UM-SJTU Joint Institute

Physical Laboratory

VP141

Laboratory Report

Exercise 4

Measurement of the Speed of Sound

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1 Introduction

The objective of the exercise is to study several methods of measuring the speed of sound in air: the resonance method, the phase comparison method, and the time difference method, including successive difference method in measurement data processing.

Sound is a mechanical wave that propagates through a compressible medium. It's a longitudinal wave since the direction of vibrations of the medium is the same as the direction of propagation. Sound with the frequency higher than $20,000\ Hz$ is called *ultrasound*, which is chosen as the signal source in this experiment because its wavelength is short enough to measure the speed precisely.

The phase speed v, the frequency f and the length λ of a wave are related by the formula

$$v = \lambda f. \tag{1}$$

For motion with constant speed v along a straight line, we have

$$v = \frac{L}{t}, \tag{2}$$

where L is the distance travelled over time t.

2 Measurement Method

The experimental setup consists of a signal source, two piezoelectric transducers S_1 and S_2, and oscilloscope arranged as shown in Figure 1.

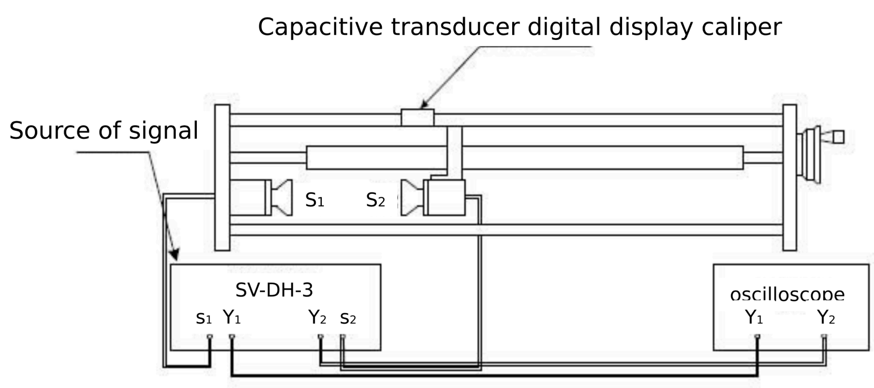

Figure 1: Experimental setup

2.0.1 Resonance method

The elements S_1 and S_2 are the wave source and the receiver (also reflector), respectively, placed a distance L from each other. If they are arranged parallel to each other, the sound wave is reflected. If

$$L = n\frac{\lambda}{2}, \tag{3}$$

where $n = 1, 2, \cdots$, *i.e.* the distance is a multiple of half-wavelength, standing waves will form, and maximum output power will be observed in the oscillograph (Figure 2). The distance between two successive maxima ($L_{i+1} - L_i$) is always $\lambda/2$. After the position corresponding to each maximum is measured, it is easy to find the wavelength and then the speed of sound by using Eq. 1. The frequency f is displayed directly on the signal generator.

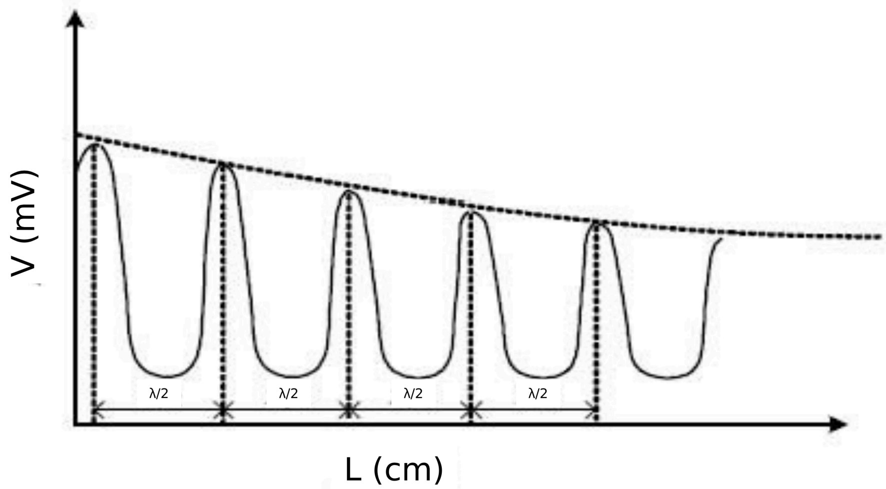

Figure 2: Relation between the signal voltage and the distance

2.0.2 Phase-comparison method

If the phase of the wave at two points on the wave propagation direction is equal, then the distance between these points L has to be a multiple of the wavelength, *i.e.*

$$L = n\lambda,$$

where $n = 1, 2, \cdots$. The experimental setup for the phase comparison method is the same as in the previous method (Figure 1). Lissajous figures are used to identify the values of L. Lissajous figures (or Lissajous curves) are trajectories of a particle that moves in a plane so that *i.e.* it moves in a harmonic motion independently along two perpendicular directions (for example the axes x and y of a Cartesian coordinate system), so that $\mathbf{r}(t) = (A_x cos(\omega_x t + \phi_x),\ A_y cos(\omega_y t + \phi_y))$. When the two superimposed harmonic motions have identical frequency $\omega x = \omega y$ and phase difference $|\phi x - \phi y| = n\pi$, where $n = 0, 1, 2, \cdots$, the Lissajous figure will show as a straight line. For other values of the phase difference the figures will have an elliptical shape.

2.0.3 Time-difference method

When an ultrasonic pulse signal emitted by S_1 arrives at S_2, it is received and returned back to the processor. By contrasting the original signal with the received one, one can measure the time needed for the sound to travel from S_1 to S_2 over a distance of L. When the values of L and t are known, the phase speed of sound can be found from Eq. 2.

2.0.4 Successive difference method

The successive difference method is an effective method to increase the accuracy of the average value calculated from a series of measurement data. In this experiment, the usual method of calculating the average value, illustrated by the formula

$$\frac{\bar{\lambda}}{2} = \frac{[(L_1 - L_0) + (L_2 - L_1) + \cdots + (L_n - L_{n-1})]}{n} = \frac{L_n - L_0}{n}, \tag{4}$$

will be modified, because as Eq. 5 shows, the average value of the wavelength is determined only by the first and the last value, L_0 and L_n.

A modification of the formula by rearranging terms as

$$n\frac{\bar{\lambda}}{2} = \frac{\Sigma_{i=1}^{n}(L_{n+i} - L_i)}{n}, \tag{5}$$

produces more accurate results, as each value contributes to the final result.

3 Measurement Procedure

3.1 Resonance method

1. Set the initial distance between S_1 and S_2 at about $1cm$.

2. Turn on the signal source and the oscilloscope. Then set the following options on the panel of the signal source.
   (a) Choose *Continuous* wave for *Method*.
   (b) Adjust Signal Strength until a $10V$ peak voltage is observed on the oscilloscope.
   (c) Adjust Signal Frequency between 34.5 kHz and 40 kHz until the peak-to-peak voltage reaches its maximum. Record the frequency.
3. Increase L gradually by moving S_2, and observe the output voltage of S_2 on the oscilloscope. Record the position of S_2 as L_2 when the output voltage reaches an maximum.
4. Repeat step 3 to record 20 values of L_2 and calculate v.

3.2 Phase-comparison method

1. Use Lissajous figures to observe the phase difference between the transmitted and the received signals. Move S_2 and record the position when the Lissajous figure becomes a straight line with the same slope.
2. Repeat step 1 to collect 12 sets of data. Use the successive difference method to process the data and calculate v.

3.3 Time-difference method (liquid)

Since the pulse wave causes damped oscillations at the receiver, there will be significant interference if S_1 and S_2 resonate. The resonance can be observed on the oscilloscope.

1. Choose Pulse Wave for Method on the panel of the signal source.
2. Adjust the frequency to 100 Hz and the width to $500\mu s$.
3. Use the cursor function of the oscilloscope to measure the time and the distance between the the starting points of neighboring periods.
4. Record the distance L_1 and the time t_1.
5. Move S_2 to another position and repeat step 3. Record L_i and t_i, $i = 2, 3, 4, \cdots$.
6. Repeat step 4 to collect 12 pairs of L_i and t_i. Plot the L_i $vs.$ t_i graph and use computer software to find a linear fit to the data. The slope of the line is the speed v_{water}.

4 Result

The frequency we set on the signal generator is $f = 34498 \pm 1$Hz. The room temperature is $23 \pm 1°C$.

4.1 Measurements for resonance method

The data are presented in Table 1 with the calculation for $L_{10+i} - L_i$.

$L_i[\times 10^{-3}m] \pm [0.01 \times 10^{-3}m]$		$L_i[\times 10^{-3}m] \pm [\times 10^{-3}m]$		$L_{10+i} - L_i[\times 10^{-3}m]$	
1	29.37	11	80.44	1	51.07
2	34.49	12	85.48	2	50.99
3	39.60	13	90.01	3	50.41
4	44.74	14	95.65	4	50.91
5	49.83	15	100.82	5	50.99
6	54.90	16	105.93	6	51.03
7	60.06	17	111.12	7	51.06
8	65.24	18	116.20	8	50.96
9	70.37	19	121.32	9	50.95
10	75.38	20	126.39	10	51.01

Table 1: Data for the resonance method

The average value of ΔL can be calculated from the results presented in Table 1 as

$$\overline{\Delta L} = \frac{1}{10}\sum_{i=1}^{10} \Delta L_i = (50.94 \pm 0.05) \times 10^{-3}m, \quad u_{r,\Delta} = 0.10\%$$

Then the wavelength λ can be calculated by

$$\lambda = \frac{2\Delta L}{n} = \frac{2 \times 50.94 \times 10^{-3}}{10} = (10.19 \pm 0.01) \times 10^{-3}m, \quad u_{r,\lambda} = 0.10\%.$$

The speed of sound in air v is

$$v = \lambda f = (10.19 \pm 0.01) \times 10^{-3}m \cdot 34498Hz = 351.45 \pm 0.4m/s, \quad u_{r,v} = 0.11\%.$$

4.2 Measurements for phase comparison method

The experiment data are shown in Table 2 with the calculation for $L_{6+i} - L_i$.

$L_i[\times 10^{-3}m] \pm [0.01 \times 10^{-3}m]$		$L_i[\times 10^{-3}m] \pm [0.01 \times 10^{-3}m]$		$L_{6+i} - L_i[\times 10^{-3}m]$	
1	23.57	7	84.39	1	60.82
2	33.78	8	94.51	2	60.73
3	43.97	9	104.62	3	60.65
4	54.17	10	114.76	4	60.59
5	64.11	11	124.72	5	60.61
6	74.24	12	134.92	6	60.68

Table 2: Data for the phase comparison method

The average value of ΔL is calculated based on the results presented in Table 2 as

$$\overline{\Delta L} = \frac{1}{6}\sum_{i=1}^{6} \Delta L_i = (60.68 \pm 0.8) \times 10^{-3}m, \quad u_{r,\Delta L} = 3\%.$$

Hence, the wavelength λ can be calculated as

$$\lambda = \frac{\Delta L}{n} = \frac{60.68 \times 10^{-3}}{6} = (10.11 \pm 0.13) \times 10^{-3}m, \quad u_{r,\lambda} = 1.3\%.$$

The speed of sound in air v is

$$v = \lambda f = (10.11 \pm 0.13) \times 10^{-3}m \cdot 34498Hz = 348.889 \pm 0.4m/s, \quad u_{r,v} = 0.11\%.$$

4.3 Measurements for time difference method (liquid)

We obtain the speed of sound in water from Table 3 by linear fitting (See Figure 3).

	$t_i[\times 10^{-6}s] \pm 0.2[\times 10^{-6}s]$	$L_i[\times 10^{-3}m] \pm 0.01[\times 10^{-3}m]$
1	113.8	160.00
2	120.6	170.00
3	127.4	180.00
4	134.2	190.00
5	141.2	200.00
6	148.2	210.00
7	153.8	220.00
8	160.6	230.00
9	167.4	240.00
10	174.2	250.00
11	181.0	260.00
12	186.8	270.00

Table 3: Data for Figure 3

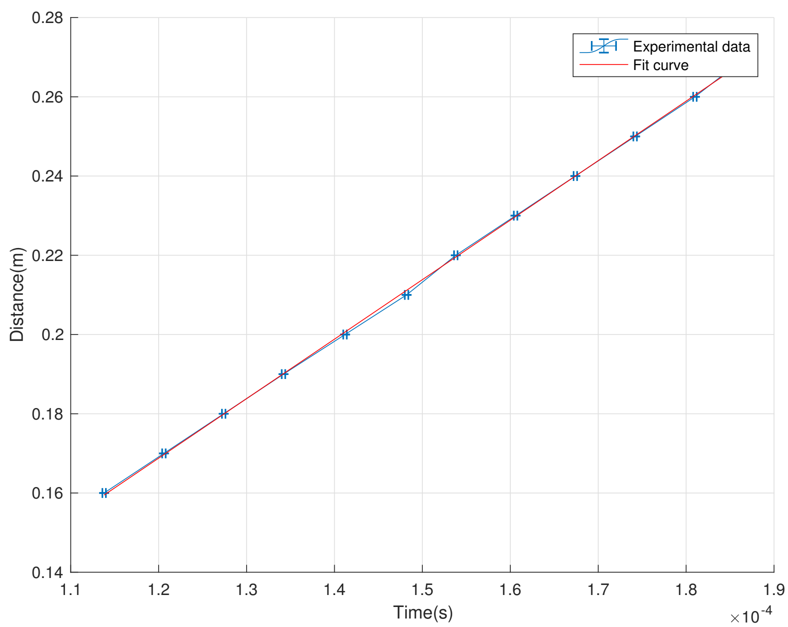

Figure 3: Fitting curve for L $vs.$ t.

Goodness of fit:

SSE: 3.331e-06

R-square: 0.9998

Adjusted R-square: 0.9997

RMSE: 0.0005771

From the slope of the fitting curve we can deduce the v_{water}

$$v_{water} = 1502 \pm 16 m/s.$$

5 Measurement Uncertainty Analysis

5.1 Uncertainty for resonance method

For type-B uncertainty of ΔL,

$$\frac{\partial \Delta L}{\partial L_{10+i}} = 1$$

$$\frac{\partial \Delta L}{\partial L_i} = -1$$

$$u_{L_{10+i}} = u_{L_i} = \Delta_{dev} = 1 \times 10^{-5} m$$

$$\begin{aligned}\Delta_{\Delta L,B} &= \sqrt{(\frac{\partial \Delta L}{\partial L_{10+i}})^2 \cdot (u_{L_{10+i}})^2 + (\frac{\partial \Delta L}{\partial L_i})^2 \cdot (u_{L_i})^2} \\ &= 0.014 \times 10^{-3} m.\end{aligned}$$

For estimate type-A uncertainty, the standard deviation of the average value can be caluculated as

$$s_{\overline{\Delta L}} = \sqrt{\frac{1}{n(n-1)} \sum_{i=1}^{n} (\Delta L_i - \overline{\Delta L})^2}.$$

From data of Table 1 we get that $s_{\overline{\Delta L}} = 0.01919 \times 10^{-3} m$. Considering $t_{0.95} = 2.26$ for $n = 10$, the type-A uncertainty can be estimated as $\Delta_{\Delta L,A} = 2.26 \times 0.01919 \times 10^{-3} = 0.0434 \times 10^{-3} m$.

Thus the combined uncertainty is

$$
\begin{aligned}
u_{\Delta L} &= \sqrt{\Delta^2_{\Delta L,A} + \Delta^2_{\Delta L,B}} \\
&= \sqrt{(0.0434 \times 10^{-3})^2 + (0.014 \times 10^{-3})^2} \\
&= 0.045 \times 10^{-3} m
\end{aligned}
$$

$$
\begin{aligned}
u_{r,\Delta L} &= \frac{u_{\Delta L}}{\overline{\Delta L}} \times 100\% = \frac{0.045 \times 10^{-3}}{50.94 \times 10^{-3}} \times 100\% \\
&= 0.09\%
\end{aligned}
$$

$$\overline{\Delta L} = (50.94 \pm 0.05) \times 10^{-3} m, u_{r,\Delta L} = 0.10\%$$

In the same way, we obtain the uncertainty of wavelength $\lambda = 2\Delta L/n$

$$\frac{\partial \lambda}{\partial \Delta L} = \frac{2}{10} = \frac{1}{5}$$

$$u_{\Delta L} = 0.05 \times 10^{-3} m$$

$$u_\lambda = \sqrt{(\frac{\partial \lambda}{\partial \Delta L})^2 \cdot (u_{\Delta L})^2} = \frac{1}{5} \times 0.05 \times 10^{-3} = 0.01 \times 10^{-3} m$$

$$u_{r,\lambda} = \frac{u_\lambda}{\overline{\lambda}} \times 100\% = \frac{0.01 \times 10^{-3}}{10.03 \times 10^{-3}} = 0.10\%$$

$$\lambda = (10.03 \pm 0.01) \times 10^{-3} m, \quad u_{r,\lambda} = 0.10\%$$

and the uncertainty of the spped of sound in air $v = \lambda f$

$$\frac{\partial v}{\partial \lambda} = f, \frac{\partial v}{\partial f} = \lambda$$

$$u_\lambda = 0.01 \times 10^{-3} m, u_f = 1Hz$$

$$u_v = \sqrt{(\frac{\partial v}{\partial \lambda})^2 \cdot (u_\lambda)^2 + (\frac{\partial v}{\partial f})^2 \cdot (u_f)^2} = \sqrt{(f)^2 \cdot (u_\lambda)^2 + (\lambda)^2 \cdot (u_f)^2}$$
$$= \sqrt{(35000)^2 \cdot (0.01 \times 10^{-3})^2 + (10.03 \times 10^{-3})^2 \cdot (1)^2} \approx 0.4 m/s$$

$$u_{r,v} = \frac{u_v}{\overline{v}} \times 100\% = \frac{0.4}{351.05} = 0.10\%$$
$$v = 351.05 \pm 0.4 m/s, \quad u_{r,v} = 0.10\%$$

5.2 Uncertainty for phase comparison method

To determine the type-B uncertainty of ΔL,

$$\frac{\partial \Delta L}{\partial L_{6+i}} = 1, \quad \frac{\partial \Delta L}{\partial L_i} = -1$$
$$u_{L_{6+i}} = u_{L_i} = \Delta_{dev} = 1 \times 10^{-5} m$$

$$\Delta_{\Delta L,B} = \sqrt{(\frac{\partial \Delta L}{\partial L_{6+i}})^2 \cdot (u_{L_{6+i}})^2 + (\frac{\partial \Delta L}{\partial L_i})^2 \cdot (u_{L_i})^2}$$
$$= \sqrt{2} \times 1 \times 10^{-5} \approx 0.014 \times 10^{-3} m.$$

For type-A uncertainty, the standard deviation of the average value is caluculated as

$$s_{\overline{\Delta L}} = \sqrt{\frac{1}{n(n-1)} \sum_{i=1}^{n} (\Delta L_i - \overline{\Delta L})^2}.$$

Uisng the data from Table 2 we find that $s_{\overline{\Delta L}} \approx 0.6485 \times 10^{-3} m$. Considering $t_{0.95} = 2.57$ for $n = 6$, the type-A uncertainty is estimated as $\Delta_{\Delta L,A} = 2.57 \times 0.6485 \times 10^{-3} \approx 1.67 \times 10^{-3} m$.

Thus the combined uncertainty is

$$u_{\Delta L} = \sqrt{\Delta_{\Delta L,A}^2 + \Delta_{\Delta L,B}^2} = \sqrt{(1.67 \times 10^{-3})^2 + (0.014 \times 10^{-3})^2} \approx 1.7 \times 10^{-3} m$$

$$u_{r,\Delta L} = \frac{u_{\Delta L}}{\overline{\Delta L}} \times 100\% = \frac{1.7 \times 10^{-3}}{59.84 \times 10^{-3}} \times 100\% \approx 3\%$$

$$\overline{\Delta L} = (59.84 \pm 1.7) \times 10^{-3} m, \quad u_{r,\Delta L} = 3\%$$

Similarly, we obtain the uncertainty of wavelength $\lambda = \Delta L/n$

$$\frac{\partial \lambda}{\partial \Delta L} = \frac{1}{6}$$

$$u_{\Delta L} = 1.7 \times 10^{-3} m$$

$$u_{\lambda} = \sqrt{(\frac{\partial \lambda}{\partial \Delta L})^2 \cdot (u_{\Delta L})^2} = \frac{1}{6} \times 1.7 \times 10^{-3} = 0.3 \times 10^{-3} m$$

$$u_{r,\lambda} = \frac{u_{\lambda}}{\overline{\lambda}} \times 100\% = \frac{0.3 \times 10^{-3}}{9.973 \times 10^{-3}} = 3\%$$

$$\lambda = (9.973 \pm 0.3) \times 10^{-3} m, \quad u_{r,\lambda} = 3\%$$

and the uncertainty of the spped of sound in air $v = \lambda f$

$$\frac{\partial v}{\partial \lambda} = f, \quad \frac{\partial v}{\partial f} = \lambda$$

$$u_{\lambda} = 0.3 \times 10^{-3} m, \quad u_f = 1Hz$$

$$u_v = \sqrt{(\frac{\partial v}{\partial \lambda})^2 \cdot (u_{\lambda})^2 + (\frac{\partial v}{\partial f})^2 \cdot (u_f)^2} = \sqrt{(f)^2 \cdot (u_{\lambda})^2 + (\lambda)^2 \cdot (u_f)^2},$$
$$= \sqrt{(35000)^2 \cdot (0.3 \times 10^{-3})^2 + (9.97 \times 10^{-3})^2 \cdot (1)^2} \approx 10 m/s,$$

$$u_{r,v} = \frac{u_v}{\overline{v}} \times 100\% = \frac{10}{348.95} = 3\%$$

$$v = 348.95 \pm 10 m/s, \quad u_{r,v} = 3\%$$

5.3 Uncertainty for the time difference method (liquid)

From MATLAB fitting toolbox, the slope of L $vs.$ t, we obtain the information of the fit.

$$v_{water} = 1502 \pm 20m/s.$$

where $\pm 20m/s$ is the 0.95-confidence uncertainty, with $t_{0.95} = 2.20$ when $n = 12$. THe relative uncertainty is that

$$u_{r,v} = \frac{u_v}{\bar{v}} \times 100\% = \frac{20}{1502} = 1.5\%.$$

Hence the speed of sound in water is

$$v_{water} = 1502 \pm 20m/s, \quad u_{r,v} = 1.5\%.$$

6 Conclusion and Discussion

For the resonance method and the comparison method

In the experiment the speed of sound in the air was measured through two ways: the resonance method and the phase comparison method. From the two means, we have

$$\begin{aligned} v &= 351.05 \pm 0.4m/s, \quad u_{r,v} = 0.10\%; \\ v &= 348.95 \pm 10m/s, \quad u_{r,v} = 3\%, \end{aligned} \tag{6}$$

respectively.

From the result after analyzing, we can conclude that the results of the resonance method has higher precision but not enough accuracy. On the other hand, the results of the phase comparison method has higher accuracy but not enough precision.

The reason account for the lack of preciseness may be the easiness to determine the time we get the distance which shares the same length of half wave length for Resonance Method, or one wave length for Phase comparison Method.

For Resonance Method, it's not very easy to observe whether the wave on oscilloscope became the maximum amplitude. For Phase-comparison Method, it is easy to observe whether the Lissajous figure becomes a straight line with a positive slope in each period.

Thus we managed to get the data that are close to each other because of the easy observation. The very uncertainty may due to random error in this experiment.

The reason for the preciseness may be that we manage to obverse the place of first resonance point, though the resonance curve is not flat enough, which may make the observation difficult. The little relative error may come from random error in this experiment and the difficulty to obverse.

In conclusion, the result produced in the experiment is relatively precise and accurate. Through the Resonance Method and the Phase-comparison Method, we can get a very precise result. Through the Time-difference Method, we can get a very accurate result, though it may only be an accident in this experiment. In order to compare these four methods, we should do the experiment for several times to reduce random error. Moreover, we are introduced the Successive Difference Method, which is a more effective way to reduce error. We can use this method in other experiments.

7 Reference

1. QIN Tian, ZENG Ming, CAO Jianjun, HAN Xugen, FENG Yaming, Mateusz KRZYZOSIAK, Physics Laboratory (Vp141/Vp241) Student Handbook Introduction to Measurement Data Analysis.

2. QIN Tian, CAO Jianjun, YI Hankun, WU Ziyou, Mateusz KRZYZOSIAK Physics Laboratory Vp141 Exercise 4 Measurement of the Speed of Sound.

8 Data Sheet