

Computer Vision for HCI

Edge Detection

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Edges in Images

- Edge pixels are pixels in an image where brightness changes sharply
 - Sharp changes in image brightness
- Interesting things happen at an edge
 - Object boundaries (light object on dark background)
 - Reflectance changes/patterns (zebra stripes, leopard spots)
 - Sharp changes in surface orientation
- Look at derivatives in image to detect edges
 - Gradients in 2-D
- Primary problem in edge detection is dealing with image noise

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Gradients and Edges

- High-contrast image pixels can be detected by computing intensity differences in local image regions
- Detect high-contrasts using neighborhood templates or masks
 - Similar to approach for noise removal
- Begin study with 1-D signals, then move onto 2-D
 - A 1-D signal could be a row or column of an image

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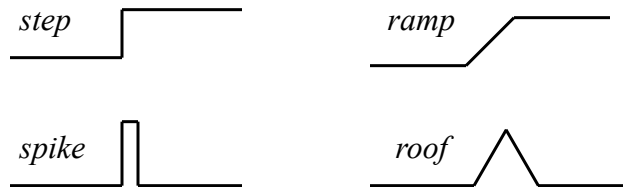
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1-D Signals

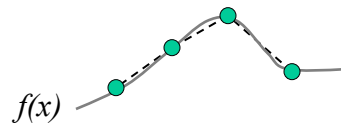
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Differencing 1-D Signals

- A few idealized types of 1-D edges



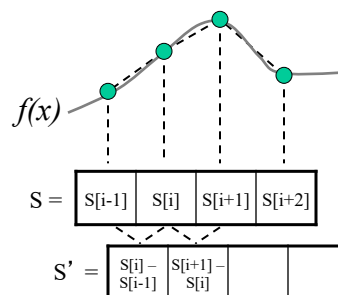
- Consider 1-D sampled signal (roof):



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Difference/Derivative Mask



$$f'(x_i) \approx \frac{f(x_i) - f(x_{i-1})}{x_i - x_{i-1}} \approx f(x_i) - f(x_{i-1})$$

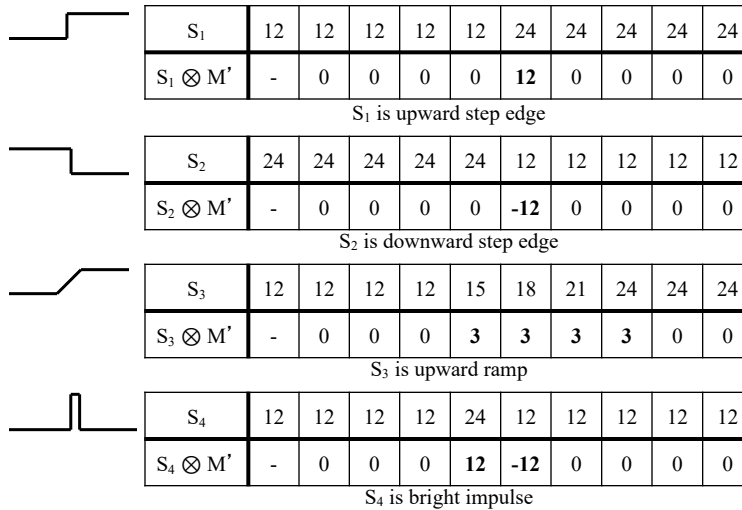
Difference Masks

$$M' = \begin{bmatrix} -1 & +1 \end{bmatrix}$$

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First Derivative Examples



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Smoothing

- Simple differences tend to give strong (unwanted) responses to **noise**
 - Poor way to estimate derivatives in real signals/images
- In practice, signal/image almost always smoothed before taking derivate
- Typically, Gaussian smoothing is used

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Derivative of Gaussian

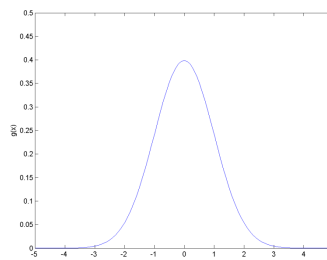
- **Smoothing then differentiating is same as convolving with the derivative of a smoothing kernel**
- Thus need only to convolve with a “derivative of the Gaussian” filter
 - Use **equation** for “derivative of Gaussian”!
- Results in smaller noise responses from derivative estimates

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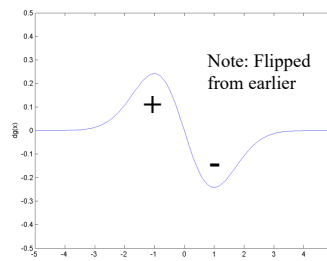
Derivative of Gaussian

$$g(x; \sigma) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-x_0)^2}{2\sigma^2}} \longrightarrow$$



$$\frac{dg(x; \sigma)}{dx} = \frac{-(x-x_0)}{\sqrt{2\pi}\sigma^3} e^{-\frac{(x-x_0)^2}{2\sigma^2}} \longrightarrow$$

(σ controls the scale/spread)

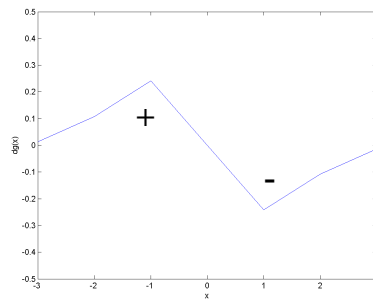


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Discrete Gaussian Derivative Mask

- Set mask size: $\text{ceil}(3\sigma)*2+1$
 - Examine values for $x = [-\text{ceil}(3\sigma) : \text{ceil}(3\sigma)]$
- For $\sigma = 1$, yields a “7-tap” filter
mask = [.01, .11, .24, 0, -.24, -.11, -.01]



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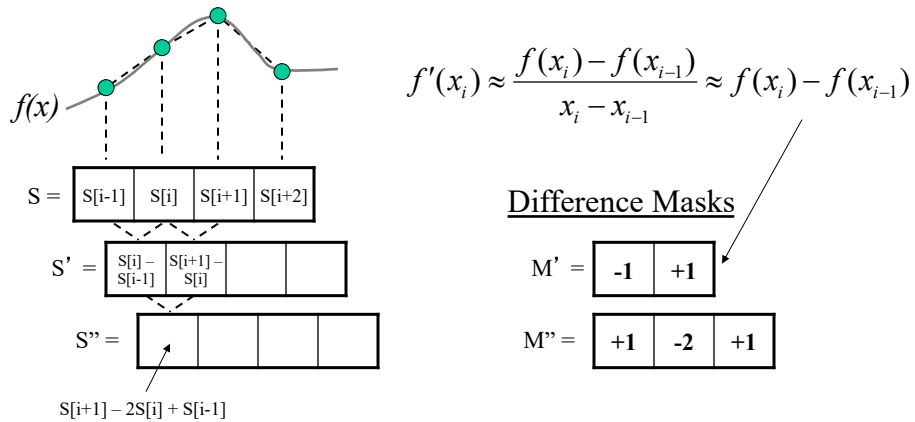
Second Derivative Option

- Second derivative is zero when derivative magnitude is extremal (at a peak/valley)
- To find large changes (edges), another good place to look is where second derivative makes “zero-crossings”
- Look for a change from “+ to -” or “- to +”
 - Can also look for “0 to +/-” or “+/- to 0”
- Produces double-sided edges

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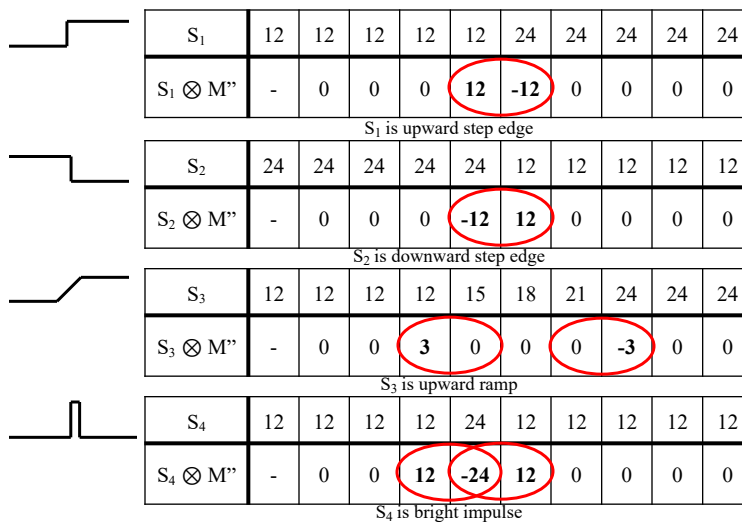
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Difference/Derivative Masks



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Second Derivative Examples

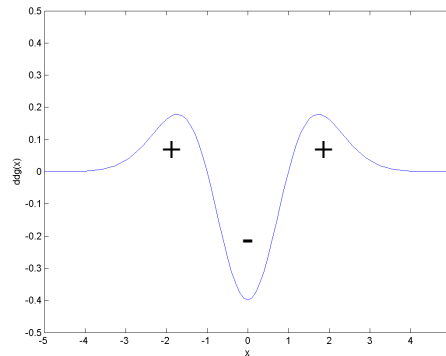


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Gaussian Second Derivative

$$\frac{\partial^2 g(x; \sigma)}{\partial x^2} = \left(\frac{(x - x_0)^2}{\sqrt{2\pi}\sigma^5} - \frac{1}{\sqrt{2\pi}\sigma^3} \right) \cdot e^{-\frac{(x - x_0)^2}{2\sigma^2}}$$

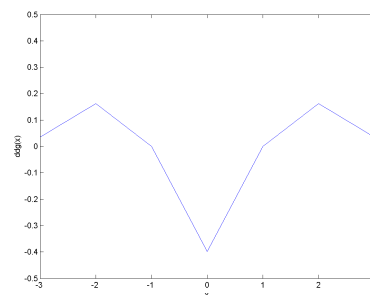


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Gaussian 2nd Derivative Mask

- Set mask size: $\text{ceil}(3\sigma) * 2 + 1$
 - Examine values for $x = [-\text{ceil}(3\sigma) : \text{ceil}(3\sigma)]$
- For $\sigma = 1$, yields a 7-tap filter
 - mask = [.04, .16, 0, -.40, 0, .16, .04]



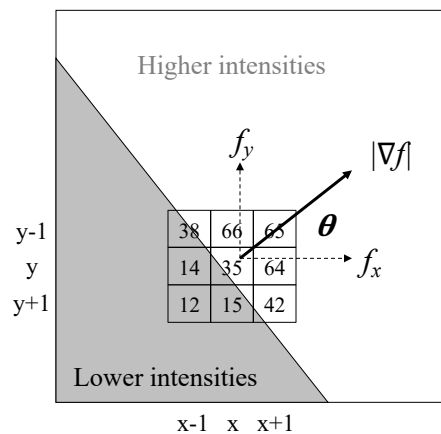
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2-D Signals (Images)

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2-D Difference “Gradient” Operators



$$\text{Gradient} \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right]$$

$$\frac{\partial f}{\partial x} \equiv f_x \approx \frac{1}{3} [(I[x+1, y] - I[x-1, y]) / 2 + (I[x+1, y-1] - I[x-1, y-1]) / 2 + (I[x+1, y+1] - I[x-1, y+1]) / 2]$$

$$\frac{\partial f}{\partial y} \equiv f_y \approx \frac{1}{3} [(I[x, y+1] - I[x, y-1]) / 2 + (I[x-1, y+1] - I[x-1, y-1]) / 2 + (I[x+1, y+1] - I[x+1, y-1]) / 2]$$

$$\theta = \text{atan}(f_y, f_x) \quad |\nabla f| = \sqrt{f_x^2 + f_y^2}$$

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Classic Gradient Masks

Prewitt: $F_x = \frac{1}{6} \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$ $F_y = \frac{1}{6} \begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

Sobel: $F_x = \frac{1}{8} \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$ $F_y = \frac{1}{8} \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$

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Separability

Prewitt: $F_x = \frac{1}{6} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 1 \end{bmatrix}$ $F_y = \frac{1}{6} \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$

Sobel: $F_x = \frac{1}{8} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 1 \end{bmatrix}$ $F_y = \frac{1}{8} \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \end{bmatrix}$

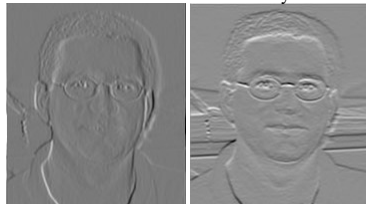
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Prewitt mask results

F_x

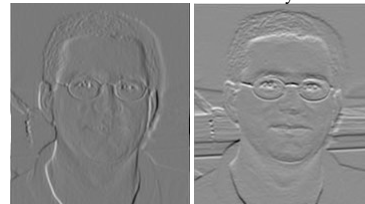
F_y



Sobel mask results

F_x

F_y



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Gradient Magnitude

$$\sqrt{F_x^2 + F_y^2} = \text{Result}$$

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Different Gradient Magnitude Strengths

(using increasing threshold to remove
weaker gradient magnitudes)



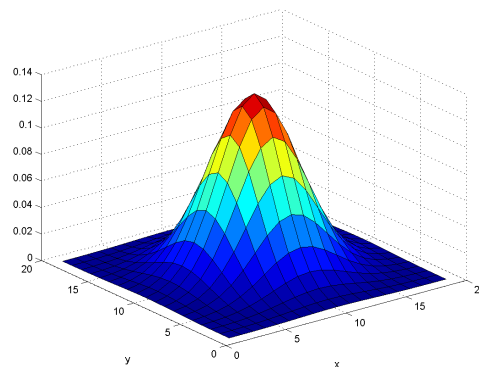
(Gradient magnitude obtained with Sobel masks)

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2-D Gaussian

$$g(x, y; \sigma) = \frac{1}{2\pi\sigma^2} e^{-\frac{(x-x_0)^2 + (y-y_0)^2}{2\sigma^2}}$$



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Gaussian Derivatives

$$\frac{\partial g(x, y; \sigma)}{\partial x} = \frac{-(x - x_0)}{2\pi\sigma^4} e^{-\frac{(x-x_0)^2 + (y-y_0)^2}{2\sigma^2}}$$

$$\frac{\partial g(x, y; \sigma)}{\partial y} = \frac{-(y - y_0)}{2\pi\sigma^4} e^{-\frac{(x-x_0)^2 + (y-y_0)^2}{2\sigma^2}}$$

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Creating Masks

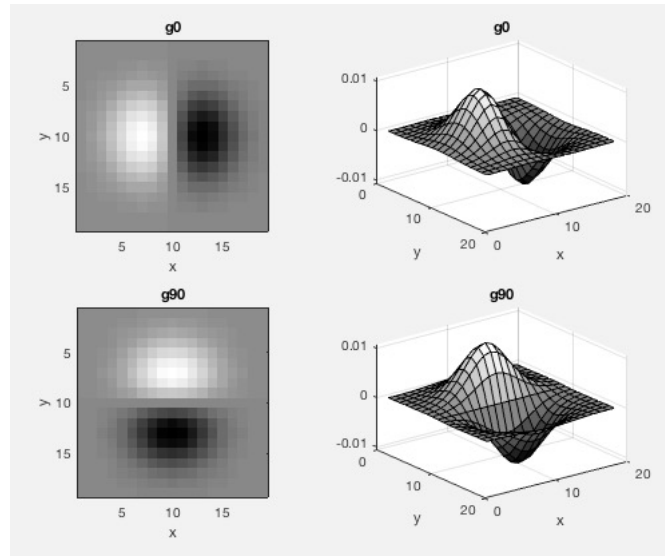
- Gx mask
 - Fill mask values with $g_x(x, y; \sigma)$ **EQUATION**
 - x-range: [-ceil(3σ) : ceil(3σ)] (could use 2σ)
 - y-range: [-ceil(3σ) : ceil(3σ)]
- Gy mask
 - Fill mask values with $g_y(x, y; \sigma)$ **EQUATION**
 - x-range: [-ceil(3σ) : ceil(3σ)] (could use 2σ)
 - y-range: [-ceil(3σ) : ceil(3σ)]

*** Do **NOT** make a Gaussian mask and take difference operator over the mask!

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Results



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Convention: Alter to Point in
Increasing Axis Dimension

$$\frac{\partial g(x, y; \sigma)}{\partial x} = \cancel{-} \frac{(x - x_0)}{2\pi\sigma^4} e^{-\frac{(x-x_0)^2 + (y-y_0)^2}{2\sigma^2}}$$

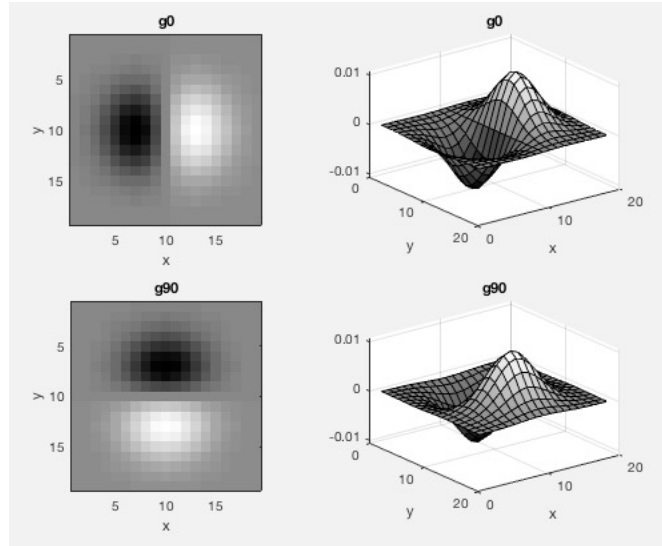
Remove negative (-) sign!

$$\frac{\partial g(x, y; \sigma)}{\partial y} = \cancel{-} \frac{(y - y_0)}{2\pi\sigma^4} e^{-\frac{(x-x_0)^2 + (y-y_0)^2}{2\sigma^2}}$$

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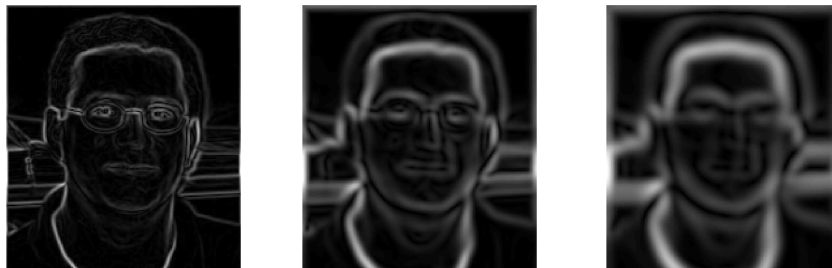
Results



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Effect of Scale

(Magnitude shown)



$\sigma = 1$

$\sigma = 3$

$\sigma = 5$

Thicker gradient ridges \longrightarrow

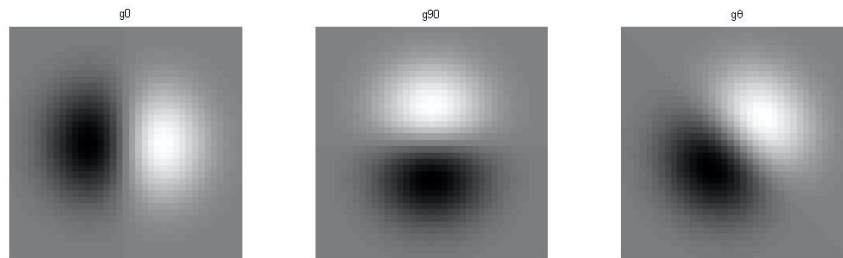
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“Steerable” Filters

- Simple version: rotate by angle θ
 - Use G_x and G_y as basis set, and synthesize filter by linear combination of G_x and G_y

$$\cos(\theta) \cdot G_x + \sin(\theta) \cdot G_y$$



“The design and use of steerable filters”, Freeman and Adelson, *IEEE PAMI*, Sept. 1991.

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Laplacian of Gaussian

- Recall second derivative function of Gaussian
 - Zero-crossings are found at edge locations
 - Smoothing used to combat noise
- Now combine two orientations into one circular filter
 - Simply sum Gaussian second derivatives in x direction and y direction
 - Non-oriented, 2nd derivative filter
- Laplacian of Gaussian operator:

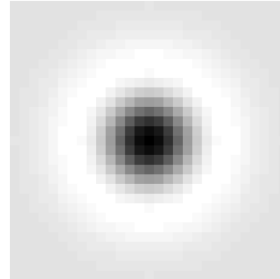
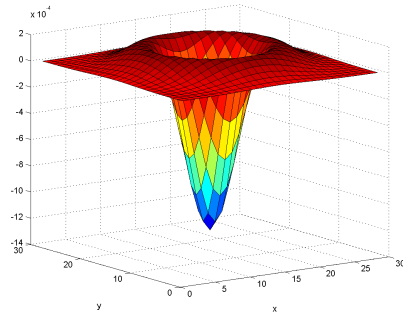
$$\nabla^2 g(x, y; \sigma) = \frac{\partial^2 g(x, y; \sigma)}{\partial x^2} + \frac{\partial^2 g(x, y; \sigma)}{\partial y^2}$$

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Laplacian of Gaussian

$$\nabla^2 g(x, y; \sigma) = \frac{\partial^2 g(x, y; \sigma)}{\partial x^2} + \frac{\partial^2 g(x, y; \sigma)}{\partial y^2}$$



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Zero-Crossings



$\sigma = 2$



Selected
zero-crossings



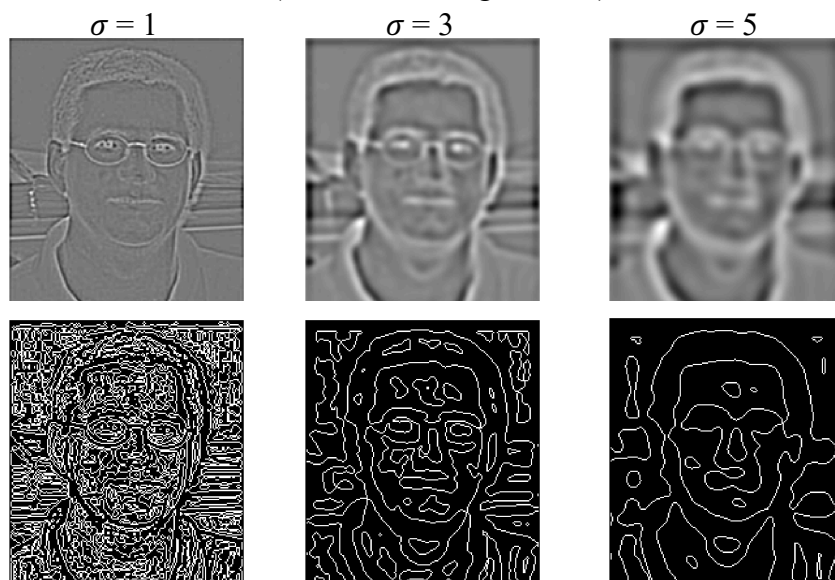
Overlay

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Effect of Scale

(all zero-crossings shown)



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Simple Laplacian Mask

- Small 3 x 3 mask approximation of LOG

$$\text{LOG} = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$



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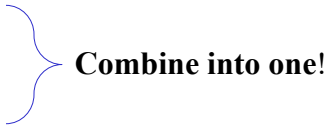
Canny Edge Detector

- Very/Most popular and effective method
- Produces extended contour/edge segments by “following” high gradient magnitudes within the smoothed image

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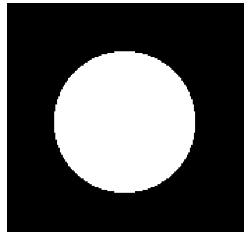
Canny Edge Detection Approach

- Step 1: Smooth the image
 - Gaussian
 - Step 2: Compute the gradients
 - Calculate magnitude and orientation at each pixel
 - Step 3: Suppress non-maximal gradients
 - Keep points where gradient magnitude is maximal along the direction of the gradient (look for “hills”)
 - Step 4: Follow edge contours (edge linking)
 - Use upper and lower thresholds (hysteresis thresholding)
-  **Combine into one!**

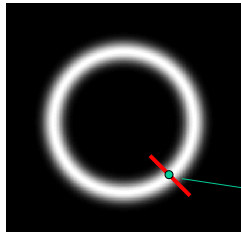
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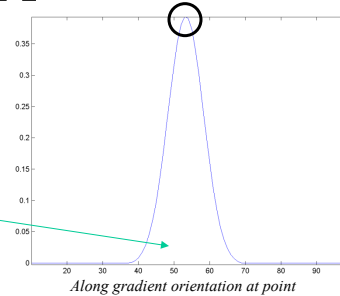
Non-Maximal Suppression



Input image



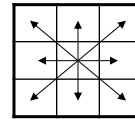
Gradient magnitude



Do for all points/pixels in gradient magnitude image (results in a thin circle)

- Detection

- Slice the gradient magnitude along gradient direction (perpendicular to edge)
 - Quantize gradient orientation by 45 degrees
 - Could also interpolate values
- Mark points along slice that are maximal



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Edge Linking

- Sequentially follow continuous contour segments
- Initiate only on edge pixels where gradient magnitude meets high threshold (T_1)
- A single threshold can cause many broken edge segments
- Once started, follow through connected pixels whose gradient magnitude meet a lower threshold (T_2)
 - In Matlab, the default is $T_2 = .4 * T_1$
- Referred to as “**hysteresis thresholding**”

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Canny Results



Original



Canny edges

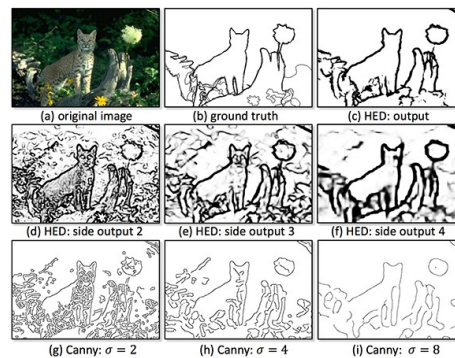
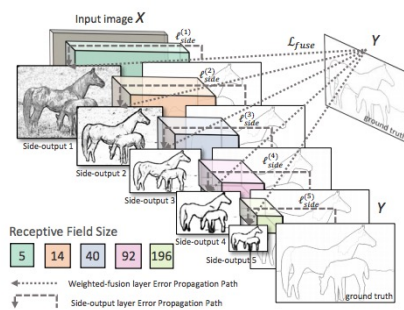


Overlay

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Deep Learning Approach Holistically-Nested Edge Detection - HED (2015)



Note: **Ground-truth** of examples required for training.
There is no “training” with Canny

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Perceptual Test: What is this???



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Summary

- Interesting things happen at an edge
 - Object boundaries
- Look for derivatives/gradients in image
 - First and second derivatives/gradients
- Classic gradient operators
 - Sobel, Prewitt
 - Gaussian derivatives
- Primary problem in edge detection is dealing with image noise
 - Smoothing and hysteresis thresholding
- Canny edge detector

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