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A New Fuzzy Propagation Model for Influence Maximization in Social Networks

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In this paper we introduce a fuzzy propagation model to deal with the influence maximization (IM) problem. The IM problem for the most of existing propagation model is NP-hard. Here, we model social networks as fuzzy directed graphs to propose an application-oriented propagation process. To this aim, we investigate an interesting relationship between zero forcing set concept in graphs and IM problem in social networks. In spite of its attractive theory, its implementation is not efficient in the real world. Thus, we improve the fuzzy zero forcing set concept and suggest our fuzzy propagation model, simultaneously. Moreover, we present a polynomial time complexity algorithm to solve IM problems under the proposed propagation process. In particular, we consider a propagation parameter to control the size of the seed set and its coverage. Also, experimental results on some real world social networks show that the propagation model finds optimal and flexible seed set.

Keywords: Social network; propagation; influence maximization; fuzzy graphs; fuzzy zero forcing set.

1. Introduction

The history of social networks reveals their critical role in progressively changing customs, patterns, social norms or even people lifestyle. Because of that, different industries use social networks as tools for influencing people and achieving their goals. Nevertheless, the number of users in social networks is too large and interacting directly with all users is impossible. It is known that users are affected and

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embrace behavior of others, and, for example, celebrities, politicians, artists are known to have a better ability to influence others. Therefore, finding a set of the most influential users facilitates the use of social networks to the industry. For instance, promotion and marketing in tourism industry aims to detect specific niche tourist groups instead of considering all users in social networks.[1]

Influence maximization problem is to find an appropriate number (k) of influential users who can widely propagate information/ideas in the network. A first result was Kempe *et al.*,[2] who suggested an optimization problem to formulate the influence maximization (IM) problem. In IM problem, spreading the influence is based on a propagation model. Thus, such propagation process should be determined at first. The most basic and common models for propagation are linear threshold,[3] the independent cascade[2] and greedy propagation model (heuristic-based model).[4]

Kempe *et al.*[2] proved that IM problems are NP-hard for linear threshold and independent cascade propagation models. To deal with this hardness, the meta-heuristic methods approximate IM solutions in polynomial time by greedy algorithms. Azaouzi *et al.*[5] reviewed meta-heuristic methods and compared their complexity. In this paper, we design a fuzzy propagation model to solve IM problems in polynomial time. We should notice that our model is not meta-heuristic.

In recent two decades, the IM problem has attracted a lot of attention and many researcheres focused on investigation IM problem from different aspects (see Refs. 5, 6, 7, 8 and references there in). Most of works have crisp background, whereas there are some studies to take advantages of fuzzy logic and game theory for social network analysis.[10–14] Atif *et al.*[14] detected some communities, then they use fuzzy concepts to choose influential users of every community. Finally, they approximate seed set for an IM problem.

All existing IM research, up to our knowledge, have modeled social networks as weighted crisp graphs where vertices correspond to the users of the network and whose relationships are represented as the edges. In this case, all users (vertices) and relationships (edges) are precisely known and have the same priority and reputation (see Refs. 5, 9 and references there in). The weights are often assigned to the edges. It means that only relationships between users are considered and the effect of users are not checked individually. In other research, such as heuristic-based methods,[5,16,17] node centrality measures are assigned as nodes weights. Detecting all weights and even thresholds aims to find most influential users and most methods determine them randomly or by probability distribution function.

In the discussion of real world networks, influencer, influenced, and influence can be considered as linguistic variables and, as such, suffer from ambiguity. To deal with this vagueness, they can be modeled by fuzzy set theory. In this paper, we simultaneously focus on the role of users and relationships among them and also the existing ambiguity to solve IM problems. More particular, we consider social networks as fuzzy graphs to deal with the mentioned ambiguity. To this end, we define a fuzzy set of active people to show the degree of activeness of each user in the network. Moreover, we define a fuzzy set to show degree strength of the relationship between users.

In our approach to discover the most influential users, we should introduce a propagation model in fuzzy graphs. Our approach is inspired in the color changing rule for detecting zero forcing set (ZFS)[15] in graphs. We suggest that spreading information in the network is repeatedly applying the color changing rule. The proposed propagation model makes an interesting relationship between solving an IM problem and detecting ZFS. We also investigate the advantages and disadvantages of the propagation process. Finally, we improve the definition of fuzzy zero forcing set and propose a new fuzzy propagation model.

The rest of the paper is organized as follows. Section 2 presents some preliminaries and mathematical background, and Sec. 3 describes our proposed fuzzy propagation model. Section 4 is devoted to description of experimental results of simulation of the propagation model on state of art data sets in IM problems. Finally, Sec. 5 includes some conclusions and suggestions for further researches.

2. Preliminaries and Mathematical Background

In this section, we provide some relevant graph theory and influence maximization concepts which are used in the paper. Also, we extend some definitions to the fuzzy case.
A simple undirected graph (no loops, no multiple edges) is denoted by $G = (V, E)$ where $V = \{v_1, \dots, v_n\}$ is the set of vertices and $E \subseteq V \times V$ is the edge set.

In what follows, firstly, we give a definition of color changing rule on a graph which is used to introduce a zero forcing set.

Definition 1.[15] Let G be a graph with all vertices initially colored either black or white. If u is a black vertex of G, with exactly one white neighbor v, then change the color of v to black. When this rule is applied, we say u forces v.

Definition 2.[15] A *Zero Forcing Set* (ZFS) of a graph G is a subset Z of V which is initially contained black-colored vertices such that it can force the entire graph G to be black by repeatedly applying the color changing rule. The zero forcing number of $G, Z(G)$, is the minimum size of a zero forcing set. Any zero forcing set of order $Z(G)$ is called a minimum zero forcing set.

Example 1.[19] We have $Z(K_n) = n - 1$ and $Z(P_n) = 1$ (see Fig. 1).

There are various ways to model social networks as graphs.[10,18] Generally, a social network can be modeled as a weighted graph $G = (V, E, W)$ where V, E, and W are the vertex, edge, and weight set, respectively. Indeed, V is a group of individuals (users) such as people or organizations who spread information, ideas or rumor. E represents relationships between individuals and their interactions.

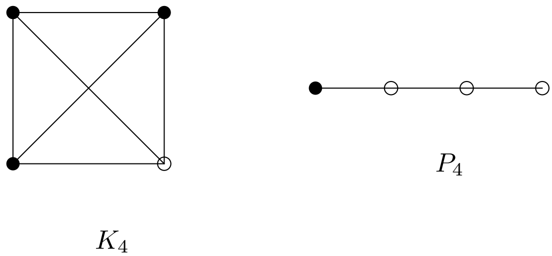

Fig. 1. $Z(K_4) = 3,\ Z(P_4) = 1.$

Moreover, $W = \{w_{u,v} \in (0, 1)|(u, v) \in E\}$ where $w_{u,v}$ is influence spread probability. Specifying weights is based on the static or dynamic analysis of the social network.[5,9]

An influence maximization problem formally defined as follows:

Definition 3.[4] Let graph $G = (V, E, W)$ and a positive integer k $(1 \leqslant k \leqslant n)$ are given. Influence maximization problem finds k most influential nodes as seed set S^*. We have

$$S^* = arg \max_{S \subseteq V, |S|=k} \sigma(S) \tag{1}$$

where, $\sigma(S)$ is the expected number of active nodes using S under a given propagation model.

In order to solve an IM problem in a social network, we model it such as finding a ZFS in the graph of social nework. To this aim, we consider active and inactive nodes in a social network are black and white, respectively. Then, information propagation or activating process is corresponding to repeatedly apply the color changing rule. Since an active node focuses only to inform whose only single inactive neighbor, then we expect that its success probability would be high. In particular, a group of active nodes can easily affect one single neighbor. Therefore, similar to the color changing rule, we let every single or group of influential nodes force only one single neighbor in each step. Proposition 1 shows that the best solution of an IM problem under above process is a ZFS on a graph.

Proposition 1. *S^* is the minimal seed set with $\sigma(S^*) = |V|$ if and only if S^* is a ZFS on graph G.*

Following example illustrates the above propagation model.

Example 2. Figure 2 shows a small social network with 4 nodes.

Case 1. Set $k = 1$, the seed set may be $S_i = \{v_i\}$ for $i = 1, \ldots, 4$. It is clear that $\sigma(S_i) = 1$ for $i = 1, \ldots, 4$ and none of influential nodes can inform even one node.

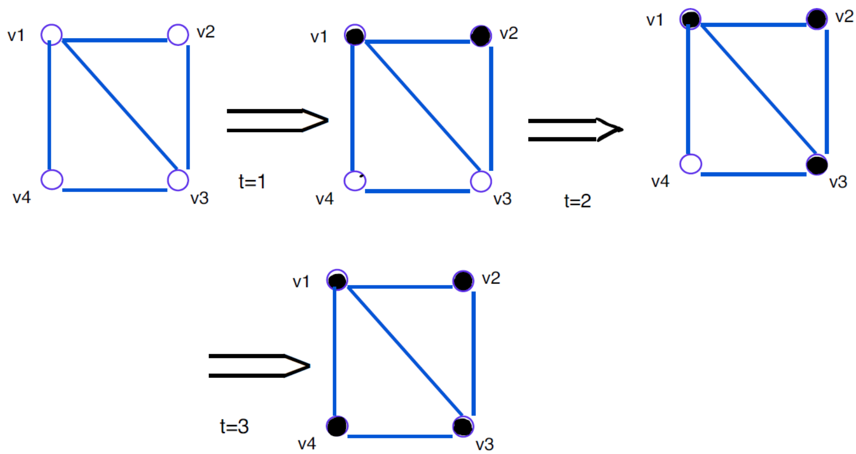

Fig. 2. Impelimentation of the propagation model.

Case 2. Set $k = 2$, $\sigma(S_{1,3}) = \sigma(S_{2,3}) = \sigma(S_{2,4}) = \sigma(S_{3,4}) = 2$ and $\sigma(S_{1,2}) = \sigma(S_{1,4}) = 4$. Its clear that the optimal seed set S^* is $S_{12} = \{v_1, v_2\}$ or $S_{14} = \{v_1, v_4\}$. Figure 2 shows propagation process with seed set S_{12}. Obviously, both of S_{12} and S_{14} are ZFS.

Case 3. Set $k = 3$, the seed set may be $S_{ijk} = \{v_i, v_j, v_k\}$ for $i, j, k = 1, \ldots, 4$, $i \neq j \neq k$. It is clear that $\sigma(S_{ijk}) = |V| = 4$ and therefor $S_{ijk} = S^*$.

The major advantages of above suggested propagation process are its strong mathematical background and finding the best solution (ZFS) for IM (if exists). However, the spreading process is too restrictive. Now, we improve this propagation process to find a practical and applicable solution for IM using fuzzy set theory.

Definition 4.[20] A fuzzy directed graph is a pair $\widetilde{G} = (\widetilde{V}, \widetilde{E})$ where $\widetilde{V}$ is a fuzzy set of vertices with membership function $\mu : V \longrightarrow [0,1]$ and $\widetilde{E}$ is a fuzzy set of edges with membership function $\rho : V \times V \longrightarrow [0,1]$ such that for all x, y in V we have $\rho(v_i, v_j) \leq \mu(v_i) \wedge \mu(v_j)$.

Justification. We model a social network as a fuzzy graph. $\mu(v_i)$ is the degree of activeness which can be determined based on the type of social network service. For instance, in Instagram, Facebook, Twitter, $\mu(v_i)$ can show the degree of reputation or activity of user v_i. $\rho(v_i, v_j)$ is the degree strength of the relationship between v_i and v_j. For instance, in Instagram if v_i just follows v_j without any more activity, $\rho(v_i, v_j)$ is a small value. Since, in social networks the strengths degree of relationships between (v_i, v_j) and (v_j, v_i) are not necessarily the same, then we consider a directed fuzzy graph.

3. A New Fuzzy Propagation Model

In this section, we deal with the influence maximization problem by presenting a new fuzzy propagation process. Proposition 1 shows an interesting relationship between IM under the color changing rule propagation process and ZFS. However, detecting ZFS in crisp graphs is NP-hard and equivalently informing the whole network with minimal seed set is NP-hard, too. In reality, we do not expect to inform the whole network and moreover, propagation process is more flexible than the color changing rule. In order to deal with these restrictions, we extend the ZFS on fuzzy graph such that by detecting fuzzy ZFS, we simultaneously suggest our propagation model.

Definition 5.[21] Let $\widetilde{G} = (\widetilde{V}, \widetilde{E})$ be a fuzzy digraph. The influence rate for every vertex v_i is defined as follows:

$$I(v_i) = \mu(v_i)\frac{\sum_{v_j} \rho(v_i, v_j)\mu(v_j)}{d_i}, \tag{2}$$

where d_i is the out degree of v_i ($d_i = \sum_{v_j} \rho(v_i, v_j)$).

Justification. In social networks, the influence rate of node v_i usually depends on its degree of activeness, $\mu(v_i)$, and also strength degree of its relationships and the degree of activeness of its neighbors.

Aliahmadipour and Rashidi[21] defined force rate of v on u. Based on their definition, the influential nodes do not effect of other nodes, while in real world, influential nodes can take effect of others, too. Therefore, in this paper, we suggest the following definition.

Definition 6. Let $\widetilde{G} = (\widetilde{V}, \widetilde{E})$ be a fuzzy graph. Force rate of v on u is denoted by $f(v, u)$ and computed as follows:

$$f(v, u) = \frac{\rho(v, u)}{\rho(v, u) + I(u)}. \tag{3}$$

Justification. The more strength degree of relationship from v_i to v_j, the more v_i can affect v_j. Moreover, we expect that the more influence rate of v_j, the more resistant to be informed by v_i.

Here, we define a black fuzzy set of digraph $\widetilde{G}$ as $\widetilde{V}^b = \{(v_i, \mu_{\widetilde{V}^b})|v_i \in V\}$ where $0 \leqslant \mu_{\widetilde{V}^b}(v_i) \leqslant 1$ shows the membership degree of v_i in the black color spectrum. For instance, if $\mu_{\widetilde{V}^b}(v_i) = 1$, then v_i is totally black vertex and if $\mu_{\widetilde{V}^b}(v_i) = 0$, then v_i is totally white vertex.

Throughout this paper, each v_i with $\mu_{\widetilde{V}^b}(v_i) \geq \alpha$ is considered black and other nodes are not black, where $0 < \alpha \leq 1$ is a given parameter. Also, for every node $v_i \in V$, the set $BN(v_i)$ contains black neighbors of v_i, i.e., $BN(v_i) = \{v_j \in N(v_i)|\mu_{\widetilde{V}^b}(v_j) \geq \alpha\}$.

Justification. α is a propagation parameter and can be set based on types of network services and the role of users in social networks. For instance, a pessimistic decision maker selects a large value of α. Indeed, she/he just lets users with high influential rate to inform others. Of course, it makes the size of seed set S^* be small. But, the coverage of the network can be poor ($\sigma(S^*) << |V|$). On the other hand, an optimistic decision maker for choosing α considers the trade off between the minimal size for seed set S^* and the percentage of network coverage. Section 4 illustrates more clearly the role of propagation parameter in practice.

In what follows, we define a fuzzy color changing rule.

Definition 7. Let $\widetilde{V}^b$ be a black fuzzy set. For every node v_i which is not black ($\mu_{\widetilde{V}^b}(v_i) < \alpha$), if

$$max_{v_j \in BN(v_i)}\{I(v_i), f(v_j, v_i)\} \geq \alpha,$$

then change the color of v_i to black and set

$$\mu_{\widetilde{V}^b}(v_i) = max_{v_j \in BN(v_i)}\{I(v_i), f(v_j, v_i)\}.$$

Justification. We assume a user with influential rate greater than or equal propagation parameter is informed without noticing which is forced or not by other influential users. Of course, we let only influential users (black nodes) inform other users. An uninformed /less informed user v_i can be affected by its informed neighbors which have strong relationships with v_i.

Definition 8. We consider $\widetilde{V}^b = \{(v_i, I(v_i))|v_i \in V\}$ as a fuzzy black set of $\widetilde{G}$. If $\widetilde{V}^b$ can force the entire graph $\widetilde{G}$ to be black by iteratively applying the fuzzy color changing rule, then we call the initially black-colored vertices of $\widetilde{V}^b$, i.e.,

$$\widetilde{Z} = \big\{(v_i, I(v_i))|I(v_i) \geq \alpha, v_i \in V\big\},$$

as fuzzy zero forcing set ($FZFS$).

Justification. When we want to propagate some information in the social network we should find influential users to do it. Based on the log histories of users we can compute their influence rate. The users with influence rate grater than propagation parameter, α, are proper candidates as seed set.

The propagation time of a fuzzy zero forcing set $\widetilde{Z}$ of a fuzzy graph is defined in.[21] Here, we clarify this definition.

Definition 9. The propagation time of a fuzzy zero forcing set $\widetilde{Z}$ in a fuzzy digraph $\widetilde{G}$, denoted by $pt(\widetilde{G}, \widetilde{Z})$, is the minimum number of steps/iterations which can change the whole graph to black.

Algorithm 1 finds FZFS on a fuzzy graph and equivalently solves an IM problem.

Algorithm 1. Constructing fuzzy zero forcing set

Input: Fuzzy graph $\widetilde{G} = (\mu, \rho)$ and α.
Output: $\widetilde{Z} = \{(v_i, \mu_{\widetilde{Z}^1}(v_i) | \mu_{\widetilde{Z}^1}(v_i) \geq \alpha, v_i \in V\}$.
Step 1. Compute $I(v_i)$ for all $v_i \in V$, $B \leftarrow \emptyset$.
Step 2. If $I(v_i) = 1$ for $i = 1, \ldots, n$, then
    the graph is crisp and STOP.
  Otherwise,
    set $t \leftarrow 1$ and $\mu_{\widetilde{Z}^t}(v_i) \leftarrow I(v_i), \ \forall v_i \in \widetilde{V}$.
Step 3. For i=1 to n
    If $\mu_{\widetilde{Z}^t}(v_i) \geq \alpha$, then
      $B \leftarrow B \cup \{v_i\}$.
    If $B = \emptyset$, then
      there is no black node, STOP.
  end For.
Step 4. *control* $\leftarrow 0$,
  While $B \neq V$
    For $u_i \in V \setminus B$
      Compute $f(v_j, u_i)$, $\forall v_j \in BN(u_i)$,
      $\mu_{\widetilde{Z}^{t+1}}(u_i) \leftarrow \max_{v_j \in BN(u_i)}\{\mu_{\widetilde{Z}^t}(u_i), f(v_j, u_i)\}$,
      If $\mu_{\widetilde{Z}^{t+1}}(u_i) \geq \alpha$, then
        $B \leftarrow B \cup \{u_i\}$ and *control* $\leftarrow 1$.
    end For
    $t \leftarrow t + 1$,
    If *control* $= 0$, then
      go to Step 6.
  end While.
Step 5. FZFS is $\widetilde{Z} = \{(v_i, \mu_{\widetilde{Z}^1}(v_i) | \mu_{\widetilde{Z}^1}(v_i) \geq \alpha, v_i \in V\}$, STOP.
Step 6. Write ("The fuzzy Graph G doesn't have FZFS."), STOP.

Justification. It should be noticed that Algorithm 1 whether stops in Step 5 or Step 6, it solves the IM problem. When it stops in Step 5, we have the ideal solution for IM which inform the whole network. Otherwise, we still have a minimal seed set but it cannot inform the entire network.

Proposition 2. *In Algorithm 1, $pt(\widetilde{G}, \widetilde{Z}) = t$.*

Time complexity of Algorithm 1. Running time of the algorithm is polynomial time of $O(tn^2 + 4t)$ and depends on the number of iterations, $t << n$ (see Table 1 for details).

Table 1. Complexity computation details.

Step	**Complexity**
Step 1.	$o(n)$: computing $I(v_i)$.
Step 2.	$o(n)$: search for finding $I(v_i) = 1$.
Step 3.	$o(n)$: in the worst case one vertex is chosen.
Step 4.	$o(t)$: "while" loop is repeated t times to make G to be black.
	$o(\frac{n}{2} \times \frac{n}{2})$: in the worst case, there are $\frac{n}{2}$ white nodes that each one has $\frac{n}{2}$ black neighbors to compute $f(v_j, u_i)$.
	$o(1)$: assigning the membership.
	$o(1)$: check $\mu_{\widetilde{Z}^{t+1}}(u_i) \geq \alpha$.
	$o(1)$: update B.
	$o(1)$: check *control*.
Final complexity	$n + n + n + t(\frac{n}{2} \times \frac{n}{2} + 1 + 1 + 1 + 1) = t\frac{n^2}{4} + 3n + 4t$.

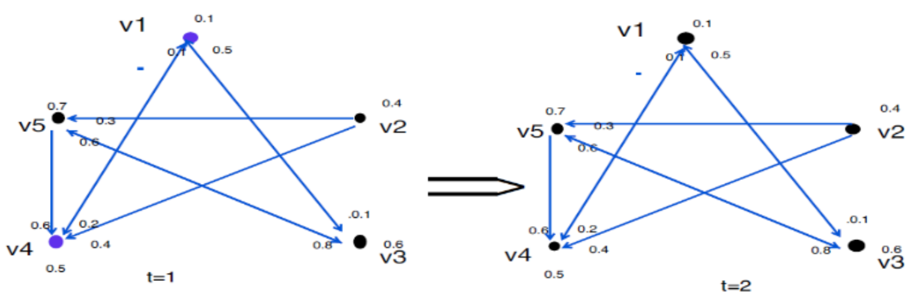

Fig. 3. Impelimentation of the propagation model.

In the following section, some examples illustrate the new definition (FZFS) and the algorithm.

Example 3. In this example, we implement Algorithm 1 on a fuzzy graph with some strong vertices in Fig. 3. We consider $\alpha = 0.2$ and find the seed set in the following steps:

- $I = \{\frac{v_i}{I(v_i)} | i = 1, \ldots, 5\} = \{\frac{v_1}{0.053}, \frac{v_2}{0.234}, \frac{v_3}{0.256}, \frac{v_4}{0.050}, \frac{v_5}{0.39}\}$, $t = 0$ and $B = \emptyset$.
- $\widetilde{Z}^1 = I$ and $\mu_{z^0}(v_i) = I(v_i)$ for each v_i.
- $B = \{v_2, v_3, v_5\}$ and $\widetilde{Z}^2 = \{\frac{v_1}{0.90}, \frac{v_2}{0.234}, \frac{v_3}{0.256}, \frac{v_4}{0.92}, \frac{v_5}{0.39}\}$, $t = 1$ and go Step 3 again.
- $B = \{v_1, v_2, v_3, v_4, v_5\}$ and $\widetilde{Z}^2 = \{\frac{v_1}{0.90}, \frac{v_2}{0.234}, \frac{v_3}{0.256}, \frac{v_4}{0.92}, \frac{v_5}{0.39}\}$, and $t = 2$ and go Step 4 again.
- Algorithm is stopped. So $\widetilde{Z} = \widetilde{Z}^1$ is a FZFS of this fuzzy graph and $\widetilde{pt}(G, \widetilde{Z}) = 2$.

Remark 1. Algorithm 1 detects the influential nodes based on the propagation parameter α. When we choose small values for α, there is more chance for nodes with small μ and ρ to be in propagation process. Hence, the size of influential nodes set is often large and it covers almost the entire network. This case obtains an upper bound for the set of influential nodes which can be used as initial approximation by other existing algorithms. On the other hand, for the large value of α, the size of seed set would be very small whereas the coverage percent would be less.

Section 4 shows the role of propagation parameter α in practice.

4. Experimental Results

This section deals with the simulation of the proposed model to detect influential nodes in four large social network datasets, available from the SNAP repository.[22] Also, we use the extracted subgraphs from Flickr data set[22] which is studied by Atif *et al.*[14] Moreover, due to the fuzzy base of Atif *et al.* method, we give a brief comparision between their results and ours on Flickr data set.[22]

A brief description of the datasets is reported in Table 2. Bitcoin, Bitcoin-alpha, and Wiki-Vote, Flickr, and CA-Hepth have 3783, 5881, 7115, 500, and 9877 nodes, respectively. Also, they have wide variations in terms of node degrees and large maximum out-degree.

We make fuzzy graphs for networks in Table 2 based on their attributes. Bitcoin and Bitcoin-alpha networks are weighted and directed graphs. Therefore, we transform their weights to the membership value of edges (ρ) with linear transformation $\frac{x-\min}{\max-\min}$. Since there is no information about the networks nodes, then the membership value of nodes (μ) are generated with uniform distribution in $[0, 1]$. For Wiki-Vote, Fickr, and CA-HepTh, both the membership value of nodes (μ) and edges (ρ) are selected uniformly in $[0, 1]$.

As a first experiment, we implement Algorithm 1 in MATLAB (2017 b) and Gephi. We initially set different values of propagation parameter α. Then, for each α we run the algorithm 10 times and present the average of results in Tables 3–6. As

Table 2. The social networks from the Stanford large network datasets.[22]

Name	Type	Nodes	Edges	Description
soc-sign-bitcoin-otc	Weighted, Directed,	5,881	35,592	Bitcoin OTC web of trust network
soc-sign-bitcoin-alpha	Weighted, Directed,	3,783	24,186	Bitcoin Alpha web of trust network
Wiki-Vote	Directed	7115	103689	Wikipedia who-votes-on-whom network
Flickr	Directed	500	7115	Image relationships on Flickr
CA-HepTh	Directed	9877	51971	High Energy Physics - Theory collaboration network

Table 3. Results of applying Algorithm 1 on Bitcoinalpha network with arbitrary α.

α	The number of influential nodes	The number of informed nodes	Propagation time	The percentage of coverage
0.4	951	3775	6	%99.79
0.5	477	3777	6	%99.80
0.6	211	3198	7	%84.54
0.7	104	2028	10	%54.8
0.73	106	1596	11	%0.4219
0.76	83	83	1	%0.01
0.78	47	47	1	%0.01
0.8	29	29	1	%0.0075

Table 4. Results of applying Algorithm 1 on Bitcoin network with several α.

α	The number of influential nodes	The number of informed nodes	Propagation time	The percentage of coverage
0.4	1178	5877	6	%99.94
0.5	691	5875	6	%99.90
0.6	221	5158	7	%88.17
0.7	154	3542	10	%60.23
0.73	148	3078	9	%52.2
0.76	80	80	1	%0.01
0.78	77	77	1	%0.01
0.8	42	42	1	%0.0071

Table 5. Results of applying Algorithm 1 on Wiki-vote network with several α.

α	The number of influential nodes	The number of informed nodes	Propagation time	The percentage of coverage
0.4	1419	6887	4	%96.8
0.5	605	6700	5	%94.17
0.6	262	6237	5	%87.66
0.7	133	4910	7	%69.01
0.73	107	4078	7	%57.32
0.76	86	3585	7	%50.39
0.78	80	80	1	%0.01
0.8	59	59	1	%0.0083

you see, Algorithm 1 has similar outcomes in all networks and the results confirm Remark 1. When α increases from 0.7 to 0.8, the coverage of all networks quickly decreases from more than %50 to less than %1. Therefore, we investigate the effect of changing α in $[0.7, 0.8]$ on the coverage of the network more precisely. We see that for each network, there is a critical range of α in which a sharp decrease occurs in the coverage rate. In spite of using random membership values, the results are independent of the size of network and only depend on the value of α.

As a second experiment, we briefly compare the results of applying Atif *et al.*[14] approach and our results on Flickr data set in Table 8. Algorithm 1 does not take k as an input parameter and it finds the optimal seed set with respect to α, while Atif *et al.* method set k as an input. Therefore, we cannot choose the same values

Table 6. Results of applying Algorithm 1 on CA-HEP-TH network with several α.

α	The number of influential nodes	The number of informed nodes	Propagation time	The percentage of coverage
0.4	2061	9626	9	%97.46
0.5	884	9351	8	%94.99
0.6	352	8635	9	%87.43
0.7	123	6295	12	%63.73
0.73	64	7030	9	%71.1
0.76	30	6017	12	%60.12
0.78	23	5031	12	%56.67
0.8	31	31	1	%0.0032

Table 7. Results of applying Algorithm 1 on Flickr network with several α.

α	The number of influential nodes	The number of informed nodes	Propagation time	The percentage of coverage
0.4	76	491	4	%98.79
0.5	33	492	4	%98.99
0.6	8	474	6	%95.3
0.7	12	337	6	%67.49
0.73	14	14	1	%0.02
0.76	11	11	1	%0.02
0.78	9	9	1	%0.01
0.8	4	4	1	%0.008

Table 8. Results of comparision on Flickr network.

Atif et. al		our proposed method	
The size of seed set (k)	Coverage	The size of seed set (k)	Coverage
10	%68	8	%95.3
15	%70	12	%67.49
35	%74	33	%98.99
50	%78	76	%98.79

of k for both methods in Table 8 and we compare the coverage of network with respect to similar values of k.

5. Conclusion

It is well known that every one is strongly influenced by others, in particular by neighbors or friends. Therefore, influence propagation plays a fundamental role in social networks. In this paper, we focus on influence maximization problems in social networks. We model the networks as fuzzy graphs and define a fuzzy color changing rule to suggest our new fuzzy propagation model. Then, we improve the existing algorithm for finding fuzzy zero forcing set to solve IM problems. The remarkable advantages of our propagation model are its mathematical background, concerning the role of both users and their relationships, and solving IM problem in polynomial time. We also provide computational experiments on five social networks, Finally, we implement the proposed algorithm on Bitcoin-alpha, Bitcoin, Wiki-vote, Flikcr, and CA-HEP-TH networks. We consider two subjects for further research concerning privacy preservation in our propagation model and also computing the the membership values of nodes in a network based on their historical logs.

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