Ch3. 확률변수와 확률분포함수 Appendix

- $\boldsymbol{E[aX+b] = aE[X]+b}$

$g(X)$ $\sum g(x)f(x)$

$E[aX+b]$

$= \sum_{all\ x}(ax+b)f(x)$

$= \sum_{all\ x} axf(x) + \sum_{all\ x} bf(x)$

$= a\sum_{all\ x} xf(x) + b\underbrace{\sum_{all\ x} f(x)}_{=1}$

$= aE[X]+b$

- $\boldsymbol{V[X] = E[(X-\mu)^2] = E[X^2]-\mu^2}$

$V[X] = E[(X-\mu)^2]$

$= E[X^2 - 2\mu X + \mu^2]$ $E[\mu^2]$

$= E[X^2] - E[2\mu X] + \mu^2$

$= E[X^2] - \underbrace{2\mu E[X]}_{-2\mu^2} + \mu^2$

$= E[X^2] - \mu^2$

- $\boldsymbol{V[aX+b] = a^2 V[X]}$

Y $E[(Y - E[Y])^2]$

$V[aX+b] = E[(aX+b-E[aX+b])^2]$

$= E[(aX+\cancel{b}-aE[X]-\cancel{b})^2]$

$= E[a^2(X-E[X])^2]$

$= a^2 E[(X-E[X])^2]$

$= a^2 V[X]$

- $\boldsymbol{COV[X,Y] = E[(X-\mu_X)(Y-\mu_Y)] = E[XY]-\mu_X\mu_Y}$

$E[(X-\mu_X)(Y-\mu_Y)]$

$= E[XY - X\mu_Y - \mu_X Y + \mu_X\mu_Y]$

$= E[XY] - E[X\mu_Y] - E[\mu_X Y] + \mu_X\mu_Y$

$= E[XY] - \mu_Y E[X] - \mu_X E[Y] + \mu_X \mu_Y$

$= E[XY] - \mu_X \mu_Y$

- $\boldsymbol{X' = aX + b}$ **이고** $\boldsymbol{Y' = cY + d}$ **일 때,** $\boldsymbol{\sigma_{X'Y'} = ac \cdot \sigma_{XY}}$

$COV[aX + b, cY + d]$

$= E[(aX + b - E[aX + b])(cY + d - E[cY + d])]$

$= E[(aX + b - (aE[X] + b))(cY + d - (cE[Y] + d))]$

$= E[(aX + b - a\mu_X - b)(cY + d - c\mu_Y - d)]$

$= E[ac(X - \mu_X)(Y - \mu_Y)]$

$= ac \cdot E[(X - \mu_X)(Y - \mu_Y)]$

- $\boldsymbol{X' = aX + b}$ **이고,** $\boldsymbol{Y' = cY + d}$**이고,** $\boldsymbol{ac > 0}$ **일 때,** $\boldsymbol{\rho_{X'Y'} = \rho_{XY}}$

$CORR[aX + b, cY + d]$

$= \frac{COV[aX+b, cY+d]}{S[aX+b]\ S[cY+d]}$

$= \frac{ac \cdot COV[X,Y]}{|a|S[X]\ |c|S[Y]}$

$= \frac{ac \cdot COV[X,Y]}{|ac| \cdot S[X]S[Y]}$

$= CORR[X, Y]$

- $\boldsymbol{E[X \pm Y] = E[X] \pm E[Y]}$

$E[X + Y] = \sum_{all\ x} \sum_{all\ y} (x + y) f(x, y)$

$= \sum_{all\ x} \sum_{all\ y} x f(x, y) + \sum_{all\ x} \sum_{all\ y} y f(x, y)$

$= \sum_{all\ x} x \sum_{all\ y} f(x, y) + \sum_{all\ y} y \sum_{all\ x} f(x, y)$

$= \sum_{all\ x} x f(x) + \sum_{all\ y} y f(y)$

$= E[X] + E[Y]$

- $\boldsymbol{V[X \pm Y] = V[X] + V[Y] \pm 2COV[X, Y]}$

$V[X+Y]$

$= E[(X+Y-E[X+Y])^2]$

$= E[(X+Y-E[X]-E[Y])^2]$

$= E[((X-E[X])+(Y-E[Y]))^2]$

$= E[(X-E[X])^2+(Y-E[Y])^2+2(X-E[X])(Y-E[Y])]$

$= E[(X-E[X])^2]+E[(Y-E[Y])^2]+2E[(X-E[X])(Y-E[Y])]$

$= V[X]+V[Y]+2COV[X,Y]$

- **X와 Y가 서로 독립인 경우 $COV[X,Y]=0$**

  X와 Y가 서로 독립 $\Leftrightarrow$ $f(x,y)=f(x)f(y)$ $\Longrightarrow$ $E[XY]=E[X]E[Y]$

  $E[XY]=\sum_{all\ x}\sum_{all\ y}xyf(x,y)$

  $=\sum_{all\ x}\sum_{all\ y}xyf(x)f(y)$

  $=\sum_{all\ x}xf(x)\sum_{all\ y}yf(y)$

  $=E[X]E[Y]$