Mid Level Image Features : Shapes

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I N D E X

Shapes

Approaches to Shape description

- Region based shape descriptors
- Boundary based descriptors
- Interest Operator + Descriptor

Applications

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Shapes

- Shape goes one step further than color and texture.

- Color and Texture are both global attributes of an image; shape is not an image attributes

  local 하다.

  - Shape tends to refer to a specific region of an image
  - Segmentation is still a crucial problem to be solved, so interests operator is emplo yed for shape description

- Two-dimensional shape recognition is an important aspect of image a nalysis (image matching/retrieval)

Shape descriptors

- There are three approaches to defining shapes

1. **Shape represented by its region descriptors – Simple !!**
2. **Shape represented by its Boundary**
3. **Shape represented by its interests points (corners)**

Region based Shape Descriptors

Geometric and Shape Properties

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- area
- centroid
- perimeter
- perimeter length
- circularity, elongation
- mean and standard deviation of radial distance
- second order moments (row, column, mixed)
- bounding box
- extremal axis length from bounding box
- lengths and orientations of axes of best-fit ellipse

Often want features independent of position, orientation, scale

Zero-order moment

- Why use moments?
  - Geometric moments of different orders represent spatial characteristics of the image distribution

- Zero-order moment

$$A = \sum_{i=1}^{n} \sum_{j=1}^{m} B[i, j]$$

  - Total intensity of image
  - For binary image → area

Centroid

- An object's position in the image determines its spatial location.
- center of area (a centroid, center of mass) : first order moment
  - Intensity centroid
  - Geometrical center in binary image

$$\begin{cases} \bar{x} = \dfrac{\sum_{i=1}^{n}\sum_{j=1}^{n} jB[i,j]}{A} & : \text{ average (mean) of } j \text{ coordinates of object (1) pixels} \\ \bar{y} = \dfrac{\sum_{i=1}^{n}\sum_{j=1}^{n} iB[i,j]}{A} & : \text{ average of } i \text{ coordinates of object (1) pixels} \end{cases}$$

- A precision of tenths of a pixel is often justifiable for the centroid.
- Centroids of regions can be interesting points for analysis and matching

Second Moments

There are three second-order spatial moments of a region

- Second-order row moment

$$\mu_{rr} = \frac{1}{A} \sum_{(r,c)\in R} (r - \bar{r})^2$$

- Second-order mixed moment

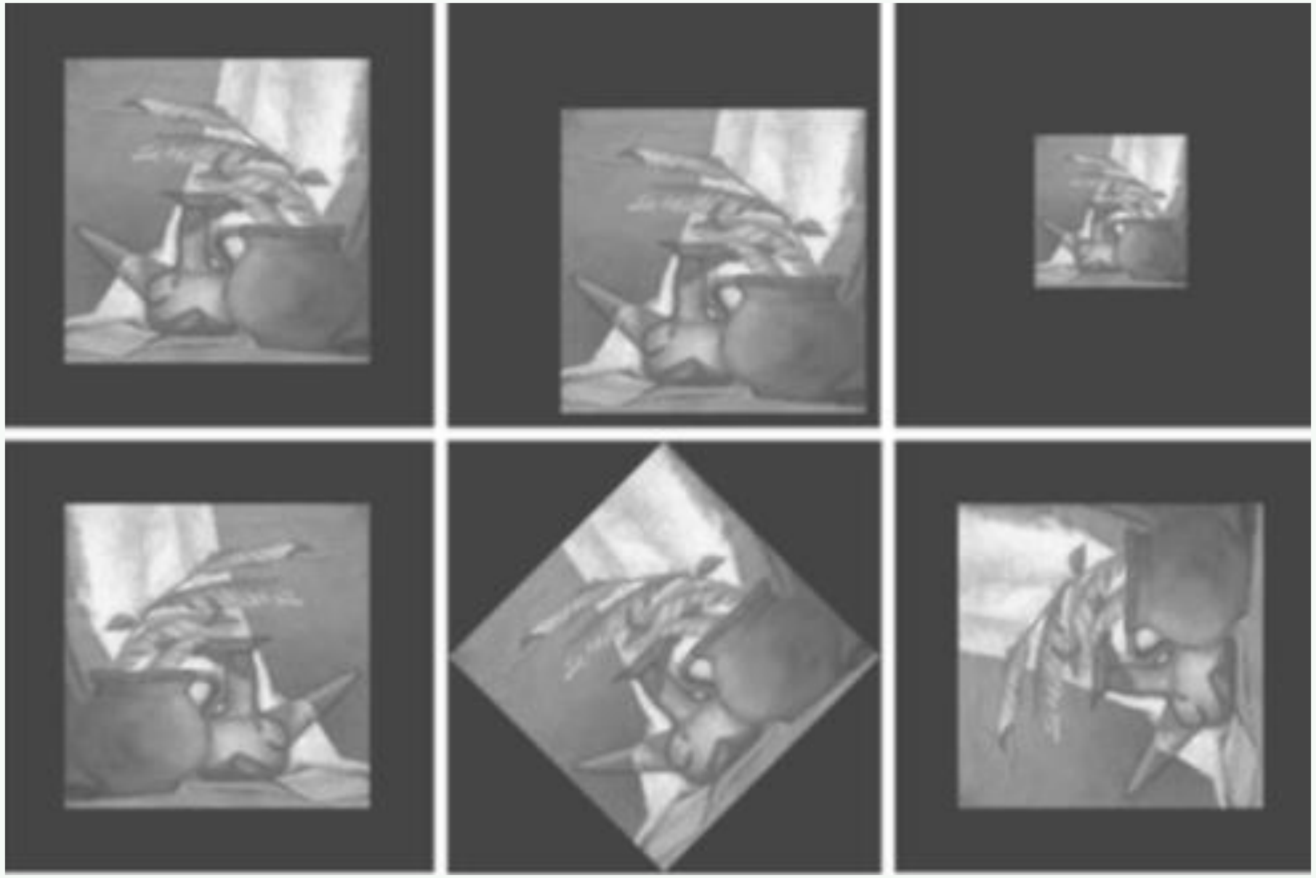
$$\mu_{rc} = \frac{1}{A} \sum_{(r,c)\in R} (r - \bar{r})(c - \bar{c})$$

- Second-order column moment

$$\mu_{cc} = \frac{1}{A} \sum_{(r,c)\in R} (c - \bar{c})^2$$

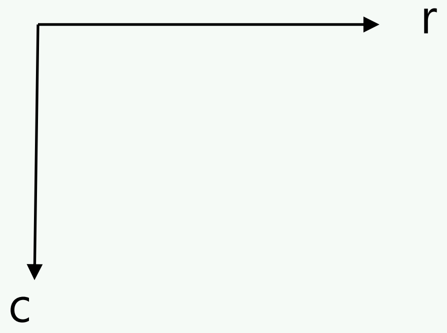
Moment Invariants

- Geometric transformation: translation, scale, mirroring, rotation



Contrast second moments

- For the letter ‘I’
- Versus the letter ‘O’
- Versus the underline ‘_’



Perimeter and Perimeter Length

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Perimeter $P_4 = \{ (r,c) \in R \mid N_8(r,c) - R \neq \phi \}$

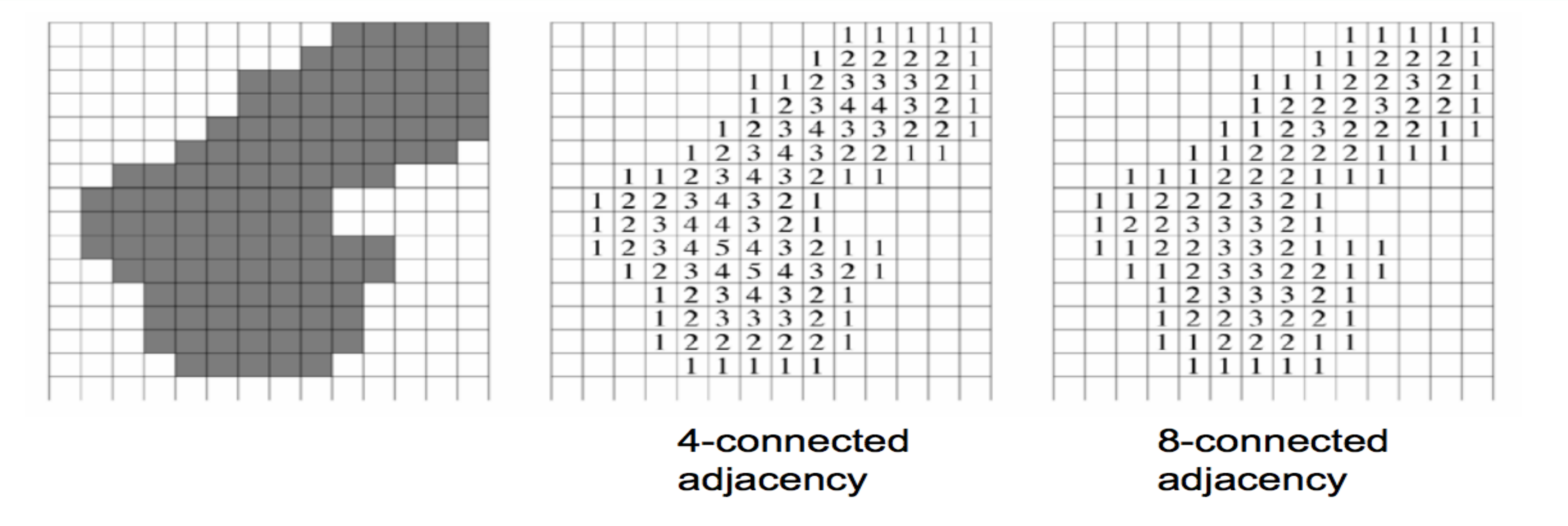
$P_8 = \{ (r,c) \in R \mid N_4(r,c) - R \neq \phi \}$

Perimeter Length

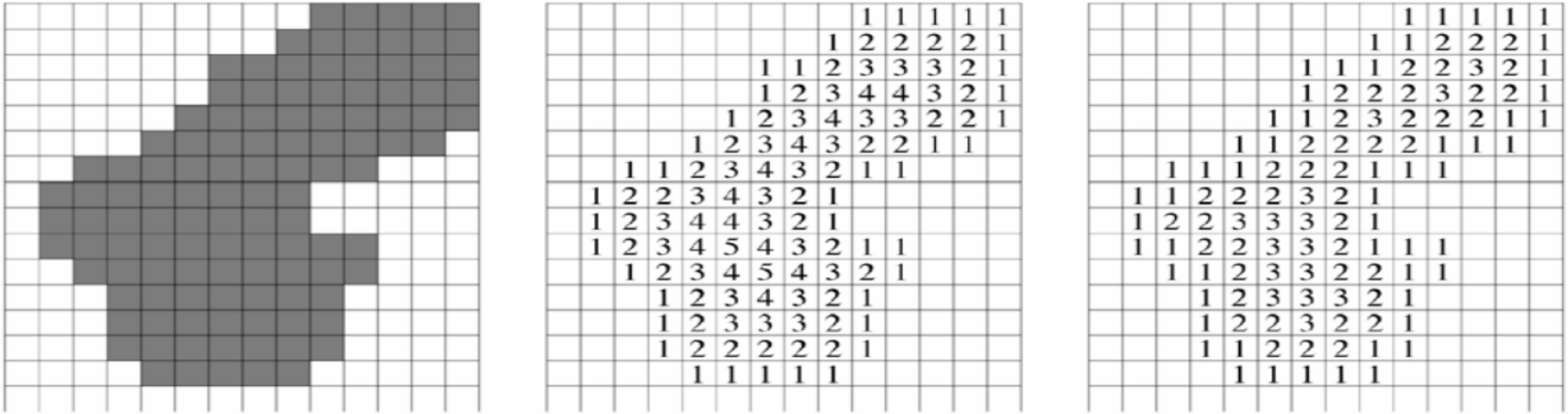
$$|P| = |\{k \mid (r_{k+1}, c_{k+1}) \in N_4(r_k, c_k)\}| + \sqrt{2}\,|\{k \mid (r_{k+1}, c_{k+1}) \in N_8(r_k, c_k) - N_4(r_k, c_k)\}|$$

Perimeter can vary significantly with object orientation

Perimeter and Perimeter Length



Perimeter and Perimeter Length



4-connected adjacency

8-connected adjacency

Circularity

- Common measure of circularity of a region is length of the perimeter squared divided by area

- Circularity (1): $$C_1 = \frac{|P|^2}{A}$$

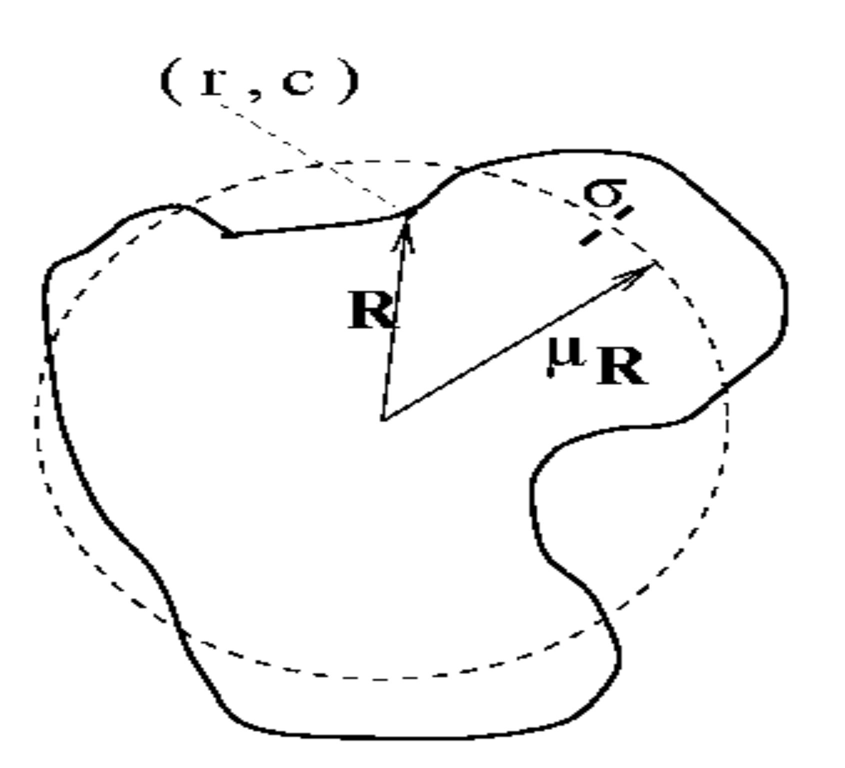
Circularity as variance of "radius"

- A second measure uses variation off of a circle
- Circularity (2): $$C_2 = \frac{\mu_R}{\sigma_R}$$

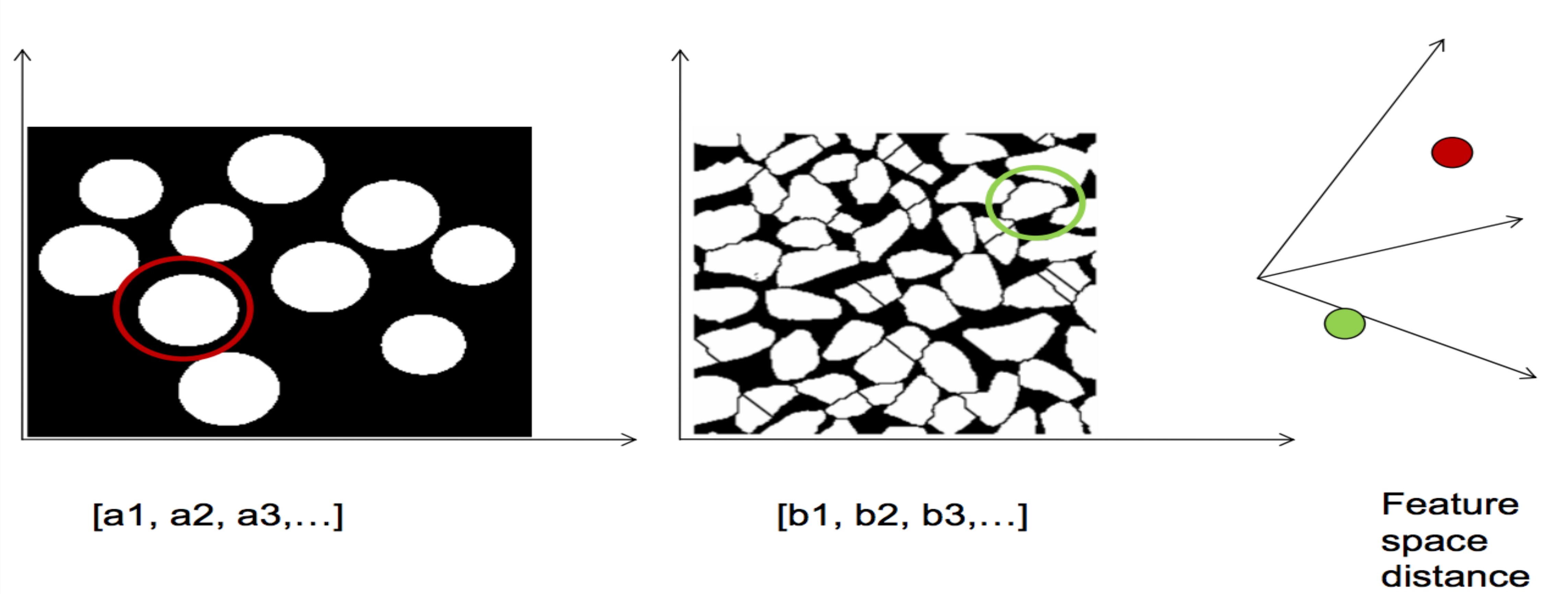
  - μ_R and σ_R^2 are the mean and variance of the distance from the centroid of the shape to the boundary pixels (r_k, c_k).

$$\mu_R = \frac{1}{K}\sum_{k=0}^{K-1} \|(r_k, c_k) - (\bar{r}, \bar{c})\|$$

$$\sigma_R^2 = \frac{1}{K}\sum_{k=0}^{K-1} [\|(r_k, c_k) - (\bar{r}, \bar{c})\| - \mu_R]^2$$



Invariant descriptors



Orientation (1)

- Define the orientation of an object as <u>the orientation of the axis of elongation</u>.

  ≡ axis of least <u>second order moment</u>

  variation(分散) = spread of data

  ≡ axis of least inertia

- The axis of least second moment for an object is the line which gives

$$\min_{line} \chi^2 = \min_{line} \sum_{i=1}^{n} \sum_{j=1}^{n} r_{ij}^2 B[i,j]$$

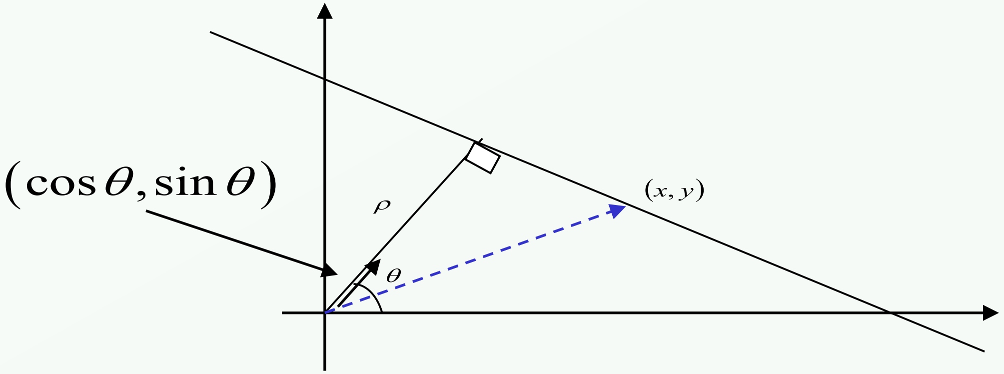
where r_{ij} the perpendicular distance from an object point $[i, j]$ to the line (axis)

Orientation (2)

Polar representation of a straight line

why polar representation instead of $y = ax + b$

cannot represent the vertical line



$$(x, y) \cdot (\cos\theta, \sin\theta) = \rho$$

$$x\cos\theta + y\sin\theta = \rho$$

projection of (x, y) onto the direction $(\cos\theta, \sin\theta)$

Then,

$$r^2 = (x\cos\theta + y\sin\theta - \rho)^2$$

$$\chi^2 = \sum_{i=1}^{n}\sum_{j=1}^{n}(x_{ij}\cos\theta + y_{ij}\sin\theta - \rho)^2 B[i, j]$$

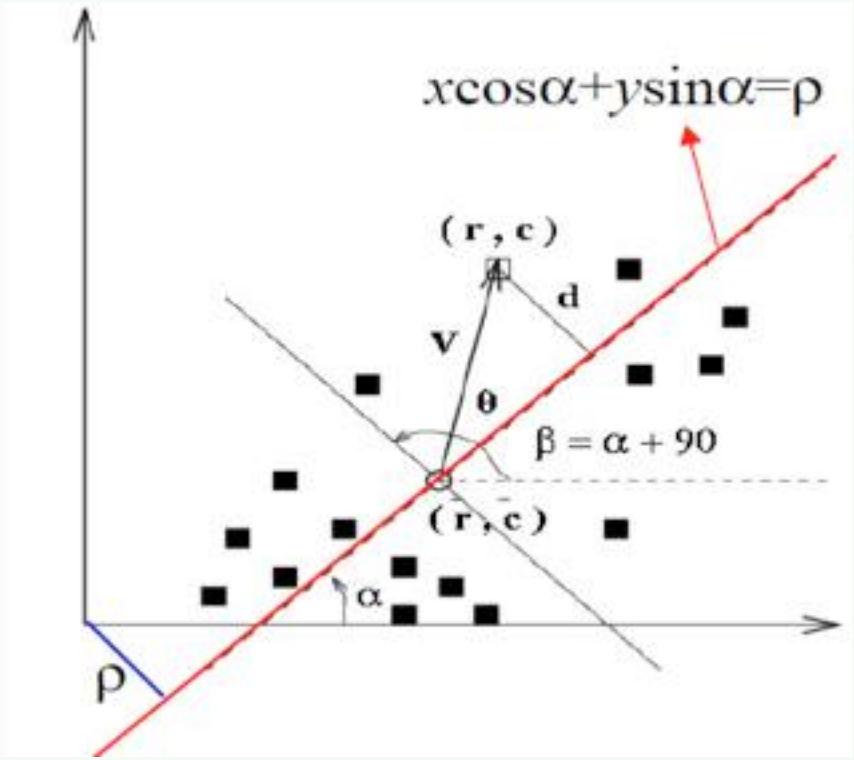
Problem: Find ρ and θ that minimizes χ^2.

Orientation (3)

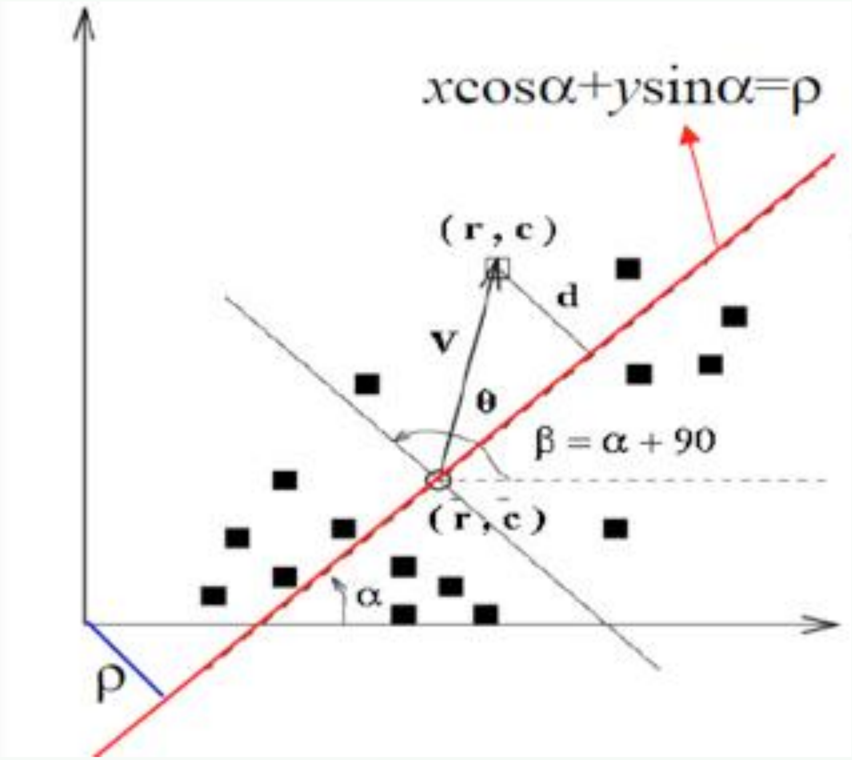
Solution: $\frac{\partial \chi^2}{\partial \rho} = 0$ and $\frac{\partial \chi^2}{\partial \theta} = 0$

- The elongation E of the object $\equiv \frac{\chi_{\max}}{\chi_{\min}}$

Orientation (4)
: Axis with Least Second Moment



Orientation (4)
: Axis with Least Second Moment



Orientation (4)
: Axis with Least Second Moment

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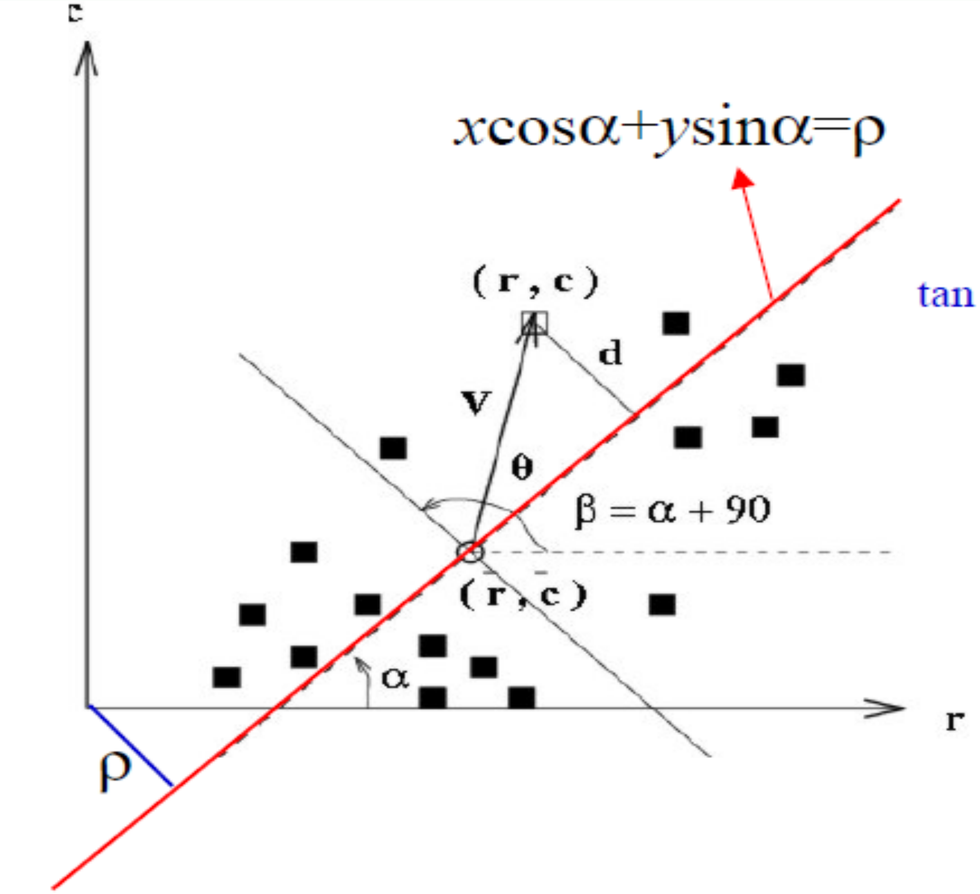
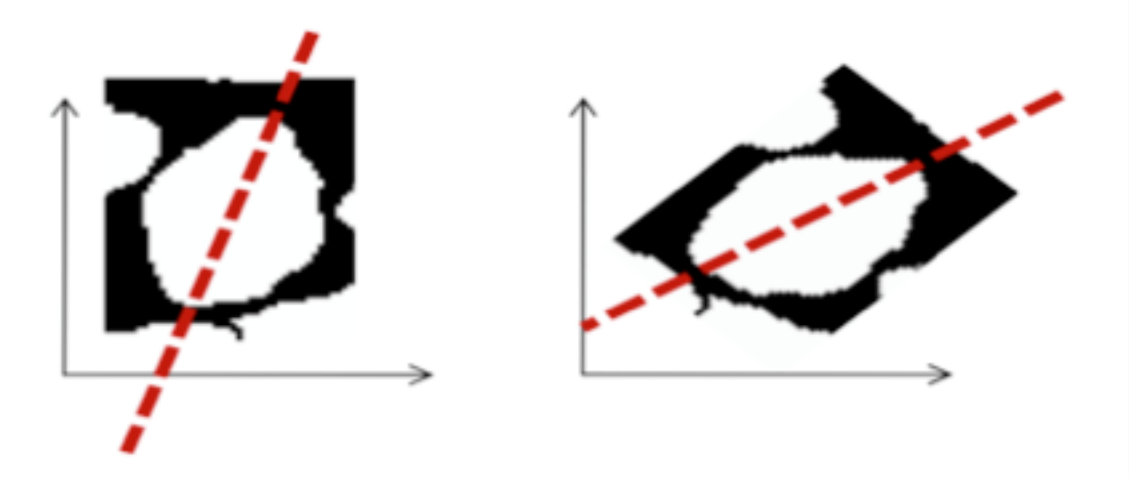


$$
\begin{aligned}
\tan 2\alpha &= \frac{2\sum(r-\bar{r})(c-\bar{c})}{\sum(r-\bar{r})(r-\bar{r})-\sum(c-\bar{c})(c-\bar{c})} \\
&= \frac{\frac{1}{A}2\sum(r-\bar{r})(c-\bar{c})}{\frac{1}{A}\sum(r-\bar{r})(r-\bar{r})-\frac{1}{A}\sum(c-\bar{c})(c-\bar{c})} \\
&= \frac{2\mu_{rc}}{\mu_{rr}-\mu_{cc}}
\end{aligned}
$$

Orientation (5)
: Axis with Least Second Moment

- Invariance to orientation?
  : Need a common alignment



Axis for which the squared distance to 2d object points is minimized

Basic Properties of a Region

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0
2	2	2	2	0	0	0	0	0	1	1	1	1	1	1	0
2	2	2	2	0	0	0	0	1	1	1	1	1	1	1	1
2	2	2	2	0	0	0	0	1	1	1	1	1	1	1	1
2	2	2	2	0	0	0	0	1	1	1	1	1	1	1	1
2	2	2	2	0	0	0	0	0	1	1	1	1	1	1	0
2	2	2	2	0	0	0	0	0	0	1	1	1	1	0	0
2	2	2	2	0	0	0	0	0	0	0	0	0	0	0	0
2	2	2	2	0	0	0	0	0	0	0	0	0	0	0	0
2	2	2	2	0	0	3	3	3	0	0	0	0	0	0	0
2	2	2	2	0	0	3	3	3	0	0	0	0	0	0	0
2	2	2	2	0	0	3	3	3	0	0	0	0	0	0	0
2	2	2	2	0	0	0	0	0	0	0	0	0	0	0	0

region num.	region area	row of center	col of center	perim. length	circu-larity$_1$	circu-larity$_2$	radius mean	radius var.
1	44	6	11.5	21.2	10.2	15.4	3.33	.05
2	48	9	1.5	28	16.3	2.5	3.80	2.28
3	9	13	7	8	7.1	5.8	1.2	0.04

Topological Region Descriptors

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- Topological properties: properties of image preserved under rubber-sheet distortions
  - # holes in the image
  - # connected components

H=2, C=1

H=0, C=3

H=1, C=1　　H=2, C=1