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Fuzzy sign-aware diffusion models for influence maximization in signed social networks

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ABSTRACT

The diffusion models in the influence maximization problem are a trending topic in many companies' viral marketing to raise their business promotion. The majority of existing diffusion models have focused only on trust relationships, and a few models have considered distrust relationships. Nevertheless, the latter models lack appropriate theoretical support to cover social influences resulting from different user-relationship types and still do not provide proper predictions. In this study, a fuzzy-based approach is first introduced to model the influence propagation for different user-relationship types. Then, four novel fuzzy sign-aware diffusion models named FSC-SB, FSC-N, FST-SB, and FST-N are proposed by the introduced fuzzy-based approach in two categories: cascade and threshold-based models. In the proposed models, the user-relationship type is determined by a fuzzy expert system in which a natural multi-trust level relationship is applied instead of a commonly used crisp relationship. Moreover, new rules and equations are defined to determine a user's state by information received from its active neighbors. The performance of proposed models was compared with some state-of-the-art models conducted by two real-world networks, Bitcoin OTC and Bitcoin Alpha. The experimental results showed that the proposed models enhance the prediction accuracy and make effective decisions in viral marketing.

1. Introduction

Nowadays, social networks play a significant role in daily life to share opinions and propagate information easily and quickly through the internet [47]. Due to this popularity, many organizations utilize these social network platforms for viral marketing [49]. This marketing strategy is based on word-of-mouth interaction and communication among consumers to advertise products or services at low cost but highly effective [37]. Considering the limitations of the advertising budget in organizations, marketing begins with a few customers as seed sets who are encouraged to propagate the advertisements among their friends and acquaintances [6]. After this propagation process, the organizations would be exposed to the sight of a large number of people, so many of them start adapting to advertise and buy their products.

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Motivated by the idea of viral marketing, the influence maximization problem was first studied in the social network as an optimization problem by Domingo and Richardson [36]. Then, Kempe et al. [15,16] introduced the formal definition of influence maximization as a discrete optimization problem. A social network in this domain is constructed as a graph where vertices indicate customers/users and edges correspond to the relationships among them. Each edge is associated with an influence probability where a user influences another one. The purpose of the influence maximization problem is to determine a small subset of the k users who can maximize the influence propagation over a social network under a specific information diffusion model.

Diffusion models can be utilized to analyze and predict information propagation patterns in social networks [48]. The current diffusion models are mostly based on the trust relationships in users' interactions, and a few of them consider the distrust relationships of users existing in the real world. In some contexts, the trust relationships are named positive or friend relationships, and the distrust relationships are referred to as negative, spite or foe relationships [13,23]. There have been proposed various diffusion models with respect only to the trust relationships such as independent cascade (IC) [15], linear threshold (LT) [15], susceptible-infected-removed (SIR) [35], linear threshold with one direction state transition (LT1DT) [45], susceptible-infected-hesitation knowledge diffusion (SIH) [50]. Recently, some research has been carried out to expand the diffusion models by adding the distrust relationships along with trust ones, like polarity-related independent cascade (IC-P) [23], independent cascade model with negative opinions (IC-N) [4], trust-based latency aware independent cascade (TLIC) [29], linear threshold with multi-level attitude (LT-MLA) [42], and heat diffusion-based polarity influence diffusion (HDPID) [43]. However, the existing diffusion models do not fulfill the need for precise and accurate predictions. One of the significant shortcomings is that the crisp logic is considered for users' social relationships in these models, while relationships do not obey the crisp logic in the real world. A crisp relationship means paying attention only to whether a relationship exists or not. Users may have several neighbors, and indeed, they will have different influences on each of their neighbors. Therefore, a social network consists of users with different relationships strength from low to high, depending on their friendship and trust level.

To overcome these shortcomings, several sign-aware diffusion models covering comprehensive fuzzy relationships among network users will be proposed in this study. The theory of fuzzy logic, introduced by Zadeh [46], has achieved a great deal of attention in various fields of social networks because of its capability to handle the uncertainty embedded in real-life situations such as influence maximization [3], link prediction [9], community detection [5], marketing analysis [8], and privacy-preserving [21]. Although the information diffusion models have been widely studied in a crisp setting, to the best of our knowledge, their extension in a multi-state relationships setup based on using a fuzzy expert system has not been considered yet. With that regard, the following points summarize the contributions of this study.

- A fuzzy-based approach is introduced using three plans to determine influence diffusion patterns through trust and distrust relationships on signed social networks.
- Four new information diffusion models based on the fuzzy expert system are proposed in two different groups of cascade and threshold diffusion models:
  - The fuzzy sign-aware cascade model including suspending and blocking users (FSC-SB) and the fuzzy sign-aware cascade model including negative users (FSC-N) are proposed in the cascade diffusion models group.
  - The fuzzy sign-aware threshold model including suspending and blocking users (FST-SB) and the fuzzy sign-aware threshold model including negative users (FST-N) are proposed in the threshold diffusion models group.
- The monotonicity and submodularity properties of the proposed diffusion models are investigated via some theorems. Moreover, it is demonstrated that the influence maximization problem is NP-hard under the proposed models.
- The performance of proposed cascade-based and threshold-based diffusion models has been evaluated by two real-world signed social networks, Bitcoin OTC and Bitcoin Alpha [19,20]. The experimental results show the effectiveness of the proposed fuzzy sign-aware diffusion models compared with the state-of-art models.
- Additionally, in this study, we introduce a standard presentation of components and notations in the signed social networks. These components and notations can be applied to influence maximization related topics to have integrity in these social networks.

The remaining sections of this study are organized as follows. Section 2 reviews some related works on information diffusion models. Section 3 describes our fuzzy-based approach, including three plans for influence propagation through trust and distrust relationships in social networks. The corresponding fuzzy diffusion models for each plan are proposed in two categories: cascade and threshold-based models. Section 4 is dedicated to demonstrating the properties of proposed diffusion models. Details of the experiments and the evaluation results of the proposed diffusion models on Bitcoin OTC and Bitcoin Alpha networks are reported in section 5. Finally, concluding remarks and future research directions are presented in Section 6.

2. Related work

Since the first introduction of social network graphs, there have been found many applications in academic research [32], marketing, and business environments [2,18,47]. Accordingly, they have been extended in different aspects such as viral marketing [24,37], rumor containment [45], recommendation systems [38], community detection [5,31], link prediction [9], and social influence analysis [40,41]. This study focuses on the viral marketing optimization process under sign-aware influence diffusion models in social networks. Diffusion models are used to describe and predict information propagation processes such as news, advertisements, opinions, and rumors among social network users for viral marketing [26]. The grey highlighted rectangles of Fig. 1 represent trends in the field of this study.

As seen in Fig. 1, information diffusion models can be classified generally into progressive and non-progressive models in viral

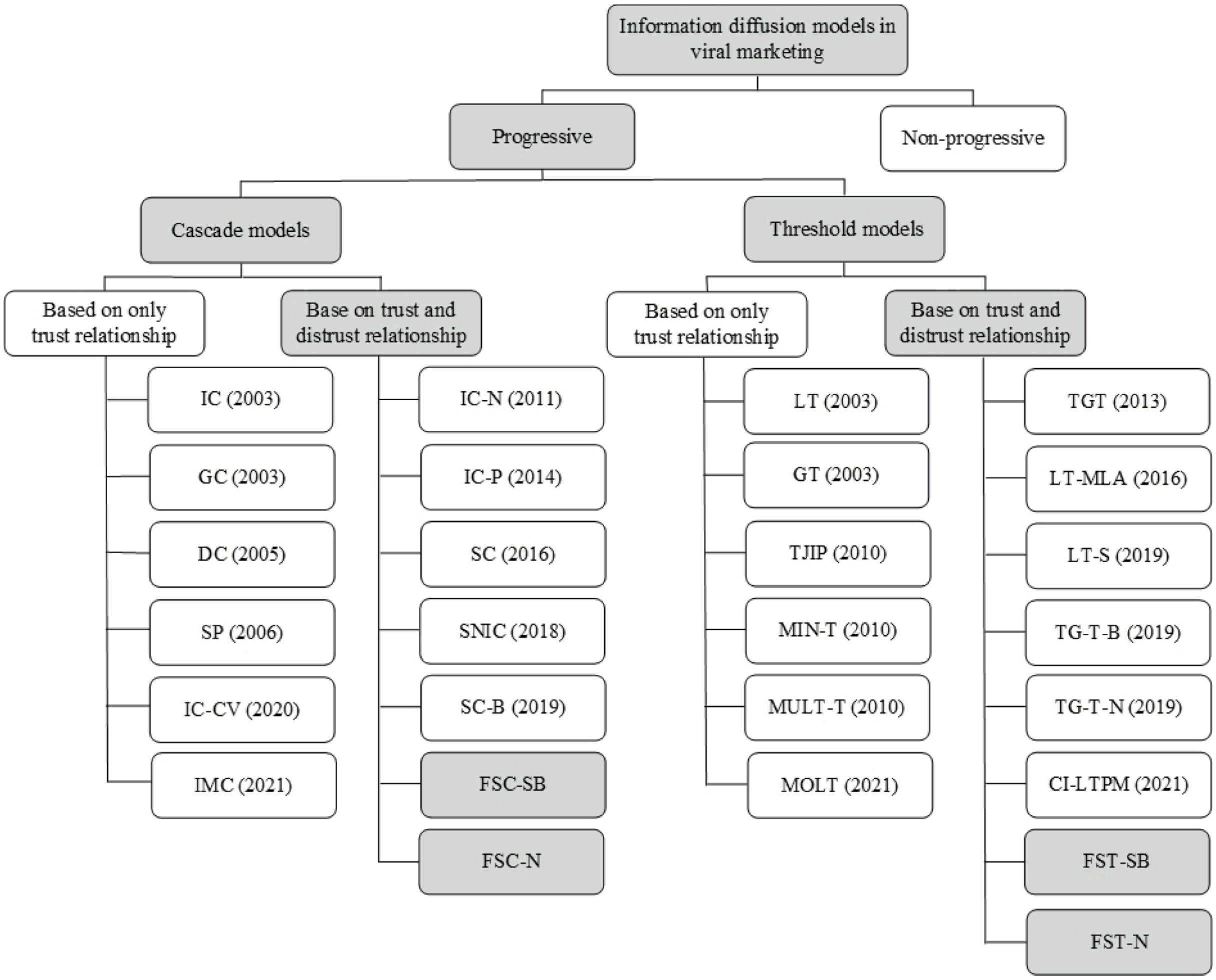


Fig. 1. A classification of information diffusion models in viral marketing.

marketing. In both models, the status of each user is considered active or inactive in a social network. The active state implies that the user has accepted the information exposed to it and may send the information to its neighbors, while the inactive state implies that the user has refused to receive the information. In the progressive models, when a user becomes active, it remains active permanently, while in non-progressive models, the active users may change their states under some mechanisms from active to inactive and vice versa periodically [30]. While most of the influence maximization algorithms based on the progressive models have been investigated extensively in the existing literature, the non-progressive models have not attracted much scientific attention [13,15]. Hence, this study only focuses on progressive diffusion models for viral marketing. As shown in Fig. 1, progressive diffusion models can be categorized into two main categories: cascade and threshold models. In the following, some well-known and recent diffusion models of these categories are reviewed.

2.1. *Progressive diffusion models based on cascade models*

Depending on the type of relationships among users in social networks, the progressive cascade models can be divided into two categories: based on only trust relationships and both trust and distrust relationships.

Since Domingos et al. [7] presented the influence maximization problem in viral marketing, various information diffusion models have been proposed in the first category. Kempe et al. [15] have firstly formalized the influence maximization problem as a discrete optimization problem under specific diffusion models. The independent cascade (IC) model is one of the most famous progressive models introduced by Kempe et al. In the IC model, each user is considered either active or inactive. To investigate a seed set S efficiency, assume that all users of S are active and the rest of the network's users are inactive. The activation process of inactive users proceeded in discrete steps. At every time step, activated users in the previous step can have just one try to activate each of their inactive neighbors based on influence probability p. The propagation process goes on until there are not any newly activated users in a time step. Finally, the overall influence of S is estimated by the total number of activated users through the propagation process.

A general cascade (GC) model [15] is an extended case of the IC model. This model is similar to the IC model except that the failed tries to activate users are considered in the success activation probability computation via an incremental function. After that study, a decreasing cascade (DC) model [16] was developed based on the GC model, where the attenuation effect of the influence probability is applied among users. A shortest-path (SP) model [17] is another particular version of the IC model such that every user is activated only by the shortest paths of the initial seed set. The independent cascade model with coupons and valuations (IC-CV) [27] was proposed in which a different profit maximization schema was introduced by offering coupons to seed set users for viral marketing. Tong et al. [39] recently introduced an independent multi-cascade (IMC) model based on the notion of resolution function. Indeed, the IMC model investigates the strategies for spreading one information cascade when multiple cascades can occur simultaneously in a social network.

Unlike the first category, the second category of progressive cascade models is supposed that relationships among social network users may not always be a trust type in the real world. The distrust relationships were firstly considered by Guha et al. [11] for modeling the influence propagation process among the network users. They introduced three models based on trust and distrust relationships: trust only where the distrust relationship is completely ignored, one-step distrust where if a user distrusts someone then it will discount all opinions made by that person, and propagated distrust where both trust and distrust relationships can be propagated together. Various diffusion models have been proposed considering distrust relationships so far. In the following, some of these existing models will be described briefly.

Chen et al. [4] introduced an independent cascade model with negative opinions (IC-N) to determine the diffusion influence when negative relationships exist in a network. In the IC-N model, a new parameter called quality factor has been defined, which is the tendency of people to have negative opinions about a product. Li et al. [23] proposed a polarity-related independent cascade (IC-P) model, which includes the positive and negative polarity of relationships among social network users. In this model, the status of the seed set users can consist of both positive and negative active states. During the propagation process, when a user takes place in each of the active states, it will not alter in the future. Liu et al. [28] presented an independent cascade diffusion model called SNIC for expressing two different relationships of influence diffusion among users in a signed social network. They used the independent dissemination paths to model the influence diffusion in the SNIC model.

Hosseini-Pozveh et al. [12] presented a sign-aware cascade (SC) model for the influence maximization problem. This model is an extension of the IC model that users assumed to be in one of the three states: positive active, negative active, and inactive. In the SC model, when users have positive relationships with their inactive neighbors, they try to activate them in accordance with their opinions. However, when the users have negative relationships with their inactive neighbors, they try to activate them in the opposite direction of their opinions. In their following-up work [13], they presented diffusion models based on the framework of Guha et al. [11] to incorporate trust/distrust relationships in social networks. In this regard, a sign-aware cascade including blocking users (SC-B) model was proposed based on the IC model in order to model the negative influence diffusion in a social network. In this model, when a social user becomes active, it is allowed to activate its inactive neighbors. If the inactive neighbor trusts active users, it can accept disseminated information. Otherwise, the neighbor user may be blocked, which means that it will not send the information to others.

2.2. Progressive diffusion models based on threshold models

Depending on the type of relationships among users in social networks, the progressive threshold models can be divided into two categories: based on only trust relationships and both trust and distrust relationships.

A well-known model of the first category is the linear threshold (LT) [15]. In this model, the social influence value of each user v from its neighbor user w is $b_{v,w}$, where these values must satisfy the equation $\sum_{w \text{ neighbor of } v} b_{v,w} \leqslant 1$. Also, for each user in a social network, a threshold θ_v should be determined to indicate a lower bound value that enables the user v to become active. For each user, the threshold value is randomly selected from the standard uniform distribution. Let S_0 denotes an initial set of active users and assumes that the propagation process is performed in discrete steps. In this process based on the LT model, an inactive user will become active if the sum of influence weights of its active neighbors exceeds its threshold value, i.e., $\sum_{w \text{ neighbor of } v} b_{v,w} \geqslant \theta_v$. The propagation process continues until there are not any newly activated users in a time step. Then Kempe et al. [15] presented the general threshold (GT) model, which is an extension of the LT model. The extension is in the threshold function. In the LT model, the threshold function is simply the sum of active neighbors' influence weights, while in the GT model, it is an arbitrary general function. An instance of the GT model with threshold function $1 - \prod_{v \in s}(1 - p_{v,u})$, where $p_{v,u}$ is influence probability of active user v on inactive user u, was introduced by Goyal [10] named TJIP, in which they exerted learning joint influence probabilities between users through the action log and the connection graph. Other classic threshold models, except for the ones mentioned above, were also proposed with different threshold values. Some of these models that can be pointed out are the minimum threshold model (MIN-T) [33] and the multiplication threshold model (MULT-T) [33]. Recently, Olivares et al. [34] proposed a multi-objective linear threshold model named MOLT. Under this model, they applied a multi-objective approach that considers two objectives aiming to the spread of influence maximization and the seed set minimization. Obviously, the optimization model involves a conflict that they concentrated on solving this problem.

The second category of the progressive threshold models considers both trust and distrust relationships in social networks. A model of this category is the trust-general threshold (TGT) which is developed by Ahmed and Ezeife [1]. The TGT model is a generalized version of the LT model that considers both positive and negative influences among users' social relationships. Wang et al. [42] offered a new LT model with attitudes entitled the LT-MLA model. This model considered both the attitudes of users and the relationships between users in the influence propagation process. Liang et al. [25] suggested a new diffusion model called LT-S in signed social networks based on the LT model integrating signed relationships and opinion information.

In a recent work, Hosseini-Pozveh et al. [13] studied a new progressive threshold model named trust-generated threshold including blocked users (TG-T-B). In this model, users are supposed to have one of the three possible states: active, blocked, and inactive. For each user, two threshold parameters θ^+ and θ^- were assigned with two random numbers between $[0,1]$ which these parameters indicate the user's tendency to be influenced by its trusted and distrusted neighbors. The propagation process proceeds in discrete steps so that each inactive user would be activated based on Eq. (1) and added to active users of previous steps.

$$\left(\left(1-\prod_{v\in s^+|v \text{ is active}}(1-p^+_{v,u})\right)>\theta^+\right) \tag{1}$$

Furthermore, each inactive user would be blocked based on Eq. (2) and added to blocked users of previous steps.

$$\left(\left(1-\prod_{v\in s^-|v \text{ is active}}(1-p^-_{v,u})\right)>\theta^-\right) \tag{2}$$

They also presented another progressive threshold model named trust-generated threshold including negative users (TG-T-N). In this model, users are supposed to have one of the three states: positive active, negative active, and inactive. For each user, two threshold parameters θ^+ and θ^- were considered with two random values in the range $[0,1]$. The propagation process proceeds in discrete steps in such a way that each inactive user would be positively activated based on Eq. (3) and added to positive active users of previous steps.

$$\left(\left(1-\prod_{v\in s^+|v \text{ is positive\_active}}\left(1-p^+_{v,u}\right)\prod_{v\in s^-|v \text{ is negative\_active}}\left(1-p^-_{v,u}\right)\right)>\theta^+\right) \tag{3}$$

Furthermore, each inactive user would be negatively activated based on Eq. (4) and added to negative active users of previous steps.

$$\left(\left(1-\prod_{v\in s^+|v \text{ is negative\_active}}\left(1-p^+_{v,u}\right)\prod_{v\in s^-|v \text{ is positive\_active}}\left(1-p^-_{v,u}\right)\right)>\theta^-\right) \tag{4}$$

In another recent study, Ju et al. [14] first brought up the problem of blocking maximization under uncertain negative source influence and its importance. Then, to tackle this problem, they proposed an extended linear threshold model called the competitive influence linear threshold propagation model (CI-LTPM). In order to estimate the influence propagation under the proposed model, they suggested using the propagation tree in the live-edge sub-graph.

Although the different diffusion models which incorporate both trust and distrust relationships have been proposed so far, their models have not considered all the various relationships among social network users. Therefore, this study introduces a new fuzzy-based approach to model the realistic behaviors of users more accurately. Moreover, using the introduced fuzzy-based approach, four new fuzzy sign-aware diffusion models are proposed in two categories of cascade and threshold-based models.

3. The proposed fuzzy sign-aware diffusion models

In the proposed FSC-SB, FSC-N, FST-SB, and FST-N models, a fuzzy user-relationship expert system (FUES) is provided to determine the user-relationship types based on the trust level. The FUES can tolerate the unreliability and imprecision of defined explicit relationship types among social network users because it is more similar to human thinking and reasoning than crisp logic. The fuzzy logic makes interactions more intuitive and natural, which is expected to increase the prediction quality of diffusion models to effectively analyze the different users' opinions in the signed social networks.

The FUES contains four main components: a fuzzification interface, a knowledge base, a fuzzy inference engine, and a defuzzification interface. The selection of high-performance membership functions for fuzzy linguistic variables is a significant challenge in fuzzification. The fuzzification interface maps crisp input values to membership degrees of the corresponding fuzzy sets. In the proposed models, the user-relationship types are modeled as a linguistic variable in three values: "distrust", "mediocre trust", and "trust".

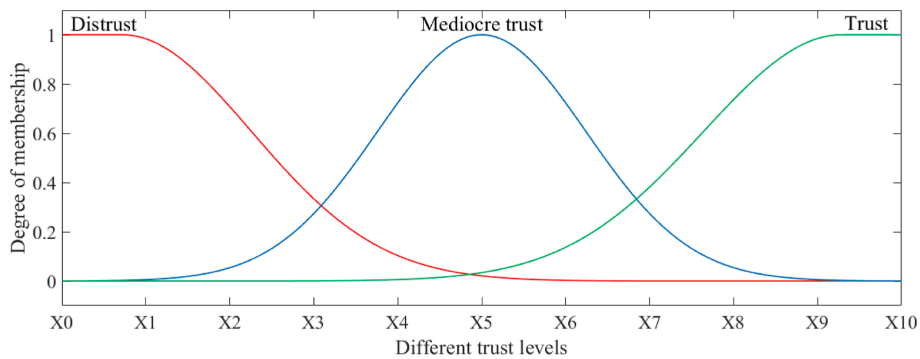


Fig. 2. Membership functions for different user-relationship types (distrust, mediocre trust, and trust).

The fuzzy membership functions are also considered as shown in Fig. 2. The fuzzy rules are defined based on the membership functions. In the fuzzy inference engine, the rules are stored in a knowledge base to compute the fuzzy output out of the fuzzy input. This component plays a key role in generating the quality of results in the information diffusion models and correctly predicting the influence propagation process among the network users. Finally, the defuzzification interface converts the fuzzy sets' results to a user-relationship type as the crisp output. In the following, the introduced fuzzy-based approach and proposed FSC-SB, FSC-N, FST-SB, and FST-N models are described in detail.

3.1. The introduced fuzzy-based approach, its plans, and user states

In the signed social networks, the modeling of influence diffusions through distrust relationships is not based on step-by-step information propagation as the same as while it is modeled by only trust relationships. Since the influence maximization in the signed social networks is an NP-hard problem, a few researchers introduced a framework-based approach [11,13] to improve the performance of diffusion models. However, their models suffer from a lack of appropriate theoretical support to cover all the different relationships among network users. Hence, we introduce a fuzzy-based approach including three following plans to model different trust levels based on the fuzzy expert system for influence maximization in viral marketing.

Plan 1: The influence diffusions are only considered as trust relationships with membership function degrees greater than or equal to $\alpha - cut$, and other fuzzy relationships are ignored in the signed social network.

Plan 2: All users' relationships with different trust levels are considered in the diffusion models. In the information propagation process, if a user γ performs an action and propagates its influence to the neighbors in step t, then the neighbor η, who is unreliable to γ, performs one of the following reactions:

- The user η tends not to do this action for all following steps.
- In the step $t+1$, the user η is not influenced by the action of user γ.

As a result, there is at least one step of negative influence propagation in the social network; therefore, it should be considered in the modeling of influence maximization.

Plan 3: All users' social relationships are considered in the diffusion models. The propagation process occurs when a user γ performs an action and propagates its influence to the neighbors in step t. Then, the neighbor η, who is unreliable to γ, performs one of the following reactions:

- The user η performs the opposite action of user γ in the step $t + 1$.
- The user η is not influenced by the action of user γ in the step $t + 1$.

In this plan, the propagation of negative influence is step-by-step on the social network; thus, it should be considered to model influence maximization.

All the available trust-based diffusion models with crisp relationships can be applied in the first plan. In the following subsections, we propose the progressive fuzzy diffusion models based on the second and third plans for two main categories: cascade and threshold-based models.

In the proposed models, the information is propagated by active users in the signed social network. The information receivers react differently based on their trust level to distribute the information in the social network as shown in Table 1. Social users' states are determined based on the influence acceptance notion, and each state may change according to some predefined transition mechanisms. The user states used in the proposed information diffusion models are defined as follows:

- An inactive state implies that a user has not been exposed to diffusion of information in the social network.
- An active state implies that a user has accepted the information propagated by other users and sends the information to its neighbors [15,48].
- An active$^+$ state implies that a user with a positive opinion accepts the diffusion influence and sends the received influence without any changes to other users in the social network [13,28].
- An active$^-$ state implies that a user with a negative opinion accepts the diffusion influence but sends the opposite of received influence to other users in the social network [13,28].

Table 1
Different user states in the proposed fuzzy sign-aware diffusion models.

Diffusion models approaches	Sender state	Receiver state		
		Trust	Mediocre trust	Distrust
The second plan	Active	Active	Blocked/Suspended/ Active	Blocked
The third plan	Active$^+$	Active$^+$	Active$^+$/Suspended/ Active$^-$	Active$^-$
	Active$^-$	Active$^-$	Active$^+$/Suspended/ Active$^-$	Active$^+$

- A blocked state implies that a user has received the diffusion information, but the user will not send it to other users for all of the next time steps in the social network [13].
- A suspended state implies that a user is in a state of uncertainty and dilemma when receiving the influences. The suspended user does not send the received influences to other network users in the next time step, but may wait for a new event to be activated in the future time steps.

3.2. Fuzzy sign-aware cascade-based (FSC) models

Since the cascade-based models considering an acceptable trust level of the fuzzy membership degree can be applied for the first plan, in the following, two fuzzy sign-aware cascade models, FSC-SB and FSC-N, are introduced for the second and third plans, respectively.

3.2.1. FSC model including suspending and blocking users (FSC-SB)

The FSC-SB is one of the progressive diffusion models generalized by the IC model based on the second plan. In this model, it is supposed that users can take one of four states including active, suspended, blocked, and inactive in a social network graph G. In addition, it is assumed that the activation process of the users takes place in discrete steps. We introduce some graphical notations in Table 2 to standardize the presentation of components of the signed social network. In this table, there are trust, mediocre trust, and distrust links between users in the social network that are symbolized in the third column. Moreover, to denote the propagation of information from one user to another, similar link notations but with two arrowheads are shown in the last row of the third column for trust, mediocre trust, and distrust links with propagation.

Using the Table 2 notations, Fig. 3 shows the general structure of the information propagation process through the FSC-SB model in which a fuzzy expert system is deployed and considered the influence of users on each other via both trust and distrust relationships. Fig. 4 provides the pseudocode of the algorithm FSC-SB for the fuzzy sign-aware cascade model including suspending and blocking users initiated by a social network consisting of a S_0 set of active users. In the first step, the active users in S_0 try to activate their inactive direct neighbors. As the diffusion continues, the activated users in the step t have an opportunity to activate their inactive and suspended neighbors as well. Meanwhile, as shown in Fig. 5, the fuzzy user-relationship expert system (FUES) is used to determine the user-relationship type based on the trust level values extracted from the trust level matrix for each neighbor η of the active user γ. Then,

Table 2
Introduced notations to present the signed social networks.

Different users	Different states	Different social links	Propagation status
Seed user	Active state	Trust link	Non-propagating
Active neighbor	Blocked state	Mediocre trust link	Propagating
Blocked neighbor	Suspended state	Distrust link	
Suspended neighbor	Inactive state	Trust link with propagation	
Inactive neighbor	Undetermined state	Mediocre trust link with propagation Distrust link with propagation	

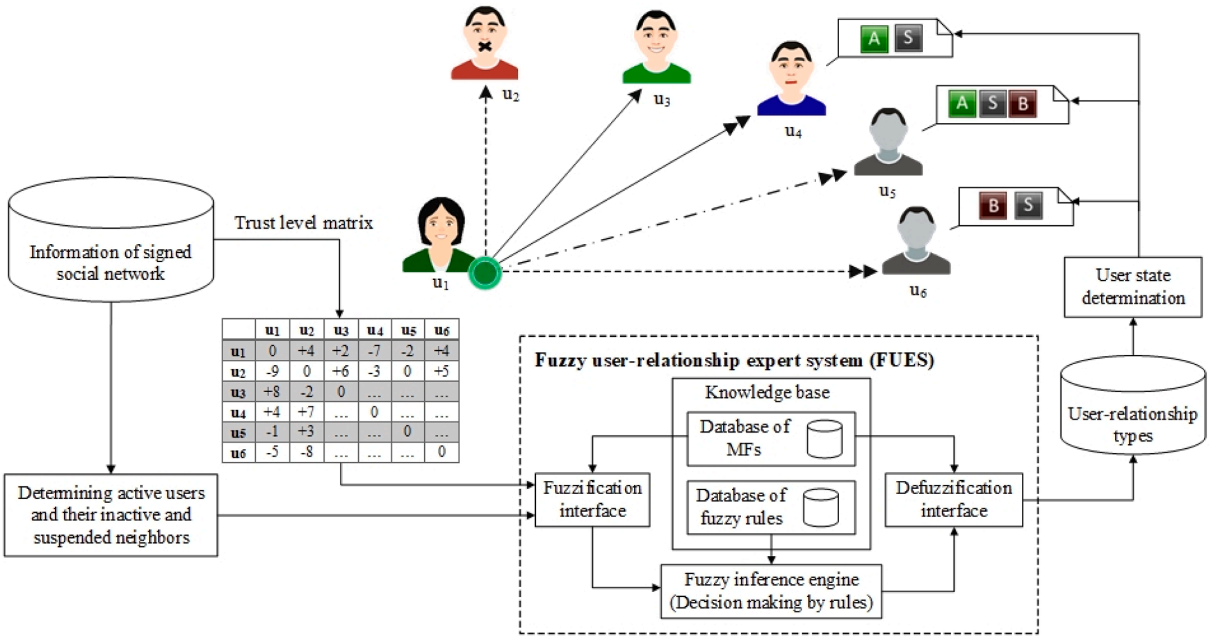


Fig. 3. The proposed fuzzy sign-aware cascade model including suspending and blocking users (FSC-SB).

Algorithm FSC-SB

Input: G (V, E) (a graph of a signed social network where V is the set of all users and E is the set of directed edges), $p_{\gamma,\eta}$(the influence probability of user γ on user η), S_0 (seed set), and TLM (trust level matrix)

Output: Active (the final set of activated users), Blocked (the final set of blocked users), and Suspended (the final set of suspended users)

Begin

Initialize Active = S_0, Blocked =Ǿ, and Suspended = Ǿ

For each user γ in S_0

  For each user η in inactive or suspended neighbors of γ in G

    Generate a random number R in [0,1]

    RelationshipType$_\eta$= call FUES (γ, η, TLM)

    If RelationshipType$_\eta$ = "trust" and R ≤ $P^{+}_{\gamma,\eta}$

      Active = Active +{η}

    else if RelationshipType$_\eta$ = "distrust" and R ≤ $P^{-}_{\gamma,\eta}$

      Blocked = Blocked +{η}

    else if RelationshipType$_\eta$ = "mediocre trust"

      Suspended = Suspended +{η}

    End if

  End for

End for

Return Active, Blocked, Suspended

End

Fig. 4. The pseudocode of the algorithm FSC-SB.

Algorithm FUES

Crisp input: γ (current active user), η (current neighbor user), and TLM (trust level matrix)
Output: RelationshipType

1. **Function** FUES
2. Selecting fuzzy membership functions (MFs)
3. Fuzzification of crisp inputs based on MFs and TLM
4. Generating fuzzy rules
5. Evaluating fuzzy rules
6. Aggregating the results of fuzzy rules
7. Determining RelationshipType by defuzzification the aggregated results
8. **Return** RelationshipType

Fig. 5. The pseudocode of the algorithm FUES.

their membership degrees are determined based on membership functions as shown in Fig. 2. The fuzzy inference engine evaluates fuzzy rules and aggregates their results to generate outputs. Then, the output is defuzzified to obtain the ultimate output of user-relationship types. Finally, the FSC-SB model predicts one of the active, suspended, and blocked states. The state of neighbor η is determined as follows:

- If the neighbor η has a trust relationship with the active user γ, then η will be activated by γ with probability $p^{+}_{\gamma,\eta}$.
- If the neighbor η has a distrust relationship with the active user γ, then η will be blocked by γ with probability $p^{-}_{\gamma,\eta}$.
- If the neighbor η has a mediocre relationship with the active user γ, then η goes to the suspended state. But, the neighbor η has other opportunities to be activated by its active neighbors in the next steps.

It should be noted that if the inactive user η has a group of newly activated neighbors and they tend to activate this user simultaneously, then a random permutation of them is selected to change the state of user η. During the propagation process, each active user can only influence its inactive and suspended neighbors once. In other words, it is assumed that the active user γ is in the neighborhood of the inactive user η. If the user γ attempts to the activate user η, then the state of user η changes to one of the active, blocked, and suspended states. After that, the user γ cannot influence the user η in the next steps. Also, if the user η is in the suspended state and the effort of user γ to activate it was unsuccessful. Therefore, the user η remains in the suspended state, and the user γ will not have another opportunity to activate this neighbor. These steps continue until any other users cannot be activated in the social network.

3.2.2. *FSC model including negative users (FSC-N)*

The FSC-N is a progressive diffusion model generalized by the IC model using the third plan. The pseudocode of the algorithm FSC-N is shown in Fig. 6, in which the users can be in one of four states including active^{+}, active^{-}, suspended, and inactive in a social network graph G. In the initial stage of the propagation process, the social network includes only active^{+} and inactive users. The set of available active^{+} users is denoted by S_0. Users in S_0 try to activate their inactive direct neighbors in the first step of the diffusion. In the following, due to the influence of users on each other via both types of trust and distrust relationships, some social users become active^{+}, suspended, and active^{-}. In the step t, each active user with a positive or negative opinion, say γ^{+} and γ^{-}, respectively, has an opportunity to activate each of its inactive and also suspended neighbors, say η. In the FSC-N model, the user-relationship type of each neighbor η with its γ users is determined by the proposed fuzzy user-relationship expert system (FUES). Finally, this model predicts the state of the neighbor η as follows:

- If the neighbor η has a trust relationship with the active^{+} user γ^{+}, then η will be active^{+} by γ^{+} with probability $p^{+}_{\gamma^{+},\eta}$.
- If the neighbor η has a distrust relationship with the active^{+} user γ^{+}, then η will be active^{-} by γ^{+} with probability $p^{-}_{\gamma^{+},\eta}$.
- If the neighbor η has a mediocre relationship with the active user γ having a positive or negative opinion, then η goes to the suspended state. However, there may be opportunities for its other active neighbors to activate it in the next steps.
- If the neighbor η has a trust relationship with the active^{-} user γ^{-}, then η will be active^{-} by γ^{-} with probability $p^{+}_{\gamma^{-},\eta}$.
- If the neighbor η has a distrust relationship with the active^{-} user γ^{-}, then η will be active^{+} by γ^{-} with probability $p^{-}_{\gamma^{-},\eta}$.

It should be noted that if the inactive user η has a group of newly activated neighbors who adapts a positive or negative opinion about the received information and they tend to activate this user simultaneously, then a random permutation of them is selected to change the state of user η. During the propagation process, each active user with the positive or negative opinion can only influence on its inactive and suspended neighbor once. In other words, it is assumed that the active user γ is in the neighborhood of the inactive user η. If the user γ attempts to activate the user η, then the state of user η changes to one of the active^{+}, active^{-}, and suspended states. After that, the user γ cannot influence the user η in the next steps. Also, if the user η is in the suspended state and the effort of user γ to activate it was unsuccessful. Therefore, the user η remains in the suspended state, and the user γ will not have another opportunity to activate this neighbor. These steps continue until any other users cannot be activated in the social network.

Algorithm FSC-N

Input: G (V, E) (a graph of a signed social network where V is the set of all users and E is the set of directed edges), $p_{\gamma,\eta}$ (the influence probability of user γ on user η), S_0 (seed set), and TLM (trust level matrix)
Output: $Active^+$ (the final set of positively activated users), $Active^-$ (the final set of negatively activated users), and Suspended (the final set of suspended users)
Begin
Initialize $Active^+ = S_0$, $Active^- = \emptyset$, and Suspended $= \emptyset$
For each user γ in S_0
    If γ is positive active //denoted by γ^+
      For each user η in inactive or suspended neighbors of γ^+ in G
        Generate a random number R in [0,1]
        $RelationshipType_\eta$ = call FUES (γ^+, η, TLM)
        If $RelationshipType_\eta$ = "trust" and $R \le P^+_{\gamma+,\eta}$
          $Active^+ = Active^+ + \{\eta\}$
        else if $RelationshipType_\eta$ = "distrust" and $R \le P^-_{\gamma+,\eta}$
          $Active^- = Active^- + \{\eta\}$
        else if $RelationshipType_\eta$ = "mediocre trust"
          Suspended = Suspended + {η}
        End if
      End for
    else if γ is negative active //denoted by γ^-
      For each user η in inactive or suspended neighbors of γ^- in G
        Generate a random number R in [0,1]
        $RelationshipType_\eta$ = call FUES (γ^-, η, TLM)
        If $RelationshipType_\eta$ = "trust" and $R \le P^+_{\gamma-,\eta}$
          $Active^- = Active^- + \{\eta\}$
        else if $RelationshipType_\eta$ = "distrust" and $R \le P^-_{\gamma-,\eta}$
          $Active^+ = Active^+ + \{\eta\}$
        else if $RelationshipType_\eta$ = "mediocre trust"
          Suspended = Suspended + {η}
        End if
      End for
    End if
End for
Return $Active^+$, $Active^-$, Suspended
End

Fig. 6. The pseudocode of the algorithm FSC-N.

3.3. Fuzzy sign-aware threshold-based (FST) models

Since the threshold-based models considering an acceptable trust level of the fuzzy membership degree can be applied for the first plan, therefore, in the following, two fuzzy sign-aware threshold models, FST-SB and FST-N are introduced for the second and third plans, respectively.

3.3.1. FST model including suspending and blocking users (FST-SB)

The FST-SB is one of the progressive diffusion models and an extension of the TG-T-B model [13] based on the second plan. The general structure of the FST-SB model is shown in Fig. 7. Also, the pseudocode of the algorithm FST-SB is illustrated in Fig. 8. In this model, it is supposed that users can take one of four states including active, suspended, blocked, and inactive in a social network graph G. In addition, it is assumed that the activation process of social users takes place in discrete steps. Each user in this model has two random threshold parameters, θ^+ and θ^-, in the interval $[0,1]$. θ^+ indicates the user's tendency to be influenced by its trusted neighbors and θ^- shows the user's tendency to be influenced by its distrusted neighbors. In the first step, the active users of set S_0 try to activate their inactive direct neighbors. During the propagation process, the users become active, suspended, and blocked due to the influence of users on each other via both types of trust and distrust relationships. In the step t, each active user, called γ, has an opportunity to activate each of its inactive and also suspended neighbors, called η. Each of η users has its own set of active neighbors, S, but with different types of relationships. In the FST-SB model, the user-relationship type of each user η with its active neighbors is determined by the proposed fuzzy user-relationship expert system (FUES). Finally, in step t, some other users will be activated or blocked based on Definitions 1 and 2, respectively, as follows:

Definition 1. *Consider the inactive or suspended user η in the social network G. Let η has a set of active neighbors S classified into three categories: trusted neighbors set S^+, distrusted neighbors set S^-, and neighbors set with mediocre trust $S^\pm$. In this case, the user η will be activated by these neighbors if Eq.* (5) *or* (6) *holds.*

$$\left(\left(1-\prod_{\gamma\in S^+|\gamma\text{ is active}}\left(1-p^+_{\gamma,\eta}\right)\right)\geqslant\theta^+\right)\text{ and }\left(\left(1-\prod_{\gamma\in S^-|\gamma\text{ is active}}\left(1-p^-_{\gamma,\eta}\right)\right)<\theta^-\right)\tag{5}$$

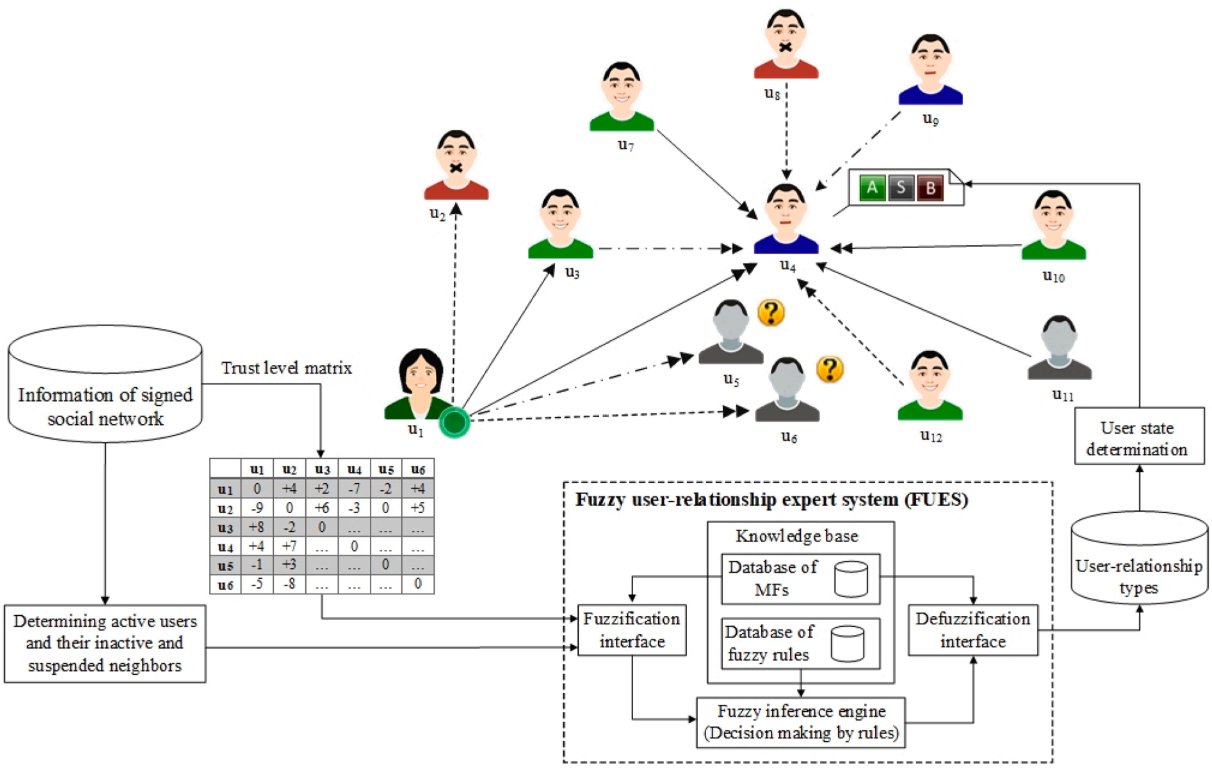


Fig. 7. The proposed fuzzy sign-aware threshold model including suspending and blocking users (FST-SB).

Algorithm FST-SB

Input: G (V, E) (a graph of a signed social network where V is the set of all users and E is the set of directed edges), $p_{\gamma,\eta}$(the influence probability of user γ on user η), S_0 (seed set), and TLM (trust level matrix)
Output: Active (the final set of activated users), Blocked (the final set of blocked users), and Suspended (the final set of suspended users)
Begin
Initialize Active = S_0, Blocked=∅, and Suspended=∅
Initialize Current_θ_η^+=1, Current_θ_η^-=1, Current_$\theta_\eta^{P\pm}$=1, Current_$\theta_\eta^{N\pm}$= 1, θ_η^+ and θ_η^- random values within [0,1] for all users $\eta \epsilon V$
For each user δ in S_0
  For each user η in inactive or suspended neighbors of δ in G
    For each user γ in active neighbors of η
      RelationshipType$_\eta$= call FUES (γ, η, TLM)
        Case RelationshipType$_\eta$ **of**
          "trust": Current_θ_η^+= (1-$p_{\gamma,\eta}^+$) × Current_θ_η^+
          "distrust": Current_θ_η^-= (1-$p_{\gamma,\eta}^-$) × Current_θ_η^-
          "mediocre trust": Current_$\theta_\eta^{P\pm}$= (1-$p_{\gamma,\eta}^+$) × Current_$\theta_\eta^{P\pm}$
                    Current_$\theta_\eta^{N\pm}$= (1-$p_{\gamma,\eta}^-$) × Current_$\theta_\eta^{N\pm}$
        End case
    End for
    If (1- Current_θ_η^+) ≥ θ_η^+ and (1- Current_θ_η^-) < θ_η^-
      Active = Active +{η}
    else if (1- Current_θ_η^+) < θ_η^+ and (1- Current_θ_η^-) ≥ θ_η^-
      Blocked = Blocked +{η}
    else if (1- Current_θ_η^+ Current_$\theta_\eta^{P\pm}$) ≥ θ_η^+ and (1- Current_θ_η^- Current_$\theta_\eta^{N\pm}$) < θ_η^-
      Active = Active +{η}
    else if (1- Current_θ_η^+ Current_$\theta_\eta^{P\pm}$) < θ_η^+ and (1- Current_θ_η^- Current_$\theta_\eta^{N\pm}$) ≥ θ_η^-
      Blocked = Blocked +{η}
    else
      Suspended = Suspended +{η}
    End if
  End for
End for
Return Active, Blocked, Suspended
End

Fig. 8. The pseudocode of the algorithm FST-SB.

$$\left(\left(1-\prod_{\gamma\in S^+|\gamma\ \text{is active}}(1-p^+_{\gamma,\eta})\prod_{\gamma\in S^\pm|\gamma\ \text{is active}}(1-p^+_{\gamma,\eta})\right)\geqslant\theta^+\right)$$
$$\text{and}$$
$$\left(\left(1-\prod_{\gamma\in S^-|\gamma\ \text{is active}}(1-p^-_{\gamma,\eta})\prod_{\gamma\in S^\pm|\gamma\ \text{is active}}(1-p^-_{\gamma,\eta})\right)<\theta^-\right) \tag{6}$$

Definition 2. *Consider the inactive or suspended user η in the social network G. Let η has a set of active neighbors S classified into three categories: trusted neighbors set S^+, distrusted neighbors set S^-, and neighbors set with mediocre trust $S^\pm$. In this case, the user η will be blocked by these neighbors if Eq.* (7) *or* (8) *holds.*

$$\left(\left(1-\prod_{\gamma\in S^+|\gamma\ \text{is active}}(1-p^+_{\gamma,\eta})\right)<\theta^+\right)\ \text{and}\left(\left(1-\prod_{\gamma\in S^-|\gamma\ \text{is active}}(1-p^-_{\gamma,\eta})\right)\geqslant\theta^-\right) \tag{7}$$

$$\left(\left(1-\prod_{\gamma\in S^{+}|\gamma\ \text{is active}}(1-p^{+}_{\gamma,\eta})\prod_{\gamma\in S^{\pm}|\gamma\ \text{is active}}(1-p^{+}_{\gamma,\eta})\right)<\theta^{+}\right)$$
$$\text{and}$$
$$\left(\left(1-\prod_{\gamma\in S^{-}|\gamma\ \text{is active}}(1-p^{-}_{\gamma,\eta})\prod_{\gamma\in S^{\pm}|\gamma\ \text{is active}}(1-p^{-}_{\gamma,\eta})\right)\geqslant\theta^{-}\right) \tag{8}$$

Remark 1. *If the user η has a trust relationship with the active user γ, then $p^{-}_{\gamma,\eta}$ is zero. This means that the active user γ will not have a negative effect on the user η. If the user η has a distrust relationship with the active user γ, then $p^{+}_{\gamma,\eta}=0$. In other words, the user γ will not have a positive effect on the user η. Also, if the user η has a mediocre trust relationship with the active user γ, then both positive and negative influence probabilities (i.e., $p^{+}_{\gamma,\eta}$ and $p^{-}_{\gamma,\eta}$) should be considered.*

If neither conditions of Definitions 1 nor 2 hold, then the user goes to the suspended state. This influence process continues until any other users cannot be activated in the social network.

3.3.2. FST model including negative users (FST-N)

The FST-N is a progressive diffusion model and an extension of the TG-T-N model [13] based on the third plan. The pseudocode of the algorithm FST-N is shown in Fig. 9. In this model, it is supposed that the users can be in one of four states including active^{+}, active^{-}, suspended, and inactive in a social network graph G. In addition, it is assumed that the activation process of social users takes place in discrete steps. Each user in this model has two random threshold parameters, θ^{+} and θ^{-}, in the interval $[0,1]$. θ^{+} indicates the user's tendency to be influenced by its trusted neighbors and θ^{-} shows the user's tendency to be influenced by its distrusted neighbors. In the first step, the active users of set S_0 try to activate their inactive direct neighbors. In the consecutive steps of the propagation process, the users become active^{+}, suspended, and active^{-} due to the influence of users on each other via both types of trust and distrust relationships. In the step t, each of these active users with a positive opinion γ^{+} and a negative opinion γ^{-} is given an opportunity to activate each of its neighbors, η, which its state can be inactive and also suspended. Each of η users has its own set of active neighbors, S, but with different types of relationships. In the FST-N model, the user-relationship type of each user η with its active^{+} and active^{-} neighbors are determined by the proposed fuzzy user-relationship expert system (FUES). Finally, the propagation process proceeds in such a way that in step t, in addition to the activated users in the previous steps, some other users will be positively activated based on Definition 3. In addition to the activated users in the previous steps, some other users will be negatively activated based on Definition 4.

Definition 3. *Consider the inactive or suspended user η in the social network G. Let η has a set of active^{+} and active^{-} neighbors S classified into three categories: trusted neighbors set S^{+}, distrusted neighbors set S^{-}, and neighbors set with mediocre trust $S^{\pm}$. In this case, the user η will be positively activated by these neighbors if Eq.* (9) *or* (10) *holds.*

$$\left(\left(1-\prod_{\gamma^{+}\in S^{+}|\gamma^{+}\ \text{is positive active}}(1-p^{+}_{\gamma+,\eta})\prod_{\gamma^{-}\in S^{-}|\gamma^{-}\ \text{is negative active}}(1-p^{-}_{\gamma-,\eta})\right)\geqslant\theta^{+}\right)$$
$$\text{and}$$
$$\left(\left(1-\prod_{\gamma^{-}\in S^{+}|\gamma^{-}\ \text{is negative active}}(1-p^{+}_{\gamma-,\eta})\prod_{\gamma^{+}\in S^{-}|\gamma^{+}\ \text{is positive active}}(1-p^{-}_{\gamma+,\eta})\right)<\theta^{-}\right) \tag{9}$$

$$\left(\left(1-\prod_{\gamma^{+}\in S^{+}|\gamma^{+}\ \text{is positive active}}(1-p^{+}_{\gamma+,\eta})\prod_{\gamma^{-}\in S^{-}|\gamma^{-}\ \text{is negative active}}(1-p^{-}_{\gamma-,\eta})\prod_{\gamma^{+}\in S^{\pm}|\gamma^{+}\ \text{is positive active}}(1-p^{+}_{\gamma+,\eta})\prod_{\gamma^{-}\in S^{\pm}|\gamma^{-}\ \text{is negative active}}(1-p^{-}_{\gamma-,\eta})\right)\geqslant\theta^{+}\right)$$
$$\text{and}$$
$$\left(\left(1-\prod_{\gamma^{-}\in S^{+}|\gamma^{-}\ \text{is negative active}}(1-p^{+}_{\gamma-,\eta})\prod_{\gamma^{+}\in S^{-}|\gamma^{+}\ \text{is positive active}}(1-p^{-}_{\gamma+,\eta})\prod_{\gamma^{+}\in S^{\pm}|\gamma^{+}\ \text{is positive active}}(1-p^{-}_{\gamma+,\eta})\prod_{\gamma^{-}\in S^{\pm}|\gamma^{-}\ \text{is negative active}}(1-p^{+}_{\gamma-,\eta})\right)<\theta^{-}\right) \tag{10}$$

Definition 4. *Consider the inactive or suspended user η in the social network G. Let η has a set of active^{+} and active^{-} neighbors S classified into three categories: trusted neighbors set S^{+}, distrusted neighbors set S^{-}, and neighbors set with mediocre trust $S^{\pm}$. In this case, the user η will be negatively activated by these neighbors if Eq.* (11) *or* (12) *holds.*

$$\left(\left(1-\prod_{\gamma^{+}\in S^{+}|\gamma^{+}\ \text{is positive active}}(1-p^{+}_{\gamma+,\eta})\prod_{\gamma^{-}\in S^{-}|\gamma^{-}\ \text{is negative active}}(1-p^{-}_{\gamma-,\eta})\right)<\theta^{+}\right)$$
$$\text{and}$$
$$\left(\left(1-\prod_{\gamma^{-}\in S^{+}|\gamma^{-}\ \text{is negative active}}(1-p^{+}_{\gamma-,\eta})\prod_{\gamma^{+}\in S^{-}|\gamma^{+}\ \text{is positive active}}(1-p^{-}_{\gamma+,\eta})\right)\geqslant\theta^{-}\right) \tag{11}$$

Algorithm FST-N

Input: G (V, E) (a graph of a signed social network where V is the set of all users and E is the set of directed edges), $p_{\gamma,\eta}$(the influence probability of user γ on user η), S_0 (seed set), and TLM (trust level matrix)

Output: $Active^+$ (the final set of positively activated users), $Active^-$ (the final set of negatively activated users), and Suspended (the final set of suspended users)

Begin

Initialize $Active^+ = S_0$, $Active^- =\emptyset$, and Suspended=$\emptyset$

Initialize Current_θ_η^+=1, Current_θ_η^-=1, Current_$\theta_\eta^{P\pm}$=1, Current_$\theta_\eta^{N\pm}$= 1, θ_η^+ and θ_η^- random values within [0,1] for all users $\eta \epsilon V$

For each user δ in S_0

    For each user η in inactive or suspended neighbors of δ in G

        For each user γ in active neighbors of η

            If γ is positive active //denoted by γ^+

                $RelationshipType_\eta$= call FUES (γ^+, η, TLM)

                Case $RelationshipType_\eta$ **of**

                    "trust": Current_θ_η^+= (1-$p_{\gamma+,\eta}^+$) × Current_θ_η^+

                    "distrust": Current_θ_η^-= (1-$p_{\gamma+,\eta}^-$) × Current_θ_η^-

                    "mediocre trust": Current_$\theta_\eta^{P\pm}$= (1-$p_{\gamma+,\eta}^+$) × Current_$\theta_\eta^{P\pm}$

                                    Current_$\theta_\eta^{N\pm}$= (1-$p_{\gamma+,\eta}^-$) × Current_$\theta_\eta^{N\pm}$

                End case

            else if γ is negative active //denoted by γ^-

                $RelationshipType_\eta$= call FUES (γ^-, η, TLM)

                Case $RelationshipType_\eta$ **of**

                    "trust": Current_θ_η^-= (1-$p_{\gamma-,\eta}^+$) × Current_θ_η^-

                    "distrust": Current_θ_η^+= (1-$p_{\gamma-,\eta}^-$) × Current_θ_η^+

                    "mediocre trust": Current_$\theta_\eta^{N\pm}$= (1-$p_{\gamma-,\eta}^+$) × Current_$\theta_\eta^{N\pm}$

                                    Current_$\theta_\eta^{P\pm}$= (1-$p_{\gamma-,\eta}^-$) × Current_$\theta_\eta^{P\pm}$

                End case

            End if

        End for

        If (1- Current_θ_η^+) ≥ θ_η^+ and (1- Current_θ_η^-) < θ_η^-

            $Active^+$= $Active^+$+ {η}

        else if (1- Current_θ_η^+) < θ_η^+ and (1- Current_θ_η^-) ≥ θ_η^-

            $Active^-$ = $Active^-$ + {η}

        else if (1- Current_θ_η^+ Current_$\theta_\eta^{P\pm}$) ≥ θ_η^+ and (1- Current_θ_η^- Current_$\theta_\eta^{N\pm}$) < θ_η^-

            $Active^+$= $Active^+$+ {η}

        else if (1- Current_θ_η^+ Current_$\theta_\eta^{P\pm}$) < θ_η^+ and (1- Current_θ_η^- Current_$\theta_\eta^{N\pm}$) ≥ θ_η^-

            $Active^-$ = $Active^-$ + {η}

        else

            Suspended = Suspended +{η}

        End if

    End for

End for

Return $Active^+$, $Active^-$, Suspended

End

Fig. 9. The pseudocode of the algorithm FST-N.

$$\left(\left(1-\prod_{\gamma^{+}\in S^{+}|\gamma^{+}\text{ is positive active}}(1-p^{+}_{\gamma+,\eta})\prod_{\gamma^{-}\in S^{-}|\gamma^{-}\text{ is negative active}}(1-p^{-}_{\gamma-,\eta})\prod_{\gamma^{+}\in S^{\pm}|\gamma^{+}\text{ is positive active}}(1-p^{+}_{\gamma+,\eta})\prod_{\gamma^{-}\in S^{\pm}|\gamma^{-}\text{ is negative active}}(1-p^{-}_{\gamma-,\eta})\right)<\theta^{+}\right)$$
and
$$\left(\left(1-\prod_{\gamma^{-}\in S^{+}|\gamma^{-}\text{ is negative active}}(1-p^{+}_{\gamma-,\eta})\prod_{\gamma^{+}\in S^{-}|\gamma^{+}\text{ is positive active}}(1-p^{-}_{\gamma+,\eta})\prod_{\gamma^{+}\in S^{\pm}|\gamma^{+}\text{ is positive active}}(1-p^{-}_{\gamma+,\eta})\prod_{\gamma^{-}\in S^{\pm}|\gamma^{-}\text{ is negative active}}(1-p^{+}_{\gamma-,\eta})\right)\geqslant\theta^{-}\right) \tag{12}$$

Remark 2. *If the user η has a trust relationship with active$^+$ or active$^-$ user γ, then $p^-_{\gamma,\eta}$ is zero. This means that the active user γ will not have a negative effect on the user η. If the user η has a distrust relationship with the active$^+$ or active$^-$ user γ, then $p^+_{\gamma,\eta}=0$. In other words, the user γ will not have a positive effect on the user η. Also, if the user η. has a mediocre trust relationship with the active$^+$ or active$^-$ user γ, then both positive and negative influence probabilities (i.e., $p^+_{\gamma,\eta}$ and $p^-_{\gamma,\eta}$) should be considered.*

If neither conditions of Definitions 3 nor 4 hold, then the user will be suspended. This influence process continues until any other users cannot be activated in the network.

4. Determining properties of the proposed sign-aware diffusion models

This section is devoted to studying some important properties of proposed diffusion models, including monotonicity and submodularity properties. In the following, a brief review of these notions is provided by Definitions 5 and 6, and then we investigate whether the proposed models are monotone and submodular through some theorems or not. We also discuss that the problem of influence maximization under the proposed models is an NP-hard problem.

Definition 5. (*Monotonicity*). *A function $\sigma(.)$ is called monotone on a finite ground set U if $\forall\, \upsilon\in U$, $\sigma(S\cup\{\upsilon\})\geqslant\sigma(S)$, where S is the seed set and $S\subseteq U$ [15].*

Definition 6. (*Submodularity*). *A function $\sigma(.)$ has submodularity property if $\forall\, \upsilon\in U$, $\sigma(S\cup\{\upsilon\})-\sigma(S)\geqslant\sigma(T\cup\{\upsilon\})-\sigma(T)$, where S and T are the seed sets and $S\subseteq T\subseteq U$ [15]. In other words, the influence function σ is submodular if we add a new user to the seed sets S and T, the influence of this single user will have a reverse relation with the seed set size.*

In the context of the influence maximization problem, the total influence function of the diffusion model m on the social network graph $G(V,E)$ with the influence probability p can be shown as follow:

$$\sigma_m(S)=E[\sigma_{G'.m}(S)]=\sum_{G'\in\varsigma(G)}Pr(G').\sigma_{G'.m}(S) \tag{13}$$

where $\varsigma(G)$ represents all subgraphs G' from the graph G. $Pr(G')$ is the probability of G', and $\sigma_{G'.m}(S)$ is the ultimate number of activated users in a subgraph G' with a seed set S under the diffusion model m.

There should be noted that a nonnegative linear combination of submodular (monotone) functions is also submodular (monotone). Therefore, if $\sigma_{G'.m}(S)$ is submodular (monotone), then $\sigma_m(S)$ will also be submodular (monotone).

4.1. Properties of the FSC-SB model

Theorem 1. *Based on the FSC-SB model, there is at least a graph G that the influence function is not monotone.*

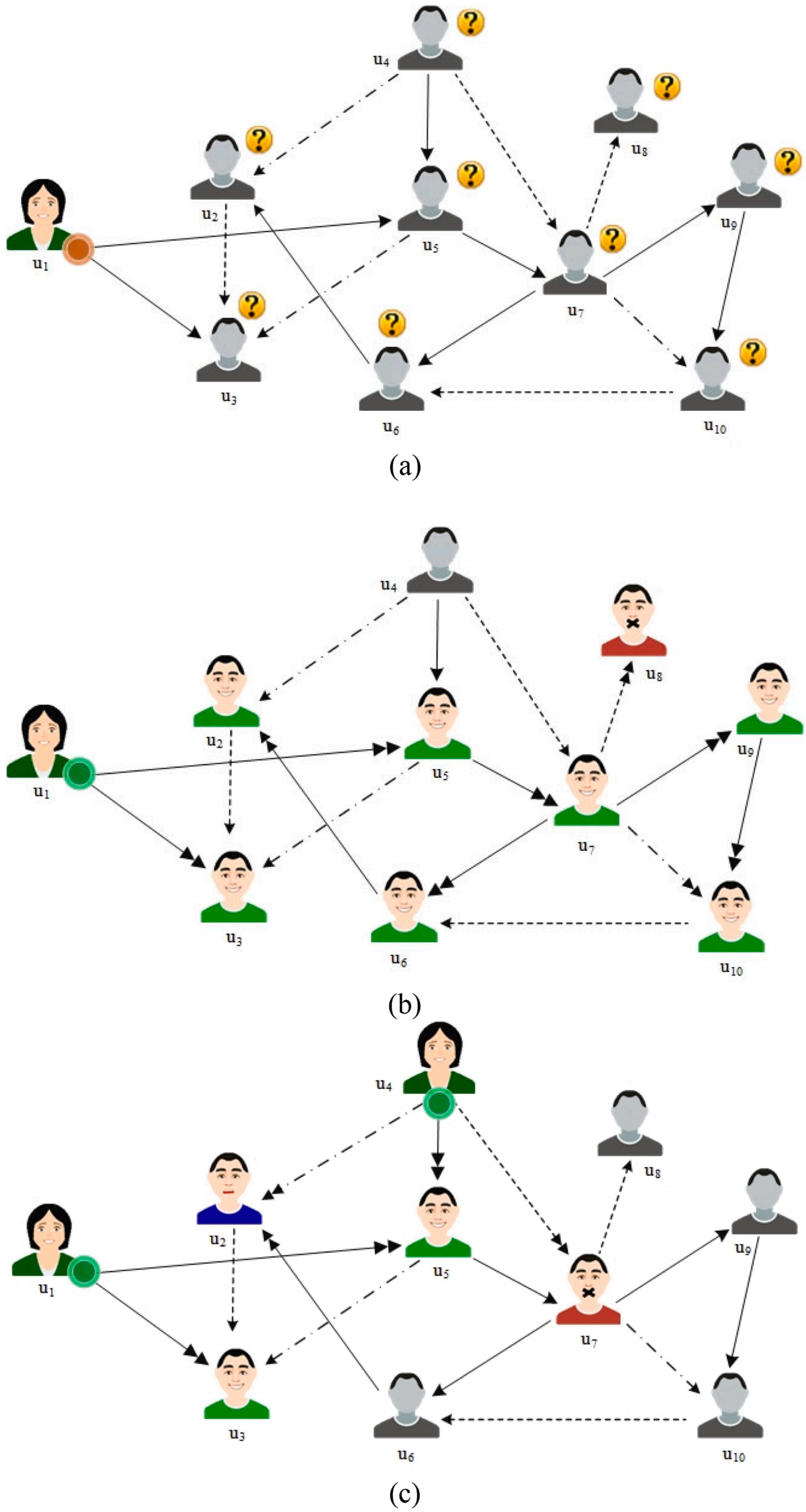
Proof. A counterexample is used to show that the influence function is not monotone under the FSC-SB model. Suppose that G' is a subgraph of social network G which is shown in Fig. 10(a), and the diffusion model is FSC-SB. As observed in Fig. 10(a), G' consists of 10 users with active and inactive initial states. There are also three types of relationship links among users, namely, trust, mediocre trust, and distrust links. Moreover, assume that the influence probability values of all links are 1. At the beginning of the information propagation process, if the seed set is a single user u_1, users $\{u_2,u_3,u_5,u_6,u_7,u_9,u_{10}\}$ will be activated by the user u_1 as shown in Fig. 10 (b); therefore, $\sigma_{FSC-SB}(G,P,\{u_1\})=7$. As seen in Fig. 10(c), if the user u_4 is added to the seed set, only users $\{u_3,u_5\}$ will be activated because u_4 blocks u_7. In this case, $\sigma_{FSC-SB}(G,P,\{u_1,u_4\})$ will be equal to 2. Consequently, we have $\sigma_{FSC-SB}(G,P,\{u_1,u_4\})\ngeq\sigma_{FSC-SB}(G,P,\{u_1\})$. This completes the proof.

Theorem 2. *Based on the FSC-SB model, there is at least a graph G that the influence function is not submodular.*

Proof. The non-submodularity of the influence function under the FSC-SB model is shown with a counterexample. Let $S=\{u_1\}$ and $T=\{u_1,u_4\}$ are the basic seed sets for information diffusion in the social network of Fig. 10(a). Then, the user $\{u_{10}\}$ is added to both seed sets. In this scenario, the users $\{u_3,u_5,u_7,u_9\}$ and $\{u_3,u_5\}$ will be activated by seed sets $\{u_1,u_{10}\}$ and $\{u_1,u_4,u_{10}\}$, respectively. Therefore, $\sigma_{FSC-SB}(G,P,\{u_1,u_{10}\})=4$ and $\sigma_{FSC-SB}(G,P,\{u_1,u_4,u_{10}\})=2$. Thus, we have $\sigma_{FSC-SB}(G,P,\{u_1,u_{10}\})-\sigma_{FSC-SB}(G,P,\{u_1\})\ngeq\sigma_{FSC-SB}(G,P,\{u_1,u_4,u_{10}\})-\sigma_{FSC-SB}(G,P,\{u_1,u_4\})$. This completes the proof.

Theorem 3. *The influence maximization under the FSC-SB model is an NP-hard problem.*

Proof. It should be noted that the influence maximization problem under the FSC-SB model is a generalization of the IC model. The



(caption on next page)

Fig. 10. A counterexample for the proof of monotonicity and submodularity in the FSC-SB model.

IC model is proved to be an NP-hard problem [15]. Since the FST-SB model is a larger class than the IC model, it will be an NP-hard problem.

4.2. Properties of the FSC-N model

Theorem 4. *Based on the FSC-N model, there is at least a graph G that the influence function is not monotone.*

Proof. We can use the same justification of Theorem 1 to prove this theorem. Consider the subgraph G' in Fig. 10(a). If the seed set includes only one user u_1, the number of positive active users will be $\sigma_{FSC-N}(G, P, \{u_1\}) = 7$. By adding u_4 to the seed set, $\sigma_{FSC-N}(G, P, \{u_1, u_4\})$ will be equal to 2 because u_7 is negatively activated by u_4. Consequently, $\sigma_{FSC-N}(.)$ is not monotone.

Theorem 5. *Based on the FSC-N model, there is at least a graph G that the influence function is not submodular.*

Proof. This theorem will be proved with the same steps as in Theorem 2. Assume that $S = \{u_1\}$ and $T = \{u_1, u_4\}$ are the basic seed sets in the subgraph G' of Fig. 10(a). By adding the positively activated user u_{10} to S and T, we will have $\sigma_{FSC-N}(G, P, \{u_1, u_{10}\}) = 4$ and $\sigma_{FSC-N}(G, P, \{u_1, u_4, u_{10}\}) = 2$, which implies the influence function is not submodular.

Theorem 6. *The influence maximization under the FSC-N model is an NP-hard problem.*

Proof. The proof is also similar to the proof of Theorem 3.

4.3. Properties of the FST-SB model

Theorem 7. *Based on the FST-SB model, there is at least a graph G that the influence function is not monotone.*

Proof. A counterexample is provided in this theorem to demonstrate that the influence function is not monotone under the FST-SB model. Consider, G' is a subgraph of social network G as shown in Fig. 11(a), also the diffusion model is FST-SB. As observed in Fig. 11 (a), G' consists of 11 users with active and inactive initial states, and also relationship links are trust, mediocre trust and distrust among users. Assume that the positive and negative threshold values for each user in the subgraph G' are the same as in the graph G. Moreover, the influence probability values of all links in the subgraph G' are similar to their equivalent values in the graph G.

Under these assumptions, the propagation process will be started by the user u_1 as the initial seed set. As it can be seen in Fig. 11(b), users $\{u_3, u_4, u_6, u_7, u_8, u_9, u_{10}, u_{11}\}$ will be activated by the user u_1 under the FST-SB model, therefore, $\sigma_{FST-SB}(G, P, \{u_1\}) = 8$. For instance, the user u_4 state is active because we have $(1-((1-0.45)\times(1-0.3)))\geqslant 0.6$ and $(1-(1-0.2)) < 0.7$. In these settings, if the user u_5 is added to the seed set, only users $\{u_2, u_6\}$ will be activated because u_5 blocks users $\{u_3, u_8\}$ (See Fig. 11(c)). Therefore, $\sigma_{FST-SB}(G, P, \{u_1, u_5\})$ is equal to 2. Consequently, we have $\sigma_{FST-SB}(G, P, \{u_1, u_5\}) \ngeq \sigma_{FST-SB}(G, P, \{u_1\})$, and the proof is completed.

Theorem 8. *Based on the FST-SB model, there is at least a graph G that the influence function is not submodular.*

Proof. A counterexample is presented to show that the influence function under the FST-SB model is not submodularity. Let $S = \{u_1\}$ and $T = \{u_1, u_5\}$. These are basic seed sets for information diffusion in the social network of Fig. 11(a). Then, the user $\{u_{11}\}$ is added to both seed sets. In such circumstances, users $\{u_6, u_8, u_9, u_{10}\}$ and $\{u_2, u_6\}$ will be activated by seed sets $\{u_1, u_{11}\}$ and $\{u_1, u_5, u_{11}\}$, respectively. Therefore, $\sigma_{FST-SB}(G, P, \{u_1, u_{11}\}) = 4$ and $\sigma_{FST-SB}(G, P, \{u_1, u_5, u_{11}\}) = 2$. Thus, we have

$\sigma_{FST-SB}(G, P, \{u_1, u_{11}\}) - \sigma_{FST-SB}(G, P, \{u_1\}) \ngeq \sigma_{FST-SB}(G, P, \{u_1, u_5, u_{11}\}) - \sigma_{FST-SB}(G, P, \{u_1, u_5\})$.

This completes the proof.

Theorem 9. *The influence maximization under the FST-SB model is NP-hard.*

Proof. The threshold model based on the joint influence probability (TJIP) [10] is a special case of the FST-SB model where trust relationships are considered in the social network. Goyal showed that the mentioned threshold model is NP-hard. Therefore, the FST-SB model is also NP-hard because it is a more general model.

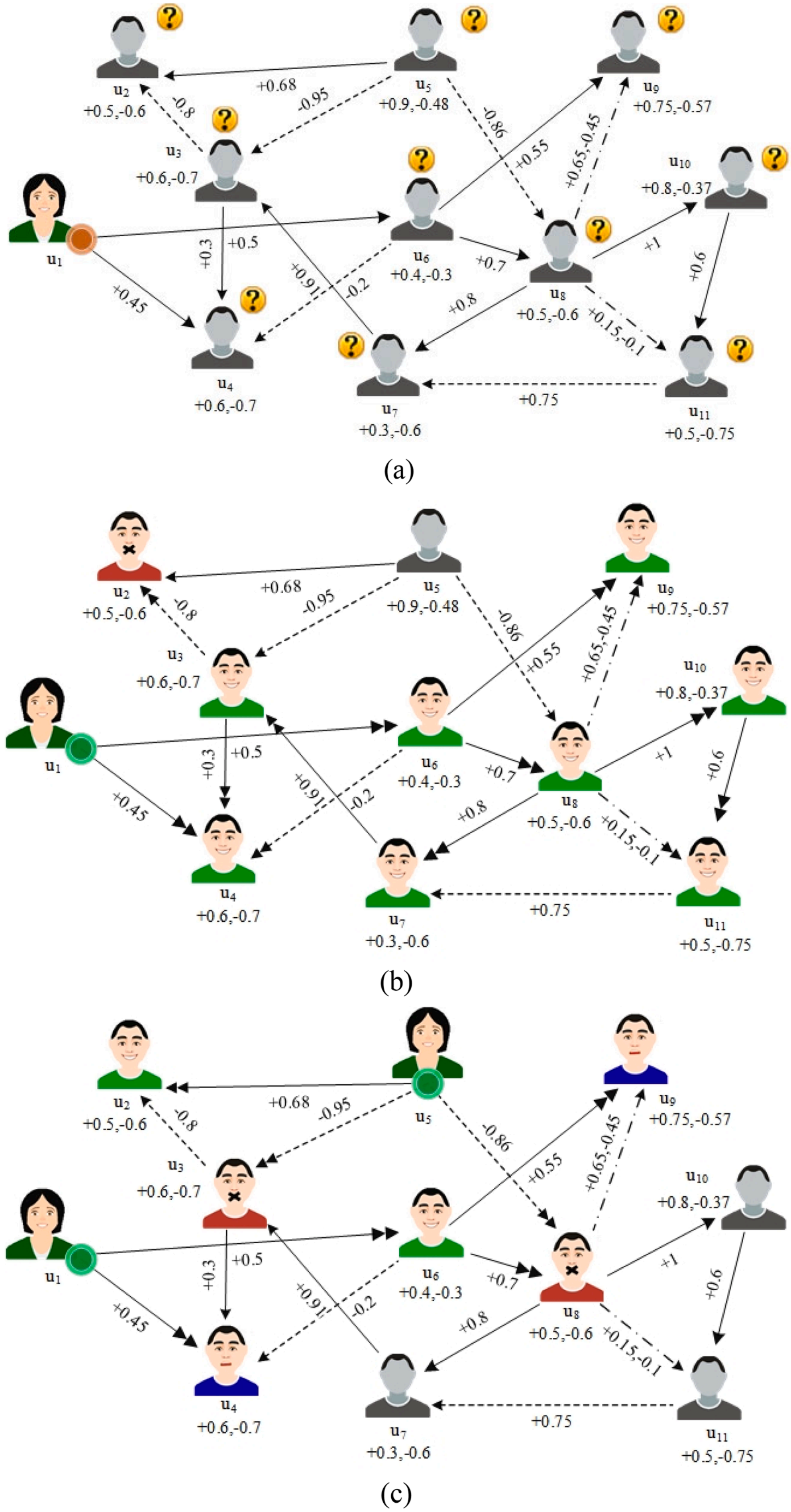
4.4. Properties of the FST-N model

Theorem 10. *Based on the FST-N model, there is at least a graph G that the influence function is not monotone.*

Proof. The same justification as given for Theorem 7 can be used to prove the theorem. Consider the subgraph G' in Fig. 11(a). If the seed set includes only one user u_1, the number of positive active users will be $\sigma_{FST-N}(G, P, \{u_1\}) = 8$. When adding u_5 to the seed set, $\sigma_{FST-N}(G, P, \{u_1, u_5\}) = 2$ because $\{u_3, u_8\}$ are negatively activated by u_5. Consequently, $\sigma_{FST-N}(.)$ is not monotone.

Theorem 11. *Based on the FST-N model, there is at least a graph G that the influence function is not submodular.*

Proof. The same steps in Theorem 8 were carried out to prove the theorem. Assume that $S = \{u_1\}$ and $T = \{u_1, u_5\}$ are the basic



(caption on next page)

Fig. 11. A counterexample for the proof of monotonicity and submodularity in the FST-SB model.

seed sets in the subgraph $G^{'}$ of Fig. 11(a). Adding the positive active user u_{11} to S and T, implies that $\sigma_{FST-N}(G, P, \{u_1, u_{11}\}) = \{u_2, u_6, u_8, u_9, u_{10}\} = 5$ and $\sigma_{FST-N}(G, P, \{u_1, u_5, u_{11}\}) = \{u_2, u_6\} = 2$ which contradicts submodularity.

Theorem 12. *The influence maximization under the FST-N model is NP-hard.*

Proof. See the proof of Theorem 9.

5. Experimental evaluation

In this section, the proposed cascade and threshold-based diffusion models are separately evaluated by several experiments through three different influence propagation plans in signed social networks. At first, the statistical information of the datasets applied in the previous studies is described in detail to select those datasets that can be used in our evaluation based on their specifications. Then, the experimental settings of this study are explained. Finally, the performance evaluation and analysis of the proposed models are presented.

5.1. Datasets

Table 3 shows the statistics of the seven well-known real-world signed social network datasets which are the most used datasets by previous works, and they are available at Standard Large Network Dataset Collection[1]. These datasets are weighted signed directed network, and their number of users, edges, positive and negative edges (pedge and nedge), and also the positive and negative ratios are shown in Table 3. As described in the last column, every directed edge (source user η, target user γ) in the network graph G indicates that a user η gives a positive, negative, or neutral opinion about the other user γ. Each edge has a weight denoting the trust level value from user η to user γ.

Since the relationships in Bitcoin OTC (OTC for short) and Bitcoin Alpha (Alpha for short) datasets are in a range of $[-10, +10]$, they can be expressed as fuzzy logic. For the other five datasets, we are not able to use the fuzzy logic due to the lack of the ratings range assigned by users. Therefore, the proposed models are evaluated by OTC and Alpha datasets.

5.2. Experimental settings

In this subsection, the experimental settings are described for the evaluation of proposed models, including evaluation metrics, baseline models, and parameter settings and implementation aspects.

5.2.1. Evaluation metrics

Before implementing extensive experiments, it is necessary to determine which metrics can effectively evaluate the performance of intended models. Thus, in order to evaluate the performance of proposed diffusion models, the three well-known evaluation metrics including precision, recall, and F-score are used that their calculations will be discussed according to the concept of each of our plans in this subsection. The evaluation metrics are defined as follows:

$$Precision = \frac{TP}{TP + FP} \tag{14}$$

$$Recall = \frac{TP}{TP + FN} \tag{15}$$

$$F - Score = \frac{2 \times Precision \times Recall}{Precision + Recall} \tag{16}$$

where TP, TN, FP, and FN are the elements of the confusion matrix standing for the true positive, true negative, false positive, and false negative, respectively. To compute these elements for models under the first and second plans, a true positive is the number of relationships created by users in the test set, and the diffusion model correctly predicted them. A true negative is the number of relationships did not create by users in the test set, and the diffusion model correctly predicted them as well. A false positive is the number of relationships created by users in the test set while the diffusion model incorrectly predicted them. Finally, a false negative is the number of relationships did not create by users in the test set while the diffusion model incorrectly predicted them. For the models based on the third plan, users' opinions are added to these predictions. Therefore, a true positive is the number of relationships and their corresponding opinions created by users in the test set, and the diffusion model correctly predicted them. A true negative is the number of relationships and their corresponding opinions that they were not created by users in the test set, and the diffusion model correctly predicted them. A false positive is the number of relationships and their corresponding opinions created by users in the test set while the diffusion model incorrectly predicted them, and a false negative is the number of relationships and their corresponding

[1] https://snap.stanford.edu/data/index.html.

Table 3
Statistical information of the signed social network datasets.

Dataset name	Num of users	Num of edges	Num of pedges	Num of nedges	Pos. ratio %	Neg. ratio %	Description
Bitcoin-OTC [19,20]	5,881	35,592	32,029	3563	89.99%	10.01%	A user of the Bitcoin OTC platform trusts or distrusts other users in the range of −10 (total distrust) to + 10 (total trust) in steps of 1 (excluding 0).
Bitcoin-Alpha [19,20]	3,783	24,186	22,650	1,536	93.65%	6.35%	The description of this dataset is as same as the description of Bitcoin OTC
Epinions [22]	131,828	841,372	717,667	123,705	85.30%	14.70%	The users' relationships in the Epinions dataset are trusts (+1) and distrusts (-1)
Wiki-RfA [44]	10,835	198,275	144,451	41,176	72.85%	20.77%	A vote is considered as support (+1), neutrality (0), and opposition (-1) in Wikipedia administrator elections by users. In this dataset, there are also 12,648 neutral votes which are about 6.38% of the total votes.
Wiki-Elec [22]	7118	107,080	83,962	23,118	78.41%	21.59%	A user votes to select adminship. Each user has 3 choices for voting: support (+1), neutral (0), and opposition (-1).
Slashdot [22]	82,140	549,202	425,072	124,130	77.40%	22.60%	A user on the technology-related news website called Slashdot is allowed to tag other users as friends (+1) and foes (-1).

opinions which were not created by users in the test set while the diffusion model incorrectly predicted them.

5.2.2. Baseline models

The proposed diffusion models are compared under two categories of cascade and threshold-based models with existing similar classic and state-of-the-art works as follows:

- Independent cascade (IC) model [15]
- Sign-aware cascade including blocking nodes (SC-B) [13]
- Sign-aware cascade (SC) [12]
- Threshold model based on the joint influence probability (TJIP) [10]
- Trust-generated threshold including blocked nodes (TG-T-B) [13]
- Trust-generated threshold including negative nodes (TG-T-N) [13]

5.2.3. Parameter settings and implementation details

Python 3.8.1 programming language is used to implement the proposed fuzzy sign-aware diffusion models and the compared diffusion models. All the experiments are performed on the PC with Intel Core i7-4790 k 4.00 GHz processor, 32.0 GB memory, and Microsoft Windows Server 2019 operating system. In order to have more precise evaluations and omit randomness of the results, the experiments are repeated 10 times and the average of results is used to compare the proposed models with state-of-the-art models.

The following describes how the fuzzy membership function used in the proposed models is determined by considering the frequency distribution of OTC and Alpha datasets. Table 4 shows the frequency distribution of the OTC dataset in which trust level can be considered as the input linguistic variable. It has three linguistic values including distrust, mediocre trust, and trust in the range of $[-10, +10]$. Although the frequency distribution of the Alpha dataset is different from the OTC dataset, its input linguistic variable is similar to the OTC dataset. Therefore, the membership functions of the fuzzy input and output are defined by the linguistic variable based on the user ratings frequency distribution analysis in both datasets as demonstrated in Fig. 12. In the proposed diffusion models, the fuzzy user-relationship expert system (FUES) aggregates all triggered fuzzy rules using the maximum operator. Finally, the aggregated fuzzy output is defuzzified using the centroid method.

In all experiments, each dataset is divided into two training (80%) and test (20%) sets according to the time priority. The training set is used to learn the influence probabilities among users in the information diffusion models. Also, the creation time of a relationship should be considered in the computations of the influence probabilities. A user rating is applied to the computations if the rate is given by the user after the creation time of their relationships. The influence probabilities computation is different based on the three introduced plans as follows:

- In the first plan, the fuzzy $\alpha - cut$ is set to 0.5. Therefore, the influence probabilities are only determined for trust relationships with membership function degrees greater than or equal to 0.5, and other fuzzy relationships are not considered in the signed social network. Since the users' activities log in OTC and Alpha datasets are available, the influence probabilities of the user γ on the user η under the first plan can be learned based on a Bernoulli trial [10] with success probability $p_{\gamma,\eta}$. This approach uses the maximum likelihood estimator to estimate the unknown parameter $p_{\gamma,\eta}$ as follows:

$$p_{\gamma,\eta} = \frac{N_{A_{\gamma,\eta}}}{N_{A_{\gamma}}} \quad (17)$$

Table 4
User ratings frequency distribution in the OTC dataset.

User ratings	Frequency	User ratings	Frequency	User ratings	Frequency
−10	2413	−3	91	+4	967
−9	20	−2	182	+5	1268
−8	31	−1	601	+6	265
−7	14	0	0	+7	208
−6	5	+1	20,048	+8	277
−5	179	+2	5562	+9	108
−4	27	+3	2561	+10	765

Eq. (17) needs to have an action log in social networks. This action log should contain the set $(\gamma, A_\gamma, T_\gamma)$, which means the user γ takes action A_γ at the time T_γ. N_{A_γ} shows the total number of actions by the user γ in the training set, and $N_{A_{\gamma,\eta}}$ denotes the number of same actions performed by the user η after the user γ. An action in OTC and Alpha datasets is any rating given by users without considering the rating value.

- In the second plan, all distrust, mediocre trust, and trust relationships are considered. Therefore, two types of influence probabilities, $p^+_{\gamma,\eta}$ and $p^-_{\gamma,\eta}$, are calculated as follows [1]:

$$p^+_{\gamma,\eta} = \frac{N_{A_{\gamma,\eta}}}{N_{A_\gamma}} \tag{18}$$

$$p^-_{\gamma,\eta} = \frac{N'_{A_{\gamma,\eta}}}{N_{A_\gamma}} \tag{19}$$

where $N'_{A_{\gamma,\eta}}$ indicates the number of same actions not made by the user η after the user γ.

- The third plan is similar to the second plan, however, in this plan, the users' rating values are counted in which the rating range reflects different users' opinions about each other. To find the influence probabilities of direct edges, the following equations extended by Hosseini-Pozveh et al. [12] are used in this study:

$$p^+_{\gamma,\eta} = \frac{N_{O_{\gamma,\eta}}}{N_{O_\gamma}} \tag{20}$$

$$p^-_{\gamma,\eta} = \frac{N'_{O_{\gamma,\eta}}}{N_{O_\gamma}} \tag{21}$$

where N_{O_γ} is the total number of given opinions by the user γ in the training set. $N_{O_{\gamma,\eta}}$ represents the number of same opinions given by the user η after the user γ. $N'_{O_{\gamma,\eta}}$ is the number of opposite opinions given by the user η after the user γ.

After the influence probabilities learning phase, experiments are implemented through users' sets which are chosen randomly from the social network members with different sizes of 3, 6, 9, 12, 18, 21, and 24. Then influence propagation process is aimed at assessing

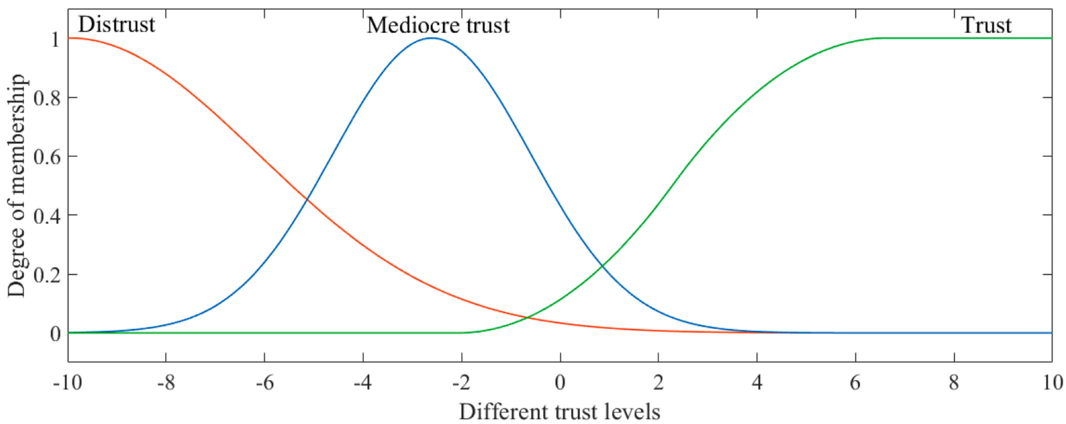


Fig. 12. Fuzzy membership functions for OTC and Alpha datasets.

the diffusion models' capabilities to predict the influence acceptance among social network users, meaning that which one of the users would give a rating to other users on the test set in the future. Also, for the diffusion models based on the third plan, users' opinions about each other are added to these predictions.

5.3. *Experimental results and analysis*

In two following subsections, several experiments are conducted by different real-world signed social networks OTC and Alpha to evaluate the proposed cascade and threshold-based diffusion models. The efficiency of the proposed diffusion models is compared with some well-known classic and state-of-the-art models.

5.3.1. *Experiments results of cascade-based diffusion models*

The proposed cascade-based diffusion models FSC-SB and FSC-N are evaluated and compared with the SC-B and IC, and with the SC and IC models, respectively. The results of their precision, recall, and F-score metrics are illustrated in Tables 5 to 8. In these tables, the maximum and average of evaluation metrics are presented based on 10 independent runs for different seed set sizes. In each experiment, the average result of the winner diffusion models is highlighted in bold font. Moreover, the total of average and the best maximum values have been reported in the last two rows of each table as the average and best behaviors of the winner model.

In Tables 5 and 6, the experimental results of the cascade diffusion models based on the second plan are shown for OTC and Alpha

Table 5
Comparison of the proposed FSC-SB with SC-B and IC cascade-based models on the OTC network.

Metrics		Precision			Recall			F-score		
User #	Index	FSC-SB	SC-B	IC	FSC-SB	SC-B	IC	FSC-SB	SC-B	IC
3	Avg	**0.046**	0.026	0.010	**0.119**	0.077	0.020	**0.066**	0.039	0.013
	Max	0.095	0.062	0.028	0.333	0.333	0.074	0.148	0.105	0.041
6	Avg	**0.140**	0.106	0.094	**0.179**	0.122	0.089	**0.157**	0.113	0.091
	Max	0.302	0.272	0.279	0.333	0.215	0.154	0.317	0.240	0.198
9	Avg	**0.124**	0.100	0.069	**0.186**	0.139	0.086	**0.149**	0.116	0.077
	Max	0.253	0.230	0.210	0.291	0.266	0.188	0.271	0.247	0.198
12	Avg	**0.202**	0.174	0.146	**0.244**	0.206	0.160	**0.221**	0.189	0.153
	Max	0.260	0.240	0.238	0.407	0.381	0.298	0.317	0.294	0.265
15	Avg	**0.211**	0.196	0.166	**0.240**	0.213	0.167	**0.225**	0.204	0.166
	Max	0.364	0.358	0.359	0.333	0.320	0.320	0.348	0.338	0.338
18	Avg	**0.238**	0.219	0.183	**0.244**	0.221	0.173	**0.241**	0.220	0.178
	Max	0.281	0.255	0.258	0.333	0.312	0.254	0.305	0.281	0.256
21	Avg	**0.256**	0.244	0.202	**0.299**	0.283	0.228	**0.276**	0.262	0.214
	Max	0.495	0.486	0.477	0.511	0.489	0.364	0.503	0.487	0.413
24	Avg	**0.324**	0.309	0.244	**0.314**	0.293	0.222	**0.319**	0.301	0.232
	Max	0.427	0.413	0.373	0.381	0.355	0.349	0.403	0.382	0.361
The avg behavior		**0.193**	0.172	0.139	**0.228**	0.194	0.143	**0.209**	0.182	0.141
The best behavior		0.495	0.486	0.477	0.511	0.489	0.364	0.503	0.487	0.413

Table 6
Comparison of the proposed FSC-SB with SC-B and IC cascade-based models on the Alpha network.

Metrics		Precision			Recall			F-score		
User #	Index	FSC-SB	SC-B	IC	FSC-SB	SC-B	IC	FSC-SB	SC-B	IC
3	Avg	**0.090**	0.089	0.058	**0.131**	0.122	0.085	**0.107**	0.103	0.069
	Max	0.118	0.120	0.090	0.301	0.274	0.208	0.170	0.167	0.126
6	Avg	**0.129**	0.102	0.062	**0.168**	0.135	0.100	**0.146**	0.116	0.077
	Max	0.263	0.257	0.156	0.329	0.300	0.241	0.292	0.277	0.189
9	Avg	**0.138**	0.117	0.090	**0.178**	0.149	0.106	**0.155**	0.131	0.097
	Max	0.284	0.265	0.250	0.292	0.333	0.208	0.288	0.295	0.227
12	Avg	**0.242**	0.224	0.152	**0.237**	0.215	0.130	**0.239**	0.219	0.140
	Max	0.371	0.357	0.288	0.317	0.310	0.211	0.342	0.332	0.244
15	Avg	**0.244**	0.218	0.184	**0.235**	0.205	0.130	**0.239**	0.211	0.152
	Max	0.355	0.331	0.304	0.345	0.333	0.255	0.350	0.332	0.277
18	Avg	**0.217**	0.197	0.155	**0.289**	0.261	0.186	**0.248**	0.225	0.169
	Max	0.323	0.296	0.286	0.349	0.330	0.264	0.335	0.312	0.275
21	Avg	**0.341**	0.323	0.277	**0.353**	0.330	0.241	**0.347**	0.326	0.258
	Max	0.480	0.466	0.476	0.430	0.410	0.370	0.454	0.436	0.416
24	Avg	**0.344**	0.330	0.275	**0.313**	0.289	0.216	**0.328**	0.308	0.242
	Max	0.450	0.438	0.383	0.417	0.349	0.349	0.433	0.388	0.365
The avg behavior		**0.218**	0.200	0.157	**0.238**	0.213	0.149	**0.228**	0.206	0.153
The best behavior		0.480	0.466	0.476	0.430	0.410	0.370	0.454	0.436	0.416

Table 7
Comparison of the proposed FSC-N with SC and IC cascade-based models on the OTC network.

Metrics		Precision			Recall			F-score		
User #	Index	FSC-N	SC	IC	FSC-N	SC	IC	FSC-N	SC	IC
3	Avg	**0.035**	0.014	0.010	**0.155**	0.052	0.020	**0.057**	0.022	0.013
	Max	0.077	0.045	0.028	0.333	0.111	0.074	0.125	0.064	0.041
6	Avg	**0.110**	0.085	0.094	**0.202**	0.126	0.089	**0.142**	0.102	0.091
	Max	0.270	0.237	0.279	0.333	0.201	0.154	0.298	0.218	0.198
9	Avg	**0.094**	0.076	0.069	**0.192**	0.157	0.086	**0.126**	0.102	0.077
	Max	0.226	0.214	0.210	0.323	0.316	0.188	0.266	0.255	0.198
12	Avg	**0.162**	0.144	0.146	**0.261**	0.243	0.160	**0.200**	0.181	0.153
	Max	0.235	0.220	0.238	0.398	0.388	0.298	0.296	0.281	0.265
15	Avg	**0.187**	0.168	0.166	**0.250**	0.229	0.167	**0.214**	0.194	0.166
	Max	0.346	0.339	0.359	0.406	0.333	0.320	0.374	0.336	0.338
18	Avg	**0.211**	0.194	0.183	**0.266**	0.238	0.173	**0.235**	0.214	0.178
	Max	0.266	0.259	0.258	0.356	0.344	0.254	0.304	0.296	0.256
21	Avg	**0.222**	0.209	0.202	**0.309**	0.299	0.228	**0.258**	0.246	0.214
	Max	0.459	0.450	0.477	0.514	0.514	0.364	0.485	0.480	0.413
24	Avg	**0.292**	0.278	0.244	**0.323**	0.312	0.222	**0.307**	0.294	0.232
	Max	0.400	0.382	0.373	0.381	0.375	0.349	0.390	0.378	0.361
The avg behavior		**0.164**	0.146	0.139	**0.245**	0.207	0.143	**0.196**	0.171	0.141
The best behavior		0.459	0.450	0.477	0.514	0.514	0.364	0.485	0.480	0.413

Table 8
Comparison of the proposed FSC-N with SC and IC cascade-based models on the Alpha network.

Metrics		Precision			Recall			F-score		
User #	Index	FSC-N	SC	IC	FSC-N	SC	IC	FSC-N	SC	IC
3	Avg	**0.108**	0.062	0.058	**0.150**	0.095	0.085	**0.126**	0.075	0.069
	Max	0.121	0.089	0.090	0.323	0.224	0.208	0.176	0.127	0.126
6	Avg	**0.109**	0.072	0.062	**0.140**	0.116	0.100	**0.123**	0.089	0.077
	Max	0.214	0.189	0.156	0.314	0.273	0.241	0.255	0.223	0.189
9	Avg	**0.119**	0.086	0.090	**0.151**	0.122	0.106	**0.133**	0.101	0.097
	Max	0.273	0.250	0.250	0.274	0.226	0.208	0.273	0.237	0.227
12	Avg	**0.220**	0.203	0.152	**0.222**	0.208	0.130	**0.221**	0.205	0.140
	Max	0.336	0.330	0.288	0.336	0.305	0.211	0.336	0.317	0.244
15	Avg	**0.221**	0.203	0.184	**0.210**	0.191	0.130	**0.215**	0.197	0.152
	Max	0.343	0.319	0.304	0.329	0.318	0.255	0.336	0.318	0.277
18	Avg	**0.191**	0.176	0.155	**0.295**	0.246	0.186	**0.232**	0.205	0.169
	Max	0.299	0.272	0.286	0.400	0.333	0.264	0.342	0.299	0.275
21	Avg	**0.326**	0.314	0.277	**0.346**	0.329	0.241	**0.336**	0.321	0.258
	Max	0.463	0.460	0.476	0.423	0.412	0.370	0.442	0.435	0.416
24	Avg	**0.320**	0.304	0.275	**0.303**	0.288	0.216	**0.311**	0.296	0.242
	Max	0.438	0.438	0.383	0.420	0.406	0.349	0.429	0.421	0.365
The avg behavior		**0.202**	0.178	0.157	**0.227**	0.199	0.149	**0.214**	0.188	0.153
The best behavior		0.463	0.460	0.476	0.423	0.412	0.370	0.442	0.435	0.416

datasets, respectively. As observed in these tables, the FSC-SB model has superior performance compared with SC-B and IC models in terms of the evaluation metrics on both datasets. According to the third plan, Tables 7 and 8 demonstrate the results of cascade-based diffusion models for OTC and Alpha datasets, respectively. The obtained results reveal that the FSC-N model has significantly better performance than the SC and IC models on both datasets.

To show the behavior of cascade models, the overall results obtained using the F-score metric are compared in Figs. 13 and 14. These results show that the proposed fuzzy sign-aware diffusion models can predict more accurately the real behavior of social network users compared with the other models. Besides, assessments on both OTC and Alpha datasets in the cascade models group show that the worst results have been achieved by the IC classic diffusion model. While the performance of diffusion models in predicting the information propagation process on the social network is improved by considering the influence propagation of distrust relationships along with the trust relationship.

5.3.2. *Experiments results of threshold-based diffusion models*

The proposed threshold-based diffusion models FTG-SB and FTG-N are assessed and their results are compared with the TG-T-B and TJIP, and with the TG-T-N and TJIP models, respectively. The results of their precision, recall, and F-score metrics are illustrated in Tables 9 to 12. In these tables, the maximum and average of evaluation metrics are presented based on 10 independent runs for different seed set sizes. In each experiment, the average result of the winner diffusion models is highlighted in bold font. Moreover, the total of average and the best maximum values have been reported in the last two rows of each table as the average and best behaviors of the winner model.

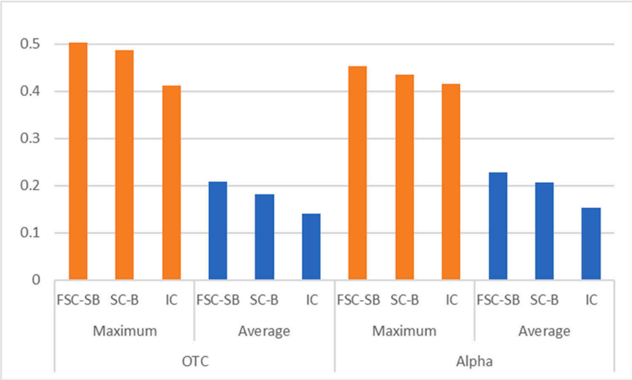


Fig. 13. The overall Comparison of the average and best F-score in cascade-based models under the second plan.

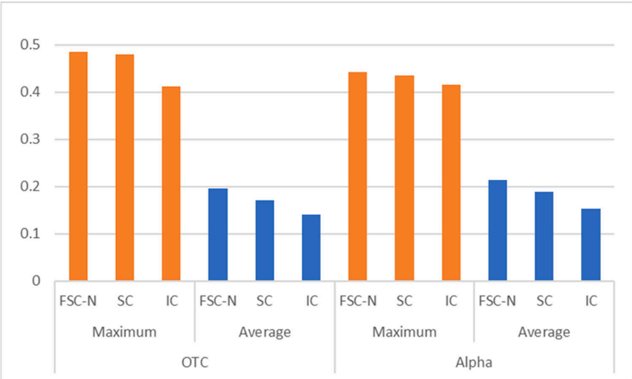


Fig. 14. The overall Comparison of the average and best F-score in cascade-based models under the third plan.

Table 9
Comparison of the proposed FST-SB with TG-T-B and TJIP threshold-based models on the OTC network.

Metrics		Precision			Recall			F-score		
User #	Index	FST-SB	TG-T-B	TJIP	FST-SB	TG-T-B	TJIP	FST-SB	TG-T-B	TJIP
3	Avg	**0.061**	0.035	0.022	**0.162**	0.094	0.037	**0.089**	0.051	0.028
	Max	0.125	0.083	0.070	0.667	0.333	0.160	0.211	0.133	0.097
6	Avg	**0.146**	0.116	0.094	**0.168**	0.123	0.091	**0.156**	0.119	0.092
	Max	0.348	0.281	0.277	0.333	0.222	0.167	0.340	0.248	0.208
9	Avg	**0.158**	0.101	0.060	**0.186**	0.133	0.090	**0.171**	0.115	0.072
	Max	0.255	0.236	0.190	0.278	0.253	0.375	0.266	0.244	0.252
12	Avg	**0.198**	0.177	0.140	**0.227**	0.207	0.158	**0.212**	0.191	0.148
	Max	0.267	0.244	0.235	0.407	0.389	0.281	0.322	0.300	0.256
15	Avg	**0.209**	0.196	0.167	**0.227**	0.213	0.171	**0.218**	0.204	0.169
	Max	0.366	0.358	0.359	0.333	0.320	0.315	0.349	0.338	0.336
18	Avg	**0.239**	0.220	0.186	**0.241**	0.221	0.178	**0.240**	0.220	0.182
	Max	0.286	0.267	0.257	0.333	0.312	0.284	0.308	0.288	0.270
21	Avg	**0.259**	0.243	0.204	**0.299**	0.281	0.238	**0.278**	0.261	0.220
	Max	0.500	0.481	0.472	0.511	0.489	0.455	0.505	0.485	0.463
24	Avg	**0.324**	0.308	0.260	**0.311**	0.290	0.244	**0.317**	0.299	0.252
	Max	0.432	0.414	0.360	0.372	0.349	0.337	0.400	0.379	0.348
The avg behavior		**0.199**	0.175	0.142	**0.228**	0.195	0.151	**0.213**	0.184	0.146
The best behavior		0.500	0.481	0.472	0.667	0.489	0.455	0.572	0.485	0.463

In Tables 9 and 10, the experimental results of the proposed threshold diffusion models and their competitors based on the second plan are shown for OTC and Alpha datasets, respectively. These results show that the FTG-SB model has superior performance compared with TG-T-B and TJIP models in terms of the evaluation metrics on both datasets. According to the third plan, Tables 11 and 12 demonstrate the results of threshold diffusion models for OTC and Alpha datasets, respectively. The obtained results reveal that the performance of the FTG-N model is more preferable than TG-T-N and TJIP models on both datasets, whereas the TJIP model shows the worst results compared with the others in threshold diffusion models. To show a comprehensive insight into the behavior of threshold

Table 10
Comparison of the proposed FST-SB with TG-T-B and TJIP threshold-based models on the Alpha network.

Metrics		Precision			Recall			F-score		
User #	Index	FST-SB	TG-T-B	TJIP	FST-SB	TG-T-B	TJIP	FST-SB	TG-T-B	TJIP
3	Avg	**0.162**	0.098	0.061	**0.215**	0.130	0.119	**0.185**	0.112	0.081
	Max	0.287	0.140	0.089	0.327	0.256	0.186	0.306	0.181	0.120
6	Avg	**0.178**	0.101	0.079	**0.249**	0.152	0.125	**0.208**	0.121	0.097
	Max	0.322	0.239	0.173	0.347	0.262	0.209	0.334	0.250	0.189
9	Avg	**0.198**	0.131	0.108	**0.261**	0.157	0.139	**0.225**	0.143	0.122
	Max	0.342	0.277	0.263	0.351	0.263	0.292	0.346	0.270	0.277
12	Avg	**0.214**	0.191	0.163	**0.275**	0.240	0.164	**0.241**	0.213	0.163
	Max	0.383	0.351	0.339	0.381	0.333	0.250	0.382	0.342	0.288
15	Avg	**0.254**	0.232	0.208	**0.258**	0.234	0.162	**0.256**	0.233	0.182
	Max	0.344	0.319	0.300	0.356	0.322	0.255	0.350	0.320	0.276
18	Avg	**0.252**	0.234	0.193	**0.289**	0.268	0.203	**0.269**	0.250	0.198
	Max	0.315	0.290	0.304	0.339	0.321	0.277	0.327	0.305	0.290
21	Avg	**0.301**	0.290	0.243	**0.314**	0.301	0.219	**0.307**	0.295	0.230
	Max	0.377	0.373	0.322	0.370	0.375	0.302	0.373	0.374	0.312
24	Avg	**0.296**	0.281	0.244	**0.309**	0.294	0.215	**0.302**	0.287	0.229
	Max	0.370	0.350	0.314	0.417	0.404	0.309	0.392	0.375	0.311
The avg behavior		**0.232**	0.195	0.162	**0.271**	0.222	0.168	**0.250**	0.208	0.165
The best behavior		0.383	0.373	0.339	0.417	0.404	0.309	0.399	0.388	0.323

Table 11
Comparison of the proposed FST-N with TG-T-N and TJIP threshold-based models on the OTC network.

Metrics		Precision			Recall			F-score		
User #	Index	FST-N	TG-T-N	TJIP	FST-N	TG-T-N	TJIP	FST-N	TG-T-N	TJIP
3	Avg	**0.098**	0.032	0.022	**0.158**	0.052	0.037	**0.121**	0.040	0.028
	Max	0.182	0.076	0.070	0.333	0.222	0.160	0.235	0.113	0.097
6	Avg	**0.114**	0.091	0.094	**0.145**	0.110	0.091	**0.128**	0.100	0.092
	Max	0.270	0.259	0.277	0.237	0.209	0.167	0.252	0.231	0.208
9	Avg	**0.127**	0.074	0.060	**0.169**	0.100	0.090	**0.145**	0.085	0.072
	Max	0.224	0.219	0.190	0.274	0.242	0.375	0.246	0.230	0.252
12	Avg	**0.182**	0.157	0.140	**0.229**	0.196	0.158	**0.203**	0.174	0.148
	Max	0.255	0.243	0.235	0.379	0.369	0.281	0.305	0.293	0.256
15	Avg	**0.197**	0.178	0.167	**0.234**	0.210	0.171	**0.214**	0.193	0.169
	Max	0.362	0.342	0.359	0.337	0.329	0.315	0.349	0.335	0.336
18	Avg	**0.218**	0.201	0.186	**0.239**	0.224	0.178	**0.228**	0.212	0.182
	Max	0.275	0.257	0.257	0.322	0.289	0.284	0.297	0.272	0.270
21	Avg	**0.232**	0.215	0.204	**0.302**	0.295	0.238	**0.262**	0.249	0.220
	Max	0.477	0.451	0.472	0.514	0.514	0.455	0.495	0.480	0.463
24	Avg	**0.305**	0.289	0.260	**0.320**	0.300	0.244	**0.312**	0.294	0.252
	Max	0.412	0.396	0.360	0.381	0.381	0.337	0.396	0.388	0.348
The avg behavior		**0.184**	0.155	0.142	**0.225**	0.186	0.151	**0.202**	0.169	0.146
The best behavior		0.477	0.451	0.472	0.514	0.514	0.455	0.495	0.480	0.463

Table 12
Comparison of the proposed FST-N with TG-T-N and TJIP threshold-based models on the Alpha network.

Metrics		Precision			Recall			F-score		
User #	Index	FST-N	TG-T-N	TJIP	FST-N	TG-T-N	TJIP	FST-N	TG-T-N	TJIP
3	Avg	**0.148**	0.020	0.061	**0.201**	0.092	0.119	**0.170**	0.033	0.081
	Max	0.242	0.045	0.089	0.281	0.233	0.186	0.260	0.075	0.120
6	Avg	**0.162**	0.071	0.079	**0.231**	0.120	0.125	**0.190**	0.089	0.097
	Max	0.277	0.194	0.173	0.309	0.231	0.209	0.292	0.211	0.189
9	Avg	**0.178**	0.066	0.108	**0.252**	0.137	0.139	**0.209**	0.089	0.122
	Max	0.298	0.129	0.263	0.339	0.258	0.292	0.317	0.172	0.277
12	Avg	**0.185**	0.148	0.163	**0.261**	0.234	0.164	**0.217**	0.181	0.163
	Max	0.348	0.331	0.339	0.353	0.315	0.250	0.350	0.323	0.288
15	Avg	**0.230**	0.215	0.208	**0.241**	0.226	0.162	**0.235**	0.220	0.182
	Max	0.324	0.307	0.300	0.329	0.318	0.255	0.326	0.312	0.276
18	Avg	**0.220**	0.206	0.193	**0.287**	0.248	0.203	**0.249**	0.225	0.198
	Max	0.295	0.261	0.304	0.333	0.289	0.277	0.313	0.274	0.290
21	Avg	**0.290**	0.248	0.243	**0.320**	0.276	0.219	**0.304**	0.261	0.230
	Max	0.368	0.306	0.322	0.379	0.364	0.302	0.373	0.332	0.312
24	Avg	**0.272**	0.230	0.244	**0.297**	0.294	0.215	**0.284**	0.258	0.229
	Max	0.350	0.267	0.314	0.413	0.406	0.309	0.379	0.322	0.311
The avg behavior		**0.211**	0.151	0.162	**0.261**	0.203	0.168	**0.233**	0.173	0.165
The best behavior		0.368	0.331	0.339	0.413	0.406	0.309	0.389	0.365	0.323

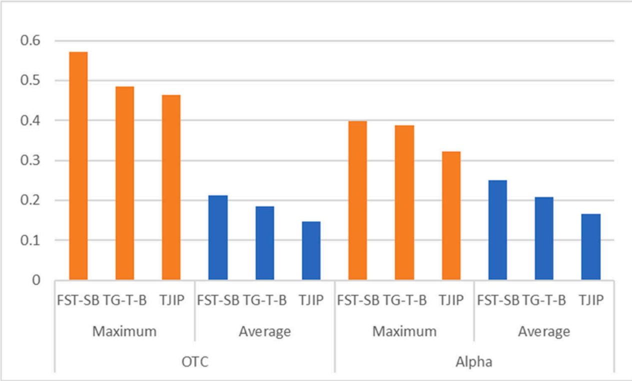


Fig. 15. The overall Comparison of the average and best F-score in threshold-based models under the second plan.

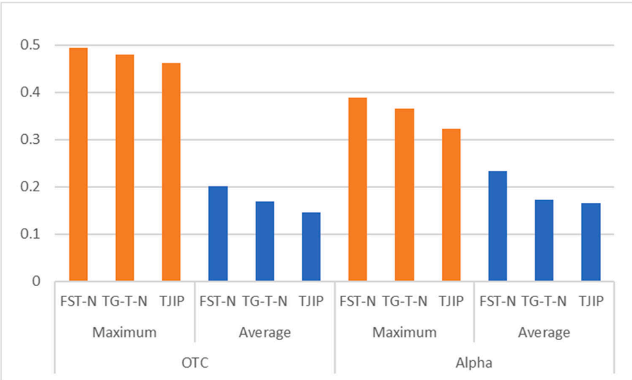


Fig. 16. The overall Comparison of the average and best F-score in threshold-based models under the third plan.

models, the overall results obtained for the average and best maximum of F-score values are drawn in Figs. 15 and 16. These results indicate that the proposed fuzzy approach significantly enhances the efficiency of the diffusion models in signed social networks.

5.3.3. *Comparative performance analysis*

This subsection provides a detailed comparative analysis based on the performance results of our cascade and threshold-based diffusion models with the existing similar works. To do so, F-score is chosen as the performance metric because it is a comprehensive reflection of both precision and recall metrics. For the purpose of conducting the comparative analysis, Table 13 summarizes the overall F-scores given in Tables 5–12 for the proposed models and other comparative models. In addition, overall F-score rank based on diffusion models of each category of cascade and threshold-based is listed in this table.

Based on the results presented in Table 13, in the cascade-based diffusion models category, the proposed FSC-SB and FSC-N models outperform all other baseline models on both OTC and Alpha datasets. For the OTC dataset, the FSC-SB model achieves an overall F-score of 0.209, representing improvements of 14.8%, 22.2%, and 48.2% compared to the SC-B, SC, and IC models, respectively. Also,

Table 13
Overall comparison of the proposed models with the competitor models.

Diffusion models	Method	OTC dataset Overall F-score	Overall F-score rank based on diffusion models	Alpha dataset Overall F-score	Overall F-score rank based on diffusion models
Cascade-based diffusion models	IC	0.141	5	0.153	5
	FSC-SB	**0.209**	**1**	**0.228**	**1**
	SC-B	0.182	3	0.206	3
	FSC-N	0.196	2	0.214	2
	SC	0.171	4	0.188	4
Threshold-based diffusion models	TJIP	0.146	5	0.165	5
	FST-SB	**0.213**	**1**	**0.250**	**1**
	TG-T-B	0.184	3	0.208	3
	FST-N	0.202	2	0.233	2
	TG-T-N	0.169	4	0.173	4

the FSC-N model yields an overall F-score of 0.196, representing improvements of 7.7%, 14.6%, and 39.0% compared to the SC-B, SC, and IC models, respectively. Regarding the Alpha dataset, the FSC-SB model achieves an overall F-score of 0.228, representing improvements of 10.7%, 21.3%, and 49.0% compared to the SC-B, SC, and IC models. Also, the FSC-N model yields an overall F-score of 0.214, representing improvements of 3.9%, 13.8%, and 39.9% compared to the SC-B, SC, and IC models.

In the category of threshold-based diffusion models, it can be observed that the proposed FST-SB and FST-N models have superior performance compared with all other baseline models on both OTC and Alpha datasets. For the OTC dataset, the FST-SB model reaches an overall F-score of 0.213, showing improvements of 15.8%, 26.0%, and 45.9% with respect to the TG-T-B, TG-T-N, and TJIP models, respectively. The FST-N model also obtains an overall F-score of 0.202, showing improvements of 9.8%, 19.5%, and 38.4% with respect to the TG-T-B, TG-T-N, and TJIP models, respectively. Regarding the Alpha dataset, the FST-SB model achieves an overall F-score of 0.250, showing improvements of 20.2%, 44.5%, and 51.5% with respect to the TG-T-B, TG-T-N, and TJIP models. Also, the FST-N model obtains an overall F-score of 0.233, showing improvements of 12.0%, 34.7%, and 41.2% with respect to the TG-T-B, TG-T-N, and TJIP models.

Finally, the different proposed models were compared with each other within their corresponding categories. In the first category, the FSC-SB model performs better in prediction compared to the FSC-N model. To be more specific, the FSC-SB model improves F-score values by 6.6% and 6.5% with respect to the FSC-N model on OTC and Alpha datasets, respectively. Moreover, in the second category, the FST-SB model gives better prediction results compared to the FST-N model. In this case, the FST-SB model improves F-score values by 5.4% and 7.3% with respect to the FST-N model on OTC and Alpha datasets, respectively.

6. Discussion

In this section, the main reasons for the superiority of the proposed fuzzy sign-aware cascade and threshold-based models over the comparative models are discussed. The results reported in Tables 5–13 demonstrate that the proposed models are competitive versus other existing similar models for IM problems. This is because all of these models utilize a fuzzy user-relationship expert system (FUES) in which a natural multi-trust level relationship is considered instead of a commonly used crisp relationship. Thus, it is more appropriate to deal with different relationships that are nearer to the natural way of human thinking and reasoning. Moreover, new rules and equations are defined in the proposed models to determine a user's state by information received from its active neighbors. In this regard, users do not immediately decide to accept or reject the promoted advertising message, and they will go to the suspended state until making their final decisions. Therefore, users have better opportunities to make their decisions based on more social interactions with other neighbors who shared the message.

The comprehensive results reported in Table 13 help to clarify differences that exist among the three proposed plans applied to cascade and threshold-based models. As compared in this table, the proposed models based on the second plan, FSC-SB and FTG-SB, achieve more accurate performance prediction than the first and third plans on both OTC and Alpha datasets. After that, FSC-N and FTG-N models based on the third plan had placed in the second rank. The results of this comparison indicate that although the FSC-N and FTG-N models have improved the prediction accuracy of the real influence propagation process, the FSC-SB and FTG-SB models have performed better than them in solving the IM problem. Thus, the users' behavior is more adapted to the proposed second plan on both datasets. In other words, these users have the tendency to ignore the social influence that they are receiving from their distrusted neighbors and hence change their states to blocked or suspended. Finally, it is noticeable that the models of the first plan, including IC and TJIP, have the worst results. This is essentially because of considering only an acceptable level of trust among network users by these models.

7. Conclusion and future studies

This study introduces a fuzzy-based approach, including three plans for modeling influence propagation in signed social networks. The main difference of this approach is applying a natural multi-level relationship instead of a commonly crisp relationship. Using the introduced approach, four novel fuzzy sign-aware diffusion models have been proposed in two categories, cascade and threshold-based. According to the first plan, the influence probabilities are only calculated for social trust relationships with fuzzy membership degrees greater than or equal to the predefined $\alpha - cut$, and other fuzzy relationships are ignored. In this plan, the classic IC and TJIP models have been discussed as well. In the second plan, two new diffusion models of FSC-SB and FST-SB have been proposed such that all user-relationship types with different trust levels have been considered in the diffusion models. Moreover, if a distrusted user acts, its neighbors tend not to do that action and change their states to blocked or suspended states. In the third plan, two new diffusion models of FSC-N and FST-N have been proposed. In this plan, all user-relationship types are considered similar to the second plan. In addition, if a distrusted user acts, its neighbors have one of two possible reactions, an opposite action or going to the suspended state. The properties of these models prove that the proposed diffusion models are not monotone and submodular. Moreover, influence maximization is an NP-hard problem under the proposed models, as proved in this study.

The performance of fuzzy sign-aware diffusion models was evaluated by comparing their results with some well-known and state-of-the-art models on two real datasets, Bitcoin OTC and Bitcoin Alpha. The proposed cascade-based diffusion models FSC-SB and FSC-N are evaluated, and their results are fairly compared with the SC-B and IC, and with the SC and IC models, respectively. In the same fashion, the proposed threshold-based diffusion models FTG-SB and FTG-N are assessed and compared with the TG-T-B and TJIP, and with the TG-T-N and TJIP models, respectively. The experimental results show that all proposed diffusion models perform considerably better than the baseline models. Moreover, a discussion has been carried out on the performance of the proposed plans. Accordingly, the proposed diffusion models based on the second plan show more accurate predictions and robust than all compared models on both datasets. After these models, the FSC-N and FTG-N models based on the third plan have had superior ranks, and the inferior ones are classic models. For further studies, identifying the most influential users by influence maximization algorithms under the proposed fuzzy sign-aware diffusion models can be an interesting practical topic that plays an essential role in the success of a company's advertising campaigns through viral marketing.

CRediT authorship contribution statement

Sohameh Mohammadi: Conceptualization, Methodology, Formal analysis, Investigation, Writing – original draft, Writing – review & editing. **Mohammad H. Nadimi-Shahraki:** Conceptualization, Methodology, Validation, Formal analysis, Investigation, Writing – review & editing, Supervision. **Zahra Beheshti:** Conceptualization, Methodology, Validation, Formal analysis, Investigation, Writing – review & editing, Supervision. **Kamran Zamanifar:** Writing – review & editing, Supervision.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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