UM-SJTU Joint Institute

Physical Laboratory

VP141

Laboratory Report

Exercise 2

Measurements of Fluid Viscosity

Name: Ren Wang 王轫 ID: 516370910177 Group: 11
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1 Introduction

Fluid viscosity is one of the most important properties of fluids, determining the fluid's flow. Motion of an object in a fluid is hindered by a drag force acting in the direction opposite to the direction of motion, i.e. opposite to the object's velocity.

The magnitude of the drag force is related to the shape and speed of the object as well as to the internal friction in the fluid. The method used in this lab is known as Stokes' method and it a common and simple method for characterizing transparent or translucent fluids with high viscosity.

Motion of an object in a fluid is hindered by a drag force acting in the direction opposite to the direction of motion, i.e. opposite to the object's velocity. The magnitude of the drag force is related to the shape and speed of the object as well as to the internal friction in the fluid.

This internal friction can be quantified by a number known as the viscosity coefficient μ. For a spherical object with radius R moving at speed v in an infinite volume of a liquid, the magnitude of the drag force is usually modeled as linear in the speed

$$F_1 = 6\pi\mu v R$$

When a spherical object falls vertically downwards in a fluid, it is being acted upon by the following three forces: The viscous force F_1 and the buoyancy force F_2 both act upwards, and the weight of the object F_3 is directed downwards. The magnitude of the buoyancy force is

$$F_2 = \frac{4}{3}\pi R^3 \rho_1 g$$

where ρ_1 is the density of the fluid and g is the acceleration due to gravity. The weight of the object

$$F_3 = \frac{4}{3}\pi R^3 \rho_2 g$$

with ρ_2 being the density of the object. After some time, the three forces will balance each other

$$F_1 + F_2 = F_3$$

so that the net force on the object will be zero and from that instant on, the object will be moving with constant speed v_t, known as the terminal speed. Applying the condition, we can find

$$\mu = \frac{2}{9} g R^2 \frac{\rho_2 - \rho_1}{v_t}$$

Therefore, the fluid viscosity can be found by measuring the terminal speed. Taking into account that the motion with terminal speed is a motion with constant velocity, the equation can be rewritten as

$$\mu = \frac{2}{9} g R^2 \frac{(\rho_2 - \rho_1)t}{s}$$

where s is the distance traveled in time t after reaching the terminal speed.

Since the volume of the fluid used in the measurement is not infinite, the results are affected by some boundary effects due to the presence of the container.

Therefore, the equation should be modified, and the formula for the corrected magnitude of the viscous force for a infinitely long cylindrical container with radius R_c is

$$F_1 = 6\pi \mu v R (1 + 2.4 \frac{R}{R_c})$$

Consequently,

$$\mu = \frac{2}{9} g R^2 \frac{(\rho_2 - \rho_1)t}{s} (1 + 2.4 \frac{R}{R_c})$$

Since the length L of the container is limited, there may be further corrections introduced, depending on the ratio on $\frac{R_c}{L}$.

2 Experimental Setup

The experimental setup consists of a Stokes' viscosity measurement device (see Figure 1) filled with castor oil in which motion of small metal balls will be observed.

Measurements of various physical quantities in the experiment are performed with a number of measurement devices: micrometer, calliper, densimeter, electronic scales, stopwatch, and thermometer.

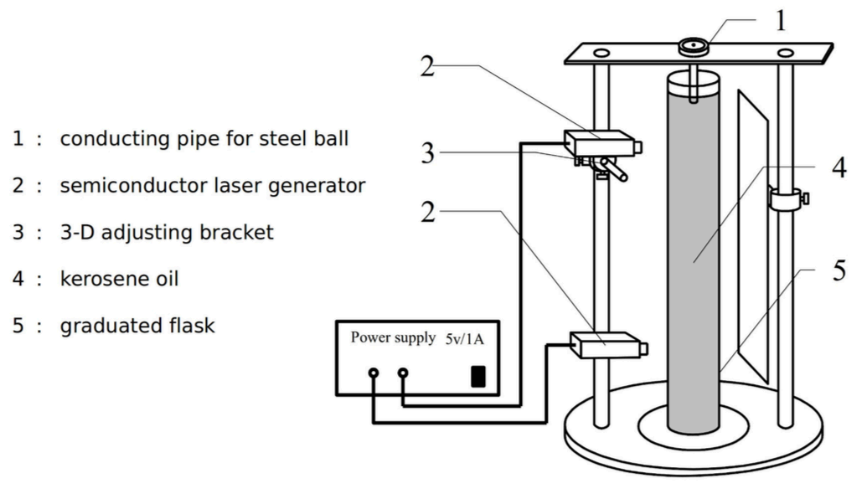

Figure 1: Stokes viscosity measurement apparatus

3 Measurement Procedue

1. Adjustment of the Stokes' viscosity measurement device.

   (1) Adjust the knobs beneath the base to make the plumb aiming at the center of the base.

   (2) Turn on the two lasers, adjust the beams so that they are parallel and aim at the plumb line.

   (3) Remove the plumb and place the graduated flask with castor oil at the center of the base.

   (4) Place the guiding pipe on the top of the viscosity measurement device.

   (5) Put a metal ball into the pipe and check whether the ball, falling down in the oil, can blocks the laser beams. If not, repeat Step 1.

2. Measurement of the (constant) velocity of a falling ball.

    (1) Measure the vertical distance s between the two laser beams at least three times.

    (2) Put a metal ball into the guiding pipe. Start the stopwatch when the ball passes through the first beam, and stop it when it passes through the second one. Record the time t and repeat the procedure for at least six times.

3. Measurement of the ball density ρ_2.

    (1) Use electronic scales to measure the mass of 40 metal balls. Calculate the average to find the mass of a single ball.

    (2) Use a micrometer to measure the diameter of the metal balls. Repeat for ten times and calculate the average value.

    (3) Calculate the ball density ρ_2.

4. Measure of the density ρ_1 of the castor oil by using the provided densimeter (one measurement). Use a calliper to measure the inner diameter D of the graduated flask for six times. Read the ambient temperature from the thermometer placed in the lab.

5. Calculate the value of viscosity coefficient μ using Eq. (5).

4 Caution

- Do not move the graduated flask during the measurement.
- Be careful with the castor oil: do not spill it on the desk.
- Do not forget to read the ambient temperature, as fluid viscosity is sensitive to the temperature.

5 Result

5.1 Distance Between Two Lasar Device

distance S [mm] $\pm$ 1 [mm]			
S	Ruler Start Point	Ruler End Point	Calculated Length
S_1	30.0	177.0	147.0
S_2	60.0	207.0	147.0
S_3	40.0	186.5	146.5

Table 1: Distance measurement data.

Then we can find

$$\bar{S} = \frac{1}{3}\sum_{k=1}^{3} S_k = 146.8mm \pm 1mm$$

$$u_{S,r} = 0.68\%$$

5.2 Time Measurement

time t [s] $\pm$ 0.01 [s]	
t_1	6.75
t_2	6.82
t_3	6.91
t_4	6.88
t_5	6.91
t_6	6.84

Table 2: Time measurement data.

Then we can find

$$\bar{t} = \frac{1}{6}\sum_{k=1}^{6} t_k = 6.85s \pm 0.01s$$

$$u_{t,r} = 0.15\%$$

5.3 The Diameters of The Balls

The initial reading of the meter is 0.38 mm. Thus, the raw data of measurement should firstly minus 0.38 mm, and is presented as following,

diameter d [mm] $\pm$ 0.005 [mm]			
d_1	1.995	d_6	1.995
d_2	1.995	d_7	2.000
d_3	2.000	d_8	1.800
d_4	1.995	d_9	1.995
d_5	2.000	d_{10}	1.995

Table 3: Ball diameter measurement data.

Then we can find

$$\bar{d} = \frac{1}{10}\sum_{k=1}^{10} d_k = 1.977mm \pm 0.005mm$$

$$u_{t,r} = 0.25\%$$

5.4 The Inner Diameter of The Flask

diameter D [mm] $\pm$ 0.02 [mm]	
D_1	61.40
D_2	61.46
D_3	61.20
D_4	61.36
D_5	61.20
D_6	61.50

Table 4: Flask diameter measurement data.

Then we can find

$$\bar{D} = \frac{1}{6}\sum_{k=1}^{6} D_k = 61.3533mm \pm 0.02mm$$

$$u_{D,r} = 0.03\%$$

5.5 Other Physical Quantities

density of the castor oil $\rho_1[g/cm^3] \pm 0.001[g/cm^3]$	0.955
mass of 40 metal balls $m[g] \pm 0.001[g]$	1.357
temperature in the lab $T[\circ C] \pm 2[\circ C]$	25
acceleration due to gravity in the lab $g[m/s^2]$	9.794

Table 5: Other Physical Quantities Measurement

5.6 Calculation of Density of One Ball

The mass of one metal ball can be calculated as

$$m_0 = \frac{m}{40} = \frac{1.357 \times 10^{-3}}{40} = 3.3925 \times 10^-5kg \pm (2.500 \times 10^-8)kg$$

We can furtherly get the density,

$$\rho_2 = \frac{m_0}{\frac{1}{6}\pi d^3} = \frac{3.3925 \times 10^-5}{\frac{1}{6} \times 3.151593 \times (1.977 \times 10^-3)^3} = 8.385 \times 10^3 kg/m^3 \pm (5.756 \times 10) kg/m^3$$

$$u_{\rho_2,r} = 0.6875\%$$

5.7 Calculation For The Viscosity Coefficient

From the last equation in introduction part

$$\mu = \frac{2}{9} g R^2 \frac{(\rho_2 - \rho_1)t}{s} (1+2.4\frac{R}{R_c}) = \frac{2 \times (1.977 \times 10^-3 \times \frac{1}{2})^2 (8.385 \times 10^3 - 0.595 \times 10^3) \times 9.974 \times 6.85}{9 \times 146.8 \times 10^-3 \times (1 + 2.4 \times \frac{1.977}{61.3533})}$$
$$= 0.7307 Pa \times s \pm (6.8329 \times 10^-3) Pa \times s$$

$$u_{\mu,r} = 0.93512\%$$

6 Measurement Uncertainty Analysis

6.1 Uncertainty of The Distance Measurement

We use formula to calculate type-A uncertainty

$$u_A = t_{0.95}\sqrt{\frac{1}{3(3-1)}\sum_{k=1}^{3}(S_k-\bar{S})^2} =$$

$$4.30 \cdot \sqrt{1/6 \cdot ((147.0-146.83)^2+(147.0-146.83)^2+(146.5-146.83)^2)} = 0.7mm$$

And type-B uncertainty is $u_B = = 1$ mm. Thus,

$$u_s = \sqrt{u_A^2+u_B^2} = \sqrt{0.7^2+1^2} = 1.2mm$$

$$u_{S,r} = 1.2/146.8 = 0.82\%$$

6.2 Uncertainty of The Time Measurement

We use formula to calculate type-A uncertainty

$$u_A = t_{0.95}\sqrt{\frac{1}{6(6-1)}\sum_{k=1}^{6}(t_k-\bar{t})^2} = 1.05 \cdot 0.0252 = 0.0492s \tag{1}$$

And type-B uncertainty is $u_B = = 0.01$ s. Thus,

$$u_t = \sqrt{u_A^2+u_B^2} = \sqrt{0.0492^2+0.01^2} = 0.0502s$$

$$u_{t,r} = 0.050/6.8517 = 0.73\%$$

6.3 Uncertainty of The Ball Diameter

We use formula to calculate type-A uncertainty

$$u_A = t_{0.95}\sqrt{\frac{1}{10(10-1)}\sum_{k=1}^{10}(d_k-\bar{d})^2} = 0.715 \cdot 0.0197 = 0.0141s \tag{2}$$

And type-B uncertainty is $u_B = = 0.005$ mm. Thus,

$$u_d = \sqrt{u_A^2 + u_B^2} = \sqrt{0.0141^2 + 0.005^2} = 0.0150mm$$

$$u_{d,r} = 0.0150/1.977 = 0.76\%$$

6.4 Uncertainty of The Inner Diameter of The Flask

We use formula to calculate type-A uncertainty

$$u_A = t_{0.95}\sqrt{\frac{1}{6(6-1)}\sum_{k=1}^{6}(D_k - \bar{D})^2} = 1.05 \cdot 0.0523 = 0.0549mm \tag{3}$$

And type-B uncertainty is $u_B = = 0.02$ mm. Thus,

$$u_D = \sqrt{u_A^2 + u_B^2} = \sqrt{0.0549^2 + 0.02^2} = 0.0584mm$$

$$u_{D,r} = 0.0584/61.3533 = 0.095\%$$

6.5 Uncertainty of The Mass of The Ball

We have u_m for 40 balls, thus

$$u_{m_0} = u_m/40 = 0.0001/40/1000 = 2.5 \cdot 10^{-8}kg$$

6.6 Uncertainty of The Density of One Ball

From

$$\rho_2 = \frac{6m_0}{\pi d^3}$$

Then we can find that

$$u_{\rho_2} = \sqrt{(\frac{\partial \rho_2}{\partial m_0})^2(u_{m_0})^2 + (\frac{\partial \rho_2}{\partial d})^2(u_d)^2} = 96.70$$

$$u_{\rho_2,r} = u_{\rho_2}/\rho_2 = 96.70/8.385 \cdot 10^3 = 1.14\%$$

The code is included as matlab/cal1.m

6.7 Uncertainty of The Viscosity Coefficient

$$\mu = \frac{2}{9} g R^2 \frac{(\rho_2 - \rho_1)t}{s}(1 + 2.4\frac{R}{R_c})$$

Thus,

$$u_\mu = \sqrt{(\frac{\partial\mu}{\partial d})^2(u_d)^2 + (\frac{\partial\mu}{\partial s})^2(u_s)^2 + (\frac{\partial\mu}{\partial t})^2(u_t)^2 + (\frac{\partial\mu}{\partial\rho_1})^2(u_{\rho_1})^2 + (\frac{\partial\mu}{\partial D})^2(u_D)^2}$$

$$u_\mu = 1.4264 \cdot 10^{-2}$$

$$u_{\mu,r} = \frac{u_\mu}{\mu} = 1.42 \cdot 10^{-2}/0.7307 = 1.94\%$$

The code is included as matlab/cal2.m

7 Conclusion and Discussion

In this lab, we use Stokes' method to measure fluid viscosity. The liquid been chosen to measure the viscosity is the castor oil in this experiment.

We measures Distance S, Time t, Diameter of the ball d, Inner diameter of the flask D, Density of one ball ρ_2, then calculate the viscosity coefficient not directly.

The relative uncertainty of each physical quantity is quite small, thus our measurement is very precise and the result is acceptable.

The reason for the accuracy is the nice design of the experiment and carefulness throughout the experiment. There are many systematical and random errors leading to the main uncertainty, but a lot of methods are used to reduce them.

We use 40 balls to calculate the average mass of the ball. For each ball's mass is quite small and very hard to measure due to the uncertainty of equipment. We choose 40 instead of 10 to maintain 4 significant numbers.

Actually the temperature has influence on μ. The higher temperature is, the more likely the molecule will move, making the viscosity lower.

My experiment setup is closer two the air conditioner than other people, so my result may be slightly affect by the temperature.

From a engineering website

`http://www.engineeringtoolbox.com/absolute-viscosity-liquids-d_1259.html` We know that the theoretical value of the viscosity of caster oil is $0.650 Pa/s$, but the experiment result is $0.7307 Pa/s$.

$$u_{\mu,r} = (0.7307 - 0.650)/0.650 \times 100\% = 12.42\%$$

The reason is the AC mentioned before cooling down the temperature of my experiment setup. We can see here lower temperature results in higher viscosity.

Furthermore, there are more problems that may cause uncertainty and error in this lab.

1. The distance between two laser device may not be measured precisely for the thread is not strictly perpendicular to the level.

2. When measuring the time, human's reponse may cause some delay between seeing the light fading and pressing down the button on the watch.

Some ways to improve this lab

1. Adjust the experimental setup carefully to the level.

2. Throw the ball in the center of the cylinder and in the same position every time.

3. Try hard to reduce response time.

8 Reference

1. The international system of units (SI) (PDF) (2008 ed.). United States Department of Commerce, NIST Special Publication 330. pp. 29 & 57.

2. "Exercise 2-Lab Manual" Qin Tian, Feng Yaming, Mateusz Krzyzosiak,Department of Physics, Shanghai Jiaotong University.

9 Data Sheet