

Example: In RSA, when  $p = 7$ ,  $q = 11$ , and  $e = 13$ , find  $d$  that is the multiplicative inverse of  $e \bmod \phi(n)$ , i.e.,  $ed = 1 \bmod \phi(n)$ , where  $n = pq = 77$ .

$$\phi(n) = (p-1)(q-1) = (7-1) \times (11-1) = 60$$

Next, we find  $d$  by using Euclid's algorithm.

$$\text{E1: } 60/13 = 4 \dots 8 \quad (\text{dividend is } \phi(n), \text{ divisor is } e)$$

$$\text{E2: } 13/8 = 1 \dots 5 \quad (\text{In the following steps, dividend is the divisor in previous step and divisor is the remainder in previous step})$$

$$\text{E3: } 8/5 = 1 \dots 3$$

$$\text{E4: } 5/3 = 1 \dots 2$$

$$\text{E5: } 3/2 = 1 \dots 1 \quad (\text{Stop when the remainder is equal to 1})$$

Then, we need to represent 1 by using  $\phi(n)$  and  $e$ .

$$1 = (3 - 2 \times 1) \bmod 60 \quad (\text{Based on E5})$$

$$= (3 - (5 - 3 \times 1)) \bmod 60 \quad (\text{Based on E4, we have } 2 = 5 - 3 \times 1)$$

$$= (3 \times 2 - 5) \bmod 60$$

$$= ((8 - 5 \times 1) \times 2 - 5) \bmod 60 \quad (\text{Based on E3, we have } 3 = 8 - 5 \times 1)$$

$$= (8 \times 2 - 5 \times 3) \bmod 60$$

$$= (8 \times 2 - (13 - 8 \times 1) \times 3) \bmod 60 \quad (\text{Based on E2, we have } 5 = 13 - 8 \times 1)$$

$$= (8 \times 5 - 13 \times 3) \bmod 60$$

$$= ((60 - 13 \times 4) \times 5 - 13 \times 3) \bmod 60 \quad (\text{Based on E1, we have } 8 = 60 - 13 \times 4)$$

$$= (60 \times 5 - 13 \times 23) \bmod 60$$

$$\begin{aligned} \text{We have } 1 &= (60 \times 5 - 13 \times 23) \bmod 60 = 13 \times (-23) \bmod 60 = 13 \times (-23 + 60) \bmod 60 \\ &= 13 \times 37 \bmod 60 \end{aligned}$$

Hence,  $d = 37$ . (Remember  $d$  must be a positive integer number since we consider modular arithmetic here. So  $d = -23$  is not correct.)