Assignment Number

Problem Statement

Program in C to find the integral of an equation within a given bound using Simpson's 1/3rd rule.

Theory

The *Simpson's* 1/3rd rule is a numerical method to find the integral $\int_{a}^{b} f(x)dx$ within some finite limits a and b. *Simpson's* $1/3^{\text{rd}}$ rule approximates $f(x)$ with a polynomial of degree two $P(x)$, ie. a parabola between the two limits a and b, and then finds the integral of that bounded parabola, and is used to represent the approximate integral $\int_{a}^{b} f(x)dx$. The integral of the approximated function is the area under the parabola bounded by the points a and b and by the positive side of the x axis. The quadratic function has three points common to the function $f(x)$, which are as follows: The end points of the approximate quadratic function $P(x)$ is the same as the function $f(x)$ at a , b. And $P(x)$ takes the same value of the function $f(x)$ at point $m=\frac{(a+b)}{2}$. Thus three points are fixed each in equal interval $a>m>b$ and a parabola is drawn through these three points $f(a), f(m), f(b)$ and the area under the parabola through these points bounded by a and b and the positive side of the X axis is found, which is used as the approximated integral value. The iterative formula on the next page can be used to find the integral of a function $f(x)$ using *Simpson's* $1/3^{\text{rd}}$ rule.

$$I_s = \frac{h}{3} * (y_0 + 4\,y_1 + y_2)$$
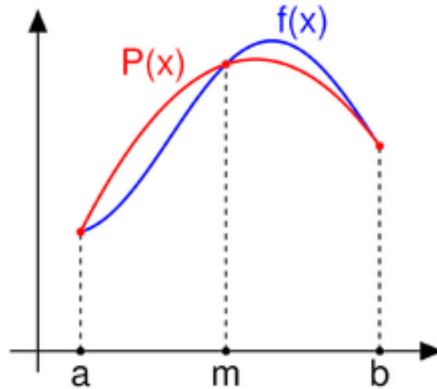
where $y_i = f(x_i)$ and $h = \frac{(x_2 - x_0)}{2}$.

Illustration 1: Simpson's rule can be derived by approximating f(x) by the quadratic function P(x)

The above formula approximates $f(x)$ with only one parabola. To represent the function more accurately with with method, the function is divided in many intervals, say n, having same width say h, such that $h = \frac{(b-a)}{n}$ and then a parabola is fit within each of the n intervals to approximate the integrals of the function within those intervals. The evaluation of the integral within each such interval using the above single interval iterative formula is used to find the overall evaluation of the integral within the limits a and b. This is known as the *Simpson's* 1/3rd *Composite Rule*. The iterative method for the *Simpson's* 1/3rd *Composite Rule* can be found by repetitively applying the single interval formula in each intervals, and is found to be :

$$I_s = \frac{h}{3}\left[(y_0 + y_n) + 2\sum_{i=even} y_i + 4\sum_{j=odd} y_j\right]$$

Algorithm

Input :
1. y = f(x) : the function to integrate
2. a : the lower limit of integration
3. b : the upper limit of integration
4. n : number of intervals

Output : The integral value of the function between the given limit, say S.

Steps :

Step 1 : At first, y = f(x) is defined

Step 2 : Input a

Step 3 : Input b

Step 4 : If b > a

        Then
1. Goto step 5

        Else
2. Goto step 2

Step 5 : Input n

Step 6 : If(n < 1)

        Then
1. Goto step 5

        Else
2. Goto step 7

Step 7 : h = $\dfrac{(b-a)}{n}$

Step 8 : S = f(a) + f(b)

Step 9 : I = 1

Step 10 : If I mod 2 = 1

        Then
1. S = 4 * f(a + I*h)

        Else
2. S = 2 * f(a + I*h)

Step 11 : I = I + 1

Step 12 : If(I <= n – 1)

        Then
1. Goto step 10

Step 13 : $S = \frac{1}{3} * h * S$

Step 14 : Print "Integral value : ", S

Step 15 : End

Source Code

```c
#include <stdio.h>
#include <math.h>

// change the definition of f(x) to evalute different functions
#define f(x) (1/(1+x))

int main(){
    double start, end, numStep;
inputlimit:
    printf("\nEnter lower limit : ");
    scanf("%lf", &start);
    printf("\nEnter upper limit : ");
    scanf("%lf", &end);
    if(end < start)
        goto inputlimit;
inputstep:
    printf("\nEnter number of steps : ");
    scanf("%lf", &numStep);
    if(numStep < 1)
        goto inputstep;

    double delta = (end - start)/numStep, temp = start + delta, sum = f(start)
+f(end);
    int count = 1;
    while(temp < end){
        double y = f(temp);
        if(count % 2 == 1)
            y *= 4;
        else
```

```
        y *= 2;
    sum += y;
    temp += delta;
    count++;
}
sum *= (delta/3);

printf("\nIntegral value : %g", sum);
return 0;
}
```

Input and Output

Set 1 : f(x) = 1/(1+x)
Enter lower limit : 0
Enter upper limit : 5
Enter number of steps : 10
Integral value : 1.79317

Set 2 : f(x) = x/(1+x)
Enter lower limit : 2
Enter upper limit : 3
Enter number of steps : 10
Integral value : 0.712318

Set 3 : f(x) = x*log(x)
Enter lower limit : 2
Enter upper limit : 3
Enter number of steps : 10
Integral value : 2.30746

Discussion

1. Simpson's 1/3rd rule gives correct result for polynomials of degree <= 3.
2. The error in calculation of Simpson's 1/3rd rule is proportional to (b – a)^4, hence for larger bound integrals, the error will be more significant.
3. Due to the implementation, the program has certain limits in terms of precision and range representation of a finite decimal number.