

Ch3. 확률변수와 확률분포함수 Appendix

- $E[aX + b] = aE[X] + b$

$$\begin{aligned} E[aX + b] &= \sum_{all\ x} (ax + b)f(x) \\ &= \sum_{all\ x} axf(x) + \sum_{all\ x} bf(x) \\ &= a \sum_{all\ x} xf(x) + b \sum_{all\ x} f(x) \\ &= aE[X] + b \end{aligned}$$

- $V[X] = E[(X - \mu)^2] = E[X^2] - \mu^2$

$$\begin{aligned} V[X] &= E[(X - \mu)^2] \\ &= E[X^2 - 2\mu X + \mu^2] \\ &= E[X^2] - E[2\mu X] + \mu^2 \\ &= E[X^2] - 2\mu E[X] + \mu^2 \\ &= E[X^2] - \mu^2 \end{aligned}$$

- $V[aX + b] = a^2V[X]$

$$\begin{aligned} V[aX + b] &= E[(aX + b - E[aX + b])^2] \\ &= E[(aX + b - aE[X] - b)^2] \\ &= E[a^2(X - E[X])^2] \\ &= a^2E[(X - E[X])^2] \\ &= a^2V[X] \end{aligned}$$

- $COV[X, Y] = E[(X - \mu_X)(Y - \mu_Y)] = E[XY] - \mu_X\mu_Y$

$$\begin{aligned} E[(X - \mu_X)(Y - \mu_Y)] &= E[XY - X\mu_Y - \mu_X Y + \mu_X\mu_Y] \\ &= E[XY] - E[X\mu_Y] - E[\mu_X Y] + \mu_X\mu_Y \end{aligned}$$

$$\begin{aligned}
&= E[XY] - \mu_Y E[X] - \mu_X E[Y] + \mu_X \mu_Y \\
&= E[XY] - \mu_X \mu_Y
\end{aligned}$$

- $X' = aX + b$ 이고 $Y' = cY + d$ 일 때, $\sigma_{X'Y'} = ac \cdot \sigma_{XY}$

$$\begin{aligned}
&COV[aX + b, cY + d] \\
&= E[(aX + b - E[aX + b])(cY + d - E[cY + d])] \\
&= E[(aX + b - (aE[X] + b))(cY + d - (cE[Y] + d))] \\
&= E[(aX + b - a\mu_X - b)(cY + d - c\mu_Y - d)] \\
&= E[ac(X - \mu_X)(Y - \mu_Y)] \\
&= ac \cdot E[(X - \mu_X)(Y - \mu_Y)]
\end{aligned}$$

- $X' = aX + b$ 이고, $Y' = cY + d$ 이고, $ac > 0$ 일 때, $\rho_{X'Y'} = \rho_{XY}$

$$\begin{aligned}
&CORR[aX + b, cY + d] \\
&= \frac{COV[aX+b, cY+d]}{S[aX+b] S[cY+d]} \\
&= \frac{ac \cdot COV[X, Y]}{|a| S[X] |c| S[Y]} \\
&= \frac{ac \cdot COV[X, Y]}{|ac| \cdot S[X] S[Y]} \\
&= CORR[X, Y]
\end{aligned}$$

- $E[X \pm Y] = E[X] \pm E[Y]$

$$\begin{aligned}
E[X + Y] &= \sum_{all\ x} \sum_{all\ y} (x + y) f(x, y) \\
&= \sum_{all\ x} \sum_{all\ y} x f(x, y) + \sum_{all\ x} \sum_{all\ y} y f(x, y) \\
&= \sum_{all\ x} x \sum_{all\ y} f(x, y) + \sum_{all\ y} y \sum_{all\ x} f(x, y) \\
&= \sum_{all\ x} x f(x) + \sum_{all\ y} y f(y) \\
&= E[X] + E[Y]
\end{aligned}$$

- $V[X \pm Y] = V[X] + V[Y] \pm 2COV[X, Y]$

$$\begin{aligned}
& V[X + Y] \\
&= E[(X + Y - E[X + Y])^2] \\
&= E[(X + Y - E[X] - E[Y])^2] \\
&= E[((X - E[X]) + (Y - E[Y]))^2] \\
&= E[(X - E[X])^2 + (Y - E[Y])^2 + 2(X - E[X])(Y - E[Y])] \\
&= E[(X - E[X])^2] + E[(Y - E[Y])^2] + 2E[(X - E[X])(Y - E[Y])] \\
&= V[X] + V[Y] + 2COV[X, Y]
\end{aligned}$$

- **X 와 Y 가 서로 독립인 경우 $COV[X, Y] = 0$**

$$X \text{와 } Y \text{가 서로 독립} \Leftrightarrow f(x, y) = f(x)f(y) \Rightarrow E[XY] = E[X]E[Y]$$

$$\begin{aligned}
E[XY] &= \sum_{all \ x} \sum_{all \ y} xyf(x, y) \\
&= \sum_{all \ x} \sum_{all \ y} xyf(x)f(y) \\
&= \sum_{all \ x} xf(x) \sum_{all \ y} yf(y) \\
&= E[X]E[Y]
\end{aligned}$$