QUANTUM AMPLITUDE DAMPING CHANNEL DISCRIMINATION

UPC POSTGRADUATE QUANTUM ENGINEERING

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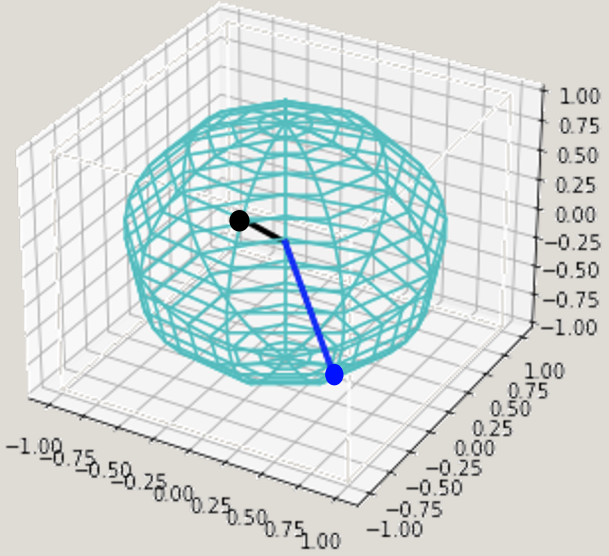
20/05/2021

ADC EFFECT?

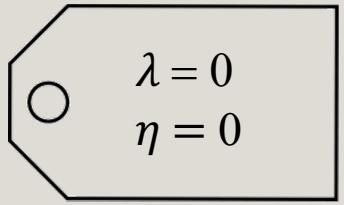
$$(X_{in}, Y_{in}, Z_{in}) \rightarrow (X_{in} \cos\eta, Y_{in} \cos\eta, sin^2\eta + Z_{in}\cos^2\eta)$$

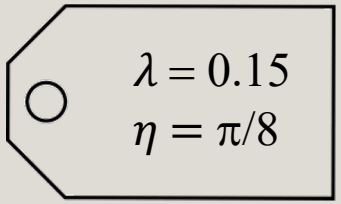


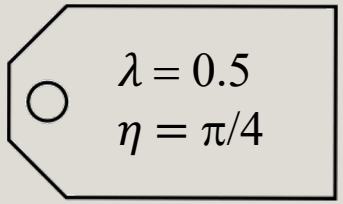
Effect on the Bloch Sphere passing through 5 different ADCs

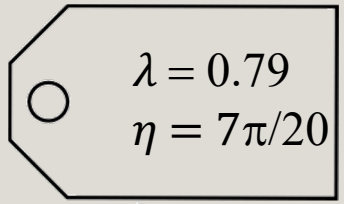
$$\lambda = sin^2\eta$$

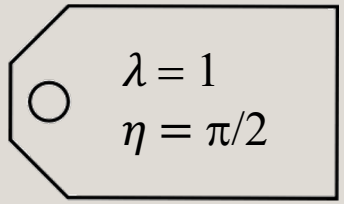
λ is defined between 0 and 1
η is defined between 0 and $\pi/2$

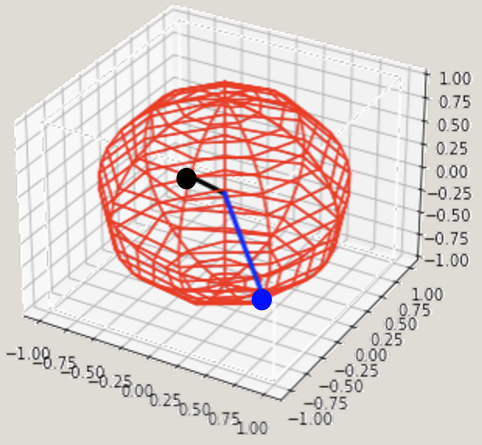


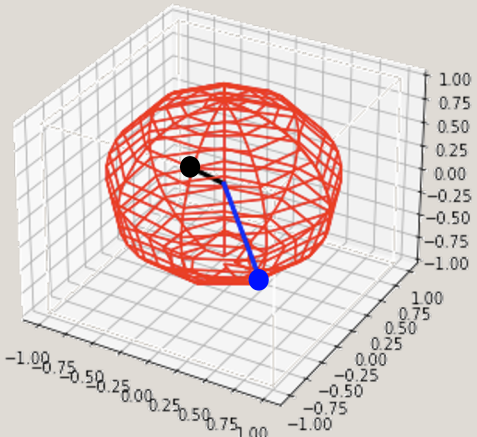


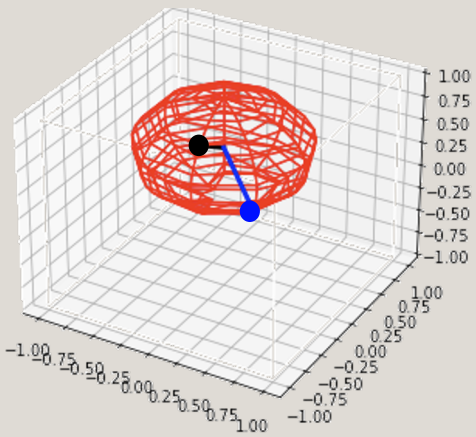


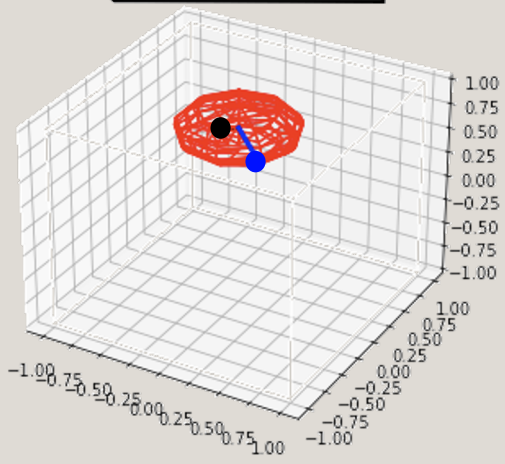


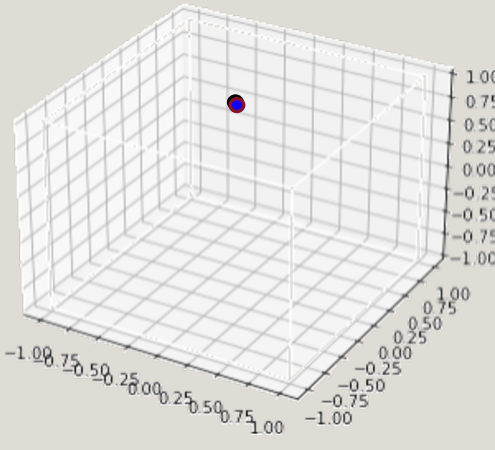




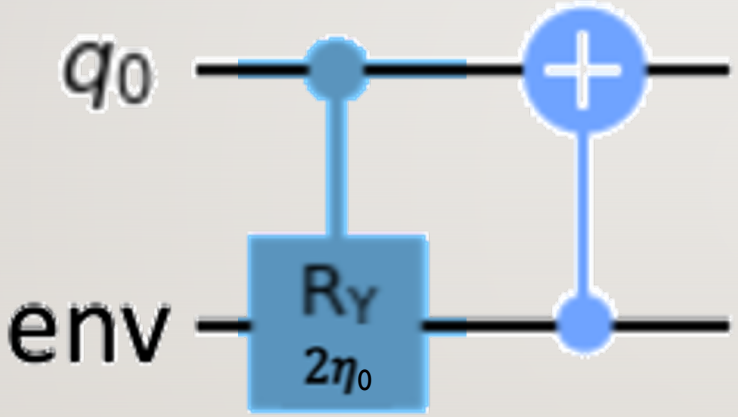


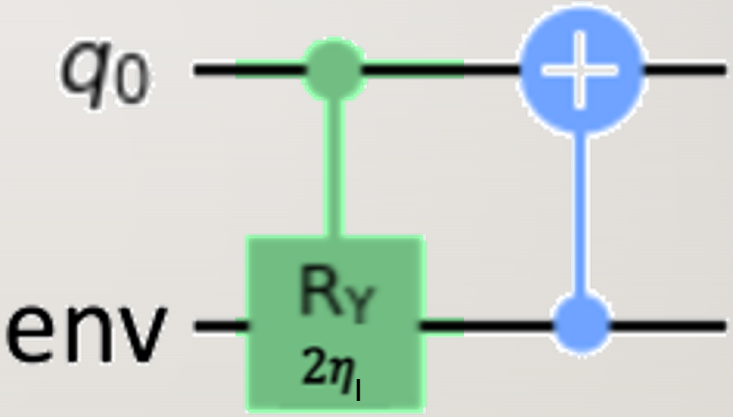


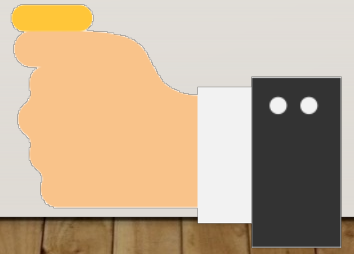




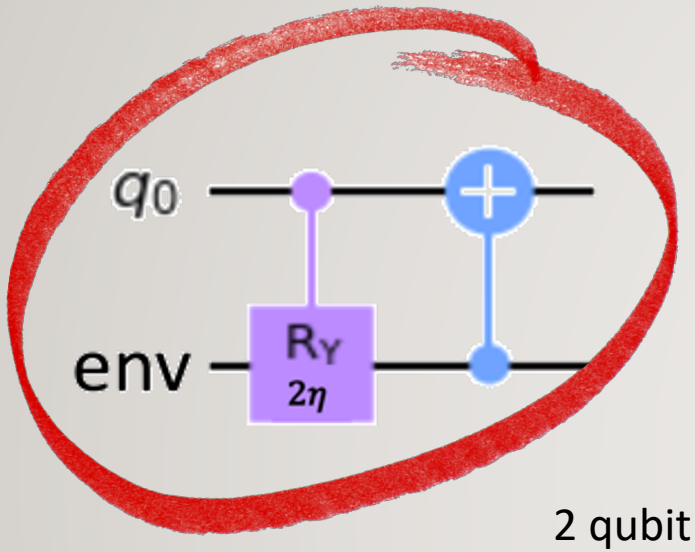
HOW TO MODEL AND DISCRIMINATE ADCs?



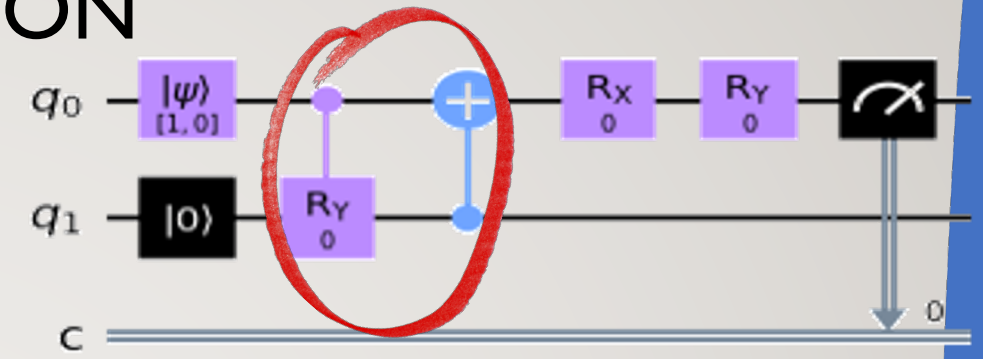




APPROACHES TO THE DISCRIMINATION PROBLEM

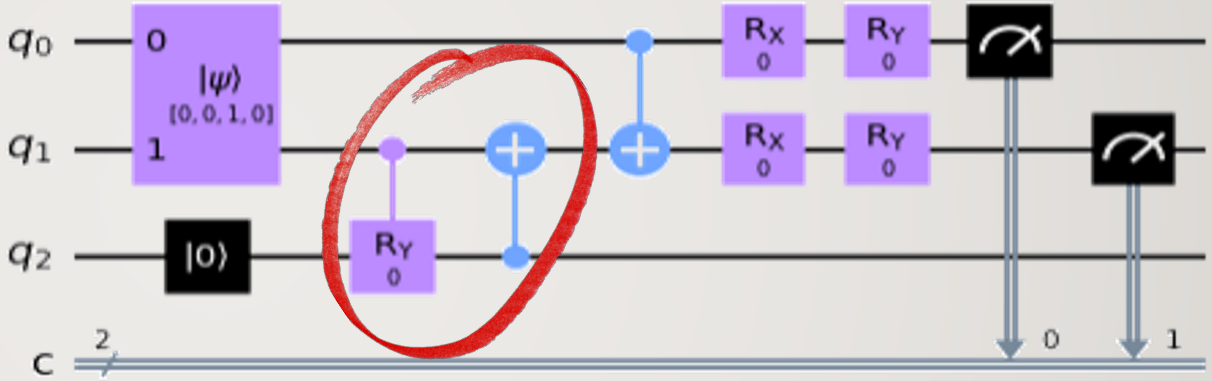


1 qubit – “Classical”



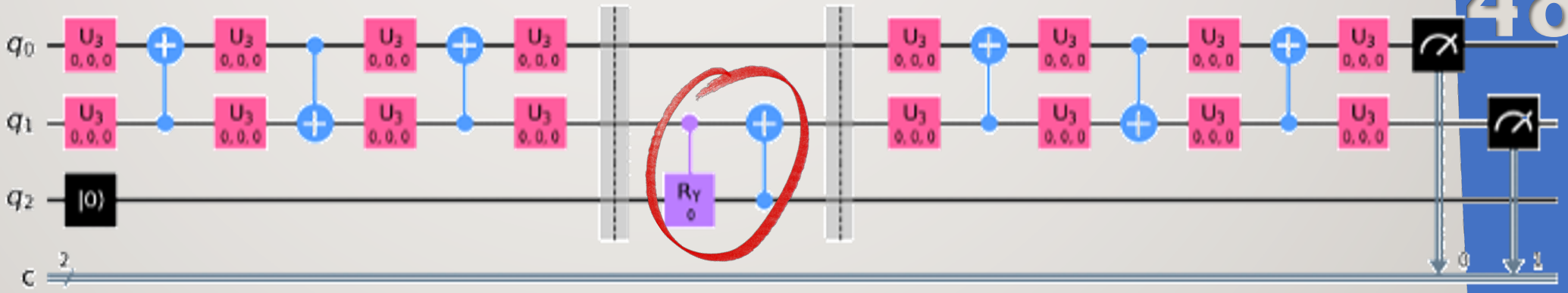
3

2 qubit –
Entanglement assisted



5

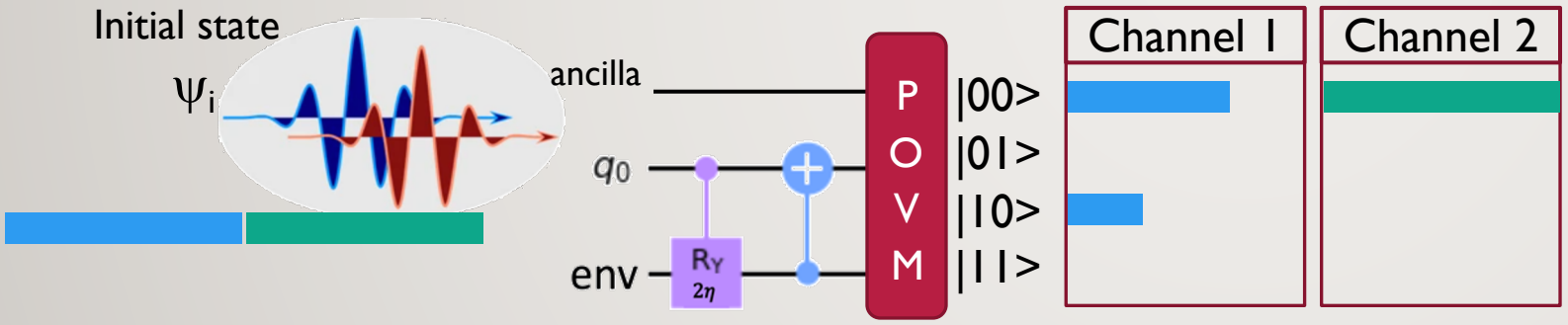
2 qubit – Entangled with universal gates



48

HOW WE DO IT?

For each pair or triplet of ADCs, create the circuit and follow 3 phases

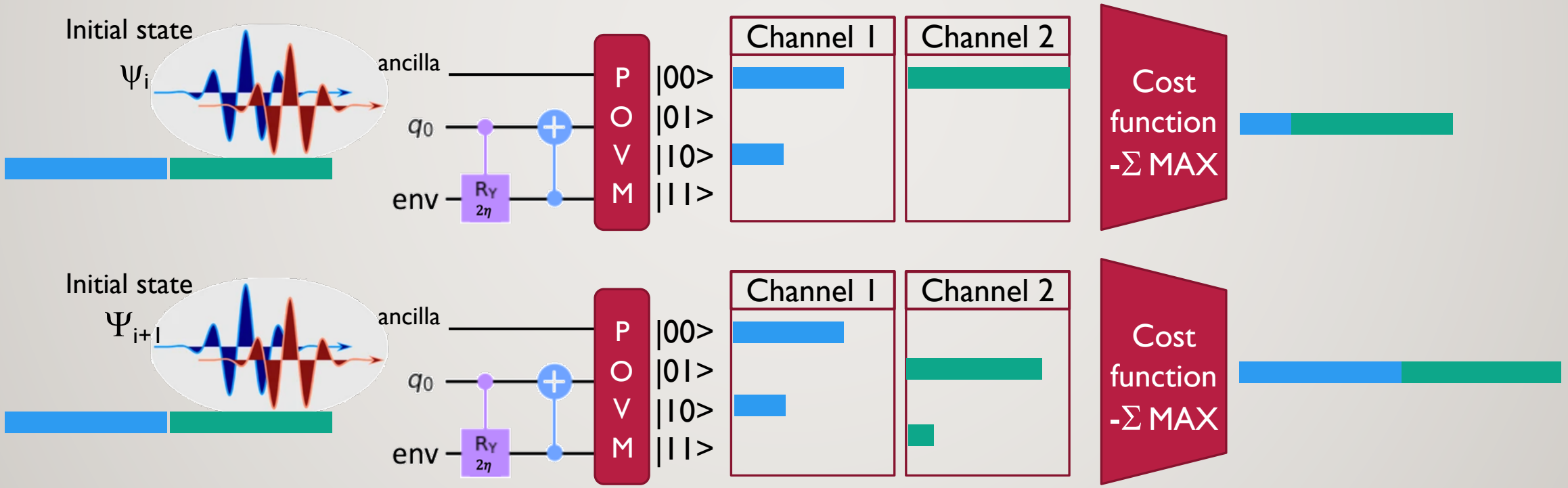


HOW WE DO IT?

For each pair or triplet of ADCs, create the circuit and follow 3 phases

1st Phase: Optimization

Cost function → Maximum likelihood → MAX



HOW WE DO IT?

For each pair or triplet of ADCs, create the circuit and follow 3 phases

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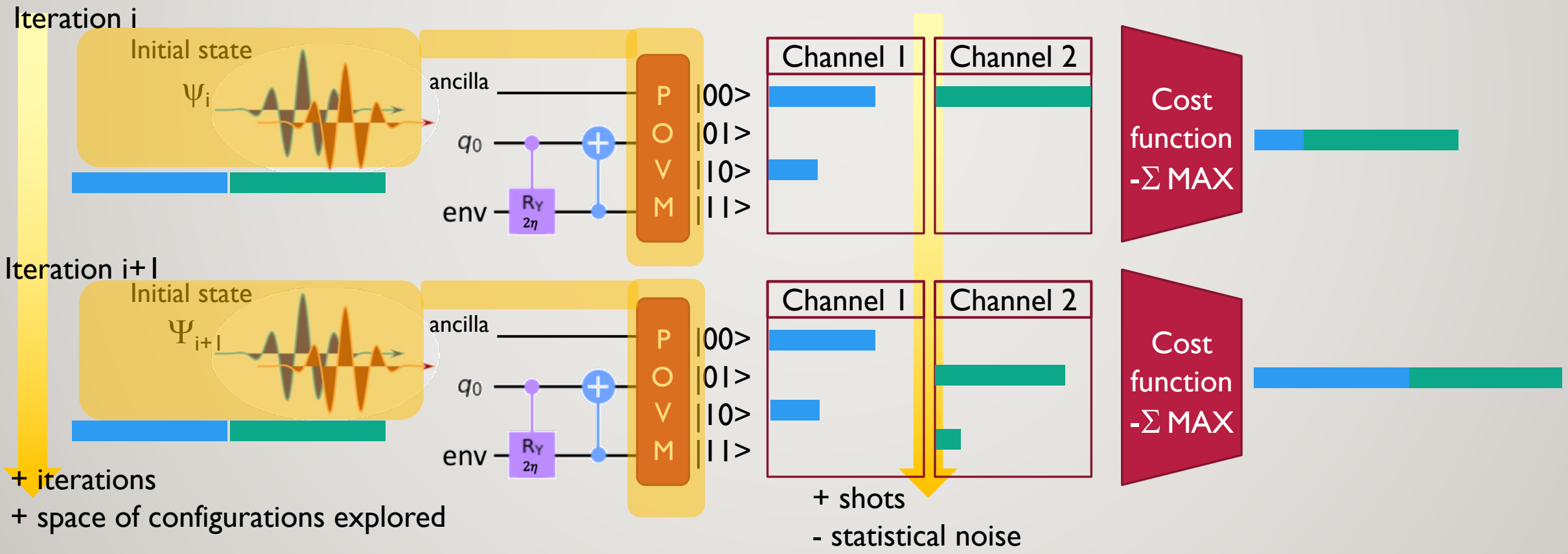
Algorithms: diversity, iterations and shots impact



HOW WE DO IT?

For each pair or triplet of ADCs, create the circuit and follow 3 phases

1st Phase: Optimization

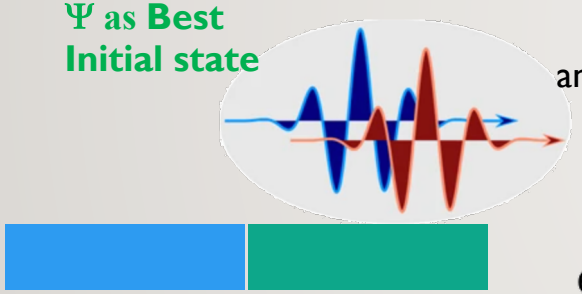
Cost function → Maximum likelihood → MAX

Algorithms: diversity, iterations and shots impact

2nd Phase: Validation

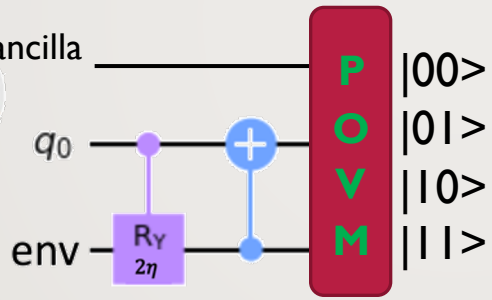
Reduce statistical noise (remove bias from optimization)

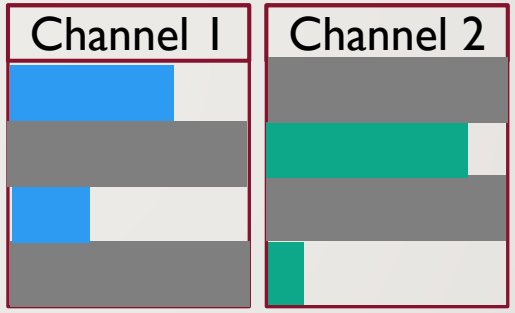
Ψ as Best Initial state



1 million shots x circuit

Best POVM





HOW WE DO IT?

For each pair or triplet of ADCs, create the circuit and follow 3 phases

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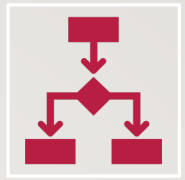
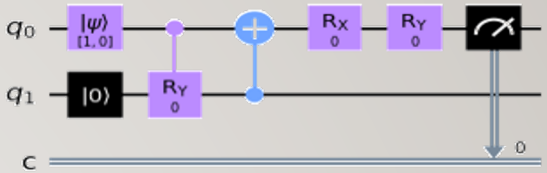


2nd Phase: Validation

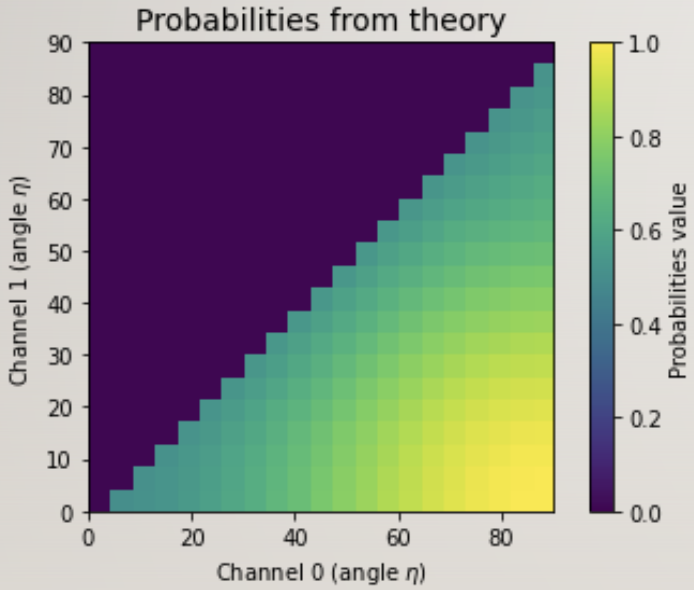
Reduce statistical noise (remove bias from optimization)

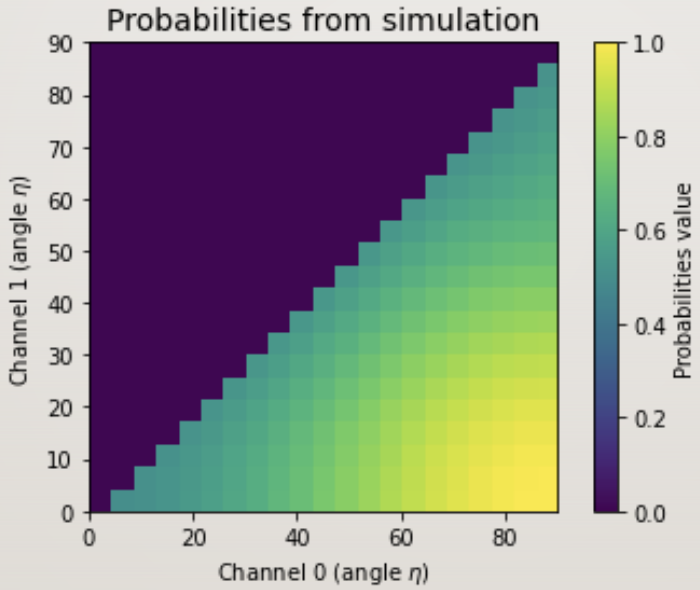
3rd Phase: Representation

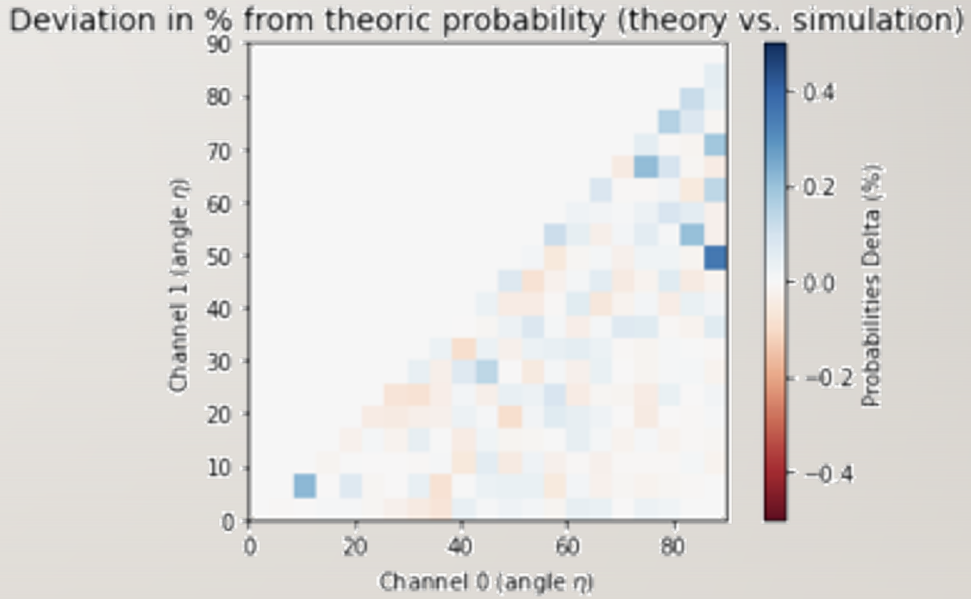
DISCRIMINATING 2 ADC: ONE-SHOT & 1 QUBIT



$$P_{succ} = \frac{1}{2}\left\{1 + (\cos\eta_1 - \cos\eta_0)\sqrt{x[1 - x(1-\gamma^2)]}\right\} \; where\; \gamma \equiv \gamma(\eta_1, \eta_0) \equiv \cos\eta_1 + \cos\eta_0$$



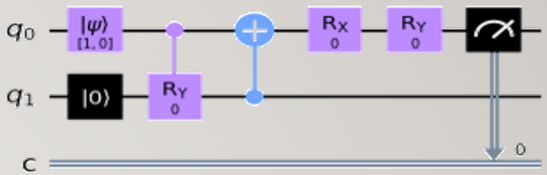




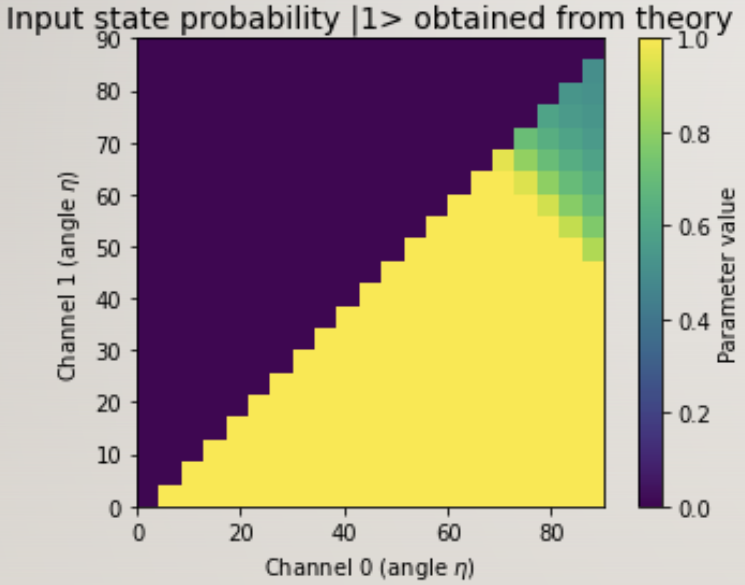
Optimization: CRS with 1,000 iterations and 10,000 shots
Validation: 2,000,000 shots

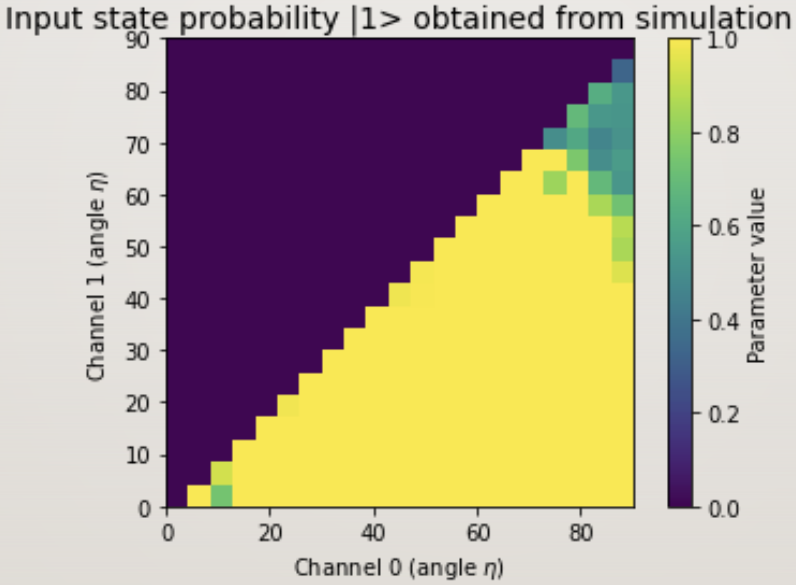
Theoretical results from: Discriminating qubit amplitude damping channels: https://arxiv.org/pdf/2009.01000.pdf

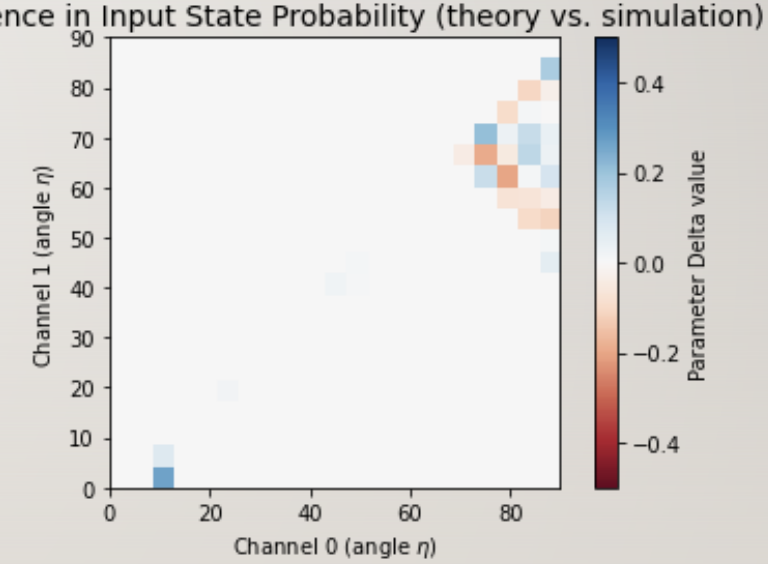
DISCRIMINATING 2 ADC: ONE-SHOT & 1 QUBIT



$$|\Psi\rangle = \sqrt{1-x}\,|0\rangle + \sqrt{x}\,|1\rangle$$

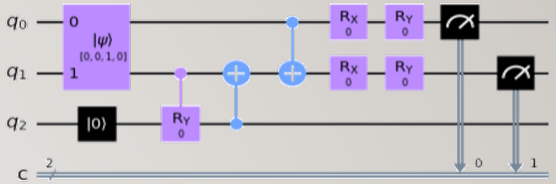




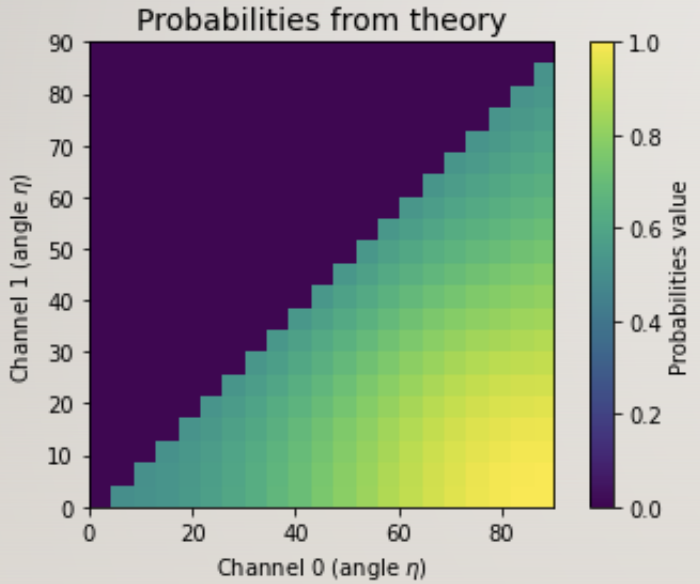


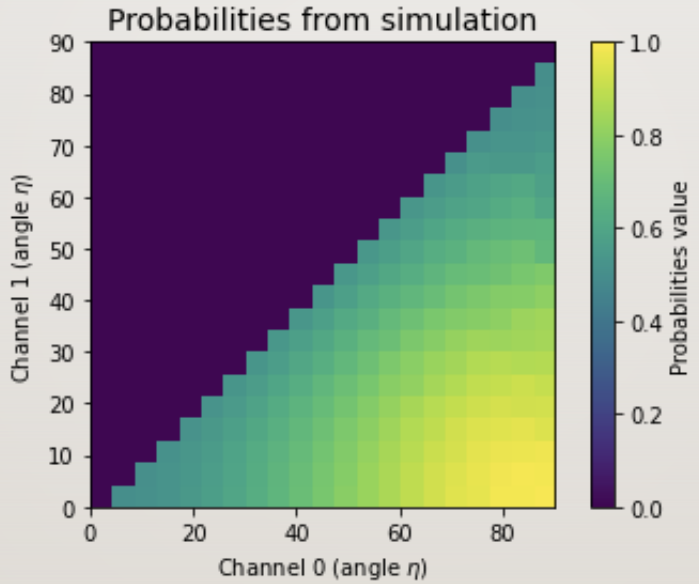
Theoretical results from: Discriminating qubit amplitude damping channels: https://arxiv.org/pdf/2009.01000.pdf

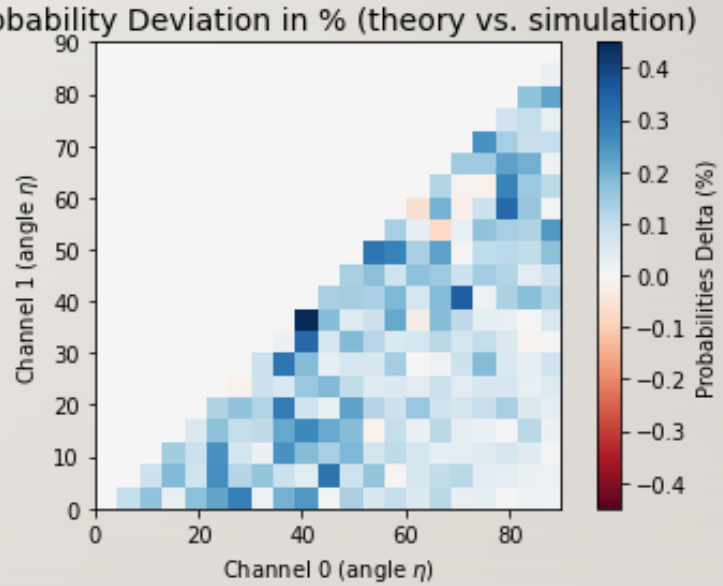
DISCRIMINATING 2 ADC: ONE-SHOT & 2 QUBITS



$$P_{succ} = \frac{1}{2} + \frac{1}{4}(\cos\eta_1 - \cos\eta_0)\left\{(1-y)\gamma + \sqrt{(1-y)[4y + (1-y)\gamma^2]}\right\}$$



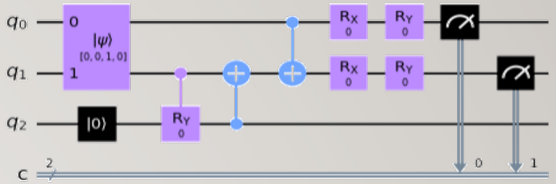




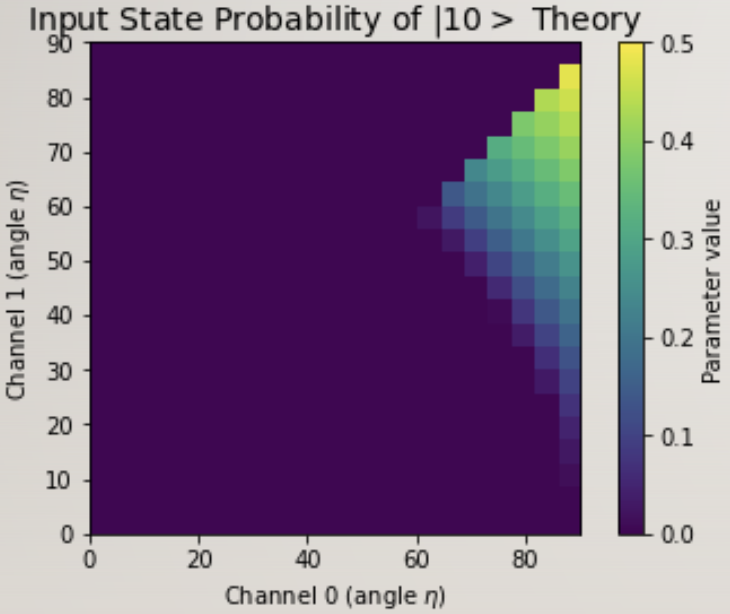
Optimization: DIRECT_L with 4,000 iterations and 100,000 shots
Validation: 2,000,000 shots

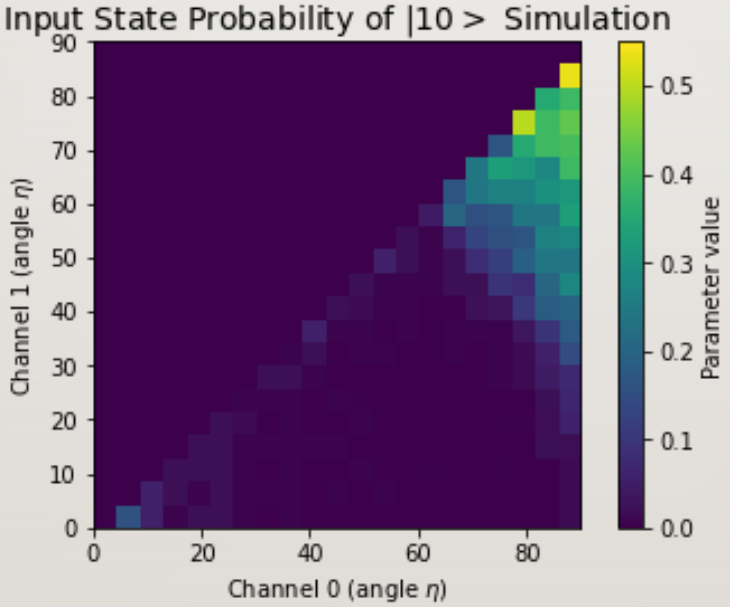
Theoretical results from: Discriminating qubit amplitude damping channels: https://arxiv.org/pdf/2009.01000.pdf

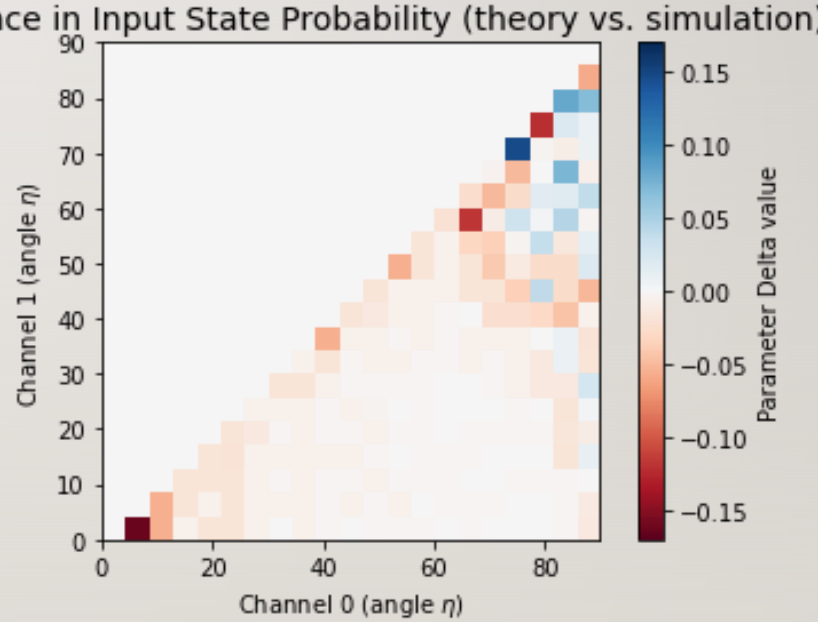
DISCRIMINATING 2 ADC: ONE-SHOT & 2 QUBITS



$$|\Psi\rangle = \sqrt{1-y}\,|01\rangle + \sqrt{y}\,|10\rangle$$







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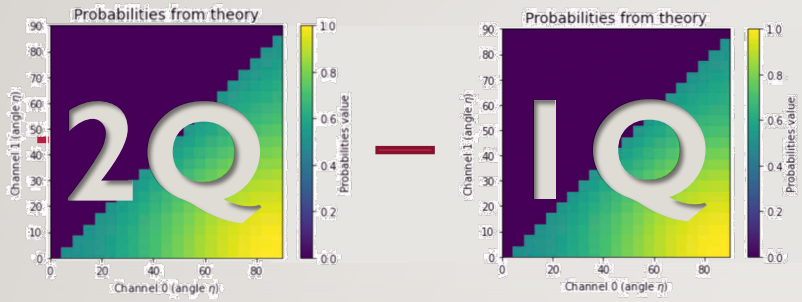
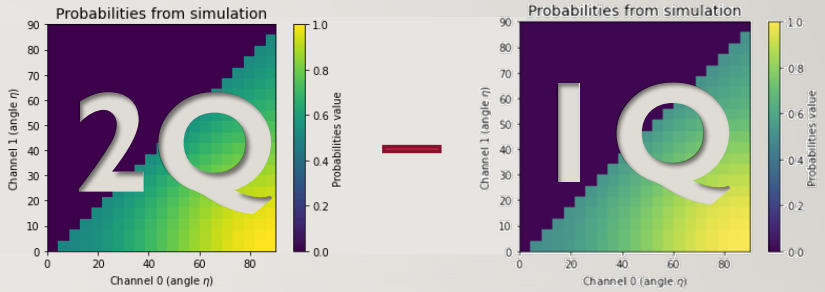
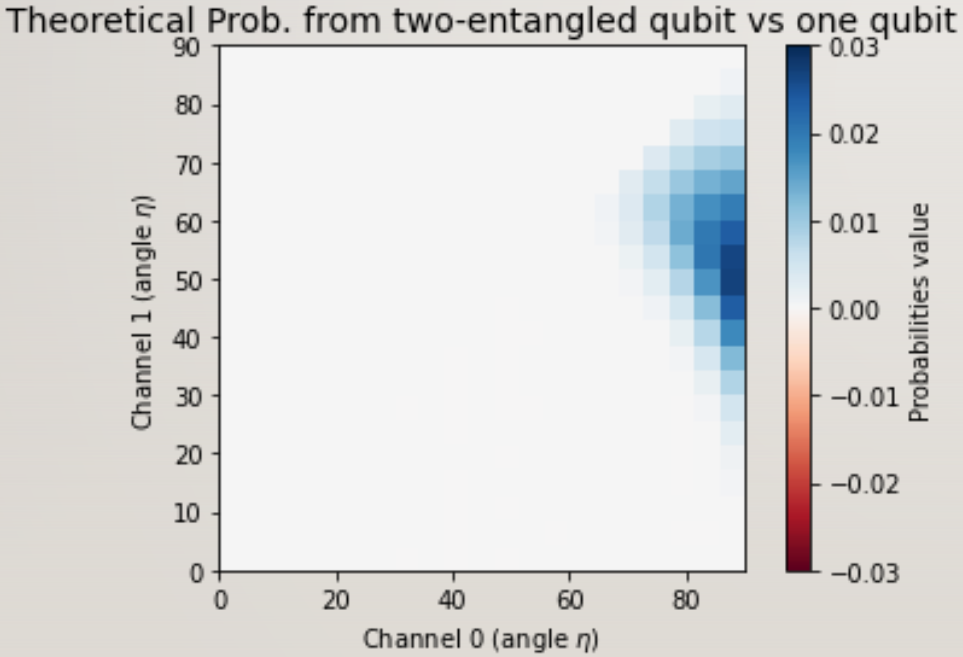
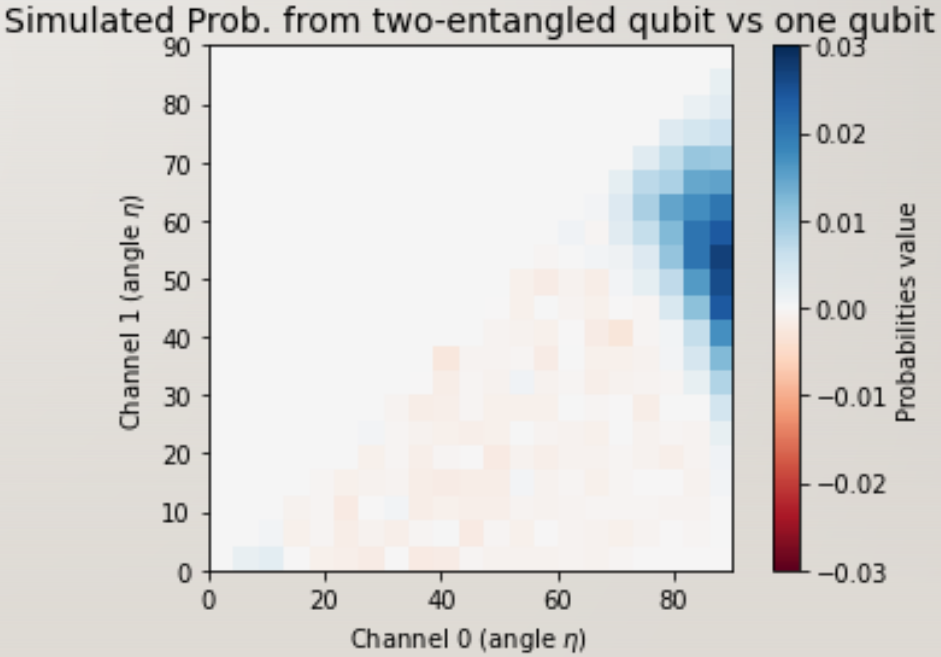
DISCRIMINATING 2 ADC: 1 vs. 2 QUBITS



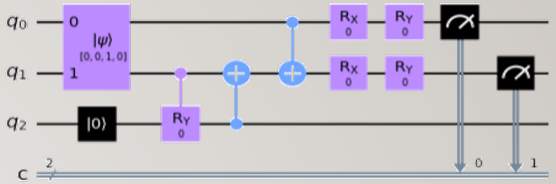




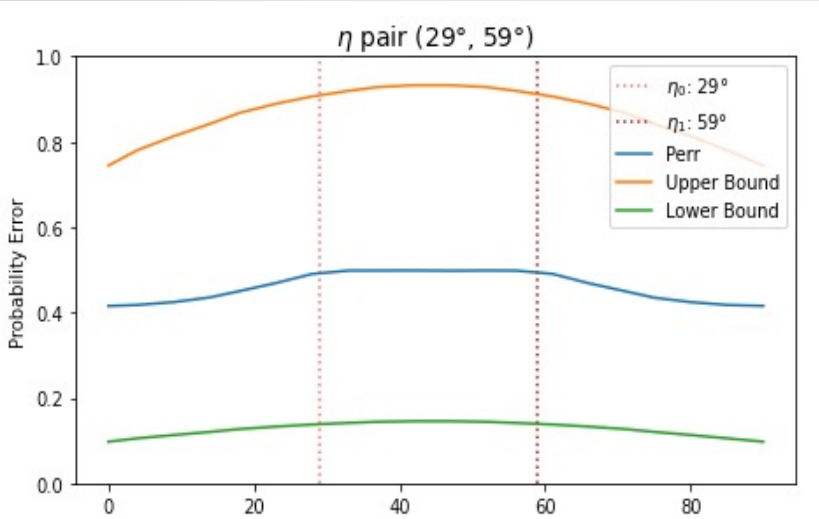


Theoretical results from: Discriminating qubit amplitude damping channels: https://arxiv.org/pdf/2009.01000.pdf

DISCRIMINATING 3 ADCs: ONE-SHOT

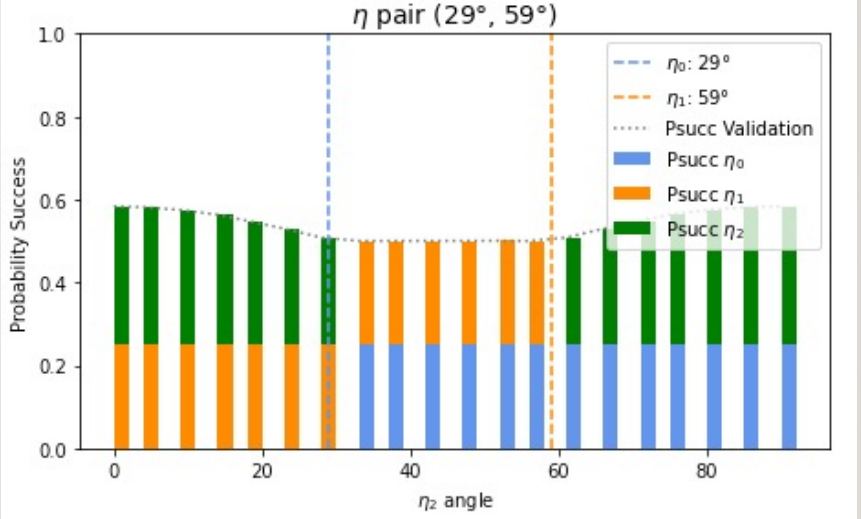


Error Probabilities



with CRS (Iterations=1,000 Shots=100,000)

Success Probabilities breakdown



validated with 1,000,000 shots per channel

P_{err} bounds:*

$$\frac{1}{2}\sum_{i\neq j}\pi_i\,\pi_j F_{i:j}^{2M} \;\leq\; p_{err} \;\leq\; \sum_{i\neq j}\sqrt{\pi_i\pi_j}\,F_{i:j}^{M}$$

*Bounds taken from this article and previous articles cited therein : https://arxiv.org/pdf/2010.03594.pdf

CONCLUSIONS, APPLICATIONS AND NEXT STEPS

Main Conclusions

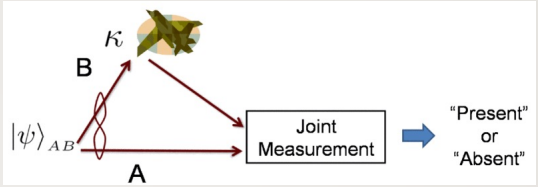
✅ Simulated results replicated from paper

⌛ Optimization process more demanding as the # of gate increases

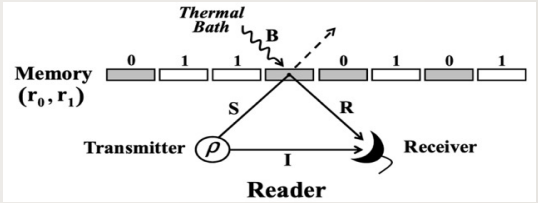
⚠️ Hit a limit of 2 ADC to discriminate +2ADCs using one-shot and 2-qubit circuit

Main Applications

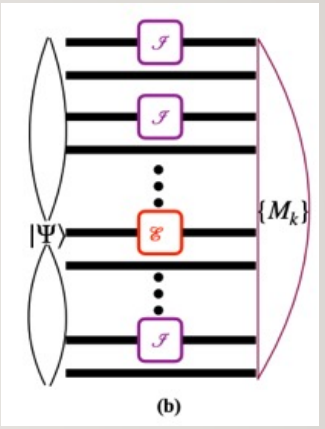
Quantum Illumination



Quantum Reading



Anomaly Detection



Next Steps

👨‍🔍 New lines of investigation to improve the discrimination of +2ADCs:

- Use POVM with more ancillae
- Use other strategies: more than one-shot, adaptive strategies…

👩‍💻 Release the project source code as an open-source library to be used for the scientific community

PROJECT FUN FACTS

Hours reading: 200h x person

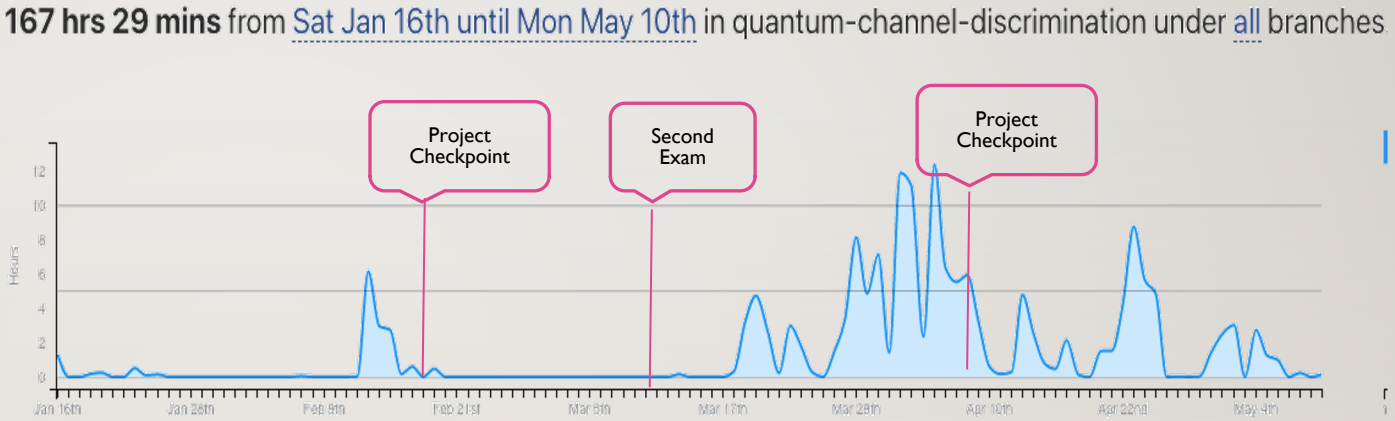
Hours coding: 167h x person

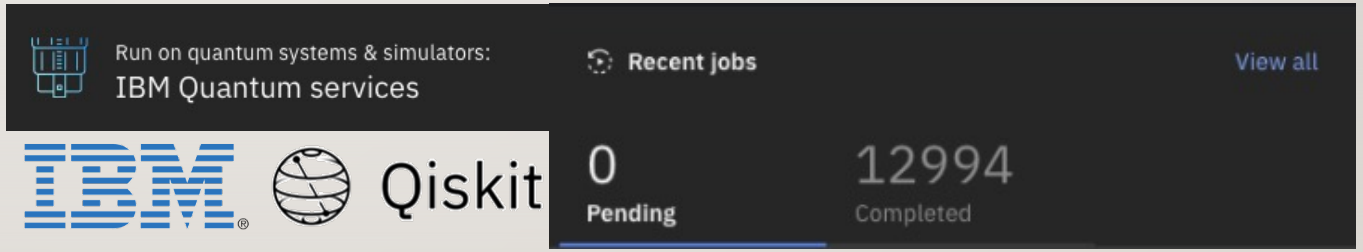
Hours with CPU > 70%: 300h

Written 73 python files with 8.416 lines of code

Used 64 jupyter notebooks with 461.327 lines

Submitted for approval at PyConES'21





Disclaimer: No cats were killed during this project preparation

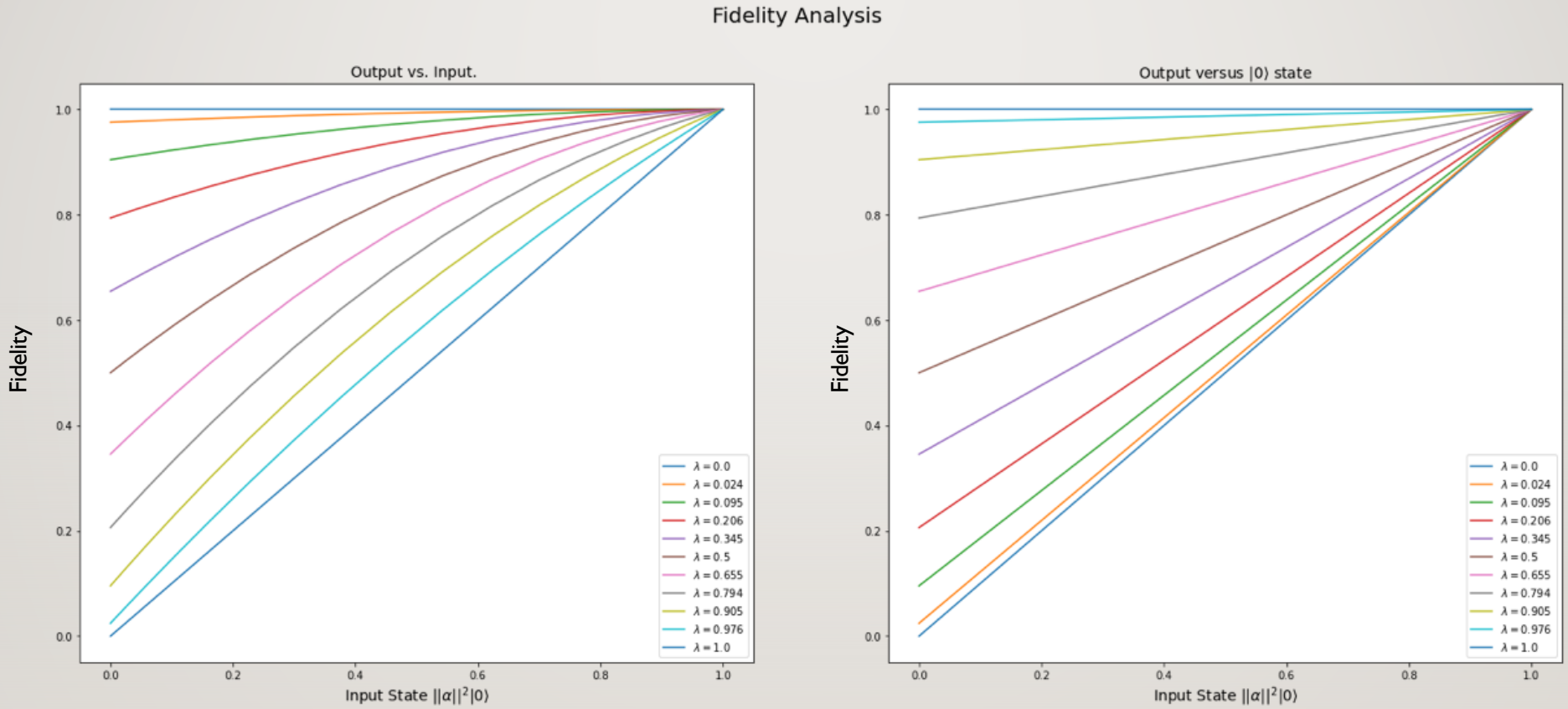
THANK YOU!

Backup

PROJECT SCOPE AND PURPOSE

We analyze and implement some strategies to discriminate between two different amplitude damping channels (ADC) with one-shot executions and reproduce and validate the results against the expected theoretical ones for each approach [1]. The implementation of the discrimination process strategies requires an optimization over the input states and the output measurements (POVM). We explore several one-shot circuit layouts to see how helpful these can be for a 2-channel discrimination and beyond [2], leading to find a limit when more than two channels are involved, where we obtain interesting results not treated in the literature.

[1] - Discriminating qubit amplitude damping channels: https://arxiv.org/pdf/2009.01000.pdf
[2] - Quantum-enhanced barcode decoding and pattern recognition: https://arxiv.org/pdf/2010.03594.pdf



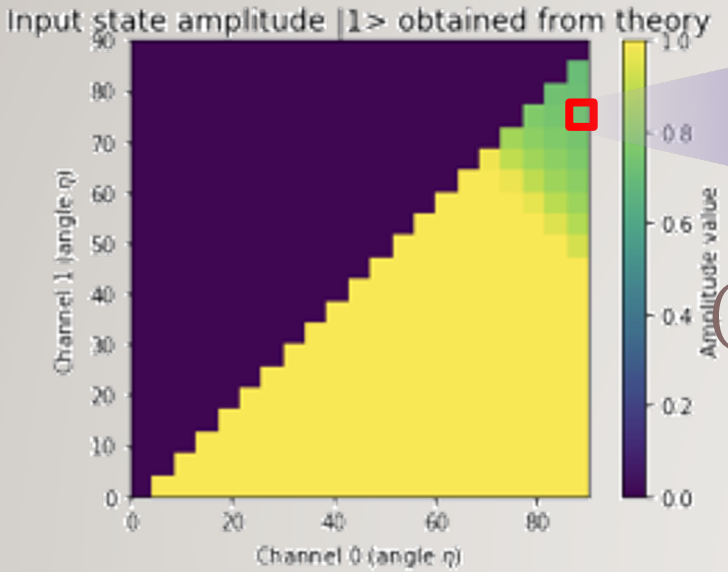
FIDELITY

$$F(\rho, \sigma) = (tr\ |\sqrt{\rho}\sigma\sqrt{\rho}\ |)^2$$

Input state: $|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle$; $\sigma = |\Psi><\Psi|$

Output state: $|\Psi'\rangle = \alpha'|0\rangle + \beta'|1\rangle$; $\rho = |\Psi'><\Psi'|$

Why *x* is different than 1?



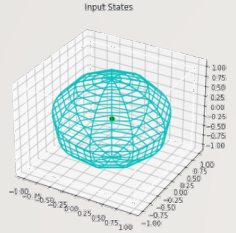
$\eta_0 = 1{,}571\ (90°)$
$\eta_1 = 1{,}335\ (76{,}5°)$
$x = 0{,}72719936$

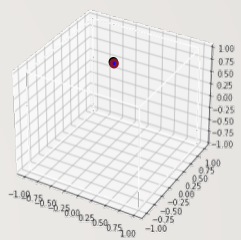
Initial State

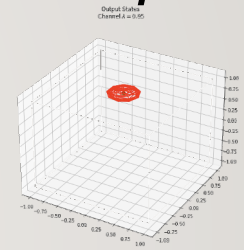
$$|\Psi\rangle = \sqrt{1-x}\,|0\rangle + \sqrt{x}\,|1\rangle$$

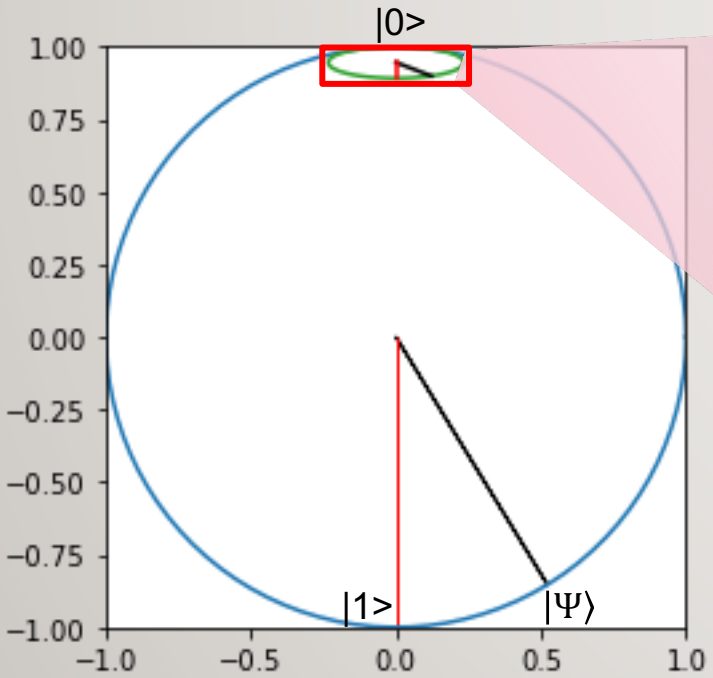
Cartesian coordinates
X = 0.5223
Y = 0
Z = -0,8528

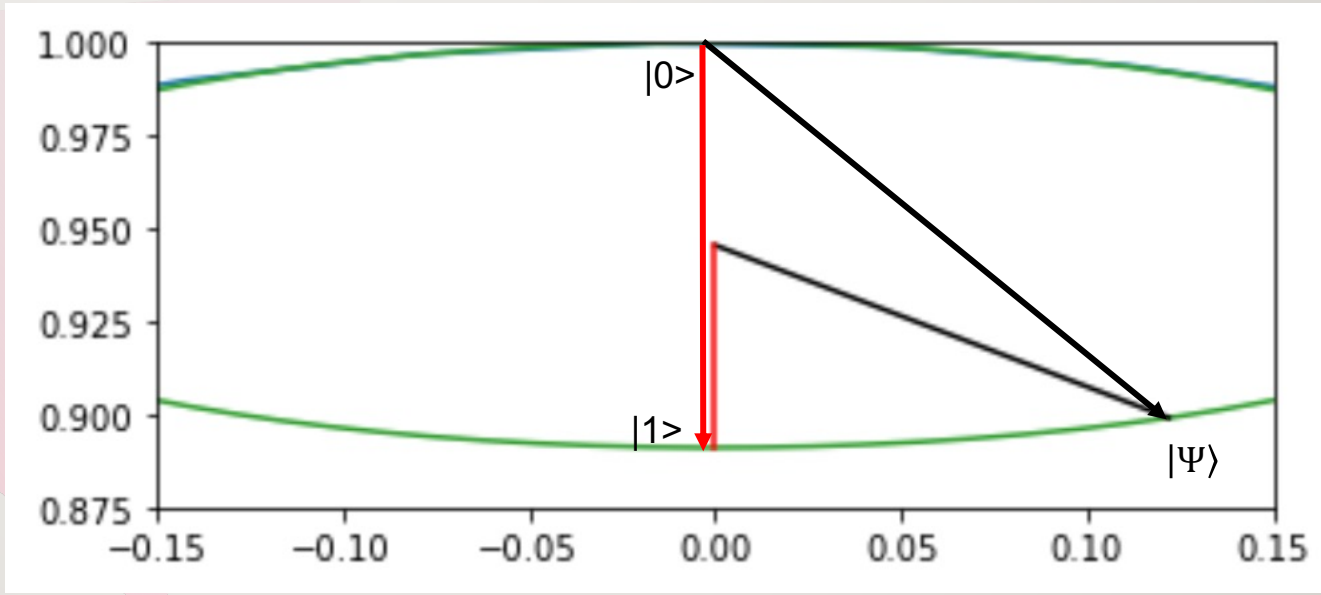
$$(X_{in}, Y_{in}, Z_{in}) \rightarrow (X_{in}\cos\eta, Y_{in}\cos\eta, \sin^2\eta + Z_{in}\cos^2\eta)$$

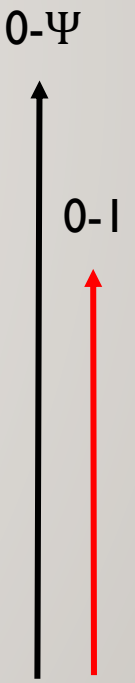




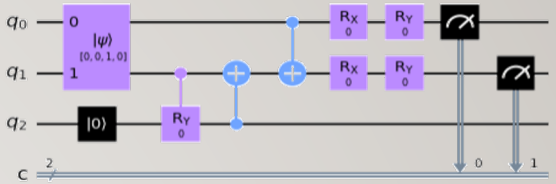






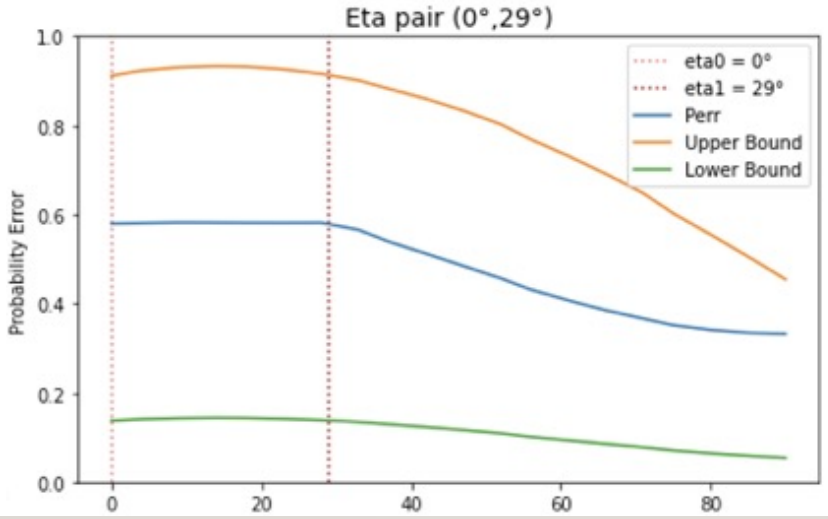


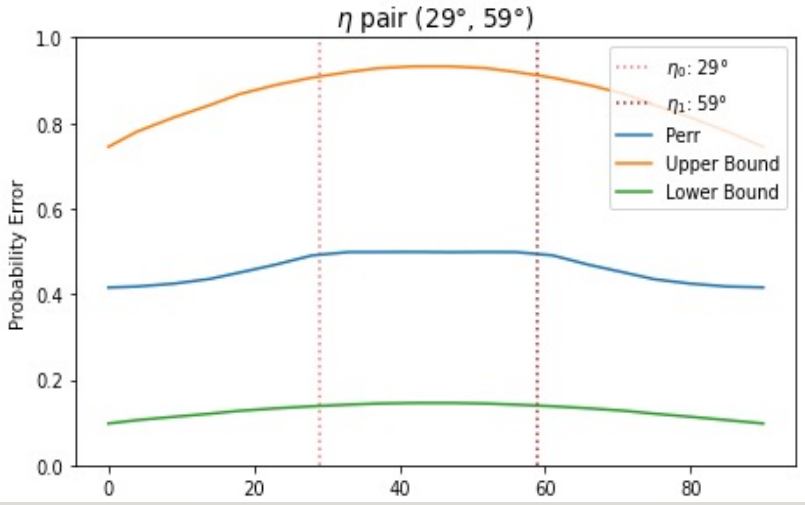
DISCRIMINATING 3 ADCs: ONE-SHOT

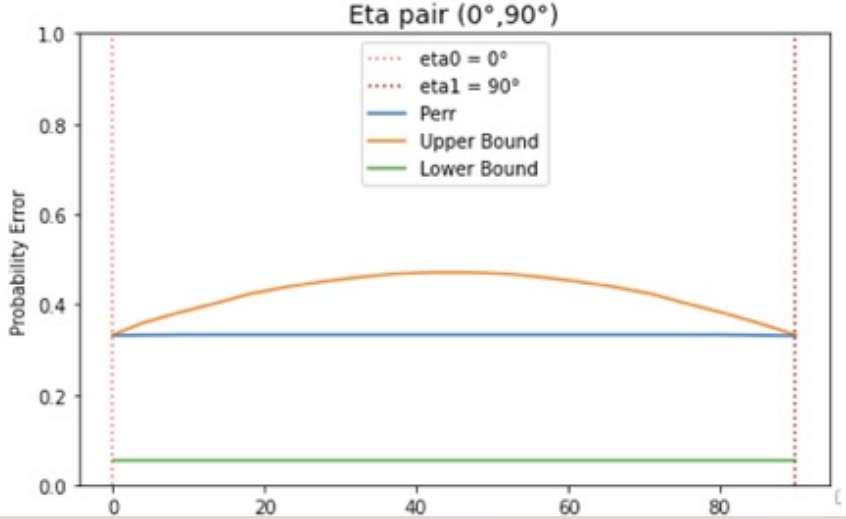


$$\frac{1}{2}\sum_{i\neq j}\pi_i\,\pi_j F_{i:j}^{2M} \;\leq\; p_{err} \;\leq\; \sum_{i\neq j}\sqrt{\pi_i\pi_j}\,F_{i:j}^{M}$$

Error Probabilities with CRS (Iterations=1,000 Shots=100,000)







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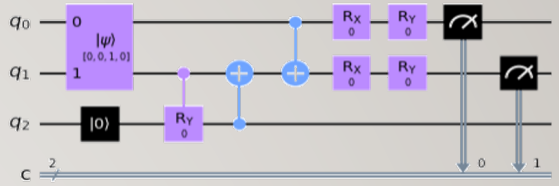
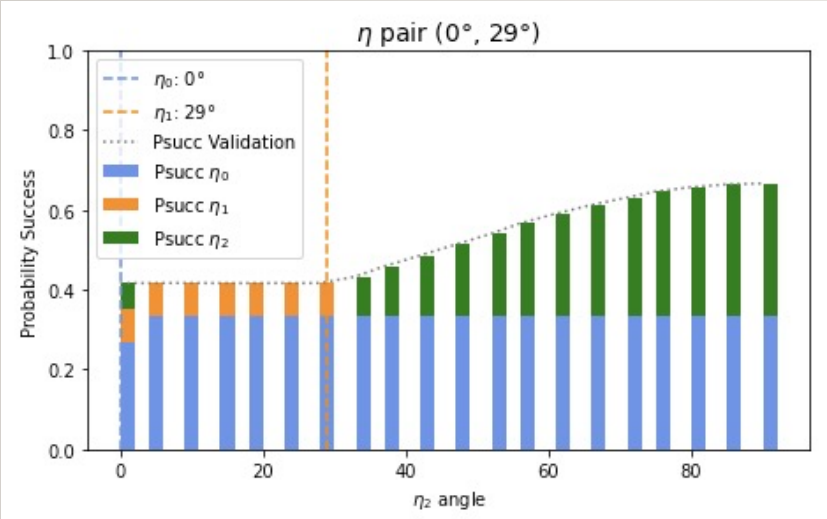
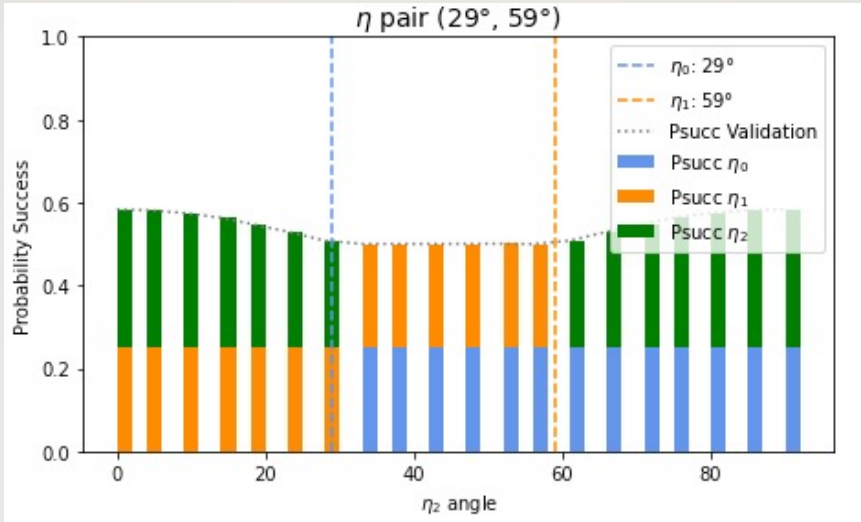
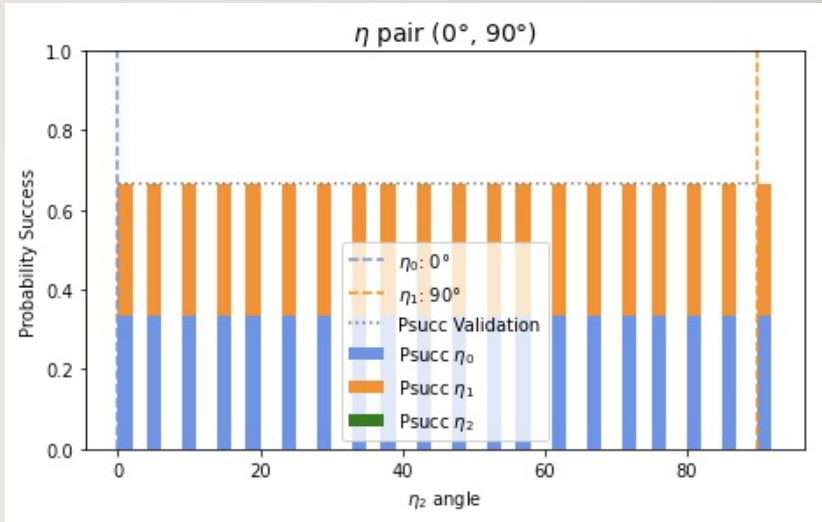
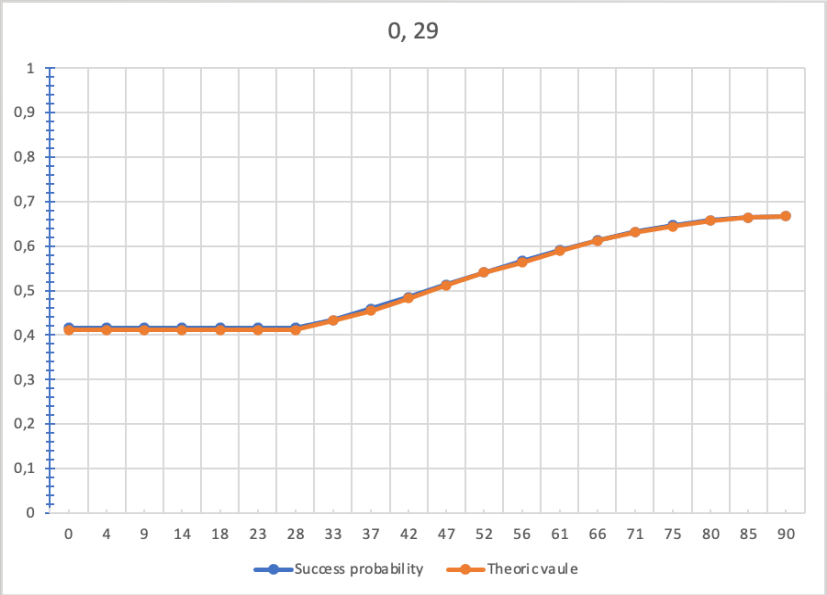
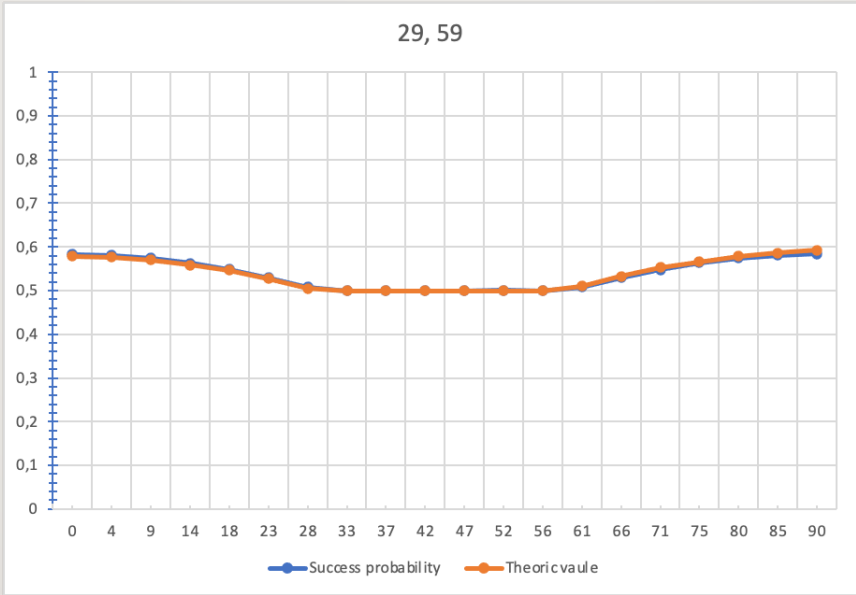


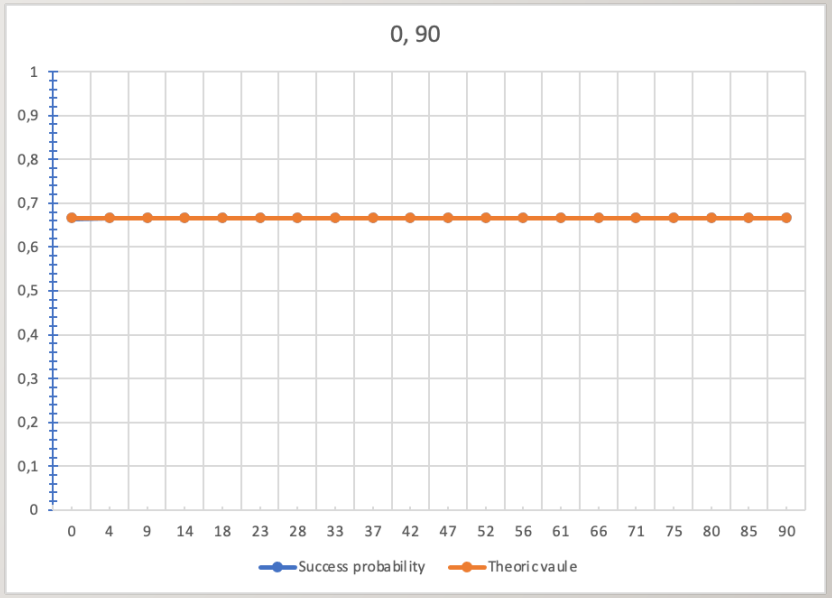




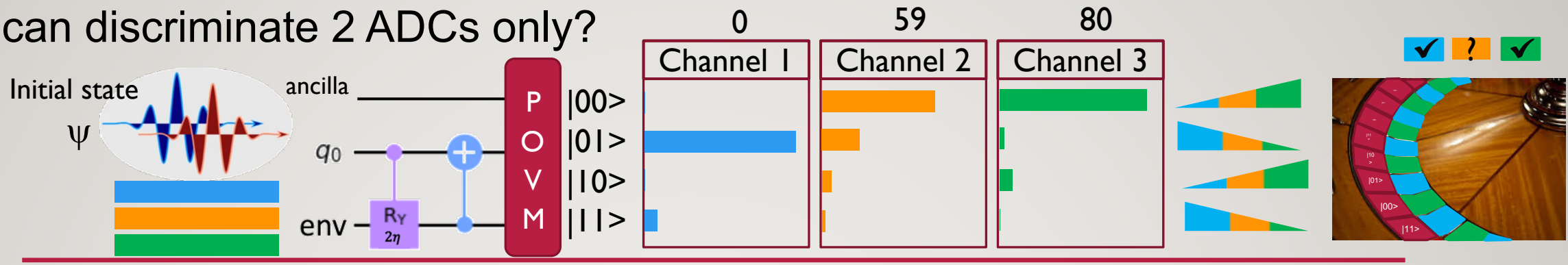


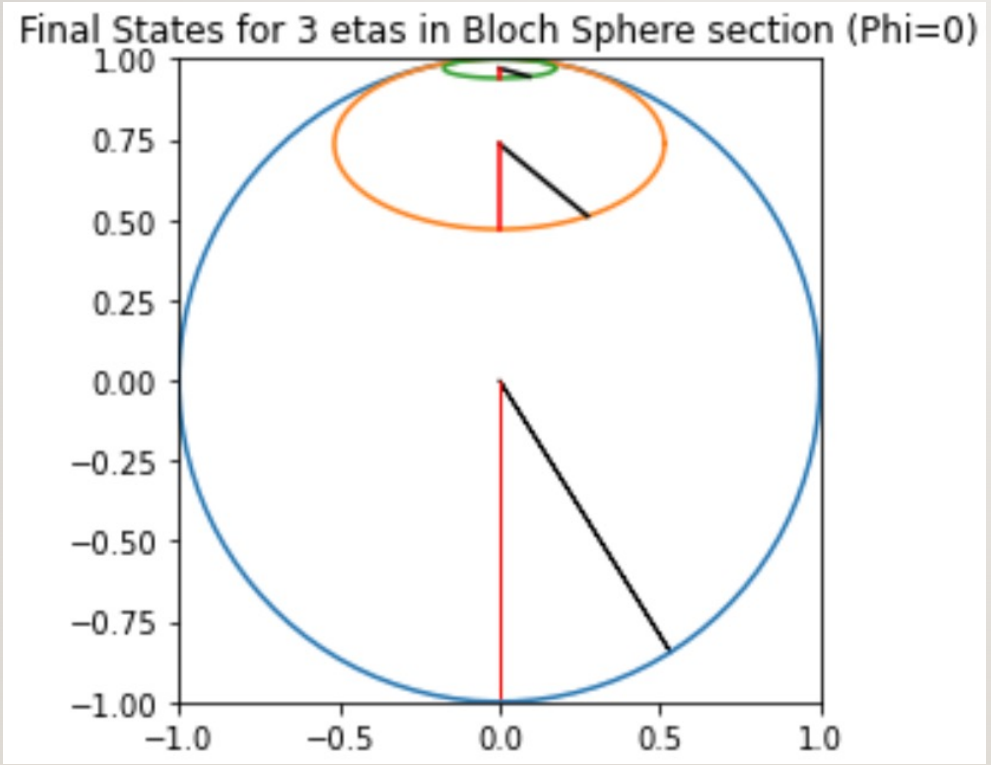






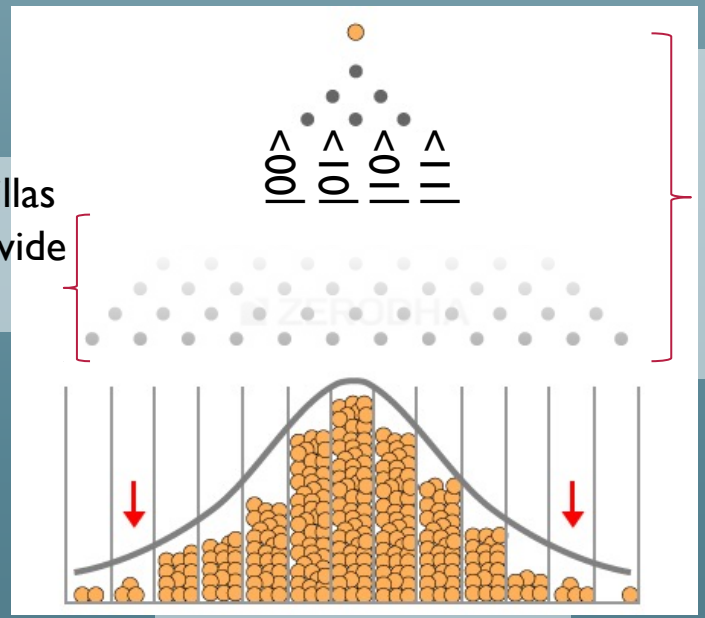
Why we can discriminate 2 ADCs only?





Analogy

More ancillas could provide +depth



Each eta (ADC) + POVM will define a different path dividers distribution

buckets = $2^{\#\ \text{qubits}}$