EE447: Assignment 1 on Minimization
Finding Roots and Minima of functions in 1-D

Harishankar Ramachandran
EE Dept, IIT Madras

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1 Reading Assignment

The root finding and minimization chapters in Numerical Recipes.

2 Finding Roots in 1-D

Find all the **positive** roots of

$$\tan x = \sqrt{\pi\alpha - x}$$

for any positive α. This type of equation typically arises in the solution of modes in a dielectric waveguide. This means that the solution should accept positive values of α supplied by the user and should return all positive roots.

1. Plot the left and right hand sides. Determine the existence of roots for $\alpha = 10$. (But note that the code created should work for any α. The value of 10 is only an example.)

2. Bracket the roots using the plots to determine the bracketing regions. Draw vertical lines through your bracketing points. This kind of visualization is extremely important to ensure that your bracketing happened correctly. Do this for $\alpha = 10$ and then generalize for any α.

3. Use bisection, secant, Brent and Newton Ralphston to see how quickly the roots are obtained.

Note: The bracketing algorithm should be independent of α. You should be able to defend the logic in the viva.

3 Finding multiple roots in 1-D

Find all the roots of

$$f(x) = (x-2+\varepsilon)(x-2-\varepsilon)(x-2)$$

numerically, using bracketting. Here ε is a small number you do not know ahead of time. Use 10^{-3} as a typical number. How will you determine how many roots are there (without using the fact that it is a third order polynomial)?

4 Minimization in 1-D

We wish to find the first ten positive minima of

$$\sin x + \frac{1}{1+x^2}$$

1. Graph the function
2. Bracket the minima by assuming that they lie between the zeros of $\sin x$. Draw vertical lines through the zeros and see if this assumption is correct.
3. Use the golden section search to zero in on the minimum.
4. Compare with Brent method.
5. Determine the “dead zone”, the range of x values around a root in which the function does not change to computer resolution.
6. Convert the method to a “find the zero” method by differentiating the above function. Then find the zeros with a root finding algorithm. **Compare the accuracy of the minimum.**