① $\frac{dm_c}{dt} = K_c A_c (C - C_i)$ — ①

$\frac{dm_c}{dt} = K_r A_c (C_i - C^*)$ — ②

$\frac{dm_c}{dt} = K_L A_L (C - C^*)$ — ③

Using eqn: ①, ②, ③ and rearranging,

$$K_L = \frac{K_L K_r}{K_L + K_r} \quad (\text{Eliminating } C_i)$$

Now, WKT,

$m_c = \rho_c \phi_v L^3$

$A_L = \phi_a L^2$

$V_c = \phi_v L^3$

$\rho_c \rightarrow$ Crystal density.

$\phi_v \rightarrow$ Volume shape factor

$\phi_a \rightarrow$ Area Shape factor

Substituting the above,

$$\frac{dm_c}{dt} = \frac{d}{dt}(\rho_c V_c) = \frac{d}{dt}(\rho_c \phi_v L^3) = K_L \phi_a L^2 (c - c_s)$$

$$3\rho_L \phi_v L^2 \frac{dL}{dt} = K_L \phi_a L^2 (c - c_s)$$

$$\frac{dL}{dt} = \frac{K_L \phi_a}{3\rho_c \phi_v}(c - c_s) = k\left(\frac{c - c_s}{c_s}\right) = ks$$

② WKT,

$$G = \frac{dL}{dt} = K_s^2 \quad \text{and} \quad B_0 = \lim_{L\to 0} \frac{d(N/V)}{dt}$$

In general, $G = K' s^n$, $B_0 = K H_T s^n$

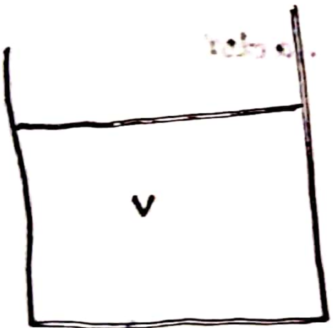
Now,

$$n_0(L) = \lim_{L\to 0}\left[\frac{d(NL/V)}{dL}\right]$$

$$= \frac{\lim_{L\to 0} \frac{d(N(L)/V)}{dt}}{dL/dt}$$

$$\therefore\ n_0(L) = \frac{B_0}{G}$$

③



$s = \text{const}^{t} = \frac{c - c^*}{c^*}$

At $t=0$, $L = L_s$, $H_s = N\phi_v L_s^3 s_c$

$t = t$, $L = L$, $H = N\phi_v L^3 s_c$

At $t = t$, $L = Gt + L_s$

$$\frac{d}{dt}(VcH_w) = -\frac{dH_c}{dt}$$

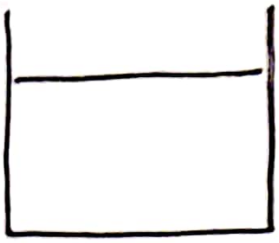
$$\frac{d}{dt}(VcH_w) = -\frac{d}{dt}(N\phi_v L^3 s_c)$$

$$c\frac{dV}{dt} = -\frac{3N\phi_v L^2 s_c}{H_w}\frac{dL}{dt}$$

$$-c\frac{dV}{dt} = \frac{3N\phi_v L_s^3 (L_s + Gt)^2 G s_c}{H_w L_s^3}$$

$$\therefore\ -c\frac{dV}{dt} = \frac{3H_s}{H_w}\left(\frac{L_s + Gt}{L_s}\right)^2 \times \frac{G}{L_s}$$

(4)



At $t=0,\ L=L_s,\ M_s = N\phi_v L_0^3 \, S_c$

$t=t,\ L=L,\ M_c = N\phi_v L_s^3 \, S_c$

$$\frac{d}{dt}(vc) = -\frac{dM_c}{dt}$$

$$v\frac{dc}{dt} = -3N\phi_v L^2 S_c \frac{dL}{dt} \qquad \left[s = \frac{c-c^*}{c^*} \right]$$

with temp. c^* value,

$$v\frac{dc}{dt} = 3N\phi_v L^2 S_c G$$

$$= 3N\phi_v (L_s + Gt)^2 S_c G$$

$$= 3m_s \left(\frac{L_s+Gt}{L_s}\right)^2 \frac{G}{L_s}$$

(5)

$$s = \frac{c-c^*}{c^*} = \frac{c}{c^*} - 1$$

$$c = c^*(s+1) = f(T)(s+1)$$

$$\frac{dc}{dT} = f'(T)(s+1)\frac{dT}{dt}$$

$$v\frac{dT}{dt} = \frac{3M_s}{f'(T)(s+1)} \left(\frac{L_s+Gt}{L_s}\right)^2 \frac{G}{L_s}$$

Let us consider $f(T) = a_1 + a_2 T$

so, $f'(T) = a_2$

so, $$v\frac{dT}{dt} = \frac{3M_s}{a_2(s+1)} \left(1 + \frac{Gt}{L_s}\right)^2 \frac{G}{L_s}$$

$$v(s+1)a_2 \frac{dT}{dt} = 3M_s \left(1 + \frac{Gt}{L_s}\right)^2 \frac{G}{L_s}$$

rate of cooling