Physics Laboratory I

VP141

Exercise II

Measurement of Fluid Viscosity

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1 Introduction

The objective of the exercise is to measure the fluid viscosity, an important property of fluids, using Stokes' method.

To analyze the free body diagram of a spherical object moving in a fluid, we find that the viscous force, the buoyancy force and the weight, where the first two forces act upwards and the last one acts downwards.

The magnitude of a drag force is related to the shape and speed of the objective as well as to the internal friction in the fluid. We use coefficient η to quantify the internal friction in the fluid. Hence we build a model for the drag force (viscous force) in an infinite volume of a liquid.

$$F_1 = 6\pi\eta vR \tag{1}$$

The magnitude of the buoyancy force is

$$F_2 = \frac{4}{3}\pi R^3\rho_1 g,$$

where ρ_1 is the density of the fluid and g is the acceleration due to gravity. The weight of the object is

$$F_3 = \frac{4}{3}\pi R^3\rho_2 g,$$

where ρ_2 is the density of the object. Since the three forces balance each other, then

$$F_1 + F_2 = F_3. \tag{2}$$

Assuming that the object is moving with constant speed v_t, we find from Eq. 2 that

$$\eta = \frac{2}{9}gR^2\frac{\rho_2 - \rho_1}{v_t}.$$

Considering that the velocity is constant, we substitute $\frac{s}{t}$ for v_t

$$\eta = \frac{2}{9}gR^2\frac{(\rho_2 - \rho_1)t}{s}, \tag{3}$$

where s is the distance travled in time t with reaching the terminal speed.

We also need to modify Eq.1 since the volume of the fluid is not infinite. To eliminate the boundary effects due to the container. Assume that the radius of the infinitely long cylindrical container is R_c, then

$$F_1 = 6\pi\eta vR(1 + 2.4\frac{R}{R_c})$$

Eventually, the viscosity coefficient can be determined as

$$\eta = \frac{2}{9} g R^2 \frac{(\rho_2 - \rho_1)t}{s} \frac{1}{1 + 2.4\frac{R}{R_c}}. \tag{4}$$

Besides, the length L may contribute to further corrections, which depends on thate ratio R_c/L.

2 Experimental setup

This exercise requires a Stokes' viscosity measurement device (see Figure 1) with castor oil and some small metal balls. The experiment also requires devices including micrometer, calliper, densimeter, electronic scales, stopwatch, and thermometer.

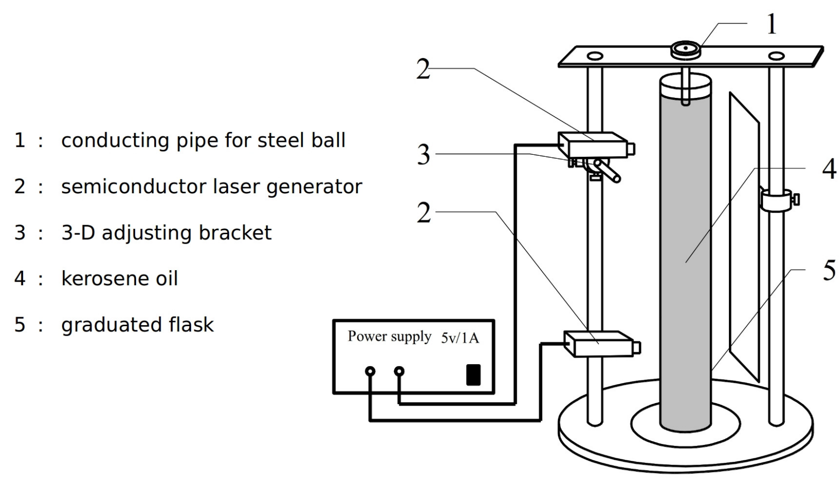

Figure 1: Stokes viscosity measurement apparatus

The micrometer is used to measure the diameters of the balls, allowing measurements with maximum uncertainty of $0.004mm$. The calliper is used to measure the inner diameter of the flask, whose maximum uncertainty is $0.02mm$.The densimeter's maximum uncertainty is $0.001g/cm^3$. The electronic scales' maximum uncertainty is $0.001g$. The stopwatch's maximum uncertainty is $0.01s$. The thermometer's maximum uncertainty is $2\,°\mathrm{C}$.

3 Measurements

3.1 Adjustment of the Stokes' viscosity measurement device

1. Make the plumb aiming at the center of the base by adjusting the knobs beneath the base.
2. Turn on the two lasers and make the beams parallel and aim at the plumb line.
3. Remove the plumb and place the flask with castor oil at the center of the base.
4. Place the guiding pipe on the top of the device.
5. Put a metal ball into it and check whether the ball can block the beams. If not, repeat.

3.2 Measurement of the (constant) velocity of a falling ball

1. Measure the vertical distance s between the two laser beams at least three times.
2. Put a metal ball into the guiding pipe. Record the time it travels between the two beams for at least six times.

3.3 Measurement of the ball density ρ_2

1. Use electronic scales to measure the mass of 40 metal balls. Calculate the average.
2. Use a micrometer to measure the diameter of the ball for ten times.
3. Calculate the density ρ_2.

3.4 Other measurements

1. Read the density of the castor oil ρ_1 by a densimeter.
2. Measure the inner diameter of the flask for six times.
3. Read the temperature from the thermometer.

3.5 Calculation of the value of viscosity coefficient

Calculate η using Eq.4.

4 Results

4.1 Measurements of the vertical distance

The average value of the vertical distance of two beams is calculated based on the results presented in Table 1 as

$$\bar{S} = \frac{1}{3}\sum_{i=1}^{3} S_i = (180.2 \pm 1.9) \times 10^{-3}m.$$

Measurement	*Distance S* [$\times 10^{-3}m$] $\pm$ 0.10 [$\times 10^{-3}m$]
S_1	179.5
S_2	180.0
S_3	181.0

Table 1: Distance measurement data

4.2 Measurements of the traveling time

Measurement	*Time t* [s] $\pm$ 0.01 [s]
t_1	8.31
t_2	8.35
t_3	8.53
t_4	8.41
t_5	8.25
t_6	8.44

Table 2: Time measurement data

The average value of the traveling time between two beams is calculated based on the results presented in Table 2 as

$$\bar{t} = \frac{1}{6}\sum_{i=1}^{6} t_i = 8.38 \pm 0.10s$$

4.3 Measurements for the diameters of the balls

The average value of the diameter of a ball is calculated based on the results presented in Table 3 as

$$\bar{d} = \frac{1}{10}\sum_{i=1}^{10} d_i = (1.985 \pm 0.005) \times 10^{-3}m$$

Measurement	*Initial readings*[$\times 10^{-3}m$]	*Diameters* $d[\times 10^{-3}m] \pm 0.004[\times 10^{-3}m]$
d_1	-0.000	1.985
d_2	-0.000	1.990
d_3	-0.000	1.980
d_4	-0.000	1.990
d_5	-0.000	1.980
d_6	-0.000	1.985
d_7	-0.000	1.980
d_8	-0.000	1.980
d_9	-0.000	1.985
d_{10}	-0.000	1.990

Table 3: Measurement data for diameters of the balls

4.4 Measurements for the inner diameters of the flask

Measurement	*Diameters* $D[\times 10^{-3}m] \pm 0.02[\times 10^{-3}m]$
D_1	62.46
D_2	62.50
D_3	62.32
D_4	62.50
D_5	62.36
D_6	62.34

Table 4: Measurement data for the inner diameter of the flask

The average value of the inner diameter of the flask is calculated based on the results presented in Table 4 as

$$\bar{D} = \frac{1}{10}\sum_{i=1}^{10} D_i = (62.41 \pm 0.21) \times 10^{-3}\ m$$

4.5 Measurements of other physical quanities

In single measurements, its combined deviation u is equal to Δ_{dev}.

Density of the castor oil	$\rho_1 = 955 \pm 1 kg/m^3$
Mass of 40 metal balls	$m = 1.357 \pm 0.001 \times 10^{-3} kg$
Temperature in the lab	$T = 24 \pm 2\,°\mathrm{C}$
Acceleration due to gravity in the lab	$g = 9.794 m/s^2$

Table 5: Values of other physical quantities

Hence the mass of a single metal ball is $m_0 = (0.03393 \pm 0.001) \times 10^{-3} kg$

5 Measurement uncertainty analysis

5.1 Uncertainty of distance measurements

The uncertainty (of type-B) of a steel ruler used to measure the vertical distence between two beams is $\Delta_{S,B} = \Delta_{dev} = 0.10 \times 10^{-3} m$. The distance is found by taking the average of 3 measurements. To estimate type-A uncertainty, the standard deviation of the average value is caluculated as

$$s_{\bar{S}} = \sqrt{\frac{1}{n(n-1)} \sum_{i=1}^{n} (S_i - \bar{S})^2}.$$

Uisng the data from Table 1 we find that $s_{\bar{S}} \approx 0.44 \times 10^{-3} m$. Considering $t_{0.95} = 4.30$ for $n = 3$, the type-A uncertainty is estimated as $\Delta_{S,A} = 4.30 \times 0.441 \times 10^{-3} m \approx 1.89 \times 10^{-3} m$. Hence the combined uncertainty is

$$u_S = \sqrt{\Delta_{S,A}^2 + \Delta_{S,B}^2} = \sqrt{(1.89 \times 10^{-3})^2 + (0.10 \times 10^{-3})^2} \approx 1.89 \times 10^{-3} m$$

and the corresponding relative uncertainty is

$$u_{rS} = \frac{u_S}{\bar{S}} \times 100\% = 1.05\%.$$

The experimentally found S is

$$S = (180.2 \pm 1.9) \times 10^{-3} m, \quad u_{rS} = 1.05\%.$$

5.2 Uncertainty of time measurements

The uncertainty (of type-B) of a stopwatch used to measure the traveling time between two beams is $\Delta_{t,B} = \Delta_{dev} = 0.01 s$. The time is found by taking the average of 6 measurements. To estimate type-A uncertainty, the standard deviation of the average value is caluculated as

$$s_{\bar{t}} = \sqrt{\frac{1}{n(n-1)} \sum_{i=1}^{n} (t_i - \bar{t})^2}.$$

Uisng the data from Table 2 we find that $s_{\bar{t}} \approx 0.04 s$. Considering $t_{0.95} = 2.57$ for $n = 6$, the type-A uncertainty is estimated as $\Delta_{t,A} = 2.57 \times 0.04 s \approx 0.10 s$.
Hence the combined uncertainty is

$$u_t = \sqrt{\Delta_{t,A}^2 + \Delta_{t,B}^2} = \sqrt{(0.10)^2 + (0.01)^2} \approx 0.10 s$$

and the corresponding relative uncertainty is

$$u_{rt} = \frac{u_t}{\bar{t}} \times 100\% = 1.19\%.$$

The experimentally found t is

$$t = 8.38 \pm 0.10 s, \quad u_{rt} = 1.19\%.$$

5.3 Uncertainty of measurements for the diameter of the balls

The uncertainty (of type-B) of a micrometer used to measure the diameter of the balls is $\Delta_{d,B} = \Delta_{dev} = 0.004 \quad \times 10^{-3} m$. The diameter is found by taking the average of 10 measurements. To estimate type-A uncertainty, the standard deviation of the average value is caluculated as

$$s_{\bar{d}} = \sqrt{\frac{1}{n(n-1)} \sum_{i=1}^{n} (d_i - \bar{d})^2}.$$

Uisng the data from Table 3 we find that $s_{\bar{d}} \approx 0.0014 \times 10^{-3} m$. Considering $t_{0.95} = 2.26$ for $n = 10$, the type-A uncertainty is estimated as $\Delta_{d,A} = 2.26 \times 0.0014 \times 10^{-3} \approx 0.0031 \times 10^{-3} m$. Hence the combined uncertainty is

$$u_d = \sqrt{\Delta_{d,A}^2 + \Delta_{d,B}^2} = \sqrt{(0.003 \times 10^{-3})^2 + (0.004 \times 10^{-3})^2} \approx 0.005 \times 10^{-3} \ m$$

and the corresponding relative uncertainty is

$$u_{rd} = \frac{u_d}{\bar{d}} \times 100\% = 0.25\%.$$

The experimentally found d is

$$d = (1.985 \pm 0.005) \times 10^{-3} m, \quad u_{rd} = 0.25\%.$$

5.4 Uncertainty of measurements for the inner diameter of the flask

The uncertainty (of type-B) of a calliper used to measure the inner diameter of the flask is $\Delta_{D,B} = \Delta_{dev} = 0.02 \times 10^{-3} m$. The diameter is found by taking the average of 6 measurements. To estimate type-A uncertainty, the standard deviation of the average value is caluculated as

$$s_{\bar{D}} = \sqrt{\frac{1}{n(n-1)} \sum_{i=1}^{n} (D_i - \bar{D})^2}.$$

Uisng the data from Table 4 we find that $s_{\bar{D}} \approx 0.08 \times 10^{-3} m$. Considering $t_{0.95} = 2.57$ for $n = 6$, the type-A uncertainty is estimated as $\Delta_{D,A} = 2.57 \times 0.08 \times 10^{-3} \approx 0.21 \times 10^{-3} m$. Hence the combined uncertainty is

$$u_D = \sqrt{\Delta_{D,A}^2 + \Delta_{D,B}^2} = \sqrt{(0.21 \times 10^{-3})^2 + (0.02 \times 10^{-3})^2} \approx 0.21 \times 10^{-3} \ m$$

and the corresponding relative uncertainty is

$$u_{rD} = \frac{u_D}{\bar{D}} \times 100\% = 0.34\%.$$

The experimentally found D is

$$D = (62.41 \pm 0.21) \times 10^{-3} m, \quad u_{rD} = 0.34\%.$$

5.5 Uncertainty of the density of the metal ball

Density of the metal balls can be found from

$$\rho_2 = \frac{m_0}{V} = \frac{m_0}{\frac{4}{3}\pi(\frac{d}{2})^3} = \frac{6m_0}{\pi d^3} = \frac{6 \times 0.03393 \times 10^{-3}}{3.14159 \times (1.985 \times 10^{-3})^3} \approx 8.285 \times 10^3 kg/m^3.$$

In order to find the propagated uncertainty, first find the partial derivatives

$$\frac{\partial \rho_2}{\partial m_0} = \frac{6}{\pi d^3}, \quad \frac{\partial \rho_2}{\partial d} = \frac{18m_0}{\pi d^4}$$

Hence, using Matlab we can obtain the propagated uncertainty by the formula

$$\begin{aligned} u_{\rho_2} &= \sqrt{(\frac{\partial \rho_2}{\partial m_0})^2 (u_{m_0})^2 + (\frac{\partial \rho_2}{\partial d})^2 (u_d)^2} \\ &= \sqrt{(\frac{6 \times 0.001 \times 10^{-3}}{3.14159 \times (1.985 \times 10^{-3})^3})^2 + (\frac{18 \times 0.03393 \times 10^{-3} \times 0.005 \times 10^{-3}}{3.14159 \times (1.985 \times 10^{-3})^4})^2} \\ &\approx 0.252 \times 10^3 kg/m^3. \end{aligned}$$

Similarly, to find the relative uncertainty, first find the partial derivatives

$$\frac{\partial \ln \rho_2}{\partial m_0} = \frac{1}{m_0}, \quad \frac{\partial \ln \rho_2}{\partial d} = -\frac{3}{d}$$

The relative uncertainty is

$$\begin{aligned} u_{r\rho_2} = \frac{u_{\rho_2}}{\bar{\rho_2}} &= \sqrt{(\frac{\partial \ln \rho_2}{\partial m_0})^2 (u_{m_0})^2 + (\frac{\partial \ln \rho_2}{\partial d})^2 (u_d)^2} \\ &= \sqrt{(\frac{0.001 \times 10^{-3}}{0.03393 \times 10^{-3}})^2 + (\frac{-3 \times 0.005 \times 10^{-3}}{1.985 \times 10-3})^2} \\ &\approx 3.04\% \end{aligned}$$

Hence the experimentally ρ_2 is

$$\rho_2 = (8.285 \pm 0.252) \times 10^3 kg/m^3, \quad u_{r\rho_2} = 3.04\%.$$

5.6 Uncertainty of the viscosity coefficient

The viscosity coeffient can be found from

$$\begin{aligned} \eta &= \frac{2}{9} g R^2 \frac{(\rho_2 - \rho_1)t}{s} \frac{1}{1 + 2.4\frac{R}{R_c}} \\ &= \frac{2}{9} \times 9.794 \times 0.0009925^2 \times \frac{(8285 - 955) \times 8.38}{0.1802} \times \frac{1}{1 + 2.4 \times \frac{0.0009925}{0.031205}} \\ &\approx 0.679 \frac{kg}{m \cdot s}. \end{aligned}$$

In order to find the propagated uncertainty, first find the partial derivatives

$$\frac{\partial\eta}{\partial R} = -\frac{8R^2gt(\rho_2-\rho_1)}{15R_cS(\frac{2.4R}{R_c}+1)^2} + \frac{4Rgt(\rho_2-\rho_1)}{9S\frac{2.4R}{R_c}+1}$$
$$\frac{\partial\eta}{\partial \rho_2} = \frac{2R^2gt}{9S(\frac{2.4R}{R_c}+1)}$$
$$\frac{\partial\eta}{\partial \rho_1} = -\frac{2R^2gt}{9S(\frac{2.4R}{R_c}+1)}$$
$$\frac{\partial\eta}{\partial t} = \frac{2R^2g(\rho_2-\rho_2)}{9S(\frac{2.4R}{R_c}+1)}$$
$$\frac{\partial\eta}{\partial S} = -\frac{2R^2gt(\rho_2-\rho_1)}{S^2(\frac{2.4R}{R_c}+1)}$$
$$\frac{\partial\eta}{\partial R_c} = \frac{8R^3gt(\rho_2-\rho_1)}{15R_c^2S(\frac{2.4R}{R_c}+1)^2}$$

Hence, using Matlab we can obtain the propagated uncertainty by the formula

$$u_\eta = \sqrt{(\frac{\partial\eta}{\partial R})^2(u_R)^2 + (\frac{\partial\eta}{\partial\rho_2})^2(u_{\rho_2})^2 + (\frac{\partial\eta}{\partial\rho_1})^2(u_{\rho_1})^2 + (\frac{\partial\eta}{\partial u_t})^2(u_t)^2 + (\frac{\partial\eta}{\partial S})^2(u_S)^2 + (\frac{\partial\eta}{\partial u_{R_c}})^2(u_{R_c})^2}$$
$$\approx 0.03\frac{kg}{m\cdot s}.$$

Similarly, to find the relative uncertainty, first find the partial derivatives

$$\frac{\partial\ln\eta}{\partial R} = \frac{9s}{2R^2gt\left(\rho_2-\rho_1\right)}\left(\frac{2.4R}{R_c}+1\right)\left(-\frac{8R^2gt\left(\rho_2-\rho_1\right)}{15R_cs\left(\frac{2.4R}{R_c}+1\right)^2} + \frac{4Rgt\left(\rho_2-\rho_1\right)}{9s\left(\frac{2.4R}{R_c}+1\right)}\right)$$
$$\frac{\partial\ln\eta}{\partial\rho_2} = \frac{1}{\rho_2-\rho_1}$$
$$\frac{\partial\ln\eta}{\partial\rho_1} = -\frac{1}{\rho_2-\rho_1}$$
$$\frac{\partial\ln\eta}{\partial t} = \frac{1}{t}$$
$$\frac{\partial\ln\eta}{\partial s} = -\frac{1}{s}$$
$$\frac{\partial\ln\eta}{\partial R_c} = \frac{2.4R}{R_c^2\left(\frac{2.4R}{R_c}+1\right)}$$

The relative uncertainty is

$$u_\eta = [(\frac{\partial \ln \eta}{\partial R})^2 (u_R)^2 + (\frac{\partial \ln \eta}{\partial \rho_2})^2 (u_{\rho_2})^2 + (\frac{\partial \ln \eta}{\partial \rho_1})^2 (u_{\rho_1})^2 + (\frac{\partial \ln \eta}{\partial u_t})^2 (u_t)^2 + (\frac{\partial \ln \eta}{\partial S})^2 (u_S)^2 + (\frac{\partial \ln \eta}{\partial u_{R_c}})^2 (u_{R_c})^2]^{\frac{1}{2}} \approx 3.91\%.$$

Hence the experimentally η is

$$\eta = 0.679 \pm 0.03 \frac{kg}{m \cdot s}, \quad u_{r_\eta} = 3.91\%.$$

6 Conclusions and discussion

In this experiment the viscosity coefficient η is measured by Stokes' method. We obtain the density of the metal balls by measuring its diameter and mass. We obtain the constant velocity from the traveling distance and time. We measure the inner diameter of the flask and the read the density of castor oil. Finally, the experimentally found η of castor oil in the environment of $24\,°\mathrm{C}, 1atm$ is

$$\eta = 0.679 \pm 0.03 \frac{kg}{m \cdot s}, \quad u_{r_\eta} = 3.91\%.$$

After the experiment, I find there are some operations probably leading to deviations in the fundamental Stokes' method. Firstly, I find it hard to confirm that the two beams are parallel. It'll be better if there exist a fixed equipment to measure the distance between the transmitting and the receiving end. Secondly, I find that among the directly measured data, the relative uncertainty of traveling time is the biggest. Stopwatch is indeed not accurate enough to eliminate the deviation. Therefore,the results can be more accurate if the time is recorded by computer when the ball blocks the beams.

References

[1] Qin Tian, Zeng Ming, Cao Jianjun, Han Xugen, Feng Yaming, Mateusz Krzyzosiak, Physics Laboratory (Vp141/Vp241) Student Handbook Introduction to Measurement Data Analysis.

[2] Qin Tian, Feng Yaming, Mateusz Krzyzosiak Physics Laboratory Vp141 Exercise 2 Measurement of Fluid Viscosity.