ECE 6500 **Assignment # 2**

Issued: Jan 31, 2023 Due: February 14, 2023

Cumulative Reading Assignments:

- Read Boyd & Vandenberghe, Chapter 2, Sections 2.1-2.3
- Read Boyd & Vandenberghe, Chapter 3, Sections 3.1-3.2, and 3.5.
- Read Boyd & Vandenberghe, Chapter 4, Sections 4.1-4.2
- Read Boyd & Vandenberghe, Chapter 5, Sections 5.1-5.6
- Read Boyd & Vandenberghe, Chapter 9, Sections 9.1-9.4

(1) (Necessary/Sufficient Conditions for Local/Global Optimality) Consider the unconstrained minimization of the following function

$$f(\mathbf{x}) = x_1^2 - x_1x_2 + x_2^2 - 3x_2.$$

Find a local minimum. Is this local minimum also a global minimum?

(2) (Positive/Negative (Semi)Definiteness of Symmetric Matrices) Identify which of the following symmetric matrices are positive/negative definite, positive/negative semi-definite, or none.

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 5 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}, \quad C = \begin{bmatrix} -3 & -3 & 0 \\ -3 & -10 & -7 \\ 0 & -7 & -8 \end{bmatrix}, \quad D = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}.$$

(3) (Convex functions) Problem 3.17 from Boyd & Vanderberghe.

(4) (Optimality Conditions for Constrained Optimization) Express the optimality conditions for the following problem (note that this has the form of the power control problem we encountered in the introduction lecture(s)):

$$\begin{aligned} \max \quad & \sum_{i=1}^{n} \log(\alpha_i + p_i) \\ s.t. \quad & p_i \geq 0, \forall i; \quad \text{and} \quad \sum_{i=1}^{n} p_i \leq P^{max}, \end{aligned}$$

where $\alpha_i > 0, \forall i$ and $P^{max} < \infty$ are given constants. Then, use the optimality conditions to write the optimal solution $\mathbf{p}^\star$ as a (potentially implicity) function of $(\alpha_i)_i$ and P^{max}.

(5) (Equivalent Problems) Problem 4.58 from Boyd & Vanderberghe.

(6) (Convex Problem Formulation) Problem 4.62 from Boyd & Vanderberghe.

(7) (Uniqueness of Projection onto a Convex set) Prove that the projection

$$P_C[x] = \arg\min_{\mathbf{y} \in C} \|\mathbf{x} - \mathbf{y}\|_2$$

of a point $\mathbf{x} \in \mathbf{R}^n$ onto a convex set $C \subset \mathbf{R}^n$ is unique.

(8) (About the Pure Newton Method) Problem 9.10 from Boyd & Vanderberghe.

(9) (Multi-Step Method Implementation-Investigation) Using the Matlab code you developed for PS1, implement the heavy-ball and the Nesterov's method (with constant choices of β and α) for the same quadratic function used in PS1, and compare their convergences with each other, as well as with the steepest-descent method you already implemented. Provide the plots of the trajectories of these methods from same initial conditions, and comment on the outcomes and provide your insights on the comparison.