Computer Vision for HCI

Image Registration

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Approaches

- **Image Registration = aligning two images to same coordinate frame**
- Feature-Based (this lecture)
  - Find correspondence between features (e.g., points, lines, contours)
  - Need transformation model

Some slides adapted from Robert Collins

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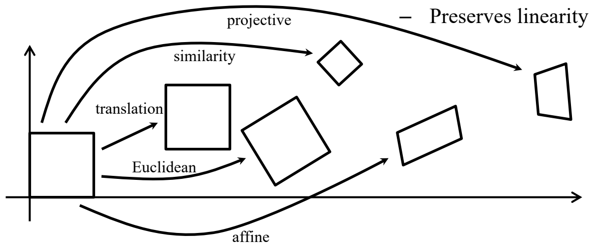
Transformations

- Isometric (Euclidean)
  - **Translation, rotation, reflection**
  - Three degrees of freedom
  - Preserves length, angle, area
- Similarity
  - Euclidean **plus isotropic scaling**
  - Four degrees of freedom
  - Preserves angle, ratios of length and area
- Affine
  - Similarity **plus skew/shear**
  - Six degrees of freedom
  - Preserves parallel lines, ratio of lengths of parallel lines, ratio of area
- **Projective (Homography)**
  - Affine **plus perspective** (non-linear effects)
  - Eight degrees of freedom
  - Preserves linearity (and others)



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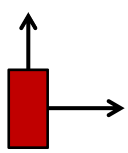
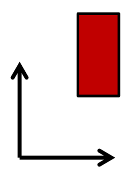
<u>Euclidean</u> Transformations

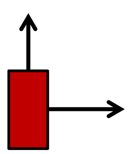
- Translation



$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & x_0 \\ 0 & 1 & y_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

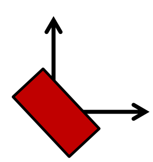


- Rotation



$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Counter-Clockwise Rotation



- Combine

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & x_0 \\ \sin\theta & \cos\theta & y_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

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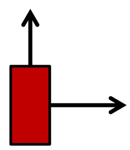
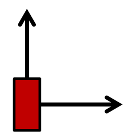
Similarity Transformations

- Scale



$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$



- Combine (with Euclidean transformations)

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} s\cos\theta & -s\sin\theta & x_0 \\ s\sin\theta & s\cos\theta & y_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

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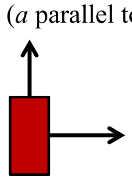
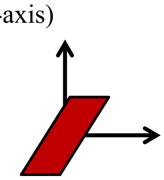
Affine Transformations

- Skew/Shear (*a* parallel to x-axis, *b* parallel to y-axis)



$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & a & 0 \\ b & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$



- Combine (with similarity transformations)

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

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Estimating Affine Transformation

- Set up system of equations

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \Rightarrow \begin{aligned} x' &= a_{11}x + a_{12}y + a_{13} \\ y' &= a_{21}x + a_{22}y + a_{23} \end{aligned}$$

- Rewrite in matrix form

$$\begin{bmatrix} x & y & 1 \end{bmatrix} \times \begin{bmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \\ a_{13} & a_{23} \end{bmatrix} = \begin{bmatrix} x' & y' \end{bmatrix}$$

- 6 DoF → need at least 3 coordinates to solve
  - Typically use more than 3

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Estimating Affine Transformation (continued)

- Write in matrix form

$$\underset{N \text{ x } 3}{\begin{bmatrix} x_1 & y_1 & 1 \\ & \vdots & \\ & & \end{bmatrix}} \underset{3 \text{ x } 2}{\begin{bmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \\ a_{13} & a_{23} \end{bmatrix}} = \underset{N \text{ x } 2}{\begin{bmatrix} x'_1 & y'_1 \\ \vdots & \\ & \end{bmatrix}}$$

$$\Downarrow$$

$$PA = P'$$

- Use pseudo-inverse to solve for A

$$A = (P^T P)^{-1} P^T P'$$

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How are Images from Multiple Viewpoints Related when Viewing a Planar Surface?

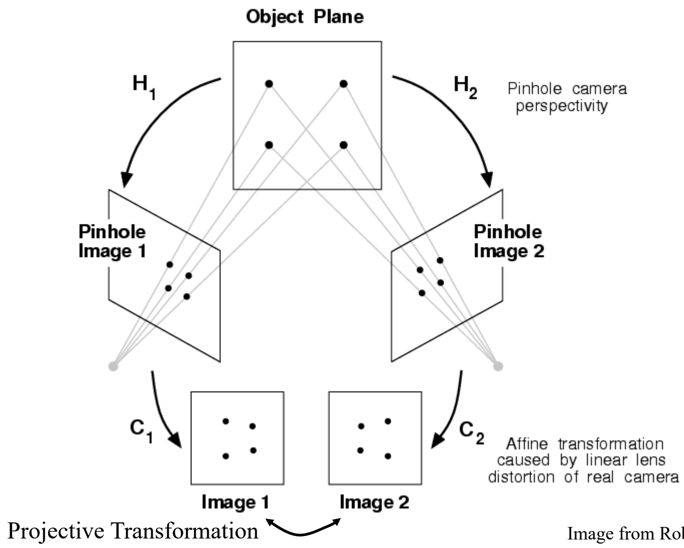

Image from Robert Collins

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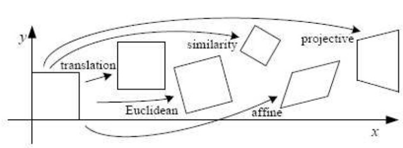
Projective Transformation

- Also called a "**homography**"
- Assuming pinhole camera model and planar surface being imaged, points on two images are related by homography
- Incorporates perspective with affine transformations



$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\Downarrow$$

$$P' = HP$$

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Estimating Planar Homography

- Matrix Form:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} \leftarrow \begin{bmatrix} \hat{x} \\ \hat{y} \\ w \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \times \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$x' = \frac{\hat{x}}{w} = \frac{h_{11}x + h_{12}y + h_{13}}{h_{31}x + h_{32}y + h_{33}}$$

$$y' = \frac{\hat{y}}{w} = \frac{h_{21}x + h_{22}y + h_{23}}{h_{31}x + h_{32}y + h_{33}}$$

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Degrees of Freedom

- 9 unknowns → 9 degrees of freedom?
- Multiplying H by nonzero constant k yields same equations → 8 degrees of freedom
  - e.g., could set h_{33} equal to constant (e.g., 1), but what if it is 0?
  - Will revisit ensure 8 DoF shortly

$$x' = \frac{kh_{11}x + kh_{12}y + kh_{13}}{kh_{31}x + kh_{32}y + kh_{33}} \qquad y' = \frac{kh_{21}x + kh_{22}y + kh_{23}}{kh_{31}x + kh_{32}y + kh_{33}}$$

$$\Rightarrow$$

$$x' = \frac{h_{11}x + h_{12}y + h_{13}}{h_{31}x + h_{32}y + h_{33}} \qquad y' = \frac{h_{21}x + h_{22}y + h_{23}}{h_{31}x + h_{32}y + h_{33}}$$

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Solving for H

$$x' = \frac{h_{11}x + h_{12}y + h_{13}}{h_{31}x + h_{32}y + h_{33}} \qquad y' = \frac{h_{21}x + h_{22}y + h_{23}}{h_{31}x + h_{32}y + h_{33}}$$

- Multiply by denominators

$$\left(h_{31}x + h_{32}y + h_{33}\right)x' = h_{11}x + h_{12}y + h_{13}$$
$$\left(h_{31}x + h_{32}y + h_{33}\right)y' = h_{21}x + h_{22}y + h_{23}$$

- Distribute and set equal to zero

$$h_{11}x + h_{12}y + h_{13} - h_{31}xx' - h_{32}yx' - h_{33}x' = 0$$
$$h_{21}x + h_{22}y + h_{23} - h_{31}xy' - h_{32}yy' - h_{33}y' = 0$$

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Solving for H (continued)

- Write in matrix form

$$\begin{array}{ccc} 2N \text{ x } 9 & 9 \text{ x } 1 & 2N \text{ x } 1 \\ \begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 & -x_1x_1' & -y_1x_1' & -x_1' \\ 0 & 0 & 0 & x_1 & y_1 & 1 & -x_1y_1' & -y_1y_1' & -y_1' \\ & & & & \vdots & & & & \end{bmatrix} & \begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \\ h_{33} \end{bmatrix} = & \begin{bmatrix} 0 \\ 0 \\ \vdots \\ \\ \end{bmatrix} \end{array}$$

$$A\mathbf{h} = \mathbf{0}$$

- 8 DoF → need at least 4 coordinates to solve for H
  - Typically use more than 4

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Solving for h

- Homogeneous Equations $$A\mathbf{h} = \mathbf{0}$$
- Multiply by both sides by A^T
$$\underset{9x9}{(A^TA)}\underset{9x1}{\mathbf{h}} = \underset{9x1}{\mathbf{0}}$$
- Recall Eigen analysis $$B\mathbf{x} = \lambda\mathbf{x}$$
- Solution of **h** given by Eigen decomposition of (A^TA)
  - Set **h** equal to Eignenvector of (A^TA) corresponding to **smallest** Eigenvalue
  - <u>Normalize</u> **h** to unit vector to enforce 8 DoF (most Eigensolvers already do)
- <u>Unrasterize</u> **h** to give homography matrix H

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Enhanced Approach
(Normalized Direct Linear Transformation)

- Increases robustness by <u>normalizing</u> data **before** computing H
- Compute similarity transformation matrices T^a and T^b for <u>each</u> point set P_i and P_i' that satisfy the following conditions
  - The points have zero mean
  - The average distance of the **shifted** points to the origin is $\sqrt{2}$ [*ave point is* $(\pm 1, \pm 1)$]

$$\hat{x} = s(x - \bar{x}) = sx - s\bar{x}$$
$$\hat{y} = s(y - \bar{y}) = sy - s\bar{y}$$

$$s = \frac{\sqrt{2}}{\frac{1}{N}\sum_{i=1}^{N}\sqrt{(x_i - \bar{x})^2 + (y_i - \bar{y})^2}}$$

$$\begin{bmatrix} \hat{x} \\ \hat{y} \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} s & 0 & -s\bar{x} \\ 0 & s & -s\bar{y} \\ 0 & 0 & 1 \end{bmatrix}}_{T} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- Compute homography $\widetilde{H}$ between transformed points (prev slide)
- Remove normalization to compute original homography between un-normalized points

$$H = (T^b)^{-1}\widetilde{H}T^a$$

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Homography Example

Image 1

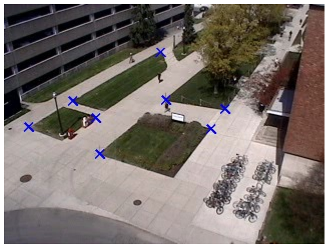
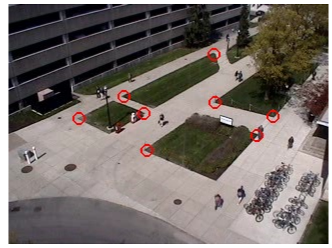
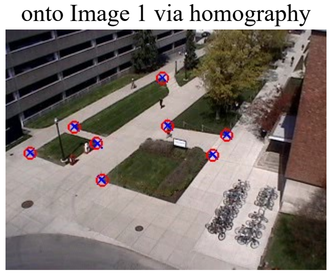


Image 2



Selected key points in images

Points from Image 2 mapped onto Image 1 via homography



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Bi-Directional

Image 1

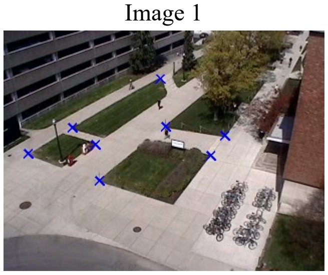
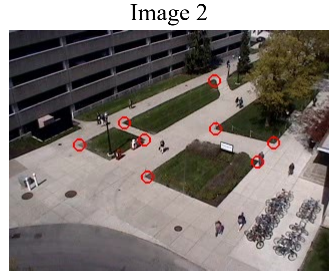


Image 2



$P_{\text{image1}} = HP_{\text{image2}}$

Image 1

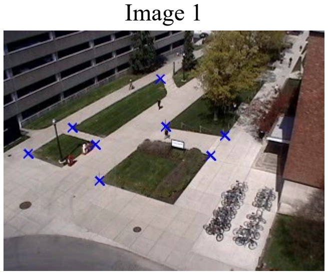
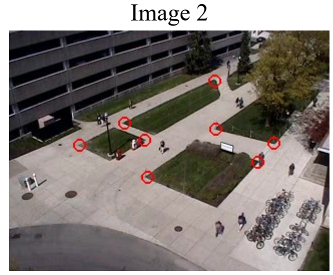


Image 2



$P_{\text{image2}} = inv(H)P_{\text{image1}}$

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RANSAC: Dealing with Outliers

- Robustly fit model to dataset containing outliers using RANdom SAmple Consensus (RANSAC)
- Algorithm:
  1. Randomly select sample of n points from dataset and estimate model (i.e., compute homography)
  2. Determine set of **inliers** X_i (points within distance d of model) from whole dataset

     "Consensus set"
  3. $|X_i| \geq T$ (i.e., have "enough" inliers), re-estimate model using points in X_i (discarding outliers) and terminate
  4. $|X_i| < T$, select new sample set and repeat
  5. If threshold not surpassed after N iterations, estimate model using largest consensus set X_i
- Need to set values for parameters n, T, d, and N

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AI Searches Public Cameras to Find When Instagram Photos Were Taken

Instagram photo (left) and a view of when it was created (right). Still frame from video by Dries Depoorter.

https://petapixel.com/2022/09/13/ai-searches-public-cameras-to-find-instagram-photos-as-they-are-taken/

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Summary

- Image registration using image transformations
- Euclidean → Similarity→ Affine → Homography
- Relationship between images viewing planar surface is defined by homography
  - Need control points to be planar
- Estimating homography
  - Normalized 8-point algorithm
  - Incorporate RANSAC
  - Gold standard

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