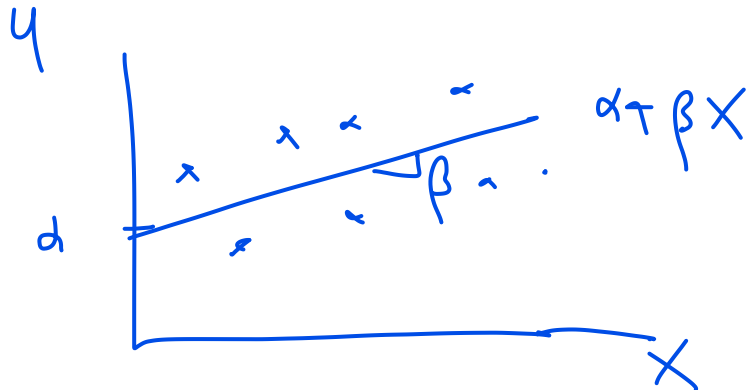


$$Y_i = \alpha + \beta x_i + \varepsilon_i$$

Diagram illustrating the components of the linear regression model. The equation is written with terms circled in red and blue. A red arrow points to the error term  $\varepsilon_i$ . The terms  $\alpha$  and  $\beta$  are circled in red, and  $x_i$  is circled in blue. The error term  $\varepsilon_i$  is circled in blue. The entire equation is underlined in red. Below the equation, the text "const." is written in red, and "r.v." is written in blue.



$$Y_i \sim N[\alpha + \beta x_i, \sigma^2]$$

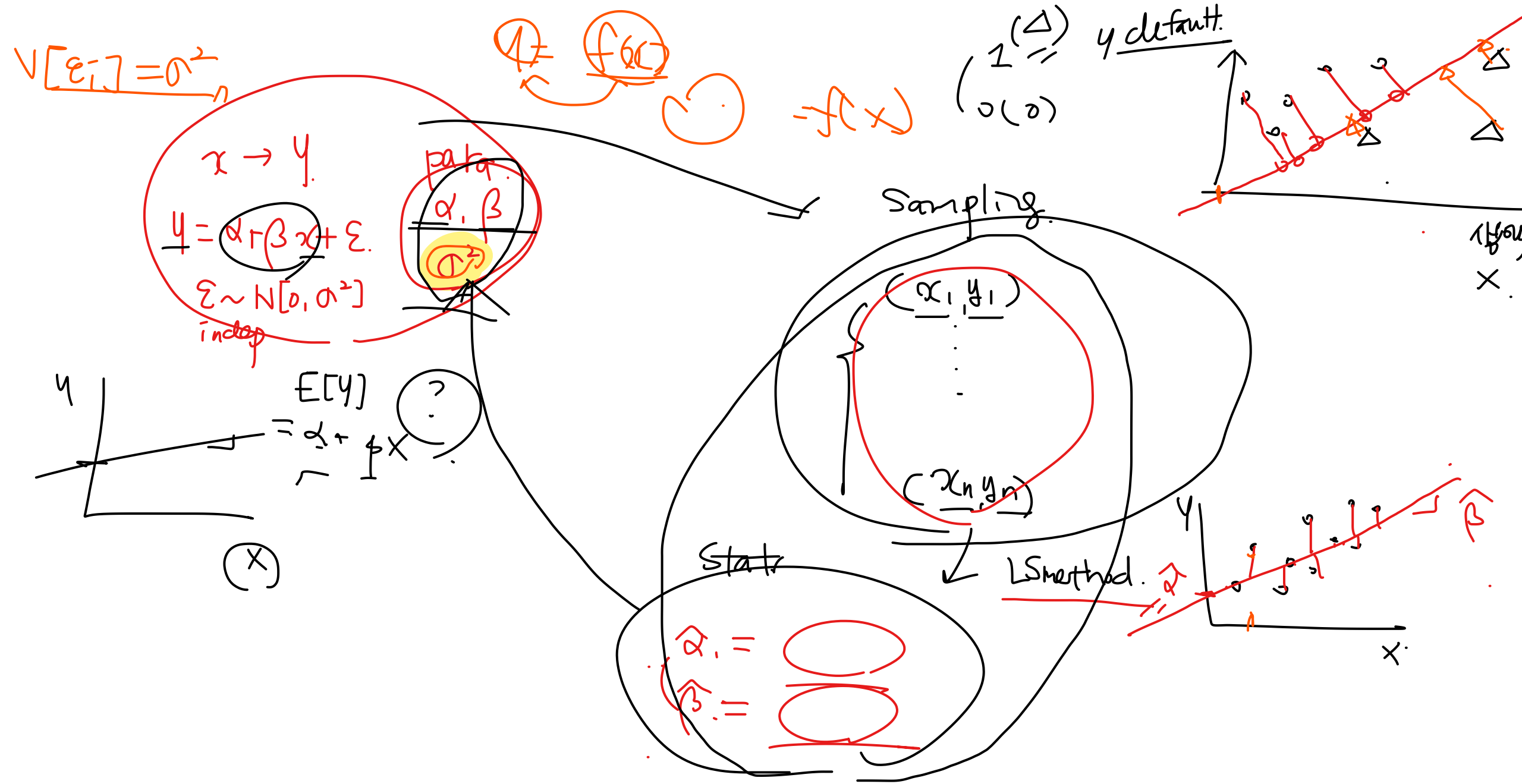
Diagram illustrating the distribution of the response variable  $Y_i$ . The equation is written with  $Y_i$  circled in red. The entire equation is enclosed in large yellow parentheses. Above the equation, the text "indep" is written in red.

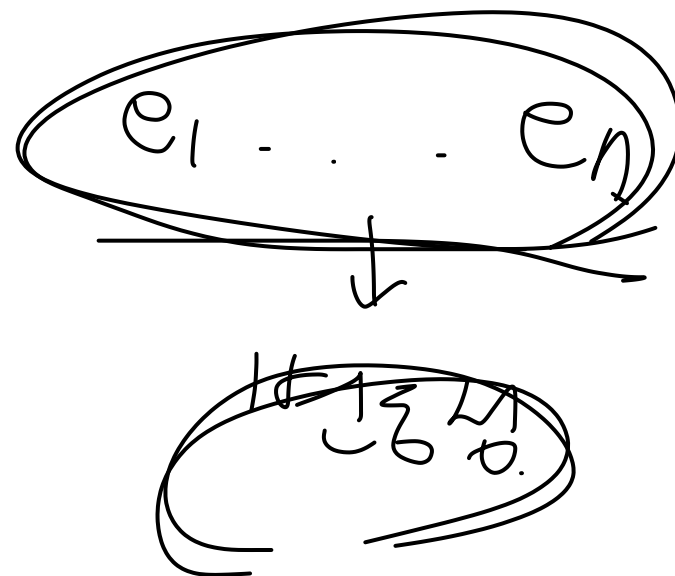
$$Y = \alpha + \beta x + \varepsilon$$

$$\varepsilon_i \sim N[0, \sigma^2]$$

Diagram illustrating the distribution of the error term  $\varepsilon_i$ . The equation is written with  $\varepsilon_i$  circled in red. The entire equation is enclosed in a red oval. Above the equation, the text "indep" is written in red. Below the equation, the text "r.v." is written in blue.

$$V[\varepsilon_i] = \sigma^2$$

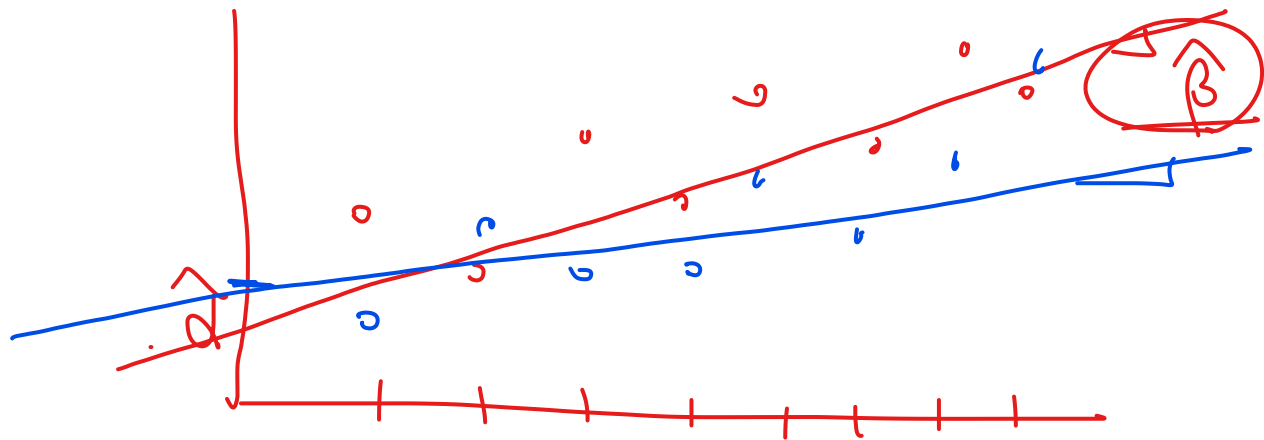




залежить від параметрів

$$N[\underline{\alpha + \beta x_i}, \sigma^2]$$

$$\begin{pmatrix} x_1, y_1 \\ \vdots \\ x_n, y_n \end{pmatrix} \sim N[\alpha + \beta x_i, \sigma^2]$$



$$S_{xx} = \sum (x_i - \bar{x})(x_i - \bar{x})$$

$$\hat{\alpha} = \bar{y} - \hat{\beta} \bar{x}$$

$$\hat{\beta} = \frac{\sum_{i=1}^n (x_i - \bar{x}) y_i}{\sum_{i=1}^n (x_i - \bar{x})^2}$$

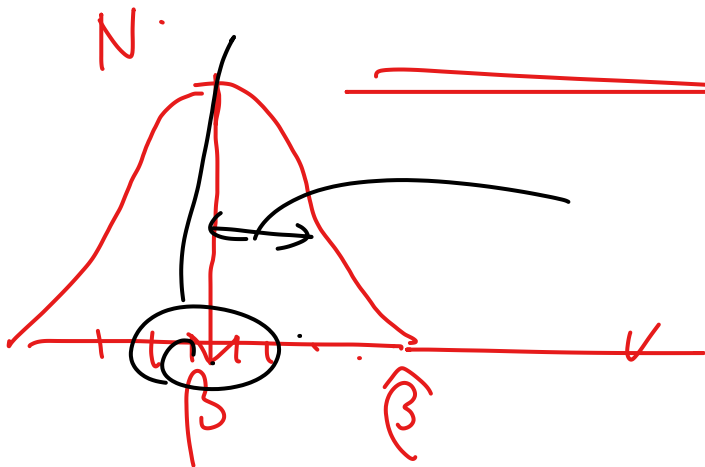
$x_1 \dots x_n$  незалежні

indep N.

$$\frac{n}{\sum_{i=1}^n} \frac{(x_i - \bar{x})}{S_{xx}} y_i$$

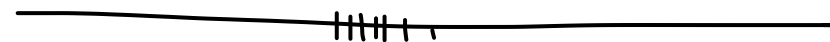
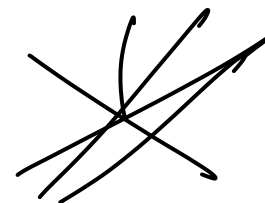
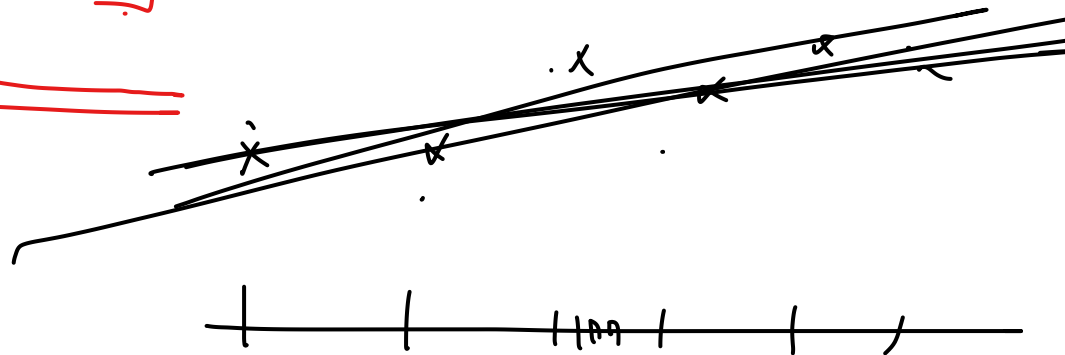
$$\hat{\beta} \sim N[\underline{\beta}, \frac{\sigma^2}{S_{xx}}]$$

$$X \sim N(\mu, \frac{\sigma^2}{n})$$



$$V[\hat{\beta}] = \frac{\sigma^2}{S_{xx}}$$

$$\sum_{i=1}^n (x_i - \bar{x})^2$$



$$\frac{E[\hat{\beta}]}{\beta}$$

$\hat{\beta} \stackrel{o}{\sim} \beta$  is the best UnB est.

X:  $\left( \frac{\text{SSE}}{\sigma^2} \sim \chi^2[n-2] \right)$

$$\sum_{i=1}^n (y_i - \hat{y}_i)^2$$

$$y_i \sim N[\alpha + \beta x_i, \sigma^2] \quad \text{i indep.}$$

$$\underline{\bar{z}_i} = \frac{(y_i - \alpha - \beta x_i)}{\sigma} \sim N[0, 1] \quad \text{i indep.}$$

$i=1 \dots n$

~~$$\sum_{i=1}^n \bar{z}_i^2$$~~

$$= \frac{\sum_{i=1}^n (y_i - \hat{\alpha} - \hat{\beta} x_i)^2}{\sigma^2} \sim \chi^2[n-2]$$

SSE

S

$$\beta$$

$$\hat{\beta} \sim N\left[\beta, \frac{\sigma^2}{S_{xx}}\right]$$

$$\underline{\underline{Z}} = \frac{\hat{\beta} - \beta}{\sqrt{\frac{\sigma^2}{S_{xx}}}} \sim \cancel{N[0, 1]}$$

$$\hat{\sigma}^2 = \frac{SSE}{n-2}$$

$$\underline{\underline{U}} = \frac{(n-2)\hat{\sigma}^2}{\sigma^2} \sim \chi^2[n-2]$$

$$\hat{\sigma}^2 = \frac{SSE}{n-2} = MSE.$$

$$\hat{\beta}, \hat{\sigma}^2 \stackrel{\text{indep.}}{\sim} \Rightarrow Z, U \stackrel{\text{indep.}}{\sim}$$

$$\frac{Z}{\sqrt{U/df}} \sim t[df]$$

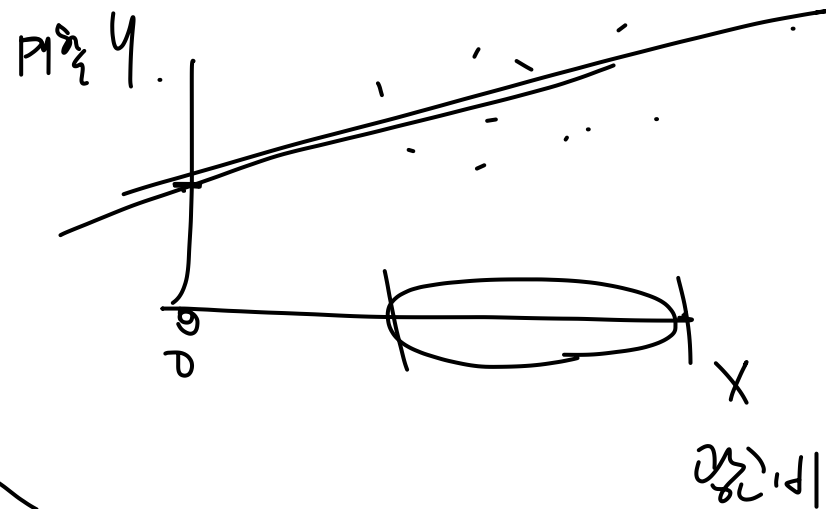
$$\frac{\frac{\hat{\beta} - \beta}{\sqrt{\cancel{\sigma^2}/S_{xx}}}}{\sqrt{\frac{(\cancel{n-2})\hat{\sigma}^2}{\cancel{\sigma^2}}}/(n-2)}} = \frac{\hat{\beta} - \beta}{\sqrt{\hat{\sigma}^2}/S_{xx}} \sim t[n-2]$$

$$\hat{y} = \hat{\alpha} + 0.8x$$

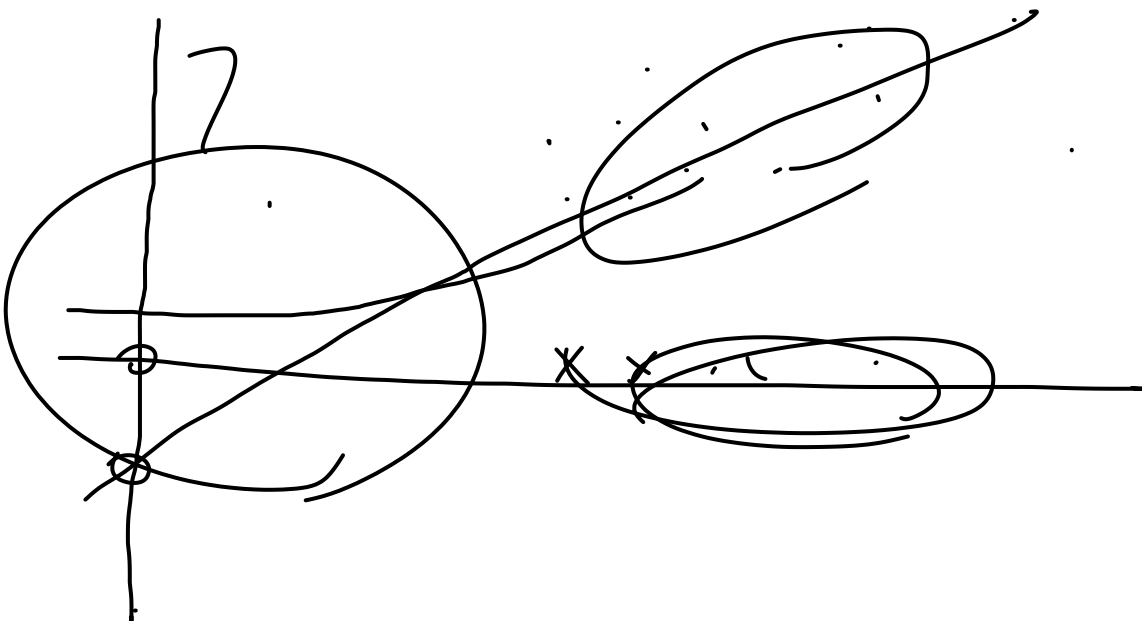
$$\hat{\alpha} = \hat{\alpha}$$

$$x=0 \rightarrow$$

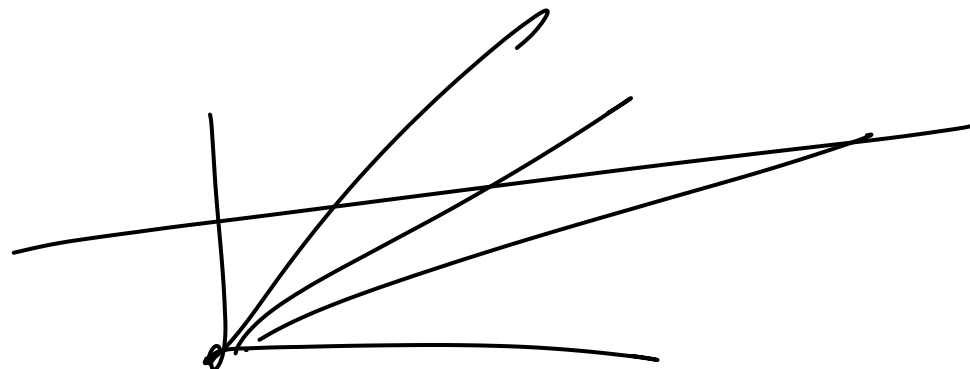
$$E[y]$$







$$\underline{y = \beta x + \varepsilon}$$



$T=1 \dots n$

$$\sum_{i=1}^n (\underbrace{y_i - \bar{y}}_{\text{}})^2 = \sum_{i=1}^n (\underbrace{\hat{y}_i - \bar{y}}_{\text{model}})^2$$

$\parallel$

SST.

$\parallel$

SSR

$$+ \sum_{i=1}^n (\underbrace{y_i - \hat{y}_i}_{\text{residual}})^2$$

$\parallel$

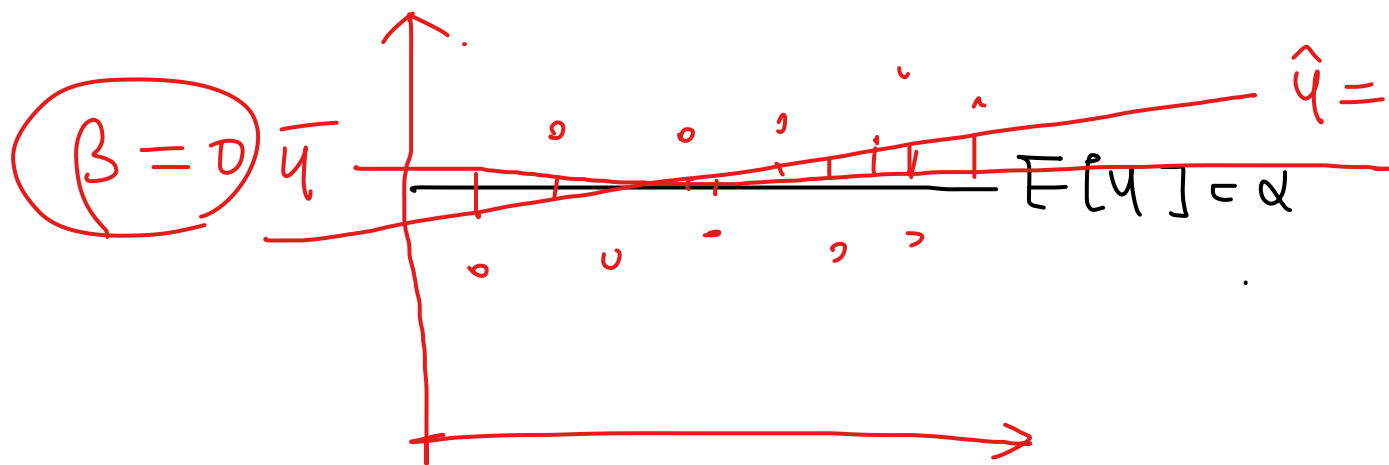
SSE.

$$SST = SSR + SSE.$$

$$\begin{array}{l} H_0: \beta = 0 \\ H_1: \beta \neq 0 \end{array}$$

$$SST = SSR + SSE$$

$$\sum (\hat{y}_i - \bar{y})^2$$



$$SSR$$

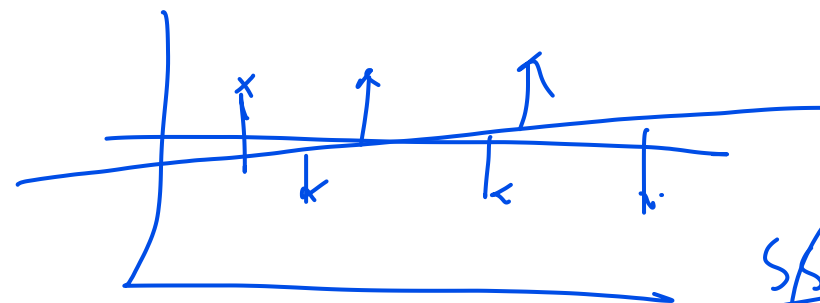
$$E[SSR] = H_0$$



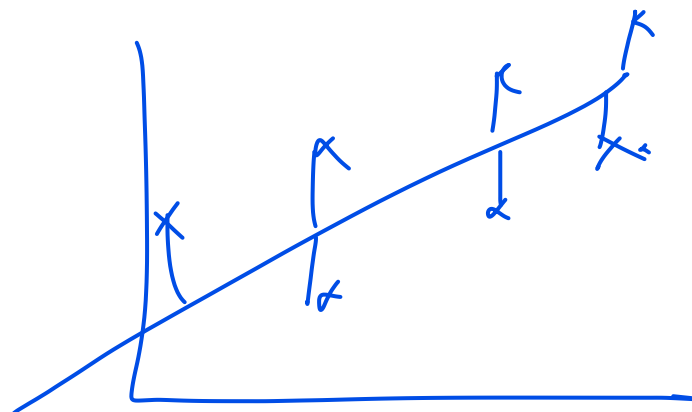


SSR,

SSR  $\uparrow$   $\text{res}$   $\uparrow$  to



$$\frac{SSE}{n-2}$$



$$\frac{SSE}{n-2}$$

$H_0: \beta = 0$   
 $H_1: \beta \neq 0$   $\rightarrow$   $SSR \uparrow$   $\rightarrow$   $\text{rej } H_0$

$$E[\text{MSR}] = \sigma^2$$

$H_0$   
 $H_1$

MSR  $\gg$   $\sigma^2$   $\Rightarrow$  rej  $H_0$

$\hat{\sigma}^2 = \text{MSE}$

$$\frac{MSR}{MSE} \approx 1 \quad H_0$$

$$\frac{MSR}{MSE} > 1. \quad \underline{H_1}$$

$H_0: \beta = 0$

$H_1: \beta \neq 0$

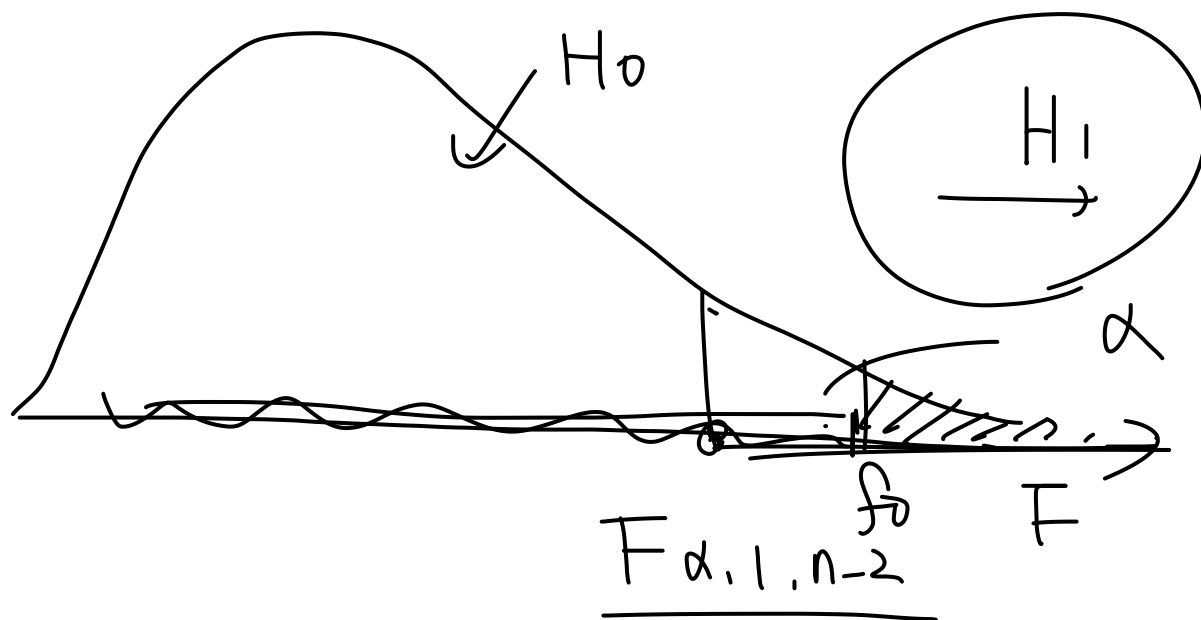
Under  $H_0$

$f_0$

$$P[F > f_0] \leq \alpha$$

$\uparrow$

$$F = \frac{MSR}{MSE} \sim F[1, n-2]$$



(X)

$$t_0 = \frac{\hat{\beta}}{\sqrt{\hat{\sigma}^2 / S_{xx}}}$$

$$T \sim t[n-2]$$

$\sum_{i=1}^n$

$$T^2 \sim F[1, n-2]$$

$$P[T > |t_0|] \times 2$$

$$= P[|T| > |t_0|]$$

$$F_0 = \frac{MSR}{MSE} = \frac{\hat{\beta}^2 S_{xx}}{\hat{\sigma}^2} = t_0^2$$

$$MSR = \sum (\hat{y}_i - \bar{y})^2$$

$$= \sum (\hat{\alpha} + \hat{\beta} x_i - \bar{y})^2$$

$$\bar{y} - \hat{\beta} \bar{x}$$

$$= \hat{\beta}^2 \sum_{i=1}^n (x_i - \bar{x})^2$$

$\sum_{i=1}^n$

$$F \sim F[1, n-2]$$



$$= P [ \underbrace{T^2}_{= F} > \underbrace{t_0^2}_{= f_0} ]$$

$$\underline{P[F > f_0]}$$



