

Problem: Find  $w$  that minimize function  $f(w) = L(w) + R(w)$   
parameter  
 $d$  = number of dimension of  $w$   
 $a$  = bound

OPE algorithm

1. Initialize  $w_0$  ( $w_0[i]$  in  $(0,1)$  for  $i = 1 \dots d$ )
2. set  $a = 0, b = 0$
3. for  $t = 1 \dots T$ 
  - 3.1 choose random  $a$  or  $b$  with probability  $\{0.5, 0.5\}$   
then increase by 1
  - 3.2  $F_t = a * L(w_t) + b * R(w_t)$
  - 3.3  $F'_t = a * L'(w_t) + b * R'(w_t)$
  - 3.4 Find  $s_t$  that minimize  $\langle F'_t, s_t \rangle$  with respect to  $\sum_{i=1}^d |(s_t[i])| < a$
  - 3.5 set  $w_t = w_{t-1} + \frac{(s_{t-1} - w_{t-1})}{t}$

**Detail:**

3.3  $R$  is non-differentiable at some point  $w$ , so at that points, we choose random **right differential** or **left differential** with probability  $\{0.5, 0.5\}$

3.4 to find  $s_t$ , we choose  $s_t$  to be a vertex of domain  $D = \{x \in R^d \text{ such that } \sum_{i=1}^d |(x[i])| < a\}$

because  $D$  is convex, and 3.4 problem is linear programming.

So one element of  $s_t$  is  $a$  or  $-a$ , other elements are zeros.

In GIST package, authors implemented loss functions :

Least :  $L(w) = 1/(2n) \|Xw - y\|^2$   $n$  is number of sample

L2SVM :  $L(w) = 1/(2n) \sum_j \max(0, 1 - y_j * x_j' * w)^2$

Logistic :  $L(w) = 1/n \sum_j \log(1 + \exp(-y_j * x_j' * w))$