Numerical Analysis

Errors and Approximation

1. What is absolute error, relative error and percentage error? **[CU 2002 2014]**
2. State the relation between significant digits and accuracy as well as precision. **[CU 2011]**
3. If $f(x) = 4\cos x - 6x$, find the relative percentage error in $f(x)$ for $x = 0$, if error in $x = 0.005$. **[CU 2012]**
4. If $y = 4x^6 - 5x$, then find the percentage error in y at $x = 1$, if the error in $x = 0.04$. **[CU 2009]**
5. If $\Delta x = 0.005, \Delta y = 0.002$ be the absolute errors in $x = 3.41$ and $y = 7.43$, find the relative error in computing the value of $x + y$. **[CU 2010]**

Calculus of finite differences

6. Define the terms – arguments and entries.
7. What do you mean by forward differences and backward differences for equidistant arguments?
8. Consider a set of values of a function f(x) as follows. Construct the forward difference table

(i)

x	0	1	2	3	4
$f(x)$	2	3	12	35	78

(ii)

x	0	1	2	3	4	5
$f(x)$	1	5	31	121	341	781

[CU 2013]

9. Show that, forward difference operator (Δ) is commutative with respect to a constant.
10. Prove that $\Delta\nabla = \Delta - \nabla$. **[SU 2016]**
11. State the fundamental theorem of Difference Calculus. Justify the theorem for the function $y = f(x) = x^4 + 2x^2 + 1$ for x = 0, 1, 2, 3, 4.
12. Define shift operator for an arbitrary function $f(x)$. Show that E. Δ = Δ.E
13. Show that $E = 1 + \Delta$
14. Derive the relation between difference operator Δ and $D = \frac{d}{dx}$ of differential calculus.
15. Find the polynomial $f(x)$ which satisfies the following data sets.

x	1	2	3	4	5
$f(x)$	4	13	34	73	136

x	-1	0	1	2
$f(x)$	1	1	1	-5

x	0	1	2	3	4	5
$f(x)$	1	5	31	121	341	781

x	0	1	2	3
$f(x)$	1	2	11	34

x	2	4	6	8
$f(x)$	5	10	17	29

16. Find the missing terms(s) from the following data:

i)

x:	0	1	2	3	4
$f(x)$:	1	3	9	-	81

ii)

x:	0	1	2	3	4	5
$f(x)$:	0	-	8	15	-	35

iii)

x:	0	5	10	15	20	25
$f(x)$:	6	10	-	17	-	31

[SU 2017]

17. Show that,

i) $\Delta^n[ke^{ax}] = k(e^{ah} - 1)^n e^{ax}$

ii) $\Delta \log f(x) = \log\left[1 + \frac{\Delta f(x)}{f(x)}\right]$

Numerical Interpolation – for equidistant & non-equidistant arguments

18. Differentiate between the terms – Interpolation and Extrapolation.

19. State Weistrass Theorem for polynomial interpolation.

20. Derive Newton's forward interpolation formula for $n + 1$ equidistant arguments.

21. State when Newton's forward interpolation technique cannot be used for computing the function f(x).

[CU 2009]

22. Derive Newton's backward interpolation formula for $n + 1$ equidistant arguments.

23. Derive Lagrange's polynomial to the nth order. **[CU 2014]**

24. Consider the following functional values for finding $f(2.5)$ and $f(19.0)$ using Newton's forward and backward interpolation formula respectively

$$f(0) = 1.0, f(5) = 1.6, f(10) = 3.8, f(15) = 8.2, f(20) = 15.4$$

25. The values of $\sin x$ are given below, for different values of x. From them find $\sin 32^\circ$

x:	30°	35°	40°	45°	50°	55°
$sinx$:	0.5000	0.5736	0.6428	0.7071	0.7660	0.8192

26. Find $y = e^{2x}$ for $x = 0.05$ and $x = 0.37$ using the given values.

x:	0.0	0.1	0.2	0.3	0.4
$f(x)$:	1.0000	1.2214	1.4918	1.8221	2.2255

27. Use the following table to find (i) $\log_{10}2.02$, (ii) $\log_{10}2.25$ and (iii) $\log_{10}2.91$. **[CU 2010 SU 2016]**

x:	2.0	2.2	2.4	2.6	2.8	3.0
$log_{10}x$	0.30103	0.34242	0.38021	0.41497	0.44716	0.47721

28. In an examination the number of candidate who secured marks between certain limit were as follows:

Marks	0 – 19	20 – 39	40 – 59	60 – 79	80 – 99
No of students	41	62	65	50	17

Estimate the number of candidates getting marks less than 70.

29. Show that the sum of Lagrangian function or coefficients is unity i.e. $\sum_{r=0}^{n} w(x) = 1$

30. Given the following table, find $f(x)$ assuming it to be a polynomial of degree three in x. **[CU 2010 SU 2016]**

x	0	1	2	3
$f(x)$	1	2	11	34

31. Given,

x_i	1	2	5	9
f_i	1	3	6	10

Compute $f(6)$ by Lagrange's interpolation technique. **[CU 2003]**

32.Compute $f(1.38)$ and $f(1.42)$ from the table by using suitable interpolation technique. **[CU 2009]**

X	1.1	1.2	1.3	1.4
f(x)	7.831	8.728	9.697	10.744

33.By suitable interpolation method, find f(2.5) from the following – **[CU 2002 SU 2016]**

x	2	3	4	5
$f(x)$	14.5	16.3	17.5	18.0

34.What do you mean by divided difference?

35.Show that the divided difference of a function $y = f(x)$ for equispaced arguments is

$$\delta(x_1, x_2, \dots, x_n) = \frac{\Delta^n y_0}{n!\,h^n}$$

36.State Newton's divided difference interpolation formula. Compute the value of $y = 0.72$ from the following table.

x	0.62	0.68	0.70	0.73	0.75
y	0.66042	0.73363	0.758584	0.796584	0.82232

Numerical Integration

37.Define the term – quadrature. What do you mean by error of approximation in Numerical Integration?

38.Derive Gauss-Legendre general quadrature formula for equidistant ordinates.

39.Derive Trapezoidal formula for numerical integration. Hence derive composite Trapezoidal rule. By drawing a suitable diagram explain the geometrical interpretation for Trapezoidal rule.

40.Derive Simpson's $\frac{1}{3}$rd formula for numerical integration. Hence derive composite Simpson's $\frac{1}{3}$rd rule.

[CU 2002 2004 2010 SU 2016]

41.By drawing a suitable diagram explain the geometrical interpretation for Simpson's $\frac{1}{3}$rd rule.

42."Simpson's 1/3rd rule gives exact value for a polynomial of degree ≤ 3" – Justify. **[CU 2010]**

43.Write an algorithm to describe –

a) Trapezoidal rule

b) Simpson's 1/3rd rule **[CU 2007 2013]**

44.Evaluate the following integrals using (i) Trapezoidal rule, (ii) Simpson's $\frac{1}{3}$rd rule and (iii) Weddle's rule by taking suitable intervals. Compute the exact value and comment on the absolute error and relative errors in each case.

i) $\int_0^1 (4x - 3x^2)dx$ **[CU 2011]**

ii) $\int_0^5 \frac{dx}{1+x}$

iii) $\int_0^1 \frac{xdx}{1+x}$

iv) $\int_0^1 \cos x \, dx$

v) $\int_0^1 \sqrt{1 - x^3}dx$ **[CU 2007]**

vi) $\int_0^{\frac{\pi}{2}} \sqrt{sinx}dx$

vii) $\int_0^{0.6} e^x dx$

viii) $\int_0^{\frac{\pi}{2}} e^{\sin x} dx$

ix) $\int_1^5 log_{10}xdx$

x) $\int_1^{1.6} \ln x \, dx$

xi) $\int_0^1 \frac{dx}{1+x^2}$ **[CU 2002 SU 2017]**

Hence find the value of π

xii) $\int_{0.1}^{0.7} (e^x + 2x)dx$

xiii) $\int_0^1 \frac{x^2+2}{x^2+1} dx$

xiv) $\int_0^4 \frac{tan^{-1}x}{1+x^2} dx$

Solution to algebraic and transcendental equations

45. Differentiate between an algebraic equation and a transcendental equation.

46. How do we determine the location or crude approximation of roots of a given equation

47. Write an algorithm for finding a root of an equation using Bisection method.

48. Find one root from each of the following equations correct up to 4 decimal places using method of Bisection.

i) $x^3 - 9x + 1 = 0$ **[CU 2009 SU 2016]**

ii) $10^x + sinx + 2x = 0$

iii) $2x - 3\,sinx - 5 = 0$

iv) $x + lnx - 2 = 0$

v) $e^x - 3x = 0$

vi) $\tan x + x = 0$

vii) $3x^2 + 5x - 40 = 0$

viii) $2x - 3\,sinx - 5 = 0$

49. Derive Newton-Raphson formula for solving an equation. Also state the convergence of this method.

[CU 2007 SU 2017]

50. By drawing a suitable diagram explain the geometrical interpretation of Newton-Raphson method.

[CU 2007]

51. State when N-R method fails? State the amount of error of this method. **[CU 2007]**

52. Use N-R method to find p^{th} root of a real number, say R. **[CU 2005]**

53. Why N-R method is called *'the method of tangents'?* **[CU 2005]**

54. Find out one root for each of the following equations correct up to 4 decimal places, using N-R method.

i) $x^3 - 4x - 1 = 0$ **[CU 2001]**

ii) $e^{-x} - x = 0$ **[CU 2011]**

iii) $2x - log_{10}x - 7 = 0$ **[CU 2002]**

iv) $x^2 + 2x - 2 = 0$

v) $3x - \cos x - 1 = 0$

vi) $x^x + x - 4 = 0$

vii) $10^x + x - 4 = 0$

viii) $e^x - 3x = 0$

ix) $x^2 + 4sinx = 0$

55. Use Hero's method to compute the following.

i) $\sqrt{27}$

ii) $\sqrt{100}$ **[CU 2005]**

iii) $\sqrt[7]{125}$

iv) $\sqrt[7]{5}$

v) $\sqrt[4]{87}$

56. By drawing a suitable diagram explain the geometrical significance of Regula-Falsi method for finding a root of an equation.

57. Comment on the convergence of Regula-Falsi method.

58. Write an algorithm to solve an equation using –

a) Newton-Raphson method

b) Regula-falsi method

c) Secant method

d) Bisection method.

59. Give the geometrical interpretation of secant method to find the root of an equation $f(x) = 0$. **[CU 2011]**

Solution of system of linear equations

60. What do you mean by a system of linear equation? When such a system is said to be homogeneous and non-homogeneous respectively?

61. When a system of linear equation is said to be diagonally dominant? When it is strictly diagonally dominant? **[SU 2017]**

62. Comment on the possibilities of the solutions of systems of linear equation.

63. Write an algorithm to solve a system of linear equations by

i) Gauss-Elimination method **[CU 2006]**

ii) Gauss-Jordan's method

iii) Gauss-Seidel's method

iv) Gauss-Jacobi's method

64. Solve the following system of linear equations by Gauss-Elimination method and Gauss-Jordan's method.

i) $x + 3y + 2z = 5$

$2x - y + z = -1$

$x + 2y + 3z = 2$

ii) $2x + 2y + 4z = 18$

$x + 3y + 2z = 13$

$3x + y + 3z = 14$ **[CU 2006 2013]**

iii) $8x + 2y - 2z = 8$

$x - 8y + 3z = -4$

$2x + y + 9z = 12$

iv) $4x + y + 2z = 16$

$x + 3y + z = 10$

$x + 2y + 5z = 12$ **[CU 2007]**

v) $10x - 7y + 3z + 5u = 6$

$-6x + 8y - z - 4u = 5$

$3x + y + 4z + 11u = 2$

$5x - 9y - 2z + 4u = 7$ **[CU 2011 SU 2016]**

vi) $3x + 4y + 2z = 15$

$5x + 2y + z = 18$

$2x + 3y + 2z = 10$

65. Solve the following system of linear equations using Gauss-Seidel's method and Gauss-Jacobi's method.

i) $12x - y + 2z = 23.78$

$x + 4y + 7z = 17.72$

$2x + 9y - z = -20.23$

ii) $5x + 3y + z = 2$

$4x + 10y + 4z = -4$

$2x + 3y + 8z = 20$

iii) $3x + y + z = 7$

$2x + y + 5z = 13$

$x + 4y + z = 9.4$

66. Find the inverse of the following matrices by using Gauss-Elimination and Gauss-Jordan's method

i) $\begin{bmatrix} 2 & 3 & -1 \\ 4 & 4 & -3 \\ 2 & -3 & 1 \end{bmatrix}$

ii) $\begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$

iii) $\begin{bmatrix} 2 & 0 & 1 \\ 3 & 2 & 5 \\ 1 & -1 & 0 \end{bmatrix}$ **[CU 2008]**

iv) $\begin{bmatrix} 1 & 2 & 6 \\ 2 & 5 & 15 \\ 6 & 15 & 46 \end{bmatrix}$

Solution of differential equations

67. Write an algorithm to find the solution of a differential equation using –

i) Runge-Kutta method of 2nd order

ii) Runge-Kutta method of 4th order

iii) Euler's method

68. Solve the following differential equations using Euler's method.

i) $\frac{dy}{dx} = x^2 + y^2, y(0) = 0. Find\ y(0.15)$

ii) $\frac{dy}{dx} = -\frac{y}{1+x}, y(0.3) = 2. Find\ y(1)$

iii) $\frac{dy}{dx} = 2xy, y(0) = 0.5. Find\ y(1)$ **[CU 2009 SU 2016]**

iv) $\frac{dy}{dx} = -xy, y(0) = 1. Find\ y(0.25)$ **[CU 2006]**

v) $\frac{dy}{dx} = \frac{x+y}{2}, y(0) = 2. Find\ y(2)$

69.Write the computational formula for 4th order Runge-Kutta method. What is the order of error in this procedure? **[CU 2012]**

70.Solve the following differential equations using Modified Euler's method, Runge-Kutta 2nd order and Runge-Kutta 4th order method.

i) $\frac{dy}{dx} = 1 - \frac{y}{x}, y(2) = 2. Find\ y(2.1)$

ii) $\frac{dy}{dx} = x + y, y(0) = 1. Find\ y(0.4)$ **[CU 2013]**

iii) $\frac{dy}{dx} = -xy, y(0) = 1. Find\ y(0.2)$ **[CU 2002]**

iv) $\frac{dy}{dx} = xy, y(0) = 2. Find\ y(0.8)$ **[CU 2010 2012]**

v) $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}, y(0) = 1\ at\ x\ 0.2.$ **[SU 2017]**

vi) $\frac{dy}{dx} = x + y, y\ (0) = 1. Find\ y\ at\ x = 0.2$ **[SU 2016]**

71.Use the Runge-Kutta method of 4th order to calculate $y(0.2)$ for the equation $\frac{dy}{dx} = \frac{t-y}{2}$ with $y(0) = 1.$

[CU 2008]

72.State the disadvantage of using Taylor's method? **[CU 2011]**

73.Find $y(0.1)$ for $\frac{dy}{dx} = 1 + xy$ with $y(0) = 1$ using Taylor's series method

74.Solve the equation $y' = x^2 + y^2, y(0) = 0$ using Taylor's method for the interval $(0, 0.4)$ using two sub-interval of size 0.2. **[CU 2011]**

75.Find $y(0.1)$ for $\frac{dy}{dx} = x - y^2$ with $y(0) = 1$ using Taylor's series method.

Method of least squares and curve fitting

76.Obtain linear regression of y on x for the equation y = $a_1x + a_0$ and derive the regression coefficients **OR,** derive the normal equations for estimating a and b for fitting a straight line of the form $y = ax + b$.

[CU 2012]

77.Derive the normal equations for estimating a, b and c for fitting a parabola of the form $y = ax^2 + bx + c$

78.Consider a set of points {(1, 2), (2, 3), (4, 1), (5, 2)}. Fit a straight line of the form $y = ax + b$. **[CU 2013]**

79. Fit a straight line of the form $y = ax + b$ for the following points $(0, 3), (1, 1), (2, 0), (4, 1), (6, 4)$

[CU 2006]

80.Fit a parabola using the method of least squares with the points (-3, 3), (0, 1), (2, 1), (4, 3)

[CU 2008 2009 SU 2016]

81.Fit a straight line of the form $y = ax + b$ for the following data.

x:	0	5	10	15	20	25	30
y:	10	14	19	25	31	36	39