UNIVERSITY OF DELAWARE

RPNG

State Estimation and SLAM

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Mechanical Engineering
Electrical and Computer Engineering
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Self-driving Cars

2

Drones

3

Underwater Vehicles

4

Autonomous navigation in the wild is hard!

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Outline

- Introduction
- Consistent state estimation and SLAM
  - Observability-Constrained (OC)-EKF
  - Deep loop closure
- Visual-inertial state estimation
  - Optimal-State-Constraint (OSC)-EKF
  - Closed-form preintegration for graph-VINS
  - Robocentric VIO
  - Schmidt EKF based VI-SLAM
  - Multi-camera/IMU VIO
  - VINS with different geometric features
  - Moving object tracking
  - Extensions to any-source aided INS
- Summary

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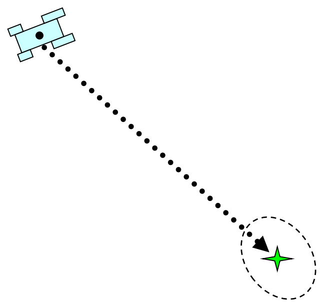
Simultaneous Localization and Mapping (SLAM)

- Detect landmarks (features) in the environment
- Jointly estimate landmark positions and robot pose



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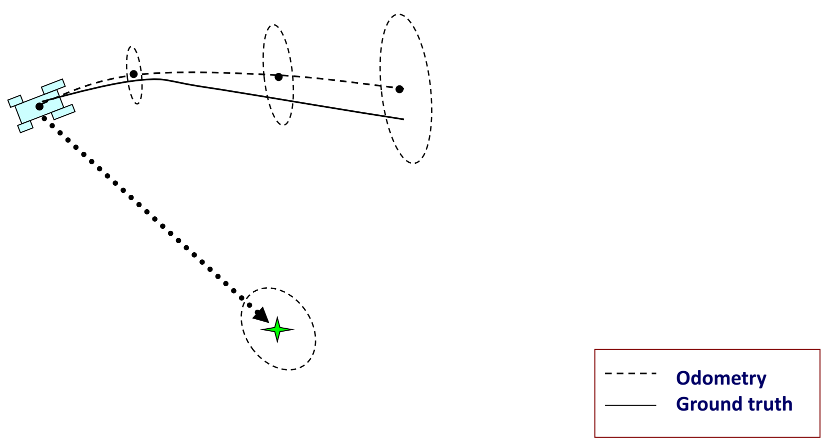

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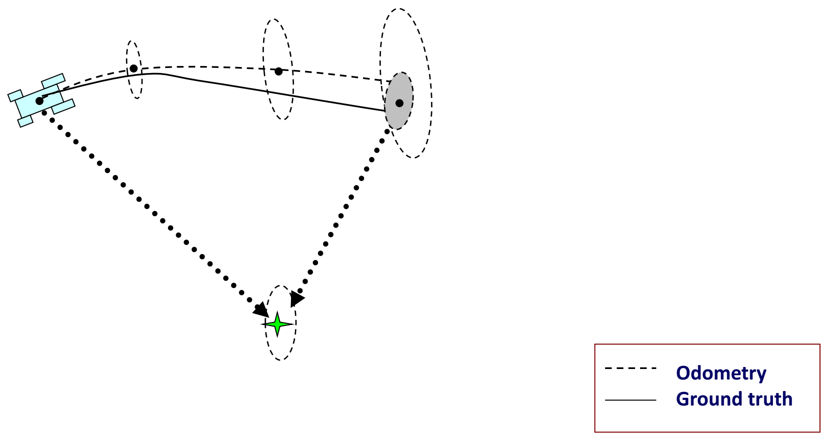

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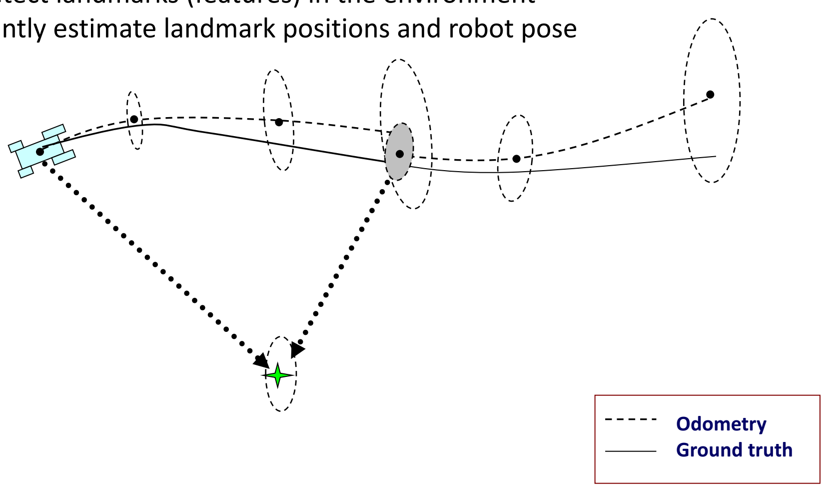

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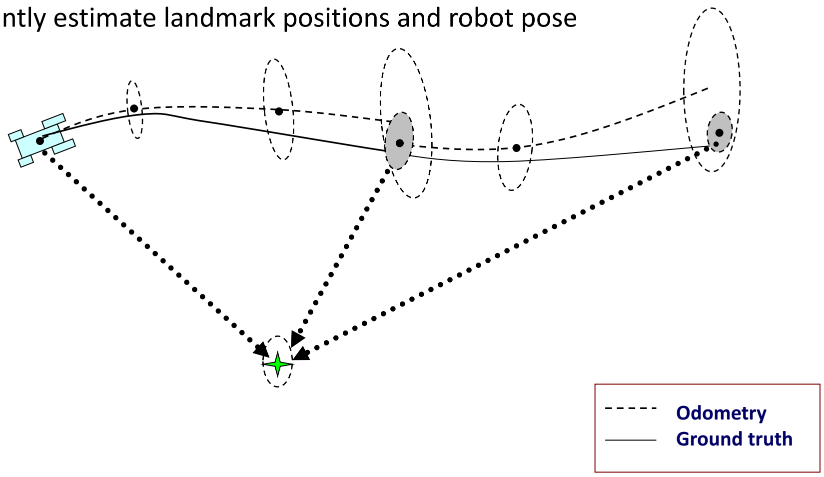

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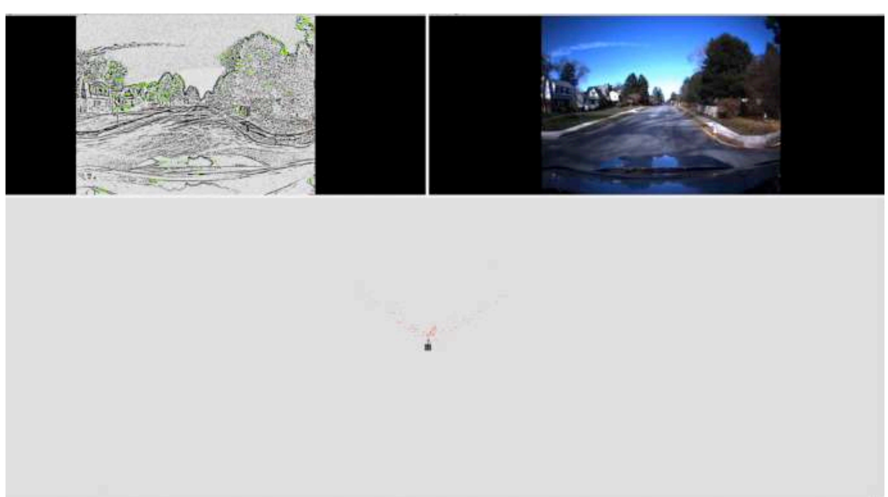
SLAM in Action: Outdoor [IROS 18a]



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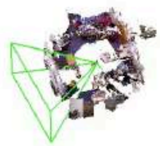
SLAM in Action: Indoor [ECMR 13a]



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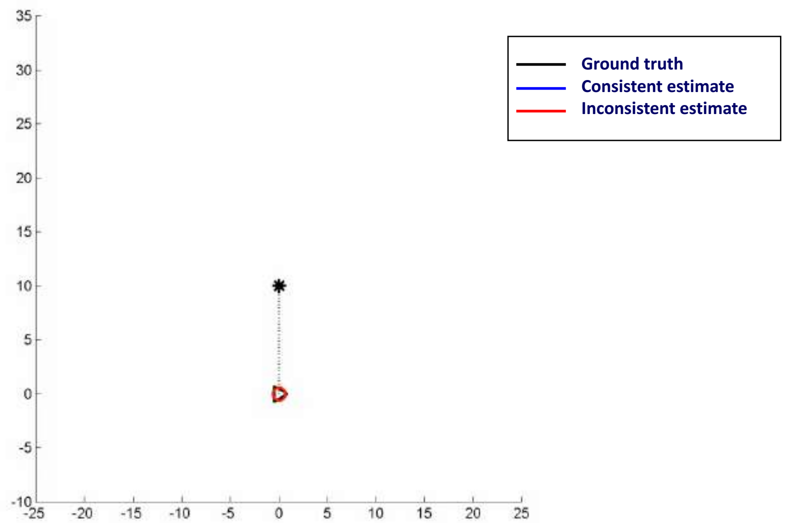
Problem: Estimation Inconsistency



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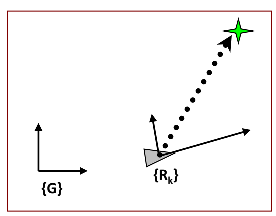
SLAM Observability Analysis [IJRR 10]

- Goal: determine the dim. of the unobservable subspace $\mathrm{M}^{\perp}$ of the EKF system model, examine its effect on consistency, and improve estimation consistency

- Nonlinear SLAM:

$$\mathbf{x}_{k+1} = \mathbf{f}(\mathbf{x}_k, \mathbf{u}_k + \mathbf{w}_k)$$
$$\mathbf{z}_k = \mathbf{h}(\mathbf{x}_k) + \mathbf{v}_k$$

$\dim(\mathrm{M}^{\perp}) = 3$
- 2dof global translation
- 1dof global rotation



- EKF linearized SLAM (error-state):

$$\tilde{\mathbf{x}}_{k+1|k} \simeq \mathbf{\Phi}_k \tilde{\mathbf{x}}_{k|k} + \mathbf{G}_k \mathbf{w}_k$$
$$\tilde{\mathbf{z}}_k \simeq \mathbf{H}_k \tilde{\mathbf{x}}_{k|k-1} + \mathbf{v}_k$$

$$\mathbf{\Phi}_k = \nabla_{\mathbf{x}_k} \mathbf{f}\Big|_{\{\mathbf{x}^\star, \mathbf{0}\}}$$
$$\mathbf{G}_k = \nabla_{\mathbf{w}_k} \mathbf{f}\Big|_{\{\mathbf{x}^\star, \mathbf{0}\}}$$
$$\mathbf{H}_k = \nabla_{\mathbf{x}_k} \mathbf{h}\Big|_{\mathbf{x}^\star}$$

$\mathbf{x}^\star$: **linearization points**

- **Key findings: when Jacobians evaluated at**
  - **true states (ideal EKF):** $\dim(\mathrm{M}^{\perp}) = 3$
  - **latest estimates (standard EKF):** $\dim(\mathrm{M}^{\perp}) = 2$

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First-Estimate Jacobian (FEJ)-EKF [ICRA 08; ISER 08]

- Lemma: **If Jacobians evaluated at *first* state estimates, i.e.,**

$$\Phi_k = \Phi_k\left(\hat{\mathbf{x}}_{R_{k|k-1}}, \hat{\mathbf{x}}_{R_{k+1|k}}\right) \qquad \mathbf{H}_k = \mathbf{H}_k\left(\hat{\mathbf{x}}_{R_{k|k-1}}, \hat{\mathbf{p}}_{L_{k_o|k_o}}\right)$$

then the observability matrix has rank of 2.

- Observability matrix:

$$\mathbf{M}_{\mathsf{FEJ}} = \underbrace{\mathbf{D}_{\mathsf{FEJ}}}_{\text{full rank}} \times \begin{bmatrix} -\mathbf{I}_2 & -\mathbf{J}\left(\hat{\mathbf{p}}_{L_{k_o|k_o}} - \hat{\mathbf{p}}_{R_{k_o|k_o-1}}\right) & \mathbf{I}_2 \\ -\mathbf{I}_2 & -\mathbf{J}\left(\hat{\mathbf{p}}_{L_{k_o|k_o}} - \hat{\mathbf{p}}_{R_{k_o|k_o-1}}\right) & \mathbf{I}_2 \\ -\mathbf{I}_2 & -\mathbf{J}\left(\hat{\mathbf{p}}_{L_{k_o|k_o}} - \hat{\mathbf{p}}_{R_{k_o|k_o-1}}\right) & \mathbf{I}_2 \\ \vdots & \vdots & \vdots \\ -\mathbf{I}_2 & -\mathbf{J}\left(\hat{\mathbf{p}}_{L_{k_o|k_o}} - \hat{\mathbf{p}}_{R_{k_o|k_o-1}}\right) & \mathbf{I}_2 \end{bmatrix}$$

$$\boxed{\begin{aligned} &\mathrm{rank}(\mathbf{M}_{\mathrm{FEJ}}) = 2 \\ \Rightarrow\ &\dim(\mathbf{M}_{\mathrm{FEJ}}^{\perp}) = 3 \end{aligned}}$$

Observability-Constrained (OC)-EKF [IJRR 10]

- Key idea: select linearization points that not only minimize linearization errors but also ensure $\dim(\mathbf{M}^{\perp}) = 3$

$$\boxed{\begin{aligned} \min_{\mathbf{x}^\star_{R_{k|k}},\ \mathbf{x}^\star_{k+1|k}} \quad & \int \left\|\mathbf{x}_{R_k} - \mathbf{x}^\star_{R_{k|k}}\right\|^2 p(\mathbf{x}_{R_k}|\mathbf{z}_{0:k})d\mathbf{x}_{R_k} + \int \left\|\mathbf{x}_{k+1} - \mathbf{x}^\star_{k+1|k}\right\|^2 p(\mathbf{x}_{k+1}|\mathbf{z}_{0:k})d\mathbf{x}_{k+1} \\ \text{s.t.} \quad & \dim(\mathbf{M}^{\perp}) = 3 \end{aligned}}$$

Open source: https://github.com/rpng/ocekf-slam

Observability-Constrained (OC)-EKF [IJRR 10]

- Key idea: select linearization points that not only minimize linearization errors but also ensure $\dim(\mathbf{M}^{\perp}) = 3$

$$\min_{\mathbf{x}^{\star}_{R_{k|k}},\, \mathbf{x}^{\star}_{k+1|k}} \int \left\| \mathbf{x}_{R_k} - \mathbf{x}^{\star}_{R_{k|k}} \right\|^2 p(\mathbf{x}_{R_k}|\mathbf{z}_{0:k}) d\mathbf{x}_{R_k} + \int \left\| \mathbf{x}_{k+1} - \mathbf{x}^{\star}_{k+1|k} \right\|^2 p(\mathbf{x}_{k+1}|\mathbf{z}_{0:k}) d\mathbf{x}_{k+1}$$

$$\text{s.t.} \quad \mathbf{H}_{k+1}\mathbf{\Phi}_k \cdots \mathbf{\Phi}_{k_o}\mathbf{U} = \mathbf{0}\ , \quad \forall k \geq k_o$$

- Optimal linearization points:

$$\mathbf{p}^{\star}_{R_{k|k}} = \hat{\mathbf{p}}_{R_{k|k}} + \frac{\boldsymbol{\lambda}_k}{2}\ , \qquad \phi^{\star}_{R_{k|k}} = \hat{\phi}_{R_{k|k}}$$

$$\mathbf{x}^{\star}_{R_{k+1|k}} = \hat{\mathbf{x}}_{R_{k+1|k}}\ , \qquad \mathbf{p}^{\star}_{L_{k+1|k}} = \hat{\mathbf{p}}_{L_{k+1|k}} - \frac{\boldsymbol{\lambda}_k}{2}$$

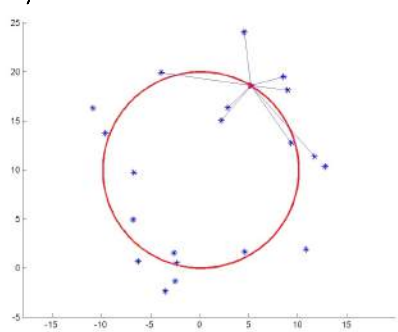
$$\boldsymbol{\lambda}_k = \left(\hat{\mathbf{p}}_{L_{k+1|k}} - \hat{\mathbf{p}}_{L_{k_o|k_o}}\right) - \left(\hat{\mathbf{p}}_{R_{k|k}} - \mathbf{p}^{\star}_{R_{k|k-1}} + \sum_{j=k_o}^{k-1} \Delta\mathbf{p}^{\star}_{R_j}\right)$$

- Propagate and update as for the standard EKF: Causal and realizable

Open source: https://github.com/rpng/ocekf-slam

SLAM Simulation

- Monte-Carlo simulations (N = 50 runs)
- Compared estimators:
  - Ideal EKF (benchmark)
  - Standard EKF
  - FEJ-EKF [ICRA 08]
  - OC-EKF [IJRR 10]
  - Robocentric mapping [Castellanos et al. 04]



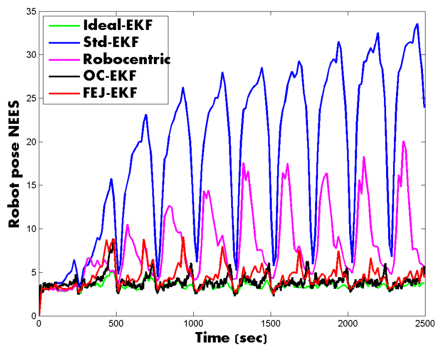
- Performance metrics:
  - Root mean squared error (RMSE)

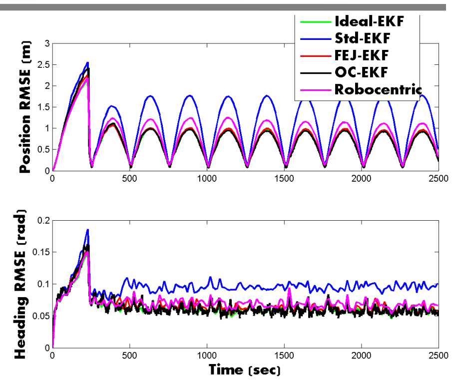
$$\text{RMSE} = \sqrt{\frac{1}{N}\sum_{j=1}^{N} \tilde{\mathbf{x}}^{(j)T}\tilde{\mathbf{x}}^{(j)}}$$

  - Normalized estimation error squared (NEES)

$$\text{NEES} = \frac{1}{N}\sum_{j=1}^{N} \tilde{\mathbf{x}}^{(j)T}\mathbf{P}^{(j)-1}\tilde{\mathbf{x}}^{(j)} \sim \chi^2 \quad \text{(for Gaussian case)}$$

Monte-Carlo Results: Robot





Avg. NEES of robot pose

Avg. RMSE of robot pose

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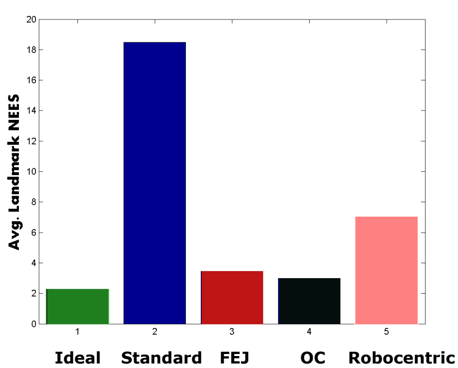
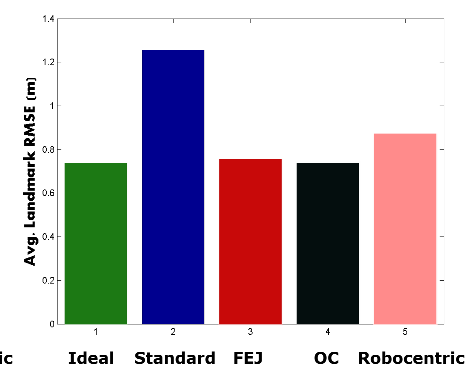
Monte-Carlo Results: Landmarks





Avg. NEES of landmark positions

Avg. RMSE of landmark positions

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Observability-based Consistent Estimator Design

- Simultaneous localization and mapping (SLAM) [ICRA 08; ISER 08; IJRR 10]
  - Observability analysis: **dim. of unobservable subspace**
    - Nonlinear system: **3**
    - Standard EKF: **2** **inconsistent**
    - FEJ/OC-EKFs: **3** **improved consistency & accuracy**
- **Observability-based methodology** has been extended to:
  - Different nonlinear estimators: UKF [ICRA 09; TRO 13], SWF [IROS 11], and incremental MAP (iMAP) [ECMR 13a; RAS 14]
  - Different navigation problems: CL [RSS 09; AURO 11; CCC 15], CLATT [ECMR 13b; RAS 14], and VINS [ICRA 14; ISRR15; ICRA 17a; IROS 17a; ICRA 18a; IROS 18a]
  - Nonlinear systems with partial-state measurements [ACC 13; SCL 17]

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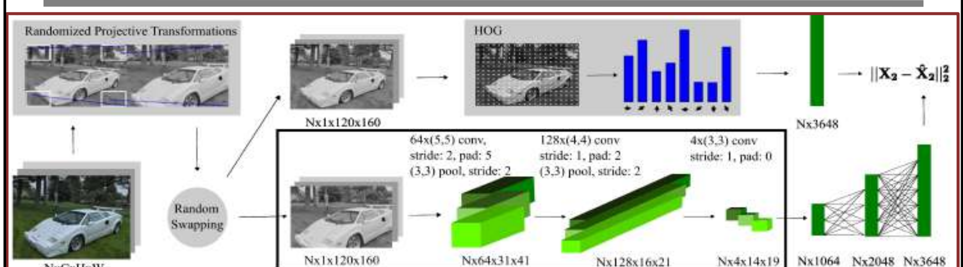
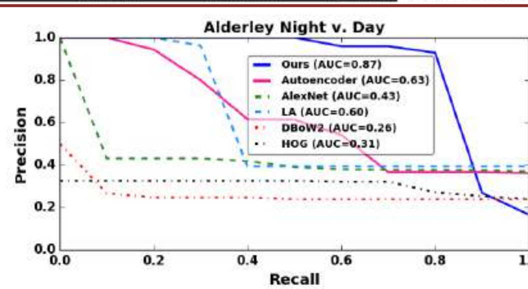
CALC: Unsupervised Deep Loop Closure [RSS 18]





Open source: https://github.com/rpng/calc

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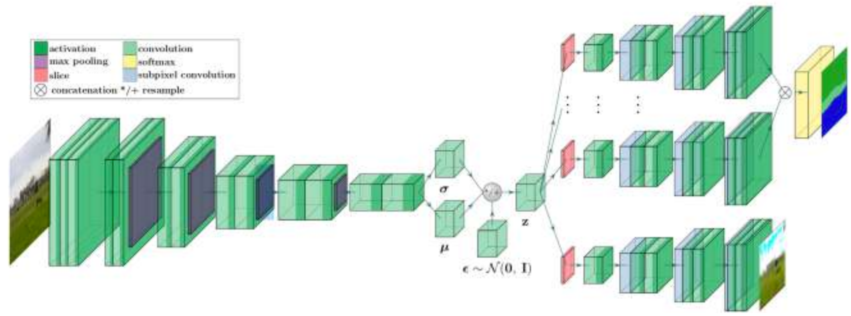
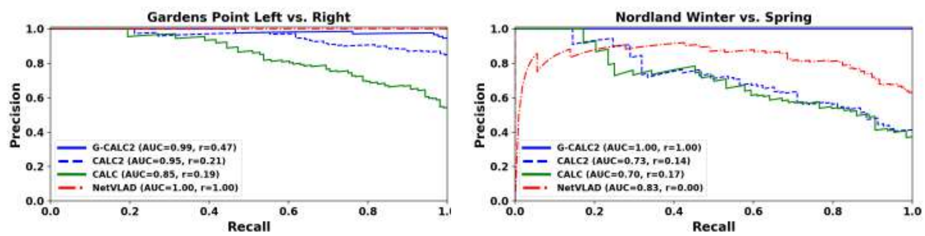
CALC2.0: Fusing Appearance, Semantics & Geometrics [IROS 19a]





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Motivating Example: AR/VR

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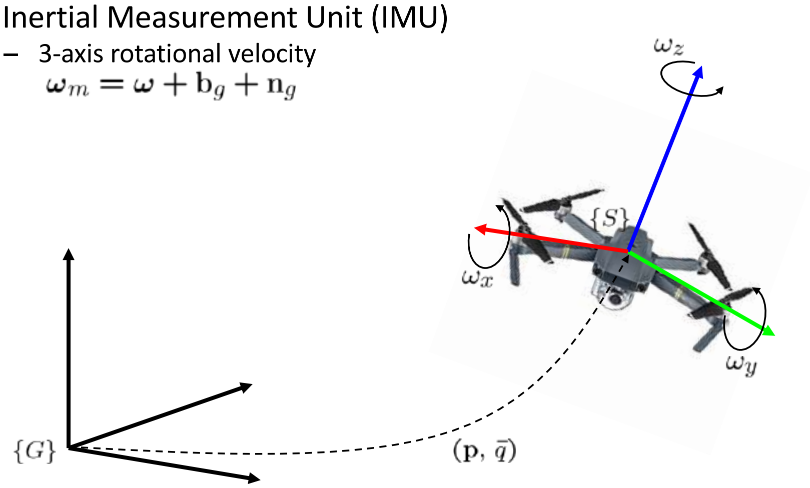
Visual-Inertial Navigation System (VINS)

- Estimate 6 DOF pose: position $\mathbf{p}$ & orientation $\bar{q}$
- Inertial Measurement Unit (IMU)
  - 3-axis rotational velocity

    $$\boldsymbol{\omega}_m = \boldsymbol{\omega} + \mathbf{b}_g + \mathbf{n}_g$$



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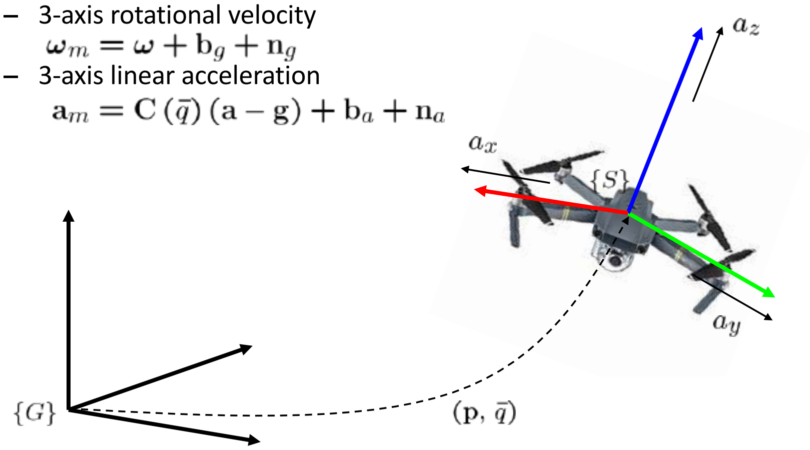
- Estimate 6 DOF pose: position $\mathbf{p}$ & orientation $\bar{q}$
- Inertial Measurement Unit (IMU)
  - 3-axis rotational velocity
    $\boldsymbol{\omega}_m = \boldsymbol{\omega} + \mathbf{b}_g + \mathbf{n}_g$
  - 3-axis linear acceleration
    $\mathbf{a}_m = \mathbf{C}(\bar{q})(\mathbf{a} - \mathbf{g}) + \mathbf{b}_a + \mathbf{n}_a$



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- Estimate 6 DOF pose: position $\mathbf{p}$ & orientation $\bar{q}$
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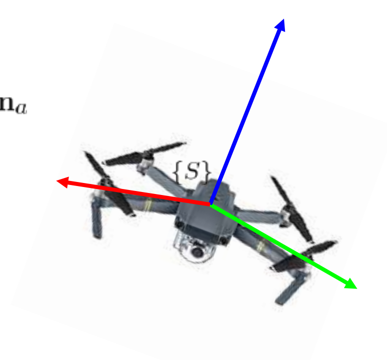
  - 3-axis rotational velocity
    $\boldsymbol{\omega}_m = \boldsymbol{\omega} + \mathbf{b}_g + \mathbf{n}_g$
  - 3-axis linear acceleration
    $\mathbf{a}_m = \mathbf{C}(\bar{q})(\mathbf{a} - \mathbf{g}) + \mathbf{b}_a + \mathbf{n}_a$
- Kinematic model of motion

$$
\boxed{
\begin{aligned}
&\dot{\bar{q}} = \frac{1}{2}\Xi(\bar{q})\,\omega \\
&\dot{\mathbf{p}} = \mathbf{v} \\
&\dot{\mathbf{v}} = \mathbf{a} \\
&\dot{\mathbf{b}} = \mathbf{w}, \quad \mathbf{w} \sim \mathcal{N}(0, \mathbf{Q})
\end{aligned}
}
$$



*However, integration of noise & bias causes **large drift** in the pose estimate!*

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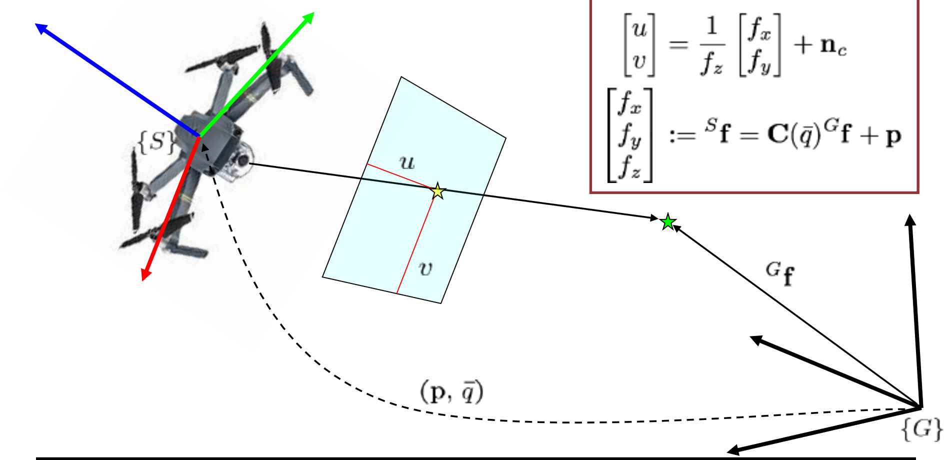
Visual-Inertial Navigation System (VINS)

- Use IMU & camera to estimate $\bar{q}$, $\mathbf{p}$, $\mathbf{v}$, $\mathbf{b}$
- Camera measurement model:



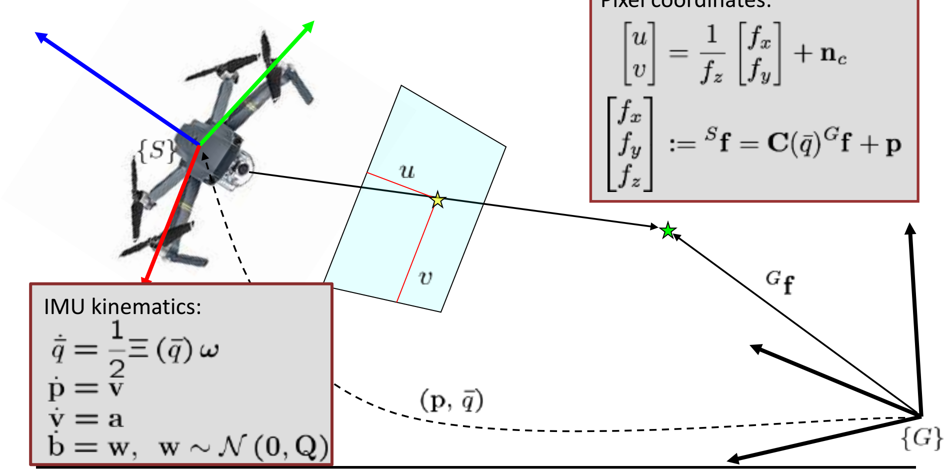
Pixel coordinates:

$$\begin{bmatrix} u \\ v \end{bmatrix} = \frac{1}{f_z} \begin{bmatrix} f_x \\ f_y \end{bmatrix} + \mathbf{n}_c$$

$$\begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix} := {}^{S}\mathbf{f} = \mathbf{C}(\bar{q})^{G}\mathbf{f} + \mathbf{p}$$

Visual-Inertial Navigation System (VINS)

- Use IMU & camera to estimate $\bar{q}$, $\mathbf{p}$, $\mathbf{v}$, $\mathbf{b}$



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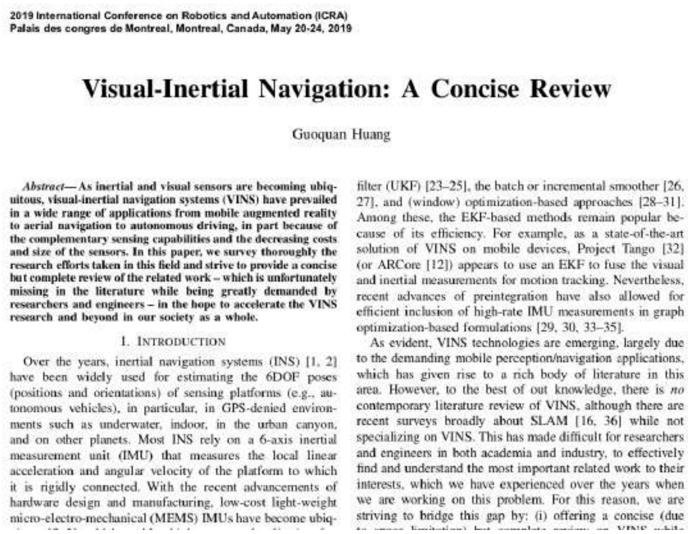
IMU kinematics:

$$\dot{\bar{q}} = \frac{1}{2}\Xi(\bar{q})\,\omega$$
$$\dot{\mathbf{p}} = \mathbf{v}$$
$$\dot{\mathbf{v}} = \mathbf{a}$$
$$\dot{\mathbf{b}} = \mathbf{w}, \quad \mathbf{w} \sim \mathcal{N}(0, \mathbf{Q})$$

VINS Review [ICRA 19]



2019 International Conference on Robotics and Automation (ICRA)
Palais des congres de Montreal, Montreal, Canada, May 20-24, 2019

Visual-Inertial Navigation: A Concise Review

Guoquan Huang

Abstract—**As inertial and visual sensors are becoming ubiquitous, visual-inertial navigation systems (VINS) have prevailed in a wide range of applications from mobile augmented reality to aerial navigation to autonomous driving, in part because of the complementary sensing capabilities and the decreasing costs and size of the sensors. In this paper, we survey thoroughly the research efforts taken in this field and strive to provide a concise but complete review of the related work – which is unfortunately missing in the literature while being greatly demanded by researchers and engineers – in the hope to accelerate the VINS research and beyond in our society as a whole.**

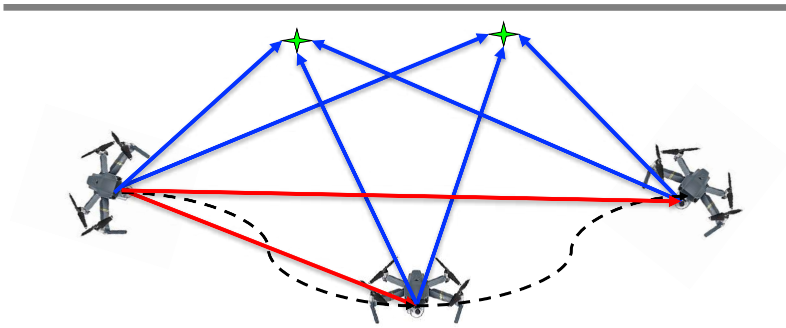
I. INTRODUCTION

Over the years, inertial navigation systems (INS) [1, 2] have been widely used for estimating the 6DOF poses (positions and orientations) of sensing platforms (e.g., autonomous vehicles), in particular, in GPS-denied environments such as underwater, indoor, in the urban canyon, and on other planets. Most INS rely on a 6-axis inertial measurement unit (IMU) that measures the local linear acceleration and angular velocity of the platform to which it is rigidly connected. With the recent advancements of hardware design and manufacturing, low-cost light-weight micro-electro-mechanical (MEMS) IMUs have become ubiq-

filter (UKF) [23–25], the batch or incremental smoother [26, 27], and (window) optimization-based approaches [28–31]. Among these, the EKF-based methods remain popular because of its efficiency. For example, as a state-of-the-art solution of VINS on mobile devices, Project Tango [32] (or ARCore [12]) appears to use an EKF to fuse the visual and inertial measurements for motion tracking. Nevertheless, recent advances of preintegration have also allowed for efficient inclusion of high-rate IMU measurements in graph optimization-based formulations [29, 30, 33–35].

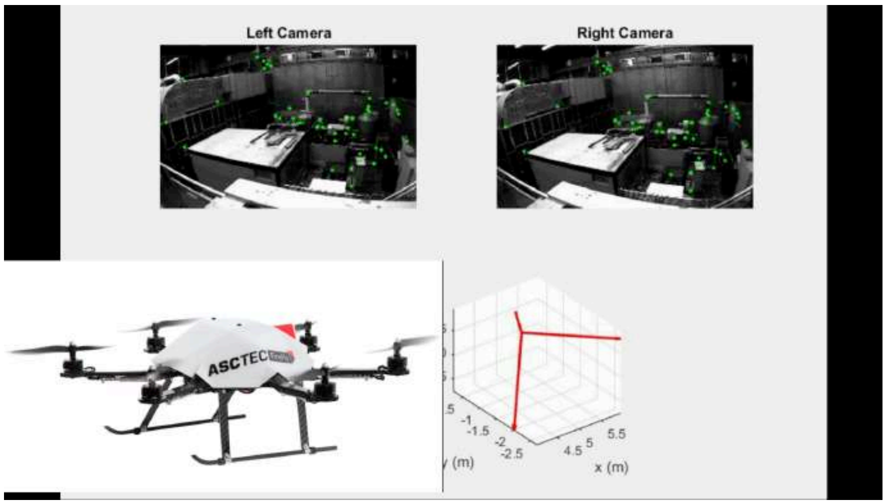
As evident, VINS technologies are emerging, largely due to the demanding mobile perception/navigation applications, which has given rise to a rich body of literature in this area. However, to the best of out knowledge, there is *no* contemporary literature review of VINS, although there are recent surveys broadly about SLAM [16, 36] while not specializing on VINS. This has made difficult for researchers and engineers in both academia and industry, to effectively find and understand the most important related work to their interests, which we have experienced over the years when we are working on this problem. For this reason, we are striving to bridge this gap by: (i) offering a concise (due

Optimal-State-Constraint (OSC)-EKF [ISRR 15; ARL 17]

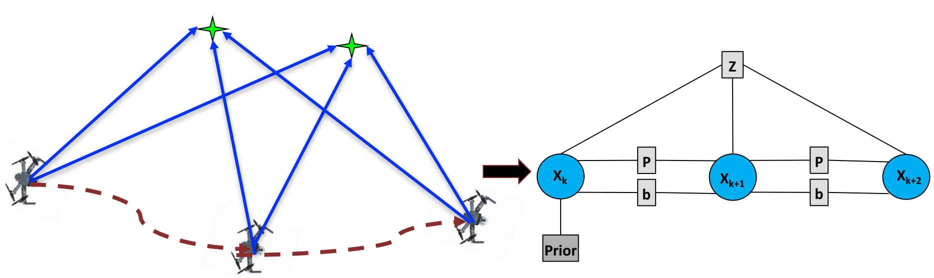


- Key ideas:
  - Process a sliding window of images *only* to infer **optimal** state constraints btw. corresponding camera poses
  - Perform EKF propagation using IMU measurements, and EKF update using the inferred state constraints
  - As a result, *no* need to store features in the state vector, yielding *constant* complexity

Results: OSC-EKF VIO



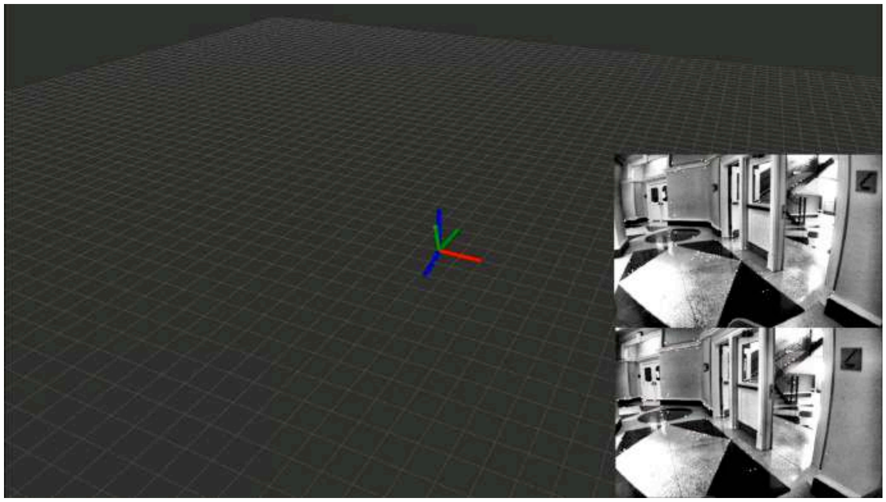
Closed-form Preintegration for Graph-VINS [WAFR 16; IJRR 19a]



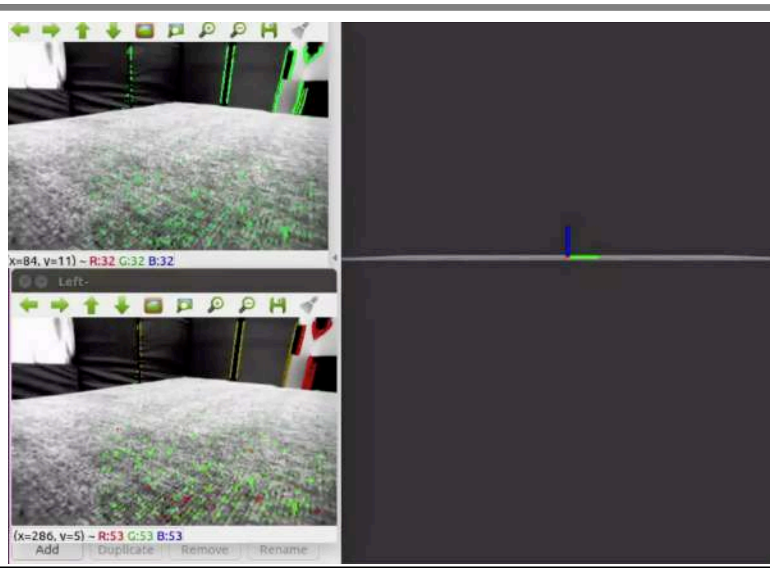
- Sliding-window BA to *tightly* fuse visual and inertial measurements
  - Analytical continuous-time IMU preintegration: (i) piecewise constant measurement, and (ii) piecewise constant local acceleration
  - Perform bundle adjustment (BA) to optimally estimate a sliding window of states and features detected
  - Marginalize out features to bound computational complexity

Open source: https://github.com/rpng/cpi

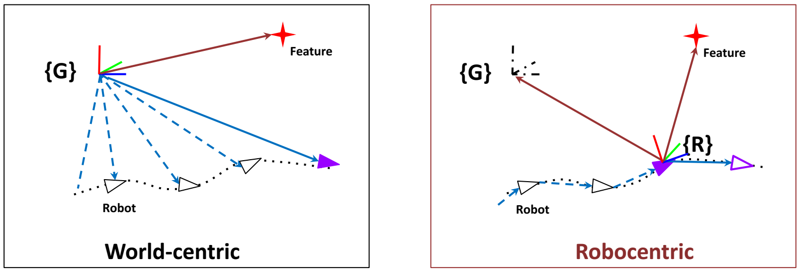
Results: Graph-VINS [IJRR 19a]



Direct VINS [ICRA 17a]



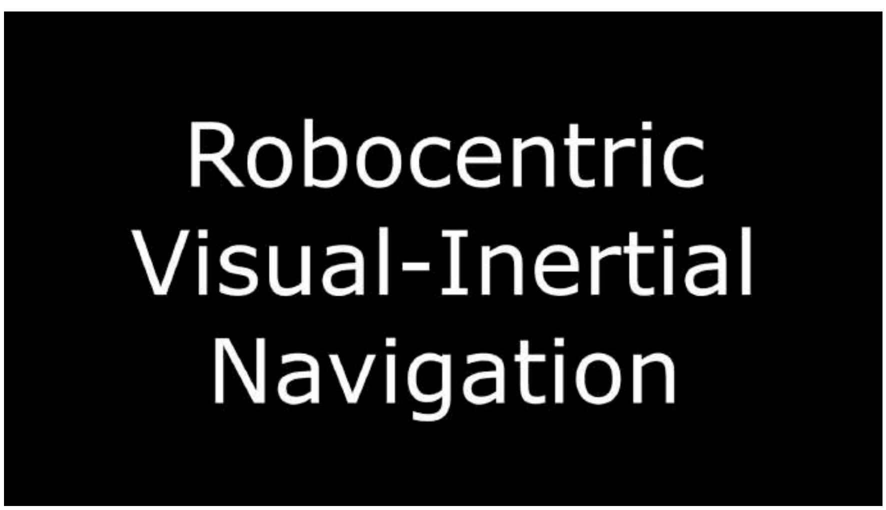
Robocentric VIO [IROS 18, IJRR19b]



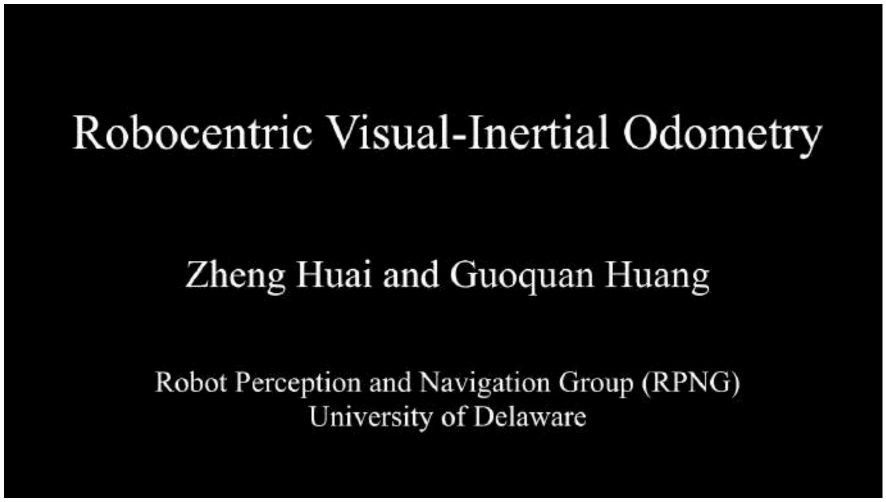
- Robocentric VIO (R-VIO) within the MSCKF framework:
  - State includes local gravity and a sliding window of relative poses
  - Close-form IMU preintegration (for propagation)
  - Inverse depth-based measurement model (for update)
  - Composition is employed to shift robocentric frame after update

Open source: https://github.com/rpng/r-vio

Results: R-VIO [IJRR 19b]

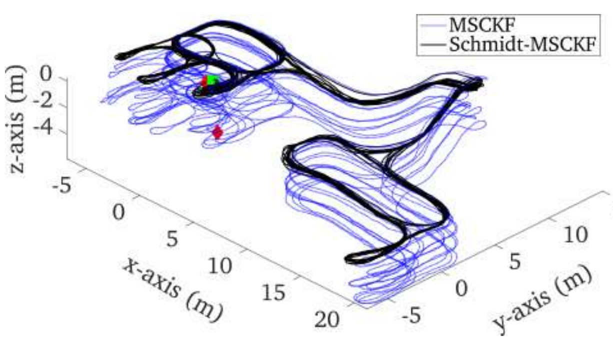


Results: R-VIO [IJRR 19b]

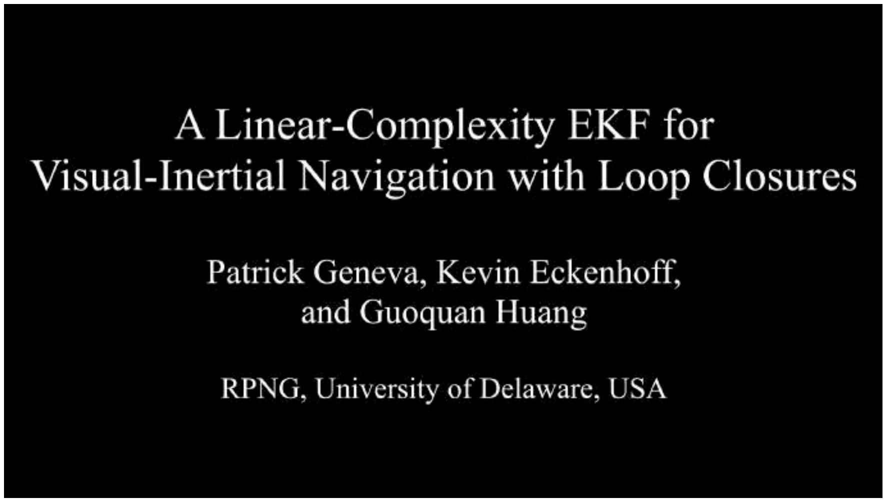


Schmidt MSCKF for VINS w/ Loop Closures [ICRA 19c]

- A novel linear-complexity algorithm for VINS with loop closures:
  - Exploit Schmidt-KF for real-time consistent inclusion of old keyframes by only updating their cross-correlations
  - Leverage MSCKF nullspace-based marginalization, allowing for efficient processing measurements of keyframe-based loop-closures



Results: Schmidt MSCKF [ICRA 19c]



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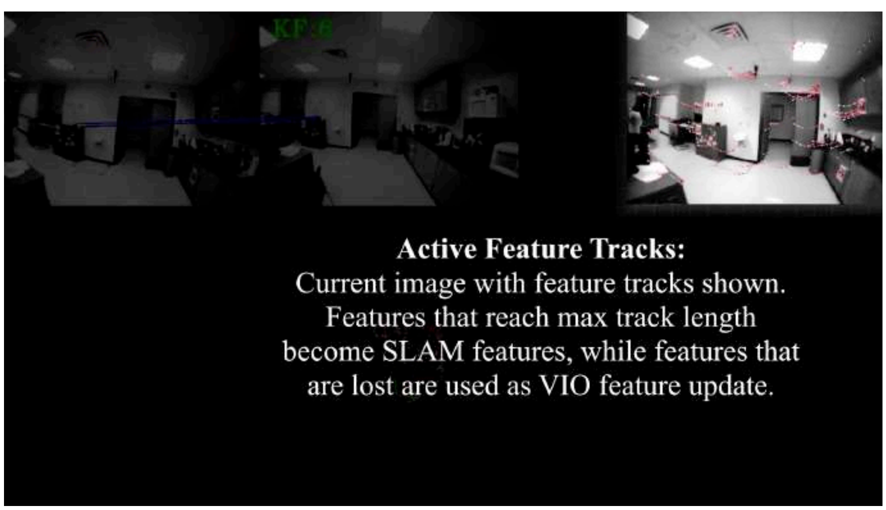
Schmidt-EKF VI-SLAM [CVPR 19]



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Multi-Camera VINS [ICRA 19a]

Multi-Camera Visual-Inertial Navigation with Online Intrinsic and Extrinsic Calibration

Kevin Eckenhoff, Patrick Geneva, Jesse Bloecker, and Guoquan Huang

RPNG, University of Delaware, USA

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Multi-IMU VINS [ICRA 19b]

Sensor-Failure-Resilient Multi-IMU Visual-Inertial Navigation

Kevin Eckenhoff, Patrick Geneva, and Guoquan Huang

RPNG, University of Delaware, USA

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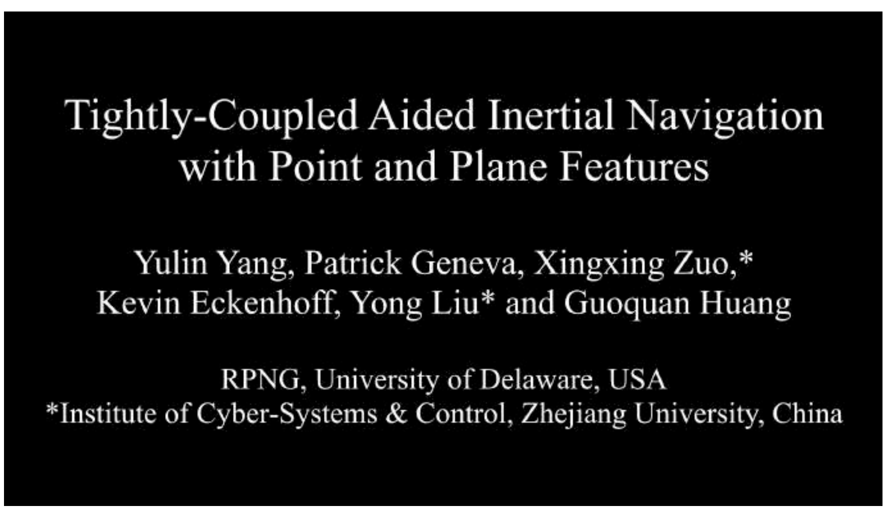
Observability Analysis and Representations [ICRA 19f; TRO 19]

Model #	Point	Error states	Line	Error states	Plane	Error states
1: General Form	f_1, f_2, f_3, f_4	not minimal	$\mathbf{n}_l, \mathbf{v}_l$	$\delta\boldsymbol{\theta}_l, \delta\phi_l$	$\pi_1, \pi_2, \pi_3, \pi_4$	not minimal
2: Geometric Form	$\mathbf{b}_f, r_f$	not minimal	$\mathbf{n}_e = \frac{\mathbf{n}_l}{\|\mathbf{n}_l\|}$ $\mathbf{v}_e = \frac{\mathbf{v}_l}{\|\mathbf{v}_l\|}$ $d_l = \frac{\|\mathbf{n}_l\|}{\|\mathbf{v}_l\|}$	not minimal	$\mathbf{n}_\pi, d_\pi$	not minimal
3: Spherical Form	θ_f, ϕ_f, r_f	$\tilde{\theta}_f, \tilde{\phi}_f, \tilde{r}_f$	$\theta_l, \phi_l, \alpha_l, d_l$	$\tilde{\theta}_l, \tilde{\phi}_l, \tilde{\alpha}_l, \tilde{d}_l$	$\theta_\pi, \phi_\pi, d_\pi$	$\tilde{\theta}_\pi, \tilde{\phi}_\pi, \tilde{d}_\pi$
4: Inverse Depth	$\theta_f, \phi_f, \lambda_f = \frac{1}{r_f}$	$\tilde{\theta}_f, \tilde{\phi}_f, \tilde{\lambda}_f$	$\theta_l, \phi_l, \alpha_l, \lambda_l = \frac{1}{d_l}$	$\tilde{\theta}_l, \tilde{\phi}_l, \tilde{\alpha}_l, \tilde{\lambda}_l$	$\theta_\pi, \phi_\pi, \lambda_\pi = \frac{1}{d_\pi}$	$\tilde{\theta}_\pi, \tilde{\phi}_\pi, \tilde{\lambda}_\pi$
5: *Quaternion*	$\bar{q}_f = \frac{1}{\sqrt{1+r_f^2}} \begin{bmatrix} \mathbf{b}_f \\ r_f \end{bmatrix}$	$\delta\boldsymbol{\theta}_f$	$\bar{q}_l, d_l$	$\delta\boldsymbol{\theta}_l, \tilde{d}_l$	$\bar{q}_\pi = \frac{1}{\sqrt{1+d_\pi^2}} \begin{bmatrix} \mathbf{n}_\pi \\ d_\pi \end{bmatrix}$	$\delta\boldsymbol{\theta}_\pi$
6: *Closest Point*	$\mathbf{p}_f = r_f \mathbf{b}_f$	$\mathbf{p}_f = \hat{\mathbf{p}}_f + \tilde{\mathbf{p}}_f$	$\mathbf{p}_l = d_l \bar{q}_l$	$\mathbf{p}_l = \hat{\mathbf{p}}_l + \tilde{\mathbf{p}}_l$	$\mathbf{p}_\pi = d_\pi \mathbf{n}_\pi$	$\mathbf{p}_\pi = \hat{\mathbf{p}}_\pi + \tilde{\mathbf{p}}_\pi$

- Unified representations for **points, lines and planes**: (i) quaternion, and (ii) closest point
- Aided INS with combination of all the geometrical features has 4 unobservable directions: global position and global yaw

Features	Unobservable Directions
Single or multiple points	4
Non-parallel lines	
Planes with non-parallel intersections	
Point and line	
Point and plane	
Single line non-parallel to planes	
Plane intersections non-parallel to lines	
Point, line and plane	
Single line	5
Single line parallel to single plane	
Two non-parallel planes	
Single plane	7

VINS w/ Point and Plane Features [ICRA 19e]



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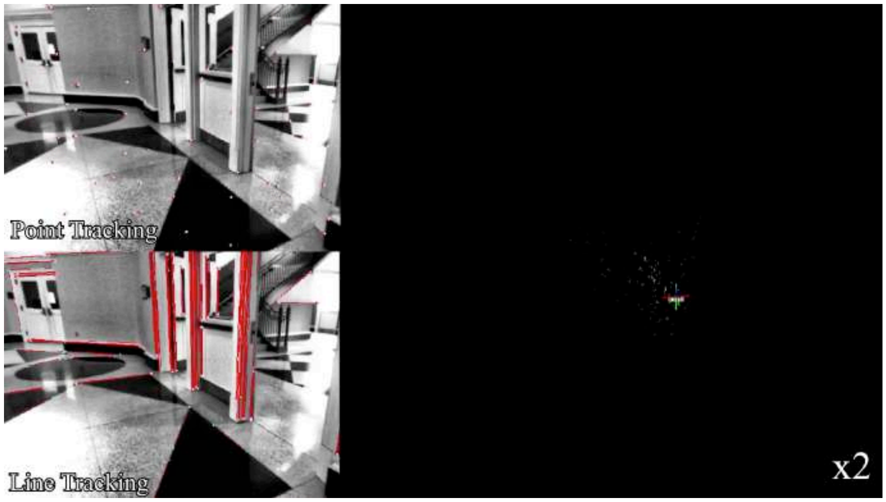
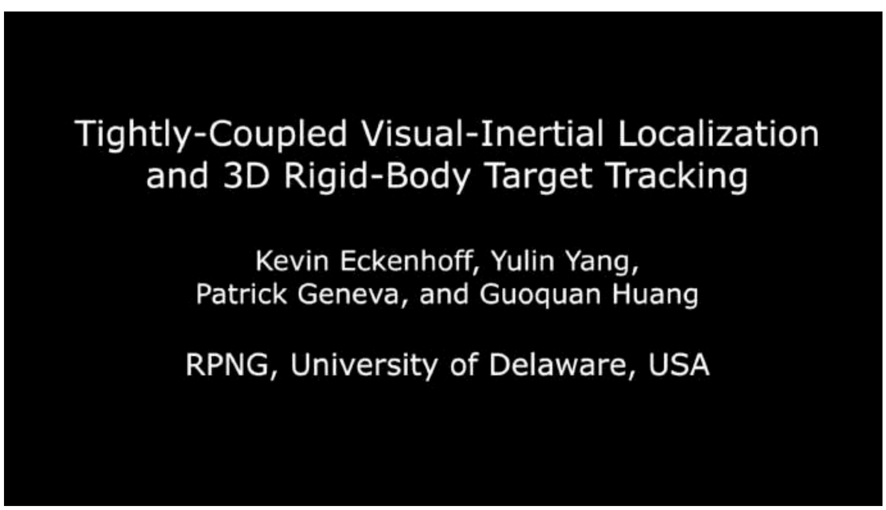
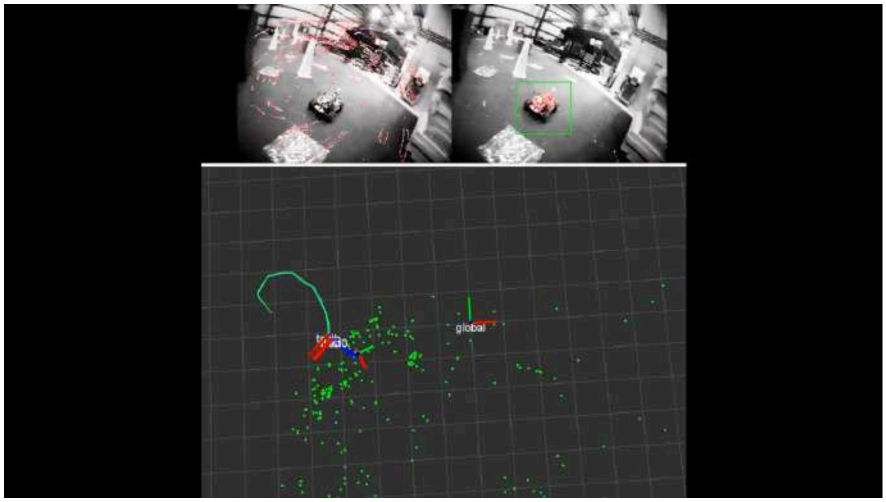
VIO w/ Point and Line Features [IROS 19b]



Visual-Inertial Localization & Moving Object Tracking [RAL 19a]





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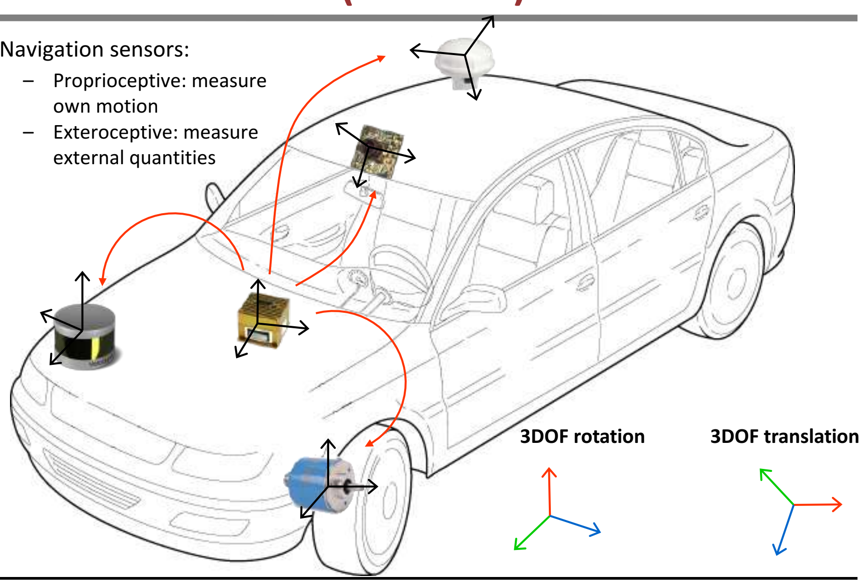
Sensor Calibration (Extrinsic)

- Navigation sensors:
  - Proprioceptive: measure own motion
  - Exteroceptive: measure external quantities



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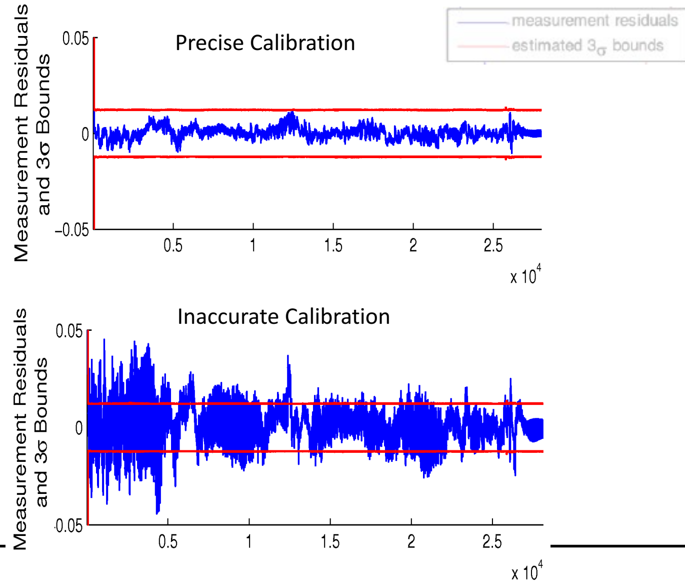
Significance of Sensor Calibration



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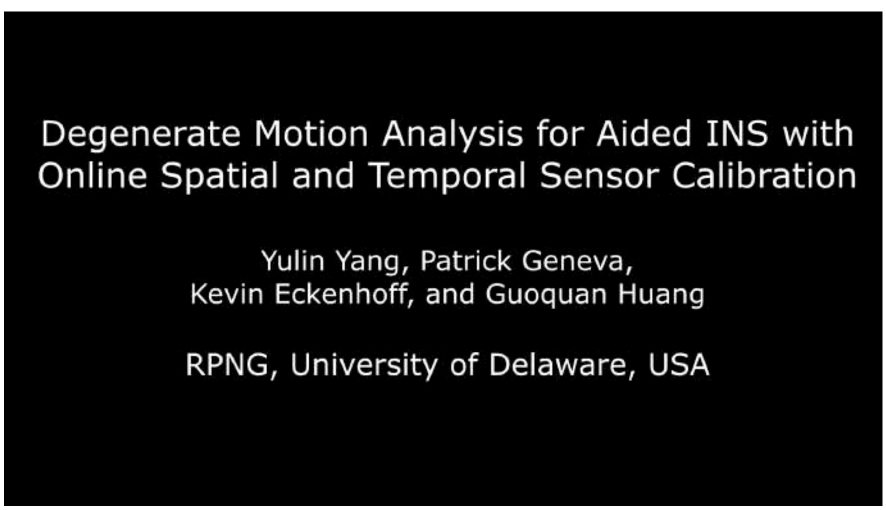
Degenerate Motions for Sensor Calibration [RAL 19b]



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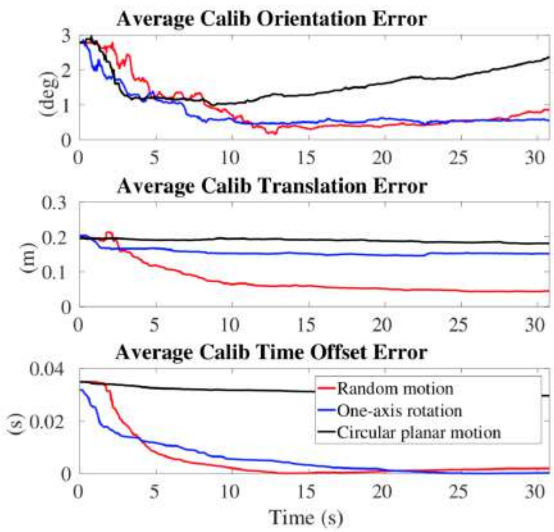
Degenerate Motions for Sensor Calibration [RAL 19b]



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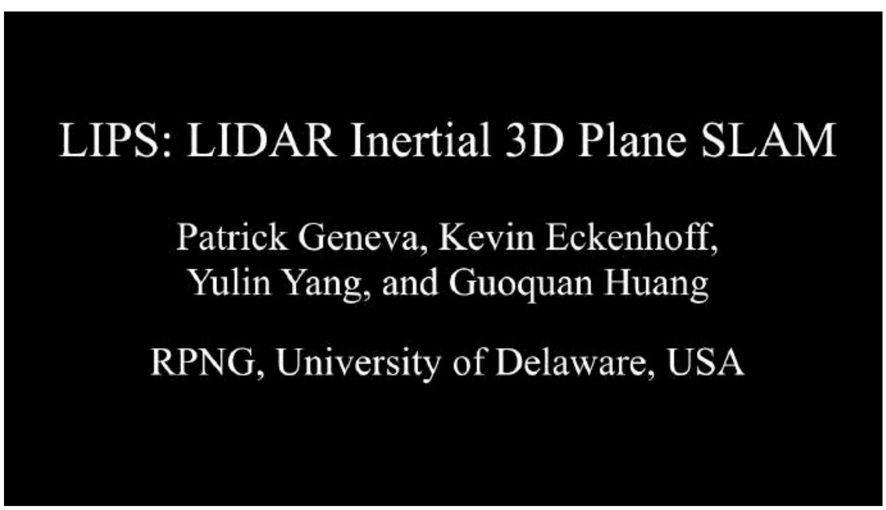
LiDAR-Inertial Plane SLAM [IROS 18b]



Open source: https://github.com/rpng/lips

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VINS w/ Prior LiDAR Map [RAL 19c]

Visual-Inertial Localization with
Prior LiDAR Map Constraints

Xingxing Zuo*, Patrick Geneva, Yulin Yang,
Wenlong Ye*, Yong Liu* and Guoquan Huang

RPNG, University of Delaware, USA
*Institute of Cyber-Systems & Control, Zhejiang University, China

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LiDAR-Inertial-Visual Odometry [IROS 19c]

LIC-Fusion: LiDAR-Inertial-Camera Odometry

Xingxing Zuo*, Patrick Geneva, Woosik Lee, Yong Liu*, Guoquan Huang

RPNG, University of Delaware, USA
*Institute of Cyber System and Control, Zhejiang University, China

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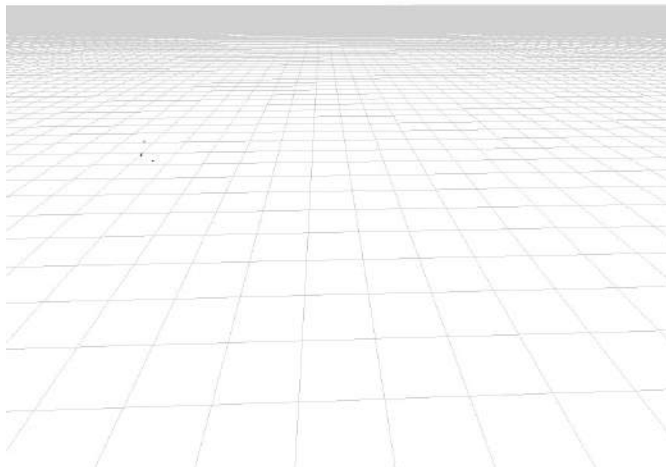
Underwater SLAM

Bluefin HAUV



[Joint work with Kaess & Leonard]

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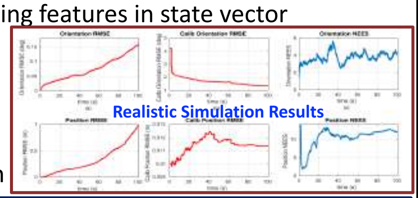
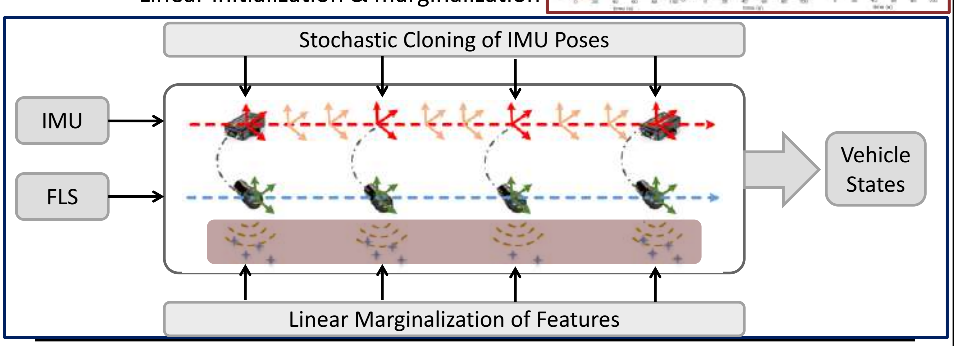
Acoustic-Inertial Navigation System (AINS) [ICRA 17b]

- **MSCKF-based acoustic-inertial odometry:**
  - Utilize all measurement info w/o having features in state vector
- **Key ideas:**
  - Stochastic cloning
  - EKF update of motion-only meas.
  - Online calibration
  - Linear initialization & marginalization





[Johannsson @ Teledyne shared BlueView MB2250 data]

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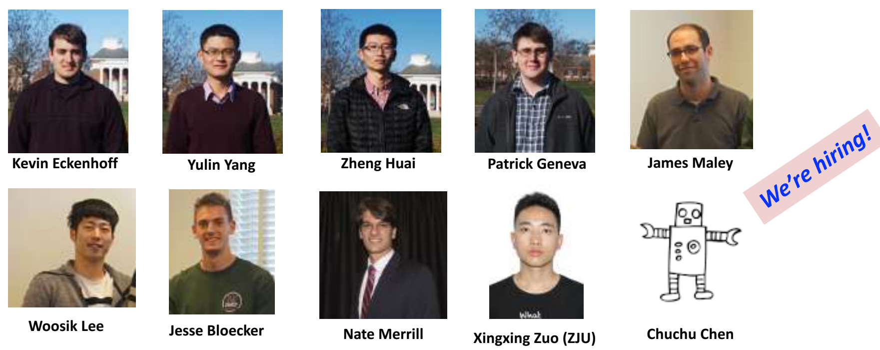
Summary

- State estimation is an enabling technology for autonomous navigation:
  - Prior/current research: state estimation for SLAM/VINS
  - Future research: distributed estimation and perception
- My lab: Robot Perception and Navigation Group (RPNG)
  http://sites.udel.edu/robot https://github.com/rpng



- Funded by NSF, DTRA, ARL, NASA, Huawei, Google, JRE/MIT, etc.

Thank you!

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