УРАВНЕНИЯ МАТЕМАТИЧЕСКОЙ ФИЗИКИ

Практическое задание № 1

Решение эллиптических краевых задач методом конечных разностей

Факультет: ПМИ   Преподаватели:
                                                      Задорожный А. Г.,
                                                      Патрушев И.И.

Группа: ПМ-81

Студенты: Ефремов А.А.,
               Ртищева К. С.

Бригада: 1

Вариант: 5

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1. Цель работы

Разработать программу решения эллиптической краевой задачи методом конечных разностей. Протестировать программу и численно оценить порядок аппроксимации.

2. Задание

Область может иметь любую форму. Предусмотреть учет первых и вторых краевых условий.

3. Анализ задачи

Эллиптическая краевая задача для функции u определяется дифференциальным уравнением

$$-\lambda(\Delta u) + \gamma u = f, \qquad \Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2},$$

Заданным в двумерной области Ω с границей S, и краевым условием

$$u|_{S_1} = u_g.$$

Для двумерного оператора Лапласа Δu дискретный аналог на неравномерной прямоугольной сетке может быть представлен пятиточечным разностным выражением

$$\Delta_h u_{i,j} = \frac{2u_{i-1,j}}{h_{i-1}^x(h_i^x + h_{i-1}^x)} + \frac{2u_{i,j-1}}{h_{j-1}^y\left(h_j^y + h_{j-1}^y\right)} + \frac{2u_{i+1,j}}{h_i^x(h_i^x + h_{i-1}^x)} +$$

$$+ \frac{2u_{i,j+1}}{h_j^y\left(h_j^y + h_{j-1}^y\right)} - \left(\frac{2}{h_{i-1}^x h_i^x} + \frac{2}{h_{j-1}^y h_j^y}\right)u_{i,j}.$$

Подставим данное разностное выражение в дифференциальное уравнение и получим:

$$-\frac{2\lambda u_{i-1,j}}{h_{i-1}^x(h_i^x + h_{i-1}^x)} - \frac{2\lambda u_{i,j-1}}{h_{j-1}^y\left(h_j^y + h_{j-1}^y\right)} - \frac{2\lambda u_{i+1,j}}{h_i^x(h_i^x + h_{i-1}^x)} -$$

$$-\frac{2\lambda u_{i,j+1}}{h_j^y\left(h_j^y + h_{j-1}^y\right)} + \left(\frac{2}{h_{i-1}^x h_i^x} + \frac{2}{h_{j-1}^y h_j^y}\right)\lambda u_{i,j} + \gamma u_{i,j} = f_{i,j}.$$

Учет первых краевых условий:

в матрице СЛАУ в $i-$й строке на место диагонального элемента ставится единица, все остальные элементы этой строки матрицы обнуляются, а $i-$й компоненте вектора правой части присваивается значение $u_g(x_i, y_i)$.

4. Структура входных данных

Первое число n в файле со входными данными - количество прямоугольных подобластей (регионов). Далее идет n наборов строк, описывающих каждый регион. В первой строке набора задаются 4 числа - координаты левой, правой, нижней и верхней границы региона. Во второй строке идут две пары чисел q и n - коэффициент разрядки и число шагов для дробления сетки по оси x и y. В третьей строке идут 4 числа m для описания условий для левой, правой, нижней и верхней границ расчетной области. Число m может быть 1 - тогда на границе будет 1 краевое условие, или номером региона, с которым граничит данное ребро, взятым с обратным знаком.

5. Текст программы

Файл "SLAE.h"

```cpp
#pragma once
#include <vector>

using namespace std;

class SLAE
{
public:
    vector<vector<double>> matrix; // Матрица системы
    vector<vector<int>> index;     // Индексы столбцов
    vector<double> f;              // Вектор правой части

    const int D = 5;              // Количество диагоналей матрицы
    int N = 0;                    // Размерность матрицы

    vector<double> xk, xk1;  // Вспомогательные векторы

    SLAE(const int& t_n)
    {
        N = t_n;

        index.resize(D);

        for(int d = 0; d < D; d++)
            index[d].resize(N);

        matrix.resize(D);

        for(int d = 0; d < D; d++)
            matrix[d].resize(N);

        xk.resize(N);
        xk1.resize(N);
        f.resize(N);
    }

    // Умножение матрицы системы на вектор vec,
    // результат в res
    void Multiplication(vector<double>& vec, vector<double>& res)
    {
        int n = vec.size(), k = 0;
        for(int j = 0; j < n; j++)
        {
            for(int d = 0; d < D; d++)
            {
                k = index[d][j];
                if(k < 0 && j + k > 0 ||
                   k > 0 && j + k < n)
                    res[j] += matrix[d][j] * vec[k + j];
            }
        }
    }

    // Норма вектора
    double Norm(const vector<double>& vec)
    {
        double res = 0;
        for(int i = 0; i < N; i++)
            res += vec[i] * vec[i];
```

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```cpp
        return sqrt(res);
    }

    // Получение относительной невязки системы
    double RelativeResidual(vector<double>& vec)
    {
        vector<double> mult(N);

        Multiplication(vec, mult);
        for(size_t i = 0; i < N; i++)
            mult[i] = f[i] - mult[i];

        return Norm(mult) / Norm(f);
    }

    // Итерационный процесс метода Гаусса-Зейделя
    void IterativeProcess(const int& j, double& sum)
    {
        int k = 0, n = xk.size();
        for(int i = 0; i < D; i++)
        {
            k = index[i][j];
            if(k + j >= 0 && k + j < n)
            {
                if(i < 3) // нижний треугольник
                    sum += matrix[i][j] * xk1[k + j];
                else // верхний треугольник
                    sum += matrix[i][j] * xk[k + j];
            }
        }
    }

    // Решение системы методом Гаусса-Зейделя
    void GaussSeidel(const int& max_iter, const double& eps,
        const double& relax)
    {
        double residual = 0.0, sum = 0.0;
        residual = RelativeResidual(xk);
        for(int k = 0; k < max_iter && residual > eps; k++)
        {
            for(int j = 0; j < N; j++)
            {
                IterativeProcess(j, sum);
                xk1[j] = xk[j] + (relax / matrix[2][j]) * (f[j] - sum);
                sum = 0.;
            }
            xk.swap(xk1);
            residual = RelativeResidual(xk);
        }
    }
};
```

Файл "Region.h"

```cpp
#pragma once
#include <vector>

using namespace std;

struct Region
{
    double left, right, top, bot;        // Границы областей

    vector<double> x_node;               // Координаты узлов по X
    vector<double> y_node;               // Координаты узлов по Y

    int n_nodes;                          // Количество узлов

    int n_x;                             // Количество узлов по X
    int n_y;                             // Количество узлов по Y

    int first, last;                     // Индексы первого и последнего
                                         // узлов в глобальной нумерации


    // Массив с информацией о краевых условиях региона
    // 0 - нижнее
    // 1 - правое
    // 2 - верхнее
    // 3 - левое
    vector<int> borders;
};
```

Файл "EllipticalProblem.h"

```cpp
#pragma once
#include <vector>
#include <fstream>
#include <string>
#include <iomanip>
#include "SLAE.h"
#include "Test.h"
#include "Region.h"

using namespace std;

class EllipticalProblem
{
public:

    vector<Region> regions;             // Регионы расчетной области

    int n_regions = 0;                  // Количество регионов
    int n_nodes = 0;                    // Общее количество узлов

    vector<vector<int>> borders;        // Информация о граничных условиях

    SLAE* slae;                         // Система
    Test test;                          // Тестовая информация

    EllipticalProblem()
    {

    }
```

```cpp
~EllipticalProblem()
{
    delete slae;
}

// Функция считывания областей из файла FILE_NAME
// и формирования сетки
void ReadFormGrid(const string& FILE_NAME)
{
    ifstream fin(FILE_NAME);

    fin >> n_regions;
    string s;

    regions.resize(n_regions);

    for(int reg_i = 0; reg_i < n_regions; reg_i++)
    {
        fin >> s;
        Region* r = &regions[reg_i];

        // Считывание границы области
        fin >> r->left;
        fin >> r->right;
        fin >> r->bot;
        fin >> r->top;

        // Генерация координат узлов по X
        int n;
        double h, q;

        fin >> q >> n;

        r->n_x = n + 1;
        r->x_node.resize(r->n_x);

        h = r->right - r->left;

        if(q != 1)
            h *= (1 - q) / (1 - pow(q, n));
        else
            h /= n;

        r->x_node[0] = r->left;

        for(int i = 0; i < n; i++)
            r->x_node[i + 1] = r->x_node[i] + h * pow(q, i);

        // Генерация координат узлов по Y

        fin >> q >> n;

        r->n_y = n + 1;
        r->y_node.resize(r->n_y);

        h = r->top - r->bot;

        if(q != 1)
            h *= (1 - q) / (1 - pow(q, n));
        else
            h /= n;

        r->y_node[0] = r->bot;
```

```cpp
        for(int i = 0; i < n; i++)
            r->y_node[i + 1] = r->y_node[i] + h * pow(q, i);

        if(reg_i != 0)
            r->first = regions[reg_i - 1].last + 1;
        else
            r->first = 0;

        r->n_nodes = r->n_x * r->n_y;

        r->last = r->first + r->n_nodes - 1;

        n_nodes += r->n_nodes;

        r->borders.resize(4);
        // Считывание информации о краевых условиях
        for(int bord_i = 0; bord_i < 4; bord_i++)
            fin >> r->borders[bord_i];
    }
    fin.close();
}

// Формирование матрицы системы
void FormMatrix()
{
    // Проход по всем регионам
    for(int reg_i = 0; reg_i < n_regions; reg_i++)
    {
        Region* r = &regions[reg_i];

        // Проход по всем узлам региона
        for(int node_i = 0; node_i < r->n_nodes; node_i++)
        {
            // Индекс узла в глобальной нумерации
            int global_i = node_i + r->first;

            // Индексы центрального узла
            int x_cent = node_i % r->n_x;
            int y_cent = floor(node_i / r->n_x);

            // Обработка некраевых узлов
            if(0 < x_cent && x_cent < r->n_x - 1 &&
               0 < y_cent && y_cent < r->n_y - 1)
            {
                // Приросты по X
                double hi = r->x_node[x_cent + 1] - r->x_node[x_cent + 0];
                double hi1 = r->x_node[x_cent - 0] - r->x_node[x_cent - 1];

                // Приросты по Y
                double hj = r->y_node[y_cent + 1] - r->y_node[y_cent + 0];
                double hj1 = r->y_node[y_cent - 0] - r->y_node[y_cent - 1];

                // Нижний узел
                slae->matrix[0][global_i] = -test.lambda() *
                    (2.0 / (hj1 * (hj + hj1)));

                // Левый узел
                slae->matrix[1][global_i] = -test.lambda() *
                    (2.0 / (hi1 * (hi + hi1)));

                // Центральный узел
                slae->matrix[2][global_i] = +test.lambda() *
                    (2.0 / (hi1 * hi) + 2.0 / (hj1 * hj)) + test.gamma();
```

```cpp
        // Правый узел
        slae->matrix[3][global_i] = -test.lambda() *
            (2.0 / (hi * (hi + hi1)));

        // Верхний узел
        slae->matrix[4][global_i] = -test.lambda() *
            (2.0 / (hj * (hj + hj1)));

        // Вектор правой части
        slae->f[global_i] = test.f(r->x_node[x_cent], r->y_node[y_cent]);
    }
    // Обработка краевых узлов
    else
    {
        int border_x = 0, border_y = 0;

        if(x_cent == 0)
            border_x = r->borders[0];
        else if(x_cent == r->n_x - 1)
            border_x = r->borders[1];

        if(y_cent == 0)
            border_y = r->borders[2];
        else if(y_cent == r->n_y - 1)
            border_y = r->borders[3];

        // Если узел на границе между соседями
        if(border_x != 1 && border_y != 1 ||
            border_x != 1 && border_y == 0 ||
            border_x == 0 && border_y  != 1)
        {
            double hi = 0, hi1 = 0, hj = 0, hj1 = 0;
            int neib_x = 0;
            int neib_y = 0;

            int neib_left, neib_right, neib_bot, neib_top;

            // Если есть сосед по X
            if(border_x != 0)
            {
                neib_x = -border_x - 1;

                // Сосед слева
                if(x_cent == 0)
                {
                    neib_left = regions[neib_x].n_x * (y_cent + 1) - 2;
                    slae->index[1][global_i] = -abs(global_i -
                        (regions[neib_x].first + neib_left));

                    hi = r->x_node[x_cent + 1] - r->x_node[x_cent + 0];
                    hi1 = r->x_node[x_cent - 0] -
                        regions[neib_x].x_node[regions[neib_x].n_x - 2];
                }
                // Сосед справа
                if(x_cent == r->n_x - 1)
                {
                    neib_right = regions[neib_x].n_x * y_cent + 1;
                    slae->index[3][global_i] = abs(global_i -
                        (regions[neib_x].first + neib_right));

                    hi = regions[neib_x].x_node[1] - r->x_node[x_cent + 0];
                    hi1 = r->x_node[x_cent - 0] - r->x_node[x_cent - 1];
                }
```

```cpp
        if(border_y == 0)
        {
            hj = r->y_node[y_cent + 1] - r->y_node[y_cent];
            hj1 = r->y_node[y_cent] - r->y_node[y_cent - 1];
        }
    }

    // Если есть сосед по Y
    if(border_y != 0)
    {
        neib_y = -border_y - 1;

        // Сосед снизу
        if(y_cent == 0)
        {
            neib_bot = regions[neib_y].n_x * (regions[neib_y].n_y - 2) +
                x_cent;
            slae->index[0][global_i] = -abs(global_i -
                (regions[neib_y].first + neib_bot));

            hj = r->y_node[y_cent + 1] - r->y_node[y_cent + 0];
            hj1 = r->y_node[y_cent - 0] -
                regions[neib_y].y_node[regions[neib_y].n_y - 2];
        }

        // Сосед сверху
        if(y_cent == r->n_y - 1)
        {
            neib_top = regions[neib_y].n_x + x_cent;
            slae->index[4][global_i] = abs(global_i -
                (regions[neib_y].first + neib_top));

            hj = regions[neib_y].y_node[1] - r->y_node[y_cent + 0];
            hj1 = r->y_node[y_cent - 0] - r->y_node[y_cent - 1];
        }

        if(border_x == 0)
        {
            hi = r->x_node[x_cent + 1] - r->x_node[x_cent + 0];
            hi1 = r->x_node[x_cent - 0] - r->x_node[x_cent - 1];
        }
    }

    // Нижний узел
slae->matrix[0][global_i] = -test.lambda() *
    (2.0 / (hj1 * (hj + hj1)));

// Левый узел
slae->matrix[1][global_i] = -test.lambda() *
    (2.0 / (hi1 * (hi + hi1)));

// Центральный узел
slae->matrix[2][global_i] = +test.lambda() *
    (2.0 / (hi1 * hi) + 2.0 / (hj1 * hj)) + test.gamma();

// Правый узел
slae->matrix[3][global_i] = -test.lambda() *
    (2.0 / (hi * (hi + hi1)));

// Верхний узел
slae->matrix[4][global_i] = -test.lambda() *
    (2.0 / (hj * (hj + hj1)));
```

```c
                // Вектор правой части
                slae->f[global_i] = test.f(r->x_node[x_cent], r->y_node[y_cent]);
            }
        }
    }
}
// Обработка первого кревого условия
    // Проход по всем регионам
    for(int reg_i = 0; reg_i < n_regions; reg_i++)
    {
        Region* r = &regions[reg_i];

        // Проход по всем узлам региона
        for(int node_i = 0; node_i < r->n_nodes; node_i++)
        {
            // Индекс узла в глобальной нумерации
            int global_i = node_i + r->first;

            // Индексы центрального узла
            int x_cent = node_i % r->n_x;
            int y_cent = floor(node_i / r->n_x);

            // Обработка некраевых узлов
            if(x_cent == 0 || x_cent == r->n_x - 1 ||
               0 == y_cent || y_cent == r->n_y - 1)
            {
                int border_x = 0, border_y = 0;

                if(x_cent == 0)
                    border_x = r->borders[0];
                else if(x_cent == r->n_x - 1)
                    border_x = r->borders[1];

                if(y_cent == 0)
                    border_y = r->borders[2];
                else if(y_cent == r->n_y - 1)
                    border_y = r->borders[3];

                // Первое краевое
                if(border_x == 1 || border_y == 1)
                {
                    slae->matrix[0][global_i] = 0;
                    slae->matrix[1][global_i] = 0;
                    slae->matrix[2][global_i] = 1.0;
                    slae->matrix[3][global_i] = 0;
                    slae->matrix[4][global_i] = 0;
                    slae->f[global_i] = test.u(r->x_node[x_cent], r->y_node[y_cent]);

                    slae->index[0][global_i] = -r->n_x;
                    slae->index[1][global_i] = -1;
                    slae->index[2][global_i] = 0;
                    slae->index[3][global_i] = 1;
                    slae->index[4][global_i] = r->n_x;
                }
            }
        }
    }
}
```

```cpp
void PrintSolution(const string& file_name)
{
    ofstream fout(file_name);
    double norm = 0.0, norm_u = 0.0;

    fout << " y            x              calc            prec";
    fout << "      dif              N   reg  location" << endl << fixed;

    // Проход по всем регионам
    for(int reg_i = 0; reg_i < n_regions; reg_i++)
    {
        Region* r = &regions[reg_i];

        // Проход по всем узлам региона
        for(int node_i = 0; node_i < r->n_nodes; node_i++)
        {
            // Индекс узла в глобальной нумерации
            int global_i = node_i + r->first;

            // Индексы центрального узла
            int x_cent = node_i % r->n_x;
            int y_cent = floor(node_i / r->n_x);

            fout << setw(9) << r->y_node[y_cent];
            fout << setw(11) << r->x_node[x_cent];

            double calc = slae->xk[global_i];
            fout << setw(15) << calc;
            double prec = test.u(r->x_node[x_cent], r->y_node[y_cent]);
            fout << setw(15) << prec;

            fout << setw(14) << scientific << abs(prec - calc);
            fout << fixed << setw(5) << global_i << setw(4) << reg_i + 1;

            // Обработка некраевых узлов
            if(0 < x_cent && x_cent < r->n_x - 1 &&
               0 < y_cent && y_cent < r->n_y - 1)
                fout << "  inner";
            else
            {
                int border_x = 0, border_y = 0;

                if(x_cent == 0)
                    border_x = r->borders[0];
                else if(x_cent == r->n_x - 1)
                    border_x = r->borders[1];

                if(y_cent == 0)
                    border_y = r->borders[2];
                else if(y_cent == r->n_y - 1)
                    border_y = r->borders[3];

                // Первое краевое
                if(border_x == 1 || border_y == 1)
                    fout << "  border";
                else
                    if(border_x != 1 && border_y != 1 ||
                       border_x != 1 && border_y == 0 ||
                       border_x == 0 && border_y != 1)
                        fout << "  inner border";
            }
            fout << endl;
            norm_u += prec * prec;
            norm += abs(calc - prec) * abs(calc - prec);
```

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```cpp
            }
        }
        fout << "||u-u*||/||u*|| = " << scientific << sqrt(norm) / sqrt(norm_u) << endl;
        fout << "||u-u*|| = " << scientific << sqrt(norm);
        fout.close();
    }
};
```

Файл "Test.h"

```cpp
#pragma once
using namespace std;

class Test
{
public:

    int N;

    Test(const int& t_N) : N(t_N) {};

    Test() : N(0) {};

    double f(const double& x, const double& y)
    {
        switch(N)
        {
        case(0): return (0)* lambda() + u(x, y) * gamma();
        case(1): return (0)* lambda() + u(x, y) * gamma();
        case(2): return (-4)* lambda() + u(x, y) * gamma();
        case(3): return (-6 * x - 6 * y) * lambda() + u(x, y) * gamma();
        case(4): return (-12 * x * x - 12 * y * y) * lambda() + u(x, y) * gamma();
        };
    }

    double lambda()
    {
        return 1;
    }

    double gamma()
    {
        return 1;
    }

    double u(const double& x, const double& y)
    {
        switch(N)
        {
        case(0): return 2.0;
        case(1): return x + y;
        case(2): return x * x + y * y;
        case(3): return x * x * x + y * y * y;
        case(4): return x * x * x * x + y * y * y * y;
        };
    }

};
```

Файл "Main.cpp"

```cpp
#include <iostream>
#include "EllipticalProblem.h"

using namespace std;

void main()
{
    EllipticalProblem ep = EllipticalProblem();

    ep.ReadFormGrid("regions.txt");

    // Инициализация СЛАУ
    ep.slae = new SLAE(ep.n_nodes);

    for(int reg_i = 0; reg_i < ep.n_regions; reg_i++)
    {
        Region* r = &ep.regions[reg_i];

        for(int node_i = 0; node_i < r->n_nodes; node_i++)
        {
            int global_i = node_i + r->first;

            ep.slae->index[0][global_i] = -r->n_x;
            ep.slae->index[1][global_i] = -1;
            ep.slae->index[2][global_i] = 0;
            ep.slae->index[3][global_i] = 1;
            ep.slae->index[4][global_i] = r->n_x;
        }
    }

    // Инициализация тестовых данных
    ep.test = Test(3);

    // Формирование матрицы системы
    ep.FormMatrix();

    ep.slae->GaussSeidel(10000, 1e-14, 0.65);

    ep.PrintSolution("res.txt");

}
```

6. Тестирование на равномерной сетке

Область имеет L-образную форму

- $\lambda = 1, \gamma = 1, u^* = x^3 + y^3, f = -6(x + y) + x^3 + y^3$

y	x	calc	prec	dif	N	reg	location
1.000000	1.000000	2.000000	2.000000	0.000000e+00	0	1	border
1.000000	1.500000	4.375000	4.375000	8.881784e-16	1	1	border
1.000000	2.000000	9.000000	9.000000	1.776357e-15	2	1	border
1.500000	1.000000	4.375000	4.375000	8.881784e-16	3	1	border
1.500000	1.500000	6.750000	6.750000	8.881784e-16	4	1	inner
1.500000	2.000000	11.375000	11.375000	1.776357e-15	5	1	inner border
2.000000	1.000000	9.000000	9.000000	1.776357e-15	6	1	border
2.000000	1.500000	11.375000	11.375000	1.776357e-15	7	1	inner border
2.000000	2.000000	16.000000	16.000000	3.552714e-15	8	1	inner border
1.000000	2.000000	9.000000	9.000000	1.776357e-15	9	2	border
1.000000	2.500000	16.625000	16.625000	3.552714e-15	10	2	border
1.000000	3.000000	28.000000	28.000000	0.000000e+00	11	2	border
1.500000	2.000000	11.375000	11.375000	0.000000e+00	12	2	inner border
1.500000	2.500000	19.000000	19.000000	3.552714e-15	13	2	inner
1.500000	3.000000	30.375000	30.375000	0.000000e+00	14	2	border
2.000000	2.000000	16.000000	16.000000	0.000000e+00	15	2	border
2.000000	2.500000	23.625000	23.625000	0.000000e+00	16	2	border
2.000000	3.000000	35.000000	35.000000	7.105427e-15	17	2	border
2.000000	1.000000	9.000000	9.000000	1.776357e-15	18	3	border
2.000000	1.500000	11.375000	11.375000	0.000000e+00	19	3	inner border
2.000000	2.000000	16.000000	16.000000	0.000000e+00	20	3	border
2.500000	1.000000	16.625000	16.625000	3.552714e-15	21	3	border
2.500000	1.500000	19.000000	19.000000	3.552714e-15	22	3	inner
2.500000	2.000000	23.625000	23.625000	0.000000e+00	23	3	border
3.000000	1.000000	28.000000	28.000000	0.000000e+00	24	3	border
3.000000	1.500000	30.375000	30.375000	0.000000e+00	25	3	border
3.000000	2.000000	35.000000	35.000000	7.105427e-15	26	3	border

$||u-u^*||/||u^*|| = 1.364848e-16$

$||u-u^*|| = 1.361552e-14$

- $\lambda = 1, \gamma = 1, u^* = x^4 + y^4, f = -12(x^2 + y^2) + x^4 + y^4$

y	x	calc	prec	dif	N	reg	location
1.000000	1.000000	2.000000	2.000000	0.000000e+00	0	1	border
1.000000	1.500000	6.062500	6.062500	0.000000e+00	1	1	border
1.000000	2.000000	17.000000	17.000000	3.552714e-15	2	1	border
1.500000	1.000000	6.062500	6.062500	0.000000e+00	3	1	border
1.500000	1.500000	10.248077	10.125000	1.230772e-01	4	1	inner
1.500000	2.000000	21.199039	21.062500	1.365390e-01	5	1	inner border
2.000000	1.000000	17.000000	17.000000	3.552714e-15	6	1	border
2.000000	1.500000	21.199039	21.062500	1.365390e-01	7	1	inner border
2.000000	2.000000	32.123077	32.000000	1.230772e-01	8	1	inner border
1.000000	2.000000	17.000000	17.000000	3.552714e-15	9	2	border
1.000000	2.500000	40.062500	40.062500	7.105427e-15	10	2	border
1.000000	3.000000	82.000000	82.000000	1.421085e-14	11	2	border
1.500000	2.000000	21.170080	21.062500	1.075797e-01	12	2	inner border
1.500000	2.500000	44.209136	44.125000	8.413639e-02	13	2	inner
1.500000	3.000000	86.062500	86.062500	1.421085e-14	14	2	border
2.000000	2.000000	32.000000	32.000000	0.000000e+00	15	2	border
2.000000	2.500000	55.062500	55.062500	0.000000e+00	16	2	border
2.000000	3.000000	97.000000	97.000000	0.000000e+00	17	2	border
2.000000	1.000000	17.000000	17.000000	3.552714e-15	18	3	border
2.000000	1.500000	21.170080	21.062500	1.075797e-01	19	3	inner border
2.000000	2.000000	32.000000	32.000000	0.000000e+00	20	3	border
2.500000	1.000000	40.062500	40.062500	7.105427e-15	21	3	border
2.500000	1.500000	44.209136	44.125000	8.413639e-02	22	3	inner
2.500000	2.000000	55.062500	55.062500	0.000000e+00	23	3	border
3.000000	1.000000	82.000000	82.000000	1.421085e-14	24	3	border
3.000000	1.500000	86.062500	86.062500	1.421085e-14	25	3	border
3.000000	2.000000	97.000000	97.000000	0.000000e+00	26	3	border

$||u-u^*||/||u^*|| = 1.256545e-03$

$||u-u^*|| = 3.238617e-01$

Область имеет Т-образную форму
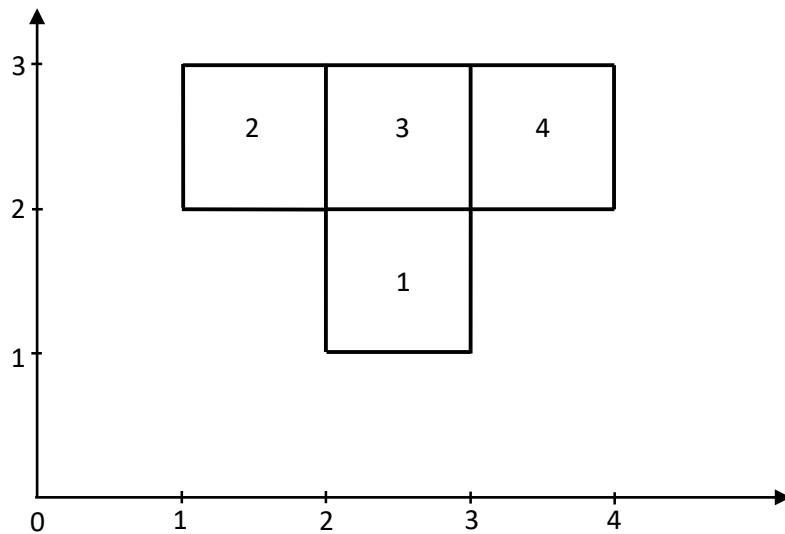
```
4
-1-
2 3 1 2
1 2 1 2
1 1 1 -3
-2-
1 2 2 3
1 2 1 2
1 -3 1 1
-3-
2 3 2 3
1 2 1 2
-2 -4 -1 1
-4-
3 4 2 3
1 2 1 2
-3 1 1 1
```

- $\lambda = 1, \gamma = 1, u^* = x^3 + y^3, f = -6(x + y) + x^3 + y^3$

y	x	calc	prec	dif	N	reg	location
1.000000	2.000000	9.000000	9.000000	1.776357e-15	0	1	border
1.000000	2.500000	16.625000	16.625000	3.552714e-15	1	1	border
1.000000	3.000000	28.000000	28.000000	0.000000e+00	2	1	border
1.500000	2.000000	11.375000	11.375000	0.000000e+00	3	1	border
1.500000	2.500000	19.000000	19.000000	0.000000e+00	4	1	inner
1.500000	3.000000	30.375000	30.375000	0.000000e+00	5	1	border
2.000000	2.000000	16.000000	16.000000	0.000000e+00	6	1	border
2.000000	2.500000	23.625000	23.625000	7.105427e-15	7	1	inner border
2.000000	3.000000	35.000000	35.000000	7.105427e-15	8	1	border
2.000000	1.000000	9.000000	9.000000	1.776357e-15	9	2	border
2.000000	1.500000	11.375000	11.375000	0.000000e+00	10	2	border
2.000000	2.000000	16.000000	16.000000	0.000000e+00	11	2	border
2.500000	1.000000	16.625000	16.625000	3.552714e-15	12	2	border
2.500000	1.500000	19.000000	19.000000	0.000000e+00	13	2	inner
2.500000	2.000000	23.625000	23.625000	7.105427e-15	14	2	inner border
3.000000	1.000000	28.000000	28.000000	0.000000e+00	15	2	border
3.000000	1.500000	30.375000	30.375000	0.000000e+00	16	2	border
3.000000	2.000000	35.000000	35.000000	7.105427e-15	17	2	border
2.000000	2.000000	16.000000	16.000000	3.552714e-15	18	3	inner border
2.000000	2.500000	23.625000	23.625000	3.552714e-15	19	3	inner border
2.000000	3.000000	35.000000	35.000000	7.105427e-15	20	3	inner border
2.500000	2.000000	23.625000	23.625000	7.105427e-15	21	3	inner border
2.500000	2.500000	31.250000	31.250000	3.552714e-15	22	3	inner
2.500000	3.000000	42.625000	42.625000	7.105427e-15	23	3	inner border
3.000000	2.000000	35.000000	35.000000	7.105427e-15	24	3	border
3.000000	2.500000	42.625000	42.625000	7.105427e-15	25	3	border
3.000000	3.000000	54.000000	54.000000	0.000000e+00	26	3	border
2.000000	3.000000	35.000000	35.000000	7.105427e-15	27	4	border
2.000000	3.500000	50.875000	50.875000	0.000000e+00	28	4	border
2.000000	4.000000	72.000000	72.000000	1.421085e-14	29	4	border
2.500000	3.000000	42.625000	42.625000	0.000000e+00	30	4	inner border
2.500000	3.500000	58.500000	58.500000	1.421085e-14	31	4	inner
2.500000	4.000000	79.625000	79.625000	1.421085e-14	32	4	border
3.000000	3.000000	54.000000	54.000000	0.000000e+00	33	4	border
3.000000	3.500000	69.875000	69.875000	1.421085e-14	34	4	border
3.000000	4.000000	91.000000	91.000000	0.000000e+00	35	4	border

||u-u*||/||u*|| = 1.554534e-16
||u-u*|| = 3.717639e-14

- $\lambda = 1, \gamma = 1, u^* = x^4 + y^4, f = -12(x^2 + y^2) + x^4 + y^4$

y	x	calc	prec	dif	N	reg	location
1.000000	2.000000	17.000000	17.000000	3.552714e-15	0	1	border
1.000000	2.500000	40.062500	40.062500	7.105427e-15	1	1	border
1.000000	3.000000	82.000000	82.000000	1.421085e-14	2	1	border
1.500000	2.000000	21.062500	21.062500	3.552714e-15	3	1	border
1.500000	2.500000	44.212148	44.125000	8.714793e-02	4	1	inner
1.500000	3.000000	86.062500	86.062500	1.421085e-14	5	1	border
2.000000	2.000000	32.000000	32.000000	0.000000e+00	6	1	border
2.000000	2.500000	55.182879	55.062500	1.203787e-01	7	1	inner border
2.000000	3.000000	97.000000	97.000000	0.000000e+00	8	1	border
2.000000	1.000000	17.000000	17.000000	3.552714e-15	9	2	border
2.000000	1.500000	21.062500	21.062500	3.552714e-15	10	2	border
2.000000	2.000000	32.000000	32.000000	0.000000e+00	11	2	border
2.500000	1.000000	40.062500	40.062500	7.105427e-15	12	2	border
2.500000	1.500000	44.212148	44.125000	8.714793e-02	13	2	inner
2.500000	2.000000	55.182879	55.062500	1.203787e-01	14	2	inner border
3.000000	1.000000	82.000000	82.000000	1.421085e-14	15	2	border
3.000000	1.500000	86.062500	86.062500	1.421085e-14	16	2	border
3.000000	2.000000	97.000000	97.000000	0.000000e+00	17	2	border
2.000000	2.000000	32.138471	32.000000	1.384710e-01	18	3	inner border
2.000000	2.500000	55.248042	55.062500	1.855415e-01	19	3	inner border
2.000000	3.000000	97.138471	97.000000	1.384710e-01	20	3	inner border
2.500000	2.000000	55.215460	55.062500	1.529601e-01	21	3	inner border
2.500000	2.500000	78.299462	78.125000	1.744616e-01	22	3	inner
2.500000	3.000000	120.215460	120.062500	1.529601e-01	23	3	inner border
3.000000	2.000000	97.000000	97.000000	0.000000e+00	24	3	border
3.000000	2.500000	120.062500	120.062500	0.000000e+00	25	3	border
3.000000	3.000000	162.000000	162.000000	2.842171e-14	26	3	border
2.000000	3.000000	97.000000	97.000000	0.000000e+00	27	4	border
2.000000	3.500000	166.062500	166.062500	2.842171e-14	28	4	border
2.000000	4.000000	272.000000	272.000000	5.684342e-14	29	4	border
2.500000	3.000000	120.182879	120.062500	1.203787e-01	30	4	inner border
2.500000	3.500000	189.212148	189.125000	8.714793e-02	31	4	inner
2.500000	4.000000	295.062500	295.062500	5.684342e-14	32	4	border
3.000000	3.000000	162.000000	162.000000	2.842171e-14	33	4	border
3.000000	3.500000	231.062500	231.062500	0.000000e+00	34	4	border
3.000000	4.000000	337.000000	337.000000	5.684342e-14	35	4	border

||u-u*||/||u*|| = 6.042730e-04
||u-u*|| = 4.650397e-01

Область имеет П-образную форму
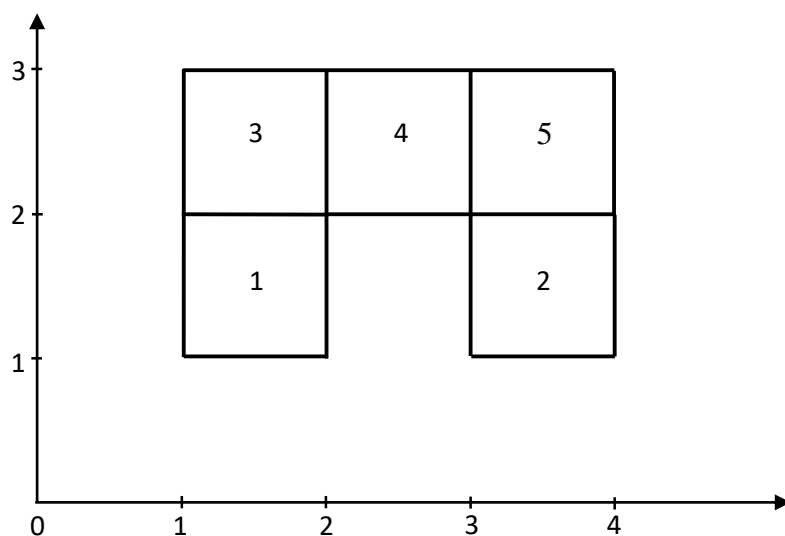
Файл regions.txt

```
5
-1-
1 2 1 2
1 2 1 2
1 1 1 -3
-2-
3 4 1 2
1 2 1 2
1 1 1 -5
-3-
1 2 2 3
1 2 1 2
1 -4 -1 1
-4-
2 3 2 3
1 2 1 2
-3 -5 1 1
-5-
3 4 2 3
1 2 1 2
-4 1 -2 1
```

- $\lambda = 1, \gamma = 1, u^* = x^3 + y^3, f = -6(x + y) + x^3 + y^3$

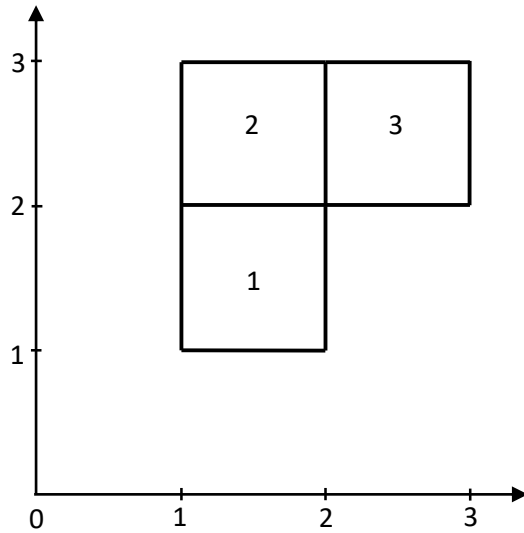| y	x	calc	prec	dif	N	reg	location
1.000000	1.000000	2.000000	2.000000	0.000000e+00	0	1	border
1.000000	1.500000	4.375000	4.375000	8.881784e-16	1	1	border
1.000000	2.000000	9.000000	9.000000	1.776357e-15	2	1	border
1.500000	1.000000	4.375000	4.375000	8.881784e-16	3	1	border
1.500000	1.500000	6.750000	6.750000	0.000000e+00	4	1	inner
1.500000	2.000000	11.375000	11.375000	0.000000e+00	5	1	border
2.000000	1.000000	9.000000	9.000000	1.776357e-15	6	1	border
2.000000	1.500000	11.375000	11.375000	0.000000e+00	7	1	inner border
2.000000	2.000000	16.000000	16.000000	0.000000e+00	8	1	border
1.000000	3.000000	28.000000	28.000000	0.000000e+00	9	2	border
1.000000	3.500000	43.875000	43.875000	7.105427e-15	10	2	border
1.000000	4.000000	65.000000	65.000000	1.421085e-14	11	2	border
1.500000	3.000000	30.375000	30.375000	0.000000e+00	12	2	border
1.500000	3.500000	46.250000	46.250000	7.105427e-15	13	2	inner
1.500000	4.000000	67.375000	67.375000	1.421085e-14	14	2	border
2.000000	3.000000	35.000000	35.000000	7.105427e-15	15	2	border
2.000000	3.500000	50.875000	50.875000	7.105427e-15	16	2	inner border
2.000000	4.000000	72.000000	72.000000	1.421085e-14	17	2	border
2.000000	1.000000	9.000000	9.000000	1.776357e-15	18	3	border
2.000000	1.500000	11.375000	11.375000	1.776357e-15	19	3	inner border
2.000000	2.000000	16.000000	16.000000	0.000000e+00	20	3	inner border
2.500000	1.000000	16.625000	16.625000	3.552714e-15	21	3	border
2.500000	1.500000	19.000000	19.000000	0.000000e+00	22	3	inner
2.500000	2.000000	23.625000	23.625000	3.552714e-15	23	3	inner border
3.000000	1.000000	28.000000	28.000000	0.000000e+00	24	3	border
3.000000	1.500000	30.375000	30.375000	0.000000e+00	25	3	border
3.000000	2.000000	35.000000	35.000000	7.105427e-15	26	3	border
2.000000	2.000000	16.000000	16.000000	0.000000e+00	27	4	border
2.000000	2.500000	23.625000	23.625000	0.000000e+00	28	4	border
2.000000	3.000000	35.000000	35.000000	7.105427e-15	29	4	border
2.500000	2.000000	23.625000	23.625000	0.000000e+00	30	4	inner border
2.500000	2.500000	31.250000	31.250000	0.000000e+00	31	4	inner
2.500000	3.000000	42.625000	42.625000	0.000000e+00	32	4	inner border
3.000000	2.000000	35.000000	35.000000	7.105427e-15	33	4	border
3.000000	2.500000	42.625000	42.625000	7.105427e-15	34	4	border
3.000000	3.000000	54.000000	54.000000	0.000000e+00	35	4	border
2.000000	3.000000	35.000000	35.000000	7.105427e-15	36	5	inner border
2.000000	3.500000	50.875000	50.875000	0.000000e+00	37	5	inner border
2.000000	4.000000	72.000000	72.000000	1.421085e-14	38	5	border
2.500000	3.000000	42.625000	42.625000	0.000000e+00	39	5	inner border
2.500000	3.500000	58.500000	58.500000	7.105427e-15	40	5	inner
2.500000	4.000000	79.625000	79.625000	1.421085e-14	41	5	border
3.000000	3.000000	54.000000	54.000000	0.000000e+00	42	5	border
3.000000	3.500000	69.875000	69.875000	1.421085e-14	43	5	border
3.000000	4.000000	91.000000	91.000000	0.000000e+00	44	5	border

||u-u*||/||u*|| = 1.511169e-16
||u-u*|| = 4.190471e-14

- $\lambda = 1, \gamma = 1, u^* = x^4 + y^4, f = -12(x^2 + y^2) + x^4 + y^4$

y	x	calc	prec	dif	N	reg	location
1.000000	1.000000	2.000000	2.000000	0.000000e+00	0	1	border
1.000000	1.500000	6.062500	6.062500	0.000000e+00	1	1	border
1.000000	2.000000	17.000000	17.000000	3.552714e-15	2	1	border
1.500000	1.000000	6.062500	6.062500	0.000000e+00	3	1	border
1.500000	1.500000	10.209257	10.125000	8.425659e-02	4	1	inner
1.500000	2.000000	21.062500	21.062500	3.552714e-15	5	1	border
2.000000	1.000000	17.000000	17.000000	3.552714e-15	6	1	border
2.000000	1.500000	21.170590	21.062500	1.080905e-01	7	1	inner border
2.000000	2.000000	32.000000	32.000000	0.000000e+00	8	1	border
1.000000	3.000000	82.000000	82.000000	1.421085e-14	9	2	border
1.000000	3.500000	151.062500	151.062500	2.842171e-14	10	2	border
1.000000	4.000000	257.000000	257.000000	5.684342e-14	11	2	border
1.500000	3.000000	86.062500	86.062500	1.421085e-14	12	2	border
1.500000	3.500000	155.209257	155.125000	8.425659e-02	13	2	inner
1.500000	4.000000	261.062500	261.062500	5.684342e-14	14	2	border
2.000000	3.000000	97.000000	97.000000	0.000000e+00	15	2	border
2.000000	3.500000	166.170590	166.062500	1.080905e-01	16	2	inner border
2.000000	4.000000	272.000000	272.000000	5.684342e-14	17	2	border
2.000000	1.000000	17.000000	17.000000	3.552714e-15	18	3	border
2.000000	1.500000	21.200032	21.062500	1.375324e-01	19	3	inner border
2.000000	2.000000	32.125128	32.000000	1.251280e-01	20	3	inner border
2.500000	1.000000	40.062500	40.062500	7.105427e-15	21	3	border
2.500000	1.500000	44.250128	44.125000	1.251280e-01	22	3	inner
2.500000	2.000000	55.206762	55.062500	1.442618e-01	23	3	inner border
3.000000	1.000000	82.000000	82.000000	1.421085e-14	24	3	border
3.000000	1.500000	86.062500	86.062500	1.421085e-14	25	3	border
3.000000	2.000000	97.000000	97.000000	0.000000e+00	26	3	border
2.000000	2.000000	32.000000	32.000000	0.000000e+00	27	4	border
2.000000	2.500000	55.062500	55.062500	0.000000e+00	28	4	border
2.000000	3.000000	97.000000	97.000000	0.000000e+00	29	4	border
2.500000	2.000000	55.177320	55.062500	1.148199e-01	30	4	inner border
2.500000	2.500000	78.237856	78.125000	1.128564e-01	31	4	inner
2.500000	3.000000	120.177320	120.062500	1.148199e-01	32	4	inner border
3.000000	2.000000	97.000000	97.000000	0.000000e+00	33	4	border
3.000000	2.500000	120.062500	120.062500	0.000000e+00	34	4	border
3.000000	3.000000	162.000000	162.000000	2.842171e-14	35	4	border
2.000000	3.000000	97.125128	97.000000	1.251280e-01	36	5	inner border
2.000000	3.500000	166.200032	166.062500	1.375324e-01	37	5	inner border
2.000000	4.000000	272.000000	272.000000	5.684342e-14	38	5	border
2.500000	3.000000	120.206762	120.062500	1.442618e-01	39	5	inner border
2.500000	3.500000	189.250128	189.125000	1.251280e-01	40	5	inner
2.500000	4.000000	295.062500	295.062500	5.684342e-14	41	5	border
3.000000	3.000000	162.000000	162.000000	2.842171e-14	42	5	border
3.000000	3.500000	231.062500	231.062500	0.000000e+00	43	5	border
3.000000	4.000000	337.000000	337.000000	5.684342e-14	44	5	border

$||u-u^*||/||u^*|| = 5.016553e-04$

$||u-u^*|| = 4.677078e-01$

Область имеет Г-образную форму

Файл regions.txt

```
3
-1-
1 2 1 2
1 2 1 2
1 1 1 -2
-2-
1 2 2 3
1 2 1 2
1 -3 -1 1
-3-
2 3 2 3
1 2 1 2
-2 1 1 1
```

- $\lambda = 1, \gamma = 1, u^* = x^3 + y^3, f = -6(x+y) + x^3 + y^3$

y	x	calc	prec	dif	N	reg	location
1.000000	1.000000	2.000000	2.000000	0.000000e+00	0	1	border
1.000000	1.500000	4.375000	4.375000	8.881784e-16	1	1	border
1.000000	2.000000	9.000000	9.000000	1.776357e-15	2	1	border
1.500000	1.000000	4.375000	4.375000	8.881784e-16	3	1	border
1.500000	1.500000	6.750000	6.750000	1.776357e-15	4	1	inner
1.500000	2.000000	11.375000	11.375000	0.000000e+00	5	1	border
2.000000	1.000000	9.000000	9.000000	1.776357e-15	6	1	border
2.000000	1.500000	11.375000	11.375000	0.000000e+00	7	1	inner border
2.000000	2.000000	16.000000	16.000000	0.000000e+00	8	1	border
2.000000	1.000000	9.000000	9.000000	1.776357e-15	9	2	border
2.000000	1.500000	11.375000	11.375000	0.000000e+00	10	2	inner border
2.000000	2.000000	16.000000	16.000000	0.000000e+00	11	2	inner border
2.500000	1.000000	16.625000	16.625000	3.552714e-15	12	2	border
2.500000	1.500000	19.000000	19.000000	3.552714e-15	13	2	inner
2.500000	2.000000	23.625000	23.625000	0.000000e+00	14	2	inner border
3.000000	1.000000	28.000000	28.000000	0.000000e+00	15	2	border
3.000000	1.500000	30.375000	30.375000	0.000000e+00	16	2	border
3.000000	2.000000	35.000000	35.000000	7.105427e-15	17	2	border
2.000000	2.000000	16.000000	16.000000	0.000000e+00	18	3	border
2.000000	2.500000	23.625000	23.625000	0.000000e+00	19	3	border
2.000000	3.000000	35.000000	35.000000	7.105427e-15	20	3	border
2.500000	2.000000	23.625000	23.625000	3.552714e-15	21	3	inner border
2.500000	2.500000	31.250000	31.250000	7.105427e-15	22	3	inner
2.500000	3.000000	42.625000	42.625000	7.105427e-15	23	3	border
3.000000	2.000000	35.000000	35.000000	7.105427e-15	24	3	border
3.000000	2.500000	42.625000	42.625000	7.105427e-15	25	3	border
3.000000	3.000000	54.000000	54.000000	0.000000e+00	26	3	border

$||u-u^*||/||u^*|| = 1.457672e-16$

$||u-u^*|| = 1.884111e-14$

- $\lambda = 1, \gamma = 1, u^* = x^4 + y^4, f = -12(x^2 + y^2) + x^4 + y^4$

y	x	calc	prec	dif	N	reg	location
1.000000	1.000000	2.000000	2.000000	0.000000e+00	0	1	border
1.000000	1.500000	6.062500	6.062500	0.000000e+00	1	1	border
1.000000	2.000000	17.000000	17.000000	3.552714e-15	2	1	border
1.500000	1.000000	6.062500	6.062500	0.000000e+00	3	1	border
1.500000	1.500000	10.209136	10.125000	8.413639e-02	4	1	inner
1.500000	2.000000	21.062500	21.062500	3.552714e-15	5	1	border
2.000000	1.000000	17.000000	17.000000	3.552714e-15	6	1	border
2.000000	1.500000	21.170080	21.062500	1.075797e-01	7	1	inner border
2.000000	2.000000	32.000000	32.000000	0.000000e+00	8	1	border
2.000000	1.000000	17.000000	17.000000	3.552714e-15	9	2	border
2.000000	1.500000	21.199039	21.062500	1.365390e-01	10	2	inner border
2.000000	2.000000	32.123077	32.000000	1.230772e-01	11	2	inner border
2.500000	1.000000	40.062500	40.062500	7.105427e-15	12	2	border
2.500000	1.500000	44.248077	44.125000	1.230772e-01	13	2	inner
2.500000	2.000000	55.199039	55.062500	1.365390e-01	14	2	inner border
3.000000	1.000000	82.000000	82.000000	1.421085e-14	15	2	border
3.000000	1.500000	86.062500	86.062500	1.421085e-14	16	2	border
3.000000	2.000000	97.000000	97.000000	0.000000e+00	17	2	border
2.000000	2.000000	32.000000	32.000000	0.000000e+00	18	3	border
2.000000	2.500000	55.062500	55.062500	0.000000e+00	19	3	border
2.000000	3.000000	97.000000	97.000000	0.000000e+00	20	3	border
2.500000	2.000000	55.170080	55.062500	1.075797e-01	21	3	inner border
2.500000	2.500000	78.209136	78.125000	8.413639e-02	22	3	inner
2.500000	3.000000	120.062500	120.062500	0.000000e+00	23	3	border
3.000000	2.000000	97.000000	97.000000	0.000000e+00	24	3	border
3.000000	2.500000	120.062500	120.062500	0.000000e+00	25	3	border
3.000000	3.000000	162.000000	162.000000	2.842171e-14	26	3	border

||u-u*||/||u*|| = 9.286118e-04
||u-u*|| = 3.238617e-01

Область имеет Ш-образную форму

- $\lambda = 1, \gamma = 1, u^* = x^3 + y^3, f = -6(x + y) + x^3 + y^3$

y	x	calc	prec	dif	N	reg	location
1.000000	1.000000	2.000000	2.000000	0.000000e+00	0	1	border
1.000000	1.500000	4.375000	4.375000	8.881784e-16	1	1	border
1.000000	2.000000	9.000000	9.000000	1.776357e-15	2	1	border
1.500000	1.000000	4.375000	4.375000	8.881784e-16	3	1	border
1.500000	1.500000	6.750000	6.750000	0.000000e+00	4	1	inner
1.500000	2.000000	11.375000	11.375000	0.000000e+00	5	1	inner border
2.000000	1.000000	9.000000	9.000000	1.776357e-15	6	1	border
2.000000	1.500000	11.375000	11.375000	3.552714e-15	7	1	inner border
2.000000	2.000000	16.000000	16.000000	3.552714e-15	8	1	inner border
1.000000	2.000000	9.000000	9.000000	1.776357e-15	9	2	border
1.000000	2.500000	16.625000	16.625000	3.552714e-15	10	2	border
1.000000	3.000000	28.000000	28.000000	0.000000e+00	11	2	border
1.500000	2.000000	11.375000	11.375000	0.000000e+00	12	2	inner border
1.500000	2.500000	19.000000	19.000000	3.552714e-15	13	2	inner
1.500000	3.000000	30.375000	30.375000	7.105427e-15	14	2	inner border
2.000000	2.000000	16.000000	16.000000	0.000000e+00	15	2	border
2.000000	2.500000	23.625000	23.625000	0.000000e+00	16	2	border
2.000000	3.000000	35.000000	35.000000	7.105427e-15	17	2	border
1.000000	3.000000	28.000000	28.000000	0.000000e+00	18	3	border
1.000000	3.500000	43.875000	43.875000	7.105427e-15	19	3	border
1.000000	4.000000	65.000000	65.000000	1.421085e-14	20	3	border
1.500000	3.000000	30.375000	30.375000	3.552714e-15	21	3	inner border
1.500000	3.500000	46.250000	46.250000	7.105427e-15	22	3	inner
1.500000	4.000000	67.375000	67.375000	0.000000e+00	23	3	inner border
2.000000	3.000000	35.000000	35.000000	0.000000e+00	24	3	inner border
2.000000	3.500000	50.875000	50.875000	7.105427e-15	25	3	inner border
2.000000	4.000000	72.000000	72.000000	1.421085e-14	26	3	inner border
1.000000	4.000000	65.000000	65.000000	1.421085e-14	27	4	border
1.000000	4.500000	92.125000	92.125000	0.000000e+00	28	4	border
1.000000	5.000000	126.000000	126.000000	0.000000e+00	29	4	border
1.500000	4.000000	67.375000	67.375000	0.000000e+00	30	4	inner border
1.500000	4.500000	94.500000	94.500000	0.000000e+00	31	4	inner
1.500000	5.000000	128.375000	128.375000	5.684342e-14	32	4	inner border
2.000000	4.000000	72.000000	72.000000	1.421085e-14	33	4	border
2.000000	4.500000	99.125000	99.125000	0.000000e+00	34	4	border
2.000000	5.000000	133.000000	133.000000	2.842171e-14	35	4	border
1.000000	5.000000	126.000000	126.000000	0.000000e+00	36	5	border
1.000000	5.500000	167.375000	167.375000	2.842171e-14	37	5	border
1.000000	6.000000	217.000000	217.000000	0.000000e+00	38	5	border
1.500000	5.000000	128.375000	128.375000	0.000000e+00	39	5	inner border
1.500000	5.500000	169.750000	169.750000	2.842171e-14	40	5	inner
1.500000	6.000000	219.375000	219.375000	0.000000e+00	41	5	border
2.000000	5.000000	133.000000	133.000000	0.000000e+00	42	5	inner border
2.000000	5.500000	174.375000	174.375000	0.000000e+00	43	5	inner border
2.000000	6.000000	224.000000	224.000000	0.000000e+00	44	5	border
2.000000	1.000000	9.000000	9.000000	1.776357e-15	45	6	border
2.000000	1.500000	11.375000	11.375000	0.000000e+00	46	6	inner border
2.000000	2.000000	16.000000	16.000000	0.000000e+00	47	6	border

y	x	calc	prec	dif	N	reg	location
2.500000	1.000000	16.625000	16.625000	3.552714e-15	48	6	border
2.500000	1.500000	19.000000	19.000000	3.552714e-15	49	6	inner
2.500000	2.000000	23.625000	23.625000	0.000000e+00	50	6	border
3.000000	1.000000	28.000000	28.000000	0.000000e+00	51	6	border
3.000000	1.500000	30.375000	30.375000	0.000000e+00	52	6	border
3.000000	2.000000	35.000000	35.000000	7.105427e-15	53	6	border
2.000000	3.000000	35.000000	35.000000	7.105427e-15	54	7	border
2.000000	3.500000	50.875000	50.875000	7.105427e-15	55	7	inner border
2.000000	4.000000	72.000000	72.000000	1.421085e-14	56	7	border
2.500000	3.000000	42.625000	42.625000	7.105427e-15	57	7	border
2.500000	3.500000	58.500000	58.500000	1.421085e-14	58	7	inner
2.500000	4.000000	79.625000	79.625000	1.421085e-14	59	7	border
3.000000	3.000000	54.000000	54.000000	0.000000e+00	60	7	border
3.000000	3.500000	69.875000	69.875000	1.421085e-14	61	7	border
3.000000	4.000000	91.000000	91.000000	0.000000e+00	62	7	border
2.000000	5.000000	133.000000	133.000000	2.842171e-14	63	8	border
2.000000	5.500000	174.375000	174.375000	0.000000e+00	64	8	inner border
2.000000	6.000000	224.000000	224.000000	0.000000e+00	65	8	border
2.500000	5.000000	140.625000	140.625000	2.842171e-14	66	8	border
2.500000	5.500000	182.000000	182.000000	2.842171e-14	67	8	inner
2.500000	6.000000	231.625000	231.625000	0.000000e+00	68	8	border
3.000000	5.000000	152.000000	152.000000	2.842171e-14	69	8	border
3.000000	5.500000	193.375000	193.375000	0.000000e+00	70	8	border
3.000000	6.000000	243.000000	243.000000	0.000000e+00	71	8	border

||u-u*||/||u*|| = 1.199623e-16
||u-u*|| = 1.051582e-13

- $\lambda = 1, \gamma = 1, u^* = x^4 + y^4, f = -12(x^2 + y^2) + x^4 + y^4$

y	x	calc	prec	dif	N	reg	location
1.000000	1.000000	2.000000	2.000000	0.000000e+00	0	1	border
1.000000	1.500000	6.062500	6.062500	0.000000e+00	1	1	border
1.000000	2.000000	17.000000	17.000000	3.552714e-15	2	1	border
1.500000	1.000000	6.062500	6.062500	0.000000e+00	3	1	border
1.500000	1.500000	10.250369	10.125000	1.253687e-01	4	1	inner
1.500000	2.000000	21.207668	21.062500	1.451680e-01	5	1	inner border
2.000000	1.000000	17.000000	17.000000	3.552714e-15	6	1	border
2.000000	1.500000	21.200149	21.062500	1.376490e-01	7	1	inner border
2.000000	2.000000	32.125369	32.000000	1.253687e-01	8	1	inner border
1.000000	2.000000	17.000000	17.000000	3.552714e-15	9	2	border
1.000000	2.500000	40.062500	40.062500	7.105427e-15	10	2	border
1.000000	3.000000	82.000000	82.000000	1.421085e-14	11	2	border
1.500000	2.000000	21.178169	21.062500	1.156694e-01	12	2	inner border
1.500000	2.500000	44.241226	44.125000	1.162264e-01	13	2	inner
1.500000	3.000000	86.190793	86.062500	1.282929e-01	14	2	inner border
2.000000	2.000000	32.000000	32.000000	0.000000e+00	15	2	border
2.000000	2.500000	55.062500	55.062500	0.000000e+00	16	2	border
2.000000	3.000000	97.000000	97.000000	0.000000e+00	17	2	border
1.000000	3.000000	82.000000	82.000000	1.421085e-14	18	3	border
1.000000	3.500000	151.062500	151.062500	2.842171e-14	19	3	border
1.000000	4.000000	257.000000	257.000000	5.684342e-14	20	3	border
1.500000	3.000000	86.223975	86.062500	1.614751e-01	21	3	inner border
1.500000	3.500000	155.304018	155.125000	1.790184e-01	22	3	inner
1.500000	4.000000	261.223975	261.062500	1.614751e-01	23	3	inner border
2.000000	3.000000	97.141024	97.000000	1.410243e-01	24	3	inner border
2.000000	3.500000	166.250378	166.062500	1.878781e-01	25	3	inner border
2.000000	4.000000	272.141024	272.000000	1.410243e-01	26	3	inner border
1.000000	4.000000	257.000000	257.000000	5.684342e-14	27	4	border
1.000000	4.500000	411.062500	411.062500	0.000000e+00	28	4	border
1.000000	5.000000	626.000000	626.000000	1.136868e-13	29	4	border
1.500000	4.000000	261.190793	261.062500	1.282929e-01	30	4	inner border
1.500000	4.500000	415.241226	415.125000	1.162264e-01	31	4	inner
1.500000	5.000000	630.178169	630.062500	1.156694e-01	32	4	inner border
2.000000	4.000000	272.000000	272.000000	5.684342e-14	33	4	border
2.000000	4.500000	426.062500	426.062500	0.000000e+00	34	4	border
2.000000	5.000000	641.000000	641.000000	1.136868e-13	35	4	border
1.000000	5.000000	626.000000	626.000000	1.136868e-13	36	5	border
1.000000	5.500000	916.062500	916.062500	0.000000e+00	37	5	border
1.000000	6.000000	1297.000000	1297.000000	2.273737e-13	38	5	border
1.500000	5.000000	630.207668	630.062500	1.451680e-01	39	5	inner border
1.500000	5.500000	920.250369	920.125000	1.253687e-01	40	5	inner
1.500000	6.000000	1301.062500	1301.062500	2.273737e-13	41	5	border
2.000000	5.000000	641.125369	641.000000	1.253687e-01	42	5	inner border
2.000000	5.500000	931.200149	931.062500	1.376490e-01	43	5	inner border
2.000000	6.000000	1312.000000	1312.000000	2.273737e-13	44	5	border
2.000000	1.000000	17.000000	17.000000	3.552714e-15	45	6	border
2.000000	1.500000	21.170650	21.062500	1.081504e-01	46	6	inner border
2.000000	2.000000	32.000000	32.000000	0.000000e+00	47	6	border

y	x	calc	prec	dif	N	reg	location
2.500000	1.000000	40.062500	40.062500	7.105427e-15	48	6	border
2.500000	1.500000	44.209271	44.125000	8.427069e-02	49	6	inner
2.500000	2.000000	55.062500	55.062500	0.000000e+00	50	6	border
3.000000	1.000000	82.000000	82.000000	1.421085e-14	51	6	border
3.000000	1.500000	86.062500	86.062500	1.421085e-14	52	6	border
3.000000	2.000000	97.000000	97.000000	0.000000e+00	53	6	border
2.000000	3.000000	97.000000	97.000000	0.000000e+00	54	7	border
2.000000	3.500000	166.184014	166.062500	1.215137e-01	55	7	inner border
2.000000	4.000000	272.000000	272.000000	5.684342e-14	56	7	border
2.500000	3.000000	120.062500	120.062500	0.000000e+00	57	7	border
2.500000	3.500000	189.212415	189.125000	8.741500e-02	58	7	inner
2.500000	4.000000	295.062500	295.062500	5.684342e-14	59	7	border
3.000000	3.000000	162.000000	162.000000	2.842171e-14	60	7	border
3.000000	3.500000	231.062500	231.062500	0.000000e+00	61	7	border
3.000000	4.000000	337.000000	337.000000	5.684342e-14	62	7	border
2.000000	5.000000	641.000000	641.000000	1.136868e-13	63	8	border
2.000000	5.500000	931.170650	931.062500	1.081504e-01	64	8	inner border
2.000000	6.000000	1312.000000	1312.000000	2.273737e-13	65	8	border
2.500000	5.000000	664.062500	664.062500	1.136868e-13	66	8	border
2.500000	5.500000	954.209271	954.125000	8.427069e-02	67	8	inner
2.500000	6.000000	1335.062500	1335.062500	2.273737e-13	68	8	border
3.000000	5.000000	706.000000	706.000000	1.136868e-13	69	8	border
3.000000	5.500000	996.062500	996.062500	0.000000e+00	70	8	border
3.000000	6.000000	1377.000000	1377.000000	2.273737e-13	71	8	border

$||u-u*||/||u*|| = 1.458105e-04$

$||u-u*|| = 6.699838e-01$

Область имеет разрывную форму
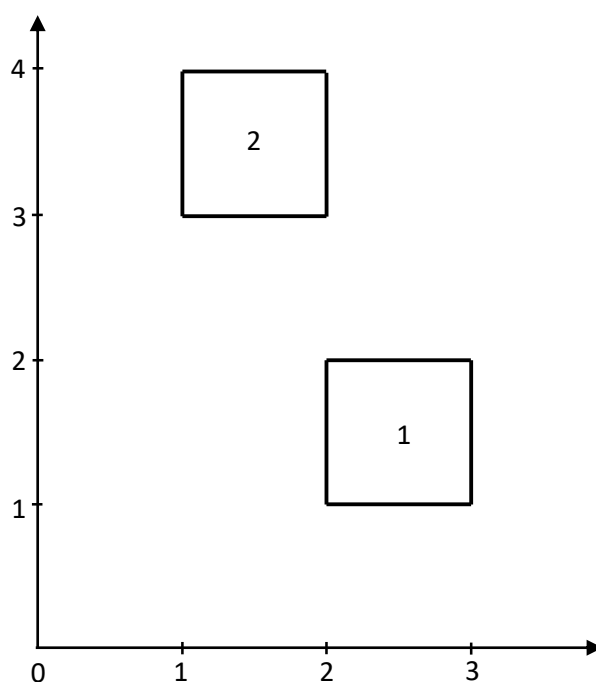
Файл regions.txt

```
2
-1-
1 2 3 4
1 2 1 2
1 1 1 1
-2-
2 3 1 2
1 2 1 2
1 1 1 1
```

- $\lambda = 1, \gamma = 1, u^* = x^3 + y^3, f = -6(x + y) + x^3 + y^3$

y	x	calc	prec	dif	N	reg	location
3.000000	1.000000	28.000000	28.000000	0.000000e+00	0	1	border
3.000000	1.500000	30.375000	30.375000	0.000000e+00	1	1	border
3.000000	2.000000	35.000000	35.000000	7.105427e-15	2	1	border
3.500000	1.000000	43.875000	43.875000	7.105427e-15	3	1	border
3.500000	1.500000	46.250000	46.250000	0.000000e+00	4	1	inner
3.500000	2.000000	50.875000	50.875000	0.000000e+00	5	1	border
4.000000	1.000000	65.000000	65.000000	1.421085e-14	6	1	border
4.000000	1.500000	67.375000	67.375000	1.421085e-14	7	1	border
4.000000	2.000000	72.000000	72.000000	1.421085e-14	8	1	border
1.000000	2.000000	9.000000	9.000000	1.776357e-15	9	2	border
1.000000	2.500000	16.625000	16.625000	3.552714e-15	10	2	border
1.000000	3.000000	28.000000	28.000000	0.000000e+00	11	2	border
1.500000	2.000000	11.375000	11.375000	0.000000e+00	12	2	border
1.500000	2.500000	19.000000	19.000000	3.552714e-15	13	2	inner
1.500000	3.000000	30.375000	30.375000	0.000000e+00	14	2	border
2.000000	2.000000	16.000000	16.000000	0.000000e+00	15	2	border
2.000000	2.500000	23.625000	23.625000	0.000000e+00	16	2	border
2.000000	3.000000	35.000000	35.000000	7.105427e-15	17	2	border

||u-u*||/||u*|| = 1.671385e-16
||u-u*|| = 2.803044e-14

- $\lambda = 1, \gamma = 1, u^* = x^4 + y^4, f = -12(x^2 + y^2) + x^4 + y^4$

y	x	calc	prec	dif	N	reg	location
3.000000	1.000000	82.000000	82.000000	1.421085e-14	0	1	border
3.000000	1.500000	86.062500	86.062500	1.421085e-14	1	1	border
3.000000	2.000000	97.000000	97.000000	0.000000e+00	2	1	border
3.500000	1.000000	151.062500	151.062500	2.842171e-14	3	1	border
3.500000	1.500000	155.183824	155.125000	5.882353e-02	4	1	inner
3.500000	2.000000	166.062500	166.062500	2.842171e-14	5	1	border
4.000000	1.000000	257.000000	257.000000	5.684342e-14	6	1	border
4.000000	1.500000	261.062500	261.062500	5.684342e-14	7	1	border
4.000000	2.000000	272.000000	272.000000	5.684342e-14	8	1	border
1.000000	2.000000	17.000000	17.000000	3.552714e-15	9	2	border
1.000000	2.500000	40.062500	40.062500	7.105427e-15	10	2	border
1.000000	3.000000	82.000000	82.000000	1.421085e-14	11	2	border
1.500000	2.000000	21.062500	21.062500	3.552714e-15	12	2	border
1.500000	2.500000	44.183824	44.125000	5.882353e-02	13	2	inner
1.500000	3.000000	86.062500	86.062500	1.421085e-14	14	2	border
2.000000	2.000000	32.000000	32.000000	0.000000e+00	15	2	border
2.000000	2.500000	55.062500	55.062500	0.000000e+00	16	2	border
2.000000	3.000000	97.000000	97.000000	0.000000e+00	17	2	border

||u-u*||/||u*|| = 1.430753e-04
||u-u*|| = 8.318903e-02

7. Тестирование на неравномерной сетке

Область имеет L-образную форму
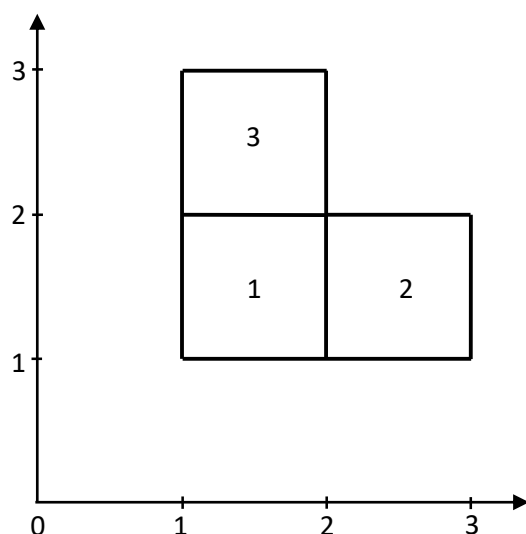
Файл regions.txt

```
3
-1-
1 2 1 2
1 2 2 2
1 -2 1 -3
-2-
2 3 1 2
1 2 2 2
-1 1 1 1
-3-
1 2 2 3
1 2 3 2
1 1 -1 1
```

- $\lambda = 1, \gamma = 1, u^* = x^2 + y^2, f = -4 + x^2 + y^2$

y	x	calc	prec	dif	N	reg	location
1.000000	1.000000	2.000000	2.000000	0.000000e+00	0	1	border
1.000000	1.500000	3.250000	3.250000	0.000000e+00	1	1	border
1.000000	2.000000	5.000000	5.000000	8.881784e-16	2	1	border
1.333333	1.000000	2.777778	2.777778	0.000000e+00	3	1	border
1.333333	1.500000	4.027778	4.027778	0.000000e+00	4	1	inner
1.333333	2.000000	5.777778	5.777778	8.881784e-16	5	1	inner border
2.000000	1.000000	5.000000	5.000000	8.881784e-16	6	1	border
2.000000	1.500000	6.250000	6.250000	8.881784e-16	7	1	inner border
2.000000	2.000000	8.000000	8.000000	0.000000e+00	8	1	inner border
1.000000	2.000000	5.000000	5.000000	8.881784e-16	9	2	border
1.000000	2.500000	7.250000	7.250000	0.000000e+00	10	2	border
1.000000	3.000000	10.000000	10.000000	1.776357e-15	11	2	border
1.333333	2.000000	5.777778	5.777778	8.881784e-16	12	2	inner border
1.333333	2.500000	8.027778	8.027778	3.552714e-15	13	2	inner
1.333333	3.000000	10.777778	10.777778	1.776357e-15	14	2	border
2.000000	2.000000	8.000000	8.000000	0.000000e+00	15	2	border
2.000000	2.500000	10.250000	10.250000	1.776357e-15	16	2	border
2.000000	3.000000	13.000000	13.000000	0.000000e+00	17	2	border
2.000000	1.000000	5.000000	5.000000	8.881784e-16	18	3	border
2.000000	1.500000	6.250000	6.250000	0.000000e+00	19	3	inner border
2.000000	2.000000	8.000000	8.000000	0.000000e+00	20	3	border
2.250000	1.000000	6.062500	6.062500	0.000000e+00	21	3	border
2.250000	1.500000	7.312500	7.312500	1.776357e-15	22	3	inner
2.250000	2.000000	9.062500	9.062500	1.776357e-15	23	3	border
3.000000	1.000000	10.000000	10.000000	1.776357e-15	24	3	border
3.000000	1.500000	11.250000	11.250000	0.000000e+00	25	3	border
3.000000	2.000000	13.000000	13.000000	0.000000e+00	26	3	border

```
||u-u*||/||u*|| = 1.498774e-16
||u-u*|| = 6.089044e-15
```

- $\lambda = 1, \gamma = 1, u^* = x^3 + y^3, f = -6(x+y) + x^3 + y^3$

y	x	calc	prec	dif	N	reg	location
1.000000	1.000000	2.000000	2.000000	0.000000e+00	0	1	border
1.000000	1.500000	4.375000	4.375000	8.881784e-16	1	1	border
1.000000	2.000000	9.000000	9.000000	1.776357e-15	2	1	border
1.333333	1.000000	3.370370	3.370370	0.000000e+00	3	1	border
1.333333	1.500000	5.790592	5.745370	4.522200e-02	4	1	inner
1.333333	2.000000	10.422651	10.370370	5.228071e-02	5	1	inner border
2.000000	1.000000	9.000000	9.000000	1.776357e-15	6	1	border
2.000000	1.500000	11.354402	11.375000	2.059783e-02	7	1	inner border
2.000000	2.000000	15.964542	16.000000	3.545829e-02	8	1	inner border
1.000000	2.000000	9.000000	9.000000	1.776357e-15	9	2	border
1.000000	2.500000	16.625000	16.625000	3.552714e-15	10	2	border
1.000000	3.000000	28.000000	28.000000	0.000000e+00	11	2	border
1.333333	2.000000	10.428561	10.370370	5.819042e-02	12	2	inner border
1.333333	2.500000	18.045339	17.995370	4.996824e-02	13	2	inner
1.333333	3.000000	29.370370	29.370370	0.000000e+00	14	2	border
2.000000	2.000000	16.000000	16.000000	0.000000e+00	15	2	border
2.000000	2.500000	23.625000	23.625000	0.000000e+00	16	2	border
2.000000	3.000000	35.000000	35.000000	7.105427e-15	17	2	border
2.000000	1.000000	9.000000	9.000000	1.776357e-15	18	3	border
2.000000	1.500000	11.361156	11.375000	1.384387e-02	19	3	inner border
2.000000	2.000000	16.000000	16.000000	0.000000e+00	20	3	border
2.250000	1.000000	12.390625	12.390625	0.000000e+00	21	3	border
2.250000	1.500000	14.810841	14.765625	4.521605e-02	22	3	inner
2.250000	2.000000	19.390625	19.390625	3.552714e-15	23	3	border
3.000000	1.000000	28.000000	28.000000	0.000000e+00	24	3	border
3.000000	1.500000	30.375000	30.375000	0.000000e+00	25	3	border
3.000000	2.000000	35.000000	35.000000	7.105427e-15	26	3	border

$||u-u^*||/||u^*|| = 1.249196e-03$

$||u-u^*|| = 1.207434e-01$

Область имеет Т-образную форму
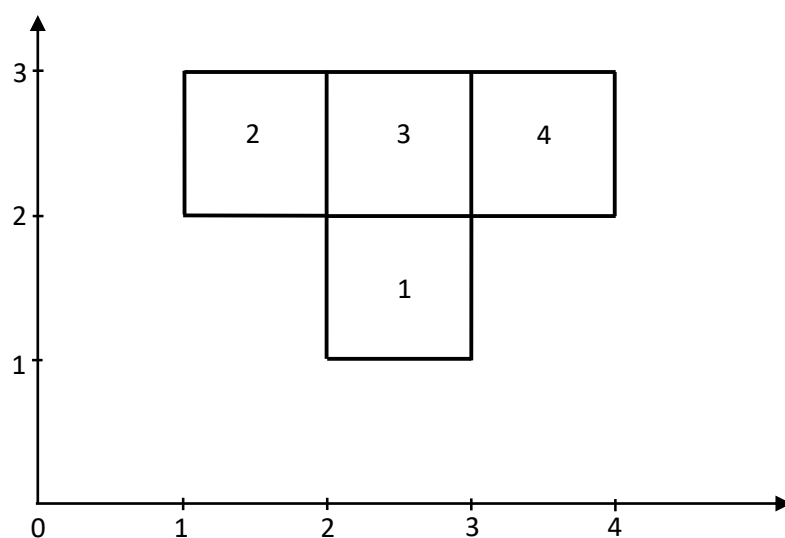
Файл regions.txt

```
4
-1-
2 3 1 2
1 2 2 2
1 1 1 -3
-2-
1 2 2 3
1 2 3 2
1 -3 1 1
-3-
2 3 2 3
1 2 3 2
-2 -4 -1 1
-4-
3 4 2 3
1 2 3 2
-3 1 1 1
```

- $\lambda = 1, \gamma = 1, u^* = x^2 + y^2, f = -4 + x^2 + y^2$

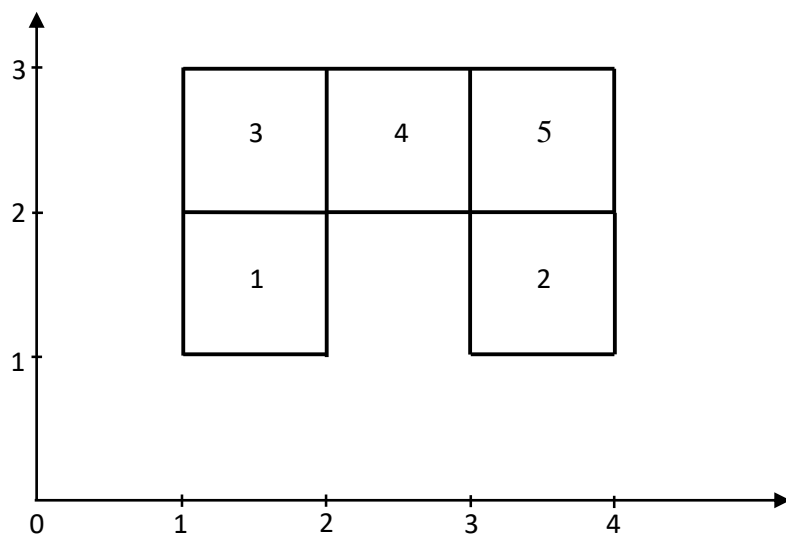| y	x	calc	prec	dif	N	reg	location
1.000000	2.000000	5.000000	5.000000	8.881784e-16	0	1	border
1.000000	2.500000	7.250000	7.250000	0.000000e+00	1	1	border
1.000000	3.000000	10.000000	10.000000	1.776357e-15	2	1	border
1.333333	2.000000	5.777778	5.777778	0.000000e+00	3	1	border
1.333333	2.500000	8.027778	8.027778	3.552714e-15	4	1	inner
1.333333	3.000000	10.777778	10.777778	1.776357e-15	5	1	border
2.000000	2.000000	8.000000	8.000000	0.000000e+00	6	1	border
2.000000	2.500000	10.250000	10.250000	1.776357e-15	7	1	inner border
2.000000	3.000000	13.000000	13.000000	0.000000e+00	8	1	border
2.000000	1.000000	5.000000	5.000000	8.881784e-16	9	2	border
2.000000	1.500000	6.250000	6.250000	0.000000e+00	10	2	border
2.000000	2.000000	8.000000	8.000000	0.000000e+00	11	2	border
2.250000	1.000000	6.062500	6.062500	0.000000e+00	12	2	border
2.250000	1.500000	7.312500	7.312500	2.664535e-15	13	2	inner
2.250000	2.000000	9.062500	9.062500	3.552714e-15	14	2	inner border
3.000000	1.000000	10.000000	10.000000	1.776357e-15	15	2	border
3.000000	1.500000	11.250000	11.250000	0.000000e+00	16	2	border
3.000000	2.000000	13.000000	13.000000	0.000000e+00	17	2	border
2.000000	2.000000	8.000000	8.000000	0.000000e+00	18	3	inner border
2.000000	2.500000	10.250000	10.250000	0.000000e+00	19	3	inner border
2.000000	3.000000	13.000000	13.000000	0.000000e+00	20	3	inner border
2.250000	2.000000	9.062500	9.062500	1.776357e-15	21	3	inner border
2.250000	2.500000	11.312500	11.312500	1.776357e-15	22	3	inner
2.250000	3.000000	14.062500	14.062500	5.329071e-15	23	3	inner border
3.000000	2.000000	13.000000	13.000000	0.000000e+00	24	3	border
3.000000	2.500000	15.250000	15.250000	0.000000e+00	25	3	border
3.000000	3.000000	18.000000	18.000000	3.552714e-15	26	3	border
2.000000	3.000000	13.000000	13.000000	0.000000e+00	27	4	border
2.000000	3.500000	16.250000	16.250000	3.552714e-15	28	4	border
2.000000	4.000000	20.000000	20.000000	3.552714e-15	29	4	border
2.250000	3.000000	14.062500	14.062500	7.105427e-15	30	4	inner border
2.250000	3.500000	17.312500	17.312500	7.105427e-15	31	4	inner
2.250000	4.000000	21.062500	21.062500	3.552714e-15	32	4	border
3.000000	3.000000	18.000000	18.000000	3.552714e-15	33	4	border
3.000000	3.500000	21.250000	21.250000	3.552714e-15	34	4	border
3.000000	4.000000	25.000000	25.000000	0.000000e+00	35	4	border

$||u-u^*||/||u^*|| = 2.061177e-16$

$||u-u^*|| = 1.606106e-14$

- $\lambda = 1, \gamma = 1, u^* = x^3 + y^3, f = -6(x + y) + x^3 + y^3$

y	x	calc	prec	dif	N	reg	location
1.000000	2.000000	9.000000	9.000000	1.776357e-15	0	1	border
1.000000	2.500000	16.625000	16.625000	3.552714e-15	1	1	border
1.000000	3.000000	28.000000	28.000000	0.000000e+00	2	1	border
1.333333	2.000000	10.370370	10.370370	1.776357e-15	3	1	border
1.333333	2.500000	18.032380	17.995370	3.700921e-02	4	1	inner
1.333333	3.000000	29.370370	29.370370	0.000000e+00	5	1	border
2.000000	2.000000	16.000000	16.000000	0.000000e+00	6	1	border
2.000000	2.500000	23.624833	23.625000	1.669378e-04	7	1	inner border
2.000000	3.000000	35.000000	35.000000	7.105427e-15	8	1	border
2.000000	1.000000	9.000000	9.000000	1.776357e-15	9	2	border
2.000000	1.500000	11.375000	11.375000	0.000000e+00	10	2	border
2.000000	2.000000	16.000000	16.000000	0.000000e+00	11	2	border
2.250000	1.000000	12.390625	12.390625	0.000000e+00	12	2	border
2.250000	1.500000	14.832959	14.765625	6.733403e-02	13	2	inner
2.250000	2.000000	19.471684	19.390625	8.105898e-02	14	2	inner border
3.000000	1.000000	28.000000	28.000000	0.000000e+00	15	2	border
3.000000	1.500000	30.375000	30.375000	0.000000e+00	16	2	border
3.000000	2.000000	35.000000	35.000000	7.105427e-15	17	2	border
2.000000	2.000000	15.992052	16.000000	7.947780e-03	18	3	inner border
2.000000	2.500000	23.621805	23.625000	3.194664e-03	19	3	inner border
2.000000	3.000000	34.992052	35.000000	7.947780e-03	20	3	inner border
2.250000	2.000000	19.468451	19.390625	7.782599e-02	21	3	inner border
2.250000	2.500000	27.096831	27.015625	8.120596e-02	22	3	inner
2.250000	3.000000	38.468451	38.390625	7.782599e-02	23	3	inner border
3.000000	2.000000	35.000000	35.000000	7.105427e-15	24	3	border
3.000000	2.500000	42.625000	42.625000	7.105427e-15	25	3	border
3.000000	3.000000	54.000000	54.000000	0.000000e+00	26	3	border
2.000000	3.000000	35.000000	35.000000	7.105427e-15	27	4	border
2.000000	3.500000	50.875000	50.875000	0.000000e+00	28	4	border
2.000000	4.000000	72.000000	72.000000	1.421085e-14	29	4	border
2.250000	3.000000	38.471684	38.390625	8.105898e-02	30	4	inner border
2.250000	3.500000	54.332959	54.265625	6.733403e-02	31	4	inner
2.250000	4.000000	75.390625	75.390625	1.421085e-14	32	4	border
3.000000	3.000000	54.000000	54.000000	0.000000e+00	33	4	border
3.000000	3.500000	69.875000	69.875000	1.421085e-14	34	4	border
3.000000	4.000000	91.000000	91.000000	0.000000e+00	35	4	border

$||u-u^*||/||u^*|| = 8.832807e-04$
$||u-u^*|| = 2.059691e-01$

Область имеет П-образную форму

Файл regions.txt

```
5
-1-
1 2 1 2
1 2 2 2
1 1 1 -3
-2-
3 4 1 2
1 2 2 2
1 1 1 -5
-3-
1 2 2 3
1 2 3 2
1 -4 -1 1
-4-
2 3 2 3
1 2 3 2
-3 -5 1 1
-5-
3 4 2 3
1 2 3 2
-4 1 -2 1
```

- $\lambda = 1, \gamma = 1, u^* = x^2 + y^2, f = -4 + x^2 + y^2$

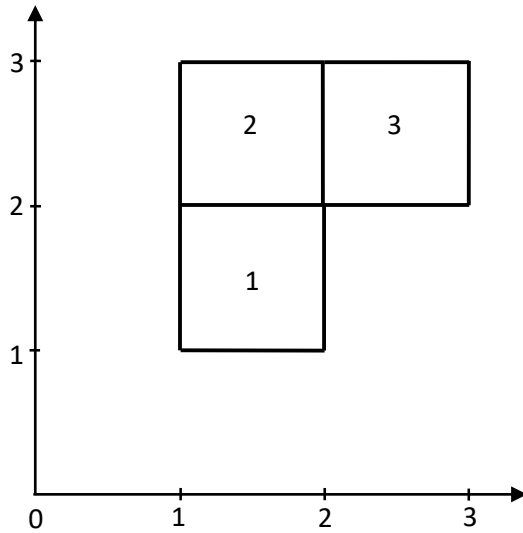| y	x	calc	prec	dif	N	reg	location
1.000000	1.000000	2.000000	2.000000	0.000000e+00	0	1	border
1.000000	1.500000	3.250000	3.250000	0.000000e+00	1	1	border
1.000000	2.000000	5.000000	5.000000	8.881784e-16	2	1	border
1.333333	1.000000	2.777778	2.777778	0.000000e+00	3	1	border
1.333333	1.500000	4.027778	4.027778	8.881784e-16	4	1	inner
1.333333	2.000000	5.777778	5.777778	0.000000e+00	5	1	border
2.000000	1.000000	5.000000	5.000000	8.881784e-16	6	1	border
2.000000	1.500000	6.250000	6.250000	0.000000e+00	7	1	inner border
2.000000	2.000000	8.000000	8.000000	0.000000e+00	8	1	border
1.000000	3.000000	10.000000	10.000000	1.776357e-15	9	2	border
1.000000	3.500000	13.250000	13.250000	0.000000e+00	10	2	border
1.000000	4.000000	17.000000	17.000000	3.552714e-15	11	2	border
1.333333	3.000000	10.777778	10.777778	1.776357e-15	12	2	border
1.333333	3.500000	14.027778	14.027778	3.552714e-15	13	2	inner
1.333333	4.000000	17.777778	17.777778	3.552714e-15	14	2	border
2.000000	3.000000	13.000000	13.000000	0.000000e+00	15	2	border
2.000000	3.500000	16.250000	16.250000	0.000000e+00	16	2	inner border
2.000000	4.000000	20.000000	20.000000	3.552714e-15	17	2	border
2.000000	1.000000	5.000000	5.000000	8.881784e-16	18	3	border
2.000000	1.500000	6.250000	6.250000	0.000000e+00	19	3	inner border
2.000000	2.000000	8.000000	8.000000	0.000000e+00	20	3	inner border
2.250000	1.000000	6.062500	6.062500	0.000000e+00	21	3	border
2.250000	1.500000	7.312500	7.312500	1.776357e-15	22	3	inner
2.250000	2.000000	9.062500	9.062500	1.776357e-15	23	3	inner border
3.000000	1.000000	10.000000	10.000000	1.776357e-15	24	3	border
3.000000	1.500000	11.250000	11.250000	0.000000e+00	25	3	border
3.000000	2.000000	13.000000	13.000000	0.000000e+00	26	3	border
2.000000	2.000000	8.000000	8.000000	0.000000e+00	27	4	border
2.000000	2.500000	10.250000	10.250000	1.776357e-15	28	4	border
2.000000	3.000000	13.000000	13.000000	0.000000e+00	29	4	border
2.250000	2.000000	9.062500	9.062500	0.000000e+00	30	4	inner border
2.250000	2.500000	11.312500	11.312500	1.776357e-15	31	4	inner
2.250000	3.000000	14.062500	14.062500	7.105427e-15	32	4	inner border
3.000000	2.000000	13.000000	13.000000	0.000000e+00	33	4	border
3.000000	2.500000	15.250000	15.250000	0.000000e+00	34	4	border
3.000000	3.000000	18.000000	18.000000	3.552714e-15	35	4	border
2.000000	3.000000	13.000000	13.000000	0.000000e+00	36	5	inner border
2.000000	3.500000	16.250000	16.250000	3.552714e-15	37	5	inner border
2.000000	4.000000	20.000000	20.000000	3.552714e-15	38	5	border
2.250000	3.000000	14.062500	14.062500	7.105427e-15	39	5	inner border
2.250000	3.500000	17.312500	17.312500	7.105427e-15	40	5	inner
2.250000	4.000000	21.062500	21.062500	3.552714e-15	41	5	border
3.000000	3.000000	18.000000	18.000000	3.552714e-15	42	5	border
3.000000	3.500000	21.250000	21.250000	3.552714e-15	43	5	border
3.000000	4.000000	25.000000	25.000000	0.000000e+00	44	5	border

||u-u*||/||u*|| = 1.997274e-16
||u-u*|| = 1.740467e-14

- $\lambda = 1, \gamma = 1, u^* = x^3 + y^3, f = -6(x + y) + x^3 + y^3$

y	x	calc	prec	dif	N	reg	location
1.000000	1.000000	2.000000	2.000000	0.000000e+00	0	1	border
1.000000	1.500000	4.375000	4.375000	8.881784e-16	1	1	border
1.000000	2.000000	9.000000	9.000000	1.776357e-15	2	1	border
1.333333	1.000000	3.370370	3.370370	0.000000e+00	3	1	border
1.333333	1.500000	5.781036	5.745370	3.566534e-02	4	1	inner
1.333333	2.000000	10.370370	10.370370	1.776357e-15	5	1	border
2.000000	1.000000	9.000000	9.000000	1.776357e-15	6	1	border
2.000000	1.500000	11.366770	11.375000	8.230195e-03	7	1	inner border
2.000000	2.000000	16.000000	16.000000	0.000000e+00	8	1	border
1.000000	3.000000	28.000000	28.000000	0.000000e+00	9	2	border
1.000000	3.500000	43.875000	43.875000	7.105427e-15	10	2	border
1.000000	4.000000	65.000000	65.000000	1.421085e-14	11	2	border
1.333333	3.000000	29.370370	29.370370	0.000000e+00	12	2	border
1.333333	3.500000	45.281036	45.245370	3.566534e-02	13	2	inner
1.333333	4.000000	66.370370	66.370370	1.421085e-14	14	2	border
2.000000	3.000000	35.000000	35.000000	7.105427e-15	15	2	border
2.000000	3.500000	50.866770	50.875000	8.230195e-03	16	2	inner border
2.000000	4.000000	72.000000	72.000000	1.421085e-14	17	2	border
2.000000	1.000000	9.000000	9.000000	1.776357e-15	18	3	border
2.000000	1.500000	11.364904	11.375000	1.009634e-02	19	3	inner border
2.000000	2.000000	15.990203	16.000000	9.797283e-03	20	3	inner border
2.250000	1.000000	12.390625	12.390625	0.000000e+00	21	3	border
2.250000	1.500000	14.827933	14.765625	6.230770e-02	22	3	inner
2.250000	2.000000	19.467164	19.390625	7.653889e-02	23	3	inner border
3.000000	1.000000	28.000000	28.000000	0.000000e+00	24	3	border
3.000000	1.500000	30.375000	30.375000	0.000000e+00	25	3	border
3.000000	2.000000	35.000000	35.000000	7.105427e-15	26	3	border
2.000000	2.000000	16.000000	16.000000	0.000000e+00	27	4	border
2.000000	2.500000	23.625000	23.625000	0.000000e+00	28	4	border
2.000000	3.000000	35.000000	35.000000	7.105427e-15	29	4	border
2.250000	2.000000	19.471149	19.390625	8.052423e-02	30	4	inner border
2.250000	2.500000	27.099228	27.015625	8.360307e-02	31	4	inner
2.250000	3.000000	38.471149	38.390625	8.052423e-02	32	4	inner border
3.000000	2.000000	35.000000	35.000000	7.105427e-15	33	4	border
3.000000	2.500000	42.625000	42.625000	7.105427e-15	34	4	border
3.000000	3.000000	54.000000	54.000000	0.000000e+00	35	4	border
2.000000	3.000000	34.990203	35.000000	9.797283e-03	36	5	inner border
2.000000	3.500000	50.864904	50.875000	1.009634e-02	37	5	inner border
2.000000	4.000000	72.000000	72.000000	1.421085e-14	38	5	border
2.250000	3.000000	38.467164	38.390625	7.653889e-02	39	5	inner border
2.250000	3.500000	54.327933	54.265625	6.230770e-02	40	5	inner
2.250000	4.000000	75.390625	75.390625	1.421085e-14	41	5	border
3.000000	3.000000	54.000000	54.000000	0.000000e+00	42	5	border
3.000000	3.500000	69.875000	69.875000	1.421085e-14	43	5	border
3.000000	4.000000	91.000000	91.000000	0.000000e+00	44	5	border

||u-u*||/||u*|| = 7.586340e-04
||u-u*|| = 2.061893e-01

Область имеет Г-образную форму

```
3
-1-
1 2 1 2
1 2 2 2
1 1 1 -2
-2-
1 2 2 3
1 2 3 2
1 -3 -1 1
-3-
2 3 2 3
2 2 3 2
-2 1 1 1
```

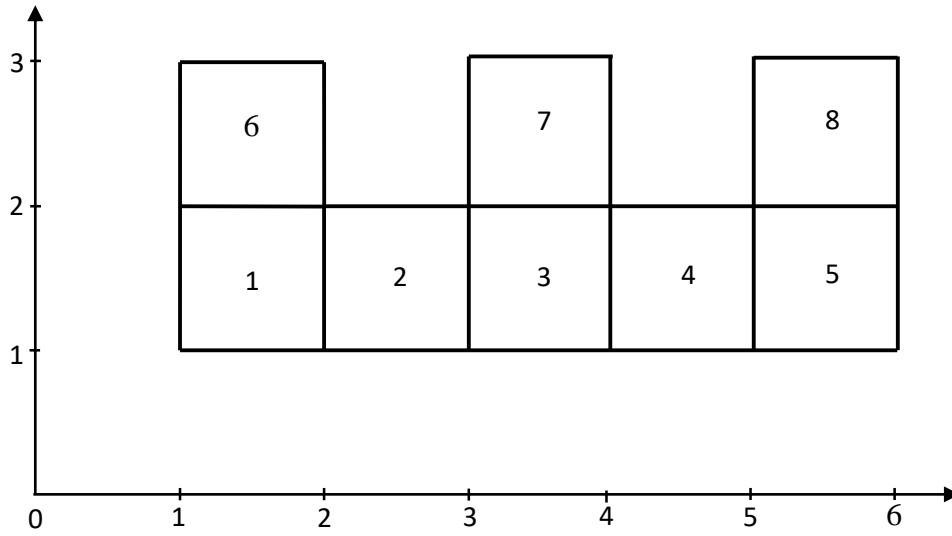- $\lambda = 1, \gamma = 1, u^* = x^2 + y^2, f = -4 + x^2 + y^2$

y	x	calc	prec	dif	N	reg	location
1.000000	1.000000	2.000000	2.000000	0.000000e+00	0	1	border
1.000000	1.500000	3.250000	3.250000	0.000000e+00	1	1	border
1.000000	2.000000	5.000000	5.000000	8.881784e-16	2	1	border
1.333333	1.000000	2.777778	2.777778	0.000000e+00	3	1	border
1.333333	1.500000	4.027778	4.027778	8.881784e-16	4	1	inner
1.333333	2.000000	5.777778	5.777778	0.000000e+00	5	1	border
2.000000	1.000000	5.000000	5.000000	8.881784e-16	6	1	border
2.000000	1.500000	6.250000	6.250000	8.881784e-16	7	1	inner border
2.000000	2.000000	8.000000	8.000000	0.000000e+00	8	1	border
2.000000	1.000000	5.000000	5.000000	8.881784e-16	9	2	border
2.000000	1.500000	6.250000	6.250000	8.881784e-16	10	2	inner border
2.000000	2.000000	8.000000	8.000000	1.776357e-15	11	2	inner border
2.250000	1.000000	6.062500	6.062500	0.000000e+00	12	2	border
2.250000	1.500000	7.312500	7.312500	8.881784e-16	13	2	inner
2.250000	2.000000	9.062500	9.062500	1.776357e-15	14	2	inner border
3.000000	1.000000	10.000000	10.000000	1.776357e-15	15	2	border
3.000000	1.500000	11.250000	11.250000	0.000000e+00	16	2	border
3.000000	2.000000	13.000000	13.000000	0.000000e+00	17	2	border
2.000000	2.000000	8.000000	8.000000	0.000000e+00	18	3	border
2.000000	2.333333	9.444444	9.444444	1.776357e-15	19	3	border
2.000000	3.000000	13.000000	13.000000	0.000000e+00	20	3	border
2.250000	2.000000	9.062500	9.062500	0.000000e+00	21	3	inner border
2.250000	2.333333	10.506944	10.506944	0.000000e+00	22	3	inner
2.250000	3.000000	14.062500	14.062500	0.000000e+00	23	3	border
3.000000	2.000000	13.000000	13.000000	0.000000e+00	24	3	border
3.000000	2.333333	14.444444	14.444444	0.000000e+00	25	3	border
3.000000	3.000000	18.000000	18.000000	3.552714e-15	26	3	border

$||u-u^*||/||u^*|| = 1.147849e-16$
$||u-u^*|| = 5.546672e-15$

- $\lambda = 1, \gamma = 1, u^* = x^3 + y^3, f = -6(x + y) + x^3 + y^3$

y	x	calc	prec	dif	N	reg	location
1.000000	1.000000	2.000000	2.000000	0.000000e+00	0	1	border
1.000000	1.500000	4.375000	4.375000	8.881784e-16	1	1	border
1.000000	2.000000	9.000000	9.000000	1.776357e-15	2	1	border
1.333333	1.000000	3.370370	3.370370	0.000000e+00	3	1	border
1.333333	1.500000	5.780635	5.745370	3.526442e-02	4	1	inner
1.333333	2.000000	10.370370	10.370370	1.776357e-15	5	1	border
2.000000	1.000000	9.000000	9.000000	1.776357e-15	6	1	border
2.000000	1.500000	11.364364	11.375000	1.063571e-02	7	1	inner border
2.000000	2.000000	16.000000	16.000000	0.000000e+00	8	1	border
2.000000	1.000000	9.000000	9.000000	1.776357e-15	9	2	border
2.000000	1.500000	11.358928	11.375000	1.607158e-02	10	2	inner border
2.000000	2.000000	15.971462	16.000000	2.853830e-02	11	2	inner border
2.250000	1.000000	12.390625	12.390625	0.000000e+00	12	2	border
2.250000	1.500000	14.822295	14.765625	5.666978e-02	13	2	inner
2.250000	2.000000	19.451395	19.390625	6.076957e-02	14	2	inner border
3.000000	1.000000	28.000000	28.000000	0.000000e+00	15	2	border
3.000000	1.500000	30.375000	30.375000	0.000000e+00	16	2	border
3.000000	2.000000	35.000000	35.000000	7.105427e-15	17	2	border
2.000000	2.000000	16.000000	16.000000	0.000000e+00	18	3	border
2.000000	2.333333	20.703704	20.703704	3.552714e-15	19	3	border
2.000000	3.000000	35.000000	35.000000	7.105427e-15	20	3	border
2.250000	2.000000	19.461041	19.390625	7.041632e-02	21	3	inner border
2.250000	2.333333	24.195417	24.094329	1.010886e-01	22	3	inner
2.250000	3.000000	38.390625	38.390625	7.105427e-15	23	3	border
3.000000	2.000000	35.000000	35.000000	7.105427e-15	24	3	border
3.000000	2.333333	39.703704	39.703704	7.105427e-15	25	3	border
3.000000	3.000000	54.000000	54.000000	0.000000e+00	26	3	border

||u-u*||/||u*|| = 1.281501e-03
||u-u*|| = 1.565605e-01

Область имеет Ш-образную форму

```
8
-1-
1 2 1 2
1 2 3 2
1 -2 1 -6
-2-
2 3 1 2
1 2 3 2
-1 -3 1 1
-3-
3 4 1 2
1 2 3 2
-2 -4 1 -7
-4-
4 5 1 2
1 2 3 2
-3 -5 1 1
-5-
5 6 1 2
1 2 3 2
-4 1 1 -8
-6-
1 2 2 3
1 2 2 2
1 1 -1 1
-7-
3 4 2 3
1 2 2 2
1 1 -3 1
-8-
5 6 2 3
1 2 2 2
1 1 -5 1
```

- $\lambda = 1, \gamma = 1, u^* = x^2 + y^2, f = -4 + x^2 + y^2$

y	x	calc	prec	dif	N	reg	location
1.000000	1.000000	2.000000	2.000000	0.000000e+00	0	1	border
1.000000	1.500000	3.250000	3.250000	0.000000e+00	1	1	border
1.000000	2.000000	5.000000	5.000000	8.881784e-16	2	1	border
1.250000	1.000000	2.562500	2.562500	4.440892e-16	3	1	border
1.250000	1.500000	3.812500	3.812500	0.000000e+00	4	1	inner
1.250000	2.000000	5.562500	5.562500	8.881784e-16	5	1	inner border
2.000000	1.000000	5.000000	5.000000	8.881784e-16	6	1	border
2.000000	1.500000	6.250000	6.250000	0.000000e+00	7	1	inner border
2.000000	2.000000	8.000000	8.000000	1.776357e-15	8	1	inner border
1.000000	2.000000	5.000000	5.000000	8.881784e-16	9	2	border
1.000000	2.500000	7.250000	7.250000	0.000000e+00	10	2	border
1.000000	3.000000	10.000000	10.000000	1.776357e-15	11	2	border
1.250000	2.000000	5.562500	5.562500	1.776357e-15	12	2	inner border
1.250000	2.500000	7.812500	7.812500	1.776357e-15	13	2	inner
1.250000	3.000000	10.562500	10.562500	3.552714e-15	14	2	inner border
2.000000	2.000000	8.000000	8.000000	0.000000e+00	15	2	border
2.000000	2.500000	10.250000	10.250000	1.776357e-15	16	2	border
2.000000	3.000000	13.000000	13.000000	0.000000e+00	17	2	border
1.000000	3.000000	10.000000	10.000000	1.776357e-15	18	3	border
1.000000	3.500000	13.250000	13.250000	0.000000e+00	19	3	border
1.000000	4.000000	17.000000	17.000000	3.552714e-15	20	3	border
1.250000	3.000000	10.562500	10.562500	1.776357e-15	21	3	inner border
1.250000	3.500000	13.812500	13.812500	3.552714e-15	22	3	inner
1.250000	4.000000	17.562500	17.562500	3.552714e-15	23	3	inner border
2.000000	3.000000	13.000000	13.000000	1.776357e-15	24	3	inner border
2.000000	3.500000	16.250000	16.250000	0.000000e+00	25	3	inner border
2.000000	4.000000	20.000000	20.000000	3.552714e-15	26	3	inner border
1.000000	4.000000	17.000000	17.000000	3.552714e-15	27	4	border
1.000000	4.500000	21.250000	21.250000	3.552714e-15	28	4	border
1.000000	5.000000	26.000000	26.000000	0.000000e+00	29	4	border
1.250000	4.000000	17.562500	17.562500	7.105427e-15	30	4	inner border
1.250000	4.500000	21.812500	21.812500	3.552714e-15	31	4	inner
1.250000	5.000000	26.562500	26.562500	3.552714e-15	32	4	inner border
2.000000	4.000000	20.000000	20.000000	3.552714e-15	33	4	border
2.000000	4.500000	24.250000	24.250000	0.000000e+00	34	4	border
2.000000	5.000000	29.000000	29.000000	0.000000e+00	35	4	border
1.000000	5.000000	26.000000	26.000000	0.000000e+00	36	5	border
1.000000	5.500000	31.250000	31.250000	0.000000e+00	37	5	border
1.000000	6.000000	37.000000	37.000000	7.105427e-15	38	5	border
1.250000	5.000000	26.562500	26.562500	7.105427e-15	39	5	inner border
1.250000	5.500000	31.812500	31.812500	1.065814e-14	40	5	inner
1.250000	6.000000	37.562500	37.562500	7.105427e-15	41	5	border
2.000000	5.000000	29.000000	29.000000	3.552714e-15	42	5	inner border
2.000000	5.500000	34.250000	34.250000	7.105427e-15	43	5	inner border
2.000000	6.000000	40.000000	40.000000	7.105427e-15	44	5	border
2.000000	1.000000	5.000000	5.000000	8.881784e-16	45	6	border
2.000000	1.500000	6.250000	6.250000	8.881784e-16	46	6	inner border
2.000000	2.000000	8.000000	8.000000	0.000000e+00	47	6	border

y	x	calc	prec	dif	N	reg	location
2.333333	1.000000	6.444444	6.444444	0.000000e+00	48	6	border
2.333333	1.500000	7.694444	7.694444	1.776357e-15	49	6	inner
2.333333	2.000000	9.444444	9.444444	1.776357e-15	50	6	border
3.000000	1.000000	10.000000	10.000000	1.776357e-15	51	6	border
3.000000	1.500000	11.250000	11.250000	0.000000e+00	52	6	border
3.000000	2.000000	13.000000	13.000000	0.000000e+00	53	6	border
2.000000	3.000000	13.000000	13.000000	0.000000e+00	54	7	border
2.000000	3.500000	16.250000	16.250000	0.000000e+00	55	7	inner border
2.000000	4.000000	20.000000	20.000000	3.552714e-15	56	7	border
2.333333	3.000000	14.444444	14.444444	0.000000e+00	57	7	border
2.333333	3.500000	17.694444	17.694444	0.000000e+00	58	7	inner
2.333333	4.000000	21.444444	21.444444	3.552714e-15	59	7	border
3.000000	3.000000	18.000000	18.000000	3.552714e-15	60	7	border
3.000000	3.500000	21.250000	21.250000	3.552714e-15	61	7	border
3.000000	4.000000	25.000000	25.000000	0.000000e+00	62	7	border
2.000000	5.000000	29.000000	29.000000	0.000000e+00	63	8	border
2.000000	5.500000	34.250000	34.250000	0.000000e+00	64	8	inner border
2.000000	6.000000	40.000000	40.000000	7.105427e-15	65	8	border
2.333333	5.000000	30.444444	30.444444	0.000000e+00	66	8	border
2.333333	5.500000	35.694444	35.694444	0.000000e+00	67	8	inner
2.333333	6.000000	41.444444	41.444444	7.105427e-15	68	8	border
3.000000	5.000000	34.000000	34.000000	7.105427e-15	69	8	border
3.000000	5.500000	39.250000	39.250000	7.105427e-15	70	8	border
3.000000	6.000000	45.000000	45.000000	0.000000e+00	71	8	border

$||u-u*||/||u*|| = 1.587415e-16$

$||u-u*|| = 2.911072e-14$

- $\lambda = 1, \gamma = 1, u^* = x^3 + y^3, f = -6(x + y) + x^3 + y^3$

y	x	calc	prec	dif	N	reg	location
1.000000	1.000000	2.000000	2.000000	0.000000e+00	0	1	border
1.000000	1.500000	4.375000	4.375000	8.881784e-16	1	1	border
1.000000	2.000000	9.000000	9.000000	1.776357e-15	2	1	border
1.250000	1.000000	2.953125	2.953125	0.000000e+00	3	1	border
1.250000	1.500000	5.388086	5.328125	5.996059e-02	4	1	inner
1.250000	2.000000	10.026685	9.953125	7.356030e-02	5	1	inner border
2.000000	1.000000	9.000000	9.000000	1.776357e-15	6	1	border
2.000000	1.500000	11.331869	11.375000	4.313110e-02	7	1	inner border
2.000000	2.000000	15.951483	16.000000	4.851684e-02	8	1	inner border
1.000000	2.000000	9.000000	9.000000	1.776357e-15	9	2	border
1.000000	2.500000	16.625000	16.625000	3.552714e-15	10	2	border
1.000000	3.000000	28.000000	28.000000	0.000000e+00	11	2	border
1.250000	2.000000	10.033264	9.953125	8.013886e-02	12	2	inner border
1.250000	2.500000	17.662180	17.578125	8.405545e-02	13	2	inner
1.250000	3.000000	29.036259	28.953125	8.313377e-02	14	2	inner border
2.000000	2.000000	16.000000	16.000000	0.000000e+00	15	2	border
2.000000	2.500000	23.625000	23.625000	0.000000e+00	16	2	border
2.000000	3.000000	35.000000	35.000000	7.105427e-15	17	2	border
1.000000	3.000000	28.000000	28.000000	0.000000e+00	18	3	border
1.000000	3.500000	43.875000	43.875000	7.105427e-15	19	3	border
1.000000	4.000000	65.000000	65.000000	1.421085e-14	20	3	border
1.250000	3.000000	29.029417	28.953125	7.629177e-02	21	3	inner border
1.250000	3.500000	44.902811	44.828125	7.468557e-02	22	3	inner
1.250000	4.000000	66.029417	65.953125	7.629177e-02	23	3	inner border
2.000000	3.000000	34.949540	35.000000	5.045972e-02	24	3	inner border
2.000000	3.500000	50.821931	50.875000	5.306924e-02	25	3	inner border
2.000000	4.000000	71.949540	72.000000	5.045972e-02	26	3	inner border
1.000000	4.000000	65.000000	65.000000	1.421085e-14	27	4	border
1.000000	4.500000	92.125000	92.125000	0.000000e+00	28	4	border
1.000000	5.000000	126.000000	126.000000	0.000000e+00	29	4	border
1.250000	4.000000	66.036259	65.953125	8.313377e-02	30	4	inner border
1.250000	4.500000	93.162180	93.078125	8.405545e-02	31	4	inner
1.250000	5.000000	127.033264	126.953125	8.013886e-02	32	4	inner border
2.000000	4.000000	72.000000	72.000000	1.421085e-14	33	4	border
2.000000	4.500000	99.125000	99.125000	0.000000e+00	34	4	border
2.000000	5.000000	133.000000	133.000000	2.842171e-14	35	4	border
1.000000	5.000000	126.000000	126.000000	0.000000e+00	36	5	border
1.000000	5.500000	167.375000	167.375000	2.842171e-14	37	5	border
1.000000	6.000000	217.000000	217.000000	0.000000e+00	38	5	border
1.250000	5.000000	127.026685	126.953125	7.356030e-02	39	5	inner border
1.250000	5.500000	168.388086	168.328125	5.996059e-02	40	5	inner
1.250000	6.000000	217.953125	217.953125	0.000000e+00	41	5	border
2.000000	5.000000	132.951483	133.000000	4.851684e-02	42	5	inner border
2.000000	5.500000	174.331869	174.375000	4.313110e-02	43	5	inner border
2.000000	6.000000	224.000000	224.000000	0.000000e+00	44	5	border
2.000000	1.000000	9.000000	9.000000	1.776357e-15	45	6	border
2.000000	1.500000	11.343285	11.375000	3.171537e-02	46	6	inner border
2.000000	2.000000	16.000000	16.000000	0.000000e+00	47	6	border

y	x	calc	prec	dif	N	reg	location
2.333333	1.000000	13.703704	13.703704	0.000000e+00	48	6	border
2.333333	1.500000	16.105169	16.078704	2.646525e-02	49	6	inner
2.333333	2.000000	20.703704	20.703704	3.552714e-15	50	6	border
3.000000	1.000000	28.000000	28.000000	0.000000e+00	51	6	border
3.000000	1.500000	30.375000	30.375000	0.000000e+00	52	6	border
3.000000	2.000000	35.000000	35.000000	7.105427e-15	53	6	border
2.000000	3.000000	35.000000	35.000000	7.105427e-15	54	7	border
2.000000	3.500000	50.845677	50.875000	2.932349e-02	55	7	inner border
2.000000	4.000000	72.000000	72.000000	1.421085e-14	56	7	border
2.333333	3.000000	39.703704	39.703704	7.105427e-15	57	7	border
2.333333	3.500000	55.605966	55.578704	2.726254e-02	58	7	inner
2.333333	4.000000	76.703704	76.703704	1.421085e-14	59	7	border
3.000000	3.000000	54.000000	54.000000	0.000000e+00	60	7	border
3.000000	3.500000	69.875000	69.875000	1.421085e-14	61	7	border
3.000000	4.000000	91.000000	91.000000	0.000000e+00	62	7	border
2.000000	5.000000	133.000000	133.000000	2.842171e-14	63	8	border
2.000000	5.500000	174.343285	174.375000	3.171537e-02	64	8	inner border
2.000000	6.000000	224.000000	224.000000	0.000000e+00	65	8	border
2.333333	5.000000	137.703704	137.703704	2.842171e-14	66	8	border
2.333333	5.500000	179.105169	179.078704	2.646525e-02	67	8	inner
2.333333	6.000000	228.703704	228.703704	0.000000e+00	68	8	border
3.000000	5.000000	152.000000	152.000000	2.842171e-14	69	8	border
3.000000	5.500000	193.375000	193.375000	0.000000e+00	70	8	border
3.000000	6.000000	243.000000	243.000000	0.000000e+00	71	8	border

||u-u*||/||u*|| = 3.577370e-04
||u-u*|| = 3.120607e-01

Область имеет разрывную форму
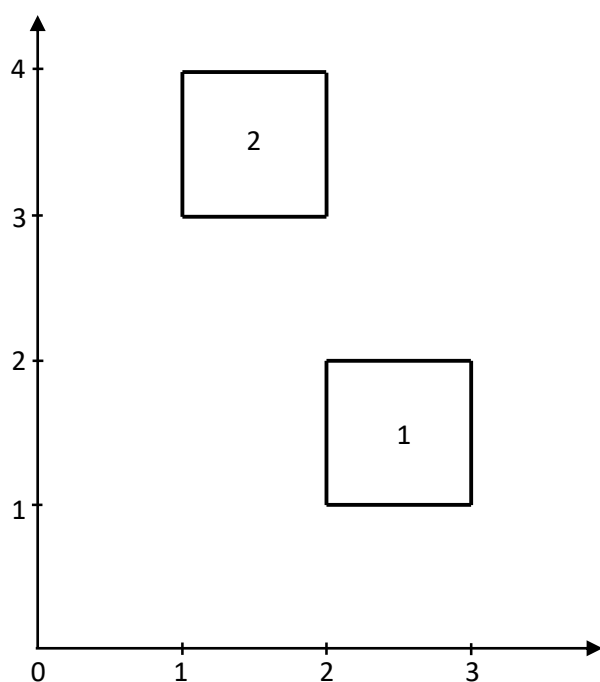
Файл regions.txt

```
2
-1-
1 2 3 4
2 2 3 2
1 1 1 1
-2-
2 3 1 2
3 2 2 2
1 1 1 1
```

- $\lambda = 1, \gamma = 1, u^* = x^2 + y^2, f = -4 + x^2 + y^2$

y	x	calc	prec	dif	N	reg	location
3.000000	1.000000	10.000000	10.000000	1.776357e-15	0	1	border
3.000000	1.333333	10.777778	10.777778	1.776357e-15	1	1	border
3.000000	2.000000	13.000000	13.000000	0.000000e+00	2	1	border
3.250000	1.000000	11.562500	11.562500	0.000000e+00	3	1	border
3.250000	1.333333	12.340278	12.340278	5.329071e-15	4	1	inner
3.250000	2.000000	14.562500	14.562500	0.000000e+00	5	1	border
4.000000	1.000000	17.000000	17.000000	3.552714e-15	6	1	border
4.000000	1.333333	17.777778	17.777778	3.552714e-15	7	1	border
4.000000	2.000000	20.000000	20.000000	3.552714e-15	8	1	border
1.000000	2.000000	5.000000	5.000000	8.881784e-16	9	2	border
1.000000	2.250000	6.062500	6.062500	0.000000e+00	10	2	border
1.000000	3.000000	10.000000	10.000000	1.776357e-15	11	2	border
1.333333	2.000000	5.777778	5.777778	0.000000e+00	12	2	border
1.333333	2.250000	6.840278	6.840278	1.776357e-15	13	2	inner
1.333333	3.000000	10.777778	10.777778	1.776357e-15	14	2	border
2.000000	2.000000	8.000000	8.000000	0.000000e+00	15	2	border
2.000000	2.250000	9.062500	9.062500	1.776357e-15	16	2	border
2.000000	3.000000	13.000000	13.000000	0.000000e+00	17	2	border

||u-u*||/||u*|| = 1.832184e-16
||u-u*|| = 9.272855e-15

- $\lambda = 1, \gamma = 1, u^* = x^3 + y^3, f = -6(x + y) + x^3 + y^3$

y	x	calc	prec	dif	N	reg	location
3.000000	1.000000	28.000000	28.000000	0.000000e+00	0	1	border
3.000000	1.333333	29.370370	29.370370	0.000000e+00	1	1	border
3.000000	2.000000	35.000000	35.000000	7.105427e-15	2	1	border
3.250000	1.000000	35.328125	35.328125	7.105427e-15	3	1	border
3.250000	1.333333	36.779141	36.698495	8.064516e-02	4	1	inner
3.250000	2.000000	42.328125	42.328125	7.105427e-15	5	1	border
4.000000	1.000000	65.000000	65.000000	1.421085e-14	6	1	border
4.000000	1.333333	66.370370	66.370370	1.421085e-14	7	1	border
4.000000	2.000000	72.000000	72.000000	1.421085e-14	8	1	border
1.000000	2.000000	9.000000	9.000000	1.776357e-15	9	2	border
1.000000	2.250000	12.390625	12.390625	0.000000e+00	10	2	border
1.000000	3.000000	28.000000	28.000000	0.000000e+00	11	2	border
1.333333	2.000000	10.370370	10.370370	1.776357e-15	12	2	border
1.333333	2.250000	13.841641	13.760995	8.064516e-02	13	2	inner
1.333333	3.000000	29.370370	29.370370	0.000000e+00	14	2	border
2.000000	2.000000	16.000000	16.000000	0.000000e+00	15	2	border
2.000000	2.250000	19.390625	19.390625	3.552714e-15	16	2	border
2.000000	3.000000	35.000000	35.000000	7.105427e-15	17	2	border

||u-u*||/||u*|| = 7.197704e-04
||u-u*|| = 1.140495e-01

8. Выводы

Результаты проверки работоспособности программы для расчетных областей различной формы на равномерной сетке:

- $\lambda = 1, \gamma = 1, u^* = x^3 + y^3, f = -6(x + y) + x^3 + y^3$

буква	$\dfrac{\|u - u^*\|}{u^*}$	$\|u - u^*\|$
L	1,364848E-16	1,361552E-14
Т	1,554534E-16	3,717639E-14
П	1,511169E-16	4,190471E-14
Г	1,457672E-16	1,884111E-14
Ш	1,199623E-16	1,051582E-13
разрывная	1,671358E-16	2,803044E-14

- $\lambda = 1, \gamma = 1, u^* = x^4 + y^4, f = -12(x^2 + y^2) + x^4 + y^4$

буква	$\dfrac{\|u - u^*\|}{u^*}$	$\|u - u^*\|$
L	1,256545E-03	3,238617E-01
Т	6,042730E-04	4,650397E-01
П	5,016553E-04	4,677078E-01
Г	9,286118E-04	3,238617E-01
Ш	1,458105E-04	6,699838E-01
разрывная	1,430753E-04	8,318903E-02

Равномерная сетка имеет третий порядок аппроксимации, поэтому при $u^* = x^4 + y^4$ наблюдается появление погрешности вычислений.

При тестировании для расчетных областей различной формы на равномерной сетке мы получили примерно одинаковую погрешность решения.

Результаты проверки работоспособности программы для расчетных областей различной формы на неравномерной сетке:

- $\lambda = 1, \gamma = 1, u^* = x^2 + y^2, f = -4 + x^2 + y^2$

буква	$\dfrac{||u - u^*||}{u^*}$	$||u - u^*||$
L	1,498774E-16	6,089044E-15
T	2,061177E-16	1,606106E-14
П	1,997274E-16	1,740467E-14
Г	1,147849E-16	5,546672E-15
Ш	1,587415E-16	2,911072E-14
разрывная	1,832184E-16	9,272855E-15

- $\lambda = 1, \gamma = 1, u^* = x^3 + y^3, f = -6(x + y) + x^3 + y^3$

буква	$\dfrac{||u - u^*||}{u^*}$	$||u - u^*||$
L	1,249196E-03	1,207434E-01
T	8,832807E-04	2,059691E-01
П	7,586340E-04	2,061893E-01
Г	1,281501E-03	1,565605E-01
Ш	3,577370E-04	3,120607E-01
разрывная	7,197704E-04	1,140495E-01

Равномерная сетка имеет второй порядок аппроксимации, поэтому при $u^* = x^3 + y^3$ наблюдается появление погрешности вычислений.

При тестировании для расчетных областей различной формы на неравномерной сетке мы получили примерно одинаковую погрешность решения.