HW1

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Part A

The scaling frequency evaluates to

$$\omega_0 = \sqrt{\frac{1(10/9 + 10)}{(10/9) \cdot 10}} = 1.$$

Then, the kinematic relations,

$$\begin{aligned} \dot{x}_1 &= x_3, && (1) \\ \dot{x}_2 &= x_4. && (2) \end{aligned}$$

Then, the remaining dynamics are

$$\begin{aligned} \dot{x}_3 &= \frac{1}{J_1}\left[-c(x_3 - x_4) - k(x_1 - x_2) + k_f I\right], && (3) \\ \dot{x}_4 &= \frac{1}{J_2}\left[-c(x_4 - x_3) - k(x_2 - x_1)\right]. && (4) \end{aligned}$$

Substituting numerical values:

$$\begin{aligned} \dot{x}_3 &= -0.9(x_1 - x_2) - 0.09(x_3 - x_4) + 0.9I, && (5) \\ \dot{x}_4 &= 0.1(x_1 - x_2) + 0.01(x_3 - x_4). && (6) \end{aligned}$$

Then,

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -0.9 & 0.9 & -0.09 & 0.09 \\ 0.1 & -0.1 & 0.01 & -0.01 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 \\ 0 \\ 0.9 \\ 0 \end{bmatrix}.$$

The output is the angular displacement of the external load:

$$y = \varphi_2 = x_2.$$

Thus,

$$C = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix}, \qquad D = \begin{bmatrix} 0 \end{bmatrix}.$$

$$\begin{aligned} \dot{x} &= Ax + BI, \\ y &= Cx \end{aligned}$$

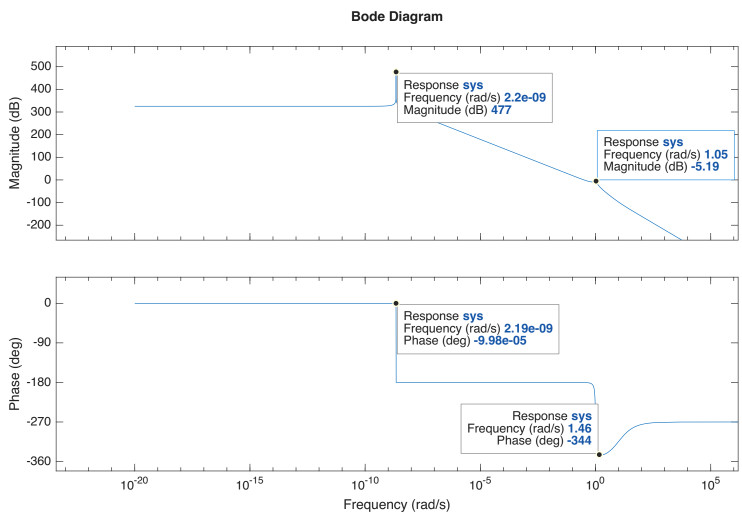
Part B

(1)

the complex conjugate eigen values $-0.5 \pm i$ indicate system will oscillate with a decaying amplitude.

the 0 eigen values indicate the system is marginally stable.

(2)



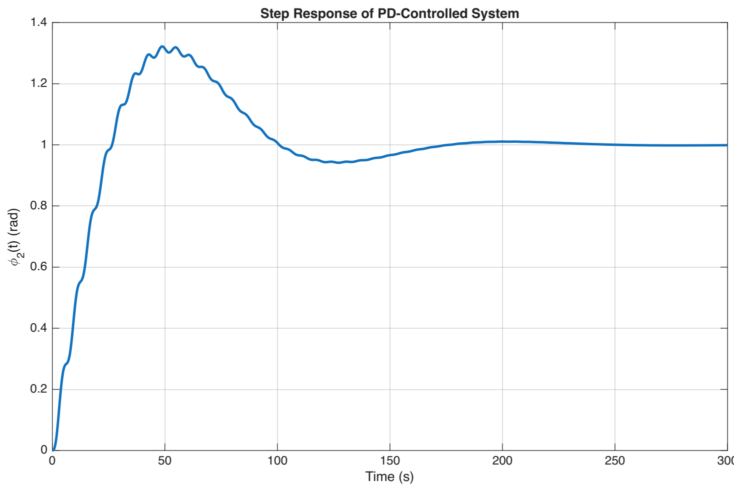
There are two peaks in the magnitude plot at $2.2e-9$ and 1.05 rad/s respectively. The peak at $2.2e-9$ rad/s is more dominant.

The first high peak is cause by two spinning disk oscillating against each other as the control frequency gradually increase from DC. Because there are two 0 eigen values, this indicate that there are two integrator in the system, so the phase shifted by 180 degrees.

Then as the frequency increases to 1.05 rad/s, which should be the resonant frequency. At this point the complex conjugate eigen values start to kick in, causing the phase to change further.

Then the zero of the system at -10 causes the phase to shift back by 90 degrees.

Part C



Rise time: 19.502 s
Settling time: 159.718 s
Percent overshoot: 32.18 %
Steady-state error: 0.000 rad

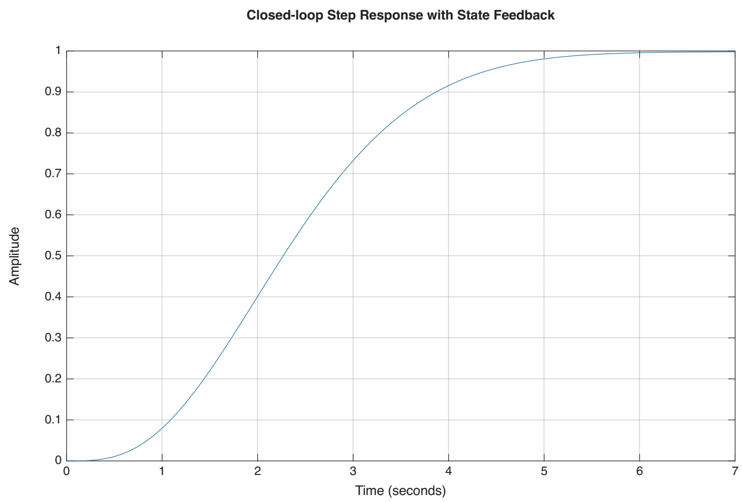
Part D

(1) $K = [8.933335.51115.4444101.2222]$

(2)

$Kr = 44.4444$

(3)



K_p	K_d	Settling Time (s)
Rise time	19.502	2.782
Settling time	159.718	4.984
Overshoot (%)	32.18	0
Steady-state error	0	0

Table 1: Settling times for different K_p and K_d values

```
disp("Part B (1)")
```

```
Part B (1)
```

```
A = [0, 0, 1, 0;
     0, 0, 0, 1;
     -0.9, 0.9, -0.09, 0.09;
     0.1, -0.1, 0.01, -0.01];
disp(A)
```

```
         0         0    1.0000         0
         0         0         0    1.0000
   -0.9000    0.9000   -0.0900    0.0900
    0.1000   -0.1000    0.0100   -0.0100
```

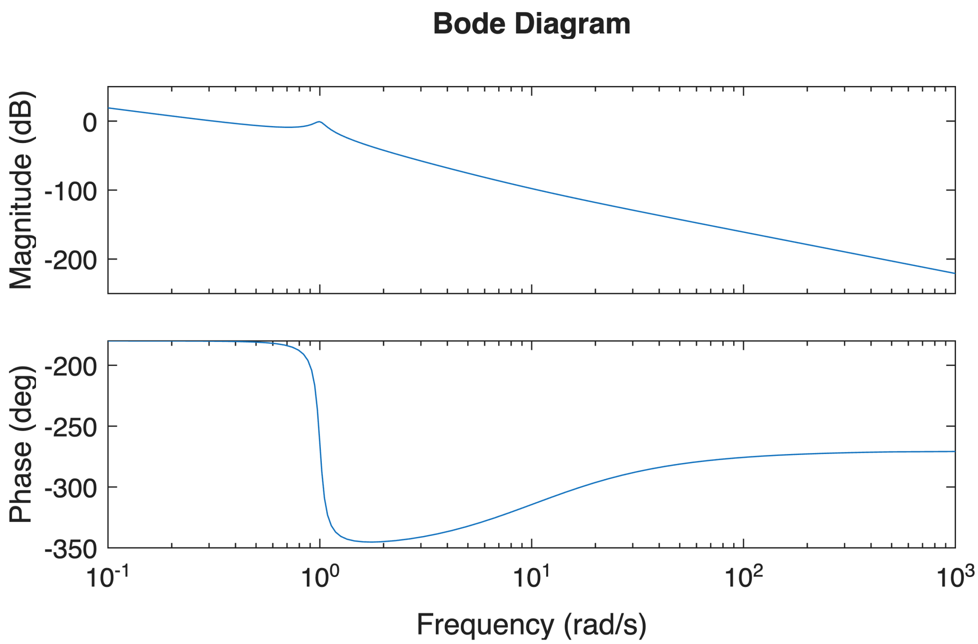
```
% Calculate the eigenvalues and eigenvectors of matrix A
[eigenVectors, eigenValues] = eig(A);

disp(diag(eigenValues))
```

```
  -0.0500 + 0.9987i
  -0.0500 - 0.9987i
  -0.0000 + 0.0000i
   0.0000 + 0.0000i
```

```
disp("Part B (2)")
```

```
Part B (2)
```

```
A = [0, 0, 1, 0;
     0, 0, 0, 1;
     -0.9, 0.9, -0.09, 0.09;
     0.1, -0.1, 0.01, -0.01];

B = [0; 0; 0.9; 0];

C = [0, 1, 0, 0];

D = 0;

sys = ss(A, B, C, D);

bode(sys)
```




```
pole(sys)
```

```
ans = 4×1 complex
  -0.0500 + 0.9987i
  -0.0500 - 0.9987i
  -0.0000 + 0.0000i
   0.0000 + 0.0000i
```

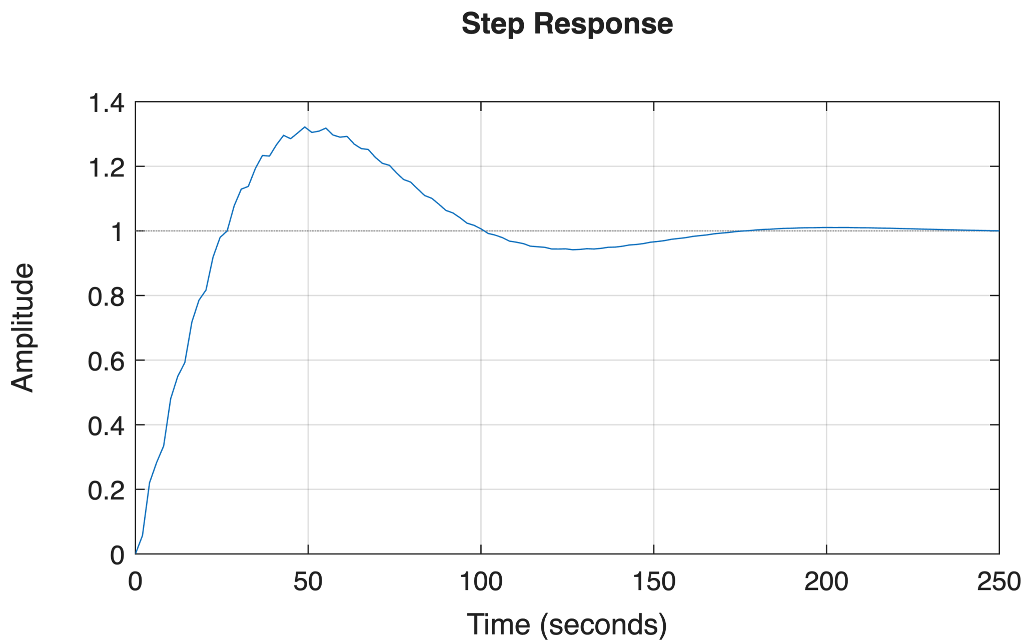
```
zero(sys)
```

```
ans =
-10.0000
```

```
disp("Part C")
```

```
Part C
```

```
Kp = 0.025;
Kd = 0.5;

C_pd = tf([Kd Kp],[1]);

sys_cl = feedback(C_pd*sys,1);

step(sys_cl)
grid on
```




```
info = stepinfo(sys_cl, 'SettlingTimeThreshold', 0.02);
ss_error = abs(1 - dcgain(sys_cl));

fprintf('Rise time: %.3f s\n', info.RiseTime);
```

```
Rise time: 19.502 s
```

```
fprintf('Settling time: %.3f s\n', info.SettlingTime);
```

```
Settling time: 159.718 s
```

```
fprintf('Percent overshoot: %.2f %%\n', info.Overshoot);
```

```
Percent overshoot: 32.18 %
```

```
fprintf('Steady-state error: %.3f rad\n', ss_error);
```

```
Steady-state error: 0.000 rad
```

```
disp("Part D (1) ")
```

```
Part D (1)
```

```
desired_poles = [-2, -1, -1+1i, -1-1i];

K = place(A, B, desired_poles);

disp(K);
```

```
    8.9333   35.5111    5.4444  101.2222
```

```
Acl = A - B*K;
eigAcl = eig(Acl);
disp(eigAcl)
```

```
  -2.0000 + 0.0000i
  -1.0000 + 1.0000i
  -1.0000 - 1.0000i
  -1.0000 + 0.0000i
```

```
disp("Part D (2)")
```

```
Part D (2)
```

```
Kr = -1 / (C * ((A - B*K) \ B))
```

```
Kr =
44.4444
```

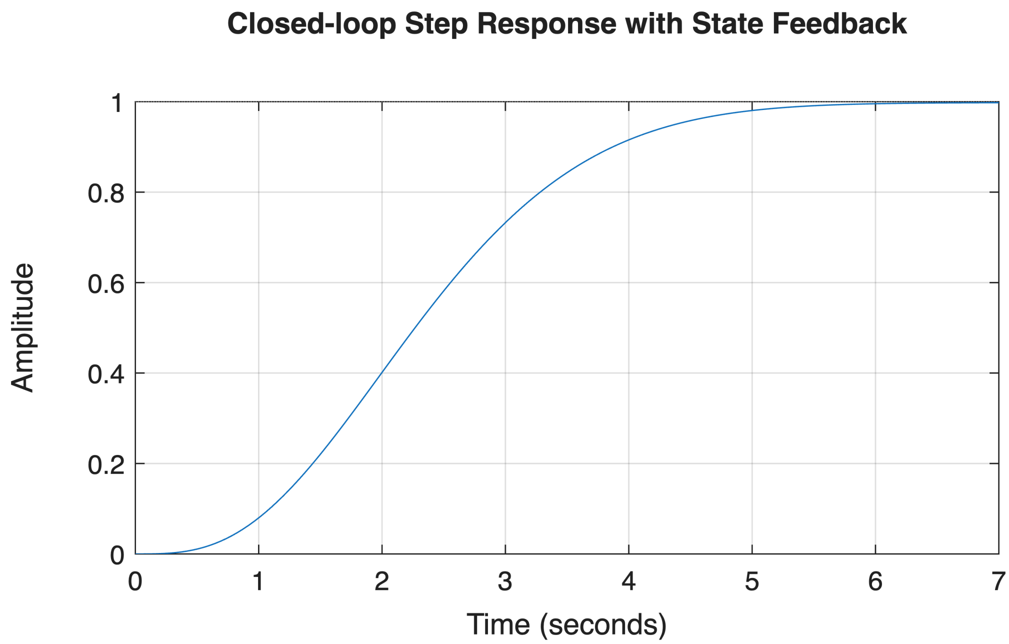
```
disp("Part D (3)")
```

```
Part D (3)
```

```
Bcl = B * Kr
```

```
Bcl = 4×1
         0
         0
   40.0000
         0
```

```
Acl = A - B*K;
Bcl = B*Kr;

sys_fb = ss(Acl, Bcl, C, 0);

step(sys_fb)
grid on
title('Closed-loop Step Response with State Feedback')
```




```
info = stepinfo(sys_fb, 'SettlingTimeThreshold', 0.02);
ss_error = abs(1 - dcgain(sys_fb));

fprintf('Rise time: %.3f s\n', info.RiseTime);
```

```
Rise time: 2.782 s
```

```
fprintf('Settling time: %.3f s\n', info.SettlingTime);
```

```
Settling time: 4.984 s
```

```
fprintf('Percent overshoot: %.2f %%\n', info.Overshoot);
```

```
Percent overshoot: 0.00 %
```

```
fprintf('Steady-state error: %.3f rad\n', ss_error);
```

```
Steady-state error: 0.000 rad
```