CSE - 5526

Homework 1

Submitted by

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Contents

1 Question 1

Give the weights and bias for a McCulloch-Pitts (M-P) neuron that has the following:

- Inputs: x, y and z
- Output: is equal to z if $x = -1$ and $y = 1$, and is equal to -1 otherwise

Solution

Table 1: Truth Table

x	y	z	Output
-1	-1	-1	-1
-1	-1	1	-1
-1	1	-1	-1
-1	1	1	1
1	-1	-1	-1
1	-1	1	-1
1	1	-1	-1
1	1	1	-1

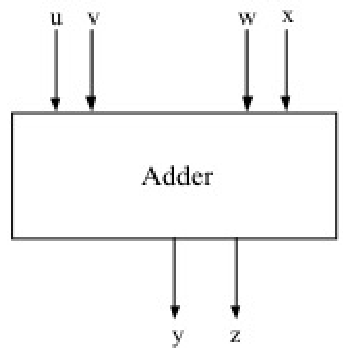
Approach: For the given set of three inputs; x, y, and z, I created a truth table which satisfied the given output constraints. Then by plugging in various values of three weights and biases, I found the combination that satisfied each input-output combination in the truth table. For inputs x, y, and z, final weights taken were -1, 1, 1, and the bias was -2.

2 Question 2

For this problem, change the definition of an M-P neuron so that both its inputs and outputs are binary (0 or 1). View uv and wx each as two-bit binary (00, 01, 10, or 11) numbers, and yz as the 2 low-order bits of the numerical addition of uv and wx (see the figure below).

(a) Give weights and biases for an M-P network which generates z.

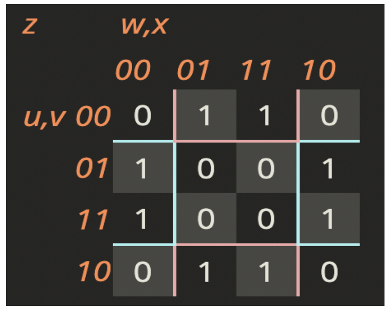
(b) Give weights and biases for an M-P network which generates y.



Solution

Table 2: Truth Table

u	v	w	x	y	z
0	0	0	0	0	0
0	0	0	1	0	1
0	0	1	0	1	0
0	0	1	1	1	1
0	1	0	0	0	1
0	1	0	1	1	0
0	1	1	0	1	1
0	1	1	1	0	0
1	0	0	0	1	0
1	0	0	1	1	1
1	0	1	0	0	0
1	0	1	1	0	1
1	1	0	0	1	1
1	1	0	1	0	0
1	1	1	0	0	1
1	1	1	1	1	0



z	w,x			
	00	01	11	10
u,v 00	0	1	1	0
01	1	0	0	1
11	1	0	0	1
10	0	1	1	0

Figure 1: Karnaugh Map for part (a)

From above K-Map, it can be inferred that the relationship between inputs and outputs for an MP network that generates z is given by:

$$z = v'x + vx' \tag{1}$$

Following neural network can be drawn that satisfies the above equation, with weights and biases mentioned:

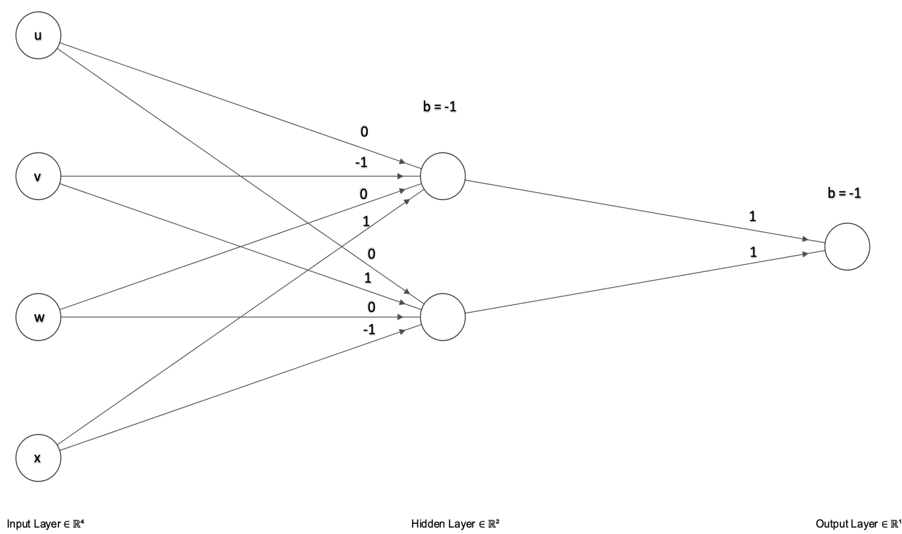


Figure 2: M-P Network for part (a)

y	w,x			
	00	01	11	10
u,v 00	0	0	1	1
01	0	1	0	1
11	1	0	1	0
10	1	1	0	0

Figure 3: Karnaugh Map for part (b)

From above K-Map, it can be inferred that the relationship between inputs and outputs for an MP network that generates y is given by:

$$y = uw'x' + uv'w' + u'v'w + u'wx' + u'vw'x + uvwx \tag{2}$$

Following neural network can be drawn that satisfies the above equation, with weights and biases mentioned:

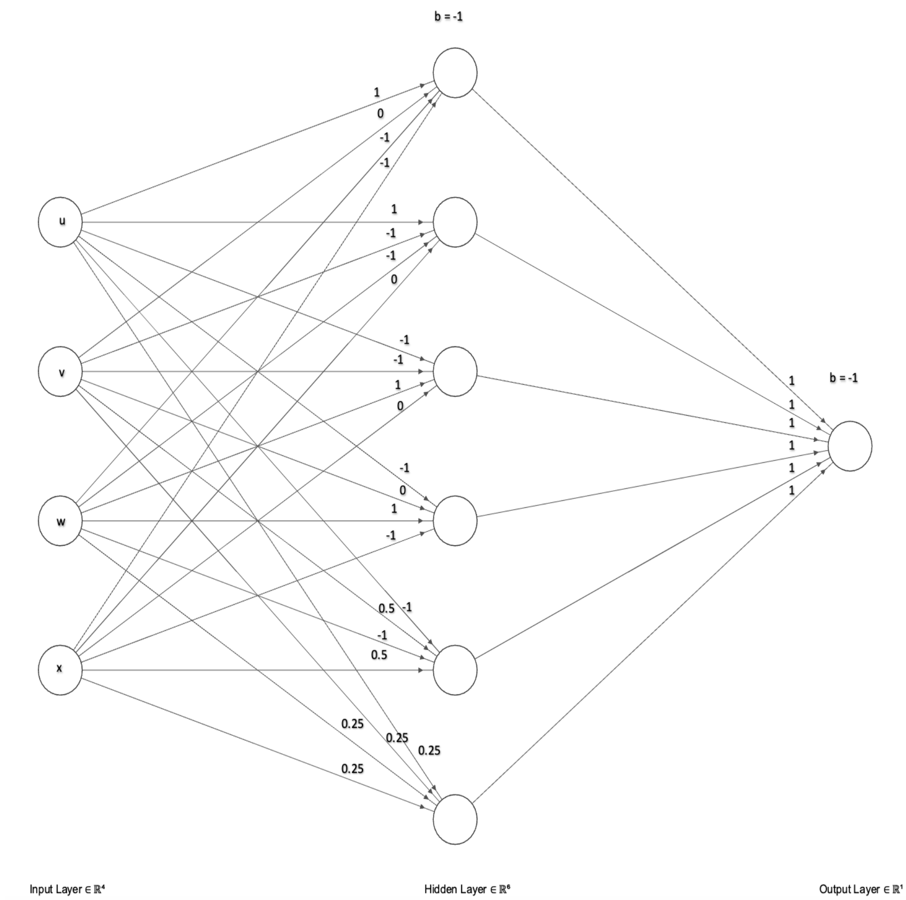
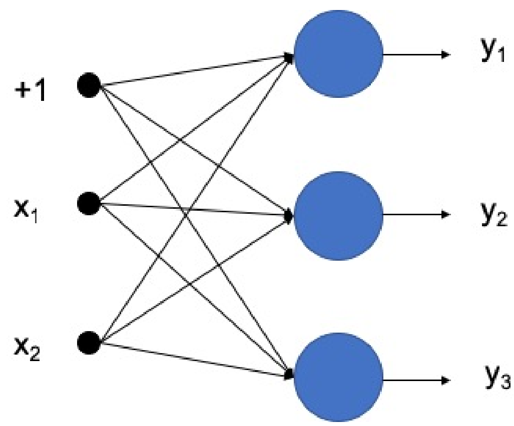


Figure 4: M-P Network for part (b)

Approach: For the given set of binary inputs; uv and wx, I created a truth table upon changing the definition of an M-P neuron so that both the inputs an outputs are binary (0 or 1). For both parts (a) and (b), I generated a Karnaugh Map to draw out a relationship between input bits and outputs (z for part (a) and y for part (b)), as shown in Figure 1 & 3. Once the input-output relationship was known, I generated M-P networks with weights and biases mentioned, as shown in Figure 2 & 4.

3 Question 3

Given the following 3-class classification problem:
C_1: {(4,1), (2,3), (3,5), (5,4), (1,6), (-1,6)}
C_2: {(0,2), (-2,2), (-3,2), (-2,4)}
C_3: {(1,-2), (3,-2)}
and the following single layer perceptron:



(a) Can the net learn to separate the samples, given that you want: if $\mathbf{x} \in C_i$ then $y_i = 1$ and $y_j = -1$ for $j \neq i$. No need to solve for the weights, but justify your answer.

(b) Remove data point (-1,6) from C_1. Repeat part (a) with this data set.

Solution

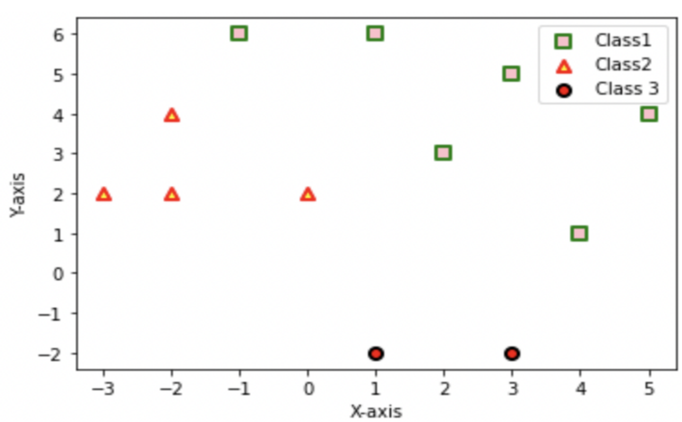


Figure 5: Scatter plot as per (a)

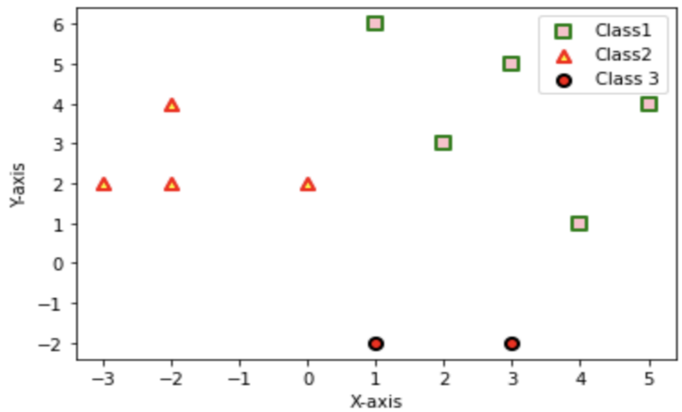


Figure 6: Scatter plot as per (b)

Approach: I plotted a graph representing all the different points from given three classes, as shown in Figure 5 & 6. For part (a), it can be seen that points in the classes C_1 and C_3 are linearly separable, hence a single layer perceptron can separate the samples in C_1 and C_3.
For part (b), as per Figure 6, after removing point (-1,6) from C_1, even the samples in C_2 become linearly separable. In this case, a single layer perceptron can separate the samples from all given classes, one at a time.