{A} \quad -②$$

Now, since the eqn: Ⓐ is non-homogenoous we need to find the particular ~~eqn:~~ soln: which is a const. Assuming $y_N = k_2$ (constant) as the particular soln: and substituting in Ⓐ

$$k_2 - \bar{A} k_2 = y_1 - \bar{A}\alpha x_0 \Rightarrow k_2 = \frac{y_1 - \bar{A}\alpha x_0}{1 - \bar{A}} \quad -③$$

Hence the general soln. of ⓐ is,

$$y_N = K_1 Z^N + K_2 = K_1 (\bar{A})^N + \frac{y_1 - \bar{A}\alpha x_N}{1-\bar{A}} \quad - (4)$$

K_1 can be evaluated by knowing the terminal cond.;
for $n=0$, $y_0 = \alpha x_0$

$$K_1 = \alpha x_0 - \frac{y_1 - \bar{A}\alpha x_0}{1-\bar{A}} = \frac{\alpha x_0 - y_1}{1-\bar{A}} \quad - (5)$$

Now, substituting the value of K_1, eqn. (4)

$$y_N = \left[\frac{\alpha x_0 - y_1}{1-\bar{A}}\right](\bar{A})^n + \frac{y_1 - \bar{A}\alpha x_0}{1-\bar{A}} \quad - (6)$$

To determine the total no. of ideal plates,
we put $n = N+1$ and $y_N = y_{N+1}$

$$y_{N+1} = \left[\frac{\alpha x_0 - y_1}{1-\bar{A}}\right](\bar{A})^{N+1} + \frac{y_1 - \bar{A}\alpha x_0}{1-\bar{A}}$$

$$\bar{A}^N = \frac{y_{N+1}\left[1/\bar{A} - 1\right] + \left[y_1/\bar{A} - \alpha x_0\right]}{\alpha x_0 - y_1}$$

Rearranging and taking log on both sides,

$$N = \frac{\log\left[\left(\frac{y_{N+1} - \alpha x_0}{y_1 - \alpha x_0}\right)\left(1 - \frac{1}{\bar{A}}\right) + \frac{1}{\bar{A}}\right]}{\log \bar{A}}$$

↳ Kremser's Eqn.