Министерство науки и высшего образования Российской Федерации

Новосибирский государственный технический университет

Кафедра ПМ

УРАВНЕНИЯ МАТЕМАТИЧЕСКОЙ ФИЗИКИ

Практическое задание № 1

Решение эллиптических краевых задач методом конечных разностей

Факультет: ПМИ

Преподаватели:
Задорожный А. Г.,
Патрушев И.И.

Группа: ПМ-81

Студенты: Ефремов А.А.,
Ртищева К. С.

Бригада: 1

Вариант: 5

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1. Цель работы

Разработать программу решения эллиптической краевой задачи методом конечных разностей. Протестировать программу и численно оценить порядок аппроксимации.

2. Задание

Область может иметь любую форму. Предусмотреть учет первых и вторых краевых условий.

3. Анализ задачи

Эллиптическая краевая задача для функции u определяется дифференциальным уравнением

$$-\lambda(\Delta u) + \gamma u = f, \qquad \Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2},$$

Заданным в двумерной области Ω с границей S, и краевым условием

$$u|_{S_1} = u_g.$$

Для двумерного оператора Лапласа Δu дискретный аналог на неравномерной прямоугольной сетке может быть представлен пятиточечным разностным выражением

$$\Delta_h u_{i,j} = \frac{2u_{i-1,j}}{h_{i-1}^x\left(h_i^x + h_{i-1}^x\right)} + \frac{2u_{i,j-1}}{h_{j-1}^y\left(h_j^y + h_{j-1}^y\right)} + \frac{2u_{i+1,j}}{h_i^x\left(h_i^x + h_{i-1}^x\right)} +$$

$$+ \frac{2u_{i,j+1}}{h_j^y\left(h_j^y + h_{j-1}^y\right)} - \left(\frac{2}{h_{i-1}^x h_i^x} + \frac{2}{h_{j-1}^y h_j^y}\right) u_{i,j}.$$

Подставим данное разностное выражение в дифференциальное уравнение и получим:

$$-\frac{2\lambda u_{i-1,j}}{h_{i-1}^x\left(h_i^x + h_{i-1}^x\right)} - \frac{2\lambda u_{i,j-1}}{h_{j-1}^y\left(h_j^y + h_{j-1}^y\right)} - \frac{2\lambda u_{i+1,j}}{h_i^x\left(h_i^x + h_{i-1}^x\right)} -$$

$$-\frac{2\lambda u_{i,j+1}}{h_j^y\left(h_j^y + h_{j-1}^y\right)} + \left(\frac{2}{h_{i-1}^x h_i^x} + \frac{2}{h_{j-1}^y h_j^y}\right)\lambda u_{i,j} + \gamma u_{i,j} = f_{i,j}.$$

Учет первых краевых условий:

в матрице СЛАУ в $i-$й строке на место диагонального элемента ставится единица, все остальные элементы этой строки матрицы обнуляются, а $i-$й компоненте вектора правой части присваивается значение $u_g(x_i, y_i)$.

4. Структура входных данных

Первое число n в файле со входными данными - количество прямоугольных подобластей (регионов). Далее идет n наборов строк, описывающих каждый регион. В первой строке набора задаются 4 числа - координаты левой, правой, нижней и верхней границы региона. Во второй строке идут две пары чисел q и n - коэффициент разрядки и число шагов для дробления сетки по оси x и y. В третьей строке идут 4 числа m для описания условий для левой, правой, нижней и верхней границ расчетной области. Число m может быть 1 - тогда на границе будет 1 краевое условие, или номером региона, с которым граничит данное ребро, взятым с обратным знаком.

5. Текст программы

Файл “SLAE.h”

```
#pragma once
#include <vector>

using namespace std;

class SLAE
{
public:
   vector<vector<double>> matrix; // Матрица системы
   vector<vector<int>> index;     // Индексы столбцов
   vector<double> f;              // Вектор правой части

   const int D = 5;               // Количество диагоналей матрицы
   int N = 0;                     // Размерность матрицы

   vector<double> xk, xk1;  // Вспомогательные векторы

   SLAE(const int& t_n)
   {
      N = t_n;

      index.resize(D);

      for(int d = 0; d < D; d++)
         index[d].resize(N);

      matrix.resize(D);

      for(int d = 0; d < D; d++)
         matrix[d].resize(N);

      xk.resize(N);
      xk1.resize(N);
      f.resize(N);
   }

   // Умножение матрицы системы на вектор vec,
   // результат в res
   void Multiplication(vector<double>& vec, vector<double>& res)
   {
      int n = vec.size(), k = 0;
      for(int j = 0; j < n; j++)
      {
         for(int d = 0; d < D; d++)
         {
            k = index[d][j];
            if(k < 0 && j + k > 0 ||
               k > 0 && j + k < n)
               res[j] += matrix[d][j] * vec[k + j];
         }
      }
   }

   // Норма вектора
   double Norm(const vector<double>& vec)
   {
      double res = 0;
      for(int i = 0; i < N; i++)
         res += vec[i] * vec[i];
```

```
        return sqrt(res);
    }

    // Получение относительной невязки системы
    double RelativeResidual(vector<double>& vec)
    {
        vector<double> mult(N);

        Multiplication(vec, mult);
        for(size_t i = 0; i < N; i++)
            mult[i] = f[i] - mult[i];

        return Norm(mult) / Norm(f);
    }

    // Итерационный процесс метода Гаусса-Зейделя
    void IterativeProcess(const int& j, double& sum)
    {
        int k = 0, n = xk.size();
        for(int i = 0; i < D; i++)
        {
            k = index[i][j];
            if(k + j >= 0 && k + j < n)
            {
                if(i < 3) // нижний треугольник
                    sum += matrix[i][j] * xk1[k + j];
                else // верхний треугольник
                    sum += matrix[i][j] * xk[k + j];
            }
        }
    }

    // Решение системы методом Гаусса-Зейделя
    void GaussSeidel(const int& max_iter, const double& eps,
        const double& relax)
    {
        double residual = 0.0, sum = 0.0;
        residual = RelativeResidual(xk);
        for(int k = 0; k < max_iter && residual > eps; k++)
        {
            for(int j = 0; j < N; j++)
            {
                IterativeProcess(j, sum);
                xk1[j] = xk[j] + (relax / matrix[2][j]) * (f[j] - sum);
                sum = 0.;
            }
            xk.swap(xk1);
            residual = RelativeResidual(xk);
        }
    }
};
```

Файл “Region.h”

```
#pragma once
#include <vector>

using namespace std;

struct Region
{
   double left, right, top, bot;       // Границы областей

   vector<double> x_node;              // Координаты узлов по X
   vector<double> y_node;              // Координаты узлов по Y

   int n_nodes;                         // Количество узлов

   int n_x;                            // Количество узлов по X
   int n_y;                            // Количество узлов по Y

   int first, last;                    // Индексы первого и последнего
                                       // узлов в глобальной нумерации

   // Массив с информацией о краевых условиях региона
   // 0 - нижнее
   // 1 - правое
   // 2 - верхнее
   // 3 - левое
   vector<int> borders;
};
```

Файл “EllipticalProblem.h”

```
#pragma once
#include <vector>
#include <fstream>
#include <string>
#include <iomanip>
#include "SLAE.h"
#include "Test.h"
#include "Region.h"

using namespace std;

class EllipticalProblem
{
public:

   vector<Region> regions;             // Регионы расчетной области

   int n_regions = 0;                  // Количество регионов
   int n_nodes = 0;                    // Общее количество узлов

   vector<vector<int>> borders;        // Информация о граничных условиях

   SLAE* slae;                         // Система
   Test test;                          // Тестовая информация

   EllipticalProblem()
   {

   }
```

```
~EllipticalProblem()
{
   delete slae;
}

// Функция считывания областей из файла FILE_NAME
// и формирования сетки
void ReadFormGrid(const string& FILE_NAME)
{
   ifstream fin(FILE_NAME);

   fin >> n_regions;
   string s;

   regions.resize(n_regions);

   for(int reg_i = 0; reg_i < n_regions; reg_i++)
   {
      fin >> s;
      Region* r = &regions[reg_i];

      // Считывание границы области
      fin >> r->left;
      fin >> r->right;
      fin >> r->bot;
      fin >> r->top;

      // Генерация координат узлов по X
      int n;
      double h, q;

      fin >> q >> n;

      r->n_x = n + 1;
      r->x_node.resize(r->n_x);

      h = r->right - r->left;

      if(q != 1)
         h *= (1 - q) / (1 - pow(q, n));
      else
         h /= n;

      r->x_node[0] = r->left;

      for(int i = 0; i < n; i++)
         r->x_node[i + 1] = r->x_node[i] + h * pow(q, i);

      // Генерация координат узлов по Y

      fin >> q >> n;

      r->n_y = n + 1;
      r->y_node.resize(r->n_y);

      h = r->top - r->bot;

      if(q != 1)
         h *= (1 - q) / (1 - pow(q, n));
      else
         h /= n;

      r->y_node[0] = r->bot;
```

```
      for(int i = 0; i < n; i++)
         r->y_node[i + 1] = r->y_node[i] + h * pow(q, i);

      if(reg_i != 0)
         r->first = regions[reg_i - 1].last + 1;
      else
         r->first = 0;

      r->n_nodes = r->n_x * r->n_y;

      r->last = r->first + r->n_nodes - 1;

      n_nodes += r->n_nodes;

      r->borders.resize(4);
      // Считывание информации о краевых условиях
      for(int bord_i = 0; bord_i < 4; bord_i++)
         fin >> r->borders[bord_i];
   }
   fin.close();
}

// Формирование матрицы системы
void FormMatrix()
{
   // Проход по всем регионам
   for(int reg_i = 0; reg_i < n_regions; reg_i++)
   {
      Region* r = &regions[reg_i];

      // Проход по всем узлам региона
      for(int node_i = 0; node_i < r->n_nodes; node_i++)
      {
         // Индекс узла в глобальной нумерации
         int global_i = node_i + r->first;

         // Индексы центрального узла
         int x_cent = node_i % r->n_x;
         int y_cent = floor(node_i / r->n_x);

         // Обработка некраевых узлов
         if(0 < x_cent && x_cent < r->n_x - 1 &&
            0 < y_cent && y_cent < r->n_y - 1)
         {
            // Приросты по X
            double hi = r->x_node[x_cent + 1] - r->x_node[x_cent + 0];
            double hi1 = r->x_node[x_cent - 0] - r->x_node[x_cent - 1];

            // Приросты по Y
            double hj = r->y_node[y_cent + 1] - r->y_node[y_cent + 0];
            double hj1 = r->y_node[y_cent - 0] - r->y_node[y_cent - 1];

            // Нижний узел
            slae->matrix[0][global_i] = -test.lambda() *
               (2.0 / (hj1 * (hj + hj1)));

            // Левый узел
            slae->matrix[1][global_i] = -test.lambda() *
               (2.0 / (hi1 * (hi + hi1)));

            // Центральный узел
            slae->matrix[2][global_i] = +test.lambda() *
               (2.0 / (hi1 * hi) + 2.0 / (hj1 * hj)) + test.gamma();
```

```
            // Правый узел
            slae->matrix[3][global_i] = -test.lambda() *
               (2.0 / (hi * (hi + hi1)));

            // Верхний узел
            slae->matrix[4][global_i] = -test.lambda() *
               (2.0 / (hj * (hj + hj1)));

            // Вектор правой части
            slae->f[global_i] = test.f(r->x_node[x_cent], r->y_node[y_cent]);
         }
         // Обработка краевых узлов
         else
         {
            int border_x = 0, border_y = 0;

            if(x_cent == 0)
               border_x = r->borders[0];
            else if(x_cent == r->n_x - 1)
               border_x = r->borders[1];

            if(y_cent == 0)
               border_y = r->borders[2];
            else if(y_cent == r->n_y - 1)
               border_y = r->borders[3];

            // Если узел на границе между соседями
            if(border_x != 1 && border_y != 1 ||
               border_x != 1 && border_y == 0 ||
               border_x == 0 && border_y  != 1)
            {
               double hi = 0, hi1 = 0, hj = 0, hj1 = 0;
               int neib_x = 0;
               int neib_y = 0;

               int neib_left, neib_right, neib_bot, neib_top;

               // Если есть сосед по X
               if(border_x != 0)
               {
                  neib_x = -border_x - 1;

                  // Сосед слева
                  if(x_cent == 0)
                  {
                     neib_left = regions[neib_x].n_x * (y_cent + 1) - 2;
                     slae->index[1][global_i] = -abs(global_i -
                        (regions[neib_x].first + neib_left));

                     hi = r->x_node[x_cent + 1] - r->x_node[x_cent + 0];
                     hi1 = r->x_node[x_cent - 0] –
                        regions[neib_x].x_node[regions[neib_x].n_x - 2];
                  }
                  // Сосед справа
                  if(x_cent == r->n_x - 1)
                  {
                     neib_right = regions[neib_x].n_x * y_cent + 1;
                     slae->index[3][global_i] = abs(global_i -
                        (regions[neib_x].first + neib_right));

                     hi = regions[neib_x].x_node[1] - r->x_node[x_cent + 0];
                     hi1 = r->x_node[x_cent - 0] - r->x_node[x_cent - 1];
                  }
```

```
    if(border_y == 0)
    {
        hj = r->y_node[y_cent + 1] - r->y_node[y_cent];
        hj1 = r->y_node[y_cent] - r->y_node[y_cent - 1];
    }
}

// Если есть сосед по Y
if(border_y != 0)
{
    neib_y = -border_y - 1;

    // Сосед снизу
    if(y_cent == 0)
    {
        neib_bot = regions[neib_y].n_x * (regions[neib_y].n_y - 2) +
            x_cent;
        slae->index[0][global_i] = -abs(global_i -
            (regions[neib_y].first + neib_bot));

        hj = r->y_node[y_cent + 1] - r->y_node[y_cent + 0];
        hj1 = r->y_node[y_cent - 0] –
            regions[neib_y].y_node[regions[neib_y].n_y - 2];
    }

    // Сосед сверху
    if(y_cent == r->n_y - 1)
    {
        neib_top = regions[neib_y].n_x + x_cent;
        slae->index[4][global_i] = abs(global_i -
            (regions[neib_y].first + neib_top));

        hj = regions[neib_y].y_node[1] - r->y_node[y_cent + 0];
        hj1 = r->y_node[y_cent - 0] - r->y_node[y_cent - 1];
    }

    if(border_x == 0)
    {
        hi = r->x_node[x_cent + 1] - r->x_node[x_cent + 0];
        hi1 = r->x_node[x_cent - 0] - r->x_node[x_cent - 1];
    }
}

    // Нижний узел
slae->matrix[0][global_i] = -test.lambda() *
    (2.0 / (hj1 * (hj + hj1)));

// Левый узел
slae->matrix[1][global_i] = -test.lambda() *
    (2.0 / (hi1 * (hi + hi1)));

// Центральный узел
slae->matrix[2][global_i] = +test.lambda() *
    (2.0 / (hi1 * hi) + 2.0 / (hj1 * hj)) + test.gamma();

// Правый узел
slae->matrix[3][global_i] = -test.lambda() *
    (2.0 / (hi * (hi + hi1)));

// Верхний узел
slae->matrix[4][global_i] = -test.lambda() *
    (2.0 / (hj * (hj + hj1)));
```

```
                    // Вектор правой части
                    slae->f[global_i] = test.f(r->x_node[x_cent], r->y_node[y_cent]);
                }
            }
        }
    }
// Обработка первого кревого условия
    // Проход по всем регионам
    for(int reg_i = 0; reg_i < n_regions; reg_i++)
    {
        Region* r = &regions[reg_i];

        // Проход по всем узлам региона
        for(int node_i = 0; node_i < r->n_nodes; node_i++)
        {
            // Индекс узла в глобальной нумерации
            int global_i = node_i + r->first;

            // Индексы центрального узла
            int x_cent = node_i % r->n_x;
            int y_cent = floor(node_i / r->n_x);

            // Обработка некраевых узлов
            if(x_cent == 0 || x_cent == r->n_x - 1 ||
               0 == y_cent || y_cent == r->n_y - 1)
            {
                int border_x = 0, border_y = 0;

                if(x_cent == 0)
                    border_x = r->borders[0];
                else if(x_cent == r->n_x - 1)
                    border_x = r->borders[1];

                if(y_cent == 0)
                    border_y = r->borders[2];
                else if(y_cent == r->n_y - 1)
                    border_y = r->borders[3];

                // Первое краевое
                if(border_x == 1 || border_y == 1)
                {
                    slae->matrix[0][global_i] = 0;
                    slae->matrix[1][global_i] = 0;
                    slae->matrix[2][global_i] = 1.0;
                    slae->matrix[3][global_i] = 0;
                    slae->matrix[4][global_i] = 0;
                    slae->f[global_i] = test.u(r->x_node[x_cent], r->y_node[y_cent]);

                    slae->index[0][global_i] = -r->n_x;
                    slae->index[1][global_i] = -1;
                    slae->index[2][global_i] = 0;
                    slae->index[3][global_i] = 1;
                    slae->index[4][global_i] = r->n_x;
                }
            }
        }
    }
}
```

```
void PrintSolution(const string& file_name)
{
   ofstream fout(file_name);
   double norm = 0.0, norm_u = 0.0;

   fout << " y          x              calc           prec";
   fout << "      dif           N   reg  location" << endl << fixed;

   // Проход по всем регионам
   for(int reg_i = 0; reg_i < n_regions; reg_i++)
   {
      Region* r = &regions[reg_i];

      // Проход по всем узлам региона
      for(int node_i = 0; node_i < r->n_nodes; node_i++)
      {
         // Индекс узла в глобальной нумерации
         int global_i = node_i + r->first;

         // Индексы центрального узла
         int x_cent = node_i % r->n_x;
         int y_cent = floor(node_i / r->n_x);

         fout << setw(9) << r->y_node[y_cent];
         fout << setw(11) << r->x_node[x_cent];

         double calc = slae->xk[global_i];
         fout << setw(15) << calc;
         double prec = test.u(r->x_node[x_cent], r->y_node[y_cent]);
         fout << setw(15) << prec;

         fout << setw(14) << scientific << abs(prec - calc);
         fout << fixed << setw(5) << global_i << setw(4) << reg_i + 1;

         // Обработка некраевых узлов
         if(0 < x_cent && x_cent < r->n_x - 1 &&
            0 < y_cent && y_cent < r->n_y - 1)
            fout << "  inner";
         else
         {
            int border_x = 0, border_y = 0;

            if(x_cent == 0)
               border_x = r->borders[0];
            else if(x_cent == r->n_x - 1)
               border_x = r->borders[1];

            if(y_cent == 0)
               border_y = r->borders[2];
            else if(y_cent == r->n_y - 1)
               border_y = r->borders[3];

            // Первое краевое
            if(border_x == 1 || border_y == 1)
               fout << "  border";
            else
               if(border_x != 1 && border_y != 1 ||
                  border_x != 1 && border_y == 0 ||
                  border_x == 0 && border_y != 1)
                  fout << "  inner border";
         }
          fout << endl;
          norm_u += prec * prec;
          norm += abs(calc - prec) * abs(calc - prec);
```

```
            }
        }
        fout << "||u-u*||/||u*|| = " << scientific << sqrt(norm) / sqrt(norm_u) << endl;
        fout << "||u-u*|| = " << scientific << sqrt(norm);
        fout.close();
    }
};
```

Файл “Test.h”

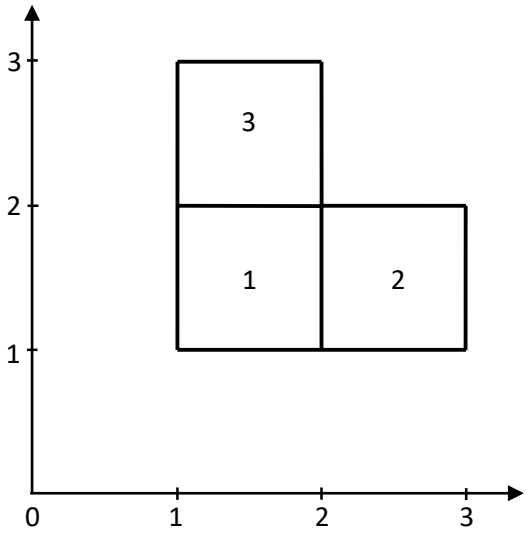
```
#pragma once
using namespace std;

class Test
{
public:

    int N;

    Test(const int& t_N) : N(t_N) {};

    Test() : N(0) {};

    double f(const double& x, const double& y)
    {
        switch(N)
        {
        case(0): return (0)* lambda() + u(x, y) * gamma();
        case(1): return (0)* lambda() + u(x, y) * gamma();
        case(2): return (-4)* lambda() + u(x, y) * gamma();
        case(3): return (-6 * x - 6 * y) * lambda() + u(x, y) * gamma();
        case(4): return (-12 * x * x - 12 * y * y) * lambda() + u(x, y) * gamma();
        };
    }

    double lambda()
    {
        return 1;
    }

    double gamma()
    {
        return 1;
    }

    double u(const double& x, const double& y)
    {
        switch(N)
        {
        case(0): return 2.0;
        case(1): return x + y;
        case(2): return x * x + y * y;
        case(3): return x * x * x + y * y * y;
        case(4): return x * x * x * x + y * y * y * y;
        };
    }

};
```

Файл “Main.cpp”

```
#include <iostream>
#include "EllipticalProblem.h"

using namespace std;

void main()
{
   EllipticalProblem ep = EllipticalProblem();

   ep.ReadFormGrid("regions.txt");

   // Инициализация СЛАУ
   ep.slae = new SLAE(ep.n_nodes);

   for(int reg_i = 0; reg_i < ep.n_regions; reg_i++)
   {
      Region* r = &ep.regions[reg_i];

      for(int node_i = 0; node_i < r->n_nodes; node_i++)
      {
         int global_i = node_i + r->first;

         ep.slae->index[0][global_i] = -r->n_x;
         ep.slae->index[1][global_i] = -1;
         ep.slae->index[2][global_i] = 0;
         ep.slae->index[3][global_i] = 1;
         ep.slae->index[4][global_i] = r->n_x;
      }
   }

   // Инициализация тестовых данных
   ep.test = Test(3);

   // Формирование матрицы системы
   ep.FormMatrix();

   ep.slae->GaussSeidel(10000, 1e-14, 0.65);

   ep.PrintSolution("res.txt");

}
```

6. Тестирование на равномерной сетке

Область имеет L-образную форму



Файл regions.txt

```
3
-1-
1 2 1 2
1 2 1 2
1 -2 1 -3
-2-
2 3 1 2
1 2 1 2
-1 1 1 1
-3-
1 2 2 3
1 2 1 2
1 1 -1 1
```

- $\lambda = 1, \gamma = 1, u^* = x^3 + y^3, f = -6(x + y) + x^3 + y^3$

y	x	calc	prec	dif	N	reg	location
1.000000	1.000000	2.000000	2.000000	0.000000e+00	0	1	border
1.000000	1.500000	4.375000	4.375000	8.881784e-16	1	1	border
1.000000	2.000000	9.000000	9.000000	1.776357e-15	2	1	border
1.500000	1.000000	4.375000	4.375000	8.881784e-16	3	1	border
1.500000	1.500000	6.750000	6.750000	8.881784e-16	4	1	inner
1.500000	2.000000	11.375000	11.375000	1.776357e-15	5	1	inner border
2.000000	1.000000	9.000000	9.000000	1.776357e-15	6	1	border
2.000000	1.500000	11.375000	11.375000	1.776357e-15	7	1	inner border
2.000000	2.000000	16.000000	16.000000	3.552714e-15	8	1	inner border
1.000000	2.000000	9.000000	9.000000	1.776357e-15	9	2	border
1.000000	2.500000	16.625000	16.625000	3.552714e-15	10	2	border
1.000000	3.000000	28.000000	28.000000	0.000000e+00	11	2	border
1.500000	2.000000	11.375000	11.375000	0.000000e+00	12	2	inner border
1.500000	2.500000	19.000000	19.000000	3.552714e-15	13	2	inner
1.500000	3.000000	30.375000	30.375000	0.000000e+00	14	2	border
2.000000	2.000000	16.000000	16.000000	0.000000e+00	15	2	border
2.000000	2.500000	23.625000	23.625000	0.000000e+00	16	2	border
2.000000	3.000000	35.000000	35.000000	7.105427e-15	17	2	border
2.000000	1.000000	9.000000	9.000000	1.776357e-15	18	3	border
2.000000	1.500000	11.375000	11.375000	0.000000e+00	19	3	inner border
2.000000	2.000000	16.000000	16.000000	0.000000e+00	20	3	border
2.500000	1.000000	16.625000	16.625000	3.552714e-15	21	3	border
2.500000	1.500000	19.000000	19.000000	3.552714e-15	22	3	inner
2.500000	2.000000	23.625000	23.625000	0.000000e+00	23	3	border
3.000000	1.000000	28.000000	28.000000	0.000000e+00	24	3	border
3.000000	1.500000	30.375000	30.375000	0.000000e+00	25	3	border
3.000000	2.000000	35.000000	35.000000	7.105427e-15	26	3	border

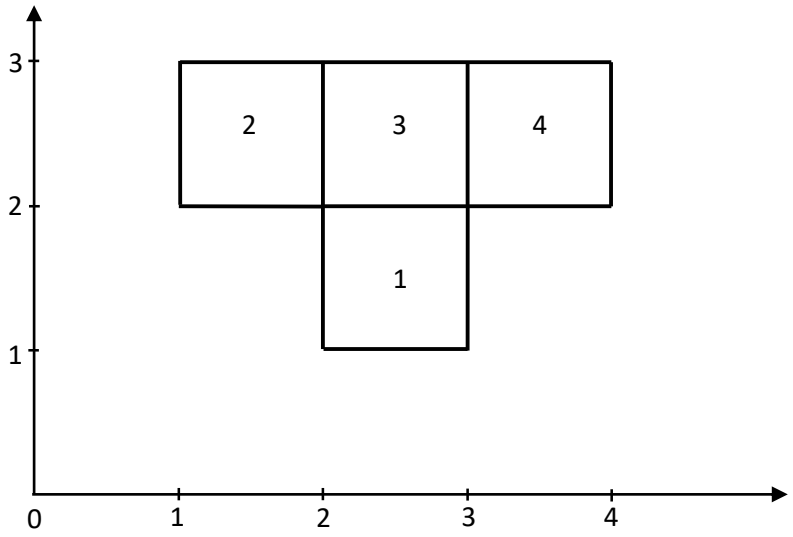
```
||u-u*||/||u*|| = 1.364848e-16
||u-u*|| = 1.361552e-14
```

- $\lambda = 1, \gamma = 1, u^* = x^4 + y^4, f = -12(x^2 + y^2) + x^4 + y^4$

y	x	calc	prec	dif	N	reg	location
1.000000	1.000000	2.000000	2.000000	0.000000e+00	0	1	border
1.000000	1.500000	6.062500	6.062500	0.000000e+00	1	1	border
1.000000	2.000000	17.000000	17.000000	3.552714e-15	2	1	border
1.500000	1.000000	6.062500	6.062500	0.000000e+00	3	1	border
1.500000	1.500000	10.248077	10.125000	1.230772e-01	4	1	inner
1.500000	2.000000	21.199039	21.062500	1.365390e-01	5	1	inner border
2.000000	1.000000	17.000000	17.000000	3.552714e-15	6	1	border
2.000000	1.500000	21.199039	21.062500	1.365390e-01	7	1	inner border
2.000000	2.000000	32.123077	32.000000	1.230772e-01	8	1	inner border
1.000000	2.000000	17.000000	17.000000	3.552714e-15	9	2	border
1.000000	2.500000	40.062500	40.062500	7.105427e-15	10	2	border
1.000000	3.000000	82.000000	82.000000	1.421085e-14	11	2	border
1.500000	2.000000	21.170080	21.062500	1.075797e-01	12	2	inner border
1.500000	2.500000	44.209136	44.125000	8.413639e-02	13	2	inner
1.500000	3.000000	86.062500	86.062500	1.421085e-14	14	2	border
2.000000	2.000000	32.000000	32.000000	0.000000e+00	15	2	border
2.000000	2.500000	55.062500	55.062500	0.000000e+00	16	2	border
2.000000	3.000000	97.000000	97.000000	0.000000e+00	17	2	border
2.000000	1.000000	17.000000	17.000000	3.552714e-15	18	3	border
2.000000	1.500000	21.170080	21.062500	1.075797e-01	19	3	inner border
2.000000	2.000000	32.000000	32.000000	0.000000e+00	20	3	border
2.500000	1.000000	40.062500	40.062500	7.105427e-15	21	3	border
2.500000	1.500000	44.209136	44.125000	8.413639e-02	22	3	inner
2.500000	2.000000	55.062500	55.062500	0.000000e+00	23	3	border
3.000000	1.000000	82.000000	82.000000	1.421085e-14	24	3	border
3.000000	1.500000	86.062500	86.062500	1.421085e-14	25	3	border
3.000000	2.000000	97.000000	97.000000	0.000000e+00	26	3	border

||u-u*||/||u*|| = 1.256545e-03
||u-u*|| = 3.238617e-01

Область имеет Т-образную форму



Файл regions.txt

```
4
-1-
2 3 1 2
1 2 1 2
1 1 1 -3
-2-
1 2 2 3
1 2 1 2
1 -3 1 1
-3-
2 3 2 3
1 2 1 2
-2 -4 -1 1
-4-
3 4 2 3
1 2 1 2
-3 1 1 1
```

- $\lambda = 1, \gamma = 1, u^* = x^3 + y^3, f = -6(x + y) + x^3 + y^3$

y	x	calc	prec	dif	N	reg	location
1.000000	2.000000	9.000000	9.000000	1.776357e-15	0	1	border
1.000000	2.500000	16.625000	16.625000	3.552714e-15	1	1	border
1.000000	3.000000	28.000000	28.000000	0.000000e+00	2	1	border
1.500000	2.000000	11.375000	11.375000	0.000000e+00	3	1	border
1.500000	2.500000	19.000000	19.000000	0.000000e+00	4	1	inner
1.500000	3.000000	30.375000	30.375000	0.000000e+00	5	1	border
2.000000	2.000000	16.000000	16.000000	0.000000e+00	6	1	border
2.000000	2.500000	23.625000	23.625000	7.105427e-15	7	1	inner border
2.000000	3.000000	35.000000	35.000000	7.105427e-15	8	1	border
2.000000	1.000000	9.000000	9.000000	1.776357e-15	9	2	border
2.000000	1.500000	11.375000	11.375000	0.000000e+00	10	2	border
2.000000	2.000000	16.000000	16.000000	0.000000e+00	11	2	border
2.500000	1.000000	16.625000	16.625000	3.552714e-15	12	2	border
2.500000	1.500000	19.000000	19.000000	0.000000e+00	13	2	inner
2.500000	2.000000	23.625000	23.625000	7.105427e-15	14	2	inner border
3.000000	1.000000	28.000000	28.000000	0.000000e+00	15	2	border
3.000000	1.500000	30.375000	30.375000	0.000000e+00	16	2	border
3.000000	2.000000	35.000000	35.000000	7.105427e-15	17	2	border
2.000000	2.000000	16.000000	16.000000	3.552714e-15	18	3	inner border
2.000000	2.500000	23.625000	23.625000	3.552714e-15	19	3	inner border
2.000000	3.000000	35.000000	35.000000	7.105427e-15	20	3	inner border
2.500000	2.000000	23.625000	23.625000	7.105427e-15	21	3	inner border
2.500000	2.500000	31.250000	31.250000	3.552714e-15	22	3	inner
2.500000	3.000000	42.625000	42.625000	7.105427e-15	23	3	inner border
3.000000	2.000000	35.000000	35.000000	7.105427e-15	24	3	border
3.000000	2.500000	42.625000	42.625000	7.105427e-15	25	3	border
3.000000	3.000000	54.000000	54.000000	0.000000e+00	26	3	border
2.000000	3.000000	35.000000	35.000000	7.105427e-15	27	4	border
2.000000	3.500000	50.875000	50.875000	0.000000e+00	28	4	border
2.000000	4.000000	72.000000	72.000000	1.421085e-14	29	4	border
2.500000	3.000000	42.625000	42.625000	0.000000e+00	30	4	inner border
2.500000	3.500000	58.500000	58.500000	1.421085e-14	31	4	inner
2.500000	4.000000	79.625000	79.625000	1.421085e-14	32	4	border
3.000000	3.000000	54.000000	54.000000	0.000000e+00	33	4	border
3.000000	3.500000	69.875000	69.875000	1.421085e-14	34	4	border
3.000000	4.000000	91.000000	91.000000	0.000000e+00	35	4	border

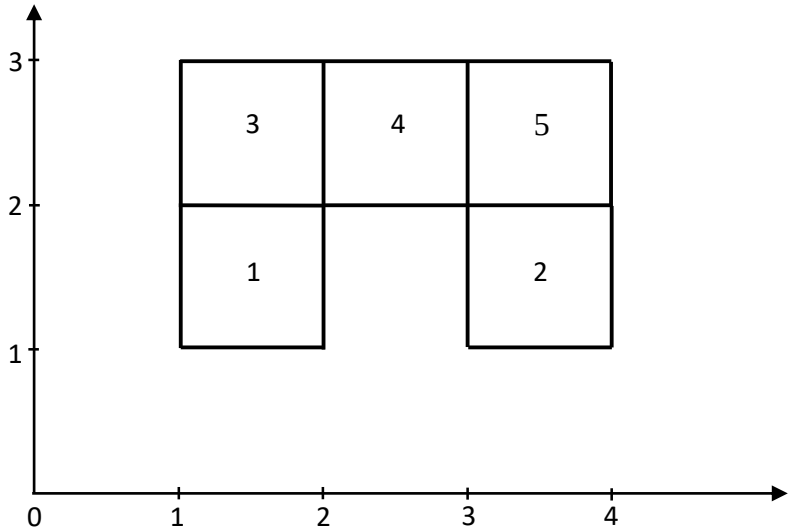
```
||u-u*||/||u*|| = 1.554534e-16
||u-u*|| = 3.717639e-14
```

- $\lambda = 1, \gamma = 1, u^* = x^4 + y^4, f = -12(x^2 + y^2) + x^4 + y^4$

y	x	calc	prec	dif	N	reg	location
1.000000	2.000000	17.000000	17.000000	3.552714e-15	0	1	border
1.000000	2.500000	40.062500	40.062500	7.105427e-15	1	1	border
1.000000	3.000000	82.000000	82.000000	1.421085e-14	2	1	border
1.500000	2.000000	21.062500	21.062500	3.552714e-15	3	1	border
1.500000	2.500000	44.212148	44.125000	8.714793e-02	4	1	inner
1.500000	3.000000	86.062500	86.062500	1.421085e-14	5	1	border
2.000000	2.000000	32.000000	32.000000	0.000000e+00	6	1	border
2.000000	2.500000	55.182879	55.062500	1.203787e-01	7	1	inner border
2.000000	3.000000	97.000000	97.000000	0.000000e+00	8	1	border
2.000000	1.000000	17.000000	17.000000	3.552714e-15	9	2	border
2.000000	1.500000	21.062500	21.062500	3.552714e-15	10	2	border
2.000000	2.000000	32.000000	32.000000	0.000000e+00	11	2	border
2.500000	1.000000	40.062500	40.062500	7.105427e-15	12	2	border
2.500000	1.500000	44.212148	44.125000	8.714793e-02	13	2	inner
2.500000	2.000000	55.182879	55.062500	1.203787e-01	14	2	inner border
3.000000	1.000000	82.000000	82.000000	1.421085e-14	15	2	border
3.000000	1.500000	86.062500	86.062500	1.421085e-14	16	2	border
3.000000	2.000000	97.000000	97.000000	0.000000e+00	17	2	border
2.000000	2.000000	32.138471	32.000000	1.384710e-01	18	3	inner border
2.000000	2.500000	55.248042	55.062500	1.855415e-01	19	3	inner border
2.000000	3.000000	97.138471	97.000000	1.384710e-01	20	3	inner border
2.500000	2.000000	55.215460	55.062500	1.529601e-01	21	3	inner border
2.500000	2.500000	78.299462	78.125000	1.744616e-01	22	3	inner
2.500000	3.000000	120.215460	120.062500	1.529601e-01	23	3	inner border
3.000000	2.000000	97.000000	97.000000	0.000000e+00	24	3	border
3.000000	2.500000	120.062500	120.062500	0.000000e+00	25	3	border
3.000000	3.000000	162.000000	162.000000	2.842171e-14	26	3	border
2.000000	3.000000	97.000000	97.000000	0.000000e+00	27	4	border
2.000000	3.500000	166.062500	166.062500	2.842171e-14	28	4	border
2.000000	4.000000	272.000000	272.000000	5.684342e-14	29	4	border
2.500000	3.000000	120.182879	120.062500	1.203787e-01	30	4	inner border
2.500000	3.500000	189.212148	189.125000	8.714793e-02	31	4	inner
2.500000	4.000000	295.062500	295.062500	5.684342e-14	32	4	border
3.000000	3.000000	162.000000	162.000000	2.842171e-14	33	4	border
3.000000	3.500000	231.062500	231.062500	0.000000e+00	34	4	border
3.000000	4.000000	337.000000	337.000000	5.684342e-14	35	4	border

```
||u-u*||/||u*|| = 6.042730e-04
||u-u*|| = 4.650397e-01
```

Область имеет П-образную форму



Файл regions.txt

```
5
-1-
1 2 1 2
1 2 1 2
1 1 1 -3
-2-
3 4 1 2
1 2 1 2
1 1 1 -5
-3-
1 2 2 3
1 2 1 2
1 -4 -1 1
-4-
2 3 2 3
1 2 1 2
-3 -5 1 1
-5-
3 4 2 3
1 2 1 2
-4 1 -2 1
```

- $\lambda = 1, \gamma = 1, u^* = x^3 + y^3, f = -6(x + y) + x^3 + y^3$

y	x	calc	prec	dif	N	reg	location
1.000000	1.000000	2.000000	2.000000	0.000000e+00	0	1	border
1.000000	1.500000	4.375000	4.375000	8.881784e-16	1	1	border
1.000000	2.000000	9.000000	9.000000	1.776357e-15	2	1	border
1.500000	1.000000	4.375000	4.375000	8.881784e-16	3	1	border
1.500000	1.500000	6.750000	6.750000	0.000000e+00	4	1	inner
1.500000	2.000000	11.375000	11.375000	0.000000e+00	5	1	border
2.000000	1.000000	9.000000	9.000000	1.776357e-15	6	1	border
2.000000	1.500000	11.375000	11.375000	0.000000e+00	7	1	inner border
2.000000	2.000000	16.000000	16.000000	0.000000e+00	8	1	border
1.000000	3.000000	28.000000	28.000000	0.000000e+00	9	2	border
1.000000	3.500000	43.875000	43.875000	7.105427e-15	10	2	border
1.000000	4.000000	65.000000	65.000000	1.421085e-14	11	2	border
1.500000	3.000000	30.375000	30.375000	0.000000e+00	12	2	border
1.500000	3.500000	46.250000	46.250000	7.105427e-15	13	2	inner
1.500000	4.000000	67.375000	67.375000	1.421085e-14	14	2	border
2.000000	3.000000	35.000000	35.000000	7.105427e-15	15	2	border
2.000000	3.500000	50.875000	50.875000	7.105427e-15	16	2	inner border
2.000000	4.000000	72.000000	72.000000	1.421085e-14	17	2	border
2.000000	1.000000	9.000000	9.000000	1.776357e-15	18	3	border
2.000000	1.500000	11.375000	11.375000	1.776357e-15	19	3	inner border
2.000000	2.000000	16.000000	16.000000	0.000000e+00	20	3	inner border
2.500000	1.000000	16.625000	16.625000	3.552714e-15	21	3	border
2.500000	1.500000	19.000000	19.000000	0.000000e+00	22	3	inner
2.500000	2.000000	23.625000	23.625000	3.552714e-15	23	3	inner border
3.000000	1.000000	28.000000	28.000000	0.000000e+00	24	3	border
3.000000	1.500000	30.375000	30.375000	0.000000e+00	25	3	border
3.000000	2.000000	35.000000	35.000000	7.105427e-15	26	3	border
2.000000	2.000000	16.000000	16.000000	0.000000e+00	27	4	border
2.000000	2.500000	23.625000	23.625000	0.000000e+00	28	4	border
2.000000	3.000000	35.000000	35.000000	7.105427e-15	29	4	border
2.500000	2.000000	23.625000	23.625000	0.000000e+00	30	4	inner border
2.500000	2.500000	31.250000	31.250000	0.000000e+00	31	4	inner
2.500000	3.000000	42.625000	42.625000	0.000000e+00	32	4	inner border
3.000000	2.000000	35.000000	35.000000	7.105427e-15	33	4	border
3.000000	2.500000	42.625000	42.625000	7.105427e-15	34	4	border
3.000000	3.000000	54.000000	54.000000	0.000000e+00	35	4	border
2.000000	3.000000	35.000000	35.000000	7.105427e-15	36	5	inner border
2.000000	3.500000	50.875000	50.875000	0.000000e+00	37	5	inner border
2.000000	4.000000	72.000000	72.000000	1.421085e-14	38	5	border
2.500000	3.000000	42.625000	42.625000	0.000000e+00	39	5	inner border
2.500000	3.500000	58.500000	58.500000	7.105427e-15	40	5	inner
2.500000	4.000000	79.625000	79.625000	1.421085e-14	41	5	border
3.000000	3.000000	54.000000	54.000000	0.000000e+00	42	5	border
3.000000	3.500000	69.875000	69.875000	1.421085e-14	43	5	border
3.000000	4.000000	91.000000	91.000000	0.000000e+00	44	5	border

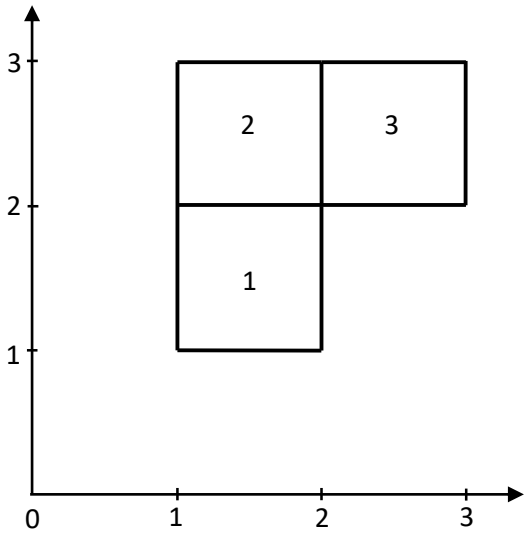
||u-u*||/||u*|| = 1.511169e-16
||u-u*|| = 4.190471e-14

- $\lambda = 1, \gamma = 1, u^* = x^4 + y^4, f = -12(x^2 + y^2) + x^4 + y^4$

y	x	calc	prec	dif	N	reg	location
1.000000	1.000000	2.000000	2.000000	0.000000e+00	0	1	border
1.000000	1.500000	6.062500	6.062500	0.000000e+00	1	1	border
1.000000	2.000000	17.000000	17.000000	3.552714e-15	2	1	border
1.500000	1.000000	6.062500	6.062500	0.000000e+00	3	1	border
1.500000	1.500000	10.209257	10.125000	8.425659e-02	4	1	inner
1.500000	2.000000	21.062500	21.062500	3.552714e-15	5	1	border
2.000000	1.000000	17.000000	17.000000	3.552714e-15	6	1	border
2.000000	1.500000	21.170590	21.062500	1.080905e-01	7	1	inner border
2.000000	2.000000	32.000000	32.000000	0.000000e+00	8	1	border
1.000000	3.000000	82.000000	82.000000	1.421085e-14	9	2	border
1.000000	3.500000	151.062500	151.062500	2.842171e-14	10	2	border
1.000000	4.000000	257.000000	257.000000	5.684342e-14	11	2	border
1.500000	3.000000	86.062500	86.062500	1.421085e-14	12	2	border
1.500000	3.500000	155.209257	155.125000	8.425659e-02	13	2	inner
1.500000	4.000000	261.062500	261.062500	5.684342e-14	14	2	border
2.000000	3.000000	97.000000	97.000000	0.000000e+00	15	2	border
2.000000	3.500000	166.170590	166.062500	1.080905e-01	16	2	inner border
2.000000	4.000000	272.000000	272.000000	5.684342e-14	17	2	border
2.000000	1.000000	17.000000	17.000000	3.552714e-15	18	3	border
2.000000	1.500000	21.200032	21.062500	1.375324e-01	19	3	inner border
2.000000	2.000000	32.125128	32.000000	1.251280e-01	20	3	inner border
2.500000	1.000000	40.062500	40.062500	7.105427e-15	21	3	border
2.500000	1.500000	44.250128	44.125000	1.251280e-01	22	3	inner
2.500000	2.000000	55.206762	55.062500	1.442618e-01	23	3	inner border
3.000000	1.000000	82.000000	82.000000	1.421085e-14	24	3	border
3.000000	1.500000	86.062500	86.062500	1.421085e-14	25	3	border
3.000000	2.000000	97.000000	97.000000	0.000000e+00	26	3	border
2.000000	2.000000	32.000000	32.000000	0.000000e+00	27	4	border
2.000000	2.500000	55.062500	55.062500	0.000000e+00	28	4	border
2.000000	3.000000	97.000000	97.000000	0.000000e+00	29	4	border
2.500000	2.000000	55.177320	55.062500	1.148199e-01	30	4	inner border
2.500000	2.500000	78.237856	78.125000	1.128564e-01	31	4	inner
2.500000	3.000000	120.177320	120.062500	1.148199e-01	32	4	inner border
3.000000	2.000000	97.000000	97.000000	0.000000e+00	33	4	border
3.000000	2.500000	120.062500	120.062500	0.000000e+00	34	4	border
3.000000	3.000000	162.000000	162.000000	2.842171e-14	35	4	border
2.000000	3.000000	97.125128	97.000000	1.251280e-01	36	5	inner border
2.000000	3.500000	166.200032	166.062500	1.375324e-01	37	5	inner border
2.000000	4.000000	272.000000	272.000000	5.684342e-14	38	5	border
2.500000	3.000000	120.206762	120.062500	1.442618e-01	39	5	inner border
2.500000	3.500000	189.250128	189.125000	1.251280e-01	40	5	inner
2.500000	4.000000	295.062500	295.062500	5.684342e-14	41	5	border
3.000000	3.000000	162.000000	162.000000	2.842171e-14	42	5	border
3.000000	3.500000	231.062500	231.062500	0.000000e+00	43	5	border
3.000000	4.000000	337.000000	337.000000	5.684342e-14	44	5	border

```
||u-u*||/||u*|| = 5.016553e-04
||u-u*|| = 4.677078e-01
```

Область имеет Г-образную форму



Файл regions.txt

```
3
-1-
1 2 1 2
1 2 1 2
1 1 1 -2
-2-
1 2 2 3
1 2 1 2
1 -3 -1 1
-3-
2 3 2 3
1 2 1 2
-2 1 1 1
```

- $\lambda = 1, \gamma = 1, u^* = x^3 + y^3, f = -6(x + y) + x^3 + y^3$

y	x	calc	prec	dif	N	reg	location
1.000000	1.000000	2.000000	2.000000	0.000000e+00	0	1	border
1.000000	1.500000	4.375000	4.375000	8.881784e-16	1	1	border
1.000000	2.000000	9.000000	9.000000	1.776357e-15	2	1	border
1.500000	1.000000	4.375000	4.375000	8.881784e-16	3	1	border
1.500000	1.500000	6.750000	6.750000	1.776357e-15	4	1	inner
1.500000	2.000000	11.375000	11.375000	0.000000e+00	5	1	border
2.000000	1.000000	9.000000	9.000000	1.776357e-15	6	1	border
2.000000	1.500000	11.375000	11.375000	0.000000e+00	7	1	inner border
2.000000	2.000000	16.000000	16.000000	0.000000e+00	8	1	border
2.000000	1.000000	9.000000	9.000000	1.776357e-15	9	2	border
2.000000	1.500000	11.375000	11.375000	0.000000e+00	10	2	inner border
2.000000	2.000000	16.000000	16.000000	0.000000e+00	11	2	inner border
2.500000	1.000000	16.625000	16.625000	3.552714e-15	12	2	border
2.500000	1.500000	19.000000	19.000000	3.552714e-15	13	2	inner
2.500000	2.000000	23.625000	23.625000	0.000000e+00	14	2	inner border
3.000000	1.000000	28.000000	28.000000	0.000000e+00	15	2	border
3.000000	1.500000	30.375000	30.375000	0.000000e+00	16	2	border
3.000000	2.000000	35.000000	35.000000	7.105427e-15	17	2	border
2.000000	2.000000	16.000000	16.000000	0.000000e+00	18	3	border
2.000000	2.500000	23.625000	23.625000	0.000000e+00	19	3	border
2.000000	3.000000	35.000000	35.000000	7.105427e-15	20	3	border
2.500000	2.000000	23.625000	23.625000	3.552714e-15	21	3	inner border
2.500000	2.500000	31.250000	31.250000	7.105427e-15	22	3	inner
2.500000	3.000000	42.625000	42.625000	7.105427e-15	23	3	border
3.000000	2.000000	35.000000	35.000000	7.105427e-15	24	3	border
3.000000	2.500000	42.625000	42.625000	7.105427e-15	25	3	border
3.000000	3.000000	54.000000	54.000000	0.000000e+00	26	3	border

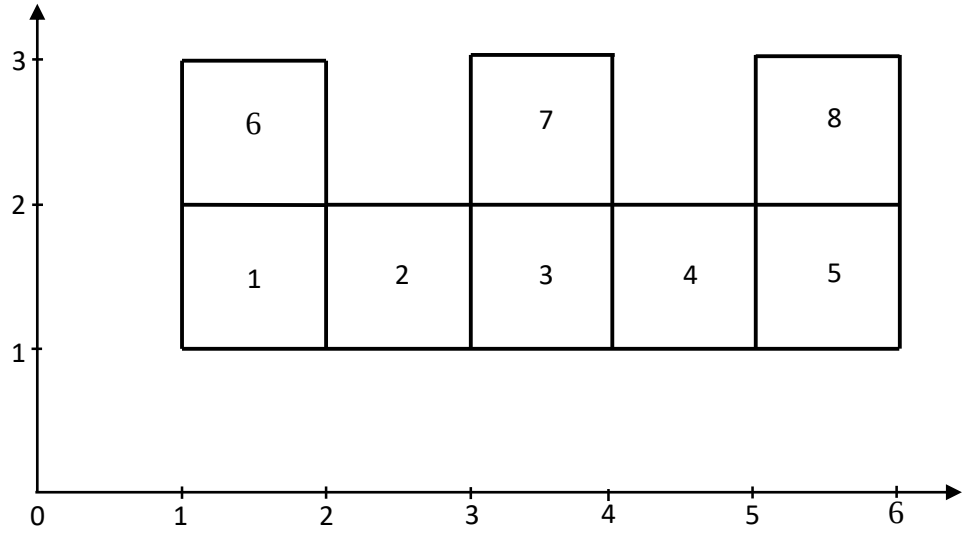
```
||u-u*||/||u*|| = 1.457672e-16
||u-u*|| = 1.884111e-14
```

- $\lambda = 1, \gamma = 1, u^* = x^4 + y^4, f = -12(x^2 + y^2) + x^4 + y^4$

y	x	calc	prec	dif	N	reg	location
1.000000	1.000000	2.000000	2.000000	0.000000e+00	0	1	border
1.000000	1.500000	6.062500	6.062500	0.000000e+00	1	1	border
1.000000	2.000000	17.000000	17.000000	3.552714e-15	2	1	border
1.500000	1.000000	6.062500	6.062500	0.000000e+00	3	1	border
1.500000	1.500000	10.209136	10.125000	8.413639e-02	4	1	inner
1.500000	2.000000	21.062500	21.062500	3.552714e-15	5	1	border
2.000000	1.000000	17.000000	17.000000	3.552714e-15	6	1	border
2.000000	1.500000	21.170080	21.062500	1.075797e-01	7	1	inner border
2.000000	2.000000	32.000000	32.000000	0.000000e+00	8	1	border
2.000000	1.000000	17.000000	17.000000	3.552714e-15	9	2	border
2.000000	1.500000	21.199039	21.062500	1.365390e-01	10	2	inner border
2.000000	2.000000	32.123077	32.000000	1.230772e-01	11	2	inner border
2.500000	1.000000	40.062500	40.062500	7.105427e-15	12	2	border
2.500000	1.500000	44.248077	44.125000	1.230772e-01	13	2	inner
2.500000	2.000000	55.199039	55.062500	1.365390e-01	14	2	inner border
3.000000	1.000000	82.000000	82.000000	1.421085e-14	15	2	border
3.000000	1.500000	86.062500	86.062500	1.421085e-14	16	2	border
3.000000	2.000000	97.000000	97.000000	0.000000e+00	17	2	border
2.000000	2.000000	32.000000	32.000000	0.000000e+00	18	3	border
2.000000	2.500000	55.062500	55.062500	0.000000e+00	19	3	border
2.000000	3.000000	97.000000	97.000000	0.000000e+00	20	3	border
2.500000	2.000000	55.170080	55.062500	1.075797e-01	21	3	inner border
2.500000	2.500000	78.209136	78.125000	8.413639e-02	22	3	inner
2.500000	3.000000	120.062500	120.062500	0.000000e+00	23	3	border
3.000000	2.000000	97.000000	97.000000	0.000000e+00	24	3	border
3.000000	2.500000	120.062500	120.062500	0.000000e+00	25	3	border
3.000000	3.000000	162.000000	162.000000	2.842171e-14	26	3	border

||u-u*||/||u*|| = 9.286118e-04
||u-u*|| = 3.238617e-01

Область имеет Ш-образную форму



Файл regions.txt

```
8
-1-
1 2 1 2
1 2 1 2
1 -2 1 -6
-2-
2 3 1 2
1 2 1 2
-1 -3 1 1
-3-
3 4 1 2
1 2 1 2
-2 -4 1 -7
-4-
4 5 1 2
1 2 1 2
-3 -5 1 1
-5-
5 6 1 2
1 2 1 2
-4 1 1 -8
-6-
1 2 2 3
1 2 1 2
1 1 -1 1
-7-
3 4 2 3
1 2 1 2
1 1 -3 1
-8-
5 6 2 3
1 2 1 2
1 1 -5 1
```

- $\lambda = 1, \gamma = 1, u^* = x^3 + y^3, f = -6(x + y) + x^3 + y^3$

y	x	calc	prec	dif	N	reg	location
1.000000	1.000000	2.000000	2.000000	0.000000e+00	0	1	border
1.000000	1.500000	4.375000	4.375000	8.881784e-16	1	1	border
1.000000	2.000000	9.000000	9.000000	1.776357e-15	2	1	border
1.500000	1.000000	4.375000	4.375000	8.881784e-16	3	1	border
1.500000	1.500000	6.750000	6.750000	0.000000e+00	4	1	inner
1.500000	2.000000	11.375000	11.375000	0.000000e+00	5	1	inner border
2.000000	1.000000	9.000000	9.000000	1.776357e-15	6	1	border
2.000000	1.500000	11.375000	11.375000	3.552714e-15	7	1	inner border
2.000000	2.000000	16.000000	16.000000	3.552714e-15	8	1	inner border
1.000000	2.000000	9.000000	9.000000	1.776357e-15	9	2	border
1.000000	2.500000	16.625000	16.625000	3.552714e-15	10	2	border
1.000000	3.000000	28.000000	28.000000	0.000000e+00	11	2	border
1.500000	2.000000	11.375000	11.375000	0.000000e+00	12	2	inner border
1.500000	2.500000	19.000000	19.000000	3.552714e-15	13	2	inner
1.500000	3.000000	30.375000	30.375000	7.105427e-15	14	2	inner border
2.000000	2.000000	16.000000	16.000000	0.000000e+00	15	2	border
2.000000	2.500000	23.625000	23.625000	0.000000e+00	16	2	border
2.000000	3.000000	35.000000	35.000000	7.105427e-15	17	2	border
1.000000	3.000000	28.000000	28.000000	0.000000e+00	18	3	border
1.000000	3.500000	43.875000	43.875000	7.105427e-15	19	3	border
1.000000	4.000000	65.000000	65.000000	1.421085e-14	20	3	border
1.500000	3.000000	30.375000	30.375000	3.552714e-15	21	3	inner border
1.500000	3.500000	46.250000	46.250000	7.105427e-15	22	3	inner
1.500000	4.000000	67.375000	67.375000	0.000000e+00	23	3	inner border
2.000000	3.000000	35.000000	35.000000	0.000000e+00	24	3	inner border
2.000000	3.500000	50.875000	50.875000	7.105427e-15	25	3	inner border
2.000000	4.000000	72.000000	72.000000	1.421085e-14	26	3	inner border
1.000000	4.000000	65.000000	65.000000	1.421085e-14	27	4	border
1.000000	4.500000	92.125000	92.125000	0.000000e+00	28	4	border
1.000000	5.000000	126.000000	126.000000	0.000000e+00	29	4	border
1.500000	4.000000	67.375000	67.375000	0.000000e+00	30	4	inner border
1.500000	4.500000	94.500000	94.500000	0.000000e+00	31	4	inner
1.500000	5.000000	128.375000	128.375000	5.684342e-14	32	4	inner border
2.000000	4.000000	72.000000	72.000000	1.421085e-14	33	4	border
2.000000	4.500000	99.125000	99.125000	0.000000e+00	34	4	border
2.000000	5.000000	133.000000	133.000000	2.842171e-14	35	4	border
1.000000	5.000000	126.000000	126.000000	0.000000e+00	36	5	border
1.000000	5.500000	167.375000	167.375000	2.842171e-14	37	5	border
1.000000	6.000000	217.000000	217.000000	0.000000e+00	38	5	border
1.500000	5.000000	128.375000	128.375000	0.000000e+00	39	5	inner border
1.500000	5.500000	169.750000	169.750000	2.842171e-14	40	5	inner
1.500000	6.000000	219.375000	219.375000	0.000000e+00	41	5	border
2.000000	5.000000	133.000000	133.000000	0.000000e+00	42	5	inner border
2.000000	5.500000	174.375000	174.375000	0.000000e+00	43	5	inner border
2.000000	6.000000	224.000000	224.000000	0.000000e+00	44	5	border
2.000000	1.000000	9.000000	9.000000	1.776357e-15	45	6	border
2.000000	1.500000	11.375000	11.375000	0.000000e+00	46	6	inner border
2.000000	2.000000	16.000000	16.000000	0.000000e+00	47	6	border

y	x	calc	prec	dif	N	reg	location
2.500000	1.000000	16.625000	16.625000	3.552714e-15	48	6	border
2.500000	1.500000	19.000000	19.000000	3.552714e-15	49	6	inner
2.500000	2.000000	23.625000	23.625000	0.000000e+00	50	6	border
3.000000	1.000000	28.000000	28.000000	0.000000e+00	51	6	border
3.000000	1.500000	30.375000	30.375000	0.000000e+00	52	6	border
3.000000	2.000000	35.000000	35.000000	7.105427e-15	53	6	border
2.000000	3.000000	35.000000	35.000000	7.105427e-15	54	7	border
2.000000	3.500000	50.875000	50.875000	7.105427e-15	55	7	inner border
2.000000	4.000000	72.000000	72.000000	1.421085e-14	56	7	border
2.500000	3.000000	42.625000	42.625000	7.105427e-15	57	7	border
2.500000	3.500000	58.500000	58.500000	1.421085e-14	58	7	inner
2.500000	4.000000	79.625000	79.625000	1.421085e-14	59	7	border
3.000000	3.000000	54.000000	54.000000	0.000000e+00	60	7	border
3.000000	3.500000	69.875000	69.875000	1.421085e-14	61	7	border
3.000000	4.000000	91.000000	91.000000	0.000000e+00	62	7	border
2.000000	5.000000	133.000000	133.000000	2.842171e-14	63	8	border
2.000000	5.500000	174.375000	174.375000	0.000000e+00	64	8	inner border
2.000000	6.000000	224.000000	224.000000	0.000000e+00	65	8	border
2.500000	5.000000	140.625000	140.625000	2.842171e-14	66	8	border
2.500000	5.500000	182.000000	182.000000	2.842171e-14	67	8	inner
2.500000	6.000000	231.625000	231.625000	0.000000e+00	68	8	border
3.000000	5.000000	152.000000	152.000000	2.842171e-14	69	8	border
3.000000	5.500000	193.375000	193.375000	0.000000e+00	70	8	border
3.000000	6.000000	243.000000	243.000000	0.000000e+00	71	8	border

||u-u*||/||u*|| = 1.199623e-16
||u-u*|| = 1.051582e-13

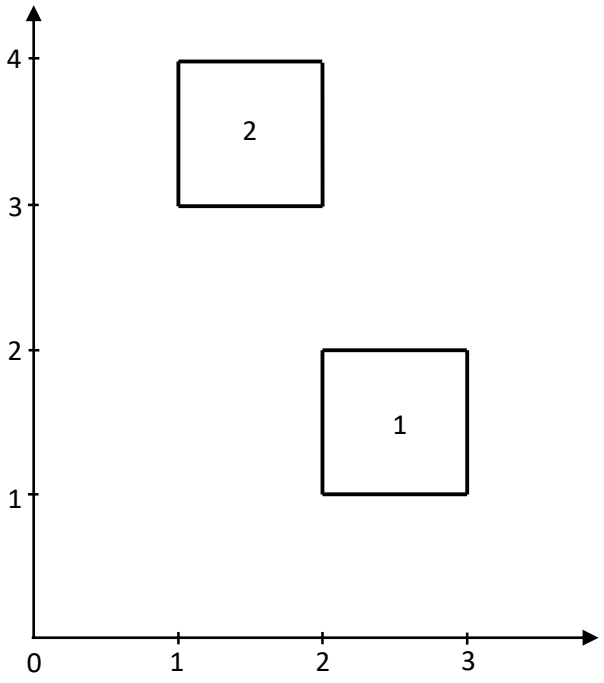
- $\lambda = 1, \gamma = 1, u^* = x^4 + y^4, f = -12(x^2 + y^2) + x^4 + y^4$

y	x	calc	prec	dif	N	reg	location
1.000000	1.000000	2.000000	2.000000	0.000000e+00	0	1	border
1.000000	1.500000	6.062500	6.062500	0.000000e+00	1	1	border
1.000000	2.000000	17.000000	17.000000	3.552714e-15	2	1	border
1.500000	1.000000	6.062500	6.062500	0.000000e+00	3	1	border
1.500000	1.500000	10.250369	10.125000	1.253687e-01	4	1	inner
1.500000	2.000000	21.207668	21.062500	1.451680e-01	5	1	inner border
2.000000	1.000000	17.000000	17.000000	3.552714e-15	6	1	border
2.000000	1.500000	21.200149	21.062500	1.376490e-01	7	1	inner border
2.000000	2.000000	32.125369	32.000000	1.253687e-01	8	1	inner border
1.000000	2.000000	17.000000	17.000000	3.552714e-15	9	2	border
1.000000	2.500000	40.062500	40.062500	7.105427e-15	10	2	border
1.000000	3.000000	82.000000	82.000000	1.421085e-14	11	2	border
1.500000	2.000000	21.178169	21.062500	1.156694e-01	12	2	inner border
1.500000	2.500000	44.241226	44.125000	1.162264e-01	13	2	inner
1.500000	3.000000	86.190793	86.062500	1.282929e-01	14	2	inner border
2.000000	2.000000	32.000000	32.000000	0.000000e+00	15	2	border
2.000000	2.500000	55.062500	55.062500	0.000000e+00	16	2	border
2.000000	3.000000	97.000000	97.000000	0.000000e+00	17	2	border
1.000000	3.000000	82.000000	82.000000	1.421085e-14	18	3	border
1.000000	3.500000	151.062500	151.062500	2.842171e-14	19	3	border
1.000000	4.000000	257.000000	257.000000	5.684342e-14	20	3	border
1.500000	3.000000	86.223975	86.062500	1.614751e-01	21	3	inner border
1.500000	3.500000	155.304018	155.125000	1.790184e-01	22	3	inner
1.500000	4.000000	261.223975	261.062500	1.614751e-01	23	3	inner border
2.000000	3.000000	97.141024	97.000000	1.410243e-01	24	3	inner border
2.000000	3.500000	166.250378	166.062500	1.878781e-01	25	3	inner border
2.000000	4.000000	272.141024	272.000000	1.410243e-01	26	3	inner border
1.000000	4.000000	257.000000	257.000000	5.684342e-14	27	4	border
1.000000	4.500000	411.062500	411.062500	0.000000e+00	28	4	border
1.000000	5.000000	626.000000	626.000000	1.136868e-13	29	4	border
1.500000	4.000000	261.190793	261.062500	1.282929e-01	30	4	inner border
1.500000	4.500000	415.241226	415.125000	1.162264e-01	31	4	inner
1.500000	5.000000	630.178169	630.062500	1.156694e-01	32	4	inner border
2.000000	4.000000	272.000000	272.000000	5.684342e-14	33	4	border
2.000000	4.500000	426.062500	426.062500	0.000000e+00	34	4	border
2.000000	5.000000	641.000000	641.000000	1.136868e-13	35	4	border
1.000000	5.000000	626.000000	626.000000	1.136868e-13	36	5	border
1.000000	5.500000	916.062500	916.062500	0.000000e+00	37	5	border
1.000000	6.000000	1297.000000	1297.000000	2.273737e-13	38	5	border
1.500000	5.000000	630.207668	630.062500	1.451680e-01	39	5	inner border
1.500000	5.500000	920.250369	920.125000	1.253687e-01	40	5	inner
1.500000	6.000000	1301.062500	1301.062500	2.273737e-13	41	5	border
2.000000	5.000000	641.125369	641.000000	1.253687e-01	42	5	inner border
2.000000	5.500000	931.200149	931.062500	1.376490e-01	43	5	inner border
2.000000	6.000000	1312.000000	1312.000000	2.273737e-13	44	5	border
2.000000	1.000000	17.000000	17.000000	3.552714e-15	45	6	border
2.000000	1.500000	21.170650	21.062500	1.081504e-01	46	6	inner border
2.000000	2.000000	32.000000	32.000000	0.000000e+00	47	6	border

y	x	calc	prec	dif	N	reg	location
2.500000	1.000000	40.062500	40.062500	7.105427e-15	48	6	border
2.500000	1.500000	44.209271	44.125000	8.427069e-02	49	6	inner
2.500000	2.000000	55.062500	55.062500	0.000000e+00	50	6	border
3.000000	1.000000	82.000000	82.000000	1.421085e-14	51	6	border
3.000000	1.500000	86.062500	86.062500	1.421085e-14	52	6	border
3.000000	2.000000	97.000000	97.000000	0.000000e+00	53	6	border
2.000000	3.000000	97.000000	97.000000	0.000000e+00	54	7	border
2.000000	3.500000	166.184014	166.062500	1.215137e-01	55	7	inner border
2.000000	4.000000	272.000000	272.000000	5.684342e-14	56	7	border
2.500000	3.000000	120.062500	120.062500	0.000000e+00	57	7	border
2.500000	3.500000	189.212415	189.125000	8.741500e-02	58	7	inner
2.500000	4.000000	295.062500	295.062500	5.684342e-14	59	7	border
3.000000	3.000000	162.000000	162.000000	2.842171e-14	60	7	border
3.000000	3.500000	231.062500	231.062500	0.000000e+00	61	7	border
3.000000	4.000000	337.000000	337.000000	5.684342e-14	62	7	border
2.000000	5.000000	641.000000	641.000000	1.136868e-13	63	8	border
2.000000	5.500000	931.170650	931.062500	1.081504e-01	64	8	inner border
2.000000	6.000000	1312.000000	1312.000000	2.273737e-13	65	8	border
2.500000	5.000000	664.062500	664.062500	1.136868e-13	66	8	border
2.500000	5.500000	954.209271	954.125000	8.427069e-02	67	8	inner
2.500000	6.000000	1335.062500	1335.062500	2.273737e-13	68	8	border
3.000000	5.000000	706.000000	706.000000	1.136868e-13	69	8	border
3.000000	5.500000	996.062500	996.062500	0.000000e+00	70	8	border
3.000000	6.000000	1377.000000	1377.000000	2.273737e-13	71	8	border

```
||u-u*||/||u*|| = 1.458105e-04
||u-u*|| = 6.699838e-01
```

Область имеет разрывную форму



Файл regions.txt

```
2
-1-
1 2 3 4
1 2 1 2
1 1 1 1
-2-
2 3 1 2
1 2 1 2
1 1 1 1
```

- $\lambda = 1, \gamma = 1, u^* = x^3 + y^3, f = -6(x + y) + x^3 + y^3$

y	x	calc	prec	dif	N	reg	location
3.000000	1.000000	28.000000	28.000000	0.000000e+00	0	1	border
3.000000	1.500000	30.375000	30.375000	0.000000e+00	1	1	border
3.000000	2.000000	35.000000	35.000000	7.105427e-15	2	1	border
3.500000	1.000000	43.875000	43.875000	7.105427e-15	3	1	border
3.500000	1.500000	46.250000	46.250000	0.000000e+00	4	1	inner
3.500000	2.000000	50.875000	50.875000	0.000000e+00	5	1	border
4.000000	1.000000	65.000000	65.000000	1.421085e-14	6	1	border
4.000000	1.500000	67.375000	67.375000	1.421085e-14	7	1	border
4.000000	2.000000	72.000000	72.000000	1.421085e-14	8	1	border
1.000000	2.000000	9.000000	9.000000	1.776357e-15	9	2	border
1.000000	2.500000	16.625000	16.625000	3.552714e-15	10	2	border
1.000000	3.000000	28.000000	28.000000	0.000000e+00	11	2	border
1.500000	2.000000	11.375000	11.375000	0.000000e+00	12	2	border
1.500000	2.500000	19.000000	19.000000	3.552714e-15	13	2	inner
1.500000	3.000000	30.375000	30.375000	0.000000e+00	14	2	border
2.000000	2.000000	16.000000	16.000000	0.000000e+00	15	2	border
2.000000	2.500000	23.625000	23.625000	0.000000e+00	16	2	border
2.000000	3.000000	35.000000	35.000000	7.105427e-15	17	2	border

||u-u*||/||u*|| = 1.671385e-16
||u-u*|| = 2.803044e-14

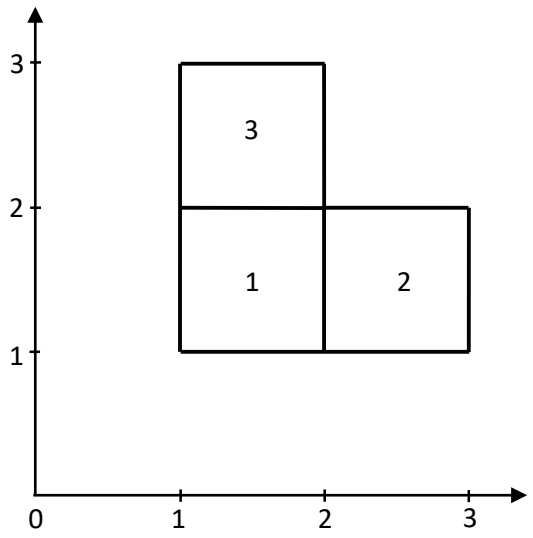
- $\lambda = 1, \gamma = 1, u^* = x^4 + y^4, f = -12(x^2 + y^2) + x^4 + y^4$

y	x	calc	prec	dif	N	reg	location
3.000000	1.000000	82.000000	82.000000	1.421085e-14	0	1	border
3.000000	1.500000	86.062500	86.062500	1.421085e-14	1	1	border
3.000000	2.000000	97.000000	97.000000	0.000000e+00	2	1	border
3.500000	1.000000	151.062500	151.062500	2.842171e-14	3	1	border
3.500000	1.500000	155.183824	155.125000	5.882353e-02	4	1	inner
3.500000	2.000000	166.062500	166.062500	2.842171e-14	5	1	border
4.000000	1.000000	257.000000	257.000000	5.684342e-14	6	1	border
4.000000	1.500000	261.062500	261.062500	5.684342e-14	7	1	border
4.000000	2.000000	272.000000	272.000000	5.684342e-14	8	1	border
1.000000	2.000000	17.000000	17.000000	3.552714e-15	9	2	border
1.000000	2.500000	40.062500	40.062500	7.105427e-15	10	2	border
1.000000	3.000000	82.000000	82.000000	1.421085e-14	11	2	border
1.500000	2.000000	21.062500	21.062500	3.552714e-15	12	2	border
1.500000	2.500000	44.183824	44.125000	5.882353e-02	13	2	inner
1.500000	3.000000	86.062500	86.062500	1.421085e-14	14	2	border
2.000000	2.000000	32.000000	32.000000	0.000000e+00	15	2	border
2.000000	2.500000	55.062500	55.062500	0.000000e+00	16	2	border
2.000000	3.000000	97.000000	97.000000	0.000000e+00	17	2	border

||u-u*||/||u*|| = 1.430753e-04
||u-u*|| = 8.318903e-02

7. Тестирование на неравномерной сетке

Область имеет L-образную форму



Файл regions.txt

```
3
-1-
1 2 1 2
1 2 2 2
1 -2 1 -3
-2-
2 3 1 2
1 2 2 2
-1 1 1 1
-3-
1 2 2 3
1 2 3 2
1 1 -1 1
```

- $\lambda = 1, \gamma = 1, u^* = x^2 + y^2, f = -4 + x^2 + y^2$

y	x	calc	prec	dif	N	reg	location
1.000000	1.000000	2.000000	2.000000	0.000000e+00	0	1	border
1.000000	1.500000	3.250000	3.250000	0.000000e+00	1	1	border
1.000000	2.000000	5.000000	5.000000	8.881784e-16	2	1	border
1.333333	1.000000	2.777778	2.777778	0.000000e+00	3	1	border
1.333333	1.500000	4.027778	4.027778	0.000000e+00	4	1	inner
1.333333	2.000000	5.777778	5.777778	8.881784e-16	5	1	inner border
2.000000	1.000000	5.000000	5.000000	8.881784e-16	6	1	border
2.000000	1.500000	6.250000	6.250000	8.881784e-16	7	1	inner border
2.000000	2.000000	8.000000	8.000000	0.000000e+00	8	1	inner border
1.000000	2.000000	5.000000	5.000000	8.881784e-16	9	2	border
1.000000	2.500000	7.250000	7.250000	0.000000e+00	10	2	border
1.000000	3.000000	10.000000	10.000000	1.776357e-15	11	2	border
1.333333	2.000000	5.777778	5.777778	8.881784e-16	12	2	inner border
1.333333	2.500000	8.027778	8.027778	3.552714e-15	13	2	inner
1.333333	3.000000	10.777778	10.777778	1.776357e-15	14	2	border
2.000000	2.000000	8.000000	8.000000	0.000000e+00	15	2	border
2.000000	2.500000	10.250000	10.250000	1.776357e-15	16	2	border
2.000000	3.000000	13.000000	13.000000	0.000000e+00	17	2	border
2.000000	1.000000	5.000000	5.000000	8.881784e-16	18	3	border
2.000000	1.500000	6.250000	6.250000	0.000000e+00	19	3	inner border
2.000000	2.000000	8.000000	8.000000	0.000000e+00	20	3	border
2.250000	1.000000	6.062500	6.062500	0.000000e+00	21	3	border
2.250000	1.500000	7.312500	7.312500	1.776357e-15	22	3	inner
2.250000	2.000000	9.062500	9.062500	1.776357e-15	23	3	border
3.000000	1.000000	10.000000	10.000000	1.776357e-15	24	3	border
3.000000	1.500000	11.250000	11.250000	0.000000e+00	25	3	border
3.000000	2.000000	13.000000	13.000000	0.000000e+00	26	3	border

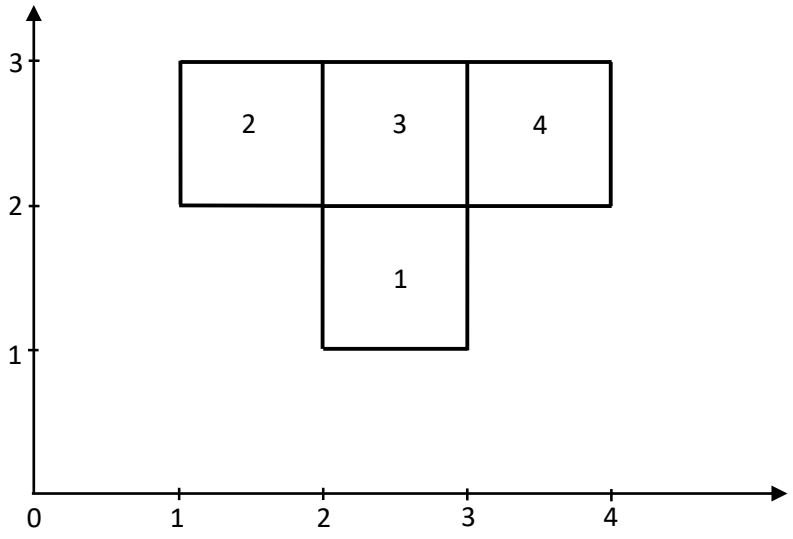
```
||u-u*||/||u*|| = 1.498774e-16
||u-u*|| = 6.089044e-15
```

- $\lambda = 1, \gamma = 1, u^* = x^3 + y^3, f = -6(x + y) + x^3 + y^3$

y	x	calc	prec	dif	N	reg	location
1.000000	1.000000	2.000000	2.000000	0.000000e+00	0	1	border
1.000000	1.500000	4.375000	4.375000	8.881784e-16	1	1	border
1.000000	2.000000	9.000000	9.000000	1.776357e-15	2	1	border
1.333333	1.000000	3.370370	3.370370	0.000000e+00	3	1	border
1.333333	1.500000	5.790592	5.745370	4.522200e-02	4	1	inner
1.333333	2.000000	10.422651	10.370370	5.228071e-02	5	1	inner border
2.000000	1.000000	9.000000	9.000000	1.776357e-15	6	1	border
2.000000	1.500000	11.354402	11.375000	2.059783e-02	7	1	inner border
2.000000	2.000000	15.964542	16.000000	3.545829e-02	8	1	inner border
1.000000	2.000000	9.000000	9.000000	1.776357e-15	9	2	border
1.000000	2.500000	16.625000	16.625000	3.552714e-15	10	2	border
1.000000	3.000000	28.000000	28.000000	0.000000e+00	11	2	border
1.333333	2.000000	10.428561	10.370370	5.819042e-02	12	2	inner border
1.333333	2.500000	18.045339	17.995370	4.996824e-02	13	2	inner
1.333333	3.000000	29.370370	29.370370	0.000000e+00	14	2	border
2.000000	2.000000	16.000000	16.000000	0.000000e+00	15	2	border
2.000000	2.500000	23.625000	23.625000	0.000000e+00	16	2	border
2.000000	3.000000	35.000000	35.000000	7.105427e-15	17	2	border
2.000000	1.000000	9.000000	9.000000	1.776357e-15	18	3	border
2.000000	1.500000	11.361156	11.375000	1.384387e-02	19	3	inner border
2.000000	2.000000	16.000000	16.000000	0.000000e+00	20	3	border
2.250000	1.000000	12.390625	12.390625	0.000000e+00	21	3	border
2.250000	1.500000	14.810841	14.765625	4.521605e-02	22	3	inner
2.250000	2.000000	19.390625	19.390625	3.552714e-15	23	3	border
3.000000	1.000000	28.000000	28.000000	0.000000e+00	24	3	border
3.000000	1.500000	30.375000	30.375000	0.000000e+00	25	3	border
3.000000	2.000000	35.000000	35.000000	7.105427e-15	26	3	border

||u-u*||/||u*|| = 1.249196e-03
||u-u*|| = 1.207434e-01

Область имеет Т-образную форму



Файл regions.txt

```
4
-1-
2 3 1 2
1 2 2 2
1 1 1 -3
-2-
1 2 2 3
1 2 3 2
1 -3 1 1
-3-
2 3 2 3
1 2 3 2
-2 -4 -1 1
-4-
3 4 2 3
1 2 3 2
-3 1 1 1
```

- $\lambda = 1, \gamma = 1, u^* = x^2 + y^2, f = -4 + x^2 + y^2$

y	x	calc	prec	dif	N	reg	location
1.000000	2.000000	5.000000	5.000000	8.881784e-16	0	1	border
1.000000	2.500000	7.250000	7.250000	0.000000e+00	1	1	border
1.000000	3.000000	10.000000	10.000000	1.776357e-15	2	1	border
1.333333	2.000000	5.777778	5.777778	0.000000e+00	3	1	border
1.333333	2.500000	8.027778	8.027778	3.552714e-15	4	1	inner
1.333333	3.000000	10.777778	10.777778	1.776357e-15	5	1	border
2.000000	2.000000	8.000000	8.000000	0.000000e+00	6	1	border
2.000000	2.500000	10.250000	10.250000	1.776357e-15	7	1	inner border
2.000000	3.000000	13.000000	13.000000	0.000000e+00	8	1	border
2.000000	1.000000	5.000000	5.000000	8.881784e-16	9	2	border
2.000000	1.500000	6.250000	6.250000	0.000000e+00	10	2	border
2.000000	2.000000	8.000000	8.000000	0.000000e+00	11	2	border
2.250000	1.000000	6.062500	6.062500	0.000000e+00	12	2	border
2.250000	1.500000	7.312500	7.312500	2.664535e-15	13	2	inner
2.250000	2.000000	9.062500	9.062500	3.552714e-15	14	2	inner border
3.000000	1.000000	10.000000	10.000000	1.776357e-15	15	2	border
3.000000	1.500000	11.250000	11.250000	0.000000e+00	16	2	border
3.000000	2.000000	13.000000	13.000000	0.000000e+00	17	2	border
2.000000	2.000000	8.000000	8.000000	0.000000e+00	18	3	inner border
2.000000	2.500000	10.250000	10.250000	0.000000e+00	19	3	inner border
2.000000	3.000000	13.000000	13.000000	0.000000e+00	20	3	inner border
2.250000	2.000000	9.062500	9.062500	1.776357e-15	21	3	inner border
2.250000	2.500000	11.312500	11.312500	1.776357e-15	22	3	inner
2.250000	3.000000	14.062500	14.062500	5.329071e-15	23	3	inner border
3.000000	2.000000	13.000000	13.000000	0.000000e+00	24	3	border
3.000000	2.500000	15.250000	15.250000	0.000000e+00	25	3	border
3.000000	3.000000	18.000000	18.000000	3.552714e-15	26	3	border
2.000000	3.000000	13.000000	13.000000	0.000000e+00	27	4	border
2.000000	3.500000	16.250000	16.250000	3.552714e-15	28	4	border
2.000000	4.000000	20.000000	20.000000	3.552714e-15	29	4	border
2.250000	3.000000	14.062500	14.062500	7.105427e-15	30	4	inner border
2.250000	3.500000	17.312500	17.312500	7.105427e-15	31	4	inner
2.250000	4.000000	21.062500	21.062500	3.552714e-15	32	4	border
3.000000	3.000000	18.000000	18.000000	3.552714e-15	33	4	border
3.000000	3.500000	21.250000	21.250000	3.552714e-15	34	4	border
3.000000	4.000000	25.000000	25.000000	0.000000e+00	35	4	border

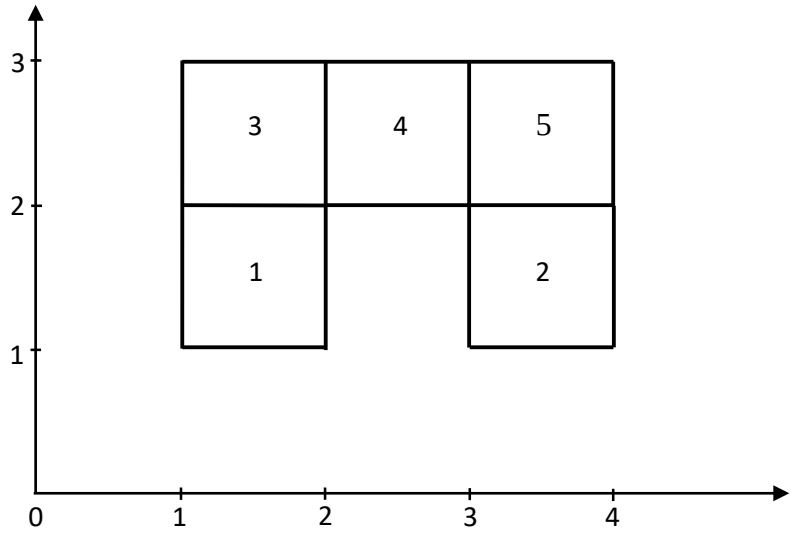
```
||u-u*||/||u*|| = 2.061177e-16
||u-u*|| = 1.606106e-14
```

- $\lambda = 1, \gamma = 1, u^* = x^3 + y^3, f = -6(x + y) + x^3 + y^3$

y	x	calc	prec	dif	N	reg	location
1.000000	2.000000	9.000000	9.000000	1.776357e-15	0	1	border
1.000000	2.500000	16.625000	16.625000	3.552714e-15	1	1	border
1.000000	3.000000	28.000000	28.000000	0.000000e+00	2	1	border
1.333333	2.000000	10.370370	10.370370	1.776357e-15	3	1	border
1.333333	2.500000	18.032380	17.995370	3.700921e-02	4	1	inner
1.333333	3.000000	29.370370	29.370370	0.000000e+00	5	1	border
2.000000	2.000000	16.000000	16.000000	0.000000e+00	6	1	border
2.000000	2.500000	23.624833	23.625000	1.669378e-04	7	1	inner border
2.000000	3.000000	35.000000	35.000000	7.105427e-15	8	1	border
2.000000	1.000000	9.000000	9.000000	1.776357e-15	9	2	border
2.000000	1.500000	11.375000	11.375000	0.000000e+00	10	2	border
2.000000	2.000000	16.000000	16.000000	0.000000e+00	11	2	border
2.250000	1.000000	12.390625	12.390625	0.000000e+00	12	2	border
2.250000	1.500000	14.832959	14.765625	6.733403e-02	13	2	inner
2.250000	2.000000	19.471684	19.390625	8.105898e-02	14	2	inner border
3.000000	1.000000	28.000000	28.000000	0.000000e+00	15	2	border
3.000000	1.500000	30.375000	30.375000	0.000000e+00	16	2	border
3.000000	2.000000	35.000000	35.000000	7.105427e-15	17	2	border
2.000000	2.000000	15.992052	16.000000	7.947780e-03	18	3	inner border
2.000000	2.500000	23.621805	23.625000	3.194664e-03	19	3	inner border
2.000000	3.000000	34.992052	35.000000	7.947780e-03	20	3	inner border
2.250000	2.000000	19.468451	19.390625	7.782599e-02	21	3	inner border
2.250000	2.500000	27.096831	27.015625	8.120596e-02	22	3	inner
2.250000	3.000000	38.468451	38.390625	7.782599e-02	23	3	inner border
3.000000	2.000000	35.000000	35.000000	7.105427e-15	24	3	border
3.000000	2.500000	42.625000	42.625000	7.105427e-15	25	3	border
3.000000	3.000000	54.000000	54.000000	0.000000e+00	26	3	border
2.000000	3.000000	35.000000	35.000000	7.105427e-15	27	4	border
2.000000	3.500000	50.875000	50.875000	0.000000e+00	28	4	border
2.000000	4.000000	72.000000	72.000000	1.421085e-14	29	4	border
2.250000	3.000000	38.471684	38.390625	8.105898e-02	30	4	inner border
2.250000	3.500000	54.332959	54.265625	6.733403e-02	31	4	inner
2.250000	4.000000	75.390625	75.390625	1.421085e-14	32	4	border
3.000000	3.000000	54.000000	54.000000	0.000000e+00	33	4	border
3.000000	3.500000	69.875000	69.875000	1.421085e-14	34	4	border
3.000000	4.000000	91.000000	91.000000	0.000000e+00	35	4	border

```
||u-u*||/||u*|| = 8.832807e-04
||u-u*|| = 2.059691e-01
```

Область имеет П-образную форму



Файл regions.txt

```
5
-1-
1 2 1 2
1 2 2 2
1 1 1 -3
-2-
3 4 1 2
1 2 2 2
1 1 1 -5
-3-
1 2 2 3
1 2 3 2
1 -4 -1 1
-4-
2 3 2 3
1 2 3 2
-3 -5 1 1
-5-
3 4 2 3
1 2 3 2
-4 1 -2 1
```

- $\lambda = 1, \gamma = 1, u^* = x^2 + y^2, f = -4 + x^2 + y^2$

y	x	calc	prec	dif	N	reg	location
1.000000	1.000000	2.000000	2.000000	0.000000e+00	0	1	border
1.000000	1.500000	3.250000	3.250000	0.000000e+00	1	1	border
1.000000	2.000000	5.000000	5.000000	8.881784e-16	2	1	border
1.333333	1.000000	2.777778	2.777778	0.000000e+00	3	1	border
1.333333	1.500000	4.027778	4.027778	8.881784e-16	4	1	inner
1.333333	2.000000	5.777778	5.777778	0.000000e+00	5	1	border
2.000000	1.000000	5.000000	5.000000	8.881784e-16	6	1	border
2.000000	1.500000	6.250000	6.250000	0.000000e+00	7	1	inner border
2.000000	2.000000	8.000000	8.000000	0.000000e+00	8	1	border
1.000000	3.000000	10.000000	10.000000	1.776357e-15	9	2	border
1.000000	3.500000	13.250000	13.250000	0.000000e+00	10	2	border
1.000000	4.000000	17.000000	17.000000	3.552714e-15	11	2	border
1.333333	3.000000	10.777778	10.777778	1.776357e-15	12	2	border
1.333333	3.500000	14.027778	14.027778	3.552714e-15	13	2	inner
1.333333	4.000000	17.777778	17.777778	3.552714e-15	14	2	border
2.000000	3.000000	13.000000	13.000000	0.000000e+00	15	2	border
2.000000	3.500000	16.250000	16.250000	0.000000e+00	16	2	inner border
2.000000	4.000000	20.000000	20.000000	3.552714e-15	17	2	border
2.000000	1.000000	5.000000	5.000000	8.881784e-16	18	3	border
2.000000	1.500000	6.250000	6.250000	0.000000e+00	19	3	inner border
2.000000	2.000000	8.000000	8.000000	0.000000e+00	20	3	inner border
2.250000	1.000000	6.062500	6.062500	0.000000e+00	21	3	border
2.250000	1.500000	7.312500	7.312500	1.776357e-15	22	3	inner
2.250000	2.000000	9.062500	9.062500	1.776357e-15	23	3	inner border
3.000000	1.000000	10.000000	10.000000	1.776357e-15	24	3	border
3.000000	1.500000	11.250000	11.250000	0.000000e+00	25	3	border
3.000000	2.000000	13.000000	13.000000	0.000000e+00	26	3	border
2.000000	2.000000	8.000000	8.000000	0.000000e+00	27	4	border
2.000000	2.500000	10.250000	10.250000	1.776357e-15	28	4	border
2.000000	3.000000	13.000000	13.000000	0.000000e+00	29	4	border
2.250000	2.000000	9.062500	9.062500	0.000000e+00	30	4	inner border
2.250000	2.500000	11.312500	11.312500	1.776357e-15	31	4	inner
2.250000	3.000000	14.062500	14.062500	7.105427e-15	32	4	inner border
3.000000	2.000000	13.000000	13.000000	0.000000e+00	33	4	border
3.000000	2.500000	15.250000	15.250000	0.000000e+00	34	4	border
3.000000	3.000000	18.000000	18.000000	3.552714e-15	35	4	border
2.000000	3.000000	13.000000	13.000000	0.000000e+00	36	5	inner border
2.000000	3.500000	16.250000	16.250000	3.552714e-15	37	5	inner border
2.000000	4.000000	20.000000	20.000000	3.552714e-15	38	5	border
2.250000	3.000000	14.062500	14.062500	7.105427e-15	39	5	inner border
2.250000	3.500000	17.312500	17.312500	7.105427e-15	40	5	inner
2.250000	4.000000	21.062500	21.062500	3.552714e-15	41	5	border
3.000000	3.000000	18.000000	18.000000	3.552714e-15	42	5	border
3.000000	3.500000	21.250000	21.250000	3.552714e-15	43	5	border
3.000000	4.000000	25.000000	25.000000	0.000000e+00	44	5	border

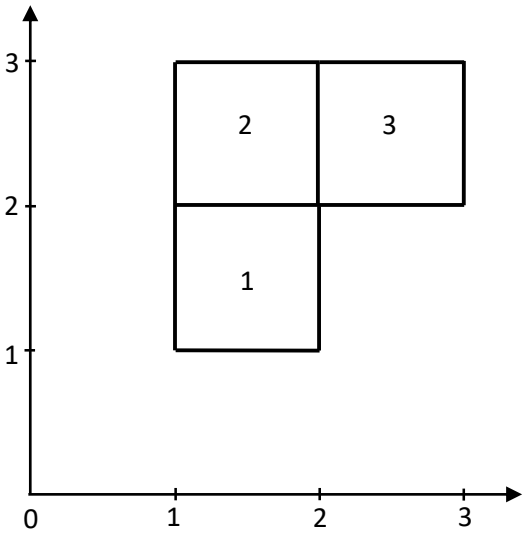
||u-u*||/||u*|| = 1.997274e-16
||u-u*|| = 1.740467e-14

- $\lambda = 1, \gamma = 1, u^* = x^3 + y^3, f = -6(x + y) + x^3 + y^3$

y	x	calc	prec	dif	N	reg	location
1.000000	1.000000	2.000000	2.000000	0.000000e+00	0	1	border
1.000000	1.500000	4.375000	4.375000	8.881784e-16	1	1	border
1.000000	2.000000	9.000000	9.000000	1.776357e-15	2	1	border
1.333333	1.000000	3.370370	3.370370	0.000000e+00	3	1	border
1.333333	1.500000	5.781036	5.745370	3.566534e-02	4	1	inner
1.333333	2.000000	10.370370	10.370370	1.776357e-15	5	1	border
2.000000	1.000000	9.000000	9.000000	1.776357e-15	6	1	border
2.000000	1.500000	11.366770	11.375000	8.230195e-03	7	1	inner border
2.000000	2.000000	16.000000	16.000000	0.000000e+00	8	1	border
1.000000	3.000000	28.000000	28.000000	0.000000e+00	9	2	border
1.000000	3.500000	43.875000	43.875000	7.105427e-15	10	2	border
1.000000	4.000000	65.000000	65.000000	1.421085e-14	11	2	border
1.333333	3.000000	29.370370	29.370370	0.000000e+00	12	2	border
1.333333	3.500000	45.281036	45.245370	3.566534e-02	13	2	inner
1.333333	4.000000	66.370370	66.370370	1.421085e-14	14	2	border
2.000000	3.000000	35.000000	35.000000	7.105427e-15	15	2	border
2.000000	3.500000	50.866770	50.875000	8.230195e-03	16	2	inner border
2.000000	4.000000	72.000000	72.000000	1.421085e-14	17	2	border
2.000000	1.000000	9.000000	9.000000	1.776357e-15	18	3	border
2.000000	1.500000	11.364904	11.375000	1.009634e-02	19	3	inner border
2.000000	2.000000	15.990203	16.000000	9.797283e-03	20	3	inner border
2.250000	1.000000	12.390625	12.390625	0.000000e+00	21	3	border
2.250000	1.500000	14.827933	14.765625	6.230770e-02	22	3	inner
2.250000	2.000000	19.467164	19.390625	7.653889e-02	23	3	inner border
3.000000	1.000000	28.000000	28.000000	0.000000e+00	24	3	border
3.000000	1.500000	30.375000	30.375000	0.000000e+00	25	3	border
3.000000	2.000000	35.000000	35.000000	7.105427e-15	26	3	border
2.000000	2.000000	16.000000	16.000000	0.000000e+00	27	4	border
2.000000	2.500000	23.625000	23.625000	0.000000e+00	28	4	border
2.000000	3.000000	35.000000	35.000000	7.105427e-15	29	4	border
2.250000	2.000000	19.471149	19.390625	8.052423e-02	30	4	inner border
2.250000	2.500000	27.099228	27.015625	8.360307e-02	31	4	inner
2.250000	3.000000	38.471149	38.390625	8.052423e-02	32	4	inner border
3.000000	2.000000	35.000000	35.000000	7.105427e-15	33	4	border
3.000000	2.500000	42.625000	42.625000	7.105427e-15	34	4	border
3.000000	3.000000	54.000000	54.000000	0.000000e+00	35	4	border
2.000000	3.000000	34.990203	35.000000	9.797283e-03	36	5	inner border
2.000000	3.500000	50.864904	50.875000	1.009634e-02	37	5	inner border
2.000000	4.000000	72.000000	72.000000	1.421085e-14	38	5	border
2.250000	3.000000	38.467164	38.390625	7.653889e-02	39	5	inner border
2.250000	3.500000	54.327933	54.265625	6.230770e-02	40	5	inner
2.250000	4.000000	75.390625	75.390625	1.421085e-14	41	5	border
3.000000	3.000000	54.000000	54.000000	0.000000e+00	42	5	border
3.000000	3.500000	69.875000	69.875000	1.421085e-14	43	5	border
3.000000	4.000000	91.000000	91.000000	0.000000e+00	44	5	border

```
||u-u*||/||u*|| = 7.586340e-04
||u-u*|| = 2.061893e-01
```

Область имеет Г-образную форму



Файл regions.txt

```
3
-1-
1 2 1 2
1 2 2 2
1 1 1 -2
-2-
1 2 2 3
1 2 3 2
1 -3 -1 1
-3-
2 3 2 3
2 2 3 2
-2 1 1 1
```

- $\lambda = 1, \gamma = 1, u^* = x^2 + y^2, f = -4 + x^2 + y^2$

y	x	calc	prec	dif	N	reg	location
1.000000	1.000000	2.000000	2.000000	0.000000e+00	0	1	border
1.000000	1.500000	3.250000	3.250000	0.000000e+00	1	1	border
1.000000	2.000000	5.000000	5.000000	8.881784e-16	2	1	border
1.333333	1.000000	2.777778	2.777778	0.000000e+00	3	1	border
1.333333	1.500000	4.027778	4.027778	8.881784e-16	4	1	inner
1.333333	2.000000	5.777778	5.777778	0.000000e+00	5	1	border
2.000000	1.000000	5.000000	5.000000	8.881784e-16	6	1	border
2.000000	1.500000	6.250000	6.250000	8.881784e-16	7	1	inner border
2.000000	2.000000	8.000000	8.000000	0.000000e+00	8	1	border
2.000000	1.000000	5.000000	5.000000	8.881784e-16	9	2	border
2.000000	1.500000	6.250000	6.250000	8.881784e-16	10	2	inner border
2.000000	2.000000	8.000000	8.000000	1.776357e-15	11	2	inner border
2.250000	1.000000	6.062500	6.062500	0.000000e+00	12	2	border
2.250000	1.500000	7.312500	7.312500	8.881784e-16	13	2	inner
2.250000	2.000000	9.062500	9.062500	1.776357e-15	14	2	inner border
3.000000	1.000000	10.000000	10.000000	1.776357e-15	15	2	border
3.000000	1.500000	11.250000	11.250000	0.000000e+00	16	2	border
3.000000	2.000000	13.000000	13.000000	0.000000e+00	17	2	border
2.000000	2.000000	8.000000	8.000000	0.000000e+00	18	3	border
2.000000	2.333333	9.444444	9.444444	1.776357e-15	19	3	border
2.000000	3.000000	13.000000	13.000000	0.000000e+00	20	3	border
2.250000	2.000000	9.062500	9.062500	0.000000e+00	21	3	inner border
2.250000	2.333333	10.506944	10.506944	0.000000e+00	22	3	inner
2.250000	3.000000	14.062500	14.062500	0.000000e+00	23	3	border
3.000000	2.000000	13.000000	13.000000	0.000000e+00	24	3	border
3.000000	2.333333	14.444444	14.444444	0.000000e+00	25	3	border
3.000000	3.000000	18.000000	18.000000	3.552714e-15	26	3	border

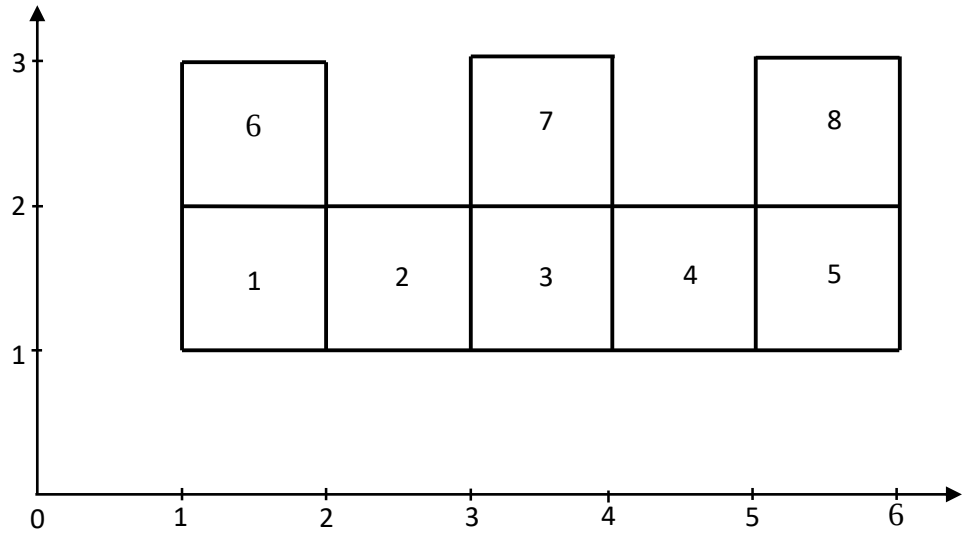
```
||u-u*||/||u*|| = 1.147849e-16
||u-u*|| = 5.546672e-15
```

- $\lambda = 1, \gamma = 1, u^* = x^3 + y^3, f = -6(x + y) + x^3 + y^3$

y	x	calc	prec	dif	N	reg	location
1.000000	1.000000	2.000000	2.000000	0.000000e+00	0	1	border
1.000000	1.500000	4.375000	4.375000	8.881784e-16	1	1	border
1.000000	2.000000	9.000000	9.000000	1.776357e-15	2	1	border
1.333333	1.000000	3.370370	3.370370	0.000000e+00	3	1	border
1.333333	1.500000	5.780635	5.745370	3.526442e-02	4	1	inner
1.333333	2.000000	10.370370	10.370370	1.776357e-15	5	1	border
2.000000	1.000000	9.000000	9.000000	1.776357e-15	6	1	border
2.000000	1.500000	11.364364	11.375000	1.063571e-02	7	1	inner border
2.000000	2.000000	16.000000	16.000000	0.000000e+00	8	1	border
2.000000	1.000000	9.000000	9.000000	1.776357e-15	9	2	border
2.000000	1.500000	11.358928	11.375000	1.607158e-02	10	2	inner border
2.000000	2.000000	15.971462	16.000000	2.853830e-02	11	2	inner border
2.250000	1.000000	12.390625	12.390625	0.000000e+00	12	2	border
2.250000	1.500000	14.822295	14.765625	5.666978e-02	13	2	inner
2.250000	2.000000	19.451395	19.390625	6.076957e-02	14	2	inner border
3.000000	1.000000	28.000000	28.000000	0.000000e+00	15	2	border
3.000000	1.500000	30.375000	30.375000	0.000000e+00	16	2	border
3.000000	2.000000	35.000000	35.000000	7.105427e-15	17	2	border
2.000000	2.000000	16.000000	16.000000	0.000000e+00	18	3	border
2.000000	2.333333	20.703704	20.703704	3.552714e-15	19	3	border
2.000000	3.000000	35.000000	35.000000	7.105427e-15	20	3	border
2.250000	2.000000	19.461041	19.390625	7.041632e-02	21	3	inner border
2.250000	2.333333	24.195417	24.094329	1.010886e-01	22	3	inner
2.250000	3.000000	38.390625	38.390625	7.105427e-15	23	3	border
3.000000	2.000000	35.000000	35.000000	7.105427e-15	24	3	border
3.000000	2.333333	39.703704	39.703704	7.105427e-15	25	3	border
3.000000	3.000000	54.000000	54.000000	0.000000e+00	26	3	border

||u-u*||/||u*|| = 1.281501e-03
||u-u*|| = 1.565605e-01

Область имеет Ш-образную форму



Файл regions.txt

```
8
-1-
1 2 1 2
1 2 3 2
1 -2 1 -6
-2-
2 3 1 2
1 2 3 2
-1 -3 1 1
-3-
3 4 1 2
1 2 3 2
-2 -4 1 -7
-4-
4 5 1 2
1 2 3 2
-3 -5 1 1
-5-
5 6 1 2
1 2 3 2
-4 1 1 -8
-6-
1 2 2 3
1 2 2 2
1 1 -1 1
-7-
3 4 2 3
1 2 2 2
1 1 -3 1
-8-
5 6 2 3
1 2 2 2
1 1 -5 1
```

- $\lambda = 1, \gamma = 1, u^* = x^2 + y^2, f = -4 + x^2 + y^2$

y	x	calc	prec	dif	N	reg	location
1.000000	1.000000	2.000000	2.000000	0.000000e+00	0	1	border
1.000000	1.500000	3.250000	3.250000	0.000000e+00	1	1	border
1.000000	2.000000	5.000000	5.000000	8.881784e-16	2	1	border
1.250000	1.000000	2.562500	2.562500	4.440892e-16	3	1	border
1.250000	1.500000	3.812500	3.812500	0.000000e+00	4	1	inner
1.250000	2.000000	5.562500	5.562500	8.881784e-16	5	1	inner border
2.000000	1.000000	5.000000	5.000000	8.881784e-16	6	1	border
2.000000	1.500000	6.250000	6.250000	0.000000e+00	7	1	inner border
2.000000	2.000000	8.000000	8.000000	1.776357e-15	8	1	inner border
1.000000	2.000000	5.000000	5.000000	8.881784e-16	9	2	border
1.000000	2.500000	7.250000	7.250000	0.000000e+00	10	2	border
1.000000	3.000000	10.000000	10.000000	1.776357e-15	11	2	border
1.250000	2.000000	5.562500	5.562500	1.776357e-15	12	2	inner border
1.250000	2.500000	7.812500	7.812500	1.776357e-15	13	2	inner
1.250000	3.000000	10.562500	10.562500	3.552714e-15	14	2	inner border
2.000000	2.000000	8.000000	8.000000	0.000000e+00	15	2	border
2.000000	2.500000	10.250000	10.250000	1.776357e-15	16	2	border
2.000000	3.000000	13.000000	13.000000	0.000000e+00	17	2	border
1.000000	3.000000	10.000000	10.000000	1.776357e-15	18	3	border
1.000000	3.500000	13.250000	13.250000	0.000000e+00	19	3	border
1.000000	4.000000	17.000000	17.000000	3.552714e-15	20	3	border
1.250000	3.000000	10.562500	10.562500	1.776357e-15	21	3	inner border
1.250000	3.500000	13.812500	13.812500	3.552714e-15	22	3	inner
1.250000	4.000000	17.562500	17.562500	3.552714e-15	23	3	inner border
2.000000	3.000000	13.000000	13.000000	1.776357e-15	24	3	inner border
2.000000	3.500000	16.250000	16.250000	0.000000e+00	25	3	inner border
2.000000	4.000000	20.000000	20.000000	3.552714e-15	26	3	inner border
1.000000	4.000000	17.000000	17.000000	3.552714e-15	27	4	border
1.000000	4.500000	21.250000	21.250000	3.552714e-15	28	4	border
1.000000	5.000000	26.000000	26.000000	0.000000e+00	29	4	border
1.250000	4.000000	17.562500	17.562500	7.105427e-15	30	4	inner border
1.250000	4.500000	21.812500	21.812500	3.552714e-15	31	4	inner
1.250000	5.000000	26.562500	26.562500	3.552714e-15	32	4	inner border
2.000000	4.000000	20.000000	20.000000	3.552714e-15	33	4	border
2.000000	4.500000	24.250000	24.250000	0.000000e+00	34	4	border
2.000000	5.000000	29.000000	29.000000	0.000000e+00	35	4	border
1.000000	5.000000	26.000000	26.000000	0.000000e+00	36	5	border
1.000000	5.500000	31.250000	31.250000	0.000000e+00	37	5	border
1.000000	6.000000	37.000000	37.000000	7.105427e-15	38	5	border
1.250000	5.000000	26.562500	26.562500	7.105427e-15	39	5	inner border
1.250000	5.500000	31.812500	31.812500	1.065814e-14	40	5	inner
1.250000	6.000000	37.562500	37.562500	7.105427e-15	41	5	border
2.000000	5.000000	29.000000	29.000000	3.552714e-15	42	5	inner border
2.000000	5.500000	34.250000	34.250000	7.105427e-15	43	5	inner border
2.000000	6.000000	40.000000	40.000000	7.105427e-15	44	5	border
2.000000	1.000000	5.000000	5.000000	8.881784e-16	45	6	border
2.000000	1.500000	6.250000	6.250000	8.881784e-16	46	6	inner border
2.000000	2.000000	8.000000	8.000000	0.000000e+00	47	6	border

y	x	calc	prec	dif	N	reg	location
2.333333	1.000000	6.444444	6.444444	0.000000e+00	48	6	border
2.333333	1.500000	7.694444	7.694444	1.776357e-15	49	6	inner
2.333333	2.000000	9.444444	9.444444	1.776357e-15	50	6	border
3.000000	1.000000	10.000000	10.000000	1.776357e-15	51	6	border
3.000000	1.500000	11.250000	11.250000	0.000000e+00	52	6	border
3.000000	2.000000	13.000000	13.000000	0.000000e+00	53	6	border
2.000000	3.000000	13.000000	13.000000	0.000000e+00	54	7	border
2.000000	3.500000	16.250000	16.250000	0.000000e+00	55	7	inner border
2.000000	4.000000	20.000000	20.000000	3.552714e-15	56	7	border
2.333333	3.000000	14.444444	14.444444	0.000000e+00	57	7	border
2.333333	3.500000	17.694444	17.694444	0.000000e+00	58	7	inner
2.333333	4.000000	21.444444	21.444444	3.552714e-15	59	7	border
3.000000	3.000000	18.000000	18.000000	3.552714e-15	60	7	border
3.000000	3.500000	21.250000	21.250000	3.552714e-15	61	7	border
3.000000	4.000000	25.000000	25.000000	0.000000e+00	62	7	border
2.000000	5.000000	29.000000	29.000000	0.000000e+00	63	8	border
2.000000	5.500000	34.250000	34.250000	0.000000e+00	64	8	inner border
2.000000	6.000000	40.000000	40.000000	7.105427e-15	65	8	border
2.333333	5.000000	30.444444	30.444444	0.000000e+00	66	8	border
2.333333	5.500000	35.694444	35.694444	0.000000e+00	67	8	inner
2.333333	6.000000	41.444444	41.444444	7.105427e-15	68	8	border
3.000000	5.000000	34.000000	34.000000	7.105427e-15	69	8	border
3.000000	5.500000	39.250000	39.250000	7.105427e-15	70	8	border
3.000000	6.000000	45.000000	45.000000	0.000000e+00	71	8	border

||u-u*||/||u*|| = 1.587415e-16
||u-u*|| = 2.911072e-14

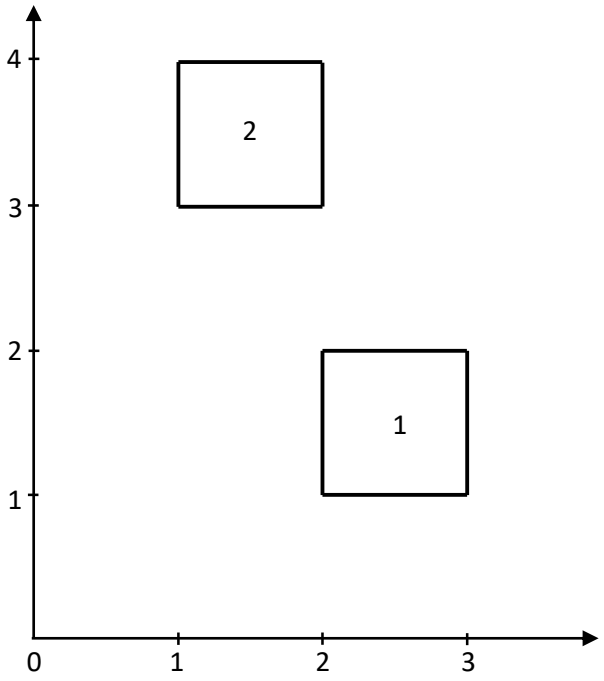
- $\lambda = 1, \gamma = 1, u^* = x^3 + y^3, f = -6(x + y) + x^3 + y^3$

y	x	calc	prec	dif	N	reg	location
1.000000	1.000000	2.000000	2.000000	0.000000e+00	0	1	border
1.000000	1.500000	4.375000	4.375000	8.881784e-16	1	1	border
1.000000	2.000000	9.000000	9.000000	1.776357e-15	2	1	border
1.250000	1.000000	2.953125	2.953125	0.000000e+00	3	1	border
1.250000	1.500000	5.388086	5.328125	5.996059e-02	4	1	inner
1.250000	2.000000	10.026685	9.953125	7.356030e-02	5	1	inner border
2.000000	1.000000	9.000000	9.000000	1.776357e-15	6	1	border
2.000000	1.500000	11.331869	11.375000	4.313110e-02	7	1	inner border
2.000000	2.000000	15.951483	16.000000	4.851684e-02	8	1	inner border
1.000000	2.000000	9.000000	9.000000	1.776357e-15	9	2	border
1.000000	2.500000	16.625000	16.625000	3.552714e-15	10	2	border
1.000000	3.000000	28.000000	28.000000	0.000000e+00	11	2	border
1.250000	2.000000	10.033264	9.953125	8.013886e-02	12	2	inner border
1.250000	2.500000	17.662180	17.578125	8.405545e-02	13	2	inner
1.250000	3.000000	29.036259	28.953125	8.313377e-02	14	2	inner border
2.000000	2.000000	16.000000	16.000000	0.000000e+00	15	2	border
2.000000	2.500000	23.625000	23.625000	0.000000e+00	16	2	border
2.000000	3.000000	35.000000	35.000000	7.105427e-15	17	2	border
1.000000	3.000000	28.000000	28.000000	0.000000e+00	18	3	border
1.000000	3.500000	43.875000	43.875000	7.105427e-15	19	3	border
1.000000	4.000000	65.000000	65.000000	1.421085e-14	20	3	border
1.250000	3.000000	29.029417	28.953125	7.629177e-02	21	3	inner border
1.250000	3.500000	44.902811	44.828125	7.468557e-02	22	3	inner
1.250000	4.000000	66.029417	65.953125	7.629177e-02	23	3	inner border
2.000000	3.000000	34.949540	35.000000	5.045972e-02	24	3	inner border
2.000000	3.500000	50.821931	50.875000	5.306924e-02	25	3	inner border
2.000000	4.000000	71.949540	72.000000	5.045972e-02	26	3	inner border
1.000000	4.000000	65.000000	65.000000	1.421085e-14	27	4	border
1.000000	4.500000	92.125000	92.125000	0.000000e+00	28	4	border
1.000000	5.000000	126.000000	126.000000	0.000000e+00	29	4	border
1.250000	4.000000	66.036259	65.953125	8.313377e-02	30	4	inner border
1.250000	4.500000	93.162180	93.078125	8.405545e-02	31	4	inner
1.250000	5.000000	127.033264	126.953125	8.013886e-02	32	4	inner border
2.000000	4.000000	72.000000	72.000000	1.421085e-14	33	4	border
2.000000	4.500000	99.125000	99.125000	0.000000e+00	34	4	border
2.000000	5.000000	133.000000	133.000000	2.842171e-14	35	4	border
1.000000	5.000000	126.000000	126.000000	0.000000e+00	36	5	border
1.000000	5.500000	167.375000	167.375000	2.842171e-14	37	5	border
1.000000	6.000000	217.000000	217.000000	0.000000e+00	38	5	border
1.250000	5.000000	127.026685	126.953125	7.356030e-02	39	5	inner border
1.250000	5.500000	168.388086	168.328125	5.996059e-02	40	5	inner
1.250000	6.000000	217.953125	217.953125	0.000000e+00	41	5	border
2.000000	5.000000	132.951483	133.000000	4.851684e-02	42	5	inner border
2.000000	5.500000	174.331869	174.375000	4.313110e-02	43	5	inner border
2.000000	6.000000	224.000000	224.000000	0.000000e+00	44	5	border
2.000000	1.000000	9.000000	9.000000	1.776357e-15	45	6	border
2.000000	1.500000	11.343285	11.375000	3.171537e-02	46	6	inner border
2.000000	2.000000	16.000000	16.000000	0.000000e+00	47	6	border

y	x	calc	prec	dif	N	reg	location
2.333333	1.000000	13.703704	13.703704	0.000000e+00	48	6	border
2.333333	1.500000	16.105169	16.078704	2.646525e-02	49	6	inner
2.333333	2.000000	20.703704	20.703704	3.552714e-15	50	6	border
3.000000	1.000000	28.000000	28.000000	0.000000e+00	51	6	border
3.000000	1.500000	30.375000	30.375000	0.000000e+00	52	6	border
3.000000	2.000000	35.000000	35.000000	7.105427e-15	53	6	border
2.000000	3.000000	35.000000	35.000000	7.105427e-15	54	7	border
2.000000	3.500000	50.845677	50.875000	2.932349e-02	55	7	inner border
2.000000	4.000000	72.000000	72.000000	1.421085e-14	56	7	border
2.333333	3.000000	39.703704	39.703704	7.105427e-15	57	7	border
2.333333	3.500000	55.605966	55.578704	2.726254e-02	58	7	inner
2.333333	4.000000	76.703704	76.703704	1.421085e-14	59	7	border
3.000000	3.000000	54.000000	54.000000	0.000000e+00	60	7	border
3.000000	3.500000	69.875000	69.875000	1.421085e-14	61	7	border
3.000000	4.000000	91.000000	91.000000	0.000000e+00	62	7	border
2.000000	5.000000	133.000000	133.000000	2.842171e-14	63	8	border
2.000000	5.500000	174.343285	174.375000	3.171537e-02	64	8	inner border
2.000000	6.000000	224.000000	224.000000	0.000000e+00	65	8	border
2.333333	5.000000	137.703704	137.703704	2.842171e-14	66	8	border
2.333333	5.500000	179.105169	179.078704	2.646525e-02	67	8	inner
2.333333	6.000000	228.703704	228.703704	0.000000e+00	68	8	border
3.000000	5.000000	152.000000	152.000000	2.842171e-14	69	8	border
3.000000	5.500000	193.375000	193.375000	0.000000e+00	70	8	border
3.000000	6.000000	243.000000	243.000000	0.000000e+00	71	8	border

```
||u-u*||/||u*|| = 3.577370e-04
||u-u*|| = 3.120607e-01
```

Область имеет разрывную форму



Файл regions.txt

```
2
-1-
1 2 3 4
2 2 3 2
1 1 1 1
-2-
2 3 1 2
3 2 2 2
1 1 1 1
```

- $\lambda = 1, \gamma = 1, u^* = x^2 + y^2, f = -4 + x^2 + y^2$

y	x	calc	prec	dif	N	reg	location
3.000000	1.000000	10.000000	10.000000	1.776357e-15	0	1	border
3.000000	1.333333	10.777778	10.777778	1.776357e-15	1	1	border
3.000000	2.000000	13.000000	13.000000	0.000000e+00	2	1	border
3.250000	1.000000	11.562500	11.562500	0.000000e+00	3	1	border
3.250000	1.333333	12.340278	12.340278	5.329071e-15	4	1	inner
3.250000	2.000000	14.562500	14.562500	0.000000e+00	5	1	border
4.000000	1.000000	17.000000	17.000000	3.552714e-15	6	1	border
4.000000	1.333333	17.777778	17.777778	3.552714e-15	7	1	border
4.000000	2.000000	20.000000	20.000000	3.552714e-15	8	1	border
1.000000	2.000000	5.000000	5.000000	8.881784e-16	9	2	border
1.000000	2.250000	6.062500	6.062500	0.000000e+00	10	2	border
1.000000	3.000000	10.000000	10.000000	1.776357e-15	11	2	border
1.333333	2.000000	5.777778	5.777778	0.000000e+00	12	2	border
1.333333	2.250000	6.840278	6.840278	1.776357e-15	13	2	inner
1.333333	3.000000	10.777778	10.777778	1.776357e-15	14	2	border
2.000000	2.000000	8.000000	8.000000	0.000000e+00	15	2	border
2.000000	2.250000	9.062500	9.062500	1.776357e-15	16	2	border
2.000000	3.000000	13.000000	13.000000	0.000000e+00	17	2	border

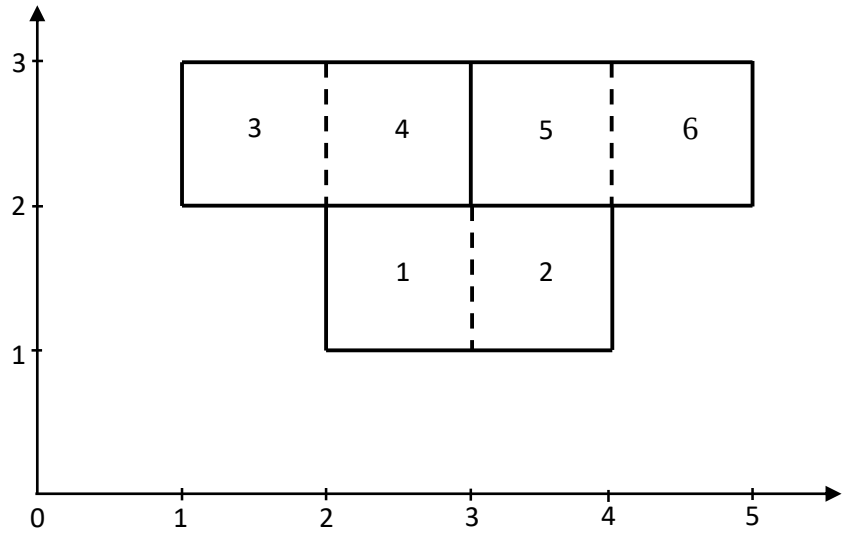
```
||u-u*||/||u*|| = 1.832184e-16
||u-u*|| = 9.272855e-15
```

- $\lambda = 1, \gamma = 1, u^* = x^3 + y^3, f = -6(x + y) + x^3 + y^3$

y	x	calc	prec	dif	N	reg	location
3.000000	1.000000	28.000000	28.000000	0.000000e+00	0	1	border
3.000000	1.333333	29.370370	29.370370	0.000000e+00	1	1	border
3.000000	2.000000	35.000000	35.000000	7.105427e-15	2	1	border
3.250000	1.000000	35.328125	35.328125	7.105427e-15	3	1	border
3.250000	1.333333	36.779141	36.698495	8.064516e-02	4	1	inner
3.250000	2.000000	42.328125	42.328125	7.105427e-15	5	1	border
4.000000	1.000000	65.000000	65.000000	1.421085e-14	6	1	border
4.000000	1.333333	66.370370	66.370370	1.421085e-14	7	1	border
4.000000	2.000000	72.000000	72.000000	1.421085e-14	8	1	border
1.000000	2.000000	9.000000	9.000000	1.776357e-15	9	2	border
1.000000	2.250000	12.390625	12.390625	0.000000e+00	10	2	border
1.000000	3.000000	28.000000	28.000000	0.000000e+00	11	2	border
1.333333	2.000000	10.370370	10.370370	1.776357e-15	12	2	border
1.333333	2.250000	13.841641	13.760995	8.064516e-02	13	2	inner
1.333333	3.000000	29.370370	29.370370	0.000000e+00	14	2	border
2.000000	2.000000	16.000000	16.000000	0.000000e+00	15	2	border
2.000000	2.250000	19.390625	19.390625	3.552714e-15	16	2	border
2.000000	3.000000	35.000000	35.000000	7.105427e-15	17	2	border

```
||u-u*||/||u*|| = 7.197704e-04
||u-u*|| = 1.140495e-01
```

8. Исследование на сходимость на равномерной сетке



Файл regions.txt

```
6
-1-
2 3 1 2
1 2 1 2
1 -2 1 -4
-2-
3 4 1 2
1 2 1 2
-1 1 1 -5
-3-
1 2 2 3
1 2 1 2
1 -4 1 1
-4-
2 3 2 3
1 2 1 2
-3 -5 -1 1
-5-
3 4 2 3
1 2 1 2
-4 -6 -2 1
-6-
4 5 2 3
1 2 1 2
-5 1 1 1
```

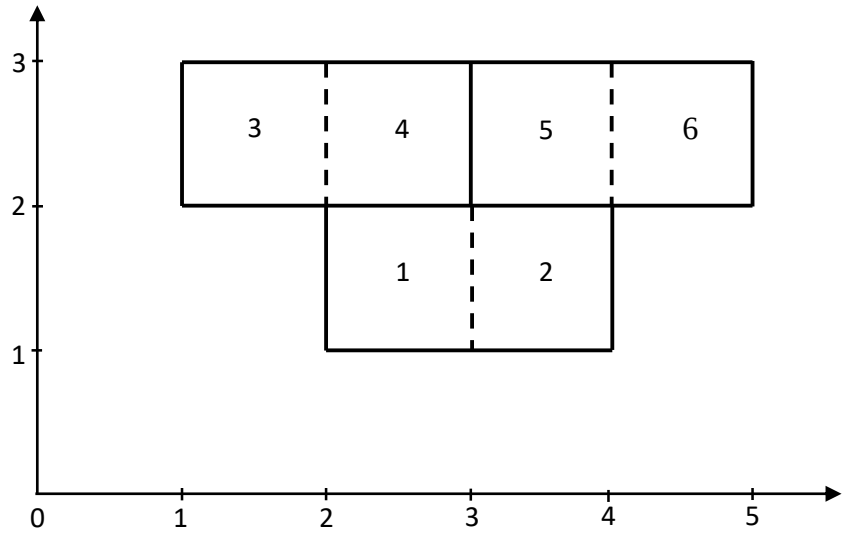
- $\lambda = 1, \gamma = 1, u^* = x^4 + y^4, f = -12(x^2 + y^2) + x^4 + y^4$

Рассмотрим значения численного и аналитического решения

n	x	y	u	u^*	$\lvert u - u^* \rvert$	$\frac{\lvert u-u^* \rvert_{n=2}}{\lvert u-u^* \rvert_{n=4}}$	$\frac{\lvert u-u^* \rvert_{n=4}}{\lvert u-u^* \rvert_{n=8}}$	$\frac{\lvert u-u^* \rvert_{n=8}}{\lvert u-u^* \rvert_{n=16}}$
2	3	2	97,249490	97	0,24949	3,815065142	3,99048084	4,002931119
4			97,065396		0,065396			
8			97,016388		0,016388			
16			97,004094		0,004094			

n	$\lvert\lvert u - u^* \rvert\rvert$	$\frac{\lvert\lvert u-u^* \rvert\rvert_{n=2}}{\lvert\lvert u-u^* \rvert\rvert_{n=4}}$	$\frac{\lvert\lvert u-u^* \rvert\rvert_{n=4}}{\lvert\lvert u-u^* \rvert\rvert_{n=8}}$	$\frac{\lvert\lvert u-u^* \rvert\rvert_{n=8}}{\lvert\lvert u-u^* \rvert\rvert_{n=16}}$
2	9,56854E-01	4,06148E+00	4,03094E+00	4,01635E+00
4	2,35593E-01			
8	5,84461E-02			
16	1,45521E-02			

9. Исследование на сходимость на неравномерной сетке



Файл regions.txt

```
6
-1-
2 3 1 2
4 2 9 2
1 -2 1 -4
-2-
3 4 1 2
4 2 9 2
-1 1 1 -5
-3-
1 2 2 3
4 2 9 2
1 -4 1 1
-4-
2 3 2 3
4 2 9 2
-3 -5 -1 1
-5-
3 4 2 3
4 2 9 2
-4 -6 -2 1
-6-
4 5 2 3
4 2 9 2
-5 1 1 1
```

- $\lambda = 1, \gamma = 1, u^* = x^4 + y^4, f = -12(x^2 + y^2) + x^4 + y^4$

Рассмотрим значения численного и аналитического решения

n	x	y	u	u^*	$\lvert u - u^* \rvert$	$\frac{\lvert u-u^* \rvert_{n=2}}{\lvert u-u^* \rvert_{n=4}}$	$\frac{\lvert u-u^* \rvert_{n=4}}{\lvert u-u^* \rvert_{n=8}}$	$\frac{\lvert u-u^* \rvert_{n=8}}{\lvert u-u^* \rvert_{n=16}}$
2	2,2	1,1	25,516022	24,8897	0,626322	4,07197051	3,829146855	3,890460048
4			25,043513		0,153813			
8			24,929869		0,040169			
16			24,900025		0,010325			

n	$\lvert\lvert u - u^* \rvert\rvert$	$\frac{\lvert\lvert u-u^* \rvert\rvert_{n=2}}{\lvert\lvert u-u^* \rvert\rvert_{n=4}}$	$\frac{\lvert\lvert u-u^* \rvert\rvert_{n=4}}{\lvert\lvert u-u^* \rvert\rvert_{n=8}}$	$\frac{\lvert\lvert u-u^* \rvert\rvert_{n=8}}{\lvert\lvert u-u^* \rvert\rvert_{n=16}}$	$\frac{\lvert\lvert u-u^* \rvert\rvert_{n=16}}{\lvert\lvert u-u^* \rvert\rvert_{n=32}}$	$\frac{\lvert\lvert u-u^* \rvert\rvert_{n=32}}{\lvert\lvert u-u^* \rvert\rvert_{n=64}}$
2	3,71357E+00	3,76258E+00	3,40017E+00	3,67662E+00	4,04650E+00	4,07278E+00
4	9,86975E-01					
8	2,90272E-01					
16	7,89507E-02					
32	1,95109E-02					
64	4,79E-03					

10. Выводы

Результаты проверки работоспособности программы:

Для расчетных областей различной формы на равномерной сетке:

- $\lambda = 1, \gamma = 1, u^* = x^3 + y^3, f = -6(x + y) + x^3 + y^3$

буква	$\frac{\|\|u - u^*\|\|}{u^*}$	$\|\|u - u^*\|\|$
L	1,364848E-16	1,361552E-14
Т	1,554534E-16	3,717639E-14
П	1,511169E-16	4,190471E-14
Г	1,457672E-16	1,884111E-14
Ш	1,199623E-16	1,051582E-13
разрывная	1,671358E-16	2,803044E-14

- $\lambda = 1, \gamma = 1, u^* = x^4 + y^4, f = -12(x^2 + y^2) + x^4 + y^4$

буква	$\frac{\|\|u - u^*\|\|}{u^*}$	$\|\|u - u^*\|\|$
L	1,256545E-03	3,238617E-01
Т	6,042730E-04	4,650397E-01
П	5,016553E-04	4,677078E-01
Г	9,286118E-04	3,238617E-01
Ш	1,458105E-04	6,699838E-01
разрывная	1,430753E-04	8,318903E-02

Равномерная сетка имеет третий порядок аппроксимации, поэтому при $u^* = x^4 + y^4$ наблюдается появление погрешности вычислений.

При тестировании для расчетных областей различной формы на равномерной сетке мы получили примерно одинаковую погрешность решения.

Для расчетных областей различной формы на неравномерной сетке:

- $\lambda = 1, \gamma = 1, u^* = x^2 + y^2, f = -4 + x^2 + y^2$

буква	$\frac{\|\|u - u^*\|\|}{u^*}$	$\|\|u - u^*\|\|$
L	1,498774E-16	6,089044E-15
Т	2,061177E-16	1,606106E-14
П	1,997274E-16	1,740467E-14
Г	1,147849E-16	5,546672E-15
Ш	1,587415E-16	2,911072E-14
разрывная	1,832184E-16	9,272855E-15

- $\lambda = 1, \gamma = 1, u^* = x^3 + y^3, f = -6(x + y) + x^3 + y^3$

буква	$\frac{\|\|u - u^*\|\|}{u^*}$	$\|\|u - u^*\|\|$
L	1,249196E-03	1,207434E-01
Т	8,832807E-04	2,059691E-01
П	7,586340E-04	2,061893E-01
Г	1,281501E-03	1,565605E-01
Ш	3,577370E-04	3,120607E-01
разрывная	7,197704E-04	1,140495E-01

Равномерная сетка имеет второй порядок аппроксимации, поэтому при $u^* = x^3 + y^3$ наблюдается появление погрешности вычислений.

При тестировании для расчетных областей различной формы на неравномерной сетке мы получили примерно одинаковую погрешность решения.

Результаты исследования порядка сходимости:

В результате исследования на равномерной сетке, мы получили, что при дроблении сетки в 2 раза погрешность решения падает в 4 раза, следовательно, порядок сходимости на равномерной сетке равен 2.

В результате исследования на неравномерной сетке, мы получили, что при дроблении сетки в 2 раза погрешность падает в 4 раза, следовательно, порядок сходимости на неравномерной сетке равен 2.