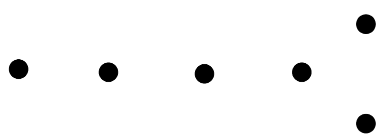
GRAPH THEORY

1. What do you mean by Graph? What is self -loop and parallel-edges? Define Simple graph. What are Trivial Graph, Multi-graph and Pseudo-graph? Define: Directed and Undirected graph.
2. Explain Konigsberg Bridge Problem.
3. Define following terms: Finite and Infinite Graph, Degree/ Valency, In-degree and Out-degree.
4. Prove that “sum of degrees of all vertices in a graph is twice the number of edges in the graph”.
5. Prove that “the number of vertices of odd degree in a graph is always even”.
6. Define: Regular graph, k-regular graph, isolated vertex, pendant vertex and NULL graph.
7. Prove that “the number of vertices in a k-regular graph is even if k is odd”.
8. You are given three vessels A, B and C of capacity 8, 5 and 3 gallons, respectively. A is filled while B and C are empty. Divide the liquid A into two equal quantities. Perform the task using minimum number of stages.
9. Prove that “an infinite graph with finite number of edges must have an infinite number of isolated vertices”.
10. Prove that “an infinite graph with finite number of vertices must have at least one pair of vertices joined by an infinite number of parallel edges or at least one vertex with infinite number of self-loops”.
11. Prove that “maximum degree of any vertex in a simple graph with N vertices is N-1”.
12. Prove that “maximum number of edges in a simple graph with N vertices is N(N-1)/2”.
13. Prove that for any directed graph G = (V, E)

$$\sum_{v \in V} \text{indegree}(v) = \sum_{v \in V} \text{outdegree}(v) = |E|$$

14. What do you mean by degree vector of a simple graph? What is graphical vector?
15. Is [5 4 4 3 3 3 2] graphical vector?
16. Find x if [8 x 7 6 6 5 4 3 3 1 1 1] is a graphical vector.
17. Define: Isomorphism. Show that following two pairs of graphs are not isomorphic.



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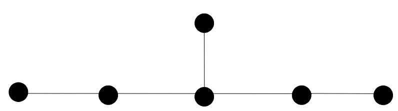

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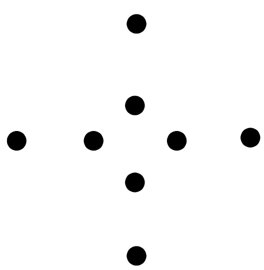

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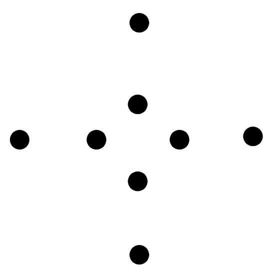

(b)

18. Define: Sub-graph, Super graph, edge-disjoint sub-graph, vertex-disjoint sub-graph, vertex-induced sub-graph, edge-induced sub-graph, walk, path, circuit, connected graph and disconnected graph.
19. Differentiate between Walk, Path and Circuit.
20. Prove that "if a graph has exactly two vertices of odd degrees, there must be a path joining these two vertices".
21. Prove that "a simple graph with n vertices and k components can have at most (n-k)(n-k+1)/2 edges".
22. Define: Euler line, Euler graph, Unicursal line, Unicursal graph.
23. Prove that "a connected graph G is an Euler graph, iff all vertices of G are of even degree".
24. Prove that "a connected graph G is an Euler graph, iff it can be decomposed into circuits".
25. Define with examples: Union, Intersection, Ring sum, Decomposition, Deletion, Fusion, Complementary graph, Self complementary graph.
26. Define: Arbitrarily traceable graph. Prove that "an Euler graph G is arbitrarily traceable from vertex v in G, iff every circuit in G contains v".
27. Define: Hamiltonian Circuit, Hamiltonian Path, Complete graph.
28. Prove that "in a complete graph with n vertices there are (n-1)/2 edge disjoint Hamiltonian circuit, if n is an odd number >=3".
29. Define: Cycle, Wheel, Bipartite graph, Complete Bipartite graph.
30. Find the number of edges in the complete bipartite graph $K_{m, n}$.
31. Prove that "if a bipartite graph G = (X, Y, E) is regular, both X and Y have the same number of elements".
32. Find maximum number of edges in a bipartite graph.
33. Define: Cut vertices/ Articulation point, Cut edge/ Bridge, Cut set.
34. What is planar graph? Prove Euler's formula i.e. r = e – v + 2; where r be the number of regions, v be the vertices and e be the number of edges.
35. Prove that: A full m-ary tree
    i) With i internal vertices contains $n = mi + 1$ vertices.
    ii) n vertices has $i = (n - 1)/m$ internal vertices and $l = [(m - 1)n + 1]/m$ leaves.
    iii) With i internal vertices has $l = (m - 1)i + 1$ leaves.
    iv) With l leaves has $n = (ml - 1)/(m - 1)$ vertices and $i = (l - 1)/(m - 1)$ internal vertices.
    v) With height h, has at most m^h leaves.
36. Define: Tree.
37. Prove that:
    i) There is one and only one path between every pair of vertices in a tree.
    ii) If in a graph G, there is one and only one path between every pair of vertices, then G is a tree.
    iii) A tree with n vertices has n – 1 edge.
    iv) Any connected graph with n vertices and n – 1 edge is a tree.
    v) A graph is tree iff it is minimally connected.
    vi) A graph G with n vertices, n-1 edge and no circuit is connected.
    vii) In any tree with two or more vertices, there are at least two pendant vertices.

38. Define: Distance between two vertices, metric, eccentricity, radius, diameter and center.
39. Prove that distance is metric.
40. Prove that a tree has either one or two centers.
41. Define: rooted tree and binary tree.
42. Prove that in a binary tree with ***n*** vertices
    i) ***n*** is always odd.
    ii) If ***p*** be the number of pendant vertices then $p = (n + 1)/2$.
    iii) Minimum level $l_{min} = \log_2 (n + 1) - 1$ and maximum level $l_{max} = (n - 1)/2$.

43. What do you mean by weighted path length? Define: spanning tree and minimum spanning tree.
44. Prove that every connected graph has at least one spanning tree.
45. What do you mean by branch and chord? Define: rank, nullity/cyclomatic number.
46. What is the nullity of a complete graph with n vertices?
47. What do you mean by distance between two spanning tree T_i and T_j?
48. Define: Adjacency matrix, adjacency list, circuit matrix, path matrix and incidence matrix.
49. Suggest three different techniques to represent a graph in memory with advantages and disadvantages of each.
50. Write steps to convert an incidence matrix to adjacency matrix and vice versa with example.
51. What is directed graph?
52. Suppose A is the adjacency matrix of a graph G and $Br = A + A^2 + \ldots.. + A^r$. Then the ij-th entry of the matrix Br gives the number of paths of length r or less from vertex v_i to v_j.

53. Write algorithm to count number of pendant vertices, number of isolated vertices, degree of each vertices, number of parallel edges and self loop (if applicable) from adjacency matrix and incidence matrix.
54. Write algorithm for the following and illustrate with example:
    i) Prim's algorithm to find MST.
    ii) Kruskal's algorithm to find MST.
    iii) Dijkstra's shortest path algorithm.
    iv) Floyd's all pair shortest path algorithm.
    v) Warshall's all pair reachability algorithm.
    vi) BFS graph traversal technique.
    vii) DFS graph traversal technique.
55. Write an algorithm to determine a graph is connected or not. If disconnected then also find the number of components in the graph.