Ch3. 확률변수와 확률분포함수 Appendix

- $\boldsymbol{E[aX+b]=aE[X]+b}$

$E[aX+b]$

$=\sum_{all\ x}(ax+b)f(x)$

$=\sum_{all\ x}axf(x)+\sum_{all\ x}bf(x)$

$=a\sum_{all\ x}xf(x)+b\sum_{all\ x}f(x)$

$=aE[X]+b$

- $\boldsymbol{V[X]=E[(X-\mu)^2]=E[X^2]-\mu^2}$

$V[X]=E[(X-\mu)^2]$

$=E[X^2-2\mu X+\mu^2]$

$=E[X^2]-E[2\mu X]+\mu^2$

$=E[X^2]-2\mu E[X]+\mu^2$

$=E[X^2]-\mu^2$

- $\boldsymbol{V[aX+b]=a^2V[X]}$

$V[aX+b]=E[(aX+b-E[aX+b])^2]$

$=E[(aX+b-aE[X]-b])^2]$

$=E[a^2(X-E[X]])^2]$

$=a^2E[(X-E[X]])^2]$

$=a^2V[X]$

- $\boldsymbol{COV[X,Y]\ =E[(X-\mu_X)(Y-\mu_Y)]=E[XY]-\mu_X\mu_Y}$

$E[(X-\mu_X)(Y-\mu_Y)]$

$=E[XY-X\mu_Y-\mu_XY+\mu_X\mu_Y]$

$=E[XY]-E[X\mu_Y]-E[\mu_XY]+\mu_X\mu_Y$

$= E[XY] - \mu_Y E[X] - \mu_X E[Y] + \mu_X \mu_Y$

$= E[XY] - \mu_X \mu_Y$

- $\boldsymbol{X' = aX + b}$ **이고** $\boldsymbol{Y' = cY + d}$ **일 때,** $\boldsymbol{\sigma_{X'Y'} = ac \cdot \sigma_{XY}}$

$COV[aX + b, cY + d]$

$= E[(aX + b - E[aX + b])(cY + d - E[cY + d])]$

$= E[(aX + \cancel{b} - (aE[X] + \cancel{b}))(cY + \cancel{d} - (cE[Y] + \cancel{d}))]$

$= E[(aX + \cancel{b} - a\mu_X - \cancel{b})(cY + \cancel{d} - c\mu_Y - \cancel{d})]$

$= E[ac(X - \mu_X)(Y - \mu_Y)]$

$= ac \cdot E[(X - \mu_X)(Y - \mu_Y)]$

E[aX]

- $\boldsymbol{X' = aX + b}$ **이고,** $\boldsymbol{Y' = cY + d}$**이고,** $\boldsymbol{ac > 0}$ **일 때,** $\boldsymbol{\rho_{X'Y'} = \rho_{XY}}$

$ac<0$ $\rho_{x'y'} = -\rho_{xy}$.

$CORR[aX + b, cY + d]$

$= \frac{COV[aX+b, cY+d]}{S[aX+b]\ S[cY+d]}$

$= \frac{ac \cdot COV[X,Y]}{|a|S[X]\ |c|S[Y]}$

$= \frac{ac\ COV[X,Y]}{|ac|\ S[X]S[Y]}$

$= CORR[X, Y]$

$S[ax+b]$

$= \sqrt{V[ax+b]}$

$= \sqrt{a^2 V[x]}$

$= |a|\ S[x]$

- $\boldsymbol{E[X \pm Y] = E[X] \pm E[Y]}$

$E[X + Y] = \Sigma_{all\ x} \Sigma_{all\ y} (x + y) f(x, y)$

$= \Sigma_{all\ x} \Sigma_{all\ y}\, x f(x, y) + \Sigma_{all\ y} \Sigma_{all\ x}\, y f(x, y)$

$= \Sigma_{all\ x}\, x \Sigma_{all\ y} f(x, y) + \Sigma_{all\ y}\, y \Sigma_{all\ x} f(x, y)$

$= \Sigma_{all\ x}\, x f(x) + \Sigma_{all\ y}\, y f(y)$

$= E[X] + E[Y]$

$E[x]$

$= \Sigma x f(x)$

$g(x)$

$E[x^2 + 2x]$

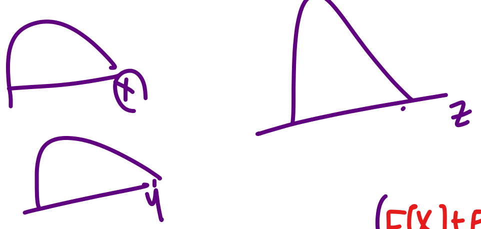
$= E[x^2] + E[2x]$

$f(x) = \sum_{all\ y} f(x, y)$

$E[① + ② + ③]$

$= E[①] + E[②] + E[③]$

- $\boldsymbol{V[X \pm Y] = V[X] + V[Y] \pm 2COV[X, Y]}$



$V[\bullet] = E[(\bullet - E[\bullet])^2]$

$E[(A_1 + \dots + A_n)^2]$

$E[(A+B+C)^2]$

$E[(A \pm B)^2] = E[A^2 + B^2 \pm 2AB]$

$= E[A^2] + E[B^2] \pm 2E[AB]$

$(E[X]+E[Y])$

$V[X \pm Y]$

$= E[(X \pm Y - E[X \pm Y])^2]$

$= E[(X \pm Y - E[X] \mp E[Y])^2]$

$= E[(\underbrace{(X - E[X])}_{A} \pm \underbrace{(Y - E[Y])}_{B})^2]$

$= E[(X - E[X])^2 + (Y - E[Y])^2 \pm 2(X - E[X])(Y - E[Y])]$

$= E[(X - E[X])^2] + E[(Y - E[Y])^2] \pm 2E[(X - E[X])(Y - E[Y])]$

$= V[X] + V[Y] \pm 2COV[X,Y]$

$Cov[X,Y] = E[XY] - E[X]E[Y]$ (= 0)

공분산 = 0.

- ***X*와 *Y*가 서로 독립인 경우 $COV[X,Y] = 0$**

X와 Y가 서로 독립 $\Leftrightarrow$ $f(x,y) = f(x)f(y)$ $\underset{\not\Leftarrow}{\overset{O}{\Rightarrow}}$ $E[XY] = E[X]E[Y]$

독립

$E[XY] = \sum_{all\ x} \sum_{all\ y} xy f(x,y)$

$= \sum_{all\ x} \sum_{all\ y} xy f(x) f(y)$

$= \sum_{all\ x} x f(x) \sum_{all\ y} y f(y)$

$= E[X]E[Y]$ $\qquad$ $E[Y]$

사건. A. B 독립.

$P(A \mid B) = P(A)$

$\Leftrightarrow$ $P(A \cap B) = P(A)P(B)$

$P(X=x, Y=y) = P(X=x)P(Y=y)$

독립.

모든 x, y에 대해 성립.

X, Y 독립.

$f(x,y) = f(x)f(y)$

독립 $\xrightarrow{O}$ Cov = 0. 선형관계 X