Tutorial on Matrices

1. Express the following as a single matrix:

(a) $\begin{pmatrix} 7 & 11 \end{pmatrix} + \begin{pmatrix} 9 & 5 \end{pmatrix}$ (b) $3\begin{pmatrix} 1 & 2 \\ -5 & 6 \end{pmatrix} - \begin{pmatrix} -5 & 6 \\ 3 & -4 \end{pmatrix} + 2\begin{pmatrix} 1 & 3 \\ -2 & 0 \end{pmatrix}$

2. Evaluate (wherever possible) the following multiplications and explain why if it is not possible:

(a) $\begin{pmatrix} 3 & -1 \\ -2 & 3 \\ 1 & 5 \end{pmatrix}\begin{pmatrix} 2 & -1 \end{pmatrix}$ (b) $\begin{pmatrix} 3 & -1 \\ -2 & 3 \\ 1 & 5 \end{pmatrix}\begin{pmatrix} 2 \\ -1 \end{pmatrix}$

3. Find the values of x, y and z given that $2\begin{pmatrix} x & 3 \\ -2 & y \end{pmatrix} - \begin{pmatrix} -5 & z \\ 3 & 6 \end{pmatrix} = \begin{pmatrix} 7 & 2 \\ -7 & -6 \end{pmatrix}$.

4. Find the transpose of $\begin{pmatrix} 15 & 12 & -1 \\ -3 & 4 & 17 \end{pmatrix}$.

5. Evaluate the following determinants:

(a) $\begin{vmatrix} -\frac{1}{3} & \frac{1}{5} \\ -\frac{2}{3} & \frac{8}{5} \end{vmatrix}$ (b) $\begin{vmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 5 & 0 & 2 \end{vmatrix}$

6. Find the inverse (wherever possible) of the following matrices:

(a) $\begin{pmatrix} 3 & -2 \\ 14 & -7 \end{pmatrix}$ (b) $\begin{pmatrix} 3 & 2 \\ 3 & 2 \end{pmatrix}$

7. It is given that $A = \begin{pmatrix} 10 & 6 \\ 3 & -2 \end{pmatrix}$, find A^{-1}.

Hence solve the system of equations :

$$10x + 6y = 5$$
$$3x - 2y = 11$$

8. Solve the following system of equations using Cramer's Rule :

$$x + y + z = 12$$
$$x - y = 2$$
$$x - z = 4$$

9*. Find the values of *k* such that $\begin{vmatrix} k & 1 & 0 \\ 1 & k & 1 \\ 0 & 1 & k \end{vmatrix} = 0$.

10. At a soccer game, the number of student tickets, adult tickets and children's tickets (below 7 years old) sold is represented by x, y and z respectively.

The number of adult tickets sold was four times that of the children tickets. Also, the total number of children's tickets and adult tickets was half the number of student tickets sold. The total number of tickets sold was 150.

(a) Write down a system of three linear equations in terms of x, y and z.
(b) Write the equations obtained in (i) in a form of a matrix.

(c) Using Cramer's rule, determine how many of each type of tickets were sold.

Answers

1. (a) $\begin{pmatrix} 16 & 16 \end{pmatrix}$ (b) $\begin{pmatrix} 10 & 6 \\ -22 & 22 \end{pmatrix}$

2. (a) The product is undefined because the number of columns in the first matrix is not equal to the number of rows in the second matrix.

   (b) $\begin{pmatrix} 7 \\ -7 \\ -3 \end{pmatrix}$

3. $x = 1$, $y = 0$ and $z = 4$ 4. $\begin{pmatrix} 15 & -3 \\ 12 & 4 \\ -1 & 17 \end{pmatrix}$ 5. (a) $-\frac{2}{5}$ (b) 45

6. (a) $\begin{pmatrix} -1 & \frac{2}{7} \\ -2 & \frac{3}{7} \end{pmatrix}$ (b) inverse does not exist

7. $x = 2$ and $y = -\frac{5}{2}$ 8. $x = 6$, $y = 4$ and $z = 2$

9*. 0, $\pm\sqrt{2}$

10.

(a) $-y + 4z = 0 \ldots.(1)$
$-x + 2y + 2z = 0 \ldots.(2)$
$x + y + z = 150 \ldots.(3)$

(b) $\begin{bmatrix} 0 & -1 & 4 \\ -1 & 2 & 2 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 150 \end{bmatrix}$

(c) $x = 100,\ y = 40,\ z = 10$