

Problem Set #1

1 Static Hopenhayn model

Firms discount flow profits according to a constant discount factor $0 < \beta < 1$. There is an unlimited number of potential entrants. On paying an entry cost $k_e > 0$ an entrant receives a once-and-for-all productivity draw $z \sim g(z)$ and then makes a once-and-for-all decision to operate or exit. On paying a fixed operating cost $k > 0$, a firm that hires n workers can produce output $y = zn^\alpha$ for $0 < \alpha < 1$. Let w denote the wage and p the price of their output. Let $w = 1$ be the numeraire.

1. Let $n(z; p)$, $y(z; p)$ and $\pi(z; p)$ denote the optimal employment policy, output, and profits of a firm with productivity z when the price is p . Solve for these functions. Let $z^*(p)$ denote the lowest level of productivity such that a firm does not exit. Solve for $z^*(p)$. How does z^* depend on p ? Explain.
2. Let $v(z; p)$ denote the value function of a firm. Solve for $v(z; p)$.
3. Use the free-entry condition and the cutoff productivity condition to derive the comparative statics of z^* and p^* with respect to k , k_e and α . Give intuition for your results.
4. Now suppose that productivity is drawn from the Pareto distribution with density

$$g(z) = \xi z^{-\xi-1}, \quad z \geq 1, \quad \xi > 1$$

Solve explicitly for z^* and p^* . How do z^* and p^* depend on the shape parameter ξ ? What is the productivity distribution of actively producing firms? Explain.

2 Numerical implementation of Hopenhayn-Rogerson (1993)

In this assignment you will numerically implement the Hopenhayn/Rogerson model discussed in class. You should feel free to use whichever programming language best suits you.

Model sketch: Please refer to the lecture notes for details; a brief sketch is provided here. Every period the representative household decides how much to consume and how much labor to supply to the market:

$$\max_{\{C, N\}} \theta \ln C - N \quad \text{s.t.} \quad pC = N + \Pi + T,$$

where p is the price of the final good. The numeraire is the wage, normalized to 1. The FOCs imply $C = \frac{\theta}{p}$, N follows from the budget constraint.

Incumbent firms produce the final good according to $y = zn^\alpha$, where n denotes the labor hired by the firm. They are subject to a per-period fixed operating cost c_f and a firing tax $g(n_t, n_{t-1}) = \tau \max\{0, n_{t-1} - n_t\}$. The productivity, z , follows an AR(1):

$$\log z_t = \mu(1 - \rho) + \rho \log z_{t-1} + \sigma \varepsilon_t \quad \varepsilon \sim N(0, 1).$$

At the beginning of each period, before the realization of z , incumbents decide whether to continue operations or exit. If the firm decides to exit, it receives $-g(0, n')$ in this period and 0 thereafter. The value function of incumbent reads:

$$V(z, n; p) = \max\{pz(n')^\alpha - n' - pc_f - g(n', n) + \beta \max[-g(0, n'), \mathbb{E}_{z'} V(z', n')]\},$$

and the wage, w , is normalized to one. Denote the firm's policy function as $n' = n^d(z, n)$ and $\chi = \{0, 1\}$, where $\chi = 1$ denotes the exit decision.

Potential entrants are ex-ante identical. They must pay an entry cost, c_e , and start producing in the next period. They draw their productivity from the distribution $G(z)$. Assume, for simplicity, that G corresponds to the invariant distribution implied by the AR(1) process.

Let $m \geq 0$ be the mass of entrants, the free-entry condition implies that in equilibrium:

$$V^e(p) = \beta \int V(z, 0; p) dG(z) - c_e \leq 0$$

with equality whenever the mass of entrants is positive, $m > 0$. Markets for the final good and labor clear.

We solve for the stationary recursive competitive equilibrium of the model.

Useful resource: You may want to consult the codes I provided to solve the Hopenhayn (1992) model.

Tasks:

1. Outline a computational algorithm to find the stationary recursive competitive equilibrium of the model. Comment: In what way is solving this model harder than solving the Hopenhayn (1992) model?
2. Solve the model on the computer using the following parameter values:
 - Discretize the productivity process using a Tauchen algorithm with 33 gridpoints and a hyperparameter equal to 6.
 - Discretize the employment state using a grid with 500 points.
 - Solve the model for $\tau = 0.2$, then calculate and report the following moments: (i) average firm size, (ii) exit/entry rate, (iii) job destruction and job creation rates.
3. Solve the model again for no tax, $\tau = 0$ and a high firing tax, $\tau = 0.5$. Let Y/N denote aggregate labor productivity (output per worker). Calculate it for each level of τ and explain your results. Why does misallocation arise in this model?