

---

# Mid Level Image Features : Shapes

Eun Yi Kim

---



Artificial Intelligence  
& Computer Vision  
L a b o r a t o r y



# Shape descriptors



- There are three approaches to defining shapes
  1. Shape represented by its region descriptors – Simple !!
  2. Shape represented by its Boundary
  3. Shape represented by its interests points (corners)





# Region based Shape Descriptors





- area
- centroid
- perimeter
- perimeter length
- circularity, elongation
- mean and standard deviation of radial distance
- second order moments (row, column, mixed)
- bounding box
- extremal axis length from bounding box
- lengths and orientations of axes of best-fit ellipse

Often want features independent of position, orientation, scale



# Topological Region Descriptors

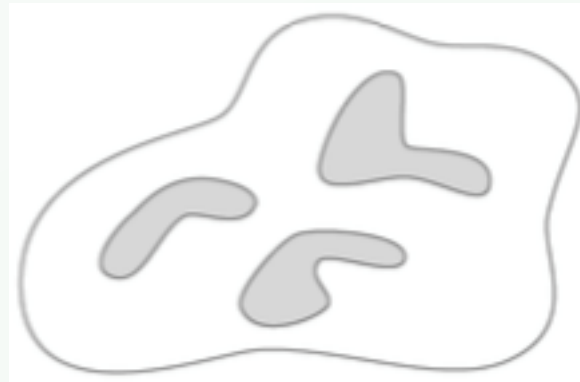


Artificial Intelligence  
& Computer Vision  
Laboratory

- Topological properties: properties of image preserved **under rubber-sheet distortions**
  - # holes in the image
  - # connected components



$H=2, C=1$



$H=0, C=3$



$H=1, C=1$

$H=2, C=1$



# Topological Region Descriptors : Hole Counting



Artificial Intelligence  
& Computer Vision  
Laboratory

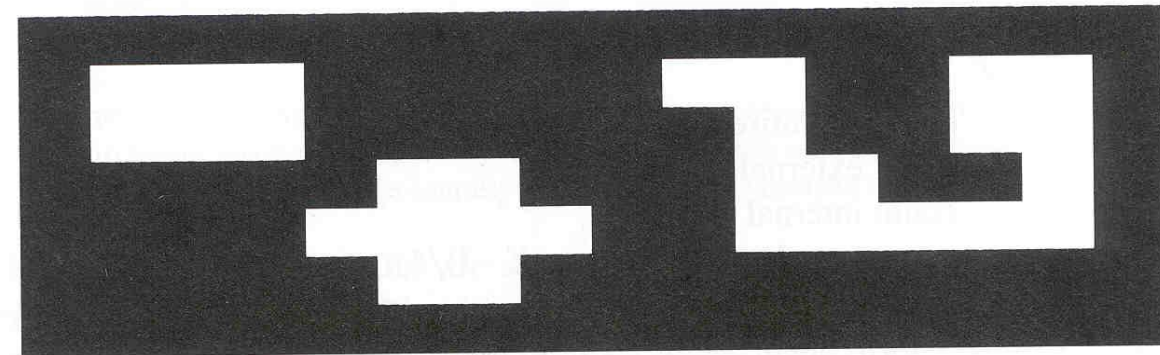
- “external corner” has 3(1)s and 1(0)
- “internal corner” has 3(0)s and 1(1)
- Holes computed from only these patterns!

1	1	1	0	0	1	1	1
1	0	1	1	1	1	0	1

(a)  $2 \times 2$  external corner patterns

0	0	0	1	1	0	0	0
0	1	0	0	0	0	1	0

(b)  $2 \times 2$  internal corner patterns



(c) Three bright holes in dark background



# Topological Region Descriptors : Hole Counting Algorithm



Artificial Intelligence  
& Computer Vision  
Laboratory

**Input a binary image and output the number of holes it contains.**

**M** is a binary image of **R** rows of **C** columns.

1 represents material through which light has not passed;

0 represents absence of material indicated by light passing.

Each region of 0s must be 4-connected and all image border pixels must be 1s.

**E** is the count of *external corners* (3 ones and 1 zero)

**I** is the count of *internal corners* (3 zeros and 1 one)

```
integer procedure Count_Holes(M)
{
    examine entire image, 2 rows at a time;
    count external corners E;
    count internal corners I;
    return(number_of_holes = (E - I)/4);
}
```





# Topological Region Descriptors : Hole Counting Example



$$(E-I)/4$$

	0	1	2	3	4	5	6	7	8	9	0	1	2	3	4	5	e	i
0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1		
1	1	0	0	0	1	1	1	1	1	0	0	1	1	0	0	1		
2	1	0	0	0	1	1	1	1	1	1	0	1	1	0	0	1		
3	1	1	1	1	1	0	0	1	1	1	0	0	1	1	0	1		
4	1	1	1	1	0	0	0	0	1	1	0	0	0	0	0	1		
5	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1		
6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1		

(d) Binary input image 7 rows high and 16 columns wide

	0	1	2	3	4	5	6	7	8	9	0	1	2	3	4	5	e	i
0	e			e					e		e		e		e		6	0
1									e	i							1	1
2	e			e	e		e				i	e	e	i			6	2
3				e	i		i	e				i		i			2	4
4				e	i		i	e		e					e		4	2
5					e		e										2	0
6																	0	0

(e) External corners marked with e; internal corners marked i







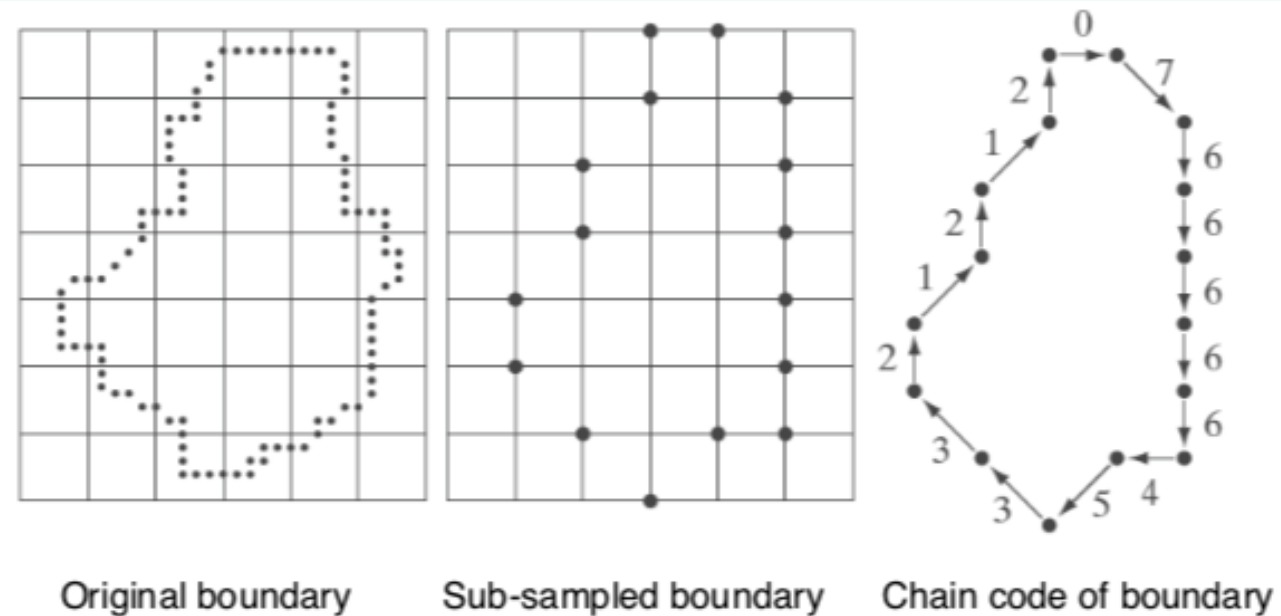
# Boundary based Shape Descriptors



# Boundary Representation



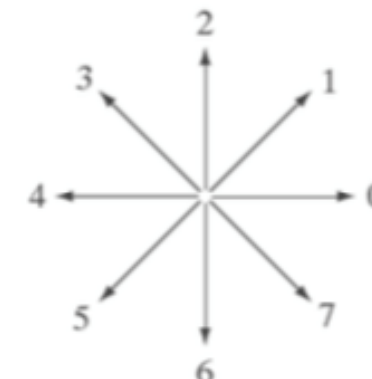
- (Freeman) Chain Code
- Boundary representation  
= 0766666453321212



Chain code for  
4-neighborhood



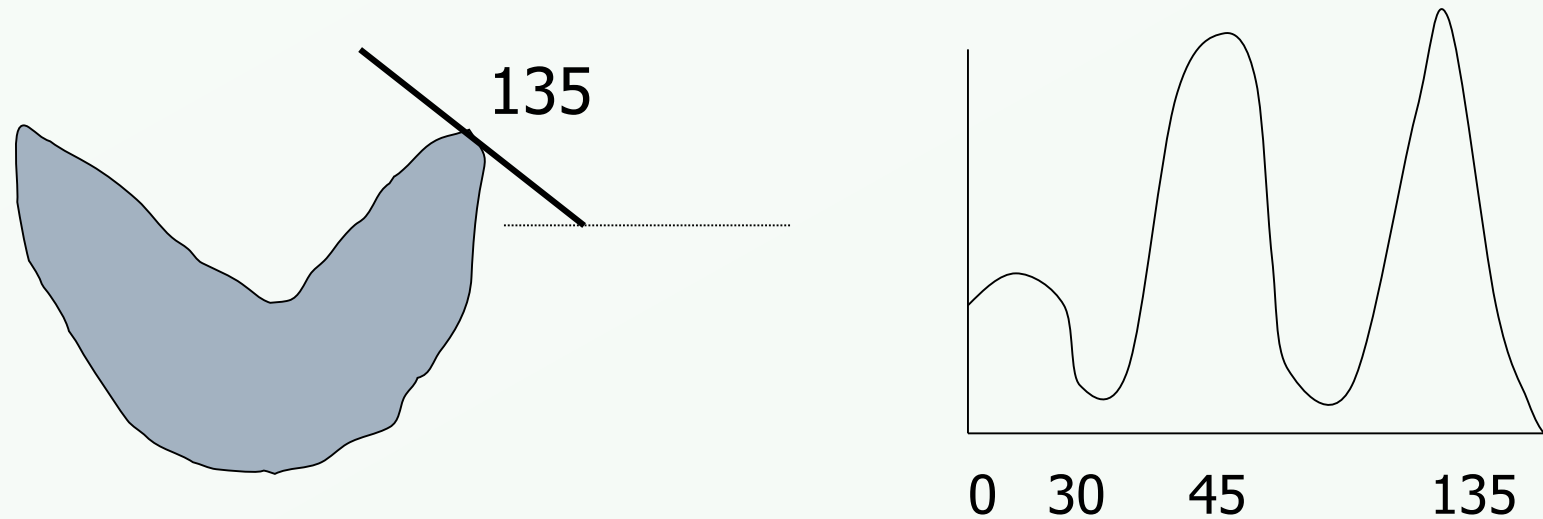
Chain code for  
8-neighborhood



# Boundary Representation



- Tangent-Angle histograms



Is this feature invariant to starting point?  
Is it invariant to size, translation, rotation?





# Interest Operator + Descriptor

- Harris operator
- Multi-scaled operator
- SIFT (scale invariant feature transform)
- HOG (histogram of oriented gradient)



# Introduction to Interest Operators



Artificial Intelligence  
& Computer Vision  
Laboratory

- Find “interesting” pieces of the image
  - E.g. corners, salient regions
  - Focus attention of algorithms
  - Speed up computation
- Many possible uses in matching/recognition
  - Search
  - Object recognition
  - Image alignment & stitching
  - Stereo
  - Tracking
  - ...

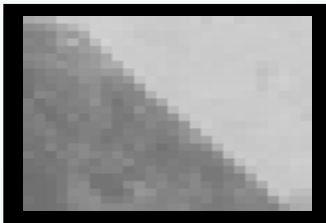


# Interest points



**0D structure:** **single points**

➡ not useful for matching



**1D structure:** **lines**

➡ edge, can be localised in 1D, subject to the aperture problem



**2D structure:** **corners**

➡ corner, or **interest point**, can be localised in 2D, good for matching

**Interest Points** have **2D** structure.

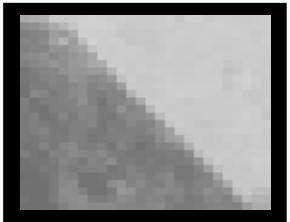


# Interest points



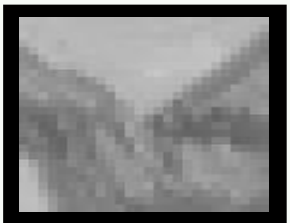
**0D structure: single points**

➡ not useful for matching



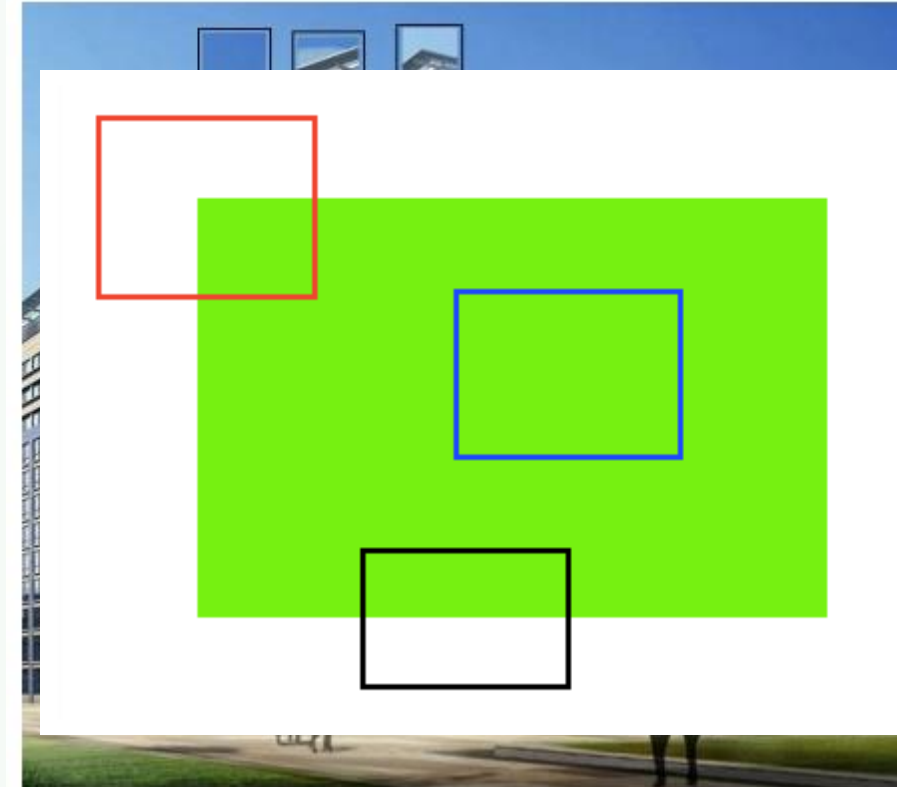
**1D structure: lines**

➡ edge, can be localised in 1D, subject to the aperture problem



**2D structure: corners**

➡ corner, or **interest point**, can be localized in 2D, good for matching

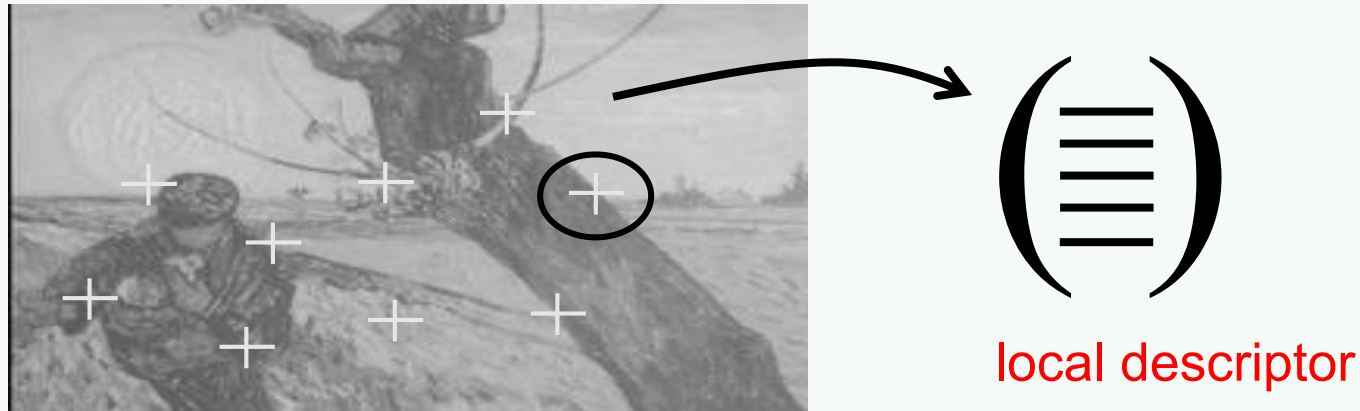


**Interest Points** have **2D** structure.





# Local invariant photometric descriptors



- *Local* : robust to occlusion/clutter + no segmentation
- *Photometric* : (use pixel values) distinctive descriptions
- *Invariant* : to image transformations + illumination changes





- Extraction of interest points with the Harris detector
- Comparison of points with cross-correlation
- Verification with the fundamental matrix

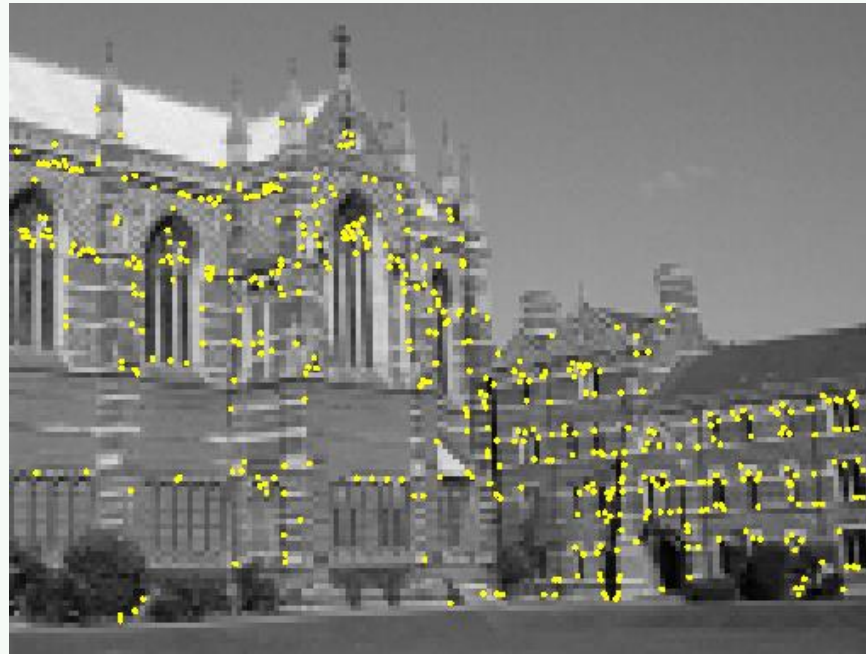
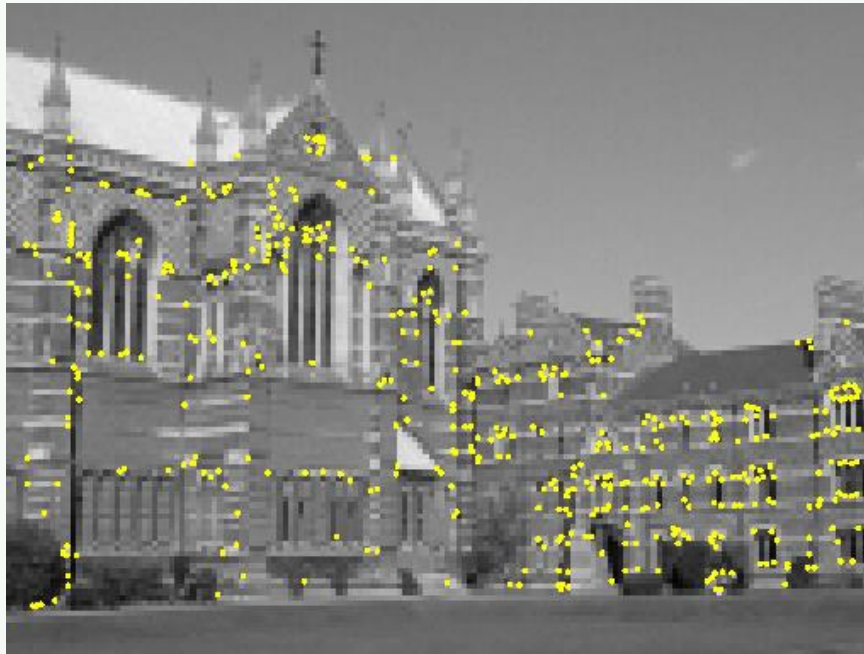
The fundamental matrix maps points from the first image to corresponding points in the second matrix using a homography, that is determined through the solution of a set of equations that usually minimizes a least square error.



# Preview: Harris detector



Artificial Intelligence  
& Computer Vision  
Laboratory



Interest points extracted with Harris ( $\sim 500$  points)



# Cross-correlation matching



Artificial Intelligence  
& Computer Vision  
Laboratory

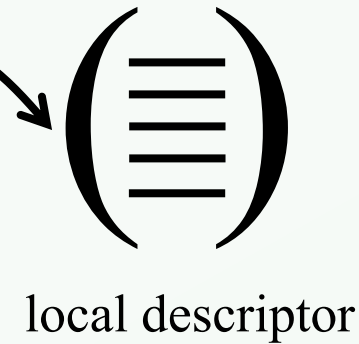
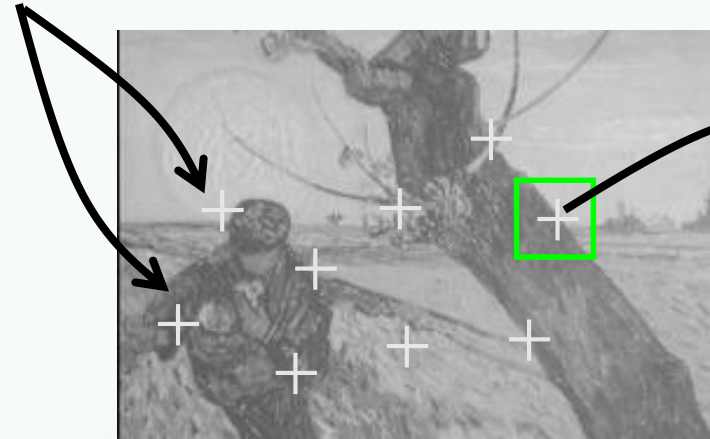


Initial matches – motion vectors (188 pairs)





interest points



local descriptor

- 1) Extraction of **interest points**
- 2) Computation of **local descriptors**
- 3) Determining **correspondences**
- 4) Selection of **similar images**



# 1. Harris detector



Based on the idea of auto-correlation



$$\rho(dr, dc) = \frac{\sum_i \sum_j I[i, j] I[i + dr, j + dc]}{\sum_i \sum_j I^2[i, j]} = \frac{I[i, j] \circ I_d[i, j]}{I[i, j] \circ I[i, j]}$$

Important difference in all directions => interest point





# Background : Moravec Corner Detector



- take a window  $w$  in the image
- shift it in four directions  
 $(1,0)$ ,  $(0,1)$ ,  $(1,1)$ ,  $(-1,1)$
- compute a difference for each
- compute the min difference at each pixel
- local maxima in the min image are the corners

$$E(x,y) = \sum_{u,v \text{ in } w} w(u,v) |I(x+u,y+v) - I(u,v)|^2$$





# Shortcomings of Moravec Operator



- Only tries 4 shifts. We'd like to consider “all” shifts.
- Uses a discrete rectangular window. We'd like to use a smooth circular (or later elliptical) window.
- Uses a simple min function. We'd like to characterize variation with respect to direction.

Result: Harris Operator





Auto-correlation fn (SSD) for a point  $(x, y)$  and a shift  $(\Delta x, \Delta y)$

$$f(x, y) = \sum_{(x_k, y_k) \in W} (I(x_k, y_k) - I(x_k + \Delta x, y_k + \Delta y))^2$$

SSD means summed square difference

Discrete shifts can be avoided with the auto-correlation matrix

with 
$$I(x_k + \Delta x, y_k + \Delta y) = \overbrace{I(x_k, y_k) + (I_x(x_k, y_k) \quad I_y(x_k, y_k))}^{\text{what is this?}} \begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix}$$

$$f(x, y) = \sum_{(x_k, y_k) \in W} \left( \begin{pmatrix} I_x(x_k, y_k) & I_y(x_k, y_k) \end{pmatrix} \begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix} \right)^2$$



# Harris detector



Rewrite as inner (dot) product

$$\begin{aligned} f(x, y) &= \sum_{(x_k, y_k) \in W} \left( \begin{bmatrix} I_x(x_k, y_k) & I_y(x_k, y_k) \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} \right)^2 \\ &= \sum_{(x_k, y_k) \in W} \begin{bmatrix} \Delta x & \Delta y \end{bmatrix} \begin{bmatrix} I_x(x_k, y_k) \\ I_y(x_k, y_k) \end{bmatrix} \begin{bmatrix} I_x(x_k, y_k) & I_y(x_k, y_k) \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} \end{aligned}$$

The center portion is a 2x2 matrix

Have we seen  
this matrix before?

$$\begin{aligned} &= \sum_W \begin{bmatrix} \Delta x & \Delta y \end{bmatrix} \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} \\ &= \begin{bmatrix} \Delta x & \Delta y \end{bmatrix} \sum_W \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} \end{aligned}$$



# Harris detector



$$= \begin{pmatrix} \Delta x & \Delta y \end{pmatrix} \begin{bmatrix} \sum_{(x_k, y_k) \in W} (I_x(x_k, y_k))^2 & \sum_{(x_k, y_k) \in W} I_x(x_k, y_k) I_y(x_k, y_k) \\ \sum_{(x_k, y_k) \in W} I_x(x_k, y_k) I_y(x_k, y_k) & \sum_{(x_k, y_k) \in W} (I_y(x_k, y_k))^2 \end{bmatrix} \begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix}$$

Auto-correlation matrix M

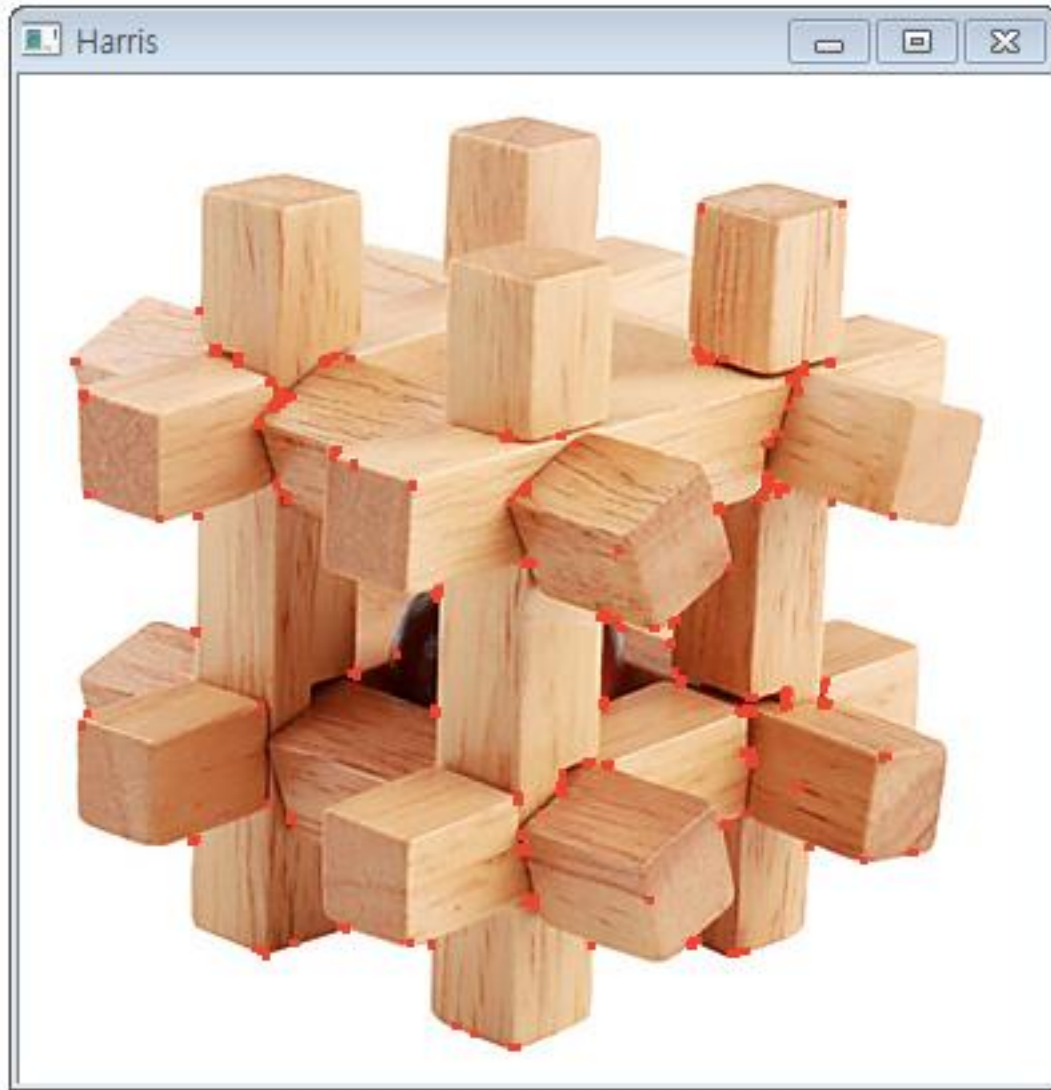




- Auto-correlation matrix
  - captures the structure of the local neighborhood
  - measure based on **eigenvalues** of  $M$ 
    - 2 strong eigenvalues  $\Rightarrow$  interest point
    - 1 strong eigenvalue  $\Rightarrow$  contour
    - 0 eigenvalue  $\Rightarrow$  uniform region
- Interest point detection
  - threshold on the eigenvalues
  - local maximum for localization



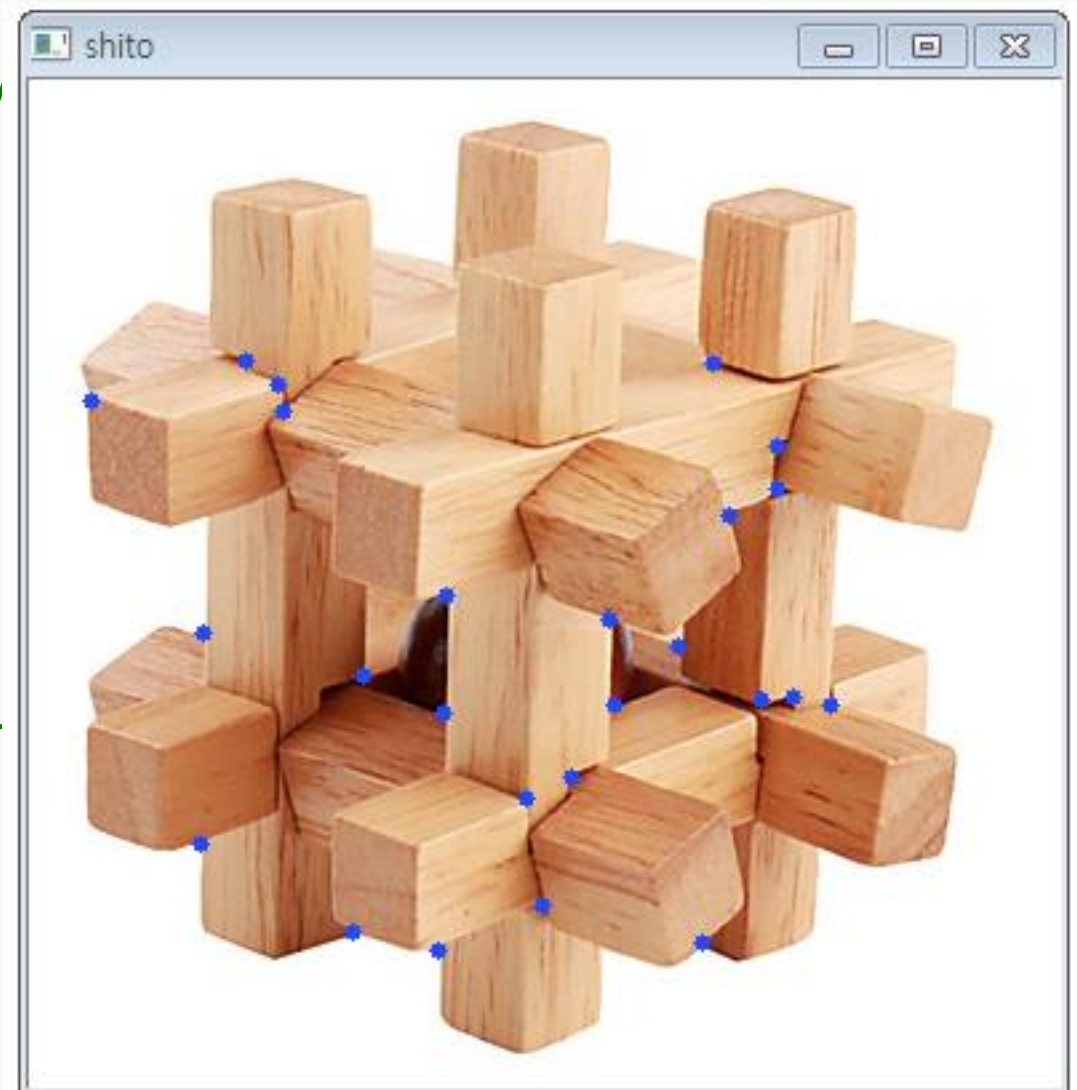
# Some Details from the Harris Paper



$\Gamma(\mathbf{M})$   
ies.

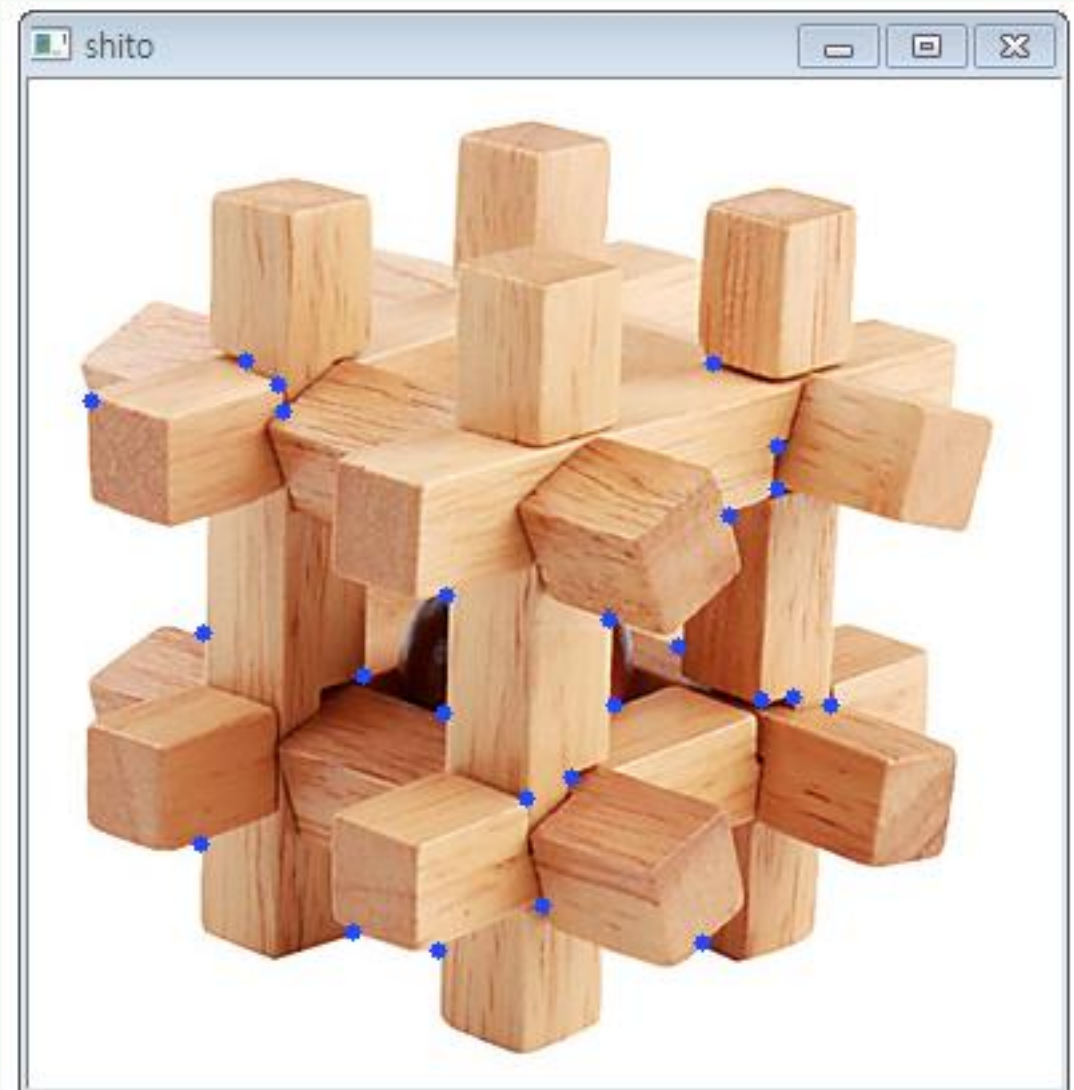
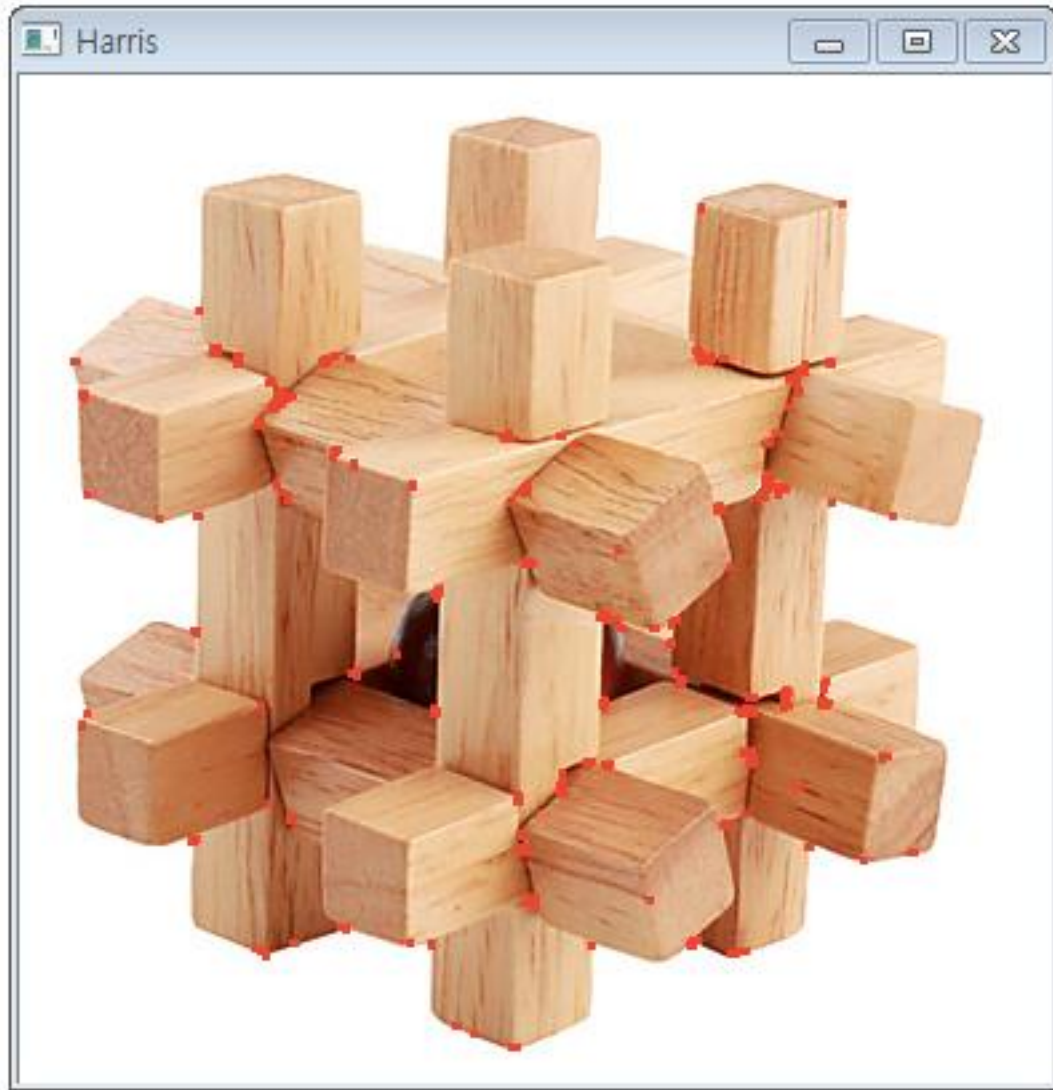
ges

$= \mathbf{r}$





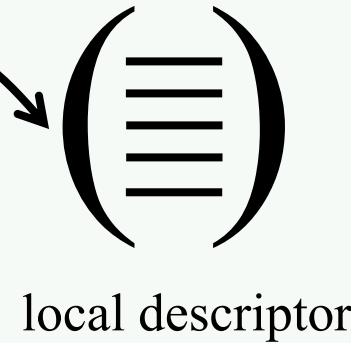
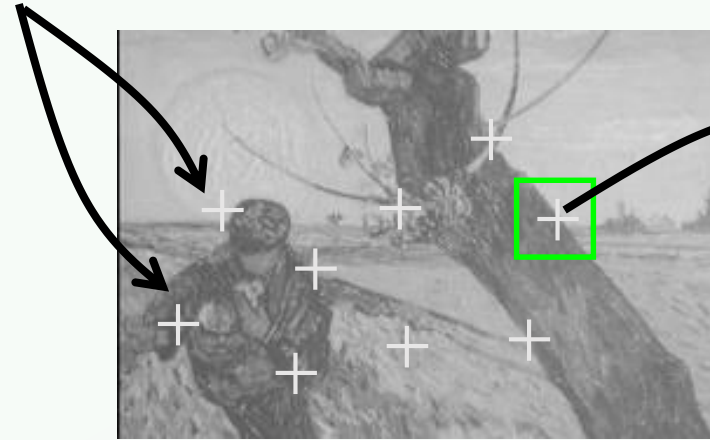
# Some Details from the Harris Paper







interest points



local descriptor

- 1) Extraction of **interest points**
- 2) Computation of **local descriptors**
- 3) Determining **correspondences**
- 4) Selection of **similar images**

