Tutorial 8

1. Find the following integrals:

(a) $\int \frac{2}{3}\, dx$ (b) $\int \pi^2\, dx$ (c) $\int x^8\, dx$ (d) $\int x^{-\frac{2}{3}}\, dx$

(e) $\int \frac{1}{x^4} dx$ (f) $\int \sqrt[3]{x}\, dx$ (g) $\int \sqrt[3]{x^2}\, dx$ (h) $\int 6x^3\, dx$

(i) $\int \frac{1}{3\pi} t\, dt$ (j) $\int \frac{\pi^2}{3}\, t^2 dt$ (k) $\int \frac{3}{x^2}\, dx$ (l) $\int 5\sqrt{x}\, dx$

(m) $\int \frac{1}{\sqrt{x}}\, dx$ (n) $\int \frac{1}{\sqrt[5]{t^2}}\, dt$

2. Find the following indefinite integrals:

(a) $\int (x^2 + x + 3) dx$ (b) $\int (7 - 5x - 3x^2)\, dx$ (c) $\int (4t^2 + 3t - 2) dt$

(d) $\int (x + 3)^2\, dx$ (e) $\int \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right) dx$ (f) $\int \left(\frac{2}{x^2} - 3x^2 + 4\right)\, dx$

(g) $\int \left(\frac{5}{x^2} - \frac{3}{x^4}\right) dx$ (h) $\int \left(8\sqrt{x} + \frac{1}{4\sqrt{x}}\right) dx$ (i) $\int \left(2x^3 + \frac{1}{\sqrt{x}} - \frac{2}{x^2}\right)\, dx$

3. Evaluate the following definite integrals:

(a) $\int_0^2 (4t^2 - t) dt$ (b) $\int_1^2 \frac{2t^2+1}{t^2}\, dt$ (c) $\int_1^3 2r(r - 2) dr$

(d) $\int_1^4 (x + 1)(2x + 1)\, dx$ (e) $\int_0^1 \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right) dx$

4. Find the following Indefinite Integrals:

(a) $\int (-6x + 1)\, dx$ (b) $\int \left(x^3 + 6\sqrt{x} - \frac{1}{x^2}\right) dx$

(c) $\int \left(\frac{x^4+7x}{x^3}\right) dx$ (d) $\int (2 - 3x)^2 dx$

5. Evaluate the followings:

(a) $\int (1 + x)^3 dx$ (b) $\int (3x - 2)^4 dx$ (c) $\int 2x(x^2 + 2)^4 dx$

(d) $\int x^2 (x^3 + 4)^8 dx$ (e) $\int (2x^2 - 1)(2x^3 - 3x + 9)^{\frac{1}{3}} dx$

(f) $\int \frac{x}{\sqrt{3-5x^2}}\, dx$ (g) $\int x\sqrt{1 - x^2}\, dx$

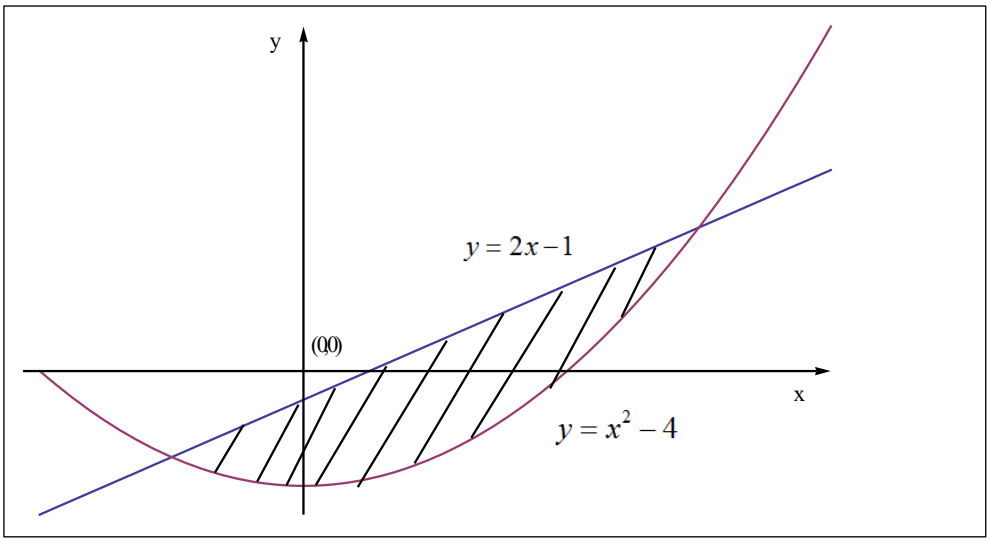
6*. Find the following indefinite integrals by substitution method.

(a) $\int 4x(x^2 + 3)^4 dx$ (b) $\int \frac{5x}{\sqrt{7-2x^2}} dx$

7.

(a) Show that the points of intersection of the curves $y = 2x - 1$ and $y = x^2 - 4$ are $(-1, -3)$and $(3,5)$.

(b) Hence, find the shaded area bounded by the curves in the figure below.



Answers

1. (a) $\frac{2}{3}x + C$ (b) $\pi^2 x + C$ (c) $\frac{1}{9}x^9 + C$ (d) $3x^{\frac{1}{3}} + C$

   (e) $-\frac{1}{3x^3} + C$ (f) $\frac{3}{4}x^{\frac{4}{3}} + C$ (g) $\frac{3}{5}x^{\frac{5}{3}} + C$ (h) $\frac{3}{2}x^4 + C$

   (i) $\frac{1}{6\pi}t^2 + C$ (j) $\frac{\pi^2}{9}t^3 + C$ (k) $-\frac{3}{x} + C$ (l) $\frac{10}{3}x^{\frac{3}{2}} + C$

   (m) $2x^{\frac{1}{2}} + C$ (n) $\frac{5}{3}t^{\frac{3}{5}} + C$

2. (a) $\frac{x^3}{3} + \frac{x^2}{2} + 3x + C$ (b) $7x - \frac{5}{2}x^2 - x^3 + C$ (c) $\frac{4}{3}t^3 + \frac{3}{2}t^2 - 2t + C$

   (d) $\frac{x^3}{3} + 3x^2 + 9x + C$ (e) $\frac{2}{3}x^{\frac{3}{2}} - 2x^{\frac{1}{2}} + C$ (f) $-\frac{2}{x} - x^3 + 4x + C$

   (g) $-\frac{5}{x} + \frac{1}{x^3} + C$ (h) $\frac{16}{3}x^{\frac{3}{2}} + \frac{1}{2}x^{\frac{1}{2}} + C$ (i) $\frac{1}{2}x^4 + 2x^{\frac{1}{2}} + \frac{2}{x} + C$

3. (a) $8\frac{2}{3}$ (b) $2\frac{1}{2}$ (c) $1\frac{1}{3}$

4. (a) $-3x^2 + x + C$ (b) $\frac{x^4}{4} + 4x^{\frac{3}{2}} + \frac{1}{x} + C$ (c) $\frac{x^2}{2} - \frac{7}{x} + C$
   (d) $4x - 6x^2 + 3x^3 + C$

5. (a) $67\frac{1}{2}$ (b) $2\frac{2}{3}$

6. a) $\frac{(1+x)^4}{4} + C$ b) $\frac{(3x-2)^5}{15} + C$ c) $\frac{(x^2+2)^5}{5} + C$

   d) $\frac{(x^3+4)^9}{27} + C$ e) $\frac{(2x^3-3x+9)^{\frac{4}{3}}}{4} + C$

   f) $\frac{-1}{5}\sqrt{3-5x^2}$+C g) $-\frac{1}{3}(1-x^2)^{\frac{3}{2}} + C$

7* (a) $\frac{2}{5}(x^2+3)^5 + C$ (b) $-\frac{5}{2}(7 - 2x^2)^{\frac{1}{2}} + C$

8. (ii) $10\frac{2}{3}$units^2