UM-SJTU Joint Institute

Physical Laboratory

VP141

Laboratory Report

Exercise 1

Measurements of the Moment of Inertia

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1 Theoretical Background

Moment of inertia of a rigid body about an axis is a quantitative characteristics that defines the body's resistance (inertia) to a change of angular velocity in rotation about that axis. This characteristics of the rigid body rotating about a fixed axis is determined not only by the mass of the body, but also by its distribution. The moment of inertia of a rigid body about a certain rotation axis can be calculated analytically. However, if the body has irregular shape or non-uniformly distributed mass, the calculation may be di cult. Experimental methods turn out to be more useful in such cases.

1.1 Laws of Physics Used

There are mainly two laws of Physics been used in the experiment. Second law of dynamics for rotational motion can find the relationship between the rotational acceleration and the torque at the object.

1.1.1 Second Law of Dynamics for Rotational Motion

The rotational motion about a fixed axis relates with the component of the torque about the axis of rotation with the moment of inertia about this axis.

$$\tau_z = I\beta_z$$

Therefore, the moment of inertia I can be found once the torque and the resulting angular acceleration are measured.

The moment of inertia is an additive quantity, the moment of inertia of the combined rigid body AB composed of A and B, about the same axis of rotation, is

$$I_{ab} = I_A + I_B$$

1.1.2 Parallel Axis Theorem

If the moment of inertia of a rigid body with mass m about an axis through the body's center of mass is I0, then for any axis parallel to that axis, the moment of inertia is

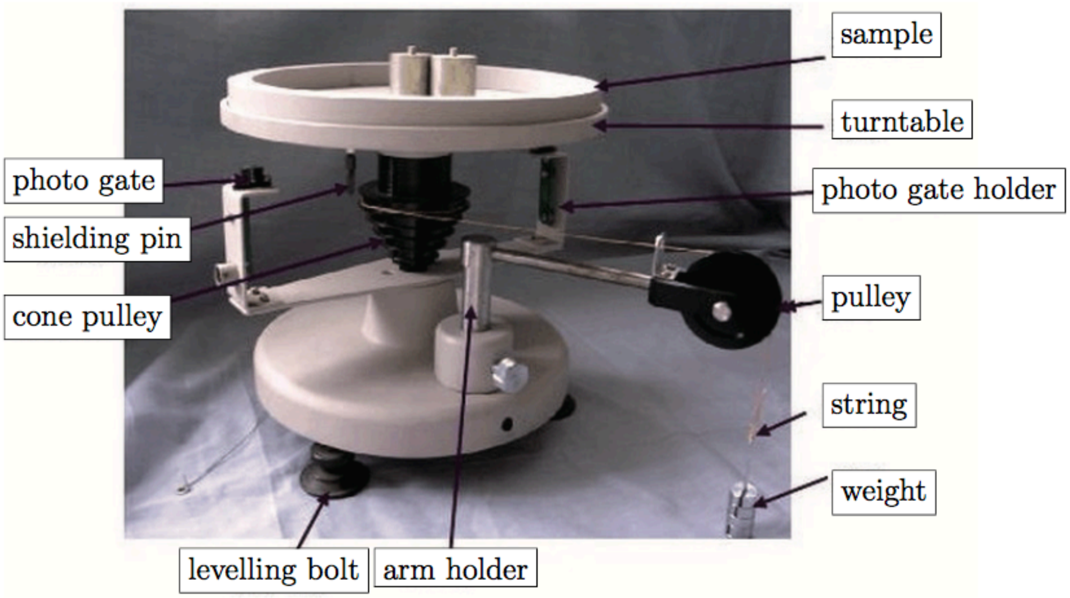
$$I = I_0 + md^2$$

where d is the distance between the axes.

2 Apparatus

The measurement setup consists of a turntable with an integrated photo-gate system used for time measurements.

- sample
- turntable
- photo gate holder
- pulley
- string
- weight
- photo gate
- shielding pin
- cone pulley
- levelling bolt
- arm holder



Since the bearings of the turntable are not frictionless, there will be a non-zero frictional torque M causing the turntable to decelerate with angular acceleration β_1, so that the second law of dynamics for rotational motion of the empty turntable reads

$$M_\mu = -I_1\beta_1$$

3 Procedure

1. Measure the mass of the weight, the hoop, the disk, and the cylinder, as well as the radius of the cone pulley and the cylinder (follow the instructor's requirements). Calculate the moment of inertia of the hoop and the disk analytically.

2. Turn the electronic timer on and switch it to mode 1-2 (single gate, multiple pulses).

3. Place the instrument close to the edge of the desk and stretch the disk pulley arm outside, so that the weight can move downwards unobstructed.

4. Level the turntable with the bubble level.

5. Make the turntable rotating and press the start button on the timer. After at least 8 signals are recorded, stop the turntable and record the data in your data sheet.

6. Attach the weight to one end of the string. Place the string on the disk pulley, thread through the hole in the arm, and wind the string around the 3rd ring of the cone pulley. Adjust the arm holder so that the string goes through the center of the hole.

7. Release the weight and start the timer. Stop the turntable when the weight hits the floor. Write down the recorded data.

8. The angular acceleration can be found by plotting $\theta = k\pi$ against t and performing a quadratic fit using data processing software. (The magnitude of the angular acceleration is equal to the coefficient next to t^2 multiplied by two. The uncertainty of the angular acceleration can be read directly from the fitting result.)

   The moment of inertia of the empty turntable is found by using the formulae in Section 3 and the data from step 5 and 7. Repeat steps 5–7 with a rigid object placed on the turntable. An equaition is used to find the moment of inertia of the rigid object.

The timer's resolution is $0.0001s$, and the error is 0.004%.

4 Calculations and Results

From the following equaitions,

$$\tau_z = I\beta_z$$

$$I_{ab} = I_A + I_B$$

$$M_\mu = -I_1\beta_1$$

$$T = m(g - a)$$

We can derive

$$m(g - R\beta_2)R - M_\mu = I_1\beta_2$$

Then we find

$$I_1 = \frac{mR(g - R\beta_2)}{\beta_2 - \beta_1}$$

Similarly, if a rigid body with an unknown moment of inertia is placed on the turntable, we may find

$$I_2 = \frac{mR(g - R\beta_4)}{\beta_4 - \beta_3}$$

Using the fact that the moment of inertia is an additive quantity, the moment of inertia of the rigid object placed on the turntable, with respect to the axis of rotation, may be found as the difference

$$I_3 = I_2 - I_1$$

4.1 Measurement of Angular Acceleration

Angular acceleration can be derive by investigating the measurement data (k,t), the corresponding angular position is

$$\theta = k\pi = \omega_0 t + \frac{1}{2}\beta t^2$$

4.2 In Lab Data

Object	1	2	3	4
Disk $[cm] \pm 0.002[cm]$	24.098	24.094	24.094	24.094
Hoop 1 $[cm] \pm 0.002[cm]$	20.982	20.918	20.966	20.976
Hoop 2 $[cm] \pm 0.002[cm]$	23.998	24.000	24.000	24.002
Cylinder A $[cm] \pm 0.002[cm]$	2.994	2.994	2.994	2.994
Cylinder B $[cm] \pm 0.002[cm]$	2.994	2.994	2.994	2.994
Cone pulley $[cm] \pm 0.002[cm]$	5.022	5.020	5.008	5.008
Hole 1 d $[cm] \pm 0.002[cm]$	3.978	5.534		
Hole 2 d $[cm] \pm 0.002[cm]$	3.982	5.540		
Hole 3 d $[cm] \pm 0.002[cm]$	5.524	6.500		
Hole 4 d $[cm] \pm 0.002[cm]$	5.520	6.520		

Table 1: Calliper measurements

Object	Mass
Disk $[g] \pm 0.1[g]$	493.1
Hoop $[g] \pm 0.1[g]$	422.5
Cylinder A $[g] \pm 0.1[g]$	165.8
Cylinder B $[g] \pm 0.1[g]$	165.8
Weight $[g] \pm 0.1[g]$	59.1

Table 2: Mass measurements

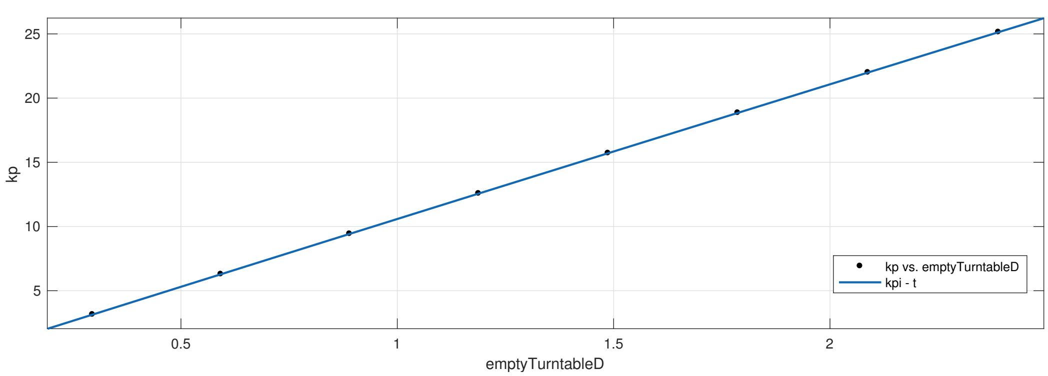
Situation	A or D	k	1	2	3	4	5	6	7	8
Empty	Dec	$t[s]$	0.2958	0.5924	0.8899	1.1881	1.4873	1.7871	2.0879	2.3895
Empty	Acc	$t[s]$	0.9005	1.5186	2.0204	2.4557	2.8448	3.1996	3.5280	3.7038
With disk	Dec	$t[s]$	0.3178	0.6362	0.9554	1.2752	1.5958	1.9170	2.2390	2.5616
With disk	Acc	$t[s]$	0.8957	1.5808	2.1582	2.6666	3.1264	3.5491	3.9428	4.3322
With hoop	Dec	$t[s]$	0.2450	0.4903	0.7359	0.9818	1.2281	1.4746	1.7216	1.9688
With hoop	Acc	$t[s]$	1.0035	1.7614	2.3967	2.9547	3.4589	3.9216	4.3521	4.7560
A 1 B 2	Dec	$t[s]$	0.4491	0.9003	1.3536	1.8089	2.2666	2.7263	3.1883	3.6524
A 1 B 2	Acc	$t[s]$	1.3586	2.0905	2.6595	3.1424	3.5692	3.9562	4.3125	4.6448
A 3 B 4	Dec	$t[s]$	0.4539	0.9099	1.3678	1.8279	2.2899	2.7541	3.2204	3.6888
A 3 B 4	Acc	$t[s]$	1.3748	2.1298	2.7179	3.2171	3.6585	4.0587	4.4273	4.7711

Table 3: Time measurements

According to United States Department of Commerce, the standard gravitational acceleration is

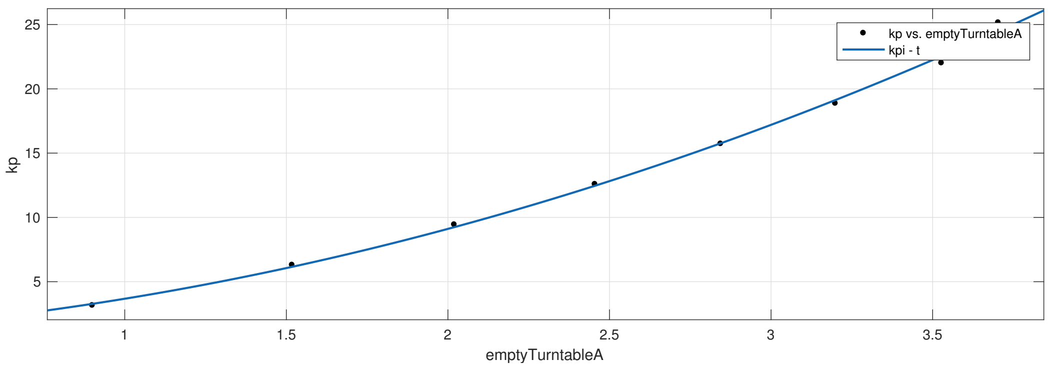
$$g = 9.80665 m/s^2$$

For the Empty turntable,



$$\beta_1 = -0.0970 radius/s^2$$

(with 95% confidence bounds)



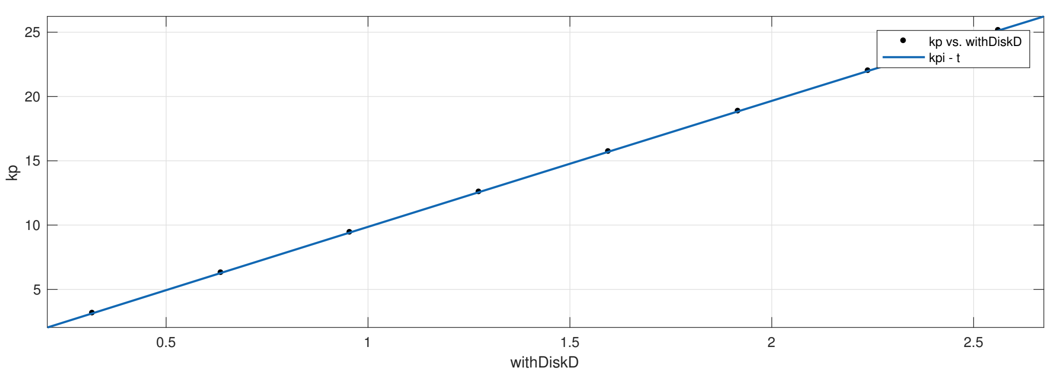
$$\beta_2 = 2.6580 radius/s^2$$

(with 95% confidence bounds)

Thus,

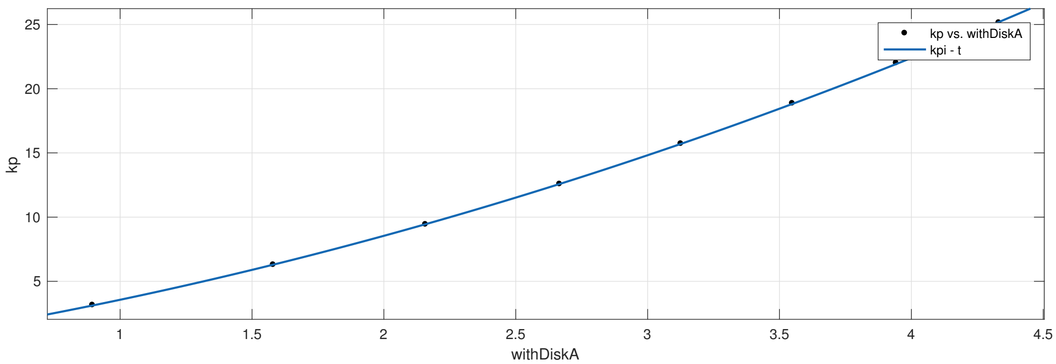
$$I_1 = \frac{59.1g \times 5.0145cm \times (9.80665m/s^2 - 5.0145cm \times (-0.0970)radius/s^2)}{2.6580radius/s^2 - (-0.0970radius/s^2)} = 0.0105kg{\times}m^2$$

For turntable with disk,



$$\beta_1 = -0.0668 radius/s^2$$

(with 95% confidence bounds)

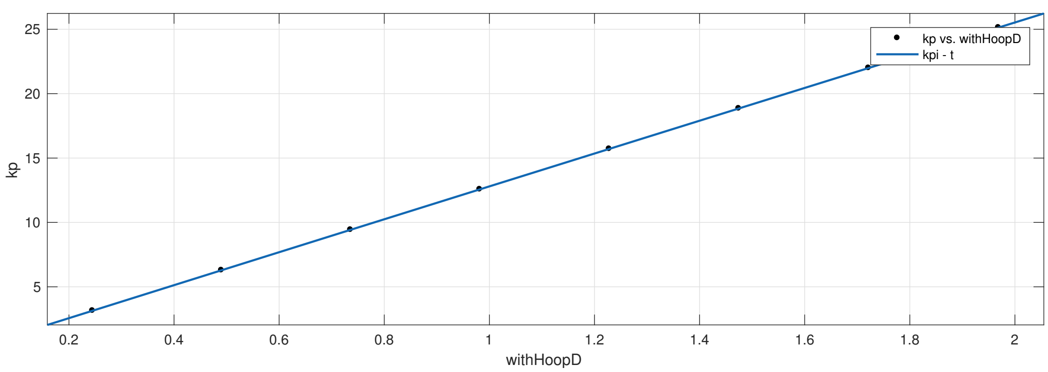


$$\beta_2 = 1.2970 radius/s^2$$

(with 95% confidence bounds)

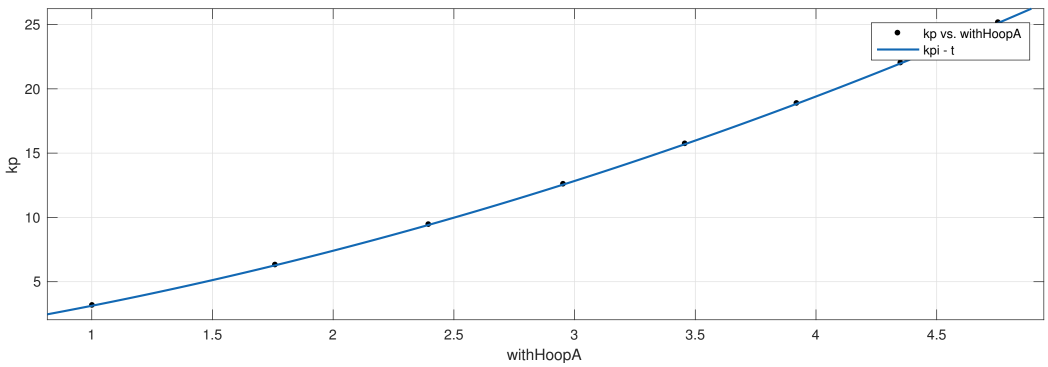
$$I_2 = \frac{59.1g \times 5.0145cm \times (9.80665m/s^2 - 5.0145cm \times (-0.0668)radius/s^2)}{1.2970radius/s^2 - (-0.0668radius/s^2)} = 0.0213kg{\times}m^2$$

For turntable with hoop,



$$\beta_1 = -0.0689 radius/s^2$$

(with 95% confidence bounds)

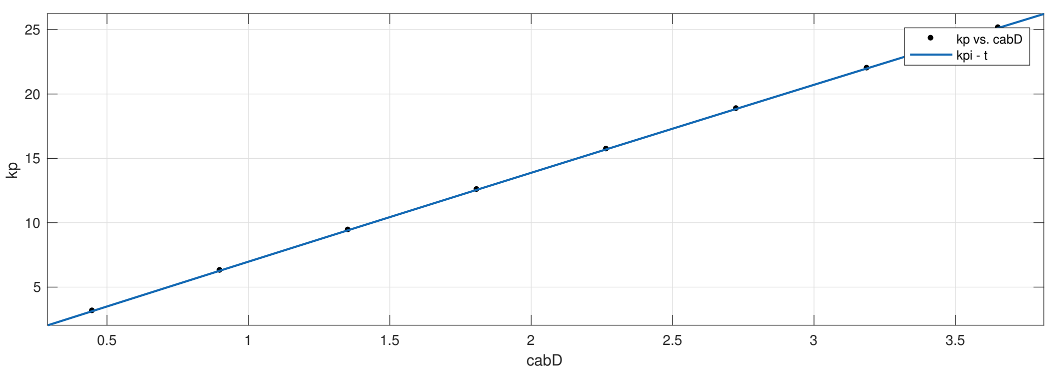


$$\beta_2 = 1.1444 radius/s^2$$

(with 95% confidence bounds)

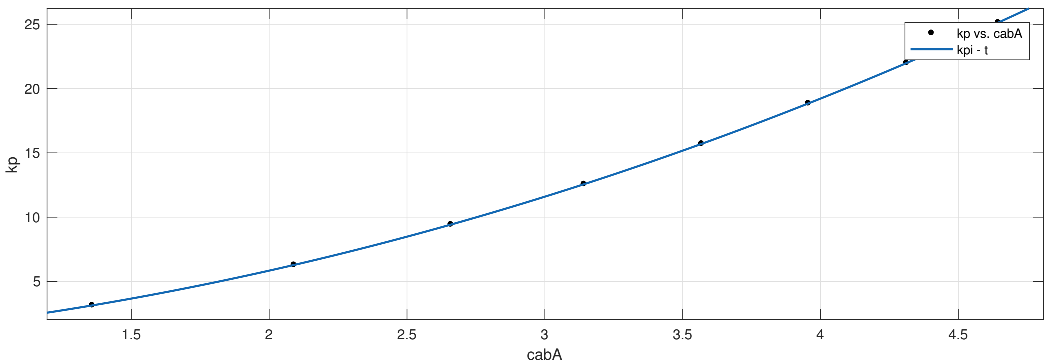
$$I_3 = \frac{59.1g \times 5.0145cm \times (9.80665m/s^2 - 5.0145cm \times (-0.0689)radius/s^2)}{1.1444radius/s^2 - (-0.0689radius/s^2)} = 0.0240kg \times m^2$$

For Cylinder A in hole 1, B in hole 2



$$\beta_1 = -0.0710 radius/s^2$$

(with 95% confidence bounds)

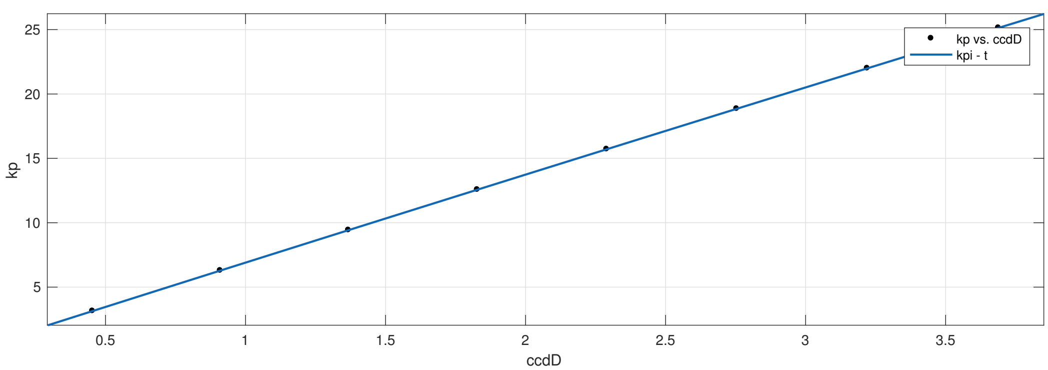


$$\beta_2 = 1.8752 radius/s^2$$

(with 95% confidence bounds)

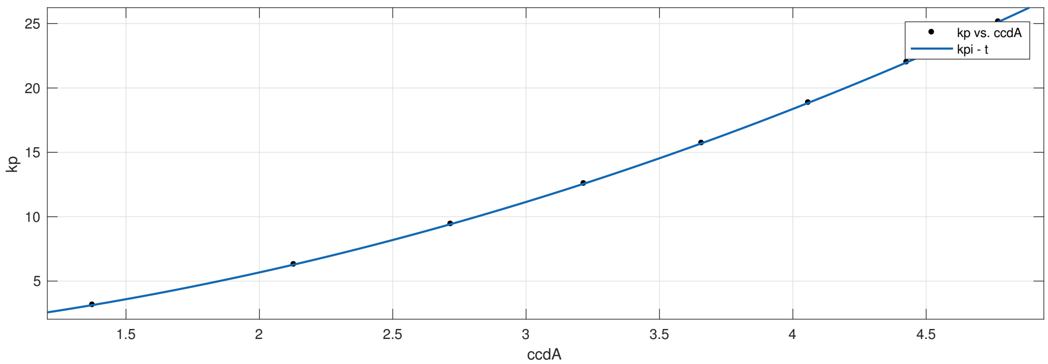
$$I_4 = \frac{59.1g \times 5.0145cm \times (9.80665m/s^2 - 5.0145cm \times (-0.0710)radius/s^2)}{1.8752radius/s^2 - (-0.0710radius/s^2)} = 0.0149kg{\times}m^2$$

For Cylinder A in hole 3, B in hole 4



$$\beta_1 = -0.0661 radius/s^2$$

(with 95% confidence bounds)



$$\beta_2 = 1.7496 radius/s^2$$

(with 95% confidence bounds)

$$I_5 = \frac{59.1g \times 5.0145cm \times (9.80665m/s^2 - 5.0145cm \times (-0.0661)radius/s^2)}{1.7496radius/s^2 - (-0.0661radius/s^2)} = 0.0160kg \times m^2$$

Thus, the moment of inertia for disk is

$$I_{disk} = I_2 - I_1 = 0.0213kg \times m^2 - 0.0105kg \times m^2 = 0.0108kg \times m^2$$

The moment of inertia for hoop is

$$I_{hoop} = I_3 - I_1 = 0.0240kg \times m^2 - 0.0105kg \times m^2 = 0.0135kg \times m^2$$

The moment of inertia for Cylinder A in hole 1 and Cylinder B in hole 2 is

$$I_{A1B2} = I_4 - I_1 = 0.0149kg \times m^2 - 0.0105kg \times m^2 = 0.0044kg \times m^2$$

The moment of inertia for Cylinder A in hole 3 and Cylinder B in hole 4 is

$$I_{A3B4} = I_5 - I_1 = 0.0160kg \times m^2 - 0.0105kg \times m^2 = 0.0055kg \times m^2$$

$$I_{A3B4} - I_{A1B2} = 0.0055kg \times m^2 - 0.0044kg \times m^2 = 0.0011kg \times m^2$$

$$md^2 = 165.8g \times (4.7560cm - 6.0120cm)^2 + 165.8g \times (4.7610cm - 6.0200cm)^2 = 0.0052436kg \times m^2$$

Thus, the relative uncertainty is

$$\frac{0.0055kg \times m^2 - 0.0052436kg \times m^2}{0.0055kg \times m^2} = 4.66\%$$

5 Measurement Uncertainty Analysis

5.1 Uncertainty of instruments

For a single measurement of the mass of the experiment setup, the uncertainty of the measurement instruments are

	Calliper	Electronic balance	Timer
Resolution	0.002cm	0.1g	0.0001s
Relative uncertainty			0.004%

Table 4: Precision of the measurement instruments

5.2 Uncertainty of Calliper Measurements

In order to estimate type–A uncertainty of the period, the standard deviation of the average value is calculated as

$$s_d = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(d_i - \bar{d})^2}$$

Thus, the standard deviation of the average value is calculated and shown below

Object	The standard deviation
Disk [cm]	0.0020
Hoop 1 [cm]	1.1003
Hoop 2 [cm]	1.1017
Cylinder A [cm]	0
Cylinder B [cm]	0
Cone pulley [cm]	0.0075

Table 5: The standard deviation

The uncertainty of Timer is $\pm 0.0001s$ and $\pm 0.004\%$

From the equation to calculate the moment of inertia,

$$I_1 = \frac{mR(g - R\beta_2)}{\beta_2 - \beta_1}$$

$$u_F = \sqrt{(\frac{\partial I}{\partial m})^2(u_m)^2 + (\frac{\partial I}{\partial R})^2(u_R)^2 + (\frac{\partial I}{\partial \beta_1})^2(u_{\beta_1})^2 + (\frac{\partial I}{\partial \beta_2})^2(u_{\beta_2})^2}$$

$$\frac{\partial I}{\partial m} = \frac{R\,(g - \beta_2 R)}{\beta_2 - \beta_1}$$

$$\frac{\partial I}{\partial R} = -\frac{\beta_2 m R' \left(g - \beta_2 R\right)}{\beta_2 - \beta_1}$$

$$\frac{\partial I}{\partial \beta_1} = \frac{m R \left(g - \beta_2 R\right)}{\left(\beta_2 - \beta_1\right)^2}$$

$$\frac{\partial I}{\partial \beta_2} = -\frac{m R R' \left(g - \beta_2 R\right)}{\beta_2 - \beta_1} - \frac{m R \left(g - \beta_2 R\right)}{\left(\beta_2 - \beta_1\right)^2}$$

$$u_m = 0.1g$$

$$u_R = 0.0075cm$$

$$u_{\beta_1} = \sqrt{(0.0001s)^2 + (0.0001 \times 95\%)^2} = 0.000137931$$

$$u_{\beta_1} = \sqrt{(0.0001s)^2 + (0.0001 \times 95\%)^2} = 0.000137931$$

Thus,

$$u_{rF} = 0.4\%$$

6 Conclusions and Discussion

In the experiment, the moment of inertia is found by measuring the angular acceleration of the turntable in different cases.

By plotting and fitting the relation between $k\pi$ and t, we found the β for each situation of the instrument. For we used MATLAB to do the fitting work, the uncertainty of the fitting curve can be easily seen as $\pm 5\%$.

By comparing the calculated value of the difference of I_A and I_B, we can use the experiment to judge whether the parallel axis theorem holds.

The calculated value of I is $md^2 = 165.8g \times (4.7560cm - 6.0120cm)^2 + 165.8g \times (4.7610cm -$

$6.0200cm)^2 = 0.0052436kg \times m^2$ and the value derived from the experiment is $I_{A3B4} - I_{A1B2} = 0.0055kg \times m^2 - 0.0044kg \times m^2 = 0.0011kg \times m^2$. The relative uncertainty is

$$\frac{0.0055kg \times m^2 - 0.0052436kg \times m^2}{0.0055kg \times m^2} = 4.66\%$$

Thus, the parallel axis theorem holds very well.

The g

7 Reference

1. The international system of units (SI) (PDF) (2008 ed.). United States Department of Commerce, NIST Special Publication 330. pp. 29 & 57.

8 Data Sheet