UM-SJTU Joint Institute

Physics Laboratory

VP141

Exercise 2

Measurements of Fluid Viscosity

Name: Fubo Qi
ID: 516370910177
Group: 11

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I Introduction

I.1 objective

The objective of the exercise is to measure the fluid viscosity, which is one of the most important properties of fluids, determining the fluids flow, by using a common and simple method called Stokes' method.

I.2 Theoretical Background

When an object moves in a fluid, its motion is hindered by a drag force acting in the opposite direction of the movement of object.Also, the magnitude of the drag force is related to serval components such as, the shape, speed of the object as well as the internal friction in the fluid. Usually, we use coefficient η to quantify the internal friction in the fluid, and the η is konwn as the viscosity coefficient. The drag force of a spherical object with radius R moving at speed v in an infinite volume of a liquid is given by,

$$F_1 = 6\pi\eta v R \tag{1}$$

And the buoyancy force is,

$$F_3 = \frac{4}{3}\pi R^3 \rho_1 g$$

Where ρ is the density of fluid and g is the acceleration due to gravity.The weight of the object is

$$F_3 = \frac{4}{3}\pi R^3 \rho_2 g$$

After sometime, the ball will achieve equilibrum speed when the three force balance each other,

$$F_1 + F_2 = F_3. \tag{2}$$

Together with the euation 1, we get,

$$\eta = \frac{2}{9} g R^2 \frac{\rho_2 - \rho_1}{v_t}. \tag{3}$$

Because of the constant velocity, the η can also be written as

$$\eta = \frac{2}{9} g R^2 \frac{(\rho_2 - \rho_1)t}{s}, \tag{4}$$

where s is the distance travled in time t with reaching the terminal speed.
Since the volume of the fluid is not infinite,the results are affected by some boundary effect

due to the presence of the container.Therefore the formula for the crrected magnitude is giben by,

$$F_1 = 6\pi\eta v R(1 + 2.4\frac{R}{R_c})$$

Where R_c is the radius of the infinitely long cylindrical container.Consequently, we get,

$$\eta = \frac{2}{9} g R^2 \frac{(\rho_2 - \rho_1)t}{s} \frac{1}{1 + 2.4\frac{R}{R_c}}. \tag{5}$$

II Apparatus and Measurement procedure

II.1 Experimental Setup

This exercise requires a Stokes' viscosity measurement device with castor oil and some small metal balls. There are also a number of measurement devices including micrometer, calliper, densimeter, electronic scales, stopwatch, and thermometer.

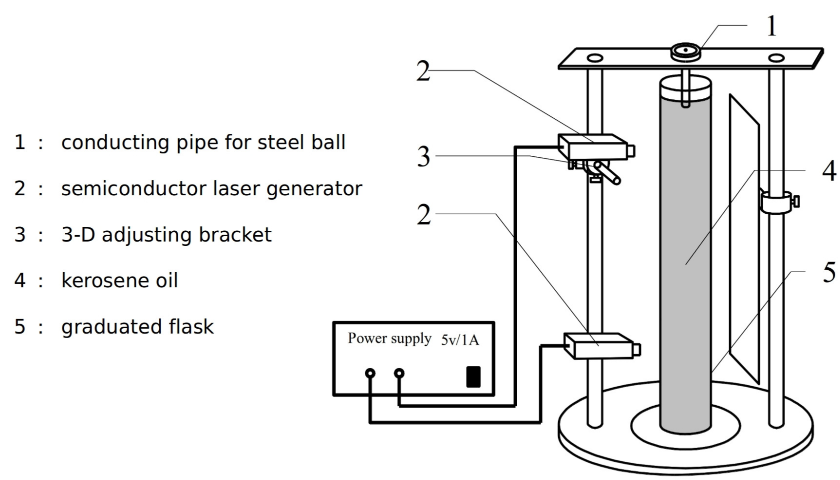

Figure 1: Stokes viscosity measurement apparatus

II.2 Measurement

II.2.1 Adjustment of the Stokes' viscosity measurement device

1. Make the plumb aiming at the center of the base by adjusting the knobs beneath the base.

2. Turn on the two lasers and make the beams parallel and aim at the plumb line.
3. Remove the plumb and place the flask with castor oil at the center of the base.
4. Place the guiding pipe on the top of the device.
5. Put a metal ball into it and check whether the ball can block the beams. If not, repeat.

II.2.2 Measurement of the (constant) velocity of a falling ball

1. Measure the vertical distance s between the two laser beams at least three times.
2. Put a metal ball into the guiding pipe. Record the time it travels between the two beams for at least six times.

II.2.3 Measurement of the ball density ρ_2

1. Use electronic scales to measure the mass of 40 metal balls. Calculate the average.
2. Use a micrometer to measure the diameter of the ball for ten times.
3. Calculate the density ρ_2.

II.2.4 Other measurements

Measure of the density ρ_1 of the castor oil by using the provided densimeter (one measurement). Use a calliper to measure the inner diameter D of the graduated flask for six times. Read the ambient temperature from the thermometer placed in the lab.

III Results

III.1 Measurement of the Distance

Distance S $[\times 10^{-3} m] \pm$ 0.10 $[\times 10^{-3} m]$			
S_1	110.0	263.0	153
S_2	140.0	292.0	152
S_3	111.0	264.5	153.5

Table 1: Distance measurement data

The average value of the vertical distance of two beams is hence calculated by,

$$\bar{S} = \frac{1}{3} \sum_{i=1}^{3} S_i = (152.8 \pm 1.55) \times 10^{-3} m$$

III.2 Measurement of the Time

Measurement	$Time\ t\ [s] \pm\ 0.01\ [s]$
t_1	6.97
t_2	7.12
t_3	7.28
t_4	7.35
t_5	6.94
t_6	7.00

Table 2: Time measurement data

The average value of the time is given as

$$\bar{t} = \frac{1}{6}\sum_{i=1}^{6} t_i = 7.11 \pm 0.157s$$

III.3 Diameter of ball

$Diameters\ d[\times 10^{-3}m] \pm 0.004[\times 10^{-3}m]$			
d_1	1.945	d_6	1.955
d_2	1.945	d_7	1.940
d_3	1.950	d_8	1.940
d_4	1.955	d_9	1.950
d_5	1.950	d_{10}	1.935

Table 3: Measurement data for diameters of the balls

With the initial reading, $d_0 = -0.040mm$ The average value of the diameter of a ball is hence calculated as

$$\bar{d} = \frac{1}{10}\sum_{i=1}^{10} d_i = (1.986\pm) \times 10^{-3}m$$

III.4 Inner Diameter of the Flask

Diameters $D[\times 10^{-3}m] \pm 0.02[\times 10^{-3}m]$	
D_1	61.70
D_2	61.78
D_3	61.68
D_4	61.70
D_5	61.64
D_6	61.68

Table 4: Measurement data for the inner diameter of the flask

The average value of the inner diameter of the flask is hence calculated

$$\bar{D} = \frac{1}{6}\sum_{i=1}^{10} D_i = (61.69 \pm 0.105) \times 10^{-3}\ m$$

III.5 Other Physicals Quantifies

Density of the castor oil	$\rho_1 = 0.956 \pm 0.001 g/cm^3$
Mass of 40 metal balls	$m = 1.359 \pm 0.001 \times 10^{-3} kg$
Temperature in the lab	$T = 26 \pm 2\,°\mathrm{C}$
Acceleration due to gravity in the lab	$g = 9.794 m/s^2$

Table 5: Values of other physical quantities

IV Calculation and Uncertainy Analysis

For a single measurement of the experiment setup, the uncertainty of the measurement instruments are For the type-A uncertainty of the distance measurement, the standard

Distance	Time	Diameter of Ball	Diameter of Flask	Density	Mass	Temperature
0.1mm	0.01s	0.004mm	0.03mm	$1 kg/m^3$	0.001g	2 °C

Table 6: Precision of the measurement instruments

deviation of the average value is given by,

$$s_X = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{X})^2} = 0.6236mm$$

and the type-A uncertainty is given by,

$$\Delta_A = \frac{s_X \times t_{0.95}}{\sqrt{n}}$$

where the $t_0.95$ is 4.3 for $n = 3$. Taking the data from Table 1, Δ_A is calculated by,

$$\Delta_A = \frac{0.6236 \times 4.3}{\sqrt{3}} = 1.48mm$$

Δ_B is equal to $0.1mm$.

$$u = \sqrt{\Delta_A^2 + \Delta_B^2} = \sqrt{1.548^2 + 0.1^2} \approx 1.55mm$$

Then, the relative uncertainty is,

$$u_r = \frac{u}{\bar{x}} \times 100\% = 1.0\%$$

We thus achieve

$$S = (152.8 \pm 1.55) \times 10^{-3}m$$

	Value	s_X	Δ_A	u	u_r
Distance $[mm]$	152.8	0.6236	1.48	1.55	1.0%
Time $[s]$	7.11	0.157	0.165	0.165	2.2%
Diameter of Ball$[mm]$	1.968	6.34×10^{-3}	4.53×10^{-3}	6×10^{-3}	0.304%
Diameter of Flask$[mm]$	61.69	0.0987	0.103	0.105	0.171%

Table 7: The uncertainty

IV.1 Uncertainy of density

Density of the mental balls can be found as,

$$\rho_2 = \frac{m_o}{V} = \frac{6m_0}{\pi d^3}$$

Where $m_0 = \frac{1.359}{40} = 0.03398 \times 10^{-3}kg$, and $d = 1.968 \times 10^{-3}m$, the desity is hence calculated by,

$$\rho_2 = \frac{6 \times 0.03398 \times 10^{-3}}{3.141593 \times (1.968 \times 10^{-3})^3} = 8.514 \times 10^3 kg/m^3$$

The uncertainty of the density is given by,

$$u_{\rho_2} = \sqrt{(\frac{\partial \rho_2}{\partial m_0})^2 (u_{m_0})^2 + (\frac{\partial \rho_2}{\partial d})^2 (u_d)^2}$$

Where the partial derivatives is given by,

$$\frac{\partial \rho_2}{\partial m_0} = \frac{6}{\pi d^3}, \quad \frac{\partial \rho_2}{\partial d} = \frac{18 m_0}{\pi d^4}$$

Also, because the $m_0 = \frac{M}{40}$, the uncertainty of m_0 is $\frac{\partial M}{\partial m_0} u_m = \frac{1 \times 0.001}{40} = 2.5 \times 10^{-5} g$ then,

$$\begin{aligned}
u_{\rho_2} &= \sqrt{(\frac{\partial \rho_2}{\partial m_0})^2 (u_{m_0})^2 + (\frac{\partial \rho_2}{\partial d})^2 (u_d)^2} \\
&= \sqrt{(\frac{6 \times 0.00025 \times 10^{-3}}{3.1415926 \times (1.968 \times 10^{-3})^3})^2 + (\frac{18 \times 0.03393 \times 10^{-3} \times 0.006 \times 10^{-3}}{3.14159 \times (1.968 \times 10^{-3})^4})^2} \\
&\approx 0.099 \times 10^3 kg/m^3.
\end{aligned}$$

The relative uncertainty is,

$$u_{r\rho 2} = \frac{u_{\rho_2}}{\bar{\rho_2}} \approx 1.16\%$$

IV.2 Uncertainy of Viscosity Coefficient

Through the calculation above, the viscosity coefficient can hence be achieved as,

$$\begin{aligned}
\eta &= \frac{2}{9} g R^2 \frac{(\rho_2 - \rho_1) t}{s} \frac{1}{1 + 2.4 \frac{R}{R_c}} \\
&= \frac{2}{9} \times 9.794 \times 0.000984^2 \times \frac{(8.514 \times 10^3 - 0.956 \times 10^3) \times 7.11}{0.153} \times \frac{1}{1 + 2.4 \times \frac{0.000984}{0.030845}} \\
&\approx 0.688 \frac{kg}{m \cdot s}.
\end{aligned}$$

The uncertainty of the Viscosity Coefficient is given by the following formula,

$$u_\eta = \sqrt{(\frac{\partial \eta}{\partial R})^2 (u_R)^2 + (\frac{\partial \eta}{\partial \rho_2})^2 (u_{\rho_2})^2 + (\frac{\partial \eta}{\partial \rho_1})^2 (u_{\rho_1})^2 + (\frac{\partial \eta}{\partial u_t})^2 (u_t)^2 + (\frac{\partial \eta}{\partial S})^2 (u_S)^2 + (\frac{\partial \eta}{\partial u_{R_c}})^2 (u_{R_c})^2}$$

And the partial derivatives is found as,

$$\frac{\partial \eta}{\partial R} = -\frac{R Rc g t}{s\,(2.4R + Rc)^2} \left(\frac{8}{15} R + \frac{4}{9} Rc \right) (\rho_1 - \rho_2) \tag{6}$$

$$\frac{\partial \eta}{\partial \rho_2} = \frac{2R^2 R_c g t}{9s\left(2.4R + R_c\right)} \tag{7}$$

$$\frac{\partial \eta}{\partial \rho_1} = -\frac{2R^2 R_c g t}{9s\left(2.4R + R_c\right)} \tag{8}$$

$$\frac{\partial \eta}{\partial t} = -\frac{2R^2 Rcg\left(\rho_1 - \rho_2\right)}{9s\left(2.4R + Rc\right)} \tag{9}$$

$$\frac{\partial \eta}{\partial S} = \frac{2R^2 R_c g t\left(\rho_1 - \rho_2\right)}{9s^2\left(2.4R + R_c\right)} \tag{10}$$

$$\frac{\partial \eta}{\partial R_c} = -\frac{8R^3 g t\left(p_1 - p_2\right)}{15s\left(2.4R + Rc\right)^2} \tag{11}$$

By the equation through (6) to (11), the uncertainty is approxmately $0.03kg/ms$. The relative uncertainty is approxmately 3.8%
Hence the experimentally η is

$$\eta = 0.688 \pm 0.03kg/ms$$

V Conclusion and Discussion

In this experiment, by using Stokes' method, the experimentally found η of castor oil in the environment of $26\,^{\circ}\mathrm{C}, 1\ atm$ is

$$\eta = 0.688 \pm 0.03kg/ms$$

$$u_{r_\eta} = 3.8\%$$

In this experiment, I find the mental ball is so small that it's hard to conver the lase, and I have to measure much more than 6 times and throw a couple of balls into the fluid. Also, it's hard to comfirm whether the two beams are parallel. When I measure the time, I always have to react some time before press the stopwatch when the laser is cover, and this will leads to uncertainty. If the equipment ensure the two beams is parallel and fixed, and the time is measured by computer when the beams is blocked, the results will be much more accurate.

References

[1] Qin Tian, Feng Yaming, Mateusz Krzyzosiak Physics Laboratory Vp141 Exercise 2 Measurement of Fluid Viscosity.