What is Logic?

- Modern logic - Reasoning with mathematical language. We use this language of logic to structure our thinking and to produce precise solutions to problems.
- This mathematical language is simpler and more precise than English.
- Aristotle developed early logic - concerned with syllogisms, what we would probably call *valid arguments*. His theory did not benefit from the notation that simplifies modern logic.

Definition: A **proposition** or **statement** is a sentence which is either true or false.

Definition: An **argument** consists of a sequence of statements called premises, and an additional statement, which is called the conclusion. In order for the argument to be **valid**, the conclusion must be true whenever the premises are true.

Example:
I will go to work or to a movie today. (premise)
I will not go to work today. (premise)
Therefore I will go to a movie today. (conclusion)

proposition P: I will go to work today.
proposition Q: I will go to a movie today.

Rewritten argument:
P or Q
not Q
Therefore P

This rule of inference is called **disjunctive syllogism**. It holds for any statements P and Q.

- In the 1800s, De Morgan and Boole began to think of logic in more mathematical terms. Their work led to what we call formal logic or symbolic logic, the study of logic using mathematical techniques.

Using logic to resolve practical issues

Example: When Pluto was discovered in 1930, it was classified as a planet. There were questions about this classification though. Perhaps Pluto should be a comet or asteroid instead? Different answers are produced depending on how "a planet" is defined and also depending on logic. [1]

One definition: Planets are larger than moons.

From this, we can get the following implication:
If a celestial object O is a planet, (premise P)
then O must be larger than every moon. (conclusion C)

This implication "if P then C" follows from the definition. The **contrapositive** statement is "if not C, then not P", and in this case says:
If a celestial object O is not larger than every moon, then O is not a planet.

Note: A statement and its contrapositive are logically equivalent. (More on this later).

[1] *Foundations of Logic and Mathematics* by Nievergelt

New developments regarding Pluto: In 1978, measurements revealed that Pluto is smaller than the moon, so by the contrapositive statement, Pluto is not a planet.

But many books classify Mercury as a planet, and Ganymeade (a moon of Jupiter) is larger than Mercury. So our implication (and our first definition) are false, so maybe Pluto can remain a planet?

This issue is dependent not just on logic, but also on the relevant definitions.