Candidate Key?

A **candidate key** is a **minimal set of attributes** that can uniquely identify all other attributes in a relation.

You need to consider **all combinations of attributes**, not just those on the **left-hand side of the FDs**, because:

- The **candidate key** may **not appear directly** on the left side of any FD.
- What matters is whether the **closure of a set of attributes** gives **all attributes** in the relation.

Why?

Functional dependencies tell us **how attributes depend on each other** —
but **candidate keys** are sets of attributes from which **you can determine the entire relation**.

1: List all attributes in the relation

Example:

R(A, B, C, D, E)

2: Identify all attributes that are dependent

→ These appear only on the **right-hand side** of FDs and **never on the left-hand side**.

These are **non-key attributes** — they can be **determined by others**, so they are **not essential** to uniquely identify tuples.

3: Identify possible starting points (attributes not determined by others)

Attributes that do not appear on the right-hand side of any FD are likely part of the candidate key, because they are not derived from anything else.

4: Try combinations of attributes using attribute closure

For each set of attributes:

- Compute its **closure** (X^+)
- If X^+ = all attributes in R → **X is a superkey**
- If X is **minimal** (no attribute can be removed) → **X is a candidate key**

Problem 1

Given Relation:

R(A, B, C, D, E)

Given Functional Dependencies (FDs):

1. $A \rightarrow B$
2. $B \rightarrow C$
3. $CD \rightarrow E$
4. $E \rightarrow A$

Find all candidate keys of the relation R using **attribute closure**.

Attribute Closure

To find a candidate key, we pick a set of attributes and compute its **closure**.

The **closure of X**, denoted X^+, is the set of all attributes functionally determined by X using the FDs.

If X^+ = all attributes in R, then X is a **superkey**.

If X is also **minimal**, it's a **candidate key**.

Let's try attribute closures of different combinations.

A^+

$A \rightarrow B$

$B \rightarrow C$

$E \rightarrow A$

$CD \rightarrow E$

Now compute **A^+**:

1. $A \rightarrow B \Rightarrow$ add B

2. $B \rightarrow C \Rightarrow$ add C
3. (A, B, C) now, but not D or E
   → So **A^+ = {A, B, C}** Not all attributes → Not a superkey

Compute B^+

Start with:

$B^+ = \{B\}$

Use FDs:

- $B \rightarrow C \Rightarrow$ Add C

Now:

$B^+ = \{B, C\}$

No more FDs apply.

$B^+ = \{B, C\} \rightarrow$ Not a superkey

Compute E^+

Start with:

$E^+ = \{E\}$

Use FDs:

- $E \rightarrow A \Rightarrow$ Add A
- $A \rightarrow B \Rightarrow$ Add B
- $B \rightarrow C \Rightarrow$ Add C

Now:

$E^+ = \{E, A, B, C\}$ Still missing D So:

$E^+ = \{E, A, B, C\}$

→ Not a superkey

Try CD^+

1. $CD \rightarrow E \Rightarrow$ add E
2. $E \rightarrow A \Rightarrow$ add A
3. $A \rightarrow B \Rightarrow$ add B
4. $B \rightarrow C \Rightarrow$ C already there

Now we have:

$CD^+ = \{C, D, E, A, B\}$ = All attributes ⇒ Superkey

Is it **minimal**? Let's test removing one attribute:

- Try **C alone**:
  - $C \rightarrow$ nothing $\Rightarrow C^+ = \{C\}$
- Try **D alone**:
  - $D \rightarrow$ nothing $\Rightarrow D^+ = \{D\}$

So **CD is minimal ⇒ Candidate Key**

Candidate Key(s):

CD

- **elation:** R(P, Q, R, S, T)
- **Functional Dependencies (FDs):**
- 1. $P \rightarrow Q$

- 2. Q → R
- 3. PR → S
- 4. S → T

Problem 2

The relation has these 5 attributes:

{P, Q, R, S, T}

We need to find a **minimal set of attribute(s)** that can determine **all 5 attributes** using the given FDs.

Try attribute closure of P

Compute **P^+** (closure of {P}):

$P^+ = \{P\}$

Now apply FDs:

- **P** → **Q** ⇒ add Q
  → {P, Q}
- **Q** → **R** ⇒ add R
  → {P, Q, R}
- **PR** → **S**: P and R are present ⇒ add S
  → {P, Q, R, S}
- **S** → **T** ⇒ add T
  → {P, Q, R, S, T} o: $P^+ = \{P, Q, R, S, T\}$ → all attributes This means **P is a superkey**.

Now check if it's **minimal**:

- P is a **single attribute** → can't remove anything.

P is a candidate key

Check for other candidate keys

Let's test other combinations to be sure there's no second candidate key.

Try Q^+

Start: {Q}

- **Q** → **R** ⇒ add R
  → {Q, R}
- No further FDs apply

$Q^+ \neq$ all attributes

Try S^+

Start: {S}

- **S** → **T** ⇒ {S, T}
  → No other FDs apply

$S^+ \neq$ all attributes

Try R^+ or T^+

- $R^+ = \{R\}$
- $T^+ = \{T\}$

No FDs begin with R or T → no useful closure
Not superkeys

Try PR⁺

Start: {P, R}

- **P → Q** ⇒ add Q
  → {P, R, Q}
- **PR → S** ⇒ add S
  → {P, R, Q, S}
- **Q → R** (R already present)
- **S → T** ⇒ add T
  → {P, R, Q, S, T}

PR^+ = all attributes
But **P alone** is already a candidate key
So **PR is not minimal , it is a super key.** Not a candidate key

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Problem 3

Relation:

R(A, B, C, D, E, F)

Functional Dependencies (FDs):

1. A → B
2. B → C
3. CD → E
4. E → F
5. F → A

Find all candidate keys for the relation R(A, B, C, D, E, F) using the attribute closure method.

1: List all attributes

R={A,B,C,D,E,F}R = \{A, B, C, D, E, F\}R={A,B,C,D,E,F}

Step 2: Identify dependent attributes (attributes appearing only on RHS)

- RHS attributes:
  - B (in 1)
  - C (in 2)
  - E (in 3)
  - F (in 4)
  - A (in 5)
  - Attributes never appearing on RHS: D

Dependent attributes={A,B,C,E,F},Independent attribute={D}

3: Attributes not on RHS must be part of every candidate key

- So every candidate key contains D.

Step 4: Find minimal sets containing D whose closure covers all attributes

We test combinations with D

Check D^+:

- Start: $\{D\}$
- No FD with LHS = D alone
- Closure = $\{D\}$ only → not a superkey

Check CD^+:

- Start: $\{C, D\}$
- $CD \rightarrow E$ → add E
- $E \rightarrow F$ → add F
- $F \rightarrow A$ → add A
- $A \rightarrow B$ → add B
- $B \rightarrow C$ C already in closure

Final: $\{A, B, C, D, E, F\}$ → superkey

Minimality:

- Remove C: $D^+ = \{D\}$ no
- Remove D: $C^+ = \{C\}$ no

Conclusion: CD is a candidate key.

Check DE^+:

- Start: $\{D, E\}$
- $E \to F$ → add F
- $F \to A$ → add A
- $A \to B$ → add B
- $B \to C$ → add C

Final: $\{A, B, C, D, E, F\}$ → superkey

Minimality:

- Remove D: $E^+ = \{E, F, A, B, C\}$ no D
- Remove E: $D^+ = \{D\}$ no

Conclusion: DE is a candidate key.

Check AD^+:

- Start: $\{A, D\}$
- $A \to B$ → add B
- $B \to C$ → add C
- $CD \to E$, we have C and D → add E
- $E \to F$ → add F

Final: $\{A, B, C, D, E, F\}$ → superkey

Minimality:

- Remove A: $D^+ = \{D\}$ no
- Remove D: $A^+ = \{A, B, C\}$ no D, E, F

Conclusion: AD is a candidate key.

Check BD^+:

- Start: $\{B, D\}$
- $B \to C$ → add C
- $CD \to E$, have C and D → add E
- $E \to F$ → add F
- $F \to A$ → add A
- $A \to B$, B already present

Final: $\{A, B, C, D, E, F\}$ → superkey

Minimality:

- Remove B: $D^+ = \{D\}$ no
- Remove D: $B^+ = \{B, C\}$ no D, E, F, A

Conclusion: BD is a candidate key.

Check FD^+:

- Start: $\{F, D\}$
- $F \to A$ → add A
- $A \to B$ → add B
- $B \to C$ → add C
- $CD \to E$, have C and D → add E
- $E \to F$, F already in closure

Final: $\{A, B, C, D, E, F\}$ → superkey

Minimality:

- Remove F: $D^+ = \{D\}$ no
- Remove D: $F^+ = \{F, A, B, C\}$ no D, E

Conclusion: FD is a candidate key.

Check single attributes without D:

- $A^+ = \{A, B, C\}$ no
- $B^+ = \{B, C\}$ no
- $E^+ = \{E, F, A, B, C\}$ no D
- $F^+ = \{F, A, B, C\}$ no D, E

None of these are keys.

Final answer:

The **candidate keys** for the relation R are:

$$\boxed{\{CD, DE, AD, BD, FD\}}$$

Step 1 – Problem Setup

Let relation **R(A, B, C, D, E)** have the following **Functional Dependencies (FDs):**

1. **A → B**
2. **B → C**
3. **A → D**
4. **D → E**

Step 2 – Find Candidate Key Using Attribute Closure

Step 2.1 – Start with possible key attributes
We check which attributes can determine all others.

Closure of {A}:

- Start: {A}
- Apply FD1 (A → B): add {B} ⇒ {A, B}
- Apply FD2 (B → C): add {C} ⇒ {A, B, C}
- Apply FD3 (A → D): add {D} ⇒ {A, B, C, D}

- Apply FD4 (D → E): add {E} ⇒ {A, B, C, D, E}
  {A}+ = {A, B, C, D, E} (all attributes) ⇒ **A is a Candidate Key**

Closure of {B}:

- Start: {B}
- Apply FD2 (B → C): add {C} ⇒ {B, C}
  Missing A, D, E ⇒ Not a key.

Closure of {D}:

- Start: {D}
- Apply FD4: add {E} ⇒ {D, E}
  Missing A, B, C ⇒ Not a key.

Result:

- **Only Candidate Key:** {A}
- **Primary Key:** {A}

Problem 5:

Relation:

R(W,X,Y,Z)

Functional Dependencies (FDs):

1. **W→XW**
2. **X→YX \to YX→Y**
3. **WY→ZWY \to ZWY→Z**
4. **Z→WZ \to WZ→W**

Step 1: List all attributes

R={W,X,Y,Z}

Step 2: Identify dependent attributes (appear only on RHS)

- **RHS attributes:**
  - **XXX (in 1)**
  - **YYY (in 2)**
  - **ZZZ (in 3)**
  - **WWW (in 4)**
- **Check which attribute(s) never appear on RHS:**

All attributes appear on RHS at least once, so no attribute is independent in this case.

3: Identify possible candidate keys (since no attribute is independent, try combinations)

Since no attribute is excluded from RHS, candidate keys may be single attributes or combinations.

4: Compute closures of attribute sets

W^+

- Start: $\{W\}$
- $W \to X$ → add X
- $X \to Y$ → add Y
- Now we have W, X, Y
- $WY \to Z$, since we have both W and Y, add Z
- $Z \to W$ already have W

Final closure: $\{W, X, Y, Z\} = R$ → **W is a superkey**

X^+

- Start: $\{X\}$
- $X \to Y$ → add Y
- Don't have W, can't use $WY \to Z$
- $Z \to W$ no Z yet

Final closure: $\{X, Y\}$ → Not all attributes → no

Y^+

- Start: $\{Y\}$
- No FD with LHS = Y

Final closure: $\{Y\}$ → no

WX^+

- Since W alone is already a superkey, no need.

WY^+

- $WY \to Z$, etc.

  Since W alone is already a key, this is not minimal.

Step 5: Check minimality

- W is a single attribute superkey, so minimal → candidate key
- Z is also a single attribute superkey, so minimal → candidate key

Final candidate keys:

$$\boxed{W, Z}$$