Tutorial on Set Theory

1. List the elements of the following sets.
   $A = \{x: x$ is an odd positive integer between 10 to 20$\}$
   $B = \{x: x \in$ set of integers, x is postive and is a multiple of 7$\}$
   $C = \{x: x \in$ set of integers, x is a solution of $2x + 1 = 2\}$

2. Given that $A = \{x|x$ is a multiple of $3\}$, $B = \{x|x$ is a multiple of $4\}$and $C = \{x|x$ is a multiple of $2\}$, find the following for values of x if x is a positive integer and less than 20:

   (a) $A \cup B$
   (b) $A \cup C$
   (c) $B \cup C$

3. Consider the set A where $A = \{x|x$ is a multiple of $2\}$, $B = \{x|x$ is a multiple of $6\}$
   Find $A \cap B$ given that x is a positive integer and less than 20.

4. Let S be a universal set consisting of the first seven letters of the English alphabets, that is, $S = \{a,b,c,d,e,f,g\}$. If $A = \{a,b,c,d\}$, find the complement of A.

5. If set $A = \{1,\ 3,\ 5,\ 7,\ 9\}$ and set $B = \{2,\ 4,\ 6,\ 8\}$, find the following if S is the universal set of integers from 1 to 9.

   (a) $A \cup B$ (b) $A^C \cap B$ (c) $A^C \cap B^C$

6. Let the universal $S = \{a, b, c, d, e\}$ with $A = \{a, b, d\}$ and $B = \{b, d, e\}$.
   Find   a)$A \cup B$ b) $B \cap A$ c)B^C d) $B - A$
              e)$A^C \cap B$ f) $A \cup B^C$ g)$A^C \cap B^C$ h) $B^C - A^C$
              i) $(A \cap B)^C$

7. Given the Universal set $S = \{x : x$ is the first 10 English alphabet$\}$, $A = \{a,b,c,d,e\}$
   and $B = \{c,d,e,f,g\}$, evaluate the following sets:
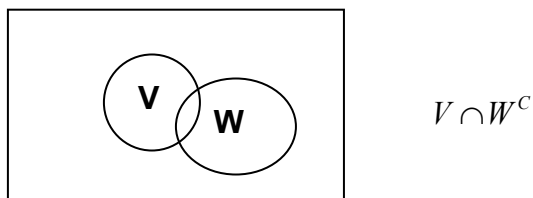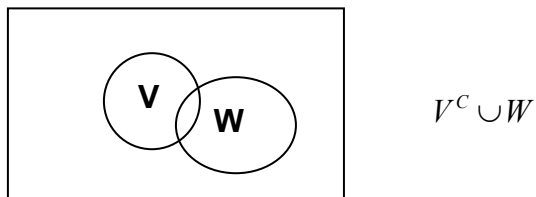   (a) $A \cup B'$ (b) $A' \cap B'$ (c) $A \cap B'$
       (d) $(A \cup B) \cap A'$ (e) $S \cap A$ (f) $S \cap (A \cap B)'$
       (g) $(A \cap B)' \cap B'$

8. In an online registration for prescribed electives, some students register for 'Accounting (A)' and some students 'Business (B)'. What events are represented by the shaded region? What does the set $A' \cup B$ represent? Use words only.

(a)   (b)   (c)

(d)
$$A \quad B$$

(e)
$$A \quad B$$

(f)
$$A \quad B$$
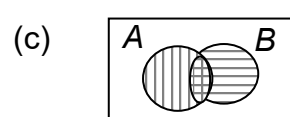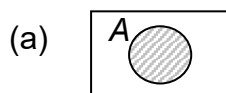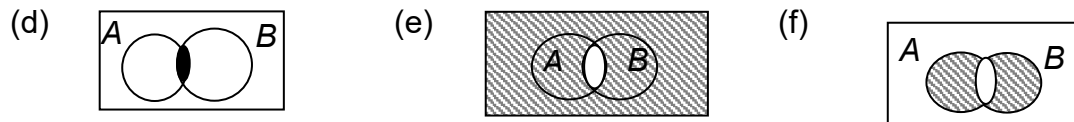
9. In the Venn diagrams below, shade $V \cap W$, $V^C \cup W$, $V \cap W^C$.

$$V \cap W$$

$$V^C \cup W$$

$$V \cap W^C$$
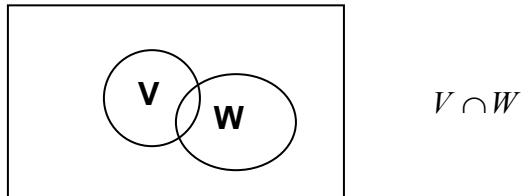
10. Prove the following the laws of algebra of sets:
   (a $(A \cup B) \cap (A \cup B^C) = A$
   (b) $(A^C \cap B) \cup (A^C \cap B^C) = A^C$
   (c) $(A \cap B) \cap (A^C \cup B^C) = \emptyset$
   (d) $(A \cup S') \cap (A \cup \emptyset'') = A$

11. Simplify the following using the laws of algebra of sets:
   (a) $(A \cap S)' \cup (B \cap \emptyset'')$
   (b) $(A' \cap B)' \cap (A \cup B)$

12. A poll of 100 students was taken at a school to find out how they travel to school. The results were as follows:
   28 mentioned car pools 9 used car pools and buses
   31 took buses 10 used car pools and sometimes their own cars
   42 drove to school 6 used buses as well as their own cars.
                                  4 used all three methods

    (a)   Draw a Venn diagram to represent the above results.
    (b)   How many students use none of the three methods?
    (c)   How many students use car pools exclusively to get to school?
    (d)   How many students use buses exclusively to get to school?

13. In XYZ Manufacturing Company, a quality inspector has inspected a sample of 28 components. Among these components, there are: 17 with assembly faults, 17 with defective parts and 18 with wrong colour coding; 7 with assembly faults and defective parts, 13 with defective parts and wrong colour coding, 9 with wrong colour coding and assembly faults. Using Venn diagram, find the number of components with all 3 faults.

Answers
1.
$A = \{11,13,15,17,19\}$
$B = \{7,14,21,28 \ldots\}$
Solution of $2x + 1 = 2$, $x = \frac{1}{2}$ (not an integer). Hence $C = \{\emptyset\}$

2.
$A \cup B = \{3,4,6,8,9,12,15,16,18\}$
$A \cup C = \{2,3,4,6,8,9,10,12,14,15,16,18\}$
$B \cup C = C = \{2,4,6,8,10,12,14,16,18\}$

3.
$A \cap B = \{6,12,18\}$

4.
$\boldsymbol{A^C = \{e, f, g\}}$

5.
$A \cup B = \{1,2,3,4,5,6,7,8,9\}$
$A^C \cap B = \{2,4,6,8\}$
$A^C \cap B^C = \emptyset$

6.
  a)$A \cup B = \{a, b, d, e\}$
  b)$B \cap A = A \cap B = \{b, d\}$
  c)$B^C = \{a, c\}$
  d)$B - A = \{e\}$
  e)$A^C \cap B = \{e\}$
  f)$A \cup B^C = \{a, b, c, d\}$
  g)$A^C \cap B^C = \{c\}$
  h)$B^C - A^C = \{a\}$
  i)$(A \cap B)^C = \{a, c, e\}$

7.
  a)$A \cup B' = \{a, b, c, d, e, h, i, j\}$
  b)$A' \cap B' = \{h, i, j\}$
  c)$A \cap B' = \{a, b\}$
  d)$(A \cup B) \cap A' = \{f, g\}$
  e)$S \cap A = A$
  f)$S \cap (A \cap B)' = \{a, b, f, g, h, i, j\}$
  g)$(A \cap B)' \cap B' = \{a, b, h, i, j\}$

8.
a) Students who registered for accounting
b) Students who did not register for accounting
c) Students who registered for accounting or for business or both
d) Students who registered for both accounting and business
e) Students who did not register for both accounting and business

f) Students who registered for only accounting or students that registered for only business.

12.
20, 13, 20

13.
5