Hidden Markov Model for POS Tagging

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1 Deriving the Viterbi Algorithm

1.1 Why Does

$$\arg \max_{t} P(t|w) = \arg \max_{t} \log P(t, w)$$

To understand why maximizing P(t|w) is equivalent to maximizing $\log P(t,w)$, we analyze the problem step-by-step:

1. Probability Maximization The goal is to find the tag sequence t that maximizes the conditional probability P(t|w):

$$\arg\max_{t} P(t|w)$$

2. Log Transformation The logarithm is a monotonic function, meaning that for any two values a and b, if a > b, then $\log(a) > \log(b)$. Thus, maximizing P(t|w) is equivalent to maximizing $\log P(t|w)$:

$$\arg\max_{t} P(t|w) = \arg\max_{t} \log P(t|w)$$

3. Bayes' Rule By Bayes' rule, the conditional probability P(t|w) can be written as:

$$P(t|w) = \frac{P(w,t)}{P(w)}$$

where:

- P(w,t) is the joint probability of the word sequence w and the tag sequence t.
- P(w) is the marginal probability of the word sequence w, which is constant for all t.

Since P(w) is constant, maximizing P(t|w) is equivalent to maximizing P(w,t):

$$\arg\max_t P(t|w) = \arg\max_t P(w,t)$$

4. Log-Space Representation To avoid numerical underflow when dealing with small probabilities, we perform computations in the log-space. Taking the logarithm:

$$\arg\max_{t} P(w, t) = \arg\max_{t} \log P(w, t)$$

This transformation simplifies computations, as probabilities that are multiplied in P(w,t) become sums in $\log P(w,t)$:

$$\log P(w,t) = \log P(w|t) + \log P(t)$$

Thus, maximizing P(w,t) or P(t|w) is equivalent to maximizing $\log P(w,t)$, which is numerically stable and computationally efficient.

Conclusion Since the logarithm is monotonic and $P(t|w) \propto P(w,t)$, we conclude that:

$$\arg\max_{t} P(t|w) = \arg\max_{t} \log P(t, w)$$

1.2 Log Probability of the Highest Scoring Sequence

The log probability of the highest scoring tag sequence of length j ending in tag t_j , denoted as $\pi_j(t_j)$, is defined as:

$$\pi_j(t_j) = \max_{t_1, \dots, t_{j-1}} \sum_{i=1}^{j} score(w, i, t_i, t_{i-1})$$

where the score function is:

$$score(w, i, t_i, t_{i-1}) = log P(w_i|t_i) + log P(t_i|t_{i-1})$$

To compute $\pi_j(t_j)$ recursively, consider the last transition in the sequence, from t_{j-1} to t_j . The best sequence up to position j ending in t_j must maximize:

$$\pi_j(t_j) = \max_{t_{j-1}} \left(\pi_{j-1}(t_{j-1}) + \mathsf{score}(w, j, t_j, t_{j-1}) \right)$$

Expanding the score function:

$$\pi_j(t_j) = \max_{t_{j-1}} \left(\pi_{j-1}(t_{j-1}) + \log P(w_j|t_j) + \log P(t_j|t_{j-1}) \right)$$

Thus, $\pi_i(t_i)$ can be expressed as:

$$\pi_j(t_j) = \max_{t_{j-1}} \left(\pi_{j-1}(t_{j-1}) + \log P(t_j|t_{j-1}) \right) + \log P(w_j|t_j)$$

This recursive formulation allows efficient computation of $\pi_j(t_j)$ for all t_j and forms the basis of the Viterbi algorithm.

1.3 Viterbi Algorithm and Complexity

This section provides the Viterbi Algorithm to compute t, as defined in the last line of Eq. (1):

$$\hat{t} = \arg\max_{t} P(t|w)$$

The algorithm identifies the most probable sequence of tags for a given sequence of words. Additionally, the time complexity is analyzed to justify its efficiency.

1.3.1 Pseudocode for Viterbi Algorithm

- Observation sequence: w[1:n] (words in the sentence)
- States: T (possible POS tags)
- Transition probabilities: $P(t_i \mid t_i)$ for all $t_i, t_i \in T$
- Emission probabilities: $P(w_k \mid t_i)$ for all w_k in Vocabulary and $t_i \in T$
- Initial probabilities: $P(t \mid START)$ for all $t \in T$

Output:

• Most probable sequence of tags: $\hat{t}[1:n]$

Algorithm:

1. Initialization:

• For each state $t \in T$:

$$v[1][t] = \log P(t \mid \mathsf{START}) + \log P(w[1] \mid$$

• Set backpointer[1][t] = NULL

2. Recursion:

• For i = 2 to n (iterate over each word):

$$\begin{aligned} v[i][t] &= \max_{t_{\text{prev}} \in T} v[i-1][t_{\text{prev}}] \\ &+ \log P(t \mid t_{\text{prev}}) \\ &+ \log P(w[i] \mid t) \end{aligned}$$

• Store the best previous state:

$$\begin{aligned} \text{backpointer}[i][t] &= \arg\max_{t_{\text{prev}} \in T} v[i-1][t_{\text{prev}}] \text{ The Viterbi algorithm can be generalized using a} \\ &+ \log P(t \mid t_{\text{prev}}) \end{aligned}$$

3. Termination:

• Compute the final log-probabilities:

$$\begin{aligned} v[n+1][\text{STOP}] &= \max_{t \in T} v[n][t] \\ &+ \log P(\text{STOP} \mid t) \end{aligned}$$

• Store the best previous state:

$$\begin{aligned} \text{backpointer}[n+1][\text{STOP}] &= \arg\max_{t \in T} v[n][t] \\ &+ \log P(\text{STOP} \mid t) \end{aligned}$$

4. Backtrace:

• Initialize:

$$\hat{t}[n+1] = STOP$$

• For i = n to 1:

$$\hat{t}[i] = \text{backpointer}[i+1][\hat{t}[i+1]]$$

5. **Return:** $\hat{t}[1:n]$

Time Complexity 1.3.2

The time complexity of the Viterbi Algorithm is derived as follows:

- The recursion step computes v[i][t] for every word i = 1 to n and every state $t \in T$. For each t, it iterates over all previous states $t_{\text{prev}} \in T$.
- Thus, the time complexity of the recursion is $O(n \cdot |T|^2)$, where n is the number of words and |T| is the number of states.
- Initialization and termination contribute O(|T|), and backtracing contributes O(n). These are negligible compared to $O(n \cdot |T|^2)$.

Overall, the time complexity is:

$$O(n \cdot |T|^2)$$

t) This quadratic dependence on |T| arises because every pair of current and previous states is considered. While computationally expensive for large |T|, this ensures globally optimal tag sequences.

1.3.3 Conclusion

The Viterbi Algorithm efficiently computes \hat{t} by combining dynamic programming with backtracking. Its $O(n \cdot |T|^2)$ complexity is acceptable for small to moderate-sized tag sets and sequence lengths.

1.4 Semiring-Based Viterbi Algorithm

semiring structure:

$$S = \langle A, \oplus, \otimes, 0_s, 1_s \rangle$$

where A is a set of elements, \oplus and \otimes are operations, and 0_s , 1_s are identity elements. In the Viterbi semiring:

• $\oplus = \max$: Selects the highest-scoring path.

- $\otimes = +$: Combines log probabilities.
- $0_s = -\infty$: Identity for \oplus .
- $1_s = 0$: Identity for \otimes .

The recursive computation of $\pi_i(t_i)$ becomes:

$$\pi_j(t_j) = \bigoplus_{t_{j-1}} (\pi_{j-1}(t_{j-1}) \otimes \operatorname{score}(w, j, t_j, t_{j-1}))$$

where:

$$score(w, j, t_j, t_{j-1}) = log P(w_j|t_j) + log P(t_j|t_{j-1})$$

Semiring Properties The semiring-based formulation relies on:

- Associativity of ⊕ and ⊗: Ensures consistent combination of paths and scores.
- Distributivity of ⊗ over ⊕: Enables efficient recursion.
- Identity elements: $0_s=-\infty$ ensures missing paths are ignored, and $1_s=0$ ensures scores add correctly.

Generalization Beyond Viterbi This semiring-based approach generalizes the Viterbi algorithm, allowing it to adapt to other tasks:

- For example, the Forward algorithm used for computing marginal probabilities can also be expressed using a semiring with ⊕ = + and ⊗ = ×.
- Other semirings can encode variations, such as computing the k-best paths or summing over all paths instead of finding only the best one.

Equivalence with Standard Viterbi Algorithm

When the max-sum semiring is used ($\oplus = \max$, $\otimes = +$), the semiring-based Viterbi algorithm is equivalent to the standard formulation:

$$\hat{t} = \arg\max_{t} \sum_{i=1}^{n} (\log P(w_i|t_i) + \log P(t_i|t_{i-1}))$$

This ensures compatibility with existing implementations while providing the flexibility to extend to other semirings.

Advantages of the Semiring-Based Approach

The semiring generalization abstracts the algorithm's operations, offering:

- Flexibility to adapt to a wide range of problems, including Forward algorithms or variants for sequence labeling.
- Numerical stability when working with log probabilities, avoiding underflow during computation.

• A unified framework to reason about dynamic programming algorithms.

2 Implementation of the HMM-Based POS Tagger

2.1 Training the HMM Model

The training process involves estimating the transition and emission probabilities with add- α smoothing to address data sparsity. The following equations summarize the process:

• Transition Probabilities:

$$P(t_j \mid t_{j-1}) = \frac{\operatorname{count}(t_{j-1}, t_j) + \alpha}{\operatorname{count}(t_{j-1}) + \alpha \cdot |T|}$$

• Emission Probabilities:

$$P(w_i \mid t_j) = \frac{\text{count}(w_i, t_j) + \alpha}{\text{count}(t_i) + \alpha \cdot |\text{Vocab}|}$$

These probabilities are stored in log-space to avoid numerical underflow during calculations.

3 Viterbi Algorithm for Decoding

The Viterbi algorithm decodes the most probable sequence of tags for a given sequence of words. It uses dynamic programming with a backtracking mechanism to ensure optimal decoding. For a sequence of n words and m tags, the algorithm has a time complexity of $O(n \cdot m^2)$.

3.1 Baseline Model

A baseline model is implemented for comparison, assigning the most frequent tag for each word based on the training set. For unseen words, the default tag NN is used.

3.2 Debugging Example from Assignment PDF

A debugging routine validated the HMM and Viterbi algorithm using a toy example from the assignment. The example involved two words with predefined probabilities:

- Transition: $P(VB \mid START) = 4$, $P(NN \mid VB) = 9$, $P(STOP \mid NN) = 1$.
- Emission: $P(\text{example_word1} \mid \text{VB}) = 1$, $P(\text{example_word2} \mid \text{NN}) = 1$.

Results:

- True and Predicted Tags: [VB, NN]
- Gold and Predicted Scores: 3.58

The routine confirms the implementation accurately computes the optimal sequence and score.

This shorter version retains all essential details while reducing repetition and simplifying the explanation.

4 Experimental Results and Analysis

4.1 Hyperparameter Tuning

The smoothing parameter α was tuned using the development set. A grid search over [0.1, 0.5, 1.0, 2.0] revealed that $\alpha = 0.1$ yields the best accuracy on the validation set (88.97%). This value was used for training the final model.

4.2 Performance Evaluation

The final HMM-based POS tagger was evaluated on the test set. Table 1 summarizes the results:

Metric	Value
Test Set Accuracy	89.16%
Baseline Accuracy	91.72%
Macro Precision	43.75%
Macro Recall	47.23%
Macro F1	43.46%

Table 1: Performance of HMM Tagger on the Test Set.

4.3 Confusion Matrix Analysis

The confusion matrix reveals that the HMM tagger performs well for high-frequency tags like NN (noun), IN (preposition), and DT (determiner). However, it struggles with systematic confusions:

- NN is often confused with JJ (848 cases) and vice versa (1147 cases).
- Plural nouns (NNS) are frequently mistaken for singular nouns (NN) in 470 instances.

These errors stem from overlapping syntactic contexts, e.g., "running" tagged as NN in "Running is fun" but as VBG in "He is running."

Performance declines further for low-frequency tags like PDT (predeterminer) and FW (foreign word), contributing to low macro-averaged metrics: precision (43.75%), recall (47.23%), and F1 (43.46%).

The full confusion matrix is provided in Appendix A.

4.4 Comparison with the Baseline Model

The HMM POS tagger achieves 89.16% accuracy, slightly below the baseline model's 91.72%. The baseline benefits from the dataset's skewed distribution, where frequent tags like NN dominate, enabling it to achieve high accuracy through simplicity.

While the HMM tagger generalizes better by incorporating transition and emission probabilities, it struggles with rare tags and exhibits systematic confusions.

Key points:

- HMM performs comparably to the baseline for frequent tags but struggles with rare ones.
- The baseline achieves higher accuracy but lacks flexibility for unseen data.

4.5 Improving the Tagger's Performance

To enhance the tagger's performance, several strategies can be employed:

- Incorporating Contextual Features: Using word embeddings like GloVe or contextual embeddings from BERT can provide richer representations of words, capturing syntactic and semantic nuances.
- Enhanced Smoothing: Implementing advanced smoothing techniques such as Good-Turing or Kneser-Ney smoothing can better address data sparsity for rare tags and transitions.
- Semi-Supervised Learning: Leveraging unlabeled data through pretraining or self-training methods can expand the model's coverage of linguistic contexts.
- Ambiguity Resolution: Annotating ambiguous sentences in the dataset with additional syntactic or semantic context can improve the model's ability to differentiate between tags like NN and JJ.

4.6 Optimizing Runtime Performance

The HMM POS tagger has a time complexity of $O(n \cdot |T|^2)$, where n is the sequence length and |T| is the number of tags. The following optimizations can reduce runtime without significant accuracy loss:

- **Beam Search:** Limiting the number of tags considered at each step can drastically reduce computations while retaining high-probability paths.
- Pruning Tag Space: Reducing the number of tags by combining semantically similar tags into broader categories can decrease the computational burden.
- **Parallelization:** Distributing computations for transition and emission probabilities across multiple threads or GPUs can speed up the Viterbi algorithm.
- Approximate Inference: Using techniques like sampling-based inference or loopy belief propagation can reduce complexity with minimal accuracy trade-offs.

Appendix

A Detailed Confusion Matrix

The complete confusion matrix, detailing both the most frequently predicted tags and the most confused tag pairs, is attached as a text document in this submission. It provides valuable insights into the model's strengths and weaknesses for individual tags.

```
True Tag
                    Predicted Tag
                                        Count
IN
          NNP
                    156
NN
          NN
                    21729
NNS
          NNS
                    9477
VBG
          VBG
                    1502
ΙN
          JJ
                    354
                    9056
,
DT
          ,
DT
                    14450
IN
          ΙN
                    16715
VBP
          VBP
                    1523
VBN
          VBN
                    2697
                    7035
\mathsf{CC}
          NNP
                    191
T0
          T0
                    3898
VB
          ۷B
                    4284
PDT
          PDT
                    50
MD
          MD
                    1651
IN
          \mathsf{DT}
                    150
VBP
          IN
                    79
                    983
          :
PRP
          PRP
                    2769
JJ
          JJ
                    8462
\mathsf{CC}
          CC
                    3847
ΕX
          ΕX
                    168
VBZ
          VBZ
                    3164
RB
          RB
                    4330
RB
          ۷B
                    22
ŹĴ
          ŹĴΚ
                    395
                    1422
NN
          JJ
                    848
JJ
          NN
                    1147
                    533
VBG
          NN
CD
          CD
                    5337
NNS
          NN
                    470
$
          $
                    1138
ĎΤ
          İΝ
                    34
NN
          UΗ
                    28
1.1
          1.1
                    1417
NNP
          NNP
                    14711
VBG
          JJ
                    365
WP
          WP
                    392
VBD
          VBD
                    4696
P<sub>0</sub>S
          P<sub>0</sub>S
                    1638
NNS
          NNP
                    167
VBD
          VΒ
                    15
PRP$
          PRP$
                    1402
                    169
RBR
          RBR
RBR
          JJR
                    104
WDT
                    713
          WDT
WDT
          DT
                    49
```

NN WRB VBN NNP EX DT NNP DT NNP IN SYM UH DT NN JJ SHB RB WPS JJ SRB WPS JJ S	201 422 773 236 79 26 126 65 202 297 205 3 122 99 74 255 185 426 10 200 115 32 10 31 259 254 496 225 145 6 47 178 20
(249 252
VB VBG NNP	213 31 5
VBP NNS NNP NNP DT NN JJ SYM PDT DT NNP	111 154 172 9 191 837 657 14 11 7
	WRB VBN NNP EX DNP NNP NNP NNP NNP NNP NNP NNP NNP NNP

NNP VBZ FW FW SJJ FRP NN NNP NNP NNP NNP NNP NNP NNP NNP NN	LS NN PDT RB FW NN RBS JJ NN LS NNPS RBR VBD NN PDT RBS JJ VBN DT PRP NN PDT NN PDT VBN PDT VBN PDT VBN PDT VBN NN PDT VBN NN PDT VBN VBN PDT VBN PDT	35 16 9 6 4 5 70 33 5 28 17 25 13 36 20 10 48 89 5 11 60 3 3 40 40 47 42 9 12 12 14 14 14 14 14 14 14 14 14 14 14 14 14
RB RB NN	IN RP VBN	510 47 42
JJ	VB	29

VBG NNP NNP JVB VBD NN VBD JN VBD DT JNN CN NN CD VBD NNP VBR NND VBR NND VBR	IN NNS MD RP RBS CD RBS VBP RBN STOP STOP STOP STOP STOP STOP STOP STOP	71 17 37 79 14 70 25 7 8 10 81 12 7 8 19 44 2 10 4 61 19 7 29 8 19 19 19 19 19 19 19 19 19 19 19 19 19
PDT	DT	13

RB PRP\$ NN CD FW NNP IN PDT JVBG NNPS VBN NNPS NNPS NNPS RBR NNS PRP CC NN NNS PRP CC NN PRP RB VBN PRP RB VBN PRP RB VBN PRP RB VBN PRP RB VBN PRP RBN RBN RBN RBN RBN RBN RBN RBN RBN RBN	SYM NNP EX , LS NNS CC CD LS RBS WP NNPS RB NN NNS LS VBN <stop> UH NN LS STOP> UH NN LS CD PRP\$ <stop> \$CD SYM VBP JJ NNS NNS NNS NNS NNS NNS NNS NNS NNS</stop></stop>	1 3 13 3 2 43 19 1 8 2 12 1 31 41 16 22 16 7 2 34 8 8 1 3 1 5 14 25 1 28 9 13 3 27
PRP RB	NN PRP	9 13 3 27 1 11 22 21 18 19 11 6 6 13

VBG CPRP NNS NNS FW # PDT VBD NNP VBD NNP VBD VBD VBD VBD VBD VBD VBD VBD VBD VBD	PRP WP\$ NNP DT CD RBS . VB . <stop> # JJ VBN JJ \$ VBD . <stop> IN RBS EX WRB VBG NN SYM PDT SYM . WDT . MD CD NNP VBP PRP VBN FW RB</stop></stop>	2 2 14 14 4 23 3 5 3 4 22 1 77 2 7 2 15 1 6 2 4 13 1 39 28 4 6 5 7 6 37 3 2 7 9 9 9 5 2 19 19 19 19 19 19 19 19 19 19 19 19 19
NNP	VBN	18
. 41 41 5	. 10	-

RBS
VB IN 13

```
JJR|RBR JJ
                   1
                    1
VBD
          WP$
JJR
                   1
          LS
                   1
VΒ
          ,
JJ
NNPS
                    2
FW
          $
                    1
ΙN
          VBP
                    1
VBG
                    3
          CD
                   1
\mathsf{CC}
VBG
          LS
                    4
         NNS | VBZ 1
VBZ
VBZ
          MD
                   1
                   1
JJ|IN
          PDT
          RB|JJ
                   1
JJ
                   2
VBN
          $
RB
          CD
                   2
                    2
JJ
          RP
CD
          IN
                   10
          IN|RB
ΙN
                   2
VBN|JJ
                   1
          VBN
                   1
VΒ
          WP$
VΒ
          MD
                    1
VΒ
          (
                   1
RB|JJ
          NN
                    1
          \mathsf{UH}
                    1
RB|JJ
VBP
                    1
JJR|RBR JJR
                   1
RBR|JJR RBR
                   1
                   1
DT
          SYM
VBP
                   1
          ΕX
UH
          DT
                    1
                    4
VBZ
          $
VBD
          CD
                   2
          WP
                    2
NNS
T0
          JJ
                   1
JJ
                   1
          RBR
                   1
VBP
LS
          NNP
                   3
          ŕź
UH
                   1
NNP
                    1
          ۷B
RBR
                   4
CD
          MD
                   4
VBG
          SYM
                   1
                   2
VB
          JJR
          $
(
ΙN
                   1
VBD
                    1
                   1
          CC
NNS
NNS
          PDT
                    1
                   2
VBN
WRB
          DT
                   1
```

JJS	\$	1
UH	NN	1
NNS	\$	2
VBP	CD	1
VBD	UH	1
VBP	DT	1
CD	PDT	2
VBD	\$	1
VB	PRP	2
JJ	JJR	2
IN	PDT	1
NNP	RP	1
NNP	P0S	10
RP	IN RB	1
NN	JJŠ	1
VB	RB	1
VBN	WP\$	1
RBS	RB	1
RBR	VBP	2
VBG	EX	2
NN	VBG NN	1
RBR	RBR JJR	1
RB	RBR JJR	1
VBZ	LS	1