University of Cologne

Practical Course M

Computational Physics

Agent Based Modelling

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Contents

Chapter 1

Asymmetric Simple Exclusion Process

The Asymmetric Simple Exclusion process (ASEP), which is popularly known as the Ising model of traffic can be described in the following manner -

1. Start with a random initial state. This could be a road containing a vector of vehicles or a binary array where the values correspond to occupation truth values.
2. With a probability q, each agent moves to its nearest neighbouring index on the right provided it is unoccupied.
3. If the road or array is periodic (PBC) then the update is done modulo the length. If the boundary is open (OBC) then,
    (a) Inject an agent into the system with probability α,
    (b) Eject an agent from the system with probabiliy β.

1.1 Periodic Boundary Conditions

We implement ASEP with PBC under the following different update schemes and look at the space-time profiles and fundamental diagrams -

- **Parallel Update -** All agents are updated simultaneously,
- **Sequential Update -** All agents are update in a fixed sequence,
- **Shuffle Update -** All agents are updated in a random order that changes in every time-step,
- **Random Sequential Update -** A random agent is chosen and updated as many times as there are agents.

PARAMETERS	**VALUES**
Length L	100 sites
Density ρ	0.2, 0.5, 0.8
Probability q	1.0, 0.5, 0.2
Time T	80 units

For fundamental diagrams, we choose a higher simulation time = 500 units for convergence.

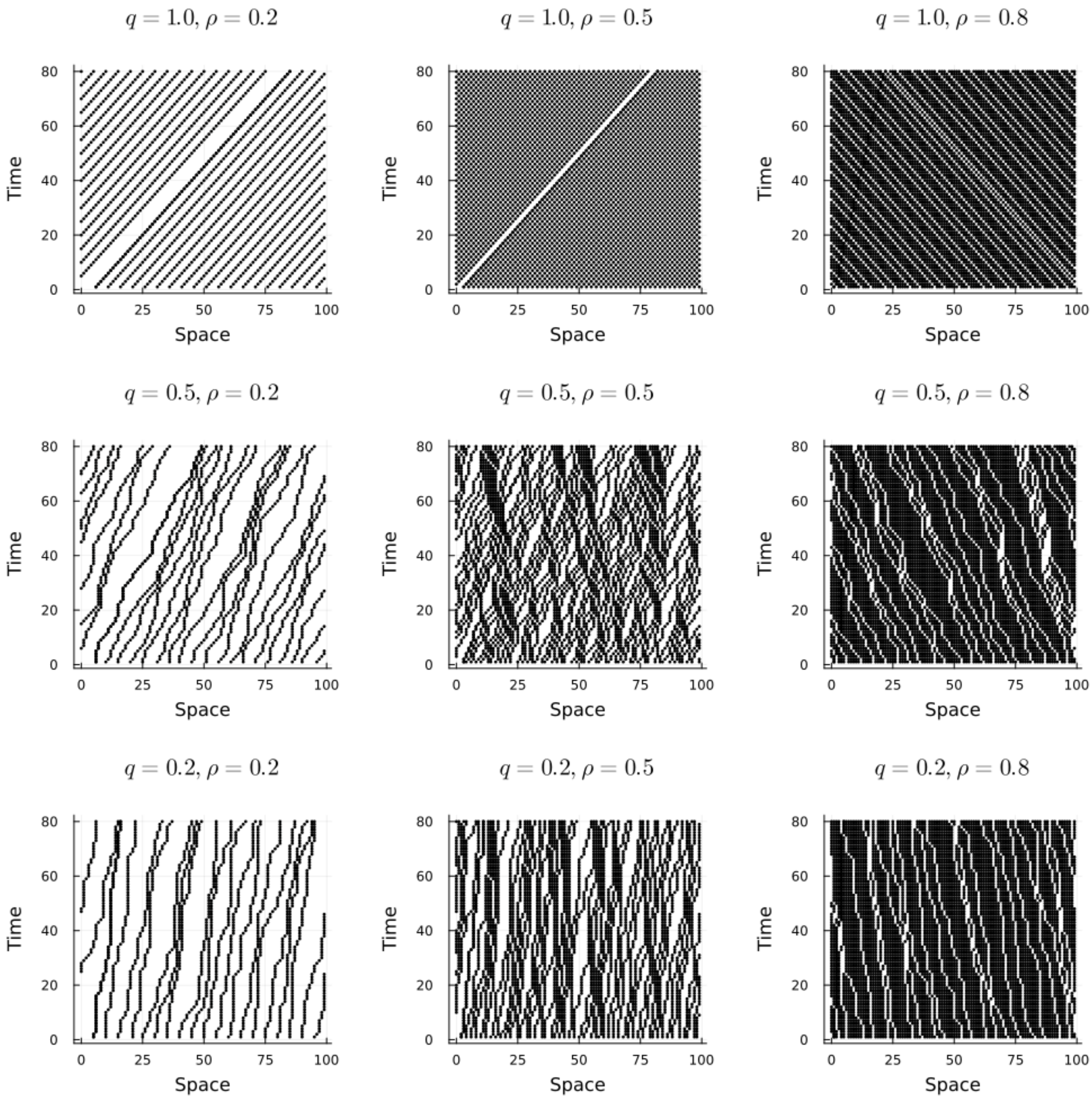


Figure 1.1: Space-Time Diagram for ASEP under PBC with Parallel Updates. The dynamics are as we would expect. There is free movement at lower densities and very ordered dynamics for $q = 1$. The spontaneous jams are characterised the dark reservoirs against the direction of motion.

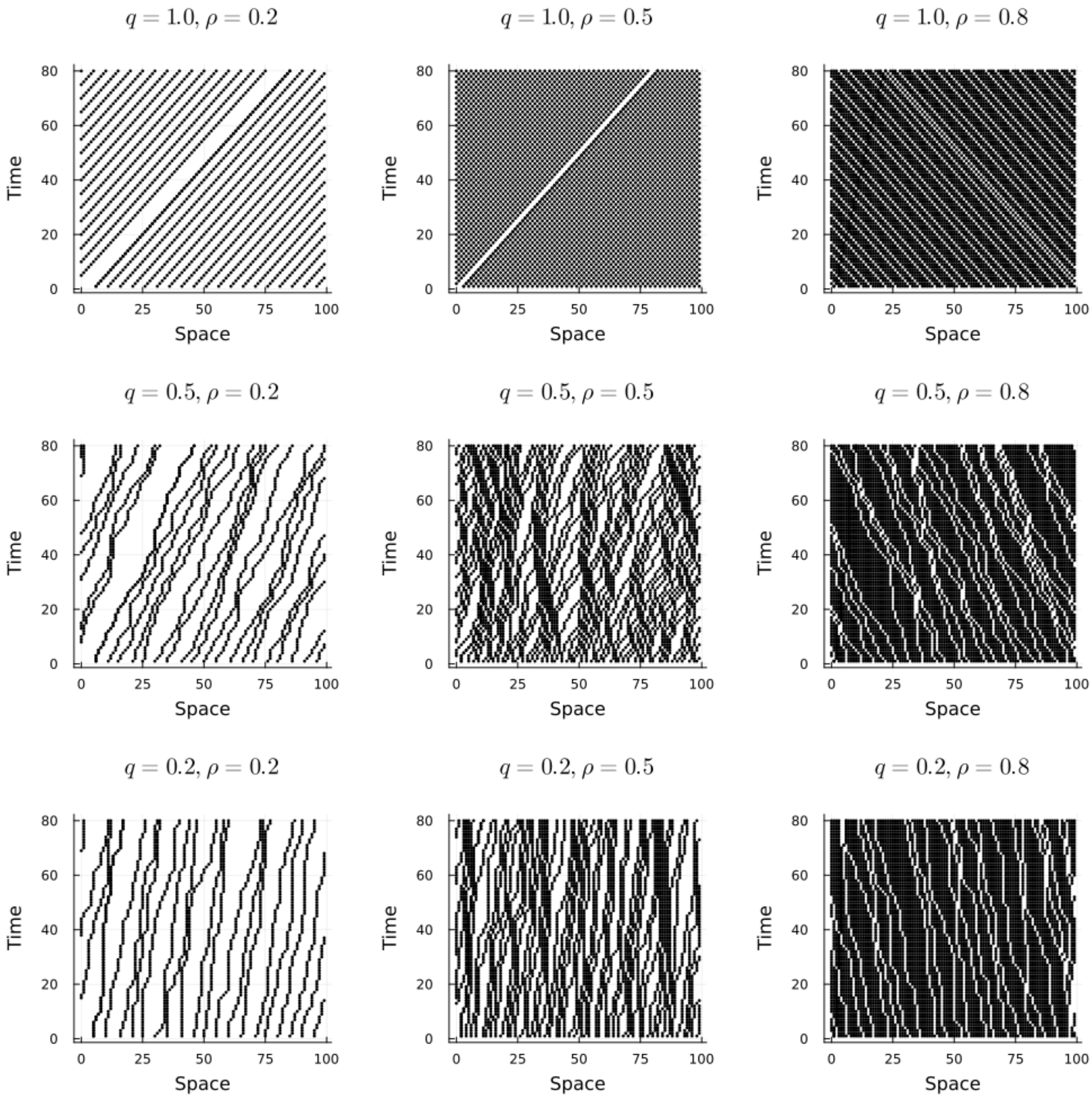


Figure 1.2: Space-Time Diagram for ASEP under PBC with Sequential Updates. As expected, the dynamics are nearly identical to the parallel update.

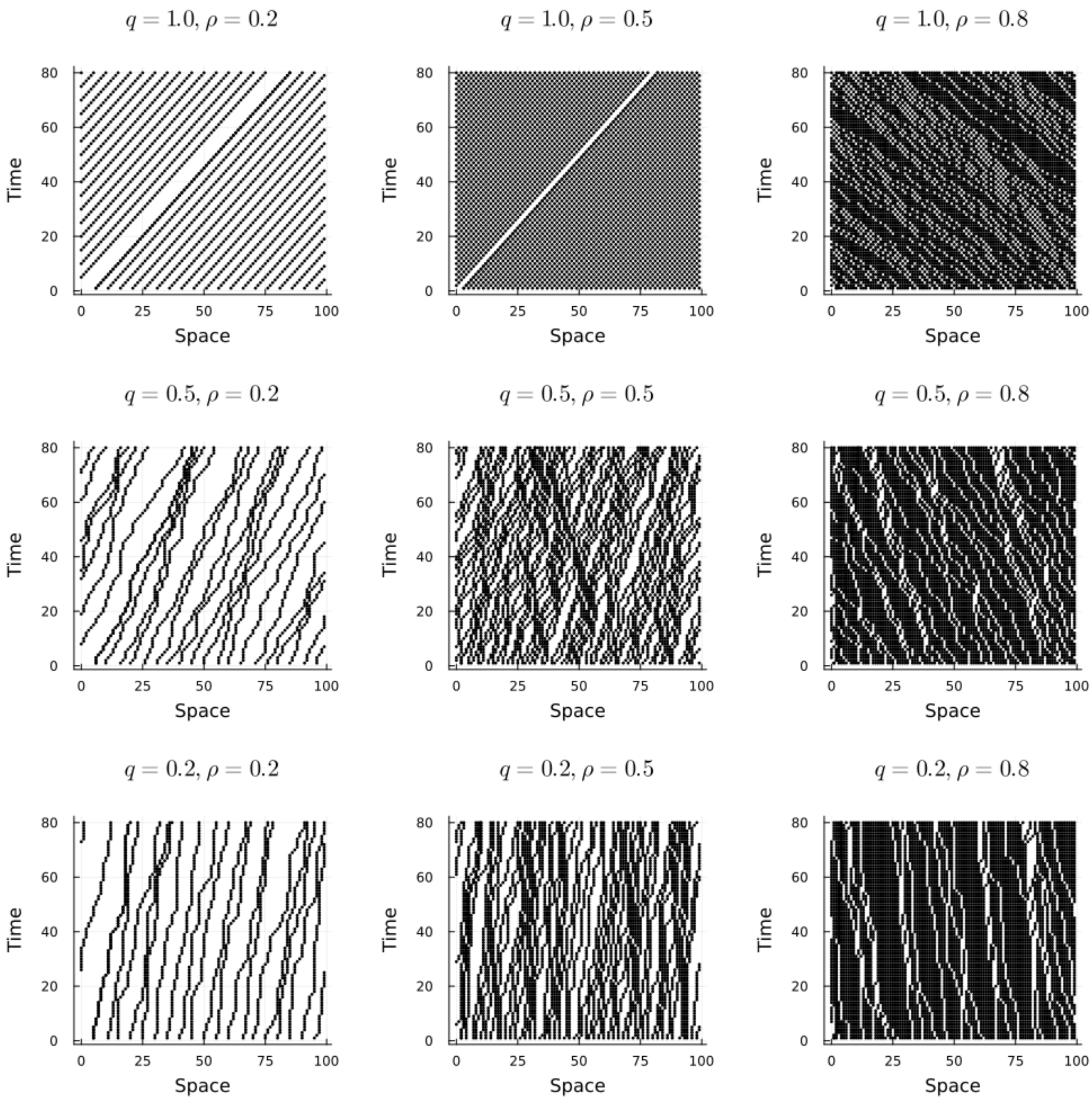


Figure 1.3: Space-Time Diagram for ASEP under PBC with Shuffle Updates. In this case as well, the dynamics are very similar to the previous update schemes, but there is a higher degree of stochasticity as a result of the randomisation of the update sequence. This is most clear when both ρ and q are high.

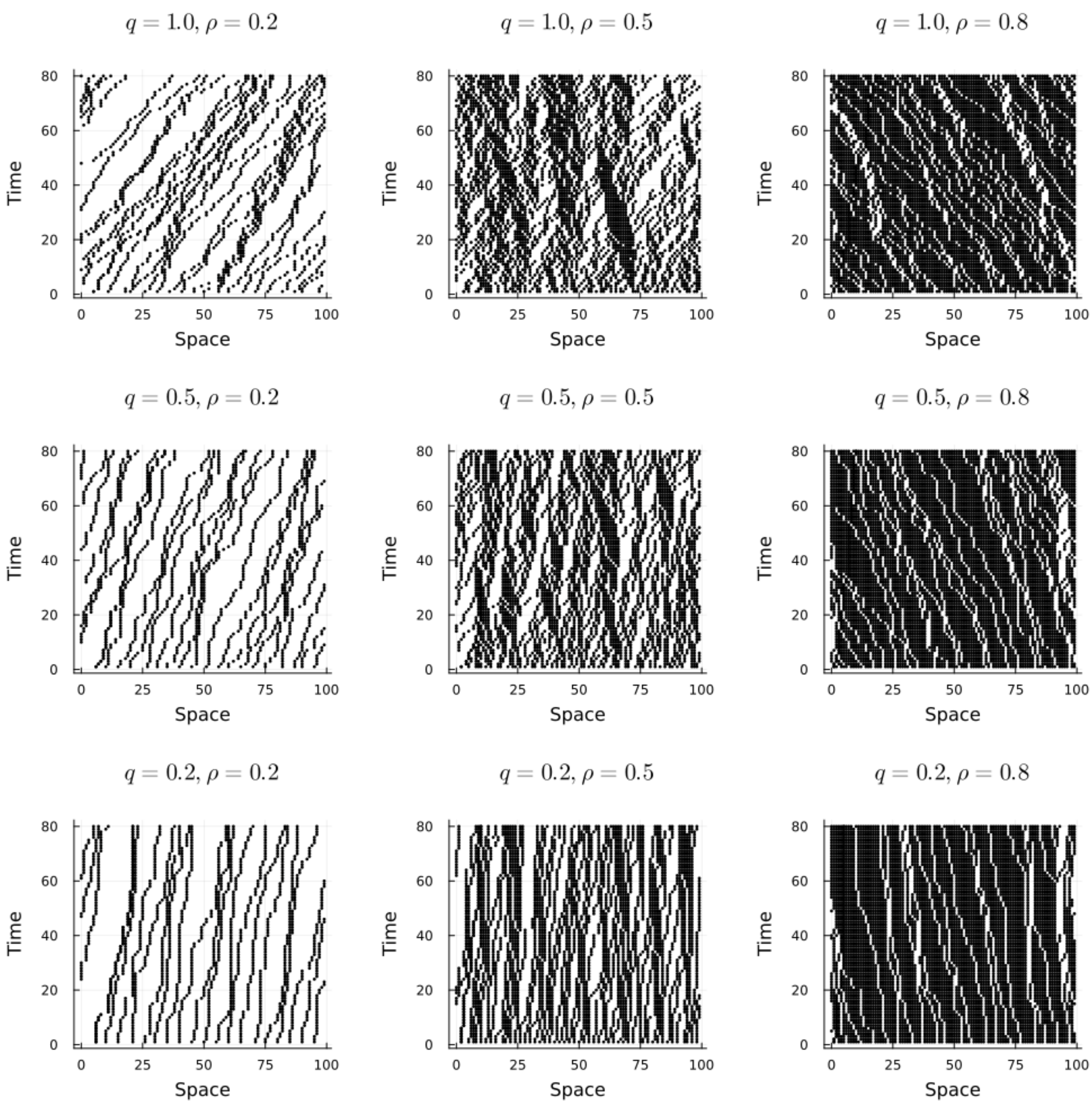
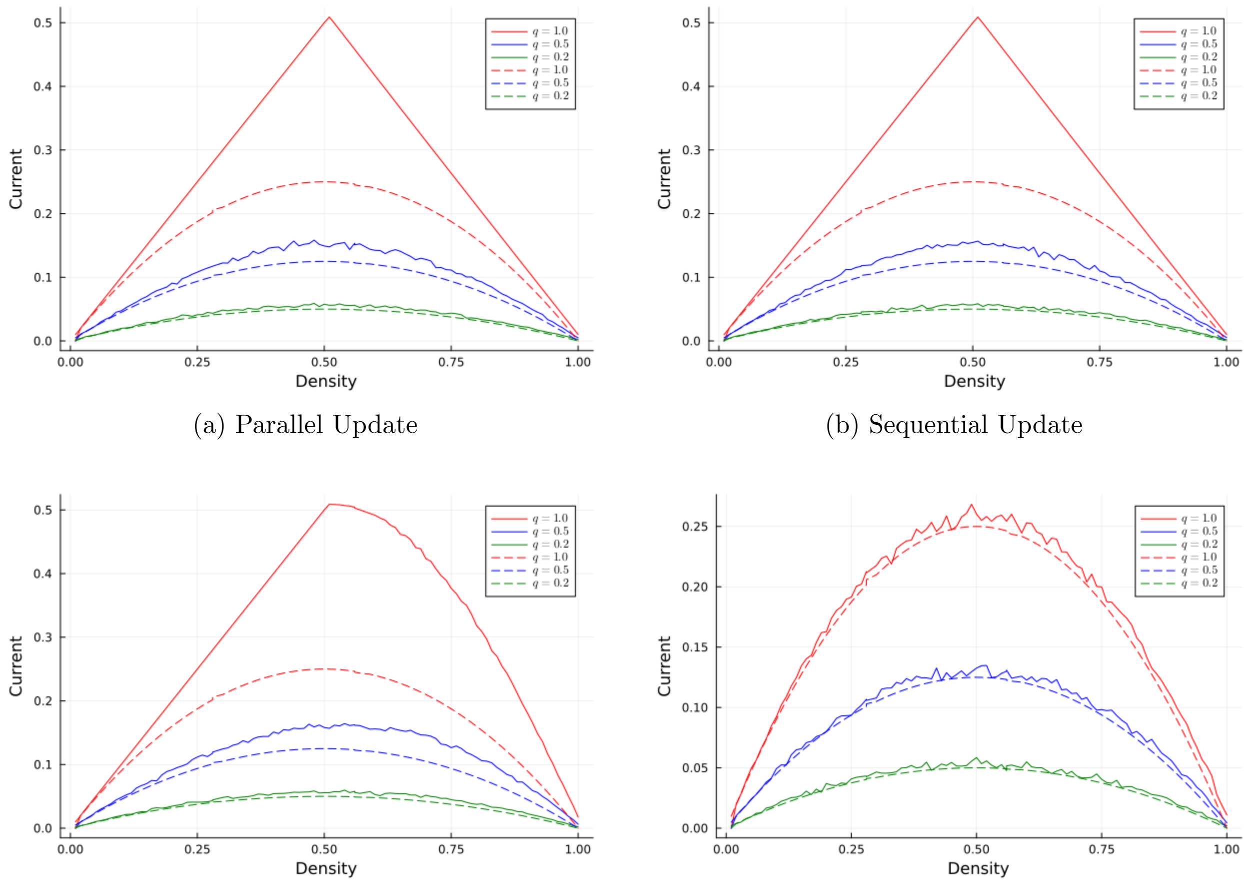


Figure 1.4: Space-Time Diagram for ASEP under PBC with Random Sequential Updates. Unsurprisingly, the randomness causes highly uncorrelated motions which is reflected by the stochastic nature of the diagrams.



(a) Parallel Update (b) Sequential Update

(c) Shuffle Update (d) Random Sequential Update

Figure 1.5: Fundamental Diagrams (FDs) for ASEP in PBC under different update schemes. The dashed lines correspond to the theoretical FDs, i.e, currents according to $J = q\rho(\rho - 1)$. As one would expect, the FDs are almost identical for parallel and sequential updates. Furthermore, it is clear that the random sequential update produces fundamental diagrams closest to what is expected due to the agents being highly uncorrelated. For all the other diagrams, this is why the lower hopping probability fits the theoretical FD best.

1.2 Open Boundary Conditions

We implement ASEP with OBC with the following parameters -

PARAMETERS	**VALUES**
Length L	100 sites
Time T	1000 (burn-in) + 9000 units
Injection Rate α	0.1, 0.3, 0.8
Ejection Rate β	0.2, 0.4, 0.7

We selected an initial density $\rho = 0.4$ and hopping probability $q = 0.9$ for this task.

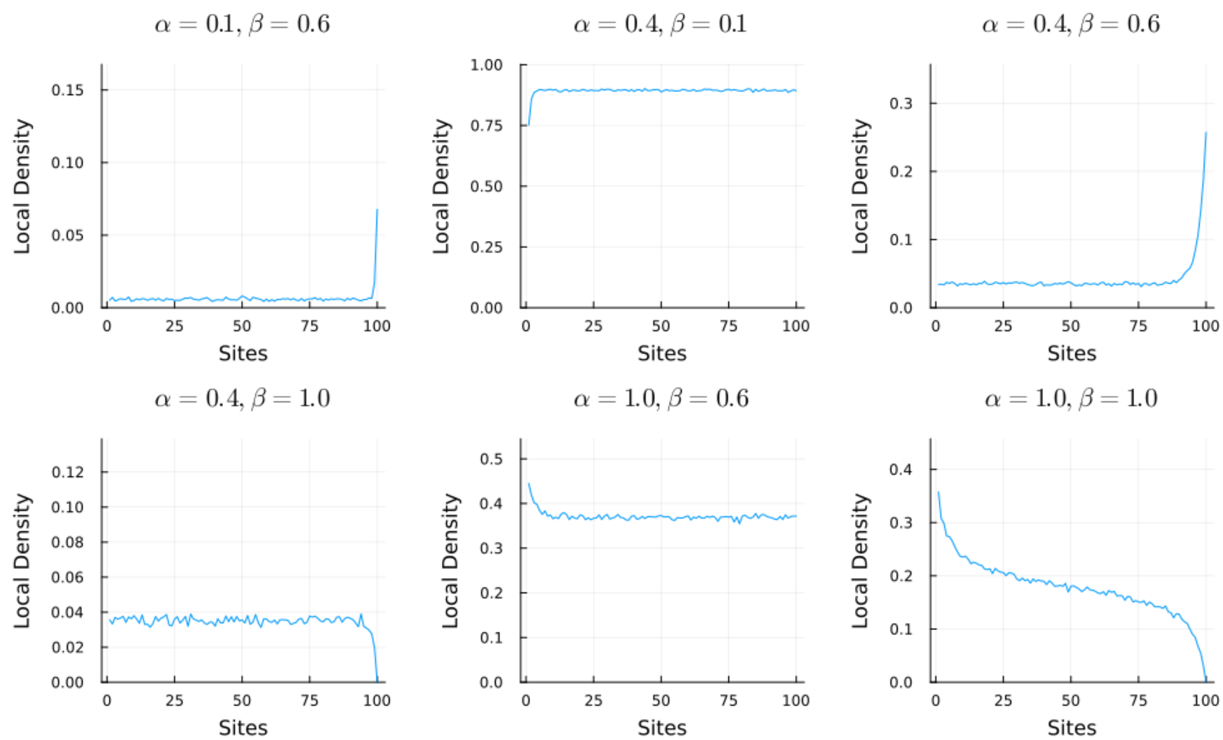
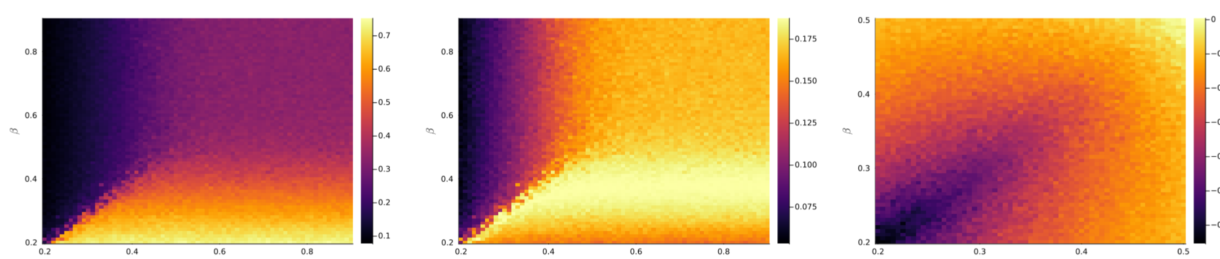


Figure 1.6: Density Profiles for ASEP in OBC. Corressponding to the phase diagram in [1], we find A ($\alpha = 0.1, \beta = 0.6$), B ($\alpha = 1.0, \beta = 0.6$) and C ($\alpha = 1.0, \beta = 1.0$) and more.



(a) Average Density Heatmap (b) Current Heatmap (c) Heatmap of $\Delta\rho = \rho_1 - \rho_N$

Figure 1.7: Phase Diagrams in an $\alpha - \beta$ grid of $[0.2, 0.9)^2$. The density heatmap clearly outlines the phase boundary between A and B, while the current heatmap outlines the phase boundary between A and C, B and C. The heatmap of $\Delta\rho$ also shows the separation between phases A and B.

Chapter 2

Nagel-Scheckenberg Model

The Nagel-Scheckenberg Model (NaSch), generalises the ASEP model of traffic and can be described in the following manner -

1. Start with a random initial state. This would be a road containing a vector of vehicles in certain positions x with velocites $v = 0$.
2. Update the velocities to accelerate upto a chosen v_{max} and decelerate such that when the position is updated in accordance to the velocities, no two vehicles occupy the same site (essentially avoid accidents). In other words, $v_k^{(t+1)} = \min\left(v_k^{(t)} + 1,\ v_{max},\ x_{k+1}^{(t)} - x_k^{(t)} - 1\right)$.
3. With a probability p, each agent randomly decelerates by unit velocity.
   (a) This process can be arbitrarily random or,
   (b) One could implement a velocity dependent randomisation (VDR), by fixing the probability $p_0 = p(v = 0)$ such that $p(v \neq 0) < p_0$.
4. Finally, we update the positions of the vehicle with $x_k^{(t+1)} = x_k^{(t)} + v_k^{(t+1)}$.

We use precisely the NaSch model with $v_{max} = 1$ to simulate the ASEP under PBC. We use the following parameters for our simulations, For fundamental diagrams, we choose a higher simulation

PARAMETERS	**VALUES**
Length L	100 sites
Density ρ	0.2, 0.5, 0.8
Probability p	0.2, 0.5, 0.7
Time T	80 units

time = 500 units for convergence. For VDR, we fix $p_0 = 0.8$. The simulations are performed under a parallel update scheme.

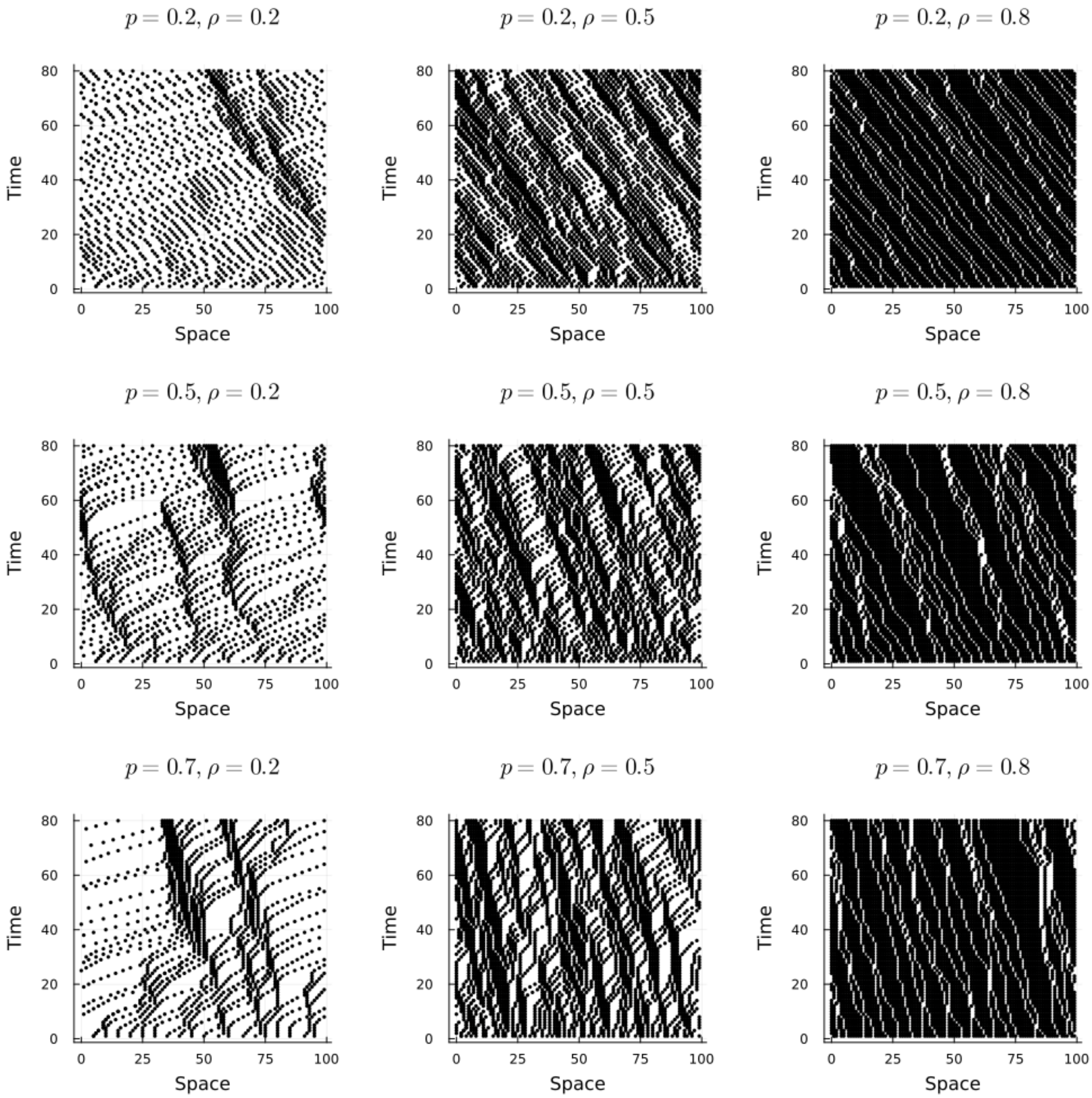


Figure 2.1: Space-time diagram for NaSch under PBC with $v_{max} = 5$. Unline ASEP, there exist stable jams even at low densities and low slow-down probability.

Now let us consider a realistic scenario.

- We can assume the sites to be of length $10m$ and the jam velocity to be -4m/s.
- Let us assume a vehicle can cover a maximum of 10 sites per iteration. In other words, the vehicle requires 10 iterations to complete the circuit of 100 sites in the ideal case.
- Realistically, a good average speed on a highway would be around 50m/s. Let us set this as the speed limit. Then, the time taken to complete the circuit is $20s$.
- Thus, 10 iterations correspond to $20s$, so every iteration is $2s$. A slope of 1 thus corresponds to a velocity of 5m/s. This means, the jam veloctiy must corresspond to a slope of -0.8.

By inverting the plots of the previous simulation, we see the slope of -0.8 matches closest to $p \approx 0.2$.

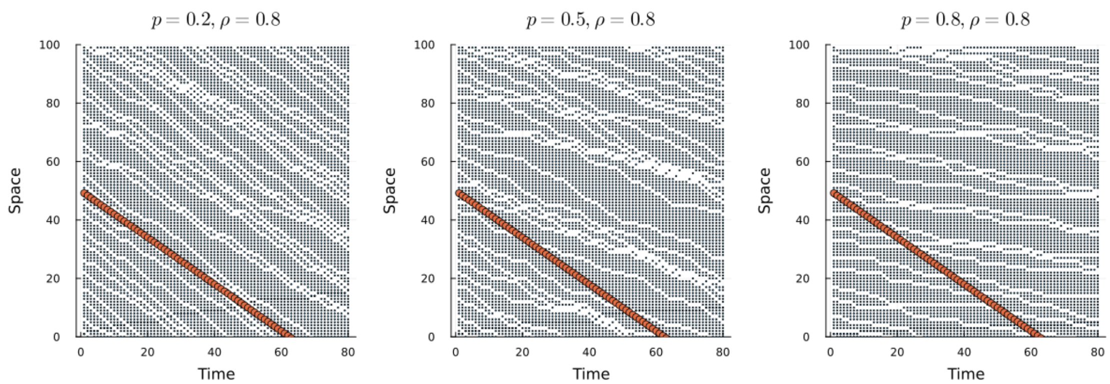


Figure 2.2: Estimating realistic value of p

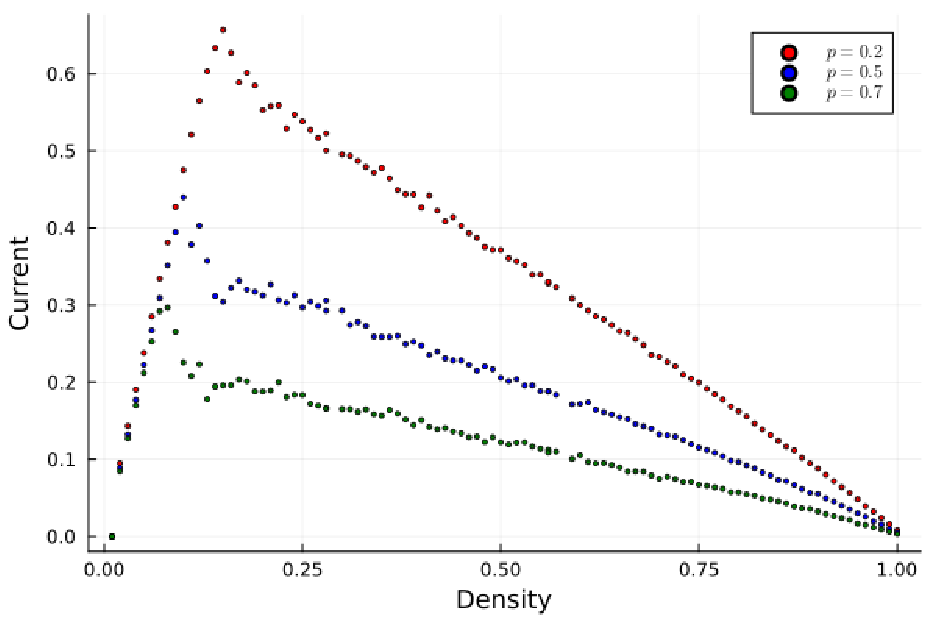


Figure 2.3: Fundamental Diagram - NaSch. Here, $J_{max} \propto p$. The left part indicates free flow.

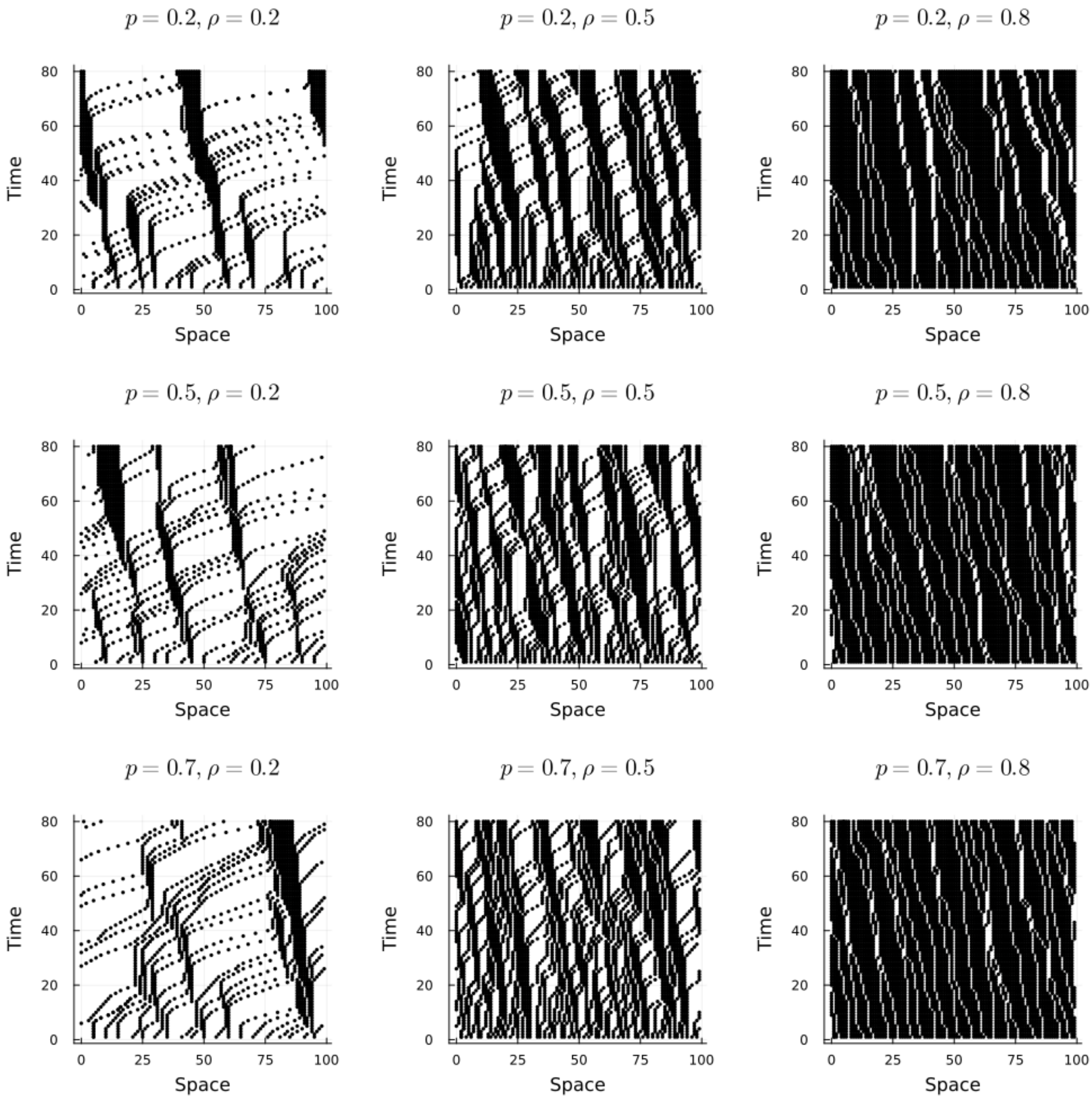


Figure 2.4: Space-time diagram for VDR under PBC with $v_{max} = 7$. Here we find that the jam velocity appears independent of both ρ and p, unlike both NaSch and ASEP. As a result, these jams are even more stabe than the ones we found in NaSch.

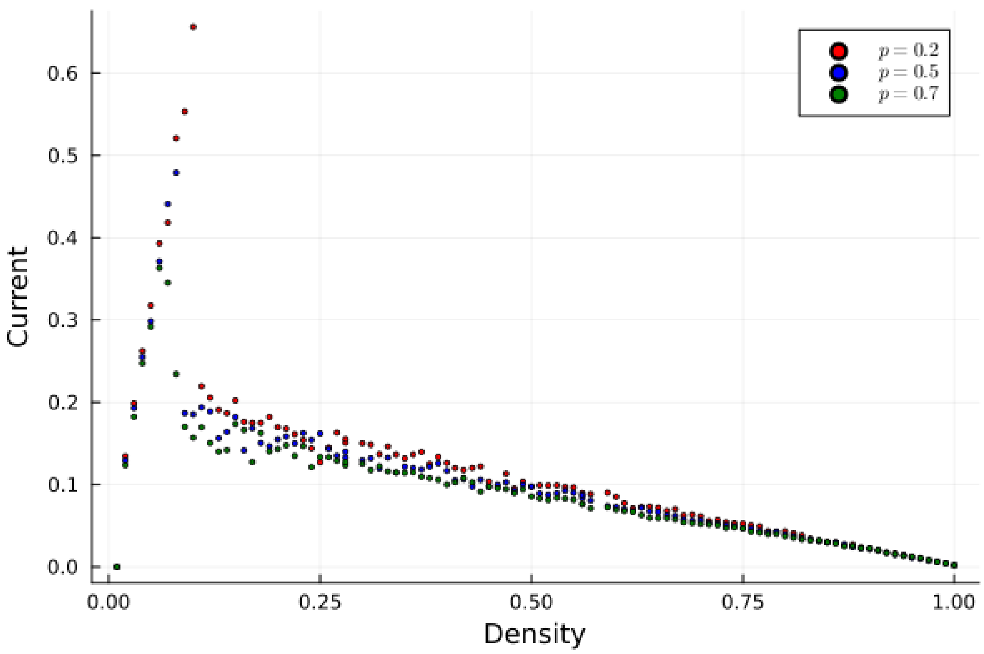


Figure 2.5: Fundamental Diagram - VDR. Here, J_{max} independent of p and the left part indicates free flow.

Bibliography

[1] University of Cologne. *MLab Computational Physics - Agent Based Modelling*, 2023.