

# Problems in MAC

# Formulas to Remember

- Pure Aloha

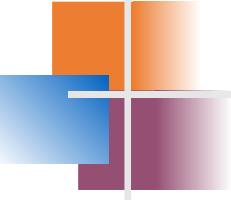
- Vulnerable Time =  $2 T_{fr}$   $T_{fr}$  is the frame transmission time
- Time out time for acknowledgment  $T_b = 2 \times T_p$ ,
- Exponential back off time  $T_b = R \times T_p$ ,  $R = 0$  to  $2^K - 1$
- Throughput =  $Gx e^{-2G}$

- Slotted Aloha

- Vulnerable time =  $T_{fr}$
- Time out time for acknowledgment  $T_b = 2 \times T_p$ ,
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- Throughput =  $Gx e^{-G}$

Or

- Throughput of aloha(slotted and Pure)= Efficiency x Bandwidth



The throughput for pure ALOHA is

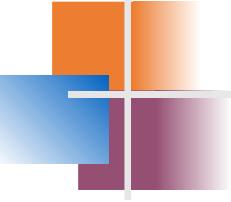
$$S = G \times e^{-2G}$$

The maximum throughput

$$S_{max} = 0.184 \text{ when } G = (1/2).$$

*How to get 100% throughput?*

Therefore, if a station generates only one frame in this vulnerable time (and no other stations generate a frame during this time), the frame will reach its destination successfully



### Note

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The throughput for slotted ALOHA is

$$S = G \times e^{-G}.$$

The maximum throughput

$$S_{max} = 0.368 \text{ when } G = 1.$$

**Legend**

$K$  : Number of attempts  
 $T_p$  : Maximum propagation time  
 $T_{fr}$  : Average transmission time  
 $T_B$  : (Backoff time):  $R \times T_p$  or  $R \times T_{fr}$   
 $R$  : (Random number): 0 to  $2^K - 1$

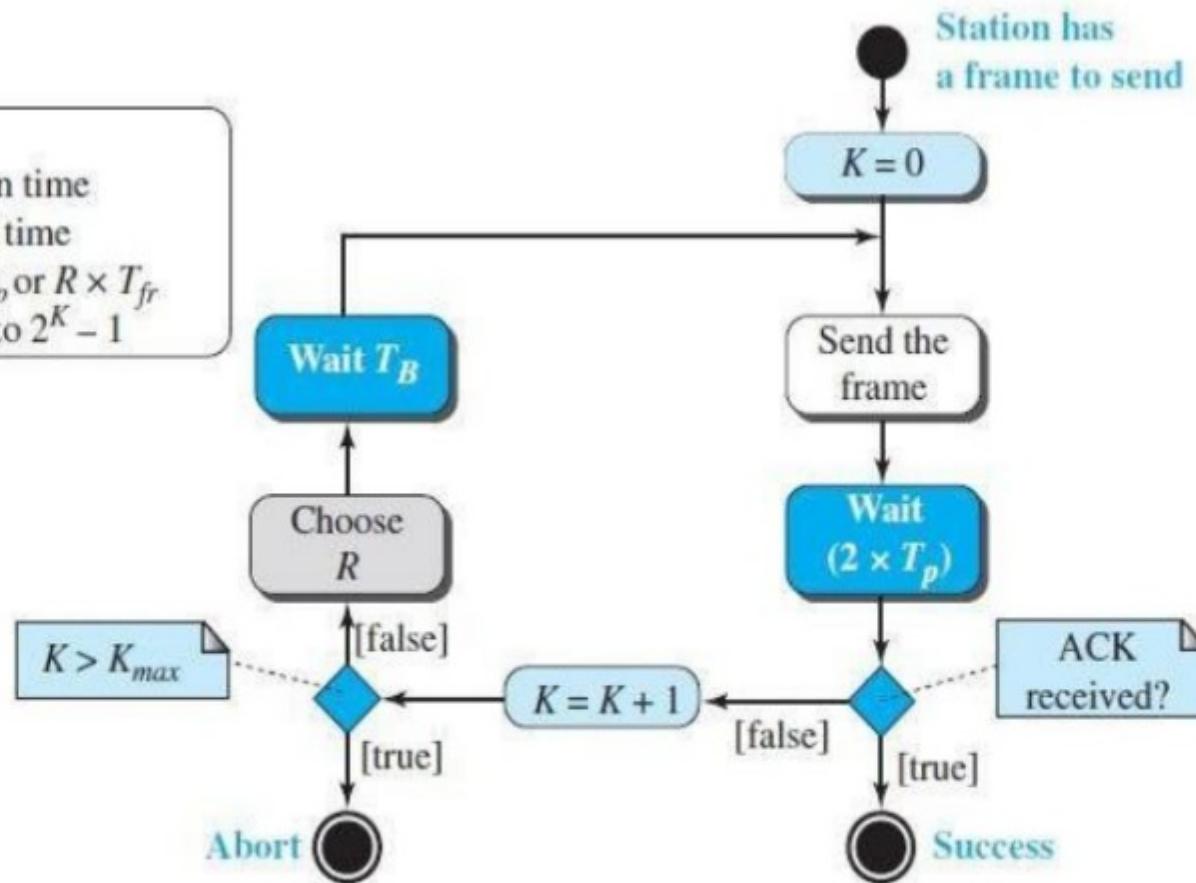
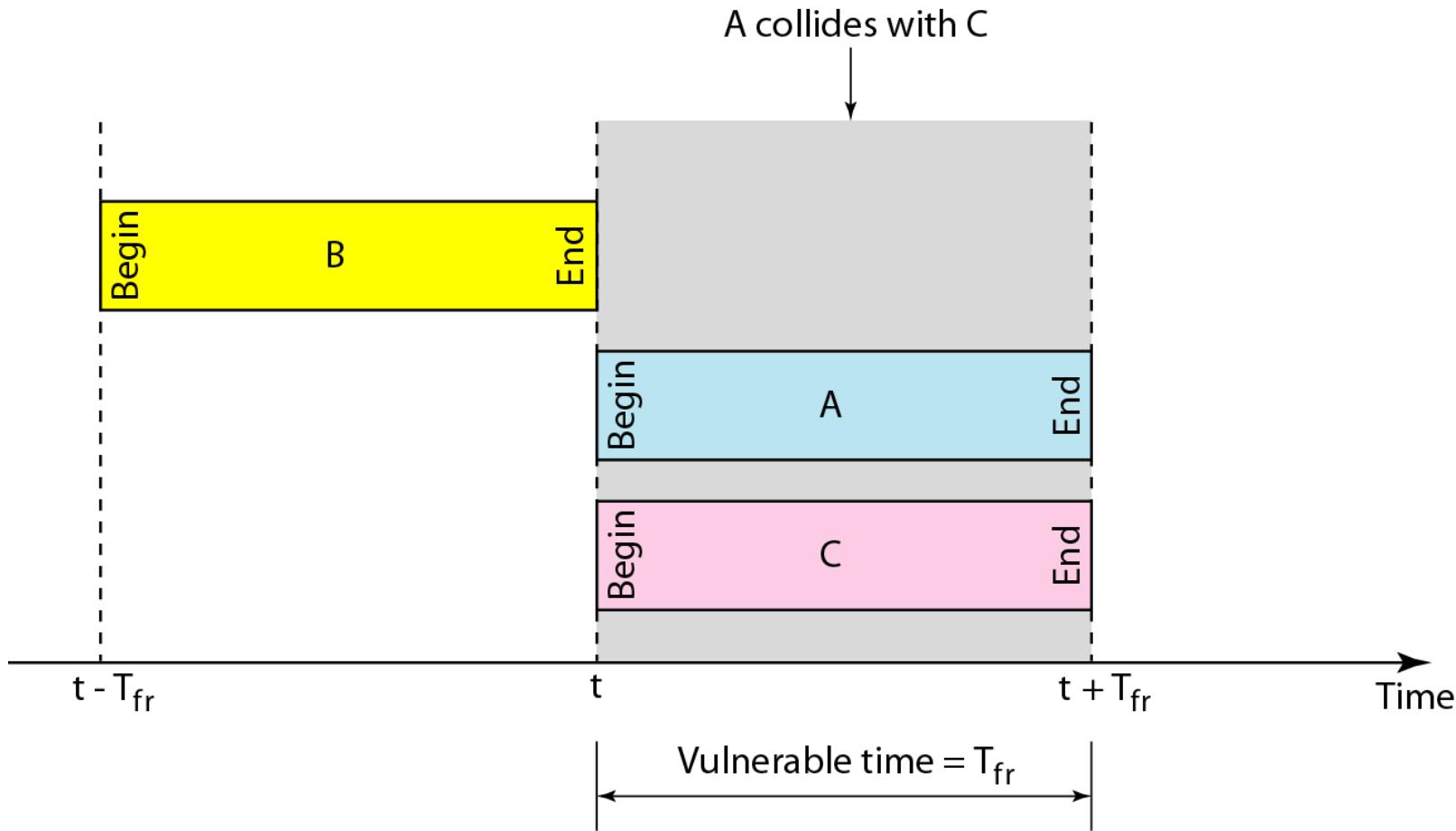
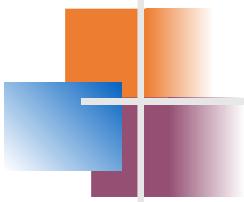


Figure 3: Procedure for pure ALOHA protocol

**Figure 12.7** Vulnerable time for slotted ALOHA protocol





## Example 1

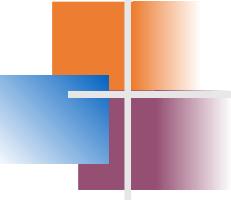
The stations on a wireless ALOHA network are a maximum of 600 km apart. If we assume that signals propagate at  $3 \times 10^8$  m/s, we find

$$Tp = (600 \times 10^3) / (3 \times 10^8) = 2 \text{ ms.}$$

find the value of TB for different values of K.

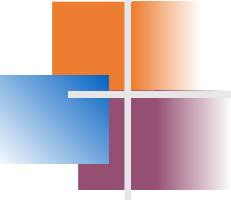
R = 0 to  $2^K - 1$  (K- No of Attempts, R- Random Number)

a. For  $K = 1$ , the range is  $\{0, 1\}$ . The station needs to generate a random number with a value of 0 or 1. This means that TB is either 0 ms ( $0 \times 2$ ) or 2 ms ( $1 \times 2$ ), based on the outcome of the random variable.



### *Example 12.1 (continued)*

- b. For  $K = 2$ , the range is  $\{0, 1, 2, 3\}$ . This means that TB can be 0, 2, 4, or 6 ms, based on the outcome of the random variable.*
- c. For  $K = 3$ , the range is  $\{0, 1, 2, 3, 4, 5, 6, 7\}$ . This means that TB can be 0, 2, 4, . . . , 14 ms, based on the outcome of the random variable.*



## Example 2

*A pure ALOHA network transmits 200-bit frames on a shared channel of 200 kbps. What is the requirement to make this frame collision-free?*

### *Solution*

*Average frame transmission time  $T_{fr} = 200 \text{ bits}/200 \text{ kbps}$  or*

*1 ms.*

*The vulnerable time is  $2 \times 1 \text{ ms} = 2 \text{ ms}$ .*

*This means no station should send later than 1 ms before this station starts transmission and no station should start sending during the one 1-ms period that this station is sending.*

**Example 3**

- A group of N stations share 100 Kbps slotted ALOHA channel. Each station output a 500 bits frame on an average of 5000 ms even if previous one has not been sent. What is the required value of N?

**Solution**

**Throughput Of One Station-**

Throughput of each station

$$\begin{aligned} &= \text{Number of bits sent per second} \\ &= 500 \text{ bits} / 5000 \text{ ms} \\ &= 500 \text{ bits} / (5000 \times 10^{-3} \text{ sec}) \\ &= 100 \text{ bits/sec} \end{aligned}$$

**Throughput Of Slotted Aloha-**

$$\begin{aligned} &\text{Throughput of slotted aloha} \\ &= \text{Efficiency} \times \text{Bandwidth} \\ &= 0.368 \times 100 \text{ Kbps} \\ &= 36.8 \text{ Kbps} \end{aligned}$$

Example 3 continued...

## Total Number Of Stations-

Throughput of slotted aloha = Total number of stations x  
Throughput of each station

Substituting the values, we get-

$$36.8 \text{ Kbps} = N \times 100 \text{ bits/sec}$$

$$\therefore N = 368$$

Thus, required value of  $N = 368$ .

**Example 3a**

- A group of N stations shares a 56-kbps pure ALOHA channel. Each station outputs a 1000-bit frame on an average of once every 100 sec, even if the previous one has not yet been sent (e.g., the stations can buffer outgoing frames). What is the maximum value of N?

**Solution**

**Throughput Of One Station-**

Throughput of each station

= Number of bits sent per second

= 1000 bits / 100sec

= = 10 bits/sec

**Throughput Of pure Aloha**

= Efficiency x Bandwidth

=  $0.18 \times 56\text{Kbps}$

= 10.3 Kbps

Example 3a continued...

### Total Number Of Stations-

Throughput of pure aloha = Total number of stations x  
Throughput of each station

Substituting the values, we get-

$$10.3 \text{ Kbps} = N \times 10 \text{ bits/sec}$$

$$\therefore N = 10300 / 10 = 1030 \text{ stations.}$$

Thus, required value of  $N = 1030$

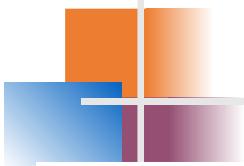
- 1000 airline reservation stations are competing for the use of a single slotted ALOHA channel. The average station makes 36 requests per hour. A slot is 100  $\mu$ sec. What is the approximate total channel load?
- Each terminal makes one request every
- $3600 \text{ sec} / 36 \text{ request} = 100 \text{ sec.}$
- Total load is **1000 requests per 100 sec or 10 requests per sec.**
- There are
- $1 \text{ sec}/100 \mu\text{sec} = 1000000 \mu\text{sec} / 100 \mu\text{sec} = 10000 \text{ slots in one second.}$
- Hence,  $G=10/10000=1/1000=0.1\%$

#### **Example 4**

Let us call  $G$  the average number of frames generated by the system during one frame transmission time. Then it can be proved that the average number of successful transmissions for pure ALOHA is

$$S = G \times e^{-2G}$$

**Throughput** Let us call  $G$  the average number of frames generated by the system during one frame transmission time. Then it can be proved that the average number of successful transmissions for pure ALOHA is  $S = G \times e^{-2G}$ . The maximum throughput  $S_{max}$  is 0.184, for  $G = \frac{1}{2}$ . In other words, if one-half a frame is generated during one frame transmission time (in other words, one frame during two frame transmission times), then 18.4 percent of these frames reach their destination successfully. This is an expected result because the vulnerable time is 2 times the frame transmission time. Therefore, if a station generates only one frame in this vulnerable time (and no other stations generate a frame during this time), the frame will reach its destination successfully.



## Example 5

A pure ALOHA network transmits 200-bit frames on a shared channel of 200 kbps. What is the throughput if the system (all stations together) produces

- a. 1000 frames per second
- b. 500 frames per second
- c. 250 frames per second.

### Solution

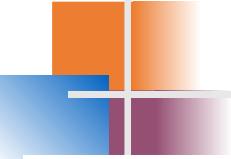
The frame transmission time is 200/200 kbps or 1 ms.

- a. If the system creates 1000 frames per second, this is 1

frame per millisecond. The capacity is G 1. In this case

$$S = G \times e^{-2G} = 1 \times e^{-2 \times 1} S = 0.135 \text{ (13.5 percent). This means that the throughput is } 1000 \times$$

$0.135 = 135$  frames. Only 135 frames out of 1000 will probably survive.



## Example 5 (continued)

b. If the system creates 500 frames per second, this is  $(1/2)$  frame per millisecond. The load is  $(1/2)$ . In this case  $S = G \times e^{-2G}$  or  $S = 0.184$  (18.4 percent). This means that the throughput is  $500 \times 0.184 = 92$  and that only 92 frames out of 500 will probably survive. Note that this is the maximum throughput case, percentagewise.

c. If the system creates 250 frames per second, this is  $(1/4)$  frame per millisecond. The load is  $(1/4)$ . In this case  $S = G \times e^{-2G}$  or  $S = 0.152$  (15.2 percent). This means that the throughput is  $250 \times 0.152 = 38$ . Only 38 frames out of 250 will probably survive.

- Propagation delay = Distance / Propagation speed
- Minimum frame size =  $2 \times$  Propagation delay  $\times$  Bandwidth

Or

Minimum frame size = Frame transmission time  $\times$  Bandwidth

Frame transmission time is  $t_{fr} = 2 \times t_p$

$T_p$  - propagation delay

- **Example 8**

- In a CSMA / CD network running at 1 Gbps over 1 km cable with no repeaters, the signal speed in the cable is 200000 km/sec. What is minimum frame size?

- **Example 9**

- A and B are the only two stations on an Ethernet. Each has a steady queue of frames to send. Both A and B attempt to transmit a frame, collide, and A wins the first backoff race. At the end of this successful transmission by A, both A and B attempt to transmit and collide. The probability that A wins the second backoff race is:

### **Example 10**

Suppose nodes A and B are on same 10 Mbps Ethernet segment and the propagation delay between two nodes is 225 bit times. Suppose A and B send frames at  $t=0$ , the frames collide then at what time, they finish transmitting a jam signal. Assume a 48 bit jam signal

- Example 8
- In a CSMA / CD network running at 1 Gbps over 1 km cable with no repeaters, the signal speed in the cable is 200000 km/sec. What is minimum frame size?

### Solution

**Propagation delay =Distance / Propagation speed - 1 km / (200000 km/sec) =  $5 \times 10^{-6}$  sec**

- Calculating Minimum Frame Size-
- **Minimum frame size =  $2 \times$  Propagation delay  $\times$  Bandwidth**
- -  **$2 \times 5 \times 10^{-6}$  sec  $\times 10^9$  bits per sec = 10000 bits**

- **Example 10**

Suppose nodes A and B are on same 10 Mbps Ethernet segment and the propagation delay between two nodes is 225 bit times. Suppose A and B send frames at  $t=0$ , the frames collide then at what time, they finish transmitting a jam signal. Assume a 48 bit jam signal

Example 10

• **Solution-**

Propagation delay ( $T_p$ )

$$= 225 \text{ bit} / 10 \text{ Mbps} = 22.5 \mu\text{sec}$$

**At t = 0,**

Nodes A and B start transmitting their frame.

Since both the stations start simultaneously, so collision occurs at the mid way.

Time after which collision occurs = Half of propagation delay.

$$\text{So, time after which collision occurs} = 22.5 \mu\text{sec} / 2 = 11.25 \mu\text{sec.}$$

**At t = 11.25 μsec,**

After collision occurs at  $t = 11.25 \mu\text{sec}$ , collided signals start travelling back.

Collided signals reach the respective nodes after time = Half of propagation delay

$$\text{Collided signals reach the respective nodes after time} = 22.5 \mu\text{sec} / 2 = 11.25 \mu\text{sec.}$$

Thus, at  $t = 22.5 \mu\text{sec}$ , collided signals reach the respective nodes.

**At t = 22.5 μsec,**

As soon as nodes discover the collision, they immediately release the jam signal.

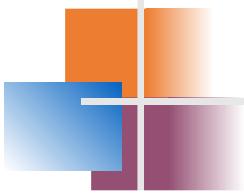
Time taken to finish transmitting the jam signal = 48 bit time = 48 bits/ 10 Mbps = 4.8  $\mu\text{sec}$ .

Thus,

Time at which the jam signal is completely transmitted

$$= 22.5 \mu\text{sec} + 4.8 \mu\text{sec}$$

$$= 27.3 \mu\text{sec} \text{ or } 273 \text{ bit times}$$



## Example 11

A network using CSMA/CD has a bandwidth of 10 Mbps. If the maximum propagation time (including the delays in the devices and ignoring the time needed to send a jamming signal, as we see later) is  $25.6 \mu\text{s}$ , what is the minimum size of the frame?

### Solution

The frame transmission time is  $T_{\text{fr}} = 2 \times T_p = 51.2 \mu\text{s}$ .

This means, in the worst case, a station needs to transmit for a period of  $51.2 \mu\text{s}$  to detect the collision.

The minimum size of the frame is  $10 \text{ Mbps} \times 51.2 \mu\text{s} = 512 \text{ bits or } 64 \text{ bytes}$ .

This is actually the minimum size of the frame for Standard Ethernet.

# Example 13

A 2 km long broadcast LAN has  $10^7$  bps bandwidth and uses CSMA / CD. The signal travels along the wire at  $2 \times 10^8$  m/sec. What is the minimum frame size that can be used on this network?

Solution-

Given-

- Distance = 2 km
- Bandwidth =  $10^7$  bps
- Speed =  $2 \times 10^8$  m/sec

### Calculating Propagation Delay-

Propagation delay ( $T_p$ )

= Distance / Propagation speed

$$= 2 \text{ km} / (2 \times 10^8 \text{ m/sec})$$

$$= 2 \times 10^3 \text{ m} / (2 \times 10^8 \text{ m/sec})$$

$$= 10^{-5} \text{ sec}$$

### Calculating Minimum Frame Size-

Minimum frame size

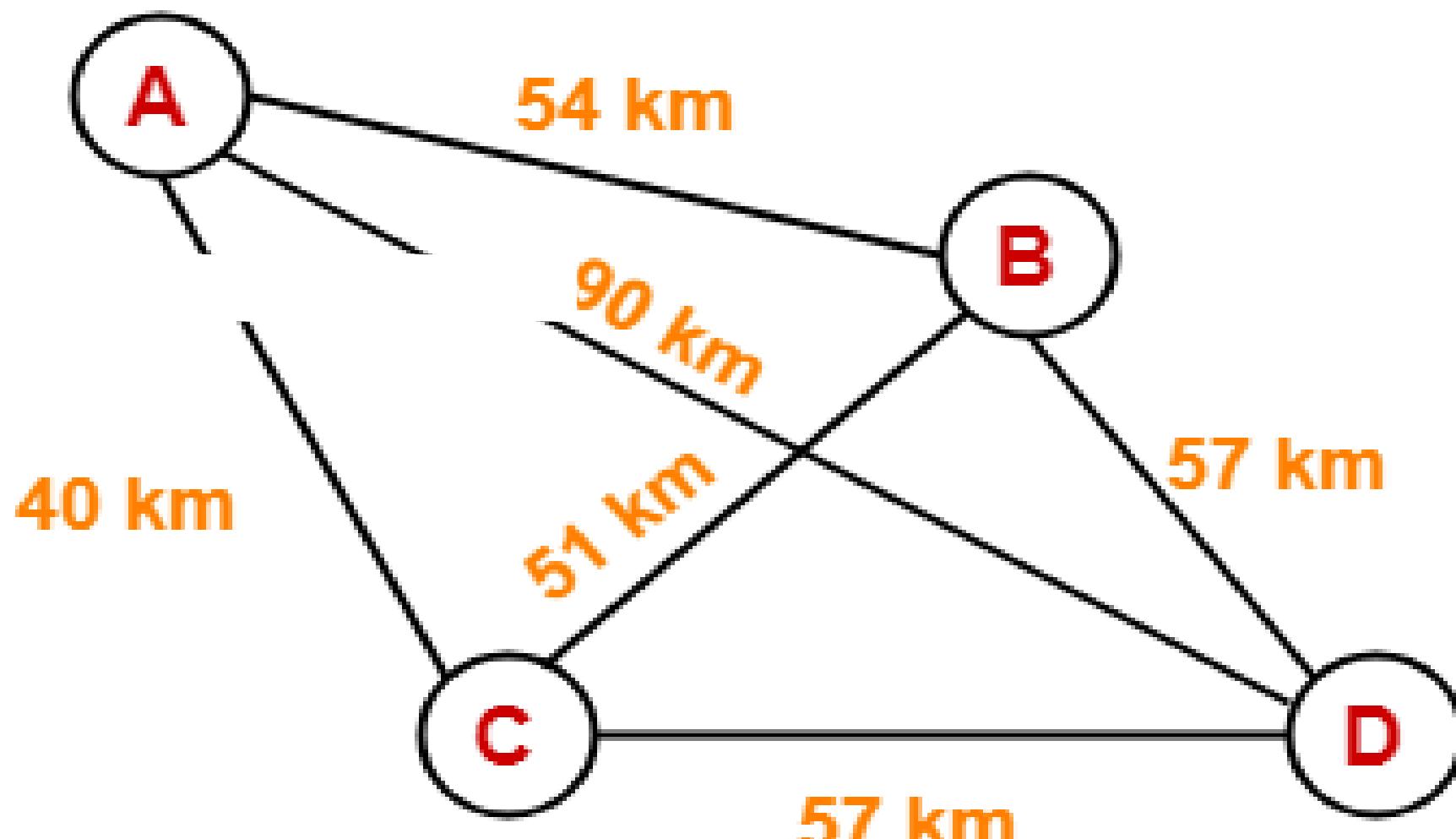
= 2 x Propagation delay x Bandwidth

$$= 2 \times 10^{-5} \text{ sec} \times 10^7 \text{ bits per sec}$$

$$= 200 \text{ bits or 25 bytes}$$

**Example 14:**

The network consists of 4 hosts distributed as shown below-



**Assume this network uses CSMA / CD and signal travels with a speed of  $3 \times 10^5$  km/sec.**

**Solution-** If Sender sends at 1 Mbps, what could be the minimum size of the packet?

- CSMA / CD is a Access Control Method.
- It is used to provide the access to stations to a broadcast link.
- In the given network, all the links are point to point.
- So, there is actually no need of implementing CSMA / CD.
- Stations can transmit whenever they want to transmit.

In CSMA / CD,

The condition to detect collision is-

Packet size  $\geq 2 \times (\text{distance} / \text{speed}) \times \text{Bandwidth}$

So, we use the values-

- Distance = 90 km
- Speed =  $3 \times 10^5$  km/sec
- Bandwidth = 1 Mbps

Substituting these values, we get-

Minimum size of data packet

$$= 2 \times (90 \text{ km} / 3 \times 10^5 \text{ km per sec}) \times 1 \text{ Mbps}$$

$$= 2 \times 30 \times 10^{-5} \text{ sec} \times 10^6 \text{ bits per sec}$$

$$= 600 \text{ bits}$$

## **Example 15**

Consider a LAN with average source and destination 100 meters apart and the round trip delay of 10  $\mu$ s. At \_\_\_\_\_ data rate does the round trip delay equal to the transmission delay for 512 B packets

$$RTT = 10 * 10^6 \text{ s}$$

$$Tt = L/B = 512 * 8 / B$$

$$\text{Given } RTT = Tt$$

$$B = 512 * 8 / 10 * 10^6$$

$$= 409.6 * 10^6 \text{ bits/s}$$

$$= 410 \text{ Mbps}$$

# Example 16

- After the  $k^{\text{th}}$  consecutive collision, each colliding station waits for a random time chosen from the interval-
  - 1.(0 to  $2^k$ ) x RTT
  - 2.(0 to  $2^k - 1$ ) x RTT
  - 3.(0 to  $2^k - 1$ ) x Maximum Propagation delay
  - 4.(0 to  $2^{k-1}$ ) x Maximum Propagation delay
  -
- **Solution-**
- Clearly, Option (B) is correct.

- A 1-km-long, 10-Mbps CSMA/CD (Carrier Sense Multiple Access with Collision Detection) LAN (not 802.3) has a propagation speed of  $200 \text{ m}/\mu\text{sec}$ . Repeaters are not allowed in this system. Data frames are 256 bits long, including 32 bits of header, checksum, and other overhead. The first bit slot after a successful transmission is reserved for the receiver to capture the channel in order to send a 32-bit acknowledgement frame. What is the effective data rate, excluding overhead, assuming that there are no collisions?

Solution

- The round-trip propagation time of the cable is  $10 \mu \text{ sec}$ . A complete transmission has six phases:
  - 1) Transmitter seizes cable ( $10 \mu \text{ sec}$ )
  - 2) Transmit data ( $25.6 \mu \text{ sec}$ )
  - 3) Delay for last bit to get to the end ( $5.0 \mu \text{ sec}$ )
  - 4) Receiver seizes cable ( $10 \mu \text{ sec}$ )
  - 5) Acknowledgement sent ( $3.2 \mu \text{ sec}$ )
  - 6) Delay for last bit to get to the end ( $5.0 \mu \text{ sec}$ )

The sum of these is  $58.8 \mu \text{ sec}$ . In this period, 224 data bits are sent, for a rate of about 3.8 Mbps

- **Problem-04:**
- A and B are the only two stations on Ethernet. Each has a steady queue of frames to send. Both A and B attempts to transmit a frame, collide and A wins first back off race. At the end of this successful transmission by A, both A and B attempt to transmit and collide. The probability that A wins the second back off race is \_\_\_\_\_

- **Solution-**
- 
- According to question, we have-
- 
- **1st Transmission Attempt-**
- 
- Both the stations A and B attempts to transmit a frame.
- A collision occurs.
- Back Off Algorithm runs.
- Station A wins and successfully transmits its 1<sup>st</sup> data packet.
- 
- **2nd Transmission Attempt-**
- 
- Station A attempts to transmit its 2<sup>nd</sup> data packet.
- Station B attempts to retransmit its 1<sup>st</sup> data packet.
- A collision occurs.

- Example 9

- A and B are the only two stations on an Ethernet. Each has a steady queue of frames to send. Both A and B attempt to transmit a frame, collide, and A wins the first back off race. At the end of this successful transmission by A, both A and B attempt to transmit and collide. The probability that A wins the second back-off race is:

- Qn2 – Ans

- 1st attempt: Value of 'k' would be  $k=0$  or  $k=1$  ( $0 \leq k \leq 2^n-1$ ; where  $n=n$ th attempt). Since A won the first race, A must have chosen  $k=0$  and B must have chosen  $k=1$  (A wins here with probability 0.25). As A won, A will again choose  $k=0$  or  $k=1$  for its 2nd frame, but B will choose  $k=0, 1, 2$  or  $3$  as B failed to send its first frame in the first attempt.
- 2nd attempt: Let  $k_A =$  value of  $k$  chosen by A and  $k_B =$  value of  $k$  chosen by B. We will use notation  $(k_A, k_B)$  to show the possible values. Now the sample space for the 2nd attempt is  $(k_A, k_B) = (0,0), (0,1), (0,2), (0,3), (1,0), (1,1), (1,2)$  or  $(1,3)$  i.e. 8 possible outcomes. For A to win,  $k_A$  should be less than  $k_B$  ( $k_A < k_B$ ). Thus, our event space is  $(k_A, k_B) = (0,1), (0,2), (0,3), (1,2), (1,3)$  i.e. 5 possible outcomes.
- Thus the probability that A wins the 2nd back-off race =  $5/8 = 0.625$

- Now,
- We have been asked the probability of station A to transmit its 2<sup>nd</sup> data packet successfully after 2<sup>nd</sup> collision.
- After the 2<sup>nd</sup> collision occurs, we have-
- 
- **At Station A-**
- 
- 2<sup>nd</sup> data packet of station A undergoes collision for the 1<sup>st</sup> time.
- So, collision number for the 2<sup>nd</sup> data packet of station A = 1.
- Now, station A randomly chooses a number from the range  $[0, 2^1-1] = [0, 1]$ .
- Then, station A waits for back off time and then attempts to retransmit its data packet.
- 
- **At Station B-**
- 
- 1<sup>st</sup> data packet of station B undergoes collision for the 2<sup>nd</sup> time.
- So, collision number for the 1<sup>st</sup> data packet of station B = 2.
- Now, station B randomly chooses a number from the range  $[0, 2^2-1] = [0, 3]$ .
- Then, station B waits for back off time and then attempts to retransmit its data packet.

Following 8 cases are possible- From here,

Probability of A winning the 2<sup>nd</sup> back off race = 5 / 8 = 0.625

Station A	Station B	Remark
0	0	Collision
0	1	A wins
0	2	A wins
0	3	A wins
1	0	B wins
1	1	Collision
1	2	A wins
1	3	A wins

- **Problem-05:**
- Suppose nodes A and B are on same 10 Mbps Ethernet segment and the propagation delay between two nodes is 225 bit times. Suppose A and B send frames at  $t=0$ , the frames collide then at what time, they finish transmitting a jam signal. Assume a 48 bit jam signal.
- Propagation delay ( $T_p$ )
- = 225 bit times
- = 225 bit / 10 Mbps
- =  $22.5 \times 10^{-6}$  sec
- = 22.5  $\mu$ sec

- At  $t = 0$ ,
- Nodes A and B start transmitting their frame.
- Since both the stations start simultaneously, so collision occurs at the mid way.
- Time after which collision occurs = Half of propagation delay.
- So, time after which collision occurs =  $22.5 \mu\text{sec} / 2 = 11.25 \mu\text{sec}$ .
- At  $t = 11.25 \mu\text{sec}$ ,
- After collision occurs at  $t = 11.25 \mu\text{sec}$ , collided signals start travelling back.
- Collided signals reach the respective nodes after time = Half of propagation delay
- Collided signals reach the respective nodes after time =  $22.5 \mu\text{sec} / 2 = 11.25 \mu\text{sec}$ .
- Thus, at  $t = 22.5 \mu\text{sec}$ , collided signals reach the respective nodes.
- At  $t = 22.5 \mu\text{sec}$ ,
- As soon as nodes discover the collision, they immediately release the jam signal.
- Time taken to finish transmitting the jam signal = 48 bit time =  $48 \text{ bits} / 10 \text{ Mbps} = 4.8 \mu\text{sec}$ .
- Time at which the jam signal is completely transmitted
- =  $22.5 \mu\text{sec} + 4.8 \mu\text{sec}$
- =  $27.3 \mu\text{sec}$  or 273 bit times

# E

- Suppose nodes A and B are attached to opposite ends of the cable with propagation delay of 12.5 ms. Both nodes attempt to transmit at  $t=0$ . Frames collide and after first collision, A draws  $k=0$  and B draws  $k=1$  in the exponential back off protocol. Ignore the jam signal. At what time (in seconds), is A's packet completely delivered at B if bandwidth of the link is 10 Mbps and packet size is 1000 bits.
- 
- **Solution-**
- 
- Given-
- Propagation delay = 12.5 ms
- Bandwidth = 10 Mbps
- Packet size = 1000 bits

- **Time At Which Collision Occurs-**

- 
- Collision occurs at the mid way after time
- = Half of Propagation delay
- =  $12.5 \text{ ms} / 2$
- = 6.25 ms
- Thus, collision occurs at time  $t = 6.25 \text{ ms}$ .

- **Time At Which Collision is Discovered-**

- 
- Collision is discovered in the time it takes the collided signals to reach the nodes
- = Half of Propagation delay
- =  $12.5 \text{ ms} / 2$
- = 6.25 ms
- Thus, collision is discovered at time  $t = 6.25 \text{ ms} + 6.25 \text{ ms} = 12.5 \text{ ms}$ .

#### **Scene After Collision-**

- 
- After the collision is discovered,
- Both the nodes wait for some random back off time.
- A chooses k=0 and then waits for back off time =  $0 \times 25 \text{ ms} = 0 \text{ ms}$ .
- B chooses k=1 and then waits for back off time =  $1 \times 25 \text{ ms} = 25 \text{ ms}$ .
- From here, A begins retransmission immediately while B waits for 25 ms.
- 

#### **Waiting Time For A-**

- 
- After winning the back off race, node A gets the authority to retransmit immediately.
- But node A does not retransmit immediately.
- It waits for the channel to clear from the last bit aborted by it on discovering the collision.
- Time taken by the last bit to get off the channel = Propagation delay = 12.5 ms.
- So, node A waits for time = 12.5 ms and then starts the retransmission.
- Thus, node A starts the retransmission at time  $t = 12.5 \text{ ms} + 12.5 \text{ ms} = 25 \text{ ms}$ .
- 

#### **Time Taken in Delivering Packet To Node B-**

- 
- Time taken to deliver the packet to node B
- = Transmission delay + Propagation delay
- =  $(1000 \text{ bits} / 10 \text{ Mbps}) + 12.5 \text{ ms}$
- =  $100 \mu\text{s} + 12.5 \text{ ms}$
- =  $0.1 \text{ ms} + 12.5 \text{ ms}$
- = 12.6 ms
- Thus, At time  $t = 25 \text{ ms} + 12.6 \text{ ms} = 37.6 \text{ ms}$ , the packet is delivered to node B
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