Computer Vision
for HCI

2-D Shape

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Region Representations/Properties

- Once regions have been identified, properties of regions can be input to higher-level decision-making procedures
  – Recognition or inspection
- Common geometric properties of region useful for simple shape description
  – Boundary, bounding box, extremal points, area, perimeter, compactness/circularity, moments, etc.
- We will examine geometric and shape representation/properties for **binary** images [(0,1) or (0, 255)]
  – Assumes have segmented out pixels for the object of interest

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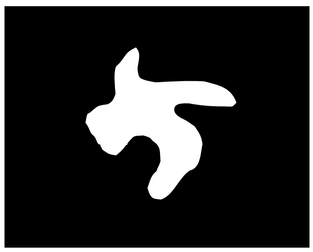
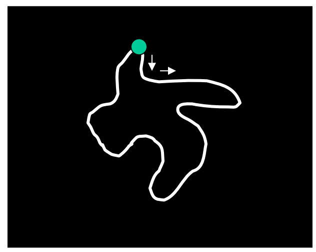
Boundary Coding

- Regions can be represented by their <u>boundaries</u> instead of an image
- Simplest form is linear list of border pixels of each region





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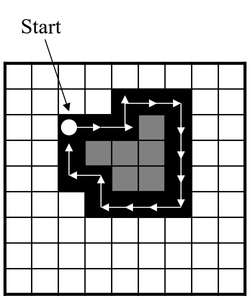
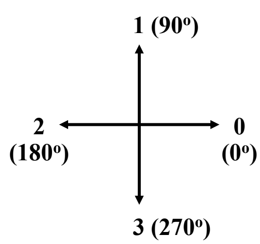
Chain Code Representation

- Simple technique for representing shape of contour
- Each directed line segment is assigned code
- <u>Chain code</u> is string of those ordered codes

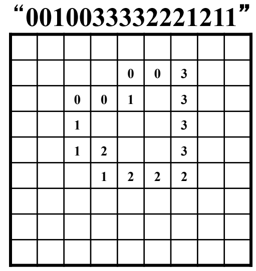


Binary image



90^{o} quantization

Chain code:
"0010033332221211"



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Shape Number

- Chain codes dependent upon:
  - Orientation and start point of contour
- Invariance to rotation by integer multiples of quantization (90^{o})
  - Take "circular" first-difference of chain code: $f(x)$-$f(x$-1)

    Chain code: 0010033332221211

    First-diff: 3013030003003130 (1-2=3!)
  - Could also align grid to main axes of object
- Invariance to starting point
  - Circular shift first-diff number to be minimal integer

    First-diff: **301303**0003003130

    Shape #: 0003003130**301303**

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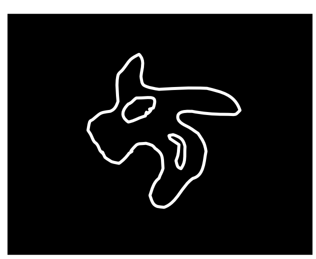
Internal/External Boundaries

- When region has one or more hole boundaries, represented by chain code for each of them
  - Spatial relation between contours?



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Quadtree Representation

- Quadtree encodes entire region (not just boundary)
  - Binary image
- Each node of quadtree represents a square region in the image
  - Has one of 3 labels: FULL, EMPTY, MIXED
- Subdivide nodes until either FULL or EMPTY

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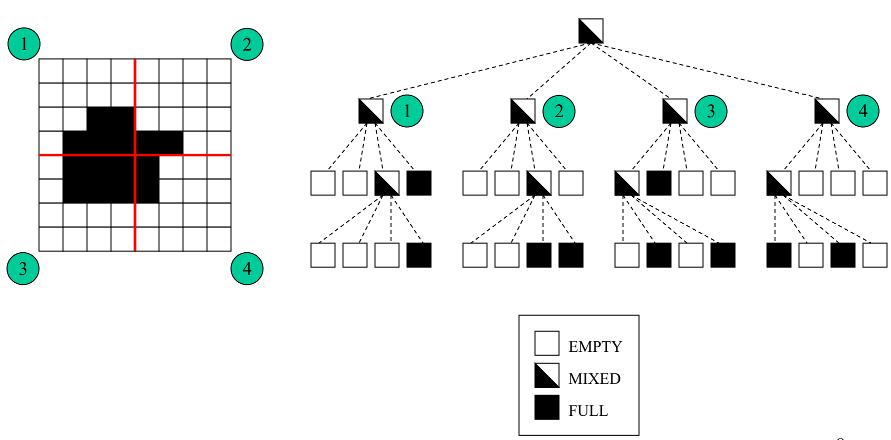
Quadtree Example



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Quadtree

- Tree/graph matching for recognition
- Previous example is "blocky"
  - Small image grid
  - Need many more levels/cells (resolution) to approximate curves
- Quadtrees have been used to represent map regions in geographic information systems (GIS)

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Medial Axis Transform

- "Skeleton" representation of binary region
- Steps
  - Original binary image is f^0
  - Iteratively compute (until no change):

$$f^k(x,y) = f^0(x,y) + \min(f^{k-1}(p,q))$$
$$\forall (p,q) \text{ 4-connected to } (x,y)$$

  - Medial axis given by all points (x,y) such that:

$$f^k(x,y) \geq f^k(p,q)$$

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Medial Axis Example

$f^0(x,y)$

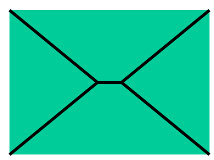
1	1	1	1	1	1
1	1	1	1	1	1
1	1	1	1	1	1
1	1	1	1	1	1
1	1	1	1	1	1

$f^1(x,y)$

1	1	1	1	1	1
1	2	2	2	2	1
1	2	2	2	2	1
1	2	2	2	2	1
1	1	1	1	1	1

$f^2(x,y)$

1	1	1	1	1	**1**
1	**2**	2	2	**2**	1
1	2	**3**	**3**	2	1
1	**2**	2	2	**2**	1
1	1	1	1	1	**1**



1					**1**
	1			**1**	
		1	**1**		
	1			**1**	
1					**1**

Medial axis

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Medial Axis Extras

- Also can recover shape from medial axis with iterative algorithm
- Matlab code available for skeletonization
  - – *Skel* function (bwmorph)
  - – Also has morphological functions

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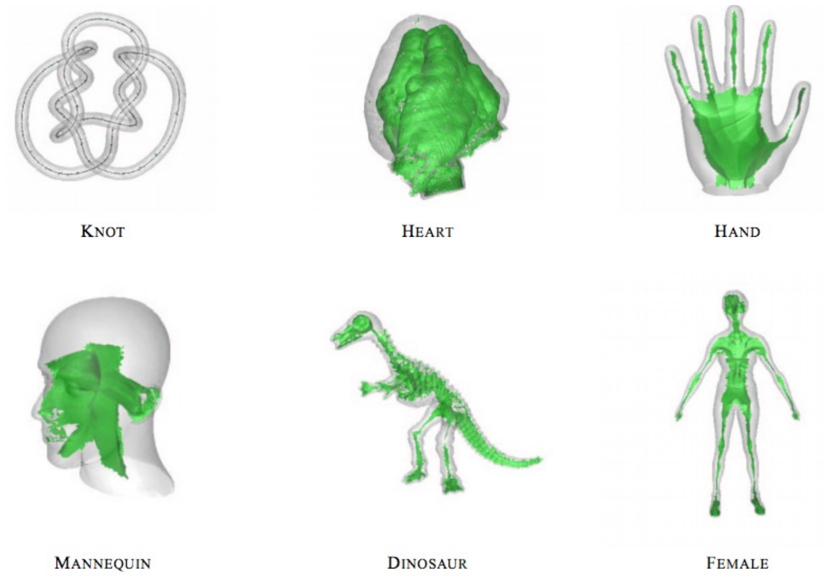
Medial Axis of 3-D Surface

(Prof. Tamal Dey)



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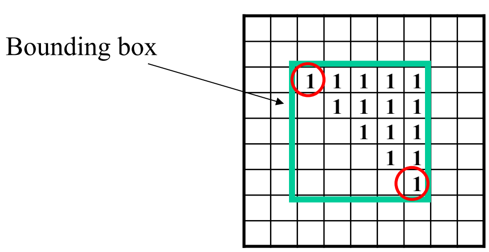
Bounding Box

- Often useful to have rough idea of where region located (e.g., for tracking)
- Bounding box is enclosing rectangle that touches topmost, bottommost, leftmost, and rightmost points in region
- As shown, can include much of background



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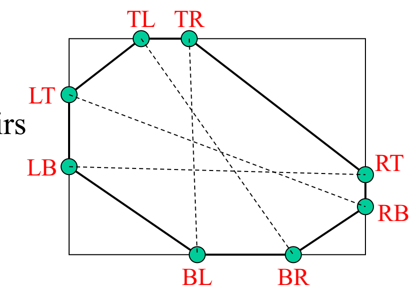
Extremal Points

- Examine where region touches bounding box
- At most 8 distinct "extremal" points/pixels for region on bounding box
  - Topmost left/right
  - Rightmost top/bottom
  - Leftmost top/bottom
  - Bottommost left/right
- Extremals occur in opposite pairs
  - Defines axis for each pair
- Compute <u>approximations</u> of
  - Axis **length**
  - Axis **orientation**



Area

- Area A of binary region R simply defined as:

$$A = \sum_{(x,y)\in R} 1$$

		1	1	1	1	1		
			1	1	1	1		
				1	1	1		
					1	1		
						1		

Binary image

Area = 15 (pixels)

Perimeter and Compactness

- Perimeter P of binary region R is sum of its border pixels
  - Border pixel has at least 1 background pixel in its neighborhood
- Compactness/Circularity C of region is defined as follows:

$$C = 4\pi \frac{A}{P^2}$$

Circle: $A = \pi r^2$ $P = 2\pi r$

$C = 1$

Square: $A = L^2$ $P = 4L$

$C = \frac{\pi}{4}$

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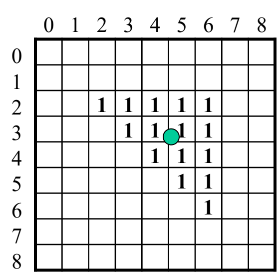
Centroid

- Centroid of binary region is average location of pixels in R:

$$x_c = \bar{x} = \frac{1}{N} \sum_{(x,y)\in R} x \qquad y_c = \bar{y} = \frac{1}{N} \sum_{(x,y)\in R} y$$



Binary image

$$\begin{aligned} x_c &= \frac{1}{15}\left[2 + (2\cdot 3) + (3\cdot 4) + (4\cdot 5) + (5\cdot 6)\right] \\ &= 4.67 \\ y_c &= \frac{1}{15}\left[(5\cdot 2) + (4\cdot 3) + (3\cdot 4) + (2\cdot 5) + 6\right] \\ &= 3.34 \end{aligned}$$

Useful for target tracking!

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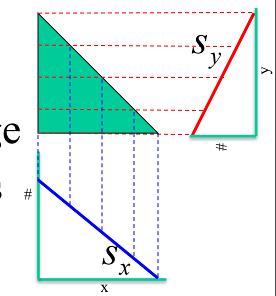
Signatures

- Useful for finding preliminary landmarks
- Horizontal signature of binary image
  - Projection of image onto the x-axis

$$s_x = \sum_y B[x,y]$$

- Vertical signature of binary image
  - Projection of image onto the y-axis

$$s_y = \sum_x B[x,y]$$



(consider figure-8 shape)

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Spatial Moments

- Spatial moments often used to describe region shape

$$m_{pq} = \sum\sum x^p y^q I[x,y]$$

"Area" [for binary (0,1) image]

$$A = m_{00} = \sum\sum x^0 y^0 I[x,y] \quad \text{"Zeroth order"}$$

$$\left.\begin{aligned} m_{10} &= \sum\sum x^1 y^0 I[x,y] \\ m_{01} &= \sum\sum x^0 y^1 I[x,y] \end{aligned}\right\} \text{"First order"}$$

"Centroid" $\longrightarrow$ $\bar{x} = \frac{m_{10}}{m_{00}} \qquad \bar{y} = \frac{m_{01}}{m_{00}}$

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Central Moments

- Central moments (translation invariant)

$$\mu_{pq} = \sum\sum (x-\bar{x})^p (y-\bar{y})^q I[x,y]$$

$$\bar{x} = \frac{m_{10}}{m_{00}} \qquad \bar{y} = \frac{m_{01}}{m_{00}}$$

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Second-Order Central Moments

- Three second-order central moments

$$\mu_{20} = \sum\sum (x-\bar{x})^2 (y-\bar{y})^0 I[x,y]$$

$$\mu_{11} = \sum\sum (x-\bar{x})^1 (y-\bar{y})^1 I[x,y]$$

$$\mu_{02} = \sum\sum (x-\bar{x})^0 (y-\bar{y})^2 I[x,y]$$

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Moment Ellipse Orientation

- If region is ellipse, second-order central moments have useful algebraic description of orientation

Let,

$$a = \mu_{20} = \sum\sum (x-\bar{x})^2 (y-\bar{y})^0 I[x,y]$$
$$b = \mu_{11} = \sum\sum (x-\bar{x})^1 (y-\bar{y})^1 I[x,y]$$
$$c = \mu_{02} = \sum\sum (x-\bar{x})^0 (y-\bar{y})^2 I[x,y]$$

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Moment Ellipse Orientation

- Resulting orientation relationship

$$\tan(2\theta) = \frac{2b}{a-c}$$

- If (b = 0) and (a = c)
  - Object is too symmetric to allow definition of axis
- *Can also use eigenvalues and eigenvectors to determine ellipse [more on this later…]*

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Similitude Moments

(Invariant to translation and scale)

$$\eta_{ij} = \frac{\mu_{ij}}{(m_{00})^{\frac{i+j}{2}+1}} = \frac{\sum\sum (x-\overline{x})^i (y-\overline{y})^j I[x,y]}{\left(\sum\sum I[x,y]\right)^{\frac{i+j}{2}+1}}$$

Not *integer* division operator

for $2 \le (i+j) \le 3$:

$$\mathrm{N} = \begin{bmatrix} \eta_{02} & \eta_{03} & \eta_{11} & \eta_{12} & \eta_{20} & \eta_{21} & \eta_{30} \end{bmatrix}$$

M.K. Hu, "Visual pattern recognition by moment invariants", Information Theory, IRE Transactions, 1962, pp. 179-187

http://www.sci.utah.edu/~gerig/CS7960-S2010/handouts/CS7960-AdvImProc-MomentInvariants.pdf

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"Per-pixel" Moment **Visualization** ONLY…

Would need to sum all of these values across the image to get the actual moment value for the image

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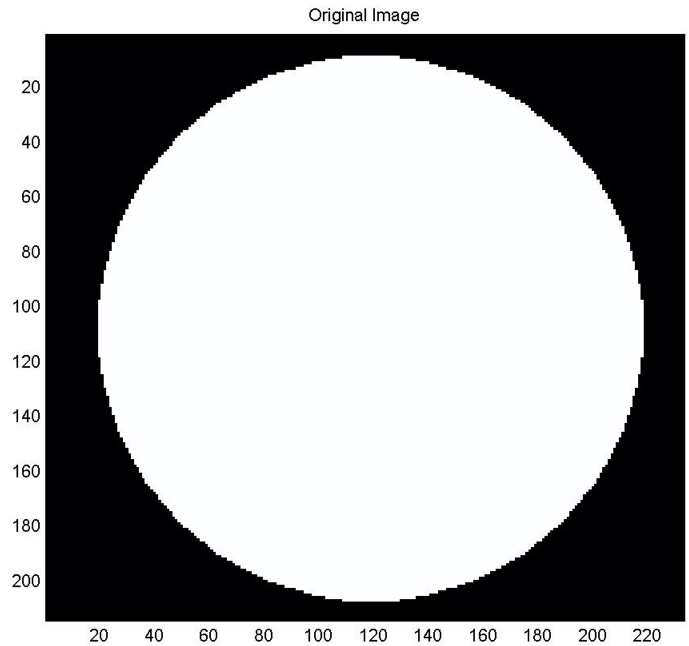

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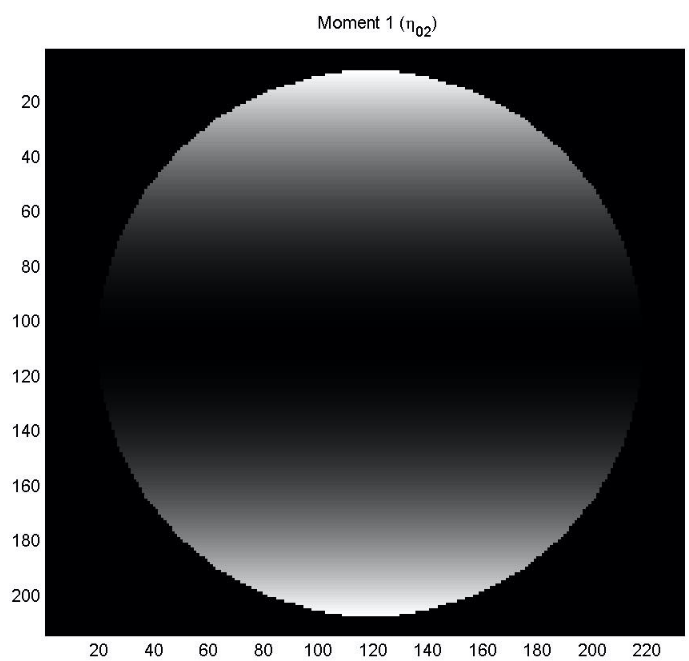

Sum <u>all</u> values to give actual single moment value!

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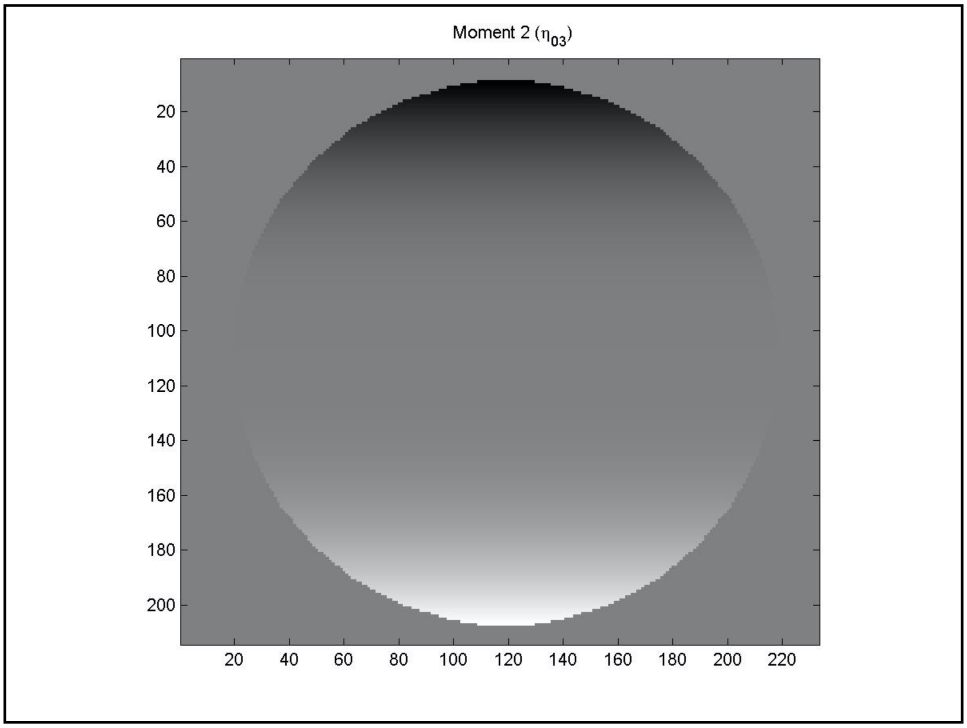
Moment 2 (η_{03})
20
40
60
80
100
120
140
160
180
200
20 40 60 80 100 120 140 160 180 200 220

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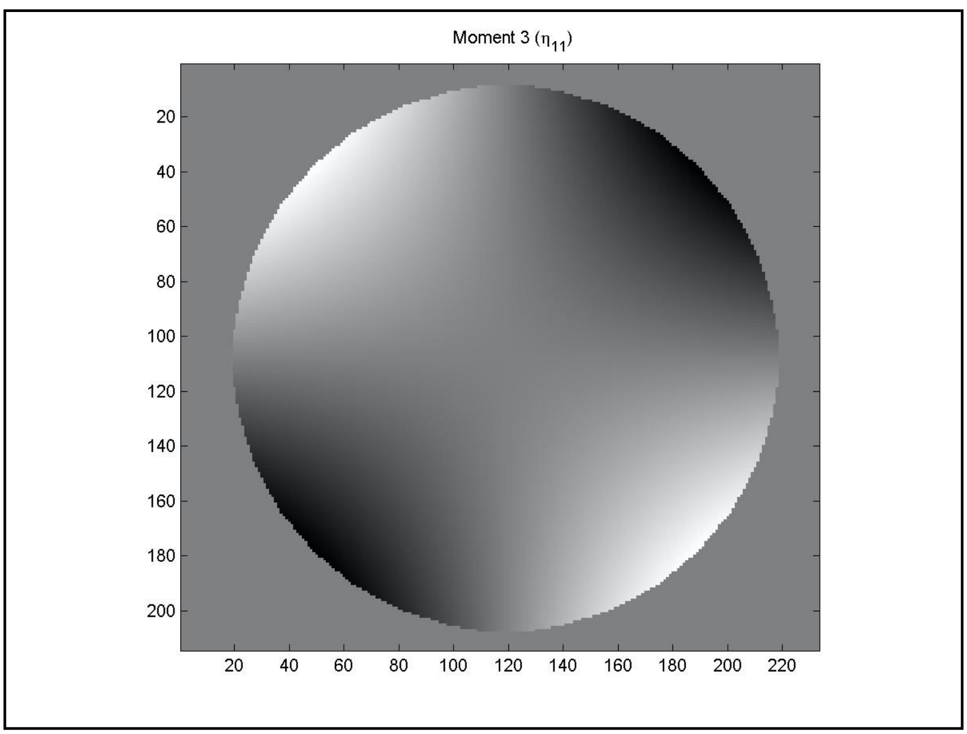
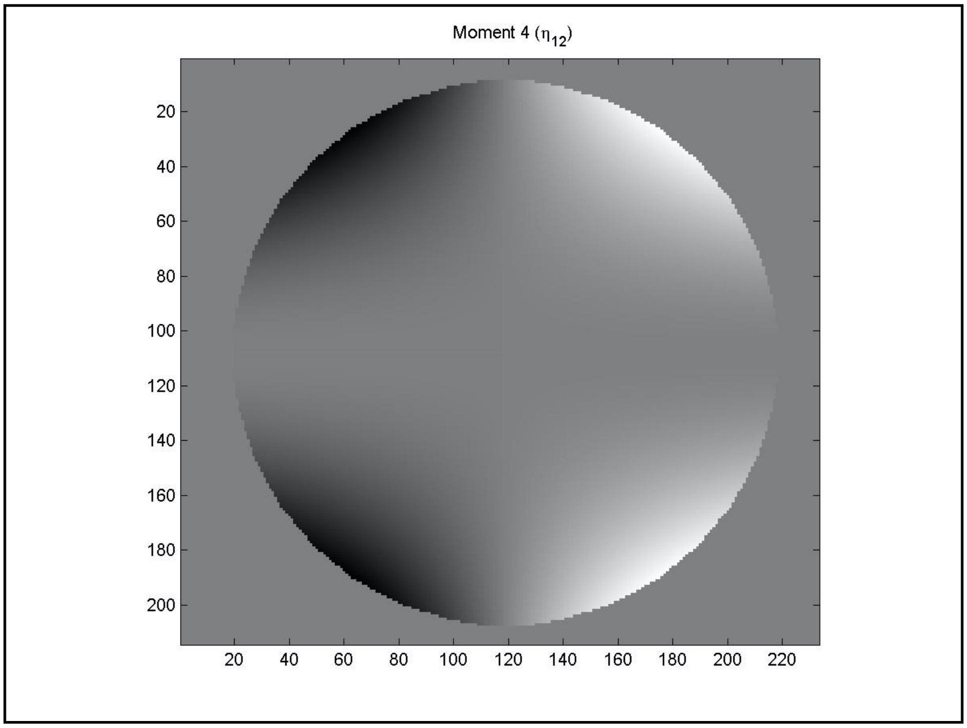

Moment 3 (η_{11})
20
40
60
80
100
120
140
160
180
200
20 40 60 80 100 120 140 160 180 200 220

30


Moment 4 (η_{12})
20
40
60
80
100
120
140
160
180
200
20
40
60
80
100
120
140
160
180
200
220

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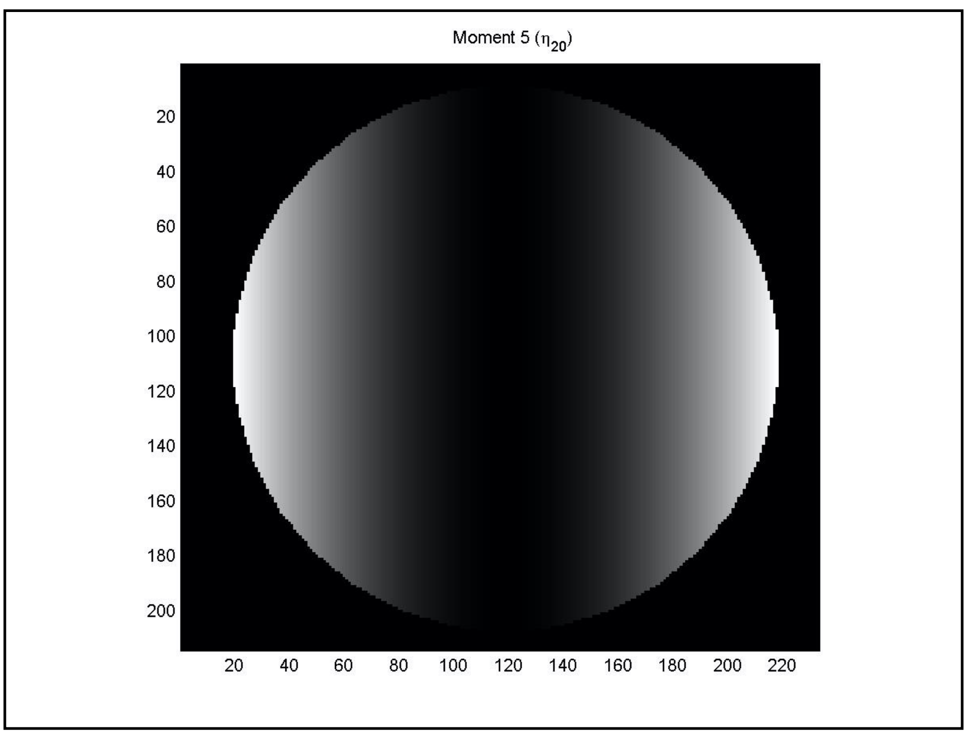
Moment 5 (η_{20})
20
40
60
80
100
120
140
160
180
200
20
40
60
80
100
120
140
160
180
200
220

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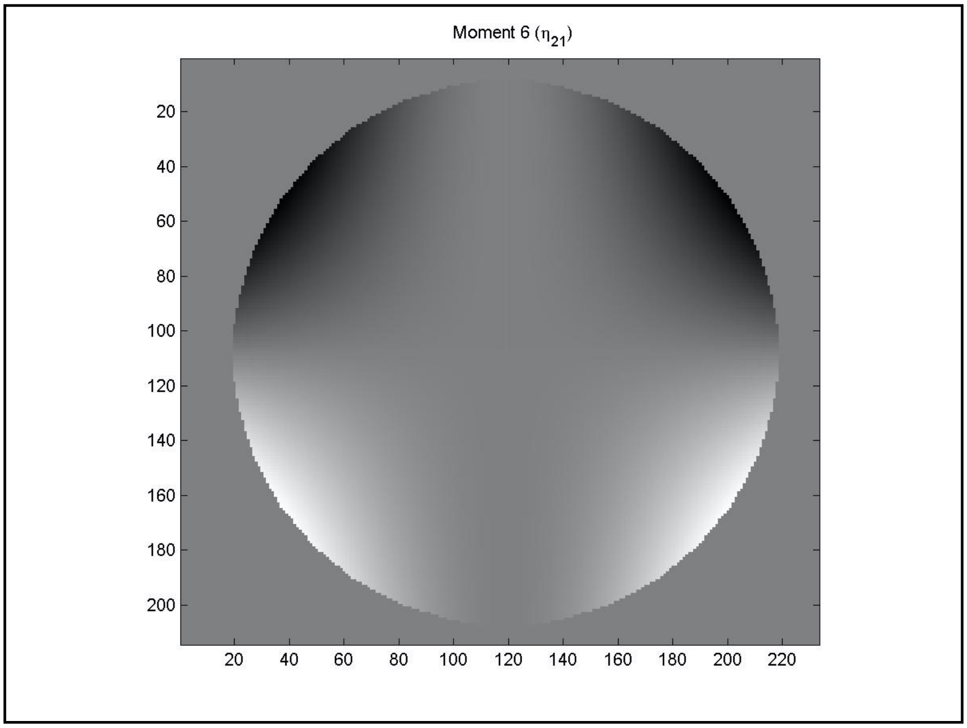
Moment 6 (η_{21})
20
40
60
80
100
120
140
160
180
200
20 40 60 80 100 120 140 160 180 200 220

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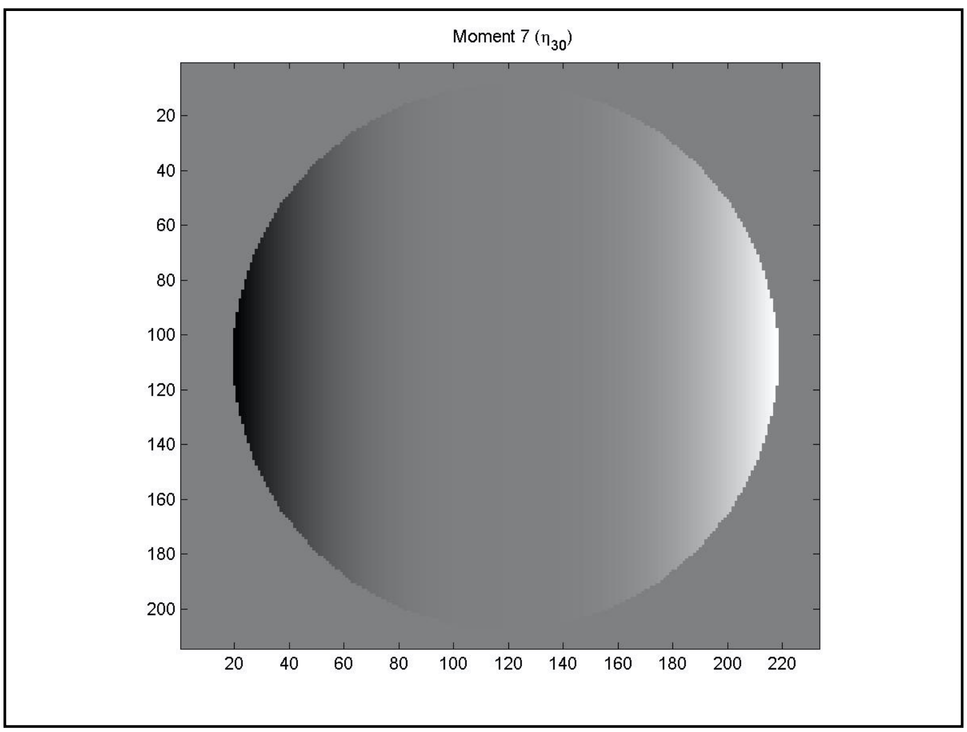
Moment 7 (η_{30})
20
40
60
80
100
120
140
160
180
200
20 40 60 80 100 120 140 160 180 200 220

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Works for Binary Images Only?

- NO!
- Can use equations for grayscale (or real-valued) image

use grayscale value

$$\eta_{ij} = \frac{\mu_{ij}}{(m_{00})^{\frac{i+j}{2}+1}} = \frac{\sum\sum (x-\bar{x})^i (y-\bar{y})^j I[x,y]}{\left(\sum\sum I[x,y]\right)^{\frac{i+j}{2}+1}}$$

for $2 \le (i+j) \le 3$:

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Summary

- Given a 2-D binary shape, use representations/properties to characterize the region
  - Recognition or matching
- Methods
  - Chain code
  - Quadtree
  - Medial axis
  - Bounding box, extremal points
  - Area, centroid
  - Perimeter, compactness, circularity
  - Signatures
  - Moments

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