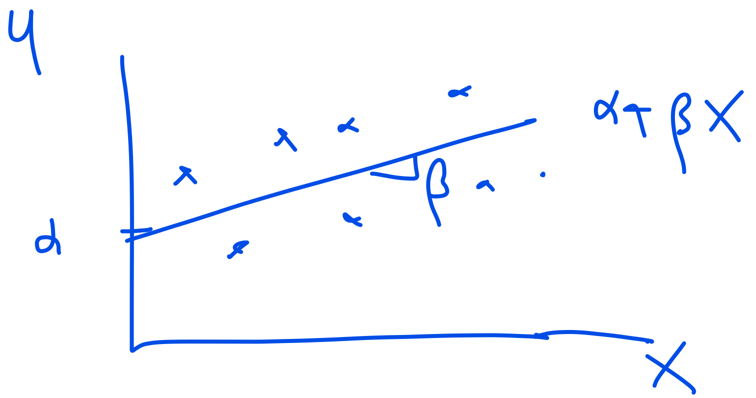
$$Y_i = \underbrace{\alpha + \beta x_i}_{\text{const.}} + \underbrace{\varepsilon_i}_{\text{r.v}}$$

$$y = \alpha + \beta x + \varepsilon .$$



$$\varepsilon_i \overset{\text{indep}}{\sim} N[0, \sigma^2]$$

$$V[\varepsilon_i] = \sigma^2$$

$$\left(Y_i \overset{\text{indep}}{\sim} N[\alpha + \beta x_i , \sigma^2] \right)$$

$V[\varepsilon_i] = \sigma^2$

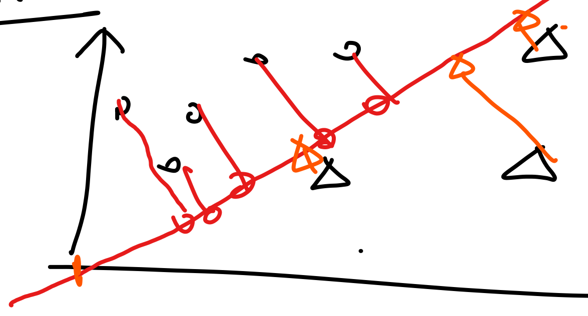
$y \neq f(x)$

$= f(x)$

$1 \, (\triangle)$ //

$0 \, (0)$

y default



$x \to y$

$y = \alpha + \beta x + \varepsilon$

$\varepsilon \sim N[0, \sigma^2]$

indep

para.

α, β

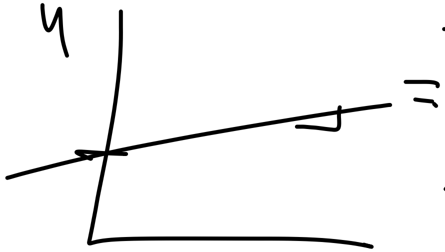
σ^2

Sampling

(x_1, y_1)

$\vdots$

(x_n, y_n)



$E[y] = \alpha + \beta X$

?

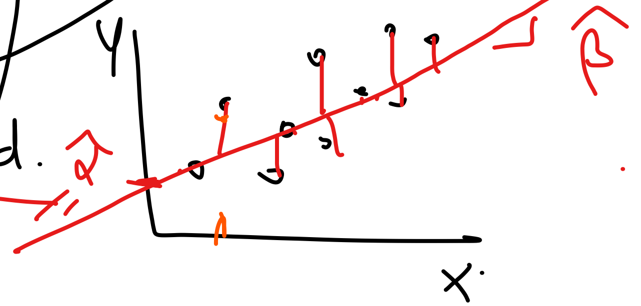
(X)

LS method

Stat

$\hat{\alpha} =$

$\hat{\beta} =$



$\hat{\alpha}$

$\hat{\beta}$

$V[\varepsilon_i] = \sigma^2$

$\cancel{\varepsilon_1 \cdots \varepsilon_n}$ $\downarrow$ 비관측

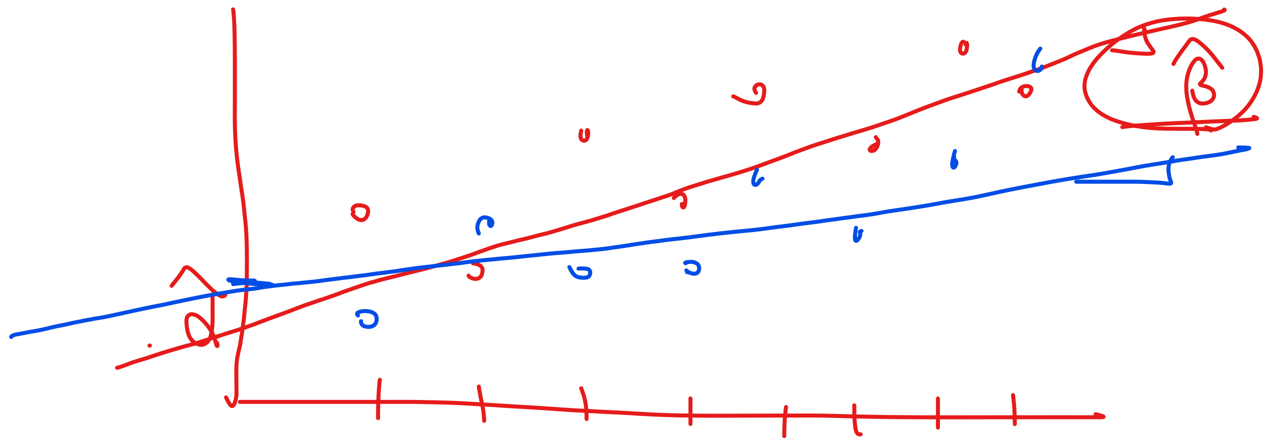
$e_1 \cdots e_n$ $\downarrow$ 비관측

공간에 매핑해

(x_1, Y_1) $\sim N[\alpha + \beta x_1, \sigma^2]$

$\vdots$

(x_n, Y_n) $\sim N[\alpha + \beta x_n, \sigma^2]$



S_{xx} $\quad \sum (x_i - \bar{x})(x_i - \bar{x})$

$$\hat{\alpha} = \bar{Y} - \hat{\beta}\bar{x} \sim ?$$

$$\hat{\beta} = \frac{\sum_{i=1}^{n} (x_i - \bar{x}) Y_i}{\sum_{i=1}^{n} (x_i - \bar{x})^2} \sim ?$$

$$\sum_{i=1}^{n} \frac{(x_i - \bar{x})}{S_{xx}} Y_i$$

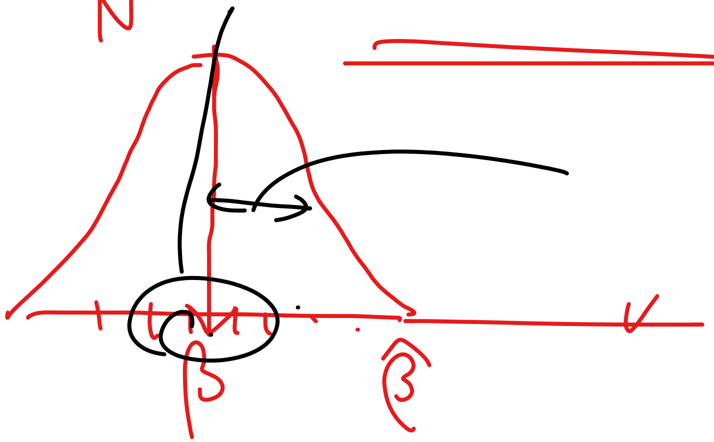
indep N.

$x_1 \ldots x_n$ 주어진다

$$\hat{\beta} \sim N\left[\beta, \frac{\sigma^2}{S_{xx}}\right]$$

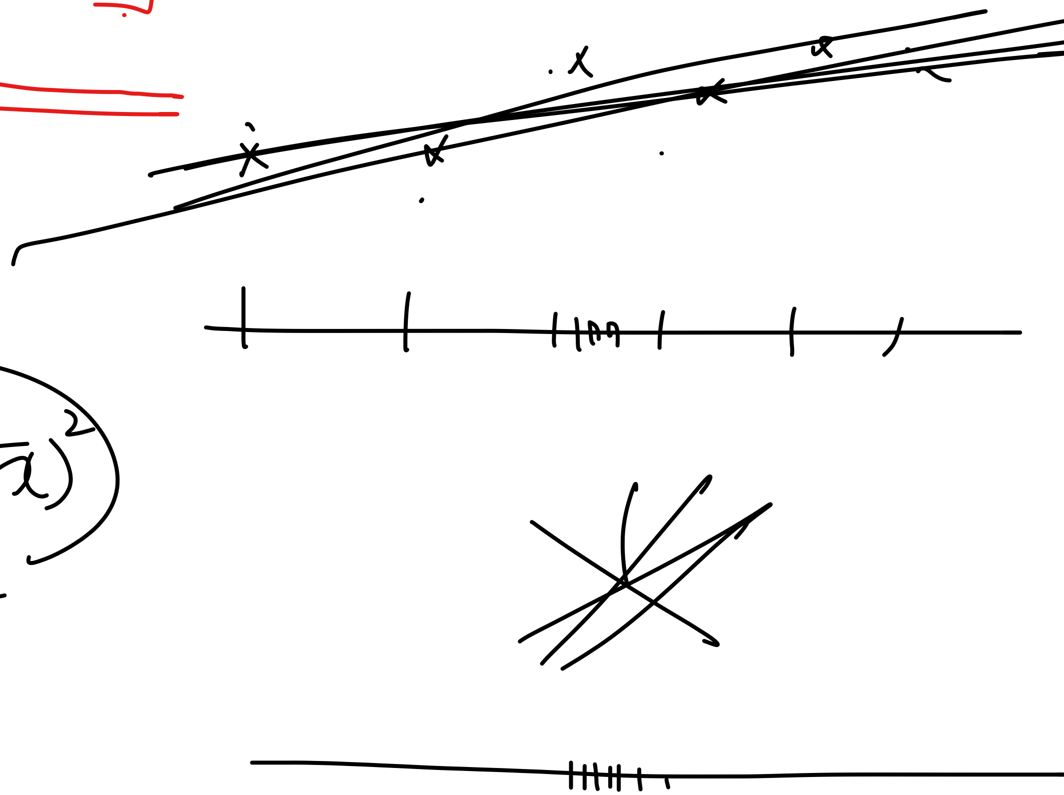
$$\bar{X} \sim N\left[\mu, \frac{\sigma^2}{n}\right]$$

N.



$$V[\hat{\beta}] = \frac{\sigma^2}{S_{xx}} \Uparrow \quad \Downarrow$$

$$S_{xx} = \sum_{i=1}^{n} (x_i - \bar{x})^2$$



$$E[\hat{\beta}] = \beta$$

$\hat{\beta}$는 β에 대한 UnB est.

$$\therefore \quad \frac{SSE}{\sigma^2} \sim \chi^2[n-2]$$

$$\sum_{i=1}^{n}(y_i - \hat{y}_i)^2$$

$$y_i \sim N[\alpha + \beta x_i, \sigma^2] \quad \text{indep.}$$

$$Z_i = \frac{y_i - \alpha - \beta x_i}{\sigma} \sim N[0, 1] \quad \text{indep.}$$

$i = 1 \ldots n$

SSE

S^2

$$\cancel{\sum_{i=1}^{n} Z_i^2} = \left(\frac{\sum_{i=1}^{n}(y_i - \hat{\alpha} - \hat{\beta} x_i)^2}{\sigma^2}\right) \sim \chi^2[n-2]$$

β

$$\hat{\beta} \sim N\left[\beta, \frac{\sigma^2}{S_{xx}}\right] \rightarrow Z = \frac{\hat{\beta} - \beta}{\sqrt{\hat{\sigma}^2 / S_{xx}}} \sim \cancel{N[0,1]}$$

$$\hat{\sigma}^2 = \frac{SSE}{n-2}$$

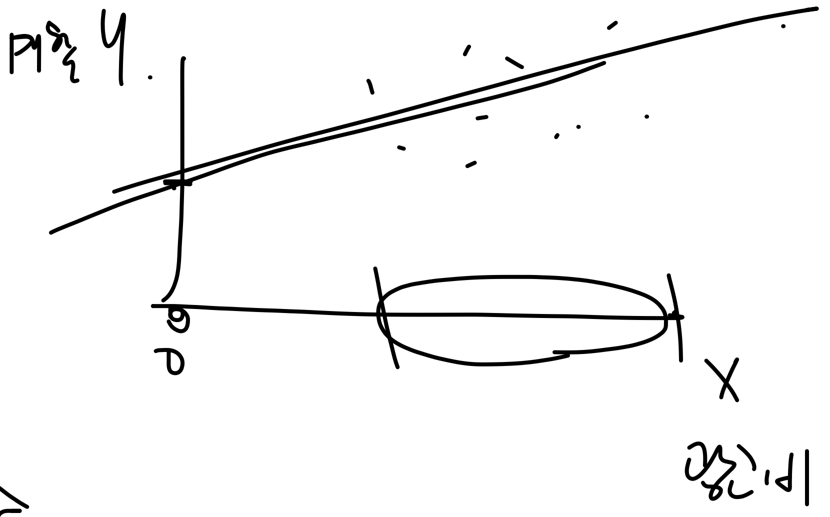
$$\hat{\sigma}^2 = \frac{SSE}{n-2} = MSE.$$

$$U = \frac{(n-2)\hat{\sigma}^2}{\sigma^2} \sim \chi^2[n-2]$$

$\hat{\beta}$, $\hat{\sigma}^2$ 독립 $\Rightarrow$ Z, U 독립

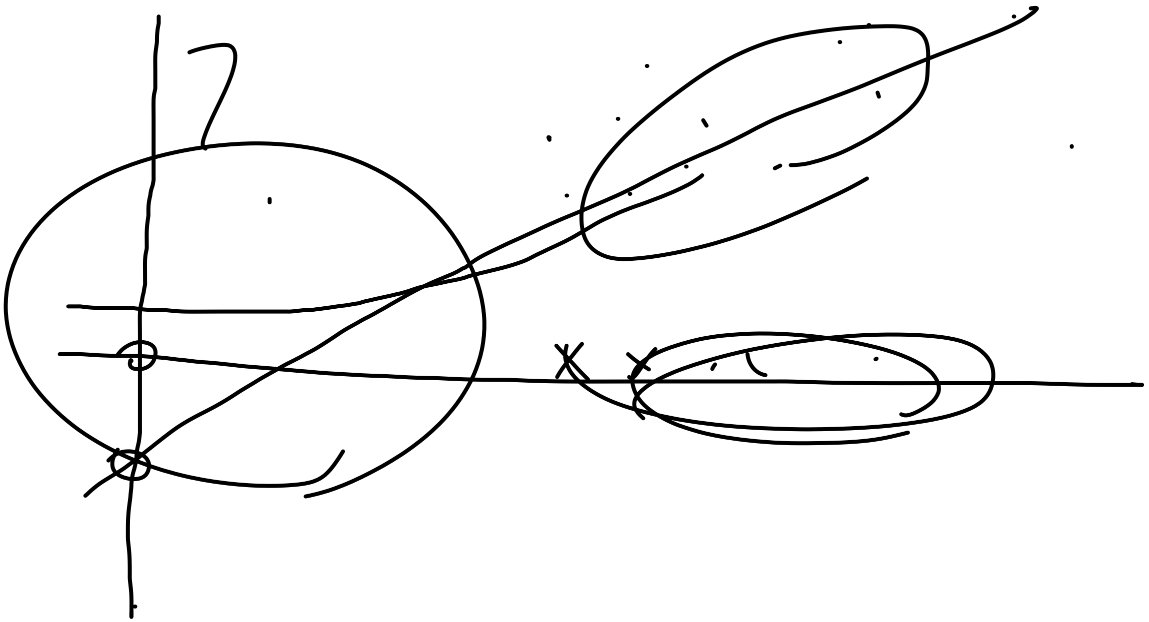
$$\frac{Z}{\sqrt{U/df}} \sim t[df]$$

$$\frac{\dfrac{\hat{\beta}-\beta}{\sqrt{\cancel{\sigma^2}/S_{xx}}}}{\sqrt{\left(\dfrac{\cancel{(n-2)}\hat{\sigma}^2}{\cancel{\sigma^2}}\right)\Big/\cancel{(n-2)}}} = \boxed{\dfrac{\hat{\beta}-\beta}{\sqrt{\hat{\sigma}^2/S_{xx}}}} \sim t[n-2]$$

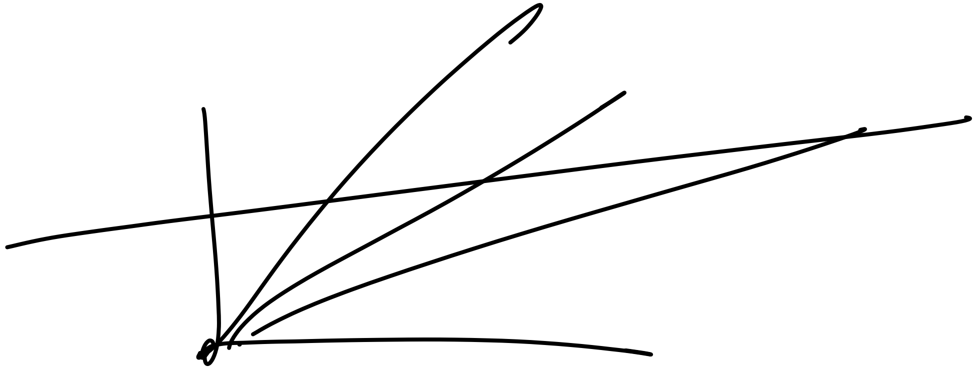


$$\hat{y} = 5 + 0.8x$$

$$\hat{\alpha} = 5 \qquad x = 0 \longrightarrow \widehat{E[y]}$$



$$y_{.} = \beta X_{.} + \varepsilon$$



$i=1 \ldots n$

$$\underbrace{\sum_{i=1}^{n}(y_i-\bar{y})^2}_{= \text{SST.}} = \underbrace{\sum_{i=1}^{n}(\hat{y}_i-\bar{y})^2}_{\substack{\text{model} \\ = \text{SSR}}} + \underbrace{\sum_{i=1}^{n}(y_i-\hat{y}_i)^2}_{\substack{\text{residual.} \\ = \text{SSE.}}}$$

$$SST = SSR + SSE.$$

$H_0: \beta = 0$

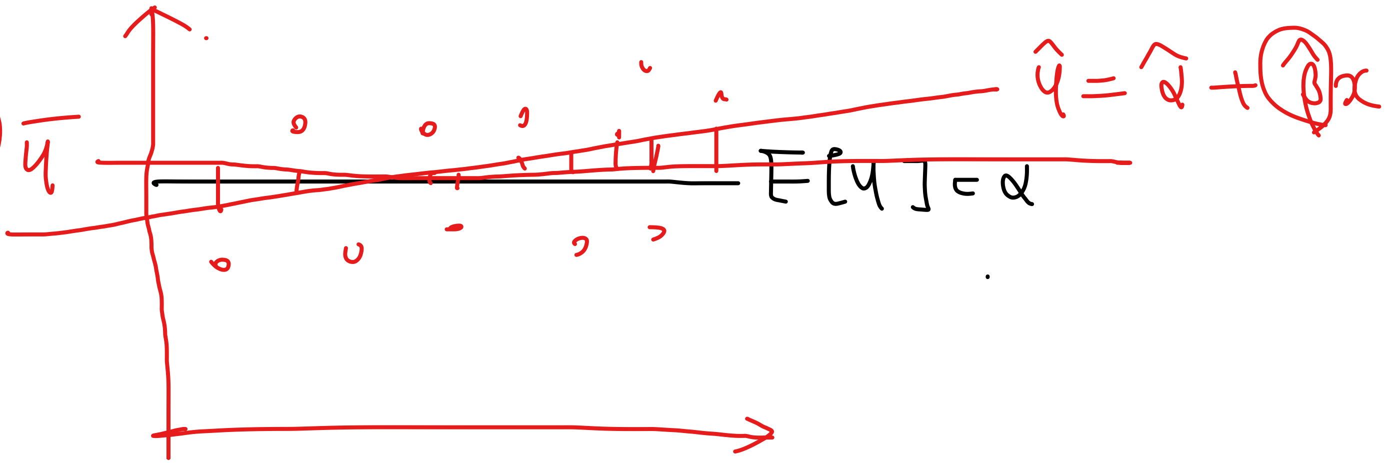
$H_1: \beta \neq 0$

$SST = SSR + SSE$

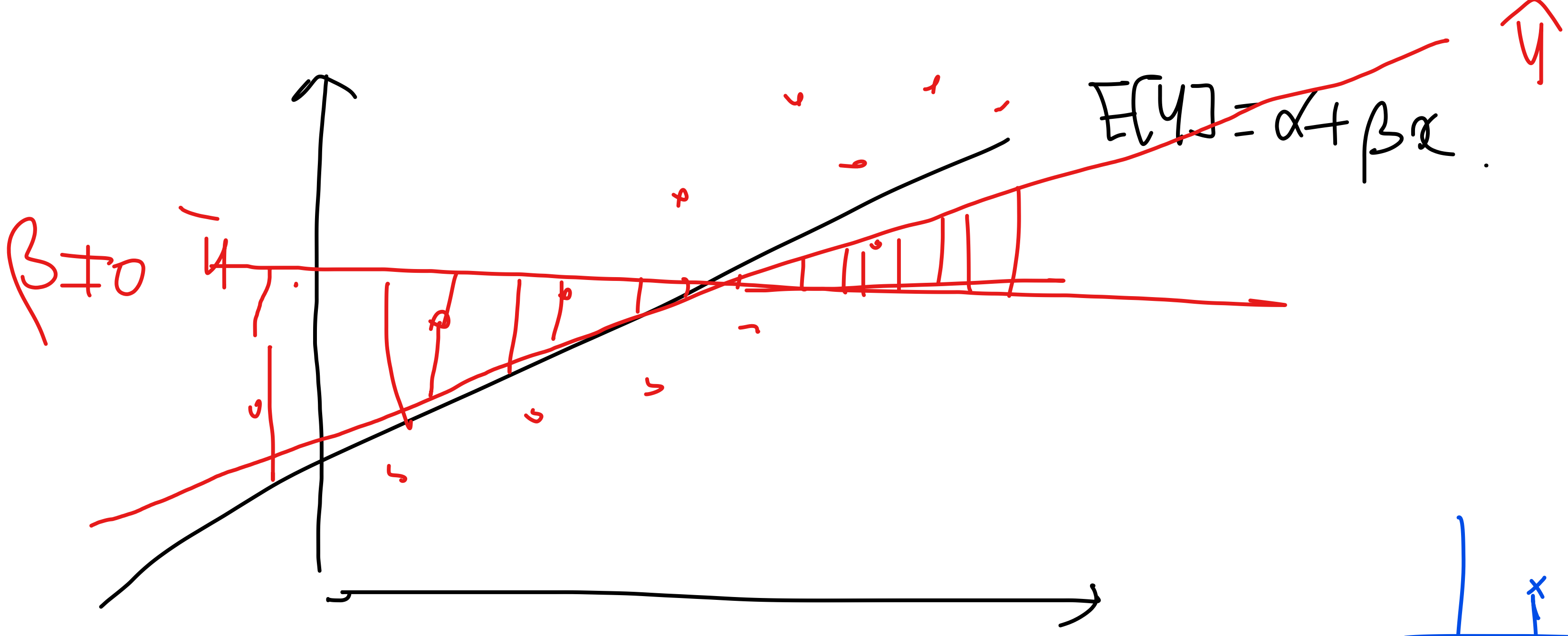
$\sum(\hat{y}_i - \bar{y})^2$

SSR

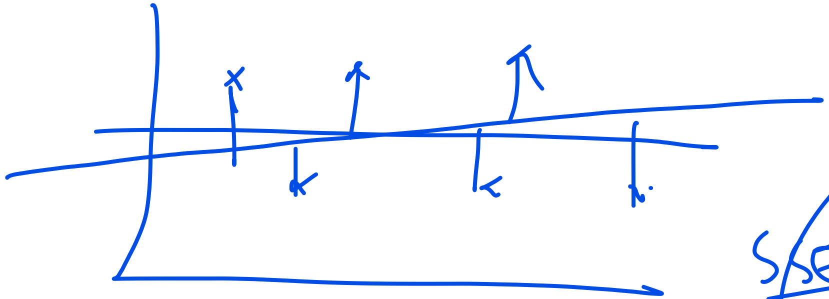
$\beta = 0$



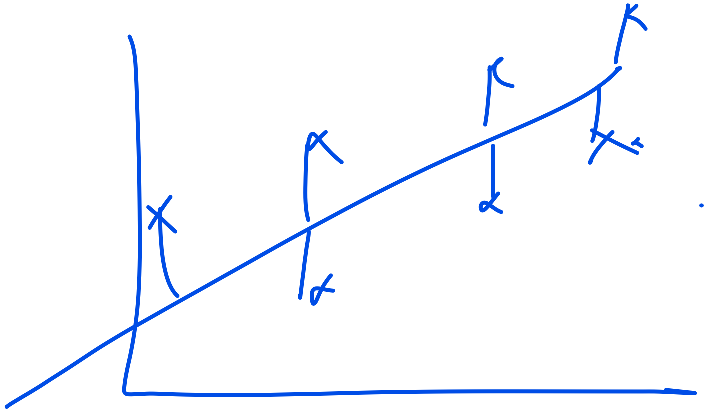
$E[SSR] \overset{H_0}{=}$



SSR



$\frac{SSE}{n-2}$



$\frac{SSE}{n-2}$

SSR $\uparrow$ rej H_0

$H_0: \beta = 0$

$H_1: \beta \neq 0$ $\longrightarrow$ SSR $\uparrow$ rej H_0

$E[MSR] = \sigma^2$ H_0

$> \sigma^2$ H_1

$\underline{MSR} \gg \sigma^2 \longrightarrow$ rej H_0

$\hat{\sigma}^2 = MSE$

$$\frac{MSR}{MSE} \approx 1 \qquad H_0$$

$$\frac{MSR}{MSE} > 1 \qquad H_1$$

$H_0: \beta = 0$

$H_1: \beta \neq 0$

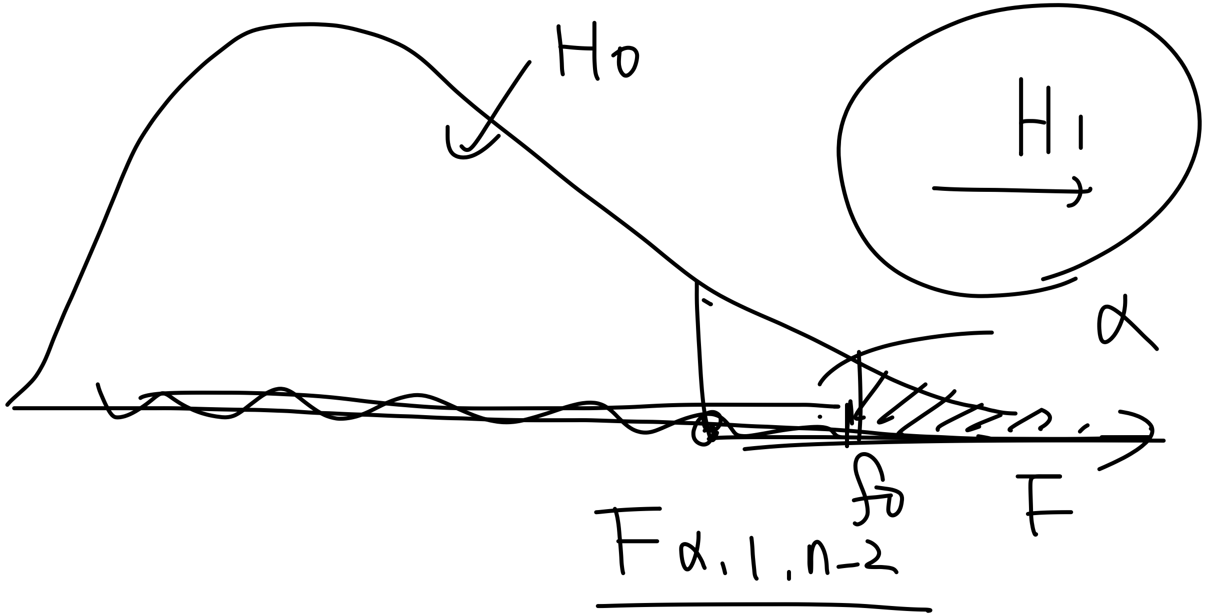
Under H_0

$\Uparrow$

$$F = \frac{MSR}{MSE} \sim F[1, n-2]$$

f_0.

$$P[F > f_0] \leq \alpha.$$



$$t_0 = \frac{\hat{\beta}}{\sqrt{\hat{\sigma}^2 / S_{xx}}}$$

$$T \sim t[n-2]$$

이므로

$$T^2 \sim F[1, n-2]$$

$$P[T > |t_0|] \times 2$$

$$= P[\,|T| > |t_0|\,]$$

$$f_0 = \frac{MSR}{MSE} = \frac{\hat{\beta}^2 S_{xx}}{\hat{\sigma}^2} = t_0^2$$

$$MSR = \sum (\hat{y}_i - \bar{y})^2$$

$$= \sum (\hat{\alpha} + \hat{\beta} x_i - \cancel{\bar{y}})^2$$

$$\hat{\alpha} = \cancel{\bar{y}} - \hat{\beta}\bar{x}$$

$$= \hat{\beta}^2 \sum_{i=1}^{n} (x_i - \bar{x})^2$$

$$\hat{\sigma}^2$$

$$F \sim F[1, n-2]$$

$$= P\,[\underbrace{T^2}_{=F} > \underbrace{t_0^2}_{=f_0}]$$

$$\frac{P[F > f_0]}{}$$