UM-SJTU Joint Institute

Physical Laboratory

VP141

Laboratory Report

Exercise 3

Simple Harmonic Motion: Oscillations in Mechanical Systems

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1 Introduction

In this lab, our goal is to study the phenomenon of simple harmonic oscillation, find the spring constant and effective mass of a spring, and how to use the air track. We will analyze the relationship between the oscillation period and the mass of the oscillator, check whether the oscillation period depends on the the amplitude, and examine the relationship between the maximum speed and the amplitude.

There are various kinds of periodic motion in nature, among which the simplest and the most fundamental one is the simple harmonic motion, where the restoring force is proportional to the displacement from the equilibrium position and as a result, the position of a particle depends on time as the sine (or cosine) function. Discussion of the simple harmonic motion is a basis for studying more complex situations.

1.0.1 Hooke's Law

Within the elastic limit of deformation, the force Fx needed to be applied in order to stretch or compress a spring by the distance x is proportional to that distance, i.e.

$$F_x = kx, \tag{1}$$

where k is a the spring constant that characterizing how easy it is to deform a spring. This constant will be found in the present exercise using a measurement device called the Jolly balance. The linear relation between the force and the deformation, is known as the Hooke's Law. According to Newton's third law of dynamics, the spring exerts a reaction force (called the elastic force) of the same magnitude but opposite direction.

1.1 Equation of Motion of the Simple Harmonic Oscillator

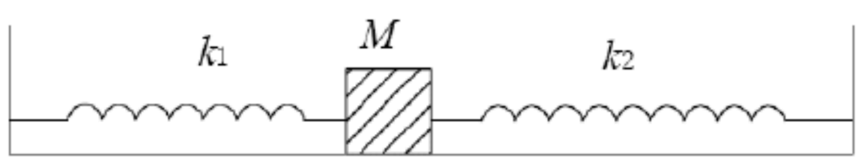

Figure 1: Mass-spring system

As shown in Figure 1, the mass with two springs is placed on an air track, which aims at eliminating the frictional forces. Assuming that the restoring force is the only force acting on mass M, the equation of motion od mass M is

$$M\frac{d^2x}{dt^2} + (k_1 + k_2)x = 0. \tag{2}$$

Hence the general solution to Eq. 2 is

$$x(t) = Acos(\omega_0 t + \phi_0), \tag{3}$$

where $\omega_0 = \sqrt{(k_1 + k_2)/M}$ is the natural angular frequency of the oscillations (determined by the parameters of the system), A is the amplitude, and ϕ_0 is the initial phase (determined by initial conditions). The natural period of oscillation is

$$T = \frac{2\pi}{\omega_0} = 2\pi\sqrt{\frac{M}{k_1 + k_2}}. \tag{4}$$

In this exercise, the relationship mentioned above will be studied.

1.2 Mass of the Spring

We take into the mass of the springs in terms of the so-called effective mass, which is the sum of the mass of the object M and the effective mass of springs m_0. The angular frequency can be expressed as

$$\omega_0 = \sqrt{\frac{k_1 + k_2}{M + m_0}} \tag{5}$$

where m_0 is 1/3 of the actual mass of the spring.

1.3 Mechanical Energy in Harmonic Motion

The elastic potential energy for a spring-mass system is $U = kx^2/2$ and the kinetic energy of an oscillating mass is $K = mv^2/2$. At the equilibrium position ($x = 0$), the speed of the mass is maximum $v = v_{\max}$. At this point the total mechanical energy is equal to maximum kinetic energy $K_{\max}$. On the other hand, at maximum displacement ($x = \pm A$) the mass is instantaneously at rest, and the contribution to the total mechanical energy is due to the potential energy only, which is at its maximum $U_{\max}$. In the absence of non-conservative forces (such as frictional forces or drag forces), the total mechanical energy is conserved and $K_{\max} = U_{\max}$, which implies

$$k = \frac{m(v_{max})^2}{A^2} \tag{6}$$

2 Experimental Setup

The measurement equipment consists of the following elements: springs, Jolly balance, air track, electronic timer, electronic balance and masses.

A: Sliding bar with metric scale;
H: Vernier for reading;
C: Small mirror with a horizontal line in the middle;
D: Fixed glass tube also with a horizontal line in the middle;
G: Knob for ascending and descending the sliding bar;
S: Spring attached to top of the bar A.

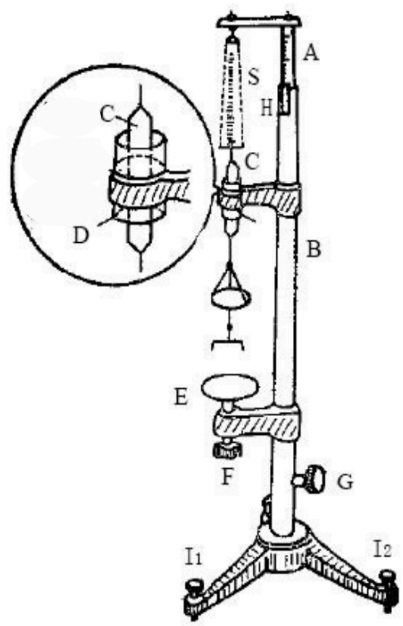

Figure 2: Jolly balance.

In order to measure the spring constant using the Jolly balance, we need to place the small mirror C (see Figure **??**) in the tube D and make three lines coincide: the line on the mirror, the line on the glass tube and its reflection in the mirror. First, without adding any weight on the bottom end of the spring, adjust the knob G and make the three lines coincide. Then read the scale L_1.

Second, add mass m to the bottom of the spring. The spring is stretched and the three lines no longer coincide. Adjust knob G to make them into one line again and read the corresponding number on scale L_2. The spring constant may be then found as

$$k = \frac{mg}{L_2 - L_1} \tag{7}$$

so that we can estimate the spring constant by finding a linear fit to the data using the least squares method.

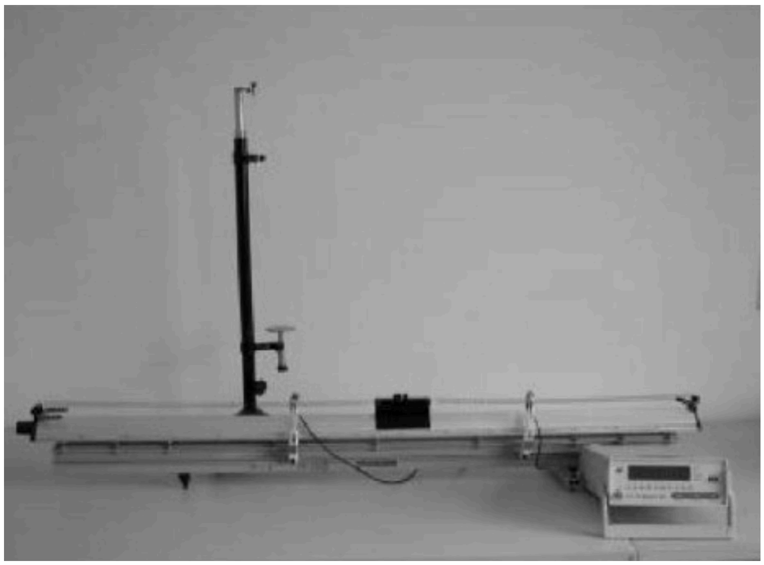

Figure 3: The experimental setup

A photoelectric measuring system consists of two photoelectric gates and an electronic timer. When a shutter on the object blocks the light, the computer will record the time. For period measurements, we use the I-shape shutter.

When measuring the instantaneous speed, the U-shape shutter is required. Δt presents the time interval when the object travels a distance of $\Delta x = (x_{in} + x_{out})/2$. Hence we estimate the speed as $v = \Delta x/\Delta t$.

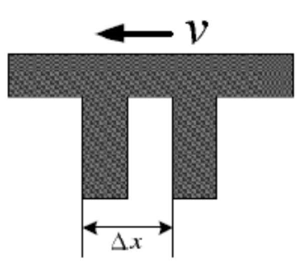

Figure 4: The U-shape shutter

3 Measurement Procedue

3.1 Spring constant

1. Adjust the Jolly balance to vertical and attach the spring Add a 20g preload and adjust knob I_1 and I_2 to make sure the mirror can move freely through the tube. Check if the balance parallel to the spring.

2. Adjust knob G to make the three lines in the tube coincide.

3. Reading the reading on the scale, add mass from 1 to 6 and record L_i in order.

4. Estimate the spring constant k_1 by fitting.

5. Repeat the measurements with spring 2. Calculate k_2.

6. Remove the preload and repeat the measurements for spring 1 and 2 connected. Calculate k_3.

3.2 Relation between oscillation period T and the mass of the oscillator M

1. Adjust the air track so that it's horizontal.

   (1) Turn on the air track and check whether there's blocked holes.

   (2) Place the cart on the track. Adjust the track with the knob on the side with a single one.

2. Horizontal air track

   (1) Attach springs to the sides of the cart and set up the I-shape shutter. Make sure that the photoelectric gate is at the equilibrium position.

   (2) Set the timer into "T" mode. Add weight in order and release it with a caliper. Record the corresponding time intervals for 10 periods.

   (3) Analyze the relation between M and T by plotting a graph.

3. Inclined air track

(1) Add three plastic plates under the air track every time. Repeat the steps in 2.

(2) Analyze the relation between M and T by plotting a graph.

3.3 Relation between the oscillation period and the amplitude

1. Keep the mass of the cart unchanged and change the amplitude (choose 6 different values). The amplitude is about 5.0/ 10.0/ 15.0/ ... /30.0 cm.

2. Apply linear fit to the data and comment on the relation between the oscillation period T and the amplitude A based on the correlation coefficient γ.

3.4 Relation between the maximum speed and the amplitude

1. Keep the mass of the cart unchanged and change the amplitude from $5.0\ to\ 30.0cm$ with gap of $5.0cm$.

2. Measure $v_{\max}$ for different amplitudes.

3.5 Mass measurement

1. Adjust the electronic balance every time before you use it. The level bubble should be in the center of the circle.

2. Add weights according to a fixed order. Weigh the cart with the I-shape shutter and with the U-shape shutter. Measure the mass of spring 1 and spring 2.

3. Record the data only after the circular symbol on the scales display disappears.

4 Caution

- Do not stretch the spring over its elastic limit, otherwise the spring will not return to its original shape.

- When using the Jolly balance, the mirror should be moving freely in the glass tube without any friction. When adding weight, hold the tray steady to avoid errors due to vibrations.

- Please use tweezers to move the weights around.

- Make sure that no air holes on the air tracks are blocked.
- Avoid scratching the cart. Do not move the cart when pressed against the air track.

5 Result

5.1 Measurements for spring constant

First we get the raw length measurement data in Table 1.

spring 1 [cm] $\pm$ 0.01 [cm]		spring 2 [cm] $\pm$ 0.01 [cm]		serial [cm] $\pm$ 0.01 [cm]	
L_0	0.00	L_0	0.00	L_0	20.00
L_1	2.00	L_1	2.01	L_1	23.96
L_2	3.96	L_2	4.00	L_2	27.94
L_3	5.96	L_3	6.03	L_3	31.90
L_4	8.02	L_4	8.06	L_4	36.40
L_5	10.04	L_5	10.08	L_5	39.94
L_6	12.02	L_6	12.10	L_6	44.02

Table 1: Spring constant measurement data

Then we can calculate the change amount of the spring length by $\Delta L_i = L_i - L_0$ and get Table 2.

spring 1 [cm] $\pm$ 0.01 [cm]		spring 2 [cm] $\pm$ 0.01 [cm]		serial [cm] $\pm$ 0.01 [cm]	
ΔL_1	2.00	ΔL_1	2.01	ΔL_1	3.96
ΔL_2	3.96	ΔL_2	4.00	ΔL_2	7.94
ΔL_3	5.96	ΔL_3	6.03	ΔL_3	11.90
ΔL_4	8.02	ΔL_4	8.06	ΔL_4	16.40
ΔL_5	10.04	ΔL_5	10.08	ΔL_5	19.94
ΔL_6	12.02	ΔL_6	12.10	ΔL_6	24.02

Table 2: Calculated Spring constant measurement data

We get the mass of the weight object for every ΔL_i in Table 3.

Since the acceleration due to gravity in Shanghai is $9.794 m/s^2$, we calculated the weights of each weight object from its mass, as shown in Table 4.

m [g] ± 0.01 [g]	
1	4.65
2	9.32
3	14.17
4	18.99
5	23.80
6	28.51

Table 3: Mass measurement data.

F [N] ± 0.0001 [N]	
1	0.0455
2	0.0913
3	0.1388
4	0.1860
5	0.2331
6	0.2792

Table 4: Weight measurement data.

For Spring 1, we can have its length change data versus the force affected on it data.

No.	length [cm] ± 0.01 [cm]	F [N] ± 0.0001 [N]
ΔL_1	2.00	0.0455
ΔL_2	3.96	0.0913
ΔL_3	5.96	0.1388
ΔL_4	8.02	0.1860
ΔL_5	10.04	0.2331
ΔL_6	12.02	0.2792

Table 5: ΔL vs. Force for Spring 1

Then we use MATLAB fit tools to find k_1

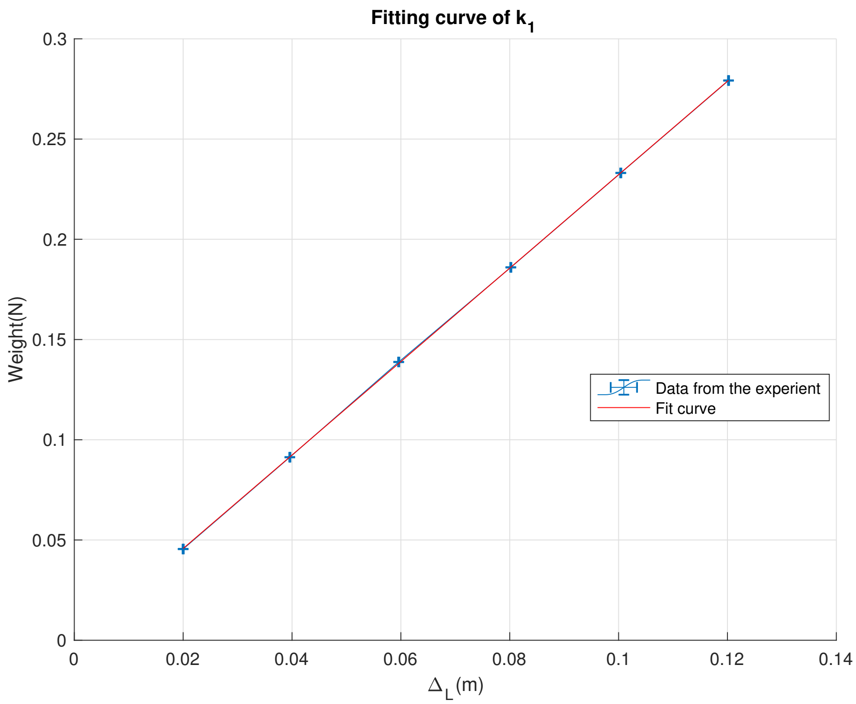

Figure 5: Fit curve of spring 1

$$k_1 = 2.3311 \pm 0.013[N/m]$$

$$u_{k_1,r} = 0.55\%$$

Goodness of fit:

SSE: 6.091e-07
R-square: 1
Adjusted R-square: 1
RMSE: 0.0003902

For Spring 2, we can have its length change data versus the force affected on it data.

No.	length [cm] $\pm$ 0.01 [cm]	F [N] $\pm$ 0.0001 [N]
ΔL_1	2.01	0.0455
ΔL_2	4.00	0.0913
ΔL_3	6.03	0.1388
ΔL_4	8.06	0.1860
ΔL_5	10.08	0.2331
ΔL_6	12.10	0.2792

Table 6: ΔL vs. Force for Spring 2

Then we use MATLAB fit tools to find k_2

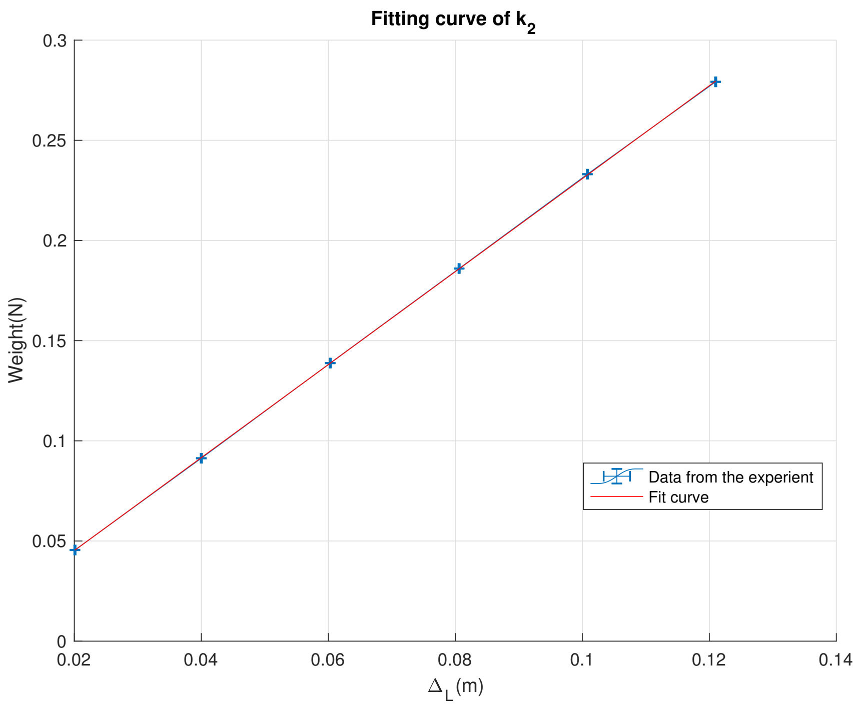

Figure 6: Fit curve of spring 2

$$k_2 = 2.3206 \pm 0.0105[N/m]$$

$$u_{k_2,r} = 0.45\%$$

Goodness of fit:

SSE: 4.305e-07

R-square: 1

Adjusted R-square: 1

RMSE: 0.0003281

For serial, we can have its length change data versus the force affected on it data.

No.	length [cm] $\pm$ 0.01 [cm]	F [N] $\pm$ 0.0001 [N]
ΔL_1	3.96	0.0455
ΔL_2	7.94	0.0913
ΔL_3	11.90	0.1388
ΔL_4	16.40	0.1860
ΔL_5	19.94	0.2331
ΔL_6	24.02	0.2792

Table 7: ΔL vs. Force for serial

Then we use MATLAB fit tools to find k_3 , in other words, the k of the serial string.

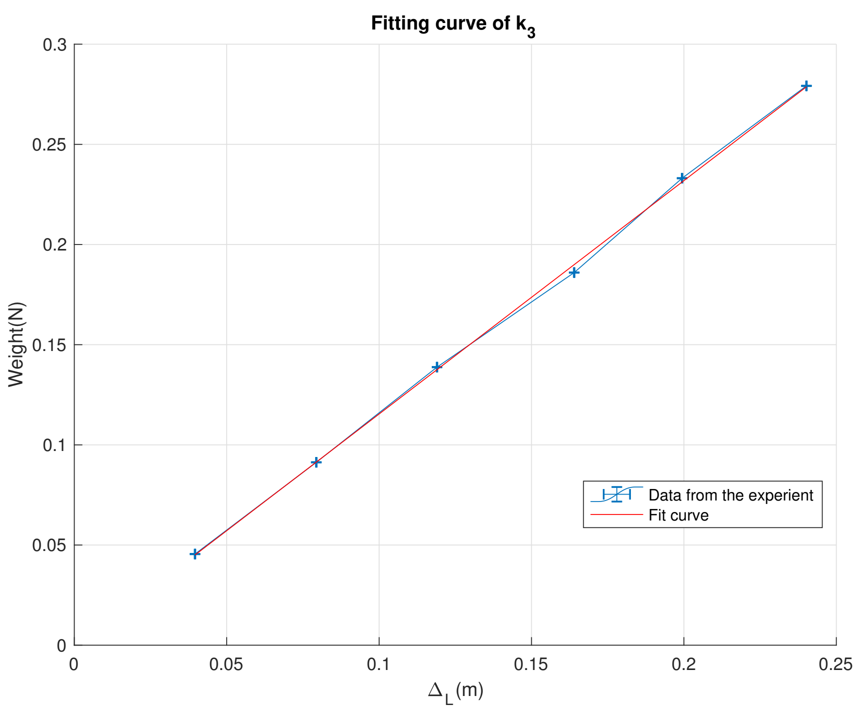

Figure 7: Fit curve of spring serial

$$k_3 = 1.165 \pm 0.0380[N/m]$$

$$u_{k_3,r} = 3.26\%$$

Goodness of fit:

SSE: 2.144e-05
R-square: 0.9994
Adjusted R-square: 0.9993
RMSE: 0.002315

By theory, we can calculate k_3, i.e. the k of the spring serial by

$$k_{3,theory} = \frac{k_1 \cdot k_2}{k_1 + k_2} = 1.1629$$

Compared with $k_3 = 1.1649$ from the experiment,

$$u_{k_{3,theory},k_3} = \frac{1.1649 - 1.1629}{1.1629} \cdot 100\% = 0.17\%$$

The theory data is close to the experiment data

5.2 Relation between the period T and the mass M

In this part, we investigate in finding the relation between the period T and the mass M. First, we need to collect the mass data needed later for the fit. Shown in Table 8.

Mass of the object [g] $\pm$ 0.01 [g]	
object with I-shape m_{I-obj}	176.87
object with U-shape m_{U-obj}	188.16
spring 1 m_{spr1}	11.23
spring 2 m_{spr2}	10.54
Mass of the object [g] $\pm$ 0.015 [g]	
equivalent mass $M_{0,I} = m_{I-obj} + \frac{1}{3}m_{spr1} + \frac{1}{3}m_{spr2}$	184.13
equivalent mass $M_{0,U} = m_{U-obj} + \frac{1}{3}m_{spr1} + \frac{1}{3}m_{spr2}$	195.42

Table 8: Mass measurement data

The the period T measured in different situations are shown in Table 9.

10 periods [ms] $\pm$ 0.1 [ms]					
horizontal		incline 1		incline 2	
m_1	12560.4	m_1	12557.8	m_1	12560.9
m_2	12718.2	m_2	12711.1	m_2	12722.0
m_3	12878.4	m_3	12872.2	m_3	12885.9
m_4	13021.2	m_4	13030.3	m_4	13034.7
m_5	13189.2	m_5	13186.0	m_5	13179.2
m_6	13326.5	m_6	13333.3	m_6	13336.9

Table 9: Measurement data for the T vs. M relation

Recall we have the mass data. We need to add the mass of object with I-shape to each m_i Thus, we can have,

m [g] $\pm$ 0.01 [g]	
1	181.52
2	186.19
3	191.04
4	195.86
5	200.67
6	205.38

Table 10: Mass measurement data with I-shape and the object.

Thus, for horizontal situation, we can get the following table.

mass [g] $\pm$ 0.01 [g]	T^2 [s^2] $\pm$ 0.00001 [s^2]
$m_1 = 181.52$	1.57763
$m_2 = 186.19$	1.61752
$m_3 = 191.04$	1.65853
$m_4 = 195.86$	1.69551
$m_5 = 200.67$	1.73954
$m_6 = 205.38$	1.77595

Table 11: T^2 vs. M for horizontal situation

Then we use MATLAB fit tools to find $slope_h$

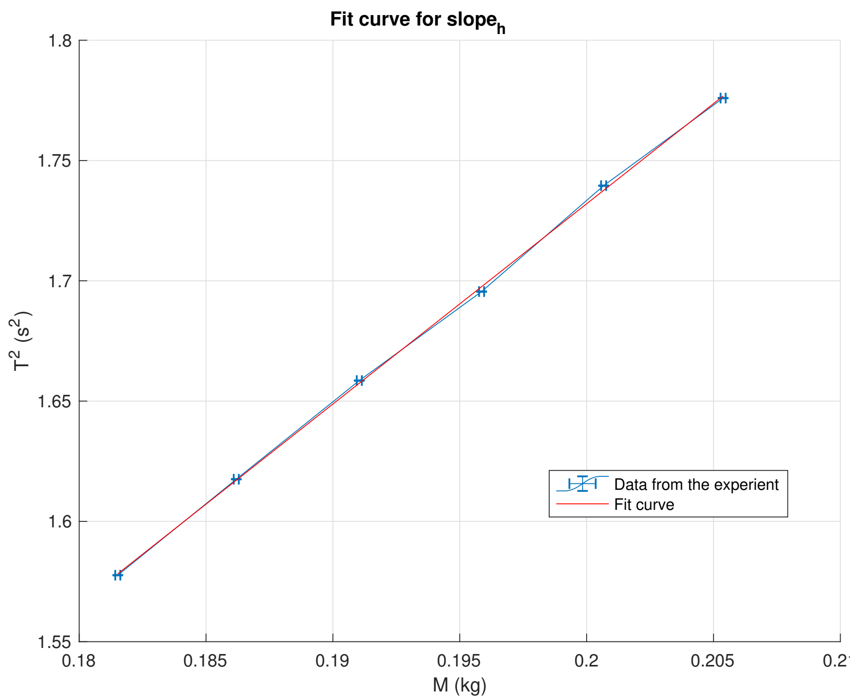

Figure 8: Fit curve of T^2 vs. M for horizontal situation

$$slope_h = 8.3233 \pm 0.2235[s^2/kg]$$

$$u_{slope_h,r} = 2.69\%$$

Goodness of fit:

SSE: 1.042e-05

R-square: 0.9996

Adjusted R-square: 0.9995

RMSE: 0.001614

For incline 1 situation, we can get the following table.

mass [g] $\pm$ 0.01 [g]	T^2 $[s^2]$ $\pm$ 0.00001 $[s^2]$
$m_1 = 181.52$	1.57698
$m_2 = 186.19$	1.61572
$m_3 = 191.04$	1.65693
$m_4 = 195.86$	1.69788
$m_5 = 200.67$	1.73870
$m_6 = 205.38$	1.77776

Table 12: T^2 vs. M for incline 1 situation

Then we use MATLAB fit tools to find $slope_{i1}$, the slope for incline 1.

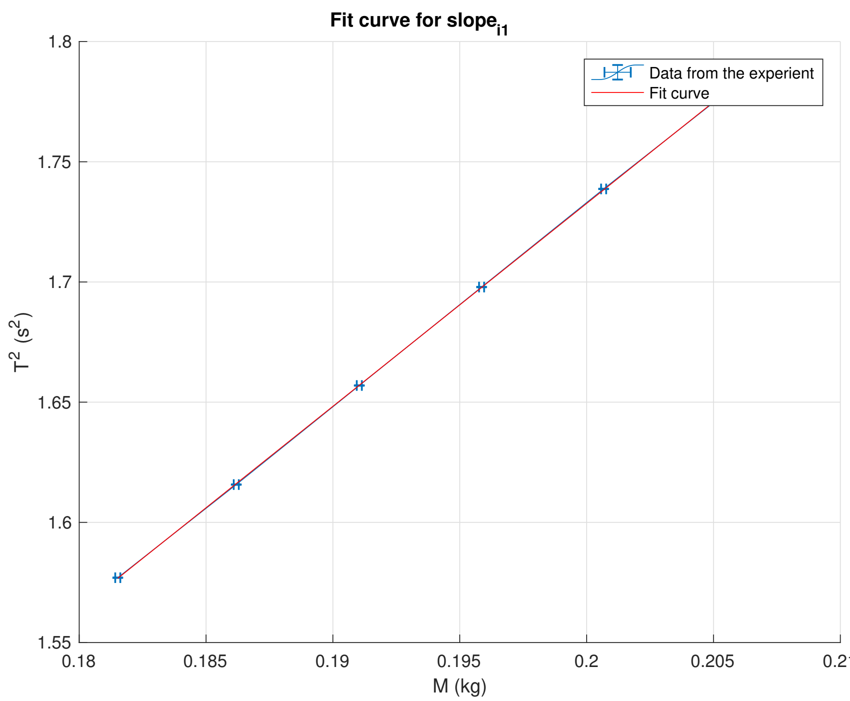

Figure 9: Fit curve of T^2 vs. M for incline 1 situation

$$slope_{i1} = 8.4380 \pm 0.0500[s^2/kg]$$

$$u_{slope_{i1},r} = 0.59\%$$

Goodness of fit:

SSE: 5.119e-07

R-square: 1

Adjusted R-square: 1

RMSE: 0.0003577

For incline 2 situation, we can get the following table.

mass [g] $\pm$ 0.01 [g]	T^2 [s^2] $\pm$ 0.00001 [s^2]
$m_1 = 181.52$	1.57776
$m_2 = 186.19$	1.61849
$m_3 = 191.04$	1.66046
$m_4 = 195.86$	1.69903
$m_5 = 200.67$	1.73691
$m_6 = 205.38$	1.77872

Table 13: T^2 vs. M for incline 2 situation

Then we use MATLAB fit tools to find $slope_{i2}$, the slope for incline 2.

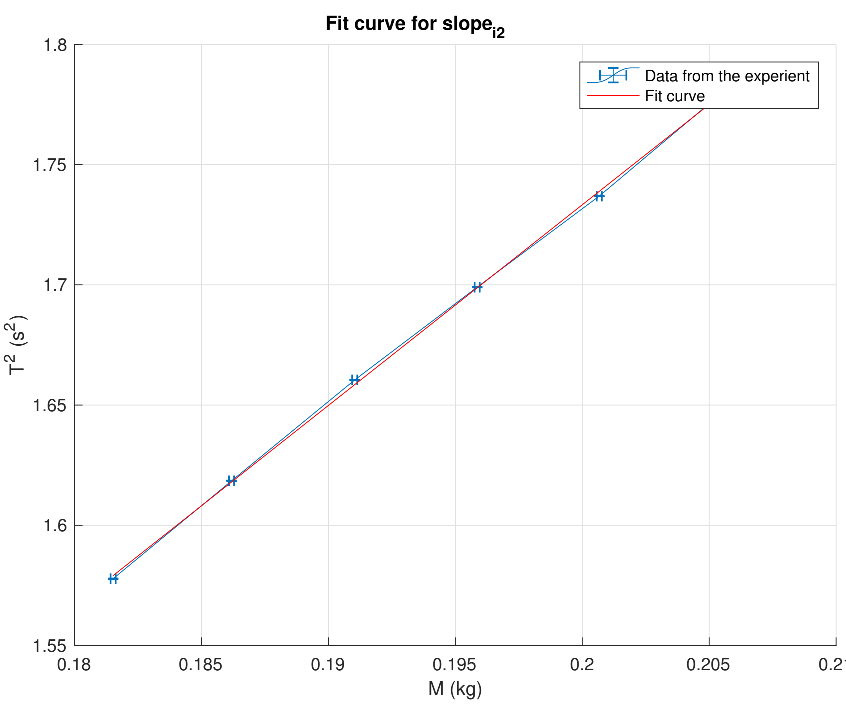

Figure 10: Fit curve of T^2 vs. M for incline 2 situation

$$slope_{i2} = 8.3467 \pm 0.2185[s^2/kg]$$

$$u_{slope_{i2},r} = 2.62\%$$

Goodness of fit:

SSE: 9.958e-06
R-square: 0.9996
Adjusted R-square: 0.9996
RMSE: 0.001578

5.3 Relation between period T and amplitude A

We get the following raw data from the experiment.

	A [cm] $\pm$ 0.1 [cm]	ten periods [ms] $\pm$ 0.1 [ms]
1	5.0	12566.3
2	10.0	12565.2
3	15.0	12560.9
4	20.0	12562.6
5	25.0	12562.8
6	30.0	12563.7

Table 14: ten periods T vs. A

Then we can drive it into T vs. A

	A [m] $\pm$ 0.001 [m]	T [s] $\pm$ 0.00001 [s]
1	0.050	1.25663
2	0.100	1.25652
3	0.150	1.25609
4	0.200	1.25626
5	0.250	1.25628
6	0.300	1.25637

Table 15: T vs. A

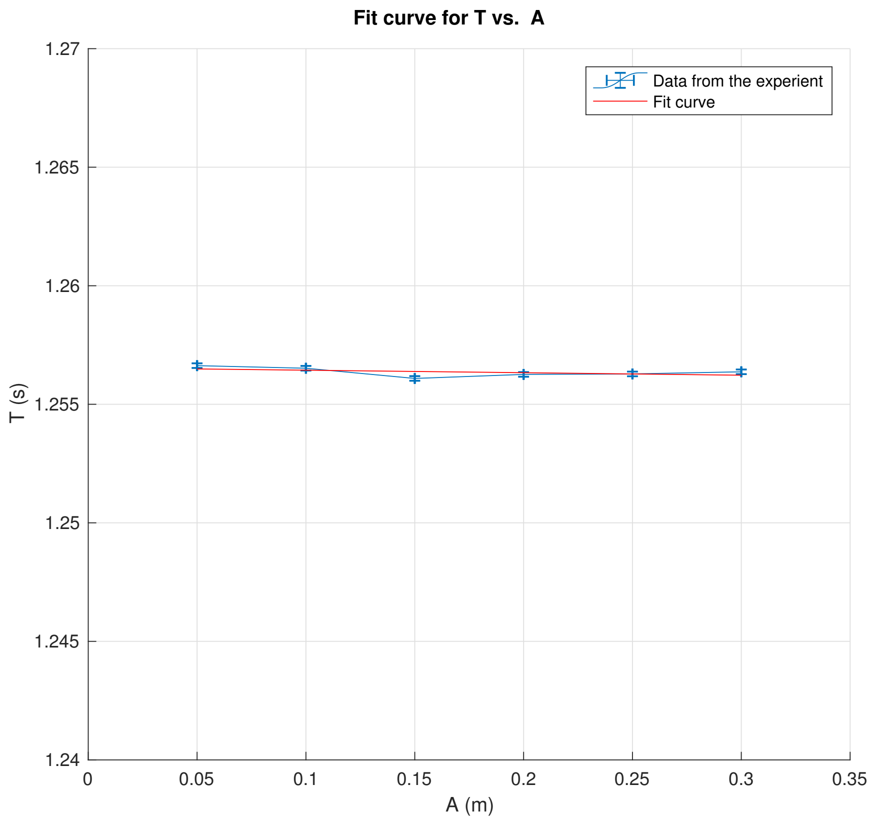

Figure 11: Fit curve of T vs. A

$$k = -0.001057 \pm 0.0025[s/m]$$

$$u_{k,r} = 236.52\%$$

Goodness of fit:

SSE: 1.39e-07

R-square: 0.2602

Adjusted R-square: 0.07529

RMSE: 0.0001864

For the relative uncertainty is quite large, and R-square is 0.2602, which is not very well, we can say that T and A do not have a clear linear relation.

5.4 Relation between $v_{\max}$ and A

x_{in} [cm] $\pm$ 0.002 [cm]	x_{out} [cm] $\pm$ 0.002 [cm]
0.472	1.542
0.482	1.540
0.480	1.530

Table 16: Data of x_{in} and x_{out} for U-shape

$$\begin{aligned} x_{in} &= \frac{1}{3}\sum_{i=1}^{3} x_{in,i} = \frac{0.472+0.482+0.480}{3} \\ &= (0.4780 \pm 0.003) \quad cm \\ u_{r,x_{in}} &= 0.63\% \\ x_{out} &= \frac{1}{3}\sum_{i=1}^{3} x_{out,i} = \frac{1.542+1.540+1.530}{3} \\ &= (1.5373 \pm 0.004) \quad cm \\ u_{r,x_{out}} &= 0.26\% \end{aligned}$$

$$\Delta L = (x_{in} + x_{out})/2 = 1.0076 \pm 0.003 \quad cm$$

A [cm] $\pm$ 0.1 [cm]		Δt [ms] $\pm$ 0.01 [ms]
1	5.0	42.60
2	10.0	20.56
3	15.0	13.57
4	20.0	10.34
5	25.0	8.25
6	30.0	6.90

Table 17: Data for the ${v_{\max}}^2$ vs. A^2 relation.

From $\Delta L = v_{max} \cdot \Delta t$, We can calculate out v_{max}

A [cm] $\pm$ 0.1 [cm]		Δt [ms] $\pm$ 0.01 [ms]	v_{max} [m/s] $\pm$ 0.0001 [m/s]
1	5.0	42.60	0.2365
2	10.0	20.56	0.4901
3	15.0	13.57	0.7425
4	20.0	10.34	0.9745
5	25.0	8.25	1.2213
6	30.0	6.90	1.4603

Table 18: Calculated Data for the ${v_{\mathrm{max}}}^2$ vs. A^2 relation.

Finally, we get the relation between${v_{\mathrm{max}}}^2$ and A^2 .

A^2 $[cm^2]$ $\pm$ 0.1 $[cm^2]$		v_{max}^2 $[m^2/s^2]$ $\pm$ 0.0001 $[m^2/s^2]$
1	25.0	0.0559
2	100.0	0.2402
3	225.0	0.5513
4	400.0	0.9496
5	625.0	1.4917
6	900.0	2.1324

Table 19: Calculated Data for the ${v_{\mathrm{max}}}^2$ vs. A^2 relation.

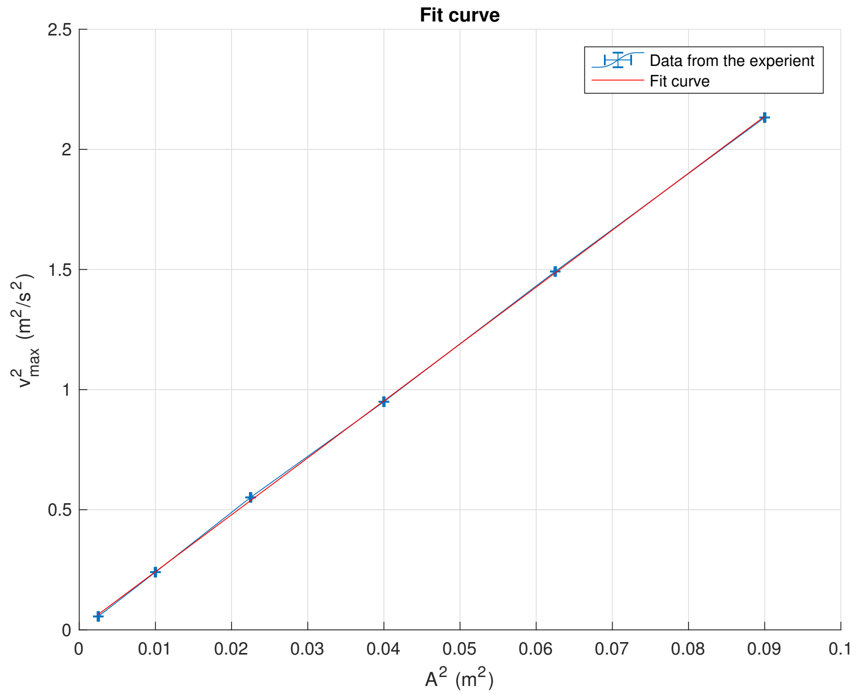

Figure 12: Fit curve of ${v_{\max}}^2$ vs. A^2

$$k = 23.6964 \pm 0.3300$$

$$u_{k,r} = 1.39\%$$

Goodness of fit:

SSE: 0.0003151
R-square: 0.9999
Adjusted R-square: 0.9999
RMSE: 0.008876

Thus, we can find that there is a linear relation between ${v_{\max}}^2$ and A^2.

6 Measurement Uncertainty Analysis

6.1 Devices Precision

The precision of each devices are shown in Table 20.

Devices	Precision	Unit
Jolly balance	0.01	[cm]
Ruler	0.1	[cm]
Timer for periods	0.1	[ms]
Timer for maximum speed	0.01	[ms]
Calliper	0.02	[mm]
Electronic scale	0.01	[g]

Table 20: Devices precision

6.2 Uncertainty in Measurement for Spring Constant

For measuring L_i, there is no type-A uncertainty. Thus,

$$u_{L_0} = 0.01 \quad [cm]$$

$$u_{L_i} = 0.01 \quad [cm]$$

$$u_{L_i,r} = \frac{0.01}{2.01} \cdot 100\% = 0.49 \quad [cm]$$

For measuring m_i, there is no type-A uncertainty. Thus,

$$u_m = 0.01 \quad [g]$$

$$u_{\Delta L} = \sqrt{(\frac{\partial \Delta L}{\partial L_i})^2 \cdot (u_{L_i})^2 + (\frac{\partial \Delta L}{\partial L_0})^2 \cdot (u_{L_0})^2} \approx 1.4 \times 10^{-4} \quad m$$

The uncertainty of the gravity force measurement u_W of $W = mg$ is

$$\frac{\partial W}{\partial m} = g = 9.794 \quad kg \cdot m/s^2$$

$$u_m = \Delta_{dev} = 0.01 \quad g$$

$$u_W = \sqrt{(\frac{\partial W}{\partial m})^2 \cdot (u_m)^2} = \sqrt{(9.794)^2 \times (1 \times 10^{-5})^2} \approx 1 \times 10^{-4} \quad N$$

$$k_1 = 2.3311 \pm 0.013 N/m, \quad u_{k_1,r} = 0.55\%$$

$$k_2 = 2.3206 \pm 0.0105 N/m, \quad u_{k_2,r} = 0.45\%$$

$$k_3 = 1.165 \pm 0.0380 N/m, \quad u_{k_3,r} = 3.26\%$$

For $k_{3,theory}$ calculated from theory,

$$k_{3,theory} = \frac{k_1 \cdot k_2}{k_1 + k_2} = 1.1629$$

$$u_{k_{3,theory}} = \sqrt{(\frac{\partial k_{3,theory}}{\partial k_1})^2 \cdot (u_{k_1})^2 + (\frac{\partial k_{3,theory}}{\partial k_2})^2 \cdot (u_{k_2})^2} = 0.0006 \quad N/m$$

$$u_{k_{3,theory},r} = \frac{0.0006}{1.1629} \cdot 100\% = 0.05\%$$

6.3 Uncertainty of the slope of T^2 *vs.* M

The uncertainty of one period is $u_T/10 = 1 \times 10^{-5} s$ and the uncertainty of mass is

$$M = m_{objI} + \frac{1}{3}m_{spr1} + \frac{1}{3}m_{spr2} + m_i,$$

$$u_M = \sqrt{(\frac{\partial M}{\partial m_{objI}})^2(u_{m_{objI}})^2 + (\frac{\partial M}{\partial m_{spr1}})^2(u_{m_{spr1}})^2 + (\frac{\partial M}{\partial m_{spr2}})^2(u_{m_{spr2}})^2 + (\frac{\partial M}{\partial m_i})^2(u_{m_i})^2}$$

$$= \sqrt{(1)^2(u_{m_{objI}})^2 + (\frac{1}{3})^2(u_{m_{spr1}})^2 + (\frac{1}{3})^2(u_{m_{spr2}})^2 + (1)^2(u_{m_i})^2}$$

As data provided $m_{objI} = (176.87 \pm 0.01) \times 10^{-3} kg$, $m_{spr1} = (11.23 \pm 0.01) \times 10^{-3} kg$, $m_{spr2} = (10.54 \pm 0.01) \times 10^{-3} kg$ and $m_1 = (4.65 \pm 0.01) \times 10^{-3} kg$

$$u_M = \sqrt{0.00001^2 + 0.00001^2/9 + 0.00001^2/9 + 0.00001^2} = 0.0000153$$
$$= 1.5 \times 10^{-5} kg.$$

For T^2,

$$u_{T^2} = \sqrt{(2T)^2(u_T)^2} = 2Tu_T$$

From the formula $T = 2\pi\sqrt{\frac{M}{k}}$ we can drive the slope of T^2 *vs.* M is

$$slope = \frac{T^2}{M} = \frac{4\pi^2}{k}$$

where k is the effective spring constant. In this experiment, the effective spring constant can be calculated by

$$F = k_1\Delta x + k_2\Delta x = k_{eff}\Delta x$$

$$u_{slope} = \sqrt{(\frac{\partial slope}{\partial slope_{hor}})^2 \cdot (u_{slope_{hor}})^2 + (\frac{\partial slope}{\partial slope_{inc1}})^2 \cdot (u_{sloinc1})^2 + (\frac{\partial slope}{\partial slope_{inc2}})^2 \cdot (u_{slope_{inc2}})^2}$$
$$= \sqrt{(\frac{1}{3})^2 \cdot (0.22)^2 + (\frac{1}{3})^2 \cdot (0.05)^2 + (\frac{1}{3})^2 \cdot (0.22)^2} = 0.18s^2/kg$$

$$u_{r,slope} = \frac{u_{slope}}{\bar{slope}} \times 100\% = \frac{0.18}{8.37} \times 100\% = 2.15\%$$

Comparing the calculated data with the experiment data,

$$\Delta slope = 8.37 - 8.45 = -0.08s^2/kg$$

$$\Delta_r slope = \frac{8.37 - 8.45}{8.45} \times 100\% = 0.95\%$$

For the deviation between theory calculation is not very large, it indicates that the experiment is quite success.

6.4 Uncertainty in the $v_{\max}^2$ *vs.* A^2 relation

For analyzing the uncertainty of Δx, First, investigate the uncertainty of x_{in} and x_{out}. The uncertainty of type-B of a calliper is

$$\Delta_{x,B} = \Delta dev = 0.02 \times 10^m$$

Take the mean value of the three time measurements of the distance. To determine type-A uncertainty, the standard deviation of the average value is

$$s_{\overline{x_{in}}} = \sqrt{\frac{1}{n(n-1)}\sum_{i=1}^{n}(x_{in,i} - \overline{x_{in}})^2}$$
$$s_{\overline{x_{in}}} = 5.3 \times 10^{-3}m$$

$$u_{x_{in}} = \sqrt{\Delta_{x_{in},A}^2 + \Delta_{x_{in},B}^2} = \sqrt{(0.02 \times 10^{-3})^2 + (5.3 \times 10^{-3})^2} = 5.3 \times 10^{-3}m$$

$$u_{r,x_{in}} = \frac{u_{x_{in}}}{\overline{x_{in}}} \times 100\% = 1.8\%$$

Similarly, we can calculate x_{out} that

$$\Delta_{x_{out},A} = t_{0.95} \cdot s_{\overline{x_{out}}} = 0.0287 \times 10^{-3} m$$
$$\Delta_{x,B} = 0.02 \times 10^{-3} m$$
$$u_{x_{out}} = 0.03 \times 10^{-3} m, \quad u_{r,x_{out}} = 0.2\%$$
$$x_{out} = (15.41 \pm 0.03) \times 10^{-3} m, \quad u_{r,x_{out}} = 0.2\%.$$

Then we can calculate the propagated uncertainty of Δx

$$\frac{\partial \Delta x}{\partial x_{in}} = \frac{\partial \Delta x}{\partial x_{out}} = \frac{1}{2}$$

$$u_{\Delta x} = \sqrt{(\frac{\partial \Delta x}{\partial x_{in}})^2 (u_{x_{in}})^2 + (\frac{\partial \Delta x}{\partial x_{out}})^2 (u_{x_{out}})^2}$$
$$= 0.04 \times 10^{-3} m,$$

$$u_{r,\Delta x} = \frac{u_{\Delta x}}{\Delta x} \times 100\% = 0.4\%$$
$$\Delta x = (9.95 \pm 0.04) \times 10^{-3} m, \quad u_{r,\Delta x} = 0.4\%$$

For the Uncertainty of the maximum speed $v_{\max}$ Then we can calculate the propagated uncertainty of $v_{\max} = \Delta x / \Delta t$. The partial derivatives are

$$\frac{\partial v_{\max}}{\partial \Delta x} = \frac{1}{\Delta t}.$$

$$\frac{\partial v_{\max}}{\partial \Delta t} = -\frac{\Delta x}{(\Delta t)^2}.$$

$$u_{v_{\max}} = \sqrt{(\frac{\partial v_{\max}}{\partial \Delta x})^2 (u_{\Delta x})^2 + (\frac{\partial v_{\max}}{\partial \Delta t})^2 (u_{\Delta t})^2}$$
$$= \sqrt{(\frac{1}{\Delta t})^2 (u_{\Delta x})^2 + (\frac{\Delta x}{(\Delta t)^2})^2 (u_{\Delta t})^2}$$

7 Conclusion and Discussion

In this lab, we have studied the harmonic oscillation and the relations between T, M, k and $v_{\max}$. Now we analyze the results.

7.1 Spring Constant

spring 1	2.3311 ± 0.013[N/m]
spring 2	2.3206 ± 0.0105[N/m]
spring series	1.165 ± 0.0380[N/m]

Table 21: The spring constant

For the relative uncertainty are all very small, the experiment in this part is very accurate.

By theory, we can calculate k_3, i.e. the k of the spring serial by

$$k_{3,theory} = \frac{k_1 \cdot k_2}{k_1 + k_2} = 1.1629$$

Compared with $k_3 = 1.1649$ from the experiment,

$$u_{k_{3,theory},k_3} = \frac{1.1649 - 1.1629}{1.1629} \cdot 100\% = 0.17\%$$

The theory data is close to the experiment data.

The accurate experimental results prove that Hooke's Law.

7.2 Relation between the period T and the mass M

From curve fitting, we find that T^2 is linearly dependent with M The slope is shown below:

horizontal	8.3233 ± 0.2235 [s2/kg]
incline 1	8.4380 ± 0.0500 [s2/kg]
incline 2	8.3467 ± 0.2185 [s2/kg]

Table 22: Slope

We find the slopes of three cases are very close to each other, so the ratio between T^2 and M are independent of the incline degree of the air track. We see all the relative errors are very small, thus the result in this part is very accurate.

7.3 The relation between T and A

From the failed fitting curve, we find there is no clear linear relation between T and A. Thus, we can conclude that T is independent of A.

7.4 Relation between $v_{\max}$ and A

From the fitting curve we find that $v_{\max}^2$ and A^2 is linearly dependent. And

$$k = 23.6964 \pm 0.3300$$

$$u_{k,r} = 1.39\%$$

The relative uncertainty of the experimental ratio is 1.39%, which is moderate. Thus the result is quite precise. Still, $v_{\max}^2$ is more easily affected by the friction force, so the relative uncertainty is quite large.

8 Reference

1. Qin Tian, Zeng Ming, Cao Jianjun, Han Xugen, Feng Yaming, Mateusz Krzy- zosiak, Physics Laboratory (Vp141/Vp241) Student Handbook Introduction to Mea- surement Data Analysis.

2. Qin Tian, Zheng Huan, Li Yingyu, Mateusz Krzyzosiak Physics Laboratory Vp141 Exercise 3 Simple Harmonic Motion: Oscillations in Mechanical Systems.

9 Data Sheet