**A Mathematical Model of Information Effectiveness, Memory, and Conscious Focus: Entropic Dynamics and Cognitive Reinforcement**

**Author: Agada Nathaniel Emmanuel**

**Affiliation: University of Ilorin**

**Abstract:**

This paper proposes a unified model for understanding the effectiveness of information, its entropic decay, and reinforcement within the context of memory and conscious experience. Drawing from principles of thermodynamics, classical mechanics, probability theory, and cognitive psychology, we derive differential equations that govern the evolution of information effectiveness (k), modulated by focus (φ), memory storage, and information structure. A feedback loop is established through a dynamic coupling between effectiveness and focus, leading to a self-regulating mechanism that simulates memory retention and decay. Extensions are made to incorporate information magnitude and sequence complexity. The model has implications for artificial memory systems, educational optimization, and theoretical neuroscience.

***Keywords:*** *memory, effectiveness, entropy, reinforcement, consciousness, information theory, focus, Lagrangian mechanics, cognitive modeling*

**1. Introduction**

Human memory and consciousness are governed by interactions between received information, the effectiveness of its encoding, and attentional focus. Classical models (Ebbinghaus, 1885; Atkinson & Shiffrin, 1968) describe decay and reinforcement separately, while recent computational approaches attempt integration (Howard & Kahana, 2002). We present a model treating effectiveness as a dynamic variable subjected to decay and reinforcement, influenced by focus and informational structure. We leverage entropy-inspired decay laws and self-reinforcing feedback loops to create a unified differential framework.

**2. Theoretical Foundations**

**2.1 Information Effectiveness (k)**

Information received is associated with an effectiveness value, denoted as k. Information effectiveness k ∈ [0,1] describes the degree to which an information unit is encoded, remembered, and available for future references. A value near 1 means fully retained memory, while k ≈ 0 corresponds to forgetfulness. There exists a threshold below which information with a k value less than this limit disintegrates almost entirely and is not encoded into memory. Above this threshold, information can be stored in memory.

The memory space functions as an information well, where each piece of information exists with a unique effectiveness value k. Information within this space is distinct and occupies a unique position.

When new information is received, the memory system searches the well for existing entries that are similar. If a match is found, no new space is created; instead, the existing information is reinforced. If the information is similar but not identical, it is stored within a reference group. In this case, accessing one piece of information will also trigger access to related entries within the reference group. Referenced information is projected into a virtual representation that can be modified for planning, ideation, and innovation. The accessibility of information from memory depends on how effectively it was originally stored.

The detailed mechanism of reference grouping will not be fully addressed in this model.

**2.2 Focus Distribution φ(t)**

A function termed focus is defined over a range of ideological time and is normally distributed across this range. The peak of the focus distribution at any given time corresponds to a particular piece of information. Focusing on information increases its effectiveness—a process referred to as reinforcement. The normal distribution of focus and its influence on the dynamics of information can be described in three ways:

**1. Time-Based Distribution**

The standard deviation represents a unit of time, with the mean aligned to the reference point of the information. This model assumes information is fixed along an ideological time axis, while focus moves across this axis at constant intensity—analogous to pointing a beam of light across a wall with information embedded at various locations. Focusing on information does not guarantee it enters consciousness, which depends on the product of information magnitude, focus intensity, and effectiveness. Information with low effectiveness yields low consciousness even when focused. However, we propose that the standard deviation must include the ideological present; an observer cannot be completely detached from the present when accessing memory.

**2. Effectiveness-Based Distribution**

Here, information in memory is separated not by time but by effectiveness. The standard deviation lies along an effectiveness axis, which ranges from 0 to 1. With constant focus intensity over time, the normal distribution becomes fixed. All information with k > 0 lies within this distribution. The height of the curve represents the combined intensity of focus and effectiveness.

**3. Centrally Referenced Model**

The most accurate model assumes a single set of referenced information at any point in time. When information is recalled, it is drawn closer to the ideological center, and its position is determined by its effectiveness. The distribution must satisfy the condition that the function φ equals 1 when k = 1 and 0 when k = 0.

Focus is modeled as a distribution φ(t) over past (P), present (R), and future (F) information. It represents the intensity of attention allocated along time. Initially modeled using a normal distribution:

In feedback models, φ becomes a function of effectiveness itself: φ(k) = kⁿ, ensuring:

• φ = 1 when k = 1 (full focus),

• φ = 0 when k = 0 (no focus),

• Adaptable control via parameter n ≥ 1.

3. Entropic Decay and Reinforcement

The natural decay of k is modeled as a first-order process:

To account for reinforcement due to focus, we add a growth term:

Where:

• λ: decay rate,

• ρ(k): reinforcement rate as a function of current effectiveness.

3.1 Reinforcement Growth Function

To reflect that stronger memories reinforce more effectively, ρ(k) is defined:

This adaptive term allows weakly encoded memories to grow slowly while stronger memories reinforce rapidly.

3.2 Feedback-Coupled Model

Let φ(k) = kⁿ. Then:

4. Information Magnitude and Sequence

We introduce two scalar descriptors:

• Information Magnitude M: The bulk or complexity of the information.

• Information Sequence S: The order, regularity, or structure of the information.

Sequence is inversely proportional to the standard deviation σ of symbolic data (e.g., ASCII codes):

The ratio M/S reflects information density or complexity:

• M/S: Information weight per unit structure (harder to memorize).

• S/M: Structural clarity per unit content (easier to memorize).

We incorporate them into λ and ρ(k):

5. Lagrangian and Hamiltonian Formalism

Following the principle of least action, we define a Lagrangian L(k, , t):

Where V(k) encodes entropy and reinforcement forces. The Euler–Lagrange equation yields the time evolution of k. The Hamiltonian H = T + V gives an energy-based interpretation of memory flow.

6. Integral Form

The differential equation is separable:

Where:

7. Interpretation of Consciousness

Consciousness is defined as:

Where I\_i is the magnitude of information i, and k\_i(t) its effectiveness at time t. The focus φ(t) governs which information dominates conscious awareness.

8. Results and Examples

Numerical simulations show:

• Reinforcement accelerates for highly effective memories.

• Random, bulky information decays rapidly.

• Structured information is more stable.

• Feedback coupling via φ(k) leads to a bistable system (forgetful vs. retained states).

9. Philosophical Implications

The model suggests that identity arises from the structured feedback between information effectiveness and attention. Self-awareness may be formalized as:

Where Φ is the vector of focus-modulated information states—i.e., the system observing itself.

10. Conclusion

This model provides a detailed, dynamic, and integrated approach to memory, focus, and consciousness. It unifies entropic decay, reinforcement, attention, and structure of information within a single mathematical framework.

Future work includes:

• Neurophysiological validation,

• Extension to multi-agent systems,

• Application to AI memory and reinforcement strategies.

References

• Ebbinghaus, H. (1885). Memory: A Contribution to Experimental Psychology.

• Atkinson, R.C., & Shiffrin, R.M. (1968). Human memory: A proposed system and its control processes.

• Howard, M.W., & Kahana, M.J. (2002). A distributed representation of temporal context.

• Cover, T.M., & Thomas, J.A. (2006). Elements of Information Theory.

Appendices

A. Derivation of φ(k)-based Feedback Model

B. Lagrangian Mechanics of Memory Dynamics

C. Numerical Integration Approach