Computer Vision for HCI

Stereo Vision

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Ambiguity in Single View

- Structure and depth are inherently ambiguous from single view

This is __________ perspective

Some slides adapted from Kristen Grauman, James Hays, Oliver Grau

Images from Lana Lazebnik

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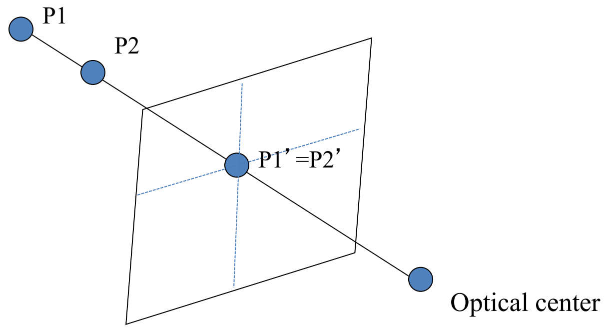
Ambiguity in Single View

- Structure and depth are inherently ambiguous from single view



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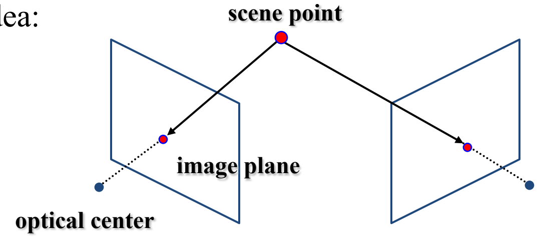
Extracting 3-D Information

- What cues help us to perceive 3-D shape and depth?
  - Shading, focus, texture, motion, …
- Stereo:
  - Shape from difference between two views
  - Infer 3-D shape of scene from two (multiple) images from different viewpoints

Main idea:



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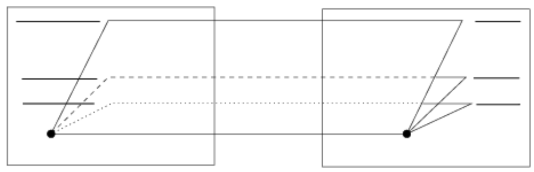
Geometry for a Simple Stereo System

- First, assume parallel optical axes, known camera parameters (i.e., calibrated cameras):



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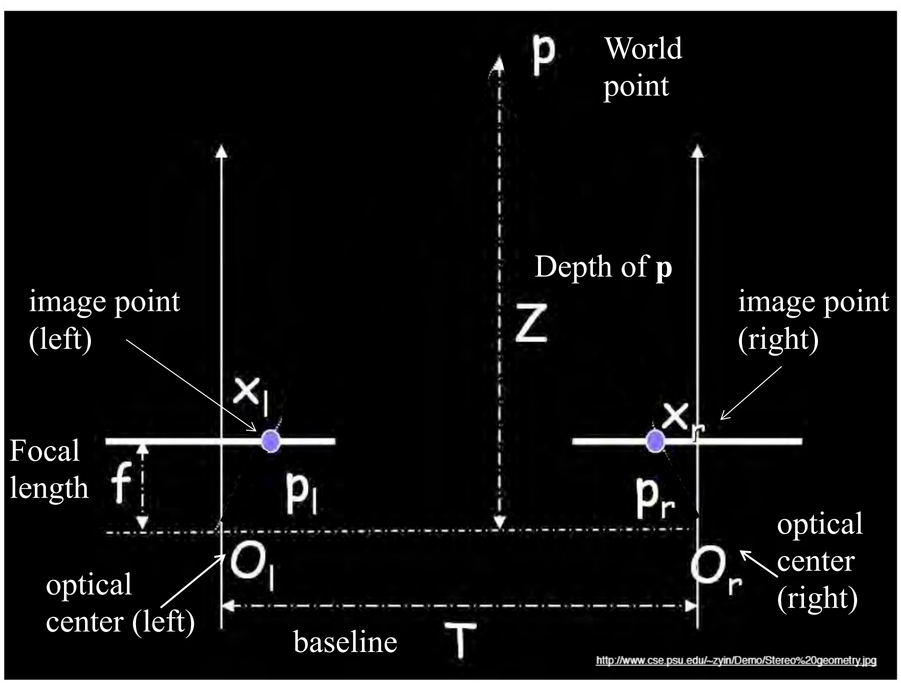

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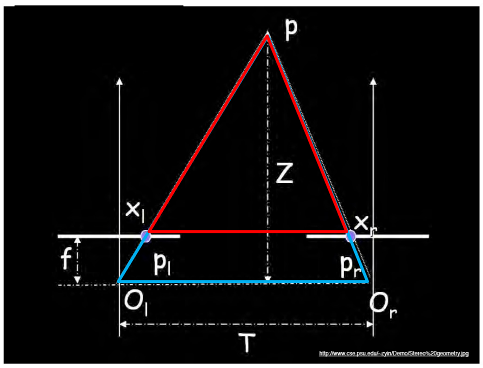
Geometry for a Simple Stereo System

- Assume parallel optical axes, known camera parameters (i.e., calibrated cameras). **What is expression for Z (depth)?**



Similar triangles (p_l, p, p_r) and (O_l, p, O_r):

$$\frac{T + x_r - x_l}{Z - f} = \frac{T}{Z}$$

$$Z = f\frac{T}{x_l - x_r}$$

disparity

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Depth from Disparity

image I(x,y)

Disparity map D(x,y)

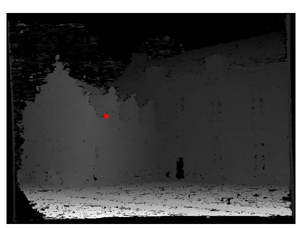

image I´(x´,y´)

So if we could find the **corresponding points** in two images, we could **estimate relative depth**…

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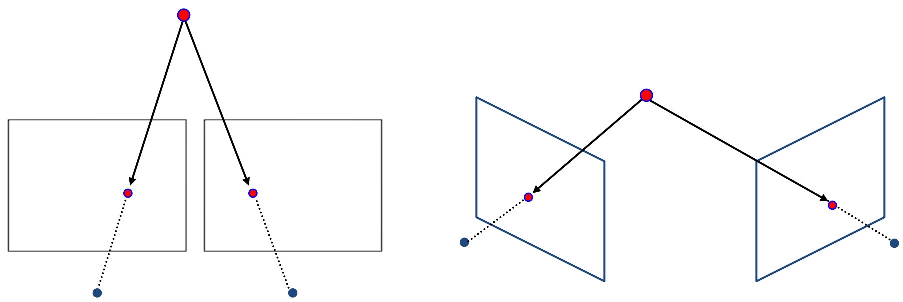
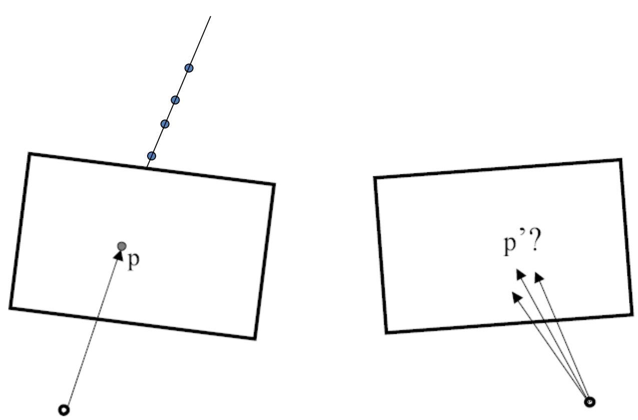
General Case, with Calibrated Cameras

- The two cameras need not have parallel optical axes.



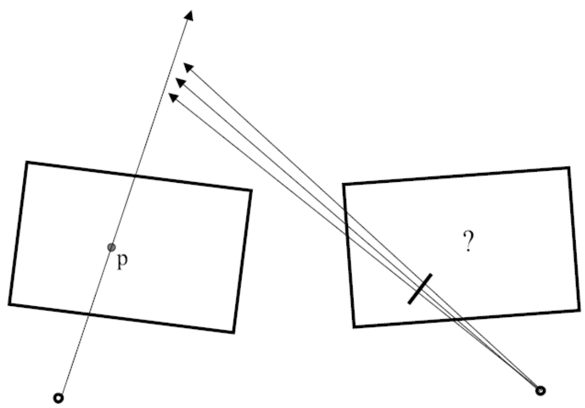
vs.

Stereo Correspondence Constraints



Given p in left image, where can corresponding point p' be?

Stereo Correspondence Constraints



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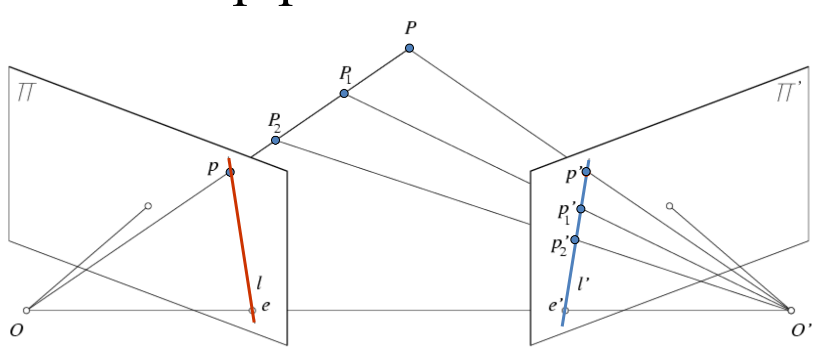
Epipolar Constraint



Geometry of two views constrains where the corresponding pixel for some image point in the first view must occur in the second view

- It must be on the line carved out by a plane connecting the world point and optical centers

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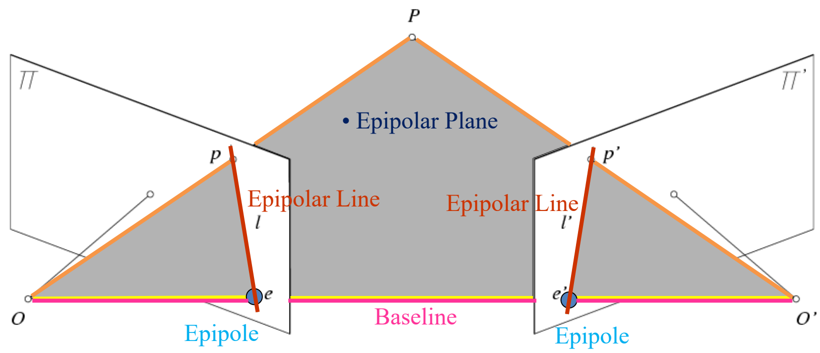
Epipolar Geometry



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Why is the epipolar constraint useful?

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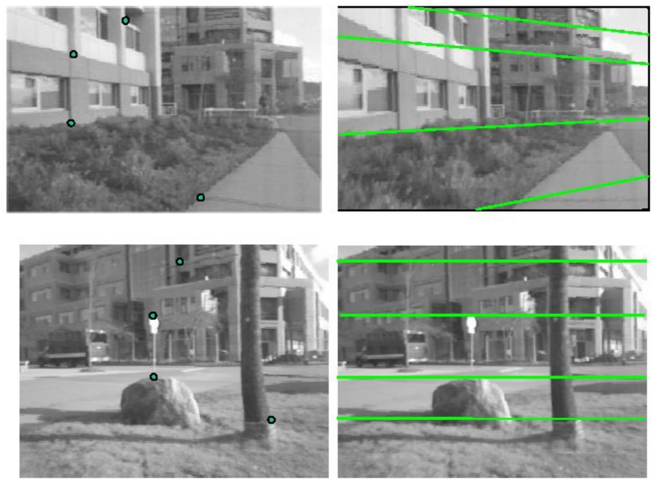
Epipolar Constraint

This is useful because it reduces the correspondence problem to a 1D search along an epipolar line.



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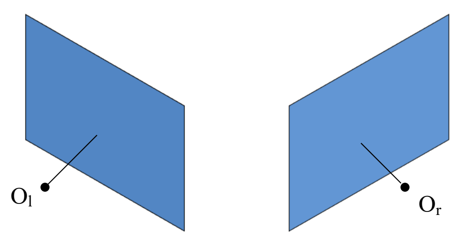
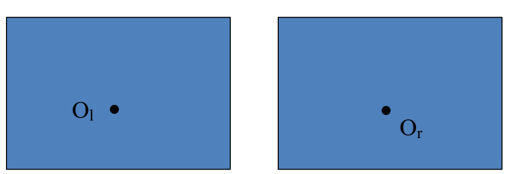
What do the Epipolar Lines Look Like?

1.



2.



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Example: Converging Cameras

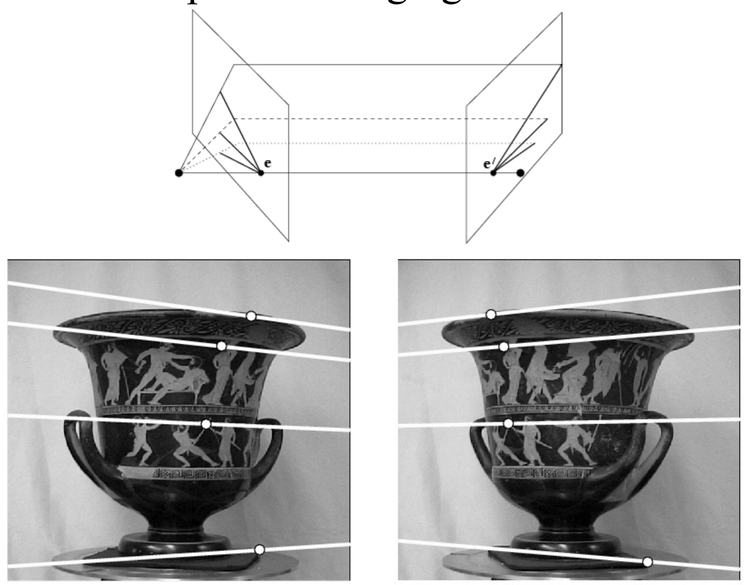

Figure from Hartley & Zisserman

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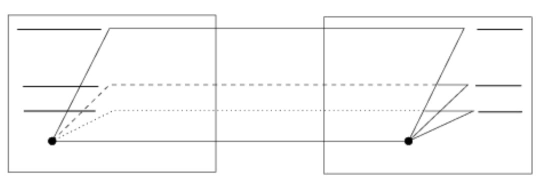
Example: Parallel Cameras



(epipoles at infinity)

Figure from Hartley & Zisserman

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Camera Motion

Consider two images from a moving camera (rather than two distinct cameras)

What would the epipolar lines look like if the camera moves directly forward?

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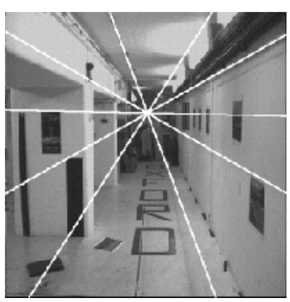
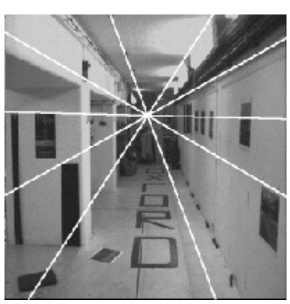
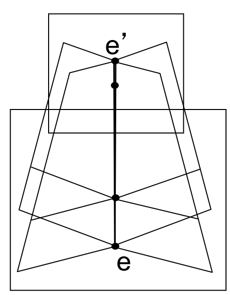
Example: Forward Motion







Epipole has same coordinates in both images.
Points move along lines radiating from e:
"Focus of expansion"

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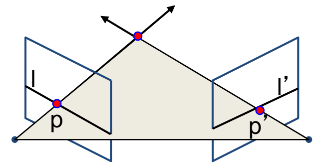
How are Two Image Planes Related?



- Let p be a point in left image, p' in right image
  - Homogeneous coordinates: [x, y, 1], [x', y', 1]
- p and p' are related by a rotation and translation of the cameras
  - Yields epipolar constraint with formula $p'^T F p = 0$
    - Longuet-Higgins 1981
- Epipolar relation
  - p maps to epipolar line l'
  - p' maps to epipolar line l
- Equation for line: ax + by + c = 0
  - Defined by parameter column vector $r = [a, b, c]^T$
  - For any point $p = [x,y,1]^T$ on the line defined by r we have $r^T p = 0$
- Recall $p'^T F p = 0$, hence the epipolar lines are

$$0 = p'^T F p = (p'^T F) p = (F^T p')^T p = l^T p \quad \rightarrow \quad l = F^T p'$$

Similarly, $l' = Fp$ Defines parameters of line (l) through point p

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Fundamental Matrix

- This matrix F is called
  - The "**Fundamental Matrix**"
    - General, uncalibrated camera case
  - The "Essential Matrix"
    - When image intrinsic parameters are known
- Can solve for F from point correspondences
  - Each (p, p') pair gives one linear equation involving elements of F

$$p'^T F p = 0$$

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Computing Fundamental Matrix

- Set up system of equations

$$\begin{bmatrix} x' & y' & 1 \end{bmatrix} \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = 0$$

$$x'xf_{11} + x'yf_{12} + x'f_{13} + y'xf_{21} + y'yf_{22} + y'f_{23} + xf_{31} + yf_{32} + f_{33} = 0$$

$$\begin{bmatrix} x'x & x'y & x' & y'x & y'y & y' & x & y & 1 \end{bmatrix} \mathbf{f} = 0$$

- Eight degrees of freedom in F (solution is good up to scale)
  - Need at least **8 (x, y) coordinates** to solve for F

$$A\mathbf{f} = \begin{bmatrix} x_1'x_1 & x_1'y_1 & x_1' & y_1'x_1 & y_1'y_1 & y_1' & x_1 & y_1 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_n'x_n & x_n'y_n & x_n' & y_n'x_n & y_n'y_n & y_n' & x_n & y_n & 1 \end{bmatrix} \mathbf{f} = \mathbf{0}$$

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Computing Fundamental Matrix
Normalized 8-Point Algorithm

- Solve for F using analogous approach as when solving for homography H
- For robustness, compute similarity transformation matrices T^a and T^b that make p_i and p_i' be:
  - Zero mean
  - Have an average distance to the origin of $\sqrt{2}$
- Set **f** equal to Eigenvector of $(A^T A)$ corresponding to smallest Eigenvalue
  - Ensure **f** is unit norm
- Unrasterize **f** to give fundamental matrix $\widetilde{F}$ (of the normalized points)
  - Noise causes rank($\widetilde{F}$) = 3 [rank is # independent cols (or rows)]
- **Due to epipolar geometry, need to enforce singularity constraint (rank($\widetilde{F}$) = 2)**
  - Take SVD of $\widetilde{F}$ ($\widetilde{F} = UDV^T$)
    - D = diag($[r,s,t]$), where singular values $r \geq s \geq t$
  - Set $\widetilde{F} = U\text{diag}([r,s,\mathbf{0}])V^T$
- Finally, remove normalization by setting $F = (T^b)^T \tilde{F} T^a$

$p'^T F p = 0$
$p'^T[(T^b)^T \tilde{F} T^a]p = 0$
$(p'^T (T^b)^T)\, \tilde{F} (T^a p) = 0$
$(T^b p')^T \tilde{F}\, (T^a p) = 0$

***Note, slightly different process (not inverse of T^b) for removing normalization than with homography!

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Building Depth Maps

- With <u>parallel optical axes</u>, one can build depth maps
- Depth maps are inversely related to disparity maps

$$Z = f \frac{T}{x_l - x_r}$$

- Need to find point correspondences to find disparities
  - Epipolar geometry constrains search for point correspondences
    - Finding correspondences is still a difficult problem

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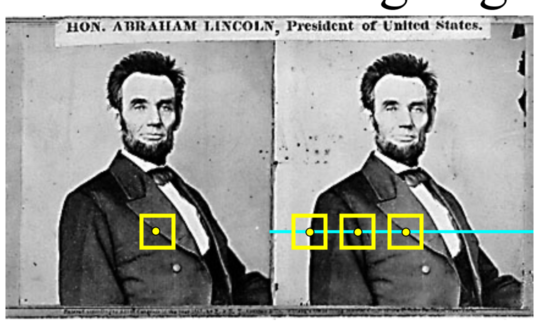
Basic Stereo Matching Algorithm



- (If necessary, rectify the two stereo images to transform epipolar lines into horizontal scanlines)
- For each pixel x in the first image
  - Find corresponding epipolar scanline in the right image
  - Examine pixels on the scanline and pick the best match x'
  - Compute disparity x-x' and set Depth(x) = $fT/(x-x')$
    - Or use relative depth as RelDepth(x) = $1/(x-x')$

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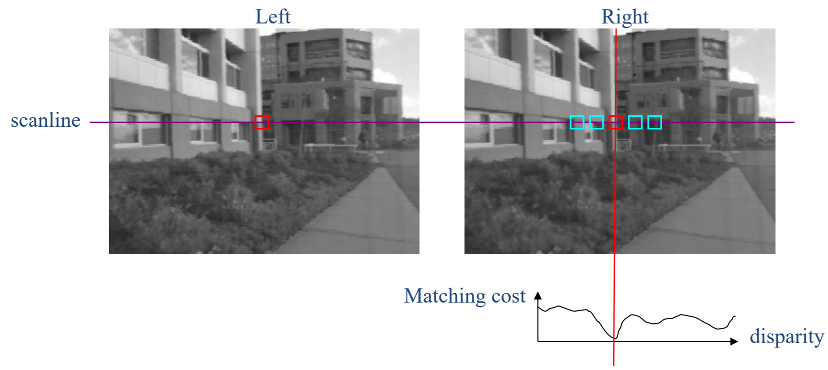
Correspondence Search



- Slide a window along the right scanline and compare contents of that window with the reference window in the left image
- Matching cost: SAD, SSD, or **normalized cross-correlation**

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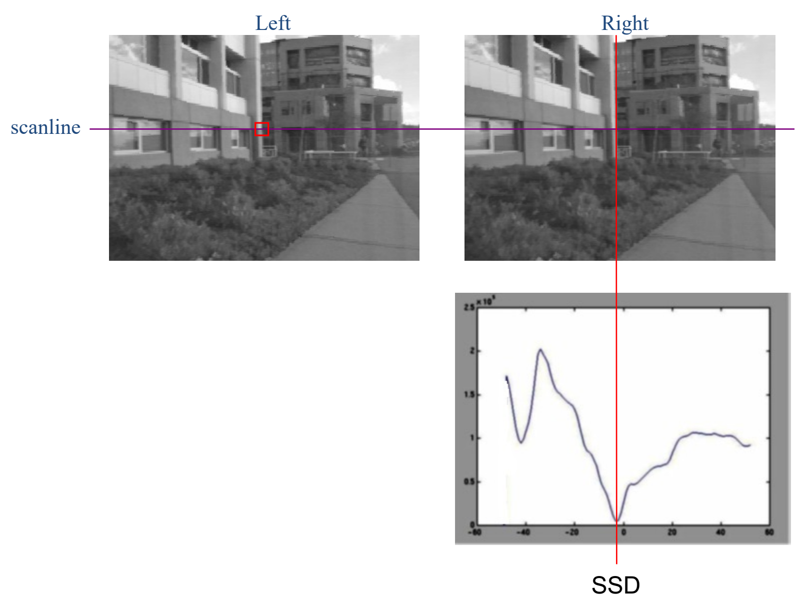
Correspondence Search



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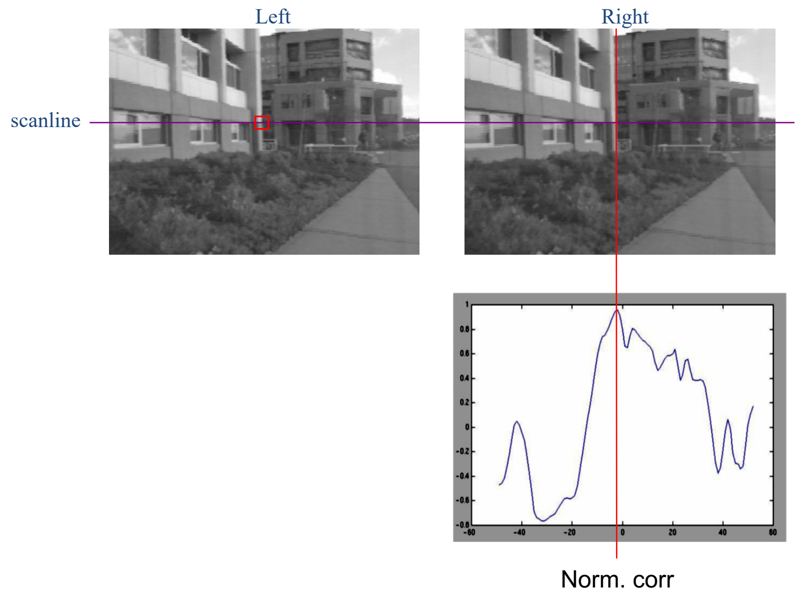
Correspondence Search



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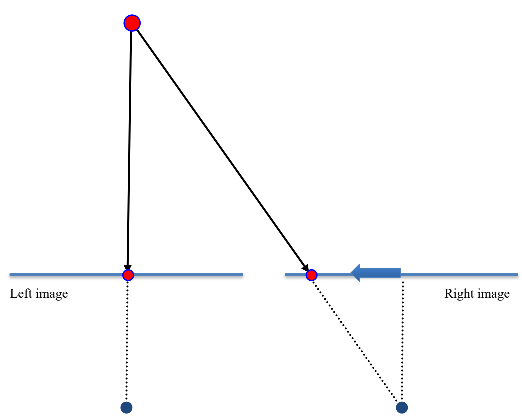
Limited Search – Leftward Only



Given the geometry, need only search "to the left" in the right image from starting point taken from the left image

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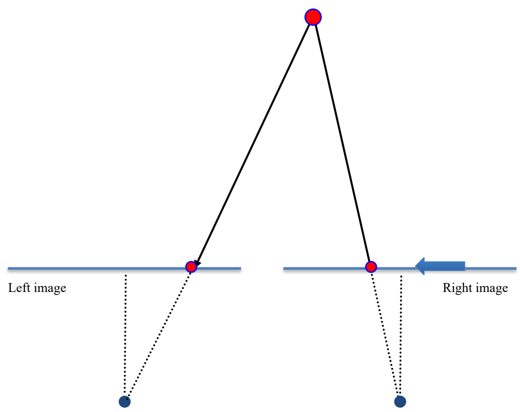
Limited Search – Leftward Only



Given the geometry, need only search "to the left" in the right image from starting point taken from the left image

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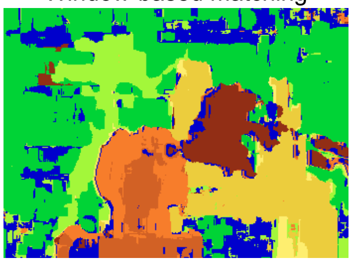
Results with Window Search

Scene

Window-based matching



Ground truth

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Effect of Window Size

W = 3 W = 20

- Smaller window
  - \+ More detail
  - – More noise
- Larger window
  - \+ Smoother disparity maps
  - – Less detail

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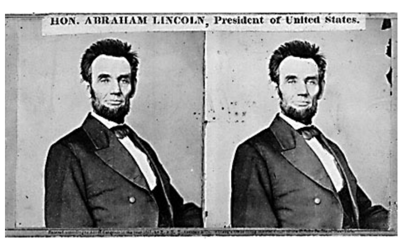
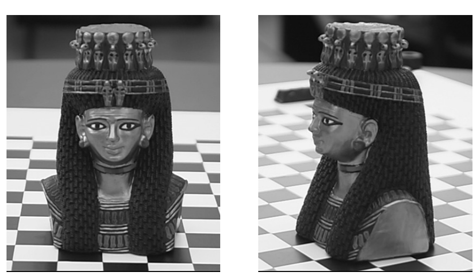
Difficulties of Correspondence Search



Textureless surfaces

Occlusions



Specularities (Non-Lambertian surfaces)

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Priors and Constraints

- Uniqueness
  - For any point in one image, there should be at most one matching point in the other image
- Ordering
  - Corresponding points should be in the same order in both views
- Smoothness
  - We expect disparity values to change slowly (for the most part)

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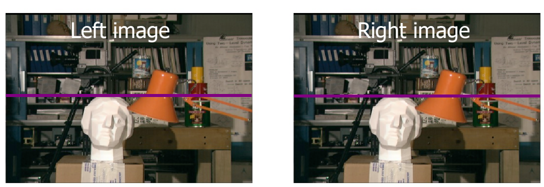
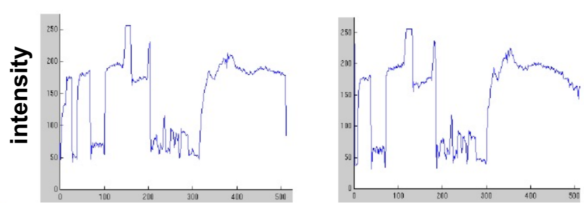
Scanline Stereo

- Try to coherently match pixels on the entire <u>scanline</u>
- Different scanlines are still optimized independently





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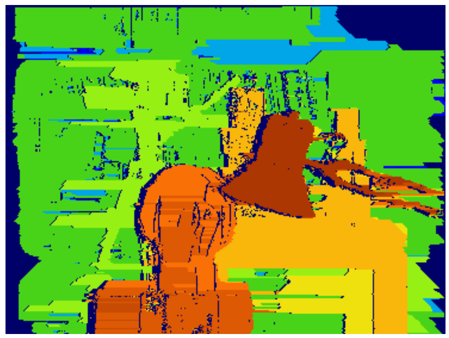
Coherent Stereo on 2-D Grid

- Scanline stereo can generate *streaking* artifacts
  - Dynamic programming approaches Ohta & Kanade '85, Cox et al. '96



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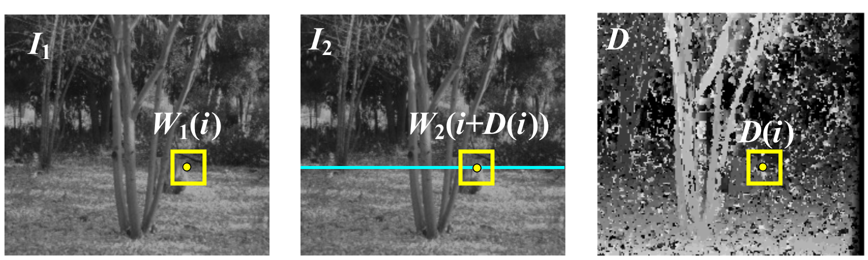
Stereo Matching as Energy Minimization



$$E(D) = \underbrace{\sum_i \left(W_1(i) - W_2(i + D(i))\right)^2}_{\text{data term}} + \lambda \underbrace{\sum_{\text{neighbors } i,j} \rho\left(D(i) - D(j)\right)}_{\text{smoothness term}}$$

- Energy functions of this form can be minimized using *graph cuts*

Y. Boykov, O. Veksler, and R. Zabih, Fast Approximate Energy Minimization via Graph Cuts, PAMI 2001

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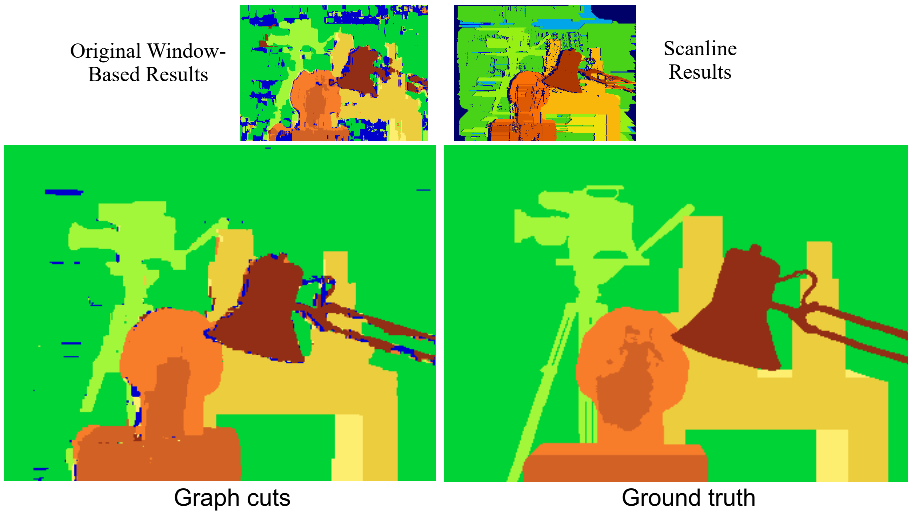
Graph Cuts Results



Y. Boykov, O. Veksler, and R. Zabih, [Fast Approximate Energy Minimization via Graph Cuts]

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Depth Estimation using Deep Learning With a Single Image

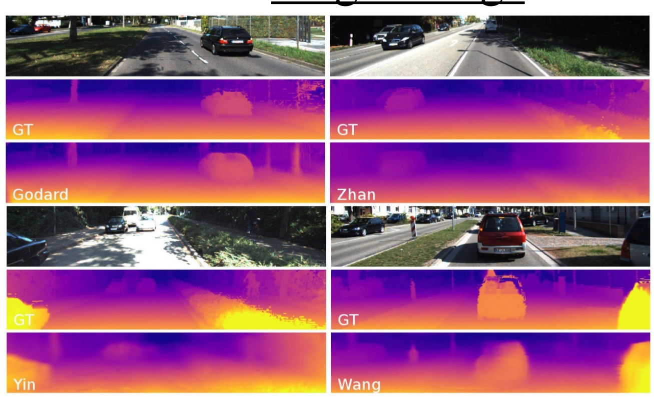

Fig. 1. An example of monocular depth estimation. The depth maps are predicted from deep neural networks proposed by Godard *et al.* [13], Zhan *et al.* [14], Yin *et al.* [15], and Wang *et al.* [16]. The results are taken from [17]. As shown in these figures, the 3D structures of objects, like trees, street and cars, can be effectively perceived from single images by deep depth networks.

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Kinect: Structured Infrared Light

http://www.youtube.com/watch?v=dTKlNGSH9Po

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Summary

- Epipolar geometry
  - Epipoles are intersection of baseline with image planes
  - Matching point in second image is on a line passing through its epipole
  - Fundamental matrix maps from a point in one image to a line (its epipolar line) in the other
  - Can solve for F given corresponding points (e.g., interest points)
- Stereo depth estimation
  - Estimate disparity by finding corresponding points
  - Depth is inversely related to disparity

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