CSE - 5526

Homework 4

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1 Question 1

Solution

At time step 0, input vector given to the network is $\mathbf{x^T} = [0.2, 0.2, 0.3, 0.4, 0.3]$.

Since external inputs set the initial conditions of neurons, y is given by $\mathbf{y^T} = [0.2, 0.2, 0.3, 0.4, 0.3]$.

For a winner-take-all network with 5 neurons, the function of each neuron is defined as:

$$y_i(t+1) = \phi((S-1)y_i(t) - \sum_{j \neq i} y_j(t)) \tag{1}$$

where S is the number of output neurons. The activation function ϕ is defined as:

$$\phi(x) = \begin{cases} 0 & \text{if } x < 0 \\ x & \text{if } 0 \leq x \leq 1 \\ 1 & \text{if } x > 1 \end{cases} \tag{2}$$

Here, $S = 5$. Therefore, for time step 1:

$$y_1(1) = \phi(4*0.2-(0.2+0.3+0.4+0.3) = \phi(0.8-1.2) = \phi(-0.4) \implies y_1(1) = 0$$

$$y_2(1) = \phi(4*0.2-(0.2+0.3+0.4+0.3) = \phi(0.8-1.2) = \phi(-0.4) \implies y_2(1) = 0$$

$$y_3(1) = \phi(4*0.3-(0.2+0.2+0.4+0.3) = \phi(1.2-1.1) = \phi(0.1) \implies y_3(1) = 0.1$$

$$y_4(1) = \phi(4*0.4-(0.2+0.2+0.3+0.3) = \phi(1.6-1) = \phi(0.6) \implies y_4(1) = 0.6$$

$$y_5(1) = \phi(4*0.3-(0.2+0.2+0.3+0.4) = \phi(1.2-1.1) = \phi(0.1) \implies y_5(1) = 0.1$$

Therefore, $\mathbf{y^T(1)} = [\mathbf{0}, \mathbf{0}, \mathbf{0.1}, \mathbf{0.6}, \mathbf{0.1}]$. Similarly, for time step 2:

$$y_1(2) = \phi(4*0-(0+0.1+0.6+0.1) = \phi(0-0.8) = \phi(-0.8) \implies y_1(2) = 0$$

$$y_2(2) = \phi(4*0-(0+0.1+0.6+0.1) = \phi(0-0.8) = \phi(-0.8) \implies y_2(2) = 0$$

$$y_3(2) = \phi(4*0.1-(0+0+0.6+0.1) = \phi(0.4-0.7) = \phi(-0.3) \implies y_3(2) = 0$$

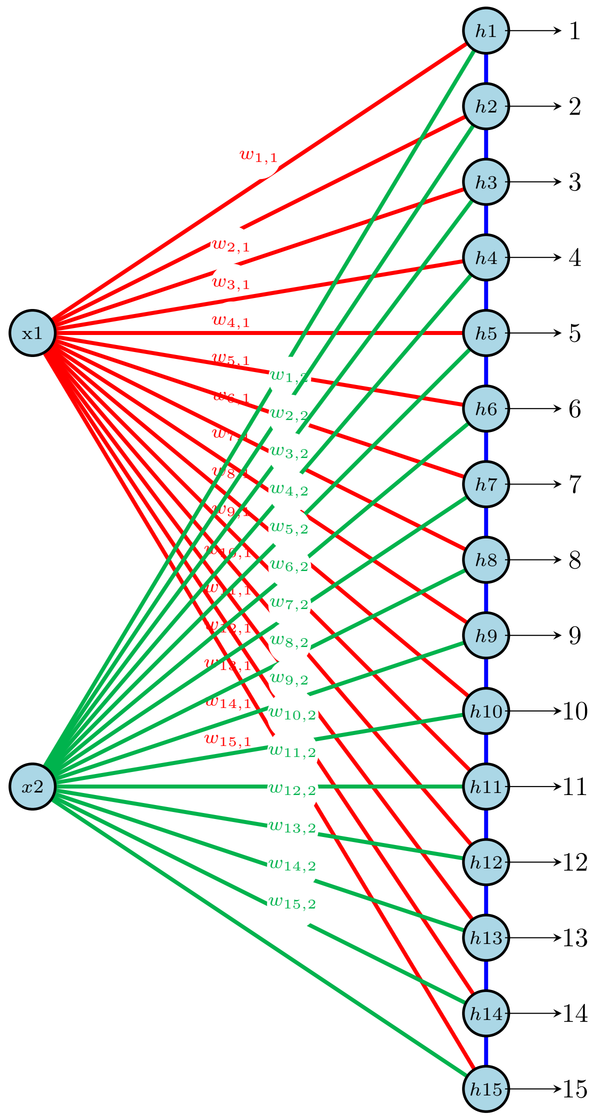
$$y_4(2) = \phi(4*0.6-(0+0+0.1+0.1) = \phi(2.4-0.2) = \phi(2.2) \implies y_4(2) = 1$$

$$y_5(2) = \phi(4*0.1-(0+0+0.1+0.6) = \phi(0.4-0.7) = \phi(-0.3) \implies y_5(2) = 0$$

Therefor, $\mathbf{y^T(2)} = [\mathbf{0}, \mathbf{0}, \mathbf{0}, \mathbf{1}, \mathbf{0}]$.

2 Question 2

Solution



Above figure shows the diagram of the network that has undergone such self-organization, wherein the SOM has been trained on two-dimensional inout vectors which were drawn from a uniform distribution over the triangular area. The network consist of an input layer that provides a 2-D input vector to the network, and 15 neurons arranged as 1-D (linear) layer. Each neuron is connected to the inputs, as well as with each other.

3 Question 3

Solution

Case 1: $x_i = -1$
Energy change resulting when neuron i flips from state $x_i(-1)$ to $-x_i(+1)$ is given by:

$\Delta E_i = E_{on} - E_{off}$

According to the property of Boltzmann distribution, the energy of a state is proportional to the negative log probability of that state. Hence,

$\Delta E_i = -k_B T \ln P_{on} - (-k_B T \ln P_{off})$

where k_B is the Boltzmann constant and is absorbed into the artificial notion of temperature T. Rearranging the terms, we get:

$\frac{\Delta E_i}{T} = -\ln P_{on} + \ln P_{off}$

$\frac{\Delta E_i}{T} = -\ln P_{on} + \ln 1 - P_{on}$

$\frac{\Delta E_i}{T} = \ln \frac{1-P_{on}}{P_{on}}$

$\frac{1-P_{on}}{P_{on}} = e^{\frac{\Delta E_i}{T}}$

$\frac{1}{P_{on}} = 1 + e^{\frac{\Delta E_i}{T}}$

$\boxed{\text{P}(\text{x}_i \rightarrow -x_i) = \frac{1}{1+e^{\frac{\Delta E_i}{T}}}}$

Case 2: $x_i = +1$
Energy change resulting when neuron i flips from state $x_i(+1)$ to $-x_i(-1)$ is given by:

$\Delta E_i = E_{off} - E_{on}$

According to the property of Boltzmann distribution, the energy of a state is proportional to the negative log probability of that state. Hence,

$\Delta E_i = -k_B T \ln P_{off} - (-k_B T \ln P_{on})$

where k_B is the Boltzmann constant and is absorbed into the artificial notion of temperature T. Rearranging the terms, we get:

$\frac{\Delta E_i}{T} = -\ln P_{off} + \ln P_{on}$

$\frac{\Delta E_i}{T} = -\ln P_{off} + \ln 1 - P_{off}$

$\frac{\Delta E_i}{T} = \ln \frac{1-P_{off}}{P_{off}}$

$\frac{1-P_{off}}{P_{off}} = e^{\frac{\Delta E_i}{T}}$

$\frac{1}{P_{off}} = 1 + e^{\frac{\Delta E_i}{T}}$

$\boxed{\mathrm{P}(\mathrm{x}_i \rightarrow -x_i) = \frac{1}{1+e^{\frac{\Delta E_i}{T}}}}$

Hence proved.