PHYSICS LABORATORY I

VP141

Exercise IV

Measurement of the Speed of Sound

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1 Introduction

The objective of the exercise is to study several methods of measuring the speed of sound in air: the resonance method, the phase comparison method, and the time difference method, including successive difference method in measurement data processing.

Sound is a mechanical wave that propagates through a compressible medium. It's a longitudinal wave since the direction of vibrations of the medium is the same as the direction of propagation. Sound with the frequency higher than $20,000$ Hz is called *ultrasound*, which is chosen as the signal source in this experiment because its wavelength is short enough to measure the speed precisely.

The phase speed v, the frequency f and the length λ of a wave are related by the formula

$$v = \lambda f. \tag{1}$$

For motion with constant speed v along a straight line, we have

$$v = \frac{L}{t}, \tag{2}$$

where L is the distance travelled over time t.

2 Experiment Setup

2.1 Apparatus

The experimental setup consists of a signal source, two piezoelectric transducers S_1 and S_2 , and oscilloscope arranged as shown in Figure 1.

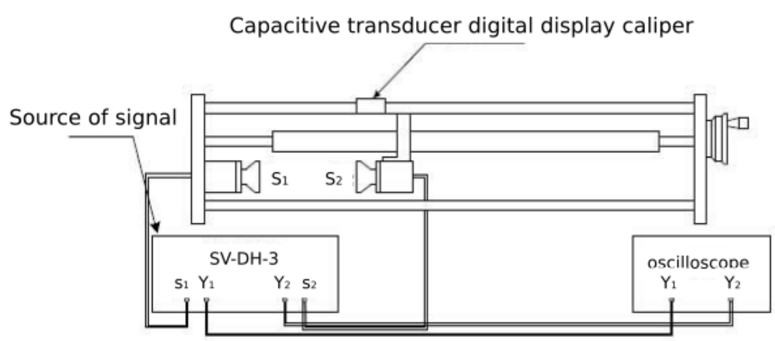

Figure 1: Experimental setup

Devices	Precision	Unit
Thermometer	1	[°C]
Signal generator for frequency	0.001	[kHz]
Calliper	0.01	[mm]
Oscilloscope for time	0.2	[μs]

Table 1: Devices precision

2.2 Devices precision

The precisions of the devices are shown in Table 1.

2.3 Measurement method

2.3.1 Resonance method

The elements S_1 and S_2 are the wave source and the receiver (also reflector), respectively, placed a distance L from each other. If they are arranged parallel to each other, the sound wave is reflected. If

$$L = n\frac{\lambda}{2}, \tag{3}$$

where $n = 1, 2, \cdots$, *i.e.* the distance is a multiple of half-wavelength, standing waves will form, and maximum output power will be observed in the oscillograph (Figure 2). The distance between two successive maxima ($L_{i+1} - L_i$) is always $\lambda/2$. After the position corresponding to each maximum is measured, it is easy to find the wavelength and then the speed of sound by using Eq. 1. The frequency f is displayed directly on the signal generator.

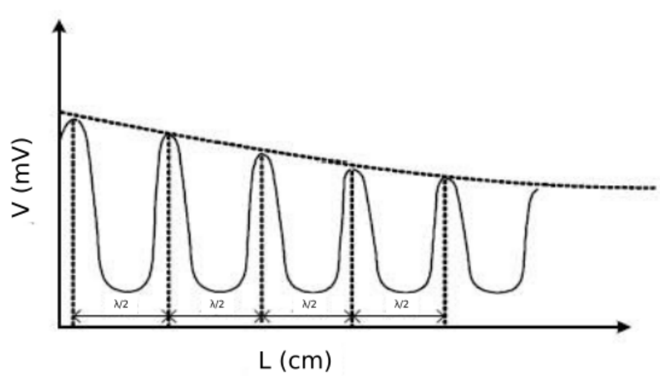

Figure 2: Relation between the signal voltage and the distance

2.3.2 Phase-comparison method

If the phase of the wave at two points on the wave propagation direction is equal, then the distance between these points L has to be a multiple of the wavelength, *i.e.*

$$L = n\lambda,$$

where $n = 1, 2, \cdots$. The experimental setup for the phase comparison method is the same as in the previous method (Figure 1). Lissajous figures are used to identify the values of L. Lissajous figures (or Lissajous curves) are trajectories of a particle that moves in a plane so that *i.e.* it moves in a harmonic motion independently along two perpendicular directions (for example the axes x and y of a Cartesian coordinate system), so that $\mathbf{r}(t) = (A_x cos(\omega_x t + \phi_x),\ A_y cos(\omega_y t + \phi_y))$. When the two superimposed harmonic motions have identical frequency $\omega x = \omega y$ and phase difference $|\phi x - \phi y| = n\pi$, where $n = 0, 1, 2, \cdots$, the Lissajous figure will show as a straight line. For other values of the phase difference the figures will have an elliptical shape.

2.3.3 Time-difference method

When an ultrasonic pulse signal emitted by S_1 arrives at S_2, it is received and returned back to the processor. By contrasting the original signal with the received one, one can measure the time needed for the sound to travel from S_1 to S_2 over a distance of L. When the values of L and t are known, the phase speed of sound can be found from Eq. 2.

2.3.4 Successive difference method

The successive difference method is an effective method to increase the accuracy of the average value calculated from a series of measurement data. In this experiment, the usual method of calculating the average value, illustrated by the formula

$$\frac{\bar{\lambda}}{2} = \frac{[(L_1 - L_0) + (L_2 - L_1) + \cdots + (L_n - L_{n-1})]}{n} = \frac{L_n - L_0}{n}, \tag{4}$$

will be modified, because as Eq. 5 shows, the average value of the wavelength is determined only by the first and the last value, L_0 and L_n .

A modification of the formula by rearranging terms as

$$n\frac{\bar{\lambda}}{2} = \frac{\Sigma_{i=1}^{n}(L_{n+i} - L_i)}{n}, \tag{5}$$

produces more accurate results, as each value contributes to the final result.

3 Measurement Procedure

3.1 Resonance method

1. Set the initial distance between S_1 and S_2 at about $1cm$.

2. Turn on the signal source and the oscilloscope. Then set the following options on the panel of the signal source.

   (1) Choose *Continuous* wave for *Method*.

   (2) Adjust Signal Strength until a $10V$ peak voltage is observed on the oscilloscope.

   (3) Adjust Signal Frequency between 34.5 kHz and 40 kHz until the peak-to-peak voltage reaches its maximum. Record the frequency.

3. Increase L gradually by moving S_2, and observe the output voltage of S_2 on the oscilloscope. Record the position of S_2 as L_2 when the output voltage reaches an maximum.

4. Repeat step 3 to record 20 values of L_2 and calculate v.

3.2 Phase-comparison method

1. Use Lissajous figures to observe the phase difference between the transmitted and the received signals. Move S_2 and record the position when the Lissajous figure becomes a straight line with the same slope.

2. Repeat step 1 to collect 12 sets of data. Use the successive difference method to process the data and calculate v.

3.3 Time-difference method (liquid)

Since the pulse wave causes damped oscillations at the receiver, there will be significant interference if S_1 and S_2 resonate. The resonance can be observed on the oscilloscope.

1. Choose Pulse Wave for Method on the panel of the signal source.

2. Adjust the frequency to 100 Hz and the width to $500\mu s$.

3. Use the cursor function of the oscilloscope to measure the time and the distance between the the starting points of neighboring periods.

4. Record the distance L_1 and the time t_1.

5. Move S_2 to another position and repeat step 3. Record L_i and t_i, $i = 2, 3, 4, \cdots$.

6. Repeat step 4 to collect 12 pairs of L_i and t_i. Plot the L_i *vs.* t_i graph and use computer software to find a linear fit to the data. The slope of the line is the speed v_{water}.

4 Results

The frequency we read from the signal generator is $f = 35000 \pm 1\text{Hz}$. The temperature is $24 \pm 1°\text{C}$.

4.1 Measurements for resonance method

The measurements are shown in Table 2 with the calculation for $L_{10+i} - L_i$. The average value

$L_i[\times 10^{-3}m] \pm [0.01 \times 10^{-3}m]$		$L_i[\times 10^{-3}m] \pm [0.01 \times 10^{-3}m]$		$L_{10+i} - L_i[\times 10^{-3}m]$	
1	9.90	11	60.08	1	50.18
2	14.92	12	65.08	2	50.16
3	19.97	13	70.06	3	50.09
4	25.02	14	75.02	4	50.00
5	30.03	15	80.25	5	50.22
6	35.06	16	85.24	6	50.18
7	40.03	17	90.24	7	50.21
8	45.08	18	95.22	8	50.14
9	49.95	19	100.13	9	50.18
10	55.01	20	105.24	10	50.23

Table 2: Data for the resonance method

of ΔL is calculated based on the results presented in Table 2 as

$$\overline{\Delta L} = \frac{1}{10}\sum_{i=1}^{10} \Delta L_i = (50.16 \pm 0.05) \times 10^{-3}m, \quad u_{r,\Delta L} = 0.10\%.$$

Hence, the wavelength λ can be calculated as

$$\lambda = \frac{2\Delta L}{n} = \frac{2 \times 50.16 \times 10^{-3}}{10} = (10.03 \pm 0.01) \times 10^{-3}m, \quad u_{r,\lambda} = 0.10\%.$$

The speed of sound in air v is

$$v = 351.05 \pm 0.4m/s, \quad u_{r,v} = 0.10\%.$$

4.2 Measurements for phase comparison method

The measurements are shown in Table 3 with the calculation for $L_{6+i} - L_i$. The average value

$L_i[\times 10^{-3}m] \pm [0.01 \times 10^{-3}m]$		$L_i[\times 10^{-3}m] \pm [0.01 \times 10^{-3}m]$		$L_{6+i} - L_i[\times 10^{-3}m]$	
1	54.00	7	113.41	1	59.41
2	64.06	8	122.04	2	57.98
3	72.97	9	131.76	3	58.79
4	81.02	10	141.83	4	60.81
5	91.17	11	153.65	5	62.46
6	103.49	12	163.08	6	59.59

Table 3: Data for the phase comparison method

of ΔL is calculated based on the results presented in Table 2 as

$$\overline{\Delta L} = \frac{1}{6}\sum_{i=1}^{6} \Delta L_i = (59.84 \pm 1.7) \times 10^{-3}m, \quad u_{r,\Delta L} = 3\%.$$

Hence, the wavelength λ can be calculated as

$$\lambda = \frac{\Delta L}{n} = \frac{\times 59.84 \times 10^{-3}}{6} = (9.973 \pm 0.3) \times 10^{-3}m, \quad u_{r,\lambda} = 3\%.$$

The speed of sound in air v is

$$v = 348.95 \pm 10m/s, \quad u_{r,v} = 3\%.$$

4.3 Measurements for time difference method (liquid)

We obtain the speed of sound in water from Table 4 by linear fitting (See Figure 3).

	$t_i[\times 10^{-6}s]$	$u_{t_i}[\times 10^{-6}s]$	$L_i[\times 10^{-3}m]$	$u_{L_i}[\times 10^{-3}m]$
1	159.6	0.2	230.00	0.01
2	154.4	0.2	220.00	0.01
3	148.2	0.2	210.00	0.01
4	141.8	0.2	200.00	0.01
5	134.0	0.2	190.00	0.01
6	128.0	0.2	180.00	0.01
7	121.4	0.2	170.00	0.01
8	114.4	0.2	160.00	0.01
9	108.0	0.2	150.00	0.01
10	101.6	0.2	140.00	0.01
11	94.8	0.2	130.00	0.01
12	87.8	0.2	120.00	0.01

Table 4: Data for Figure 3

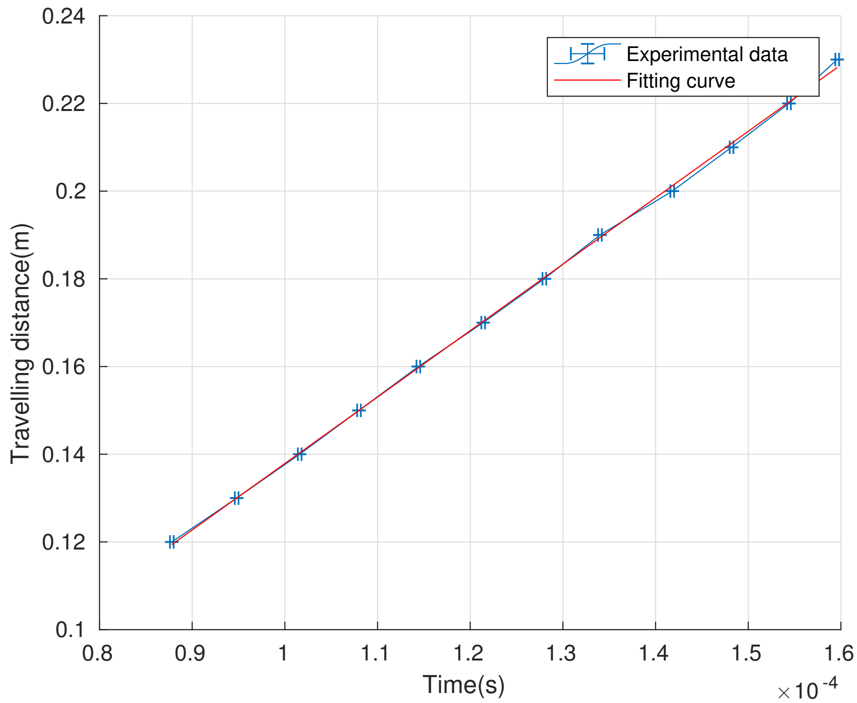

Figure 3: Fitting curve for L $vs.$ t. (The errorbar is relatively **small**)

```
Linear model Poly1:
    f(x) = p1*x + p2
Coefficients (with 95% confidence bounds):
      p1 =       1515  (1492, 1538)
      p2 =   -0.01363  (-0.01657, -0.01068)

Goodness of fit:
  SSE: 6.795e-06
  R-square: 0.9995
  Adjusted R-square: 0.9995
  RMSE: 0.0008243
```

Figure 4: Information for Fitting curve in Figure 3

From the data processing of Matalb we know about the slope that

$$v_{water} = 1515 \pm 20 m/s.$$

5 Measurement Uncetainty Analysis

5.1 Uncertainty for resonance method

To determine the type-B uncertainty of ΔL,

$$\begin{aligned}
&\frac{\partial \Delta L}{\partial L_{10+i}} = 1, \quad \frac{\partial \Delta L}{\partial L_i} = -1,\\
&u_{L_{10+i}} = u_{L_i} = \Delta_{dev} = 1 \times 10^{-5} m,\\
&\Delta_{\Delta L,B} = \sqrt{(\frac{\partial \Delta L}{\partial L_{10+i}})^2 \cdot (u_{L_{10+i}})^2 + (\frac{\partial \Delta L}{\partial L_i})^2 \cdot (u_{L_i})^2}\\
&\qquad = \sqrt{2} \times 1 \times 10^{-5} \approx 0.014 \times 10^{-3} m.
\end{aligned}$$

To estimate type-A uncertainty, the standard deviation of the average value is caluculated as

$$s_{\overline{\Delta L}} = \sqrt{\frac{1}{n(n-1)} \sum_{i=1}^{n} (\Delta L_i - \overline{\Delta L})^2}.$$

Uisng the data from Table 2 we find that $s_{\overline{\Delta L}} \approx 0.02188 \times 10^{-3} m$. Considering $t_{0.95} = 2.26$ for $n = 10$, the type-A uncertainty is estimated as $\Delta_{\Delta L,A} = 2.26 \times 0.02188 \times 10^{-3} \approx 0.0494 \times 10^{-3} m$. Hence the combined uncertainty is

$$\begin{aligned}
&u_{\Delta L} = \sqrt{\Delta^2_{\Delta L,A} + \Delta^2_{\Delta L,B}} = \sqrt{(0.0494 \times 10^{-3})^2 + (0.014 \times 10^{-3})^2} \approx 0.05 \times 10^{-3} m\\
&u_{r,\Delta L} = \frac{u_{\Delta L}}{\overline{\Delta L}} \times 100\% = \frac{0.05 \times 10^{-3}}{50.15 \times 10^{-3}} \times 100\% = 0.10\%\\
&\overline{\Delta L} = (50.16 \pm 0.05) \times 10^{-3} m, \quad u_{r,\Delta L} = 0.10\%
\end{aligned}$$

.

Similarly, we obtain the uncertainty of wavelength $\lambda = 2\Delta L/n$

$$\begin{aligned}
&\frac{\partial \lambda}{\partial \Delta L} = \frac{2}{10} = \frac{1}{5},\\
&u_{\Delta L} = 0.05 \times 10^{-3} m,\\
&u_\lambda = \sqrt{(\frac{\partial \lambda}{\partial \Delta L})^2 \cdot (u_{\Delta L})^2} = \frac{1}{5} \times 0.05 \times 10^{-3} = 0.01 \times 10^{-3} m,\\
&u_{r,\lambda} = \frac{u_\lambda}{\overline{\lambda}} \times 100\% = \frac{0.01 \times 10^{-3}}{10.03 \times 10^{-3}} = 0.10\%,\\
&\lambda = (10.03 \pm 0.01) \times 10^{-3} m, \quad u_{r,\lambda} = 0.10\%.
\end{aligned}$$

and the uncertainty of the spped of sound in air $v = \lambda f$

$$\frac{\partial v}{\partial \lambda} = f, \quad \frac{\partial v}{\partial f} = \lambda,$$
$$u_\lambda = 0.01 \times 10^{-3} m, \quad u_f = 1Hz,$$
$$u_v = \sqrt{(\frac{\partial v}{\partial \lambda})^2 \cdot (u_\lambda)^2 + (\frac{\partial v}{\partial f})^2 \cdot (u_f)^2} = \sqrt{(f)^2 \cdot (u_\lambda)^2 + (\lambda)^2 \cdot (u_f)^2},$$
$$= \sqrt{(35000)^2 \cdot (0.01 \times 10^{-3})^2 + (10.03 \times 10^{-3})^2 \cdot (1)^2} \approx 0.4m/s,$$
$$u_{r,v} = \frac{u_v}{\overline{v}} \times 100\% = \frac{0.4}{351.05} = 0.10\%,$$
$$v = 351.05 \pm 0.4m/s, \quad u_{r,v} = 0.10\%.$$

5.2 Uncertainty for phase comparison method

To determine the type-B uncertainty of ΔL,

$$\frac{\partial \Delta L}{\partial L_{6+i}} = 1, \quad \frac{\partial \Delta L}{\partial L_i} = -1,$$
$$u_{L_{6+i}} = u_{L_i} = \Delta_{dev} = 1 \times 10^{-5} m,$$
$$\Delta_{\Delta L,B} = \sqrt{(\frac{\partial \Delta L}{\partial L_{6+i}})^2 \cdot (u_{L_{6+i}})^2 + (\frac{\partial \Delta L}{\partial L_i})^2 \cdot (u_{L_i})^2}$$
$$= \sqrt{2} \times 1 \times 10^{-5} \approx 0.014 \times 10^{-3} m.$$

To estimate type-A uncertainty, the standard deviation of the average value is caluculated as

$$s_{\overline{\Delta L}} = \sqrt{\frac{1}{n(n-1)} \sum_{i=1}^{n} (\Delta L_i - \overline{\Delta L})^2}.$$

Uisng the data from Table 3 we find that $s_{\overline{\Delta L}} \approx 0.6485 \times 10^{-3} m$. Considering $t_{0.95} = 2.57$ for $n = 6$, the type-A uncertainty is estimated as $\Delta_{\Delta L,A} = 2.57 \times 0.6485 \times 10^{-3} \approx 1.67 \times 10^{-3} m$.

Hence the combined uncertainty is

$$u_{\Delta L} = \sqrt{\Delta^2_{\Delta L,A} + \Delta^2_{\Delta L,B}} = \sqrt{(1.67 \times 10^{-3})^2 + (0.014 \times 10^{-3})^2} \approx 1.7 \times 10^{-3} m$$
$$u_{r,\Delta L} = \frac{u_{\Delta L}}{\overline{\Delta L}} \times 100\% = \frac{1.7 \times 10^{-3}}{59.84 \times 10^{-3}} \times 100\% \approx 3\%$$
$$\overline{\Delta L} = (59.84 \pm 1.7) \times 10^{-3} m, \quad u_{r,\Delta L} = 3\%$$

.

Similarly, we obtain the uncertainty of wavelength $\lambda = \Delta L/n$

$$\begin{aligned}
&\frac{\partial\lambda}{\partial\Delta L} = \frac{1}{6},\\
&u_{\Delta L} = 1.7\times10^{-3}m,\\
&u_\lambda = \sqrt{(\frac{\partial\lambda}{\partial\Delta L})^2\cdot(u_{\Delta L})^2} = \frac{1}{6}\times1.7\times10^{-3} = 0.3\times10^{-3}m,\\
&u_{r,\lambda} = \frac{u_\lambda}{\bar\lambda}\times100\% = \frac{0.3\times10^{-3}}{9.973\times10^{-3}} = 3\%,\\
&\lambda = (9.973\pm0.3)\times10^{-3}m,\quad u_{r,\lambda} = 3\%.
\end{aligned}$$

and the uncertainty of the spped of sound in air $v = \lambda f$

$$\begin{aligned}
&\frac{\partial v}{\partial\lambda} = f,\quad \frac{\partial v}{\partial f} = \lambda,\\
&u_\lambda = 0.3\times10^{-3}m,\quad u_f = 1Hz,\\
&u_v = \sqrt{(\frac{\partial v}{\partial\lambda})^2\cdot(u_\lambda)^2 + (\frac{\partial v}{\partial f})^2\cdot(u_f)^2} = \sqrt{(f)^2\cdot(u_\lambda)^2+(\lambda)^2\cdot(u_f)^2},\\
&\quad = \sqrt{(35000)^2\cdot(0.3\times10^{-3})^2+(9.97\times10^{-3})^2\cdot(1)^2}\approx 10m/s,\\
&u_{r,v} = \frac{u_v}{\bar v}\times100\% = \frac{10}{348.95} = 3\%,\\
&v = 348.95\pm10m/s,\quad u_{r,v} = 3\%.
\end{aligned}$$

5.3 Uncertainty for the time difference method (liquid)

Using Matlab to plot the fitting curve of $L\ vs.\ t$, we obtain the information of the fit automatically (See Figure 4).

$$v_{water} = 1515\pm20m/s.$$

where $\pm20m/s$ is the 0.95-confidence uncertainty, with $t_{0.95} = 2.20$ when $n = 12$. THe relative uncertainty is that

$$u_{r,v} = \frac{u_v}{\bar v}\times100\% = \frac{20}{1515} = 1.5\%.$$

Hence the speed of sound in water is

$$v_{water} = 1515\pm20m/s,\quad u_{r,v} = 1.5\%.$$

6 Conclusion and Discussion

6.1 The resonance method and the comparison method

In this experiment the spped of sound in the air was found by two means: the resonance method and the phase comparison method. The two results yielded the values

$$\begin{aligned} v &= 351.05 \pm 0.4 m/s, \quad u_{r,v} = 0.10\%; \\ v &= 348.95 \pm 10 m/s, \quad u_{r,v} = 3\%, \end{aligned} \tag{6}$$

respectively. The resonance method obtains obviously smaller uncertainty. To compare these two experimentally found results, we need an objective standard.

According to Bohn Dennis A. in the report "Environmental effects on the speed of sound", *Journal of the Audio Engineering Society* P223-231, the speed of sound with respect to temperature holds the following equation

$$c = 331.45\sqrt{1 + \frac{t}{273}} \qquad [1],$$

where c is the sound of the speed and t is the temperature in degrees Celsius.

Since temperature we meausured was 24(1) °C, the "theoretical" value of sound speed should be

$$v = 331.45\sqrt{1 + \frac{24}{273}} = 345.71 m/s$$

which is within the uncertainty interval of the results with the phase comparison method.

Hence we can conclude that the results of the resonance method is **precise enough** but **not accurate enough**.

On the contrary, the results of the phase comparison method is **accurate enough** but **not precise enough**.

The possible reason for errors in the resonance method is the system error. Since the uncertainty is quite small, there should exists a certain error between the experimental value and the theoretical value, probably becasue the frequency displayed on the screen had a certain deviation.

The possible reason for errors in the phase comparison method is the reading error. We found that it was still changing in shape after we stop moving the calliper so that it's hard to tell whether it's the straight segment we want. Besides, the reading of calliper was inaccurate.

6.2 The time difference method for v_{water}

The experimentally found value is

$$v_{water} = 1515 \pm 20 m/s, \quad u_{r,v} = 1.5\%.$$

According to the experiment results of N Bilaniuk and GSK Wong in the report "Speed of sound in pure water as a function of temperature",

$t(°C)$	23.8	23.9	24.0	24.1	24.2
$v(m/s)$	1493.440	1493.717	1493.992	1494.267	1494.541

Table 5: Speed of sound in water with respect to temperature[2]

We find that our experimental value is relatively close to the standard value.

6.3 Recommendations

For the resonance method, I think it'll be more accurate if there exists somthing like a prompting light to prompt the user when the voltage begins decreasing so that the user can stop the calliper in time and read the accurate value of length.

References

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