Tutorial on Relations

1. Let R be the relation on set A where $R = \{(a, b) \quad | \, a \; = 2b\}$. List all the possible order paired of relation R if set $A = \{1,2,3,4\}$.

2. The relation R onto a set $A = \{1,2,3\}$ is given by $R = \{(1,1), (1,2), (3,2)\}$ Show the relation Rin the form of a matrix and arrow diagram.

3. The matrix below represents the relation S onto a set $B = \{a, b, c\}$. State and give the reasons on whether the relation $S$is reflexive, symmetric and/or transitive. State whether it is an equivalence relation.

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

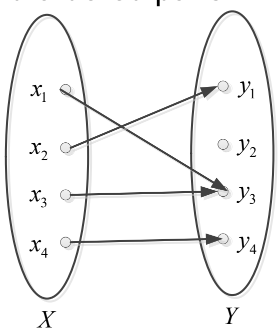
4. The matrix below represents the relation R onto a set $A = \{a, b, c\}$. State and give the reasons on whether the relation $S$is reflexive, symmetric and/or transitive. State whether it is an equivalence relation.

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

5. The matrix below represents the relation S onto a set $B = \{a, b, c\}$. State and give the reasons on whether the relation $S$is reflexive, symmetric and/or transitive. State whether it is an equivalence relation.

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

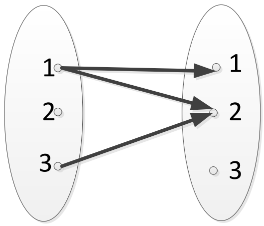
6. The relation S from set X to set Y is shown in the arrow diagram below. Show the relation Sin a form of matrix and ordered pairs.



Answers (Relations)

1. $R = \{(2,1), (4,2)\}$

2.
$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$



3. $S$is not a reflexive and not a transitive relation. $S$is a symmetric relation. Thus $S$is not an equivalence relation.

4. Relation R is an equivalence relation as it is reflexive, symmetric and transitive.

5. Relation S is not an equivalence relation as it is not reflexive, not symmetric and not transitive.

6.
$$\begin{bmatrix} 0\ 0\ 1\ 0 \\ 1\ 0\ 0\ 0 \\ 0\ 0\ 1\ 0 \\ 0\ 0\ 0\ 1 \end{bmatrix}$$

$\{(x_1, y_3), (x_2, y_1), (x_3, y_3), (x_4, y_4)\}$

Tutorial on Functions

1. Determine **whether each of the following sets of ordered pairs is a function** from the given domain to the given codomain.

(a) Domain = $\{1,2,3\}$, Codomain = $\{a, b, c, d, e\}$, $R = \{(1, a), (2, b), (3, b)\}$
(b) Domain = $\{1,2,3,4\}$, Codomain = $\{a, b, c, d\}$, $S = \{(2, d), (3, a), (4, d)\}$
(c) Domain = $\{1,2,3\}$, Codomain = $\{a, b, c, d\}$, $T = \{(1, b), (2, c), (3, a), (3, d)\}$
(d) Domain = $\{1,2,3,4\}$, Codomain = $\{a, b\}$, $T = \{(1, b), (2, b), (3, b), (4, b)\}$

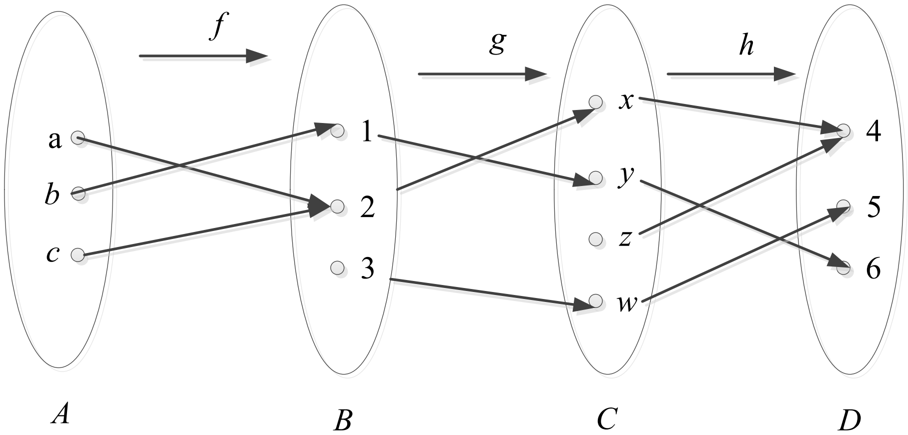
2. The function f is defined by $f(x) = x^3 + 3$ for all real values of x. Evaluate the values of the following:

(a) $f(1)$ (b) $f(-1)$ (c) $f(2a)$

3. Let the function $f: A \to B,\ g: B \to C,\ h: C \to D$ be defined by the figure below. Determine which of the functions are:

(i) injective (one-to-one) and/or

(ii) surjective (onto).

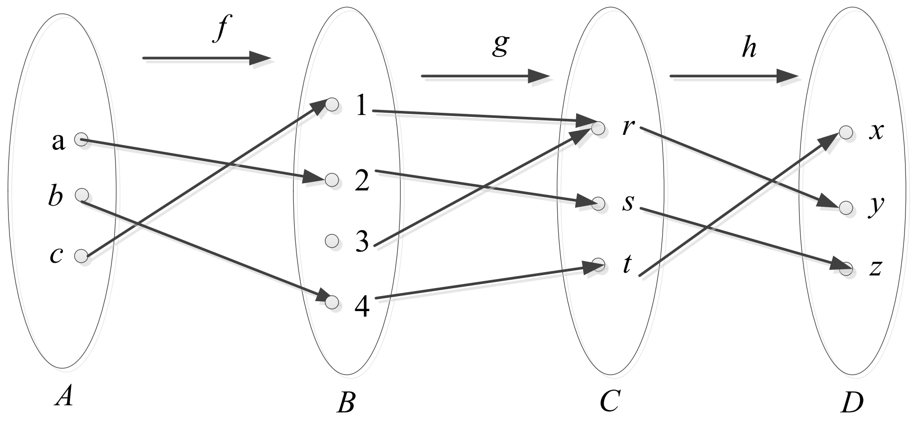


4. The functions *f* and *g* are defined for all real values of *x* as follows:

$$f(x) = 2x - 1 \text{ and } g(x) = \frac{2x+3}{x-1},\ x \neq 1$$

(a) Find the composite functions $(f \circ g)(x)$ and $(g \circ f)(x)$.
(b) Find the inverse functions $f^{-1}(x)$ and $g^{-1}(x)$.
(c) Evaluate $f \circ g^{-1}(1)$ and $g^{-1} \circ f^{-1}(-2)$.

5. Let the function $f: A \to B,\ g: B \to C,\ h: C \to D$ be defined by the figure below. Determine which of the functions are is invertible, and, if it is, find its inverse.



6. Consider the two functions $f: R \to R$ and $g: R \to R$, where $R$is the set of all real values of x, as $f(x) = x + 2$ and $g(x) = x^2$. State with reason, which of these two functions is invertible.

7. The functions f and g are defined for all real values of x as follows:

$$f(x) = \frac{8}{x-3}, \ x \neq 3 \text{ and } g(x) = 2x - 3$$

(a) Find the expressions for f^{-1}, $(f \circ g)(x)$ and $(g \circ f)(x)$.

(b) Find the value of x for which $(f \circ g)(x) = (g \circ f)(x)$.

8*. The functions *f* and *g* are defined by $f(x) = x - 3$ and $g(x) = x^2$ respectively. Find another function *h* such that $hgf(x) = x^2 - 6x + 3$

.

9*. The functions *f* and *g* are defined for all real values of *x* as follows:

$$f(x) = x^2 - 1 \text{ and } g(x) = (x-1)^2$$

(a) If $4f(x) + 3 = f(kx)$, find the value(s) of *k*.
(b) Express $g(2x + 1)$ in terms of $f(x)$.
(c) Find a function *h* such that $f(x) = g(x) + 2h(x)$.

Answers (Functions)

1. (a) Yes (b) No (c) No (d) Yes

2. (a) 4 (b) 2 (c) $8a^3 + 3$

3. f is both not injective and not surjective.
g is injective but not surjective.
h is not injective but surjective.

4. (a) $(f \circ g)(x) = \frac{3x+7}{x-1},\ x \neq 1;$ $(g \circ f)(x) = \frac{4x+1}{2x-2},\ x \neq 1$
(b) $f^{-1}(x) = \frac{1}{2}(x+1),$ $g^{-1}(x) = \frac{3+x}{x-2},\ x \neq 2$
(c) -9, -1

5. f is injective but is not surjective. Thus, f is not invertible.
g is not injective but is surjective. Thus, g is not invertible.
h is both injective and surjective. Thus, h is invertible.
Hence, $h^{-1} = \{(x, f), (y, r), (z, s)\}$

6. f is invertible, whereas g is not.

7. (a) $f^{-1}(x) = \frac{8+3x}{x},\ x \neq 0;$ $f \circ g(x) = \frac{4}{x-3},\ x \neq 3;$ $g \circ f(x) = \frac{25-3x}{x-3},\ x \neq 3$
(b) 7

8*. $h(x) = x - 6$

9*. (a) ± 2 (b) $4f(x) + 4$ (c) $h(x) = x - 1$