Assignment Number

Problem Statement

Program in C to find the root of a transcendental equation using Bisection Method.

Theory

The bisection method in mathematics is a root-finding method that repeatedly bisects an interval and then selects a subinterval in which a root must lie for further processing. It is a very simple and robust method, but it is also relatively slow. Because of this, it is often used to obtain a rough approximation to a solution which is then used as a starting point for more rapidly converging methods.

The method is applicable for numerically solving the equation $f(x) = 0$ for the real variable x, where f is a continuous function defined on an interval $[a, b]$ and where $f(a)$ and $f(b)$ have opposite signs. In this case a and b are said to bracket a root since, by the intermediate value theorem, the continuous function f must have at least one root in the interval (a, b).

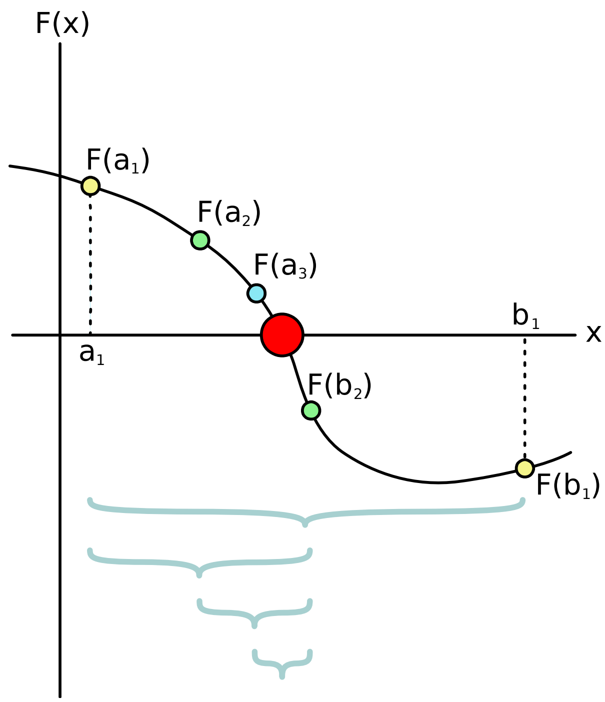
At each step the method divides the interval in two by computing the midpoint $c = (a+b) / 2$ of the interval and the value of the function $f(c)$ at that point. Unless c is itself a root (which is very unlikely, but possible) there are now only two possibilities: either $f(a)$ and $f(c)$ have opposite signs and bracket a root, or $f(c)$ and $f(b)$ have opposite signs and bracket a root. The method selects the subinterval that is guaranteed to be a bracket as the new interval to be used in the next step. In this way an interval that contains a zero of f is reduced in width by 50% at each step. The process is continued until the interval is sufficiently small.

Explicitly, if *f*(*a*) and *f*(*c*) have opposite signs, then the method sets *c* as the new value for *b*, and if *f*(*b*) and *f*(*c*) have opposite signs then the method sets *c* as the new *a*. (If *f*(*c*)=0 then *c* may be taken as the solution and the process stops.) In both cases, the new *f*(*a*) and *f*(*b*) have opposite signs, so the method is applicable to this smaller interval.

Iteration tasks

The input for the method is a continuous function *f*, an interval [*a*, *b*], and the function values *f*(*a*) and *f*(*b*). The function values are of opposite sign (there is at least one zero crossing within the interval). Each iteration performs these steps:

1. Calculate *c*, the midpoint of the interval, $c = a + b/2$.
2. Calculate the function value at the midpoint, *f*(*c*).
3. If convergence is satisfactory (that is, $c - a$ is sufficiently small, or $|f(c)|$ is sufficiently small), return *c* and stop iterating.
4. Examine the sign of *f*(*c*) and replace either (*a*, *f*(*a*)) or (*b*, *f*(*b*)) with (*c*, *f*(*c*)) so that there is a zero crossing within the new interval.



Algorithm

Input :

1. y = f(x), the function to find root of
2. The lower bound of the root, say a
3. The upper bound of the root, say b
4. A predefined small quantity, say EPSILON, which will denote the proximity of two roots to be considered as equal
5. A predefined number of steps to continue the iteration for, say STEPS

Output : The root of the function, say ROOT

Steps :

Step 1 : At first, y = f(x) is defined
Step 2 : Input a
Step 3 : Input b
Step 4 : If(f(b) * f(a) > 0)
Then
1. Goto step 2

Step 5 : prevroot = a
Step 6 : c = (b + a) / 2
Step 7 : If(c – prevroot < EPSILON)
Then
1. ROOT = c
2. Print “Root found : “, ROOT
3. Exit

Step 8 : If(f(b) * f(c) < 0)
Then
1. b = c

Step 9 : Else
1. a = c

Step 10 : prevroot = c
Step 11 : STEPS = STEPS – 1
Step 12 : If(STEPS > 0)
Then

1. Goto step 6

Step 13 : Else

1. Print “Root does not converge in given steps!”
2. Exit

Step 14 : End

Source Code

```
#include <stdio.h>
#include <math.h>

#define f(x) (x*x*x - 3*x*x + 3*x – 1) // the equation is to be defined here
#define EPSILON 0.00000001
#define STEPS 100

int main(){
   double a, b;
   printf("\nEquation : x^3 - 3*x^2 + 3*x - 1");
restart:
   printf("\nEnter initial approximation of the root : ");
   scanf("%lf%lf", &a, &b);
   if(f(a)*f(b) > 0){
      printf("The root does not lie between %g and %g!", a, b);
      goto restart;
   }
   int it = 0;
   double prevroot = a, preva = a, prevb = b;
   printf("\nIteration\t    a    \t   f(a)  \t    b    \t   f(b)  \t    c    \t   f(c)  ");
   printf("\n=========\t=========\t=========\t=========\t=========\
t=========\t=========");
   while(it < STEPS){
      double c = (a+b)/2;
      printf("\n%9d\t", (it+1));
      if(preva != a)
```

```
            printf("*%8lf", a);
        else
            printf("%9lf", a);
        printf("\t%9lf\t", f(a));
        if(prevb != b)
            printf("*%8lf", b);
        else
            printf("%9lf", b);
        printf("\t%9lf\t%9lf\t%9lf", f(b), c, f(c));
        preva = a; prevb = b;
        if(fabs(c-prevroot) < EPSILON){
            printf("\nRoot found : %10lf", prevroot);
            return 0;
        }
        if(f(b) * f(c) < 0){
            a = c;
        }
        else{
            b = c;
        }
        prevroot = c;
        it++;
    }
    printf("\nRoot does not converge in %2d steps!", it);
}
```

Input and Output

Set 1:

Equation : x^3 - 5*x + 3

Enter initial approximation of the root : 1 0

Iteration	a	f(a)	b	f(b)	c	f(c)
========	=========	========	========	=========	========	========
1	1.000000	0.000000	0.000000	-1.000000	0.500000	-0.125000
2	1.000000	0.000000	*0.500000	-0.125000	0.750000	-0.015625
3	1.000000	0.000000	*0.750000	-0.015625	0.875000	-0.001953
4	1.000000	0.000000	*0.875000	-0.001953	0.937500	-0.000244
5	1.000000	0.000000	*0.937500	-0.000244	0.968750	-0.000031
6	1.000000	0.000000	*0.968750	-0.000031	0.984375	-0.000004
7	1.000000	0.000000	*0.984375	-0.000004	0.992188	-0.000000
8	1.000000	0.000000	*0.992188	-0.000000	0.996094	-0.000000
9	1.000000	0.000000	*0.996094	-0.000000	0.998047	-0.000000
10	1.000000	0.000000	*0.998047	-0.000000	0.999023	-0.000000
11	1.000000	0.000000	*0.999023	-0.000000	0.999512	-0.000000
12	1.000000	0.000000	*0.999512	-0.000000	0.999756	-0.000000
13	1.000000	0.000000	*0.999756	-0.000000	0.999878	-0.000000
14	1.000000	0.000000	*0.999878	-0.000000	0.999939	-0.000000
15	1.000000	0.000000	*0.999939	-0.000000	0.999969	-0.000000
16	1.000000	0.000000	*0.999969	-0.000000	0.999985	-0.000000
17	1.000000	0.000000	*0.999985	-0.000000	0.999992	-0.000000
18	1.000000	0.000000	*0.999992	-0.000000	0.999996	0.000000
19	1.000000	0.000000	*0.999996	0.000000	0.999998	0.000000
20	1.000000	0.000000	*0.999998	0.000000	0.999999	0.000000
21	1.000000	0.000000	*0.999999	0.000000	1.000000	0.000000
22	1.000000	0.000000	*1.000000	0.000000	1.000000	0.000000
23	1.000000	0.000000	*1.000000	0.000000	1.000000	0.000000
24	1.000000	0.000000	*1.000000	0.000000	1.000000	0.000000
25	1.000000	0.000000	*1.000000	0.000000	1.000000	0.000000s

Root found :   1.000000

Set 2 :

Equation : x^3 - 5*x + 3

Enter initial approximation of the root : 0 1

Iteration	a	f(a)	b	f(b)	c	f(c)
========	=========	========	========	=========	========	========
1	0.000000	3.000000	1.000000	-1.000000	0.500000	0.625000
2	*0.500000	0.625000	1.000000	-1.000000	0.750000	-0.328125
3	0.500000	0.625000	*0.750000	-0.328125	0.625000	0.119141
4	*0.625000	0.119141	0.750000	-0.328125	0.687500	-0.112549
5	0.625000	0.119141	*0.687500	-0.112549	0.656250	0.001373
6	*0.656250	0.001373	0.687500	-0.112549	0.671875	-0.056080
7	0.656250	0.001373	*0.671875	-0.056080	0.664062	-0.027475
8	0.656250	0.001373	*0.664062	-0.027475	0.660156	-0.013081
9	0.656250	0.001373	*0.660156	-0.013081	0.658203	-0.005861
10	0.656250	0.001373	*0.658203	-0.005861	0.657227	-0.002246
11	0.656250	0.001373	*0.657227	-0.002246	0.656738	-0.000437
12	0.656250	0.001373	*0.656738	-0.000437	0.656494	0.000468
13	*0.656494	0.000468	0.656738	-0.000437	0.656616	0.000016
14	*0.656616	0.000016	0.656738	-0.000437	0.656677	-0.000211
15	0.656616	0.000016	*0.656677	-0.000211	0.656647	-0.000097
16	0.656616	0.000016	*0.656647	-0.000097	0.656631	-0.000041
17	0.656616	0.000016	*0.656631	-0.000041	0.656624	-0.000013
18	0.656616	0.000016	*0.656624	-0.000013	0.656620	0.000002
19	*0.656620	0.000002	0.656624	-0.000013	0.656622	-0.000006
20	0.656620	0.000002	*0.656622	-0.000006	0.656621	-0.000002
21	0.656620	0.000002	*0.656621	-0.000002	0.656621	-0.000000
22	0.656620	0.000002	*0.656621	-0.000000	0.656620	0.000001
23	*0.656620	0.000001	0.656621	-0.000000	0.656620	0.000000
24	*0.656620	0.000000	0.656621	-0.000000	0.656620	-0.000000
25	0.656620	0.000000	*0.656620	-0.000000	0.656620	0.000000
26	*0.656620	0.000000	0.656620	-0.000000	0.656620	0.000000
27	*0.656620	0.000000	0.656620	-0.000000	0.656620	-0.000000

Root found :   0.656620

Discussion

1. Bisection method is the safest and it always converges. The bisection method is the simplest of all other methods and is guaranteed to converge for a continuous function.
2. It is always possible to find the number of steps required for a given accuracy and the new methods can also be developed from bisection method and bisection method plays a very crucial role in computer science research.
3. However, bisection method is less accurate than its companions, hence it is less used where precision plays a major role.