16703 Assignment 1 - Spring 2026
State Feedback Control, Stability and PID Control

Submission Deadline: January 30th, 2026 11:59pm

Instructions:

1. The appropriate steps for each question must be shown.
2. Some questions require coding. You can use any software of your choice, but MATLAB is preferred.
3. Include the code in your report as well, at the end
4. There will be 2 submissions; one is the report, and the other is the zip file consisting of all the code. Both of them have to be submitted for getting full credit.
5. Name your report as (andrewID)_16703_HW1.pdf and the zip file of the code as (andrewID)_16703_HW1_code.zip.
6. A gradescope submission would be created. Make sure that you follow the instructions carefully while submitting this assignment. Submit the assignment as a pdf and label the pages according to the questions appropriately. Failing to do so might result in no points being awarded to the student.
7. Please, check out the attached reading material. It helps in answering some of the questions in this assignment. In addition, it also gives a slick walkthrough of PID tuning that you might actually find pretty fun!
8. Forming teams up to 3 members is highly encouraged. Refer to the following link on how to submit. Group Submission in Gradescope
9. At the end of your report, list the contributions of the individual group members. It is very important that there is an equal split in the workload among the group members.
10. Any queries can be asked through piazza or through office hours.

Introduction: Series Elastic Actuators

Series Elastic Actuators (SEAs) place an elastic element (spring) between the motor and load, providing compliance for safe human-robot interaction, force control, and shock absorption. These are the buildong blocks of legged robots including humanoids (Boston Dynamics' Atlas, Agility Robotics' Digit), exoskeletons, and prosthetics.

The spring provides:

- Force control: Torque measurement via spring deflection
- Shock absorption: Protection from impact loads
- Energy storage: Efficient locomotion in walking/running
- Safety: Compliance for human interaction

In this assignment, you will learn about controlling a basic SEA system using both PID control and full-state feedback. You'll analyze system dynamics, design controllers, and compare their performance for position tracking.

System Model

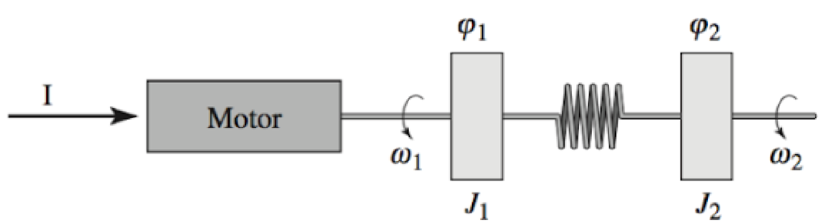

Figure 1: Motor load coupling through series spring

Fig 1 shows the diagram of the system. The motor (inertia J_1) drives an external load (J_2) through a torsional spring (stiffness k) with damping c. The motor torque is $\tau_m = k_f I$, where I is the current.

System dynamics (assuming zero disturbance, $\tau_d = 0$):

$$J_1\ddot{\varphi}_1 + c(\dot{\varphi}_1 - \dot{\varphi}_2) + k(\varphi_1 - \varphi_2) = k_f I \tag{1}$$

$$J_2\ddot{\varphi}_2 + c(\dot{\varphi}_2 - \dot{\varphi}_1) + k(\varphi_2 - \varphi_1) = 0 \tag{2}$$

Part A: System Modeling and State-Space Representation (2pts)

Task: Derive the state-space model using normalized state variables:

- $x_1 = \varphi_1$, $x_2 = \varphi_2$, $x_3 = \dot{\varphi}_1/\omega_0$, $x_4 = \dot{\varphi}_2/\omega_0$

where $\omega_0 = \sqrt{k(J_1 + J_2)/(J_1 J_2)}$ is the scaling frequency.

Given parameters: $J_1 = 10/9$ kg·m^2, $J_2 = 10$ kg·m^2, $c = 0.1$ N·m·s/rad, $k = 1$ N·m/rad, $k_f = 1$ N·m/A

Design matrices A, B, C, and D such that the output is the angular displacement of the external load (φ_2) and the input is the motor current (I).

Part B: Open-Loop Analysis (2pts)

Tasks:

1. Find the eigenvalues of the state matrix A using MATLAB and verify numerically that they are $0, 0, -0.05 \pm i$. What does this tell you about the system's stability and natural behavior?
2. Show the Bode plot (using MATLAB). Are there any visible peaks or ripples? If so, explain what you think causes them.

MATLAB functions: `eig(), bode()`

Part C: PID Control Design (5pts)

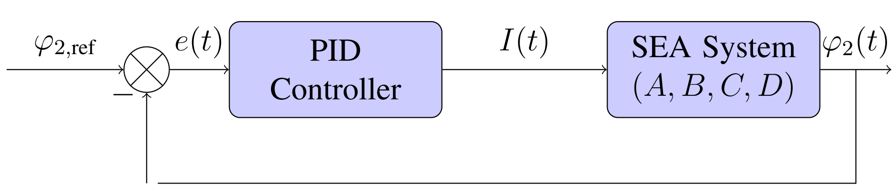

Figure 2: PID Control System Block Diagram

A robotics company developing humanoid locomotion systems has proposed the following PID controller gains for position tracking of the load angle φ_2:

$$I(t) = K_p e(t) + K_i \int_0^t e(\tau)d\tau + K_d \dot{e}(t) \tag{3}$$

where $e(t) = \varphi_{2,\text{ref}}(t) - \varphi_2(t)$ is the tracking error.

Proposed gains: $K_p = 0.025$, $K_i = 0$, $K_d = 0.5$ (the company decided to finalize on a PD controller)

Tasks:

1. Implement the PD controller (you can use the functions pid() and feedback() in MATLAB).
2. Simulate the step response for a reference step of $\varphi_{2,\text{ref}} = 1$ radian (step() function in MATLAB). Paste the plot here.
3. Compute and report the following performance metrics (you can use stepinfo() and dcgain() functions in MATLAB):
   - Rise time
   - Settling time
   - Percent overshoot
   - Steady-state error

You can read section 2.3 of the reading material for a good understanding of how these metrics are a good indicator of the performance of a control system.

Part D: State Feedback Control (6pts)

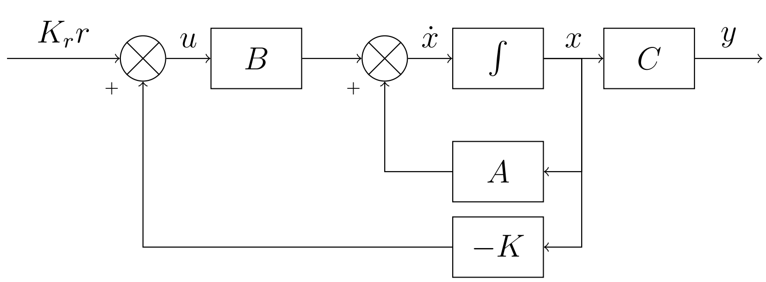

Figure 3: State Feedback Control System Block Diagram

Now your job would be to try and design a better controller. You will design a full-state feedback controller (Fig 3 - You might have come across this diagram in class)

$$I = -Kx + K_r r \tag{4}$$

where $r = \varphi_{2,\text{ref}}$ is the reference input, and K_r is a feedforward gain for reference tracking.

Modified state-space equations:

$$\dot{x} = Ax + B(-Kx + K_r r) = (A - BK)x + BK_r r \quad (5)$$

$$y = Cx \quad (6)$$

The closed-loop system matrix is $A_{\text{cl}} = A - BK$.

D1: Pole Placement (1pts)

Task: Design the feedback gain matrix K to place the closed-loop eigenvalues at:

$$\lambda = \{-2, -1, -1 + i, -1 - i\} \quad (7)$$

Use MATLAB's place() function. Verify the closed-loop eigenvalues.

D2: Reference Tracking (2pts)

The feedforward gain K_r ensures zero steady-state error for step references.

For step reference tracking, at steady state ($\dot{x} = 0$):

$$(A - BK)x_{ss} + BK_r r = 0 \quad (8)$$

$$x_{ss} = -(A - BK)^{-1}BK_r r \quad (9)$$

For the output $y = Cx$ to equal r:

$$r = Cx_{ss} = -C(A - BK)^{-1}BK_r r \quad (10)$$

Therefore:

$$K_r = -\frac{1}{C(A - BK)^{-1}B} \quad (11)$$

Task: Compute K_r using the formula above.

D3: Simulation and Comparison (3pts)

Tasks:

1. Simulate the state feedback controller response to a unit step reference. Paste the plot here.
2. Compute the same performance metrics as Part C
3. Create a comparison table:

Metric	**PD**	**State Feedback**
Rise time		
Settling time		
Overshoot (%)		
Steady-state error		

4. **Analysis:**
   - Which controller performs better and why? Were you able to achieve a better performance than the controller designed by the company (PD)?
   - What is the main limitation of state feedback that ”might” make PD preferable in practice?