UM-SJTU Joint Institute

Physical Laboratory

VP141

Laboratory Report

Exercise 5

Damped and Driven Oscillations

Mechanical Resonance

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Contents

1 Introduction

The objective of this exercise is to study damped and driven oscillations in mechanical systems using the Pohl resonator. For driven oscillations, we will also observe and quantify the mechanical resonance phenomenon.

If a periodically varying external force is applied to a damped harmonic oscillator, the resulting motion is called forced (or driven) oscillations, and the external force is called the driving force. Assuming that the driving force is of the form

$$F = F_0(sin\omega t + \delta),$$

with the amplitude F 0 and angular frequency , the resulting steady-state forced oscillations will be simple harmonic with the angular frequency equal to that of the driving force. The amplitude of these steady-state oscillations turns out to depend on the angular frequency of the the driving force, in particular on how far it is from the natural angular frequency, and the damping coefficient. The amplitude may become quite large, and this phenomenon is known as the mechanical resonance.

Another interesting property of driven steady-state oscillations is the fact that there is a phase lag between the driving force and the displacement from the equilibrium position of the oscillating particle. This phase lag reaches $\pi/2$ (a quarter of the cycle) when the system is driven at the natural angular frequency.

In this experiment, forced oscillation of a balance wheel will be studied. The corresponding quantities (such as the force and the position) will be replaced by their angular counterparts.

The driving torque $\tau_{dr} = \tau_0 cos\omega t$ and a damping torque $\tau_f = -b\frac{d\theta}{dt}$, Also, we know the restoring torque $\tau = -k\theta$, its equation od motion is of the form

$$I\frac{d^2\theta}{dt^2} = -k\theta - b\frac{d\theta}{dt} + \tau_0 cos\omega t, \tag{1}$$

where I is the moment of inertia of the balance wheel, τ_0 is the amplitude of the driving torque, and ω is the angular frequency of the driving torque. Introducing the symbols

$$\omega_0 = \frac{k}{I}, \quad 2\beta = \frac{b}{I}, \quad \mu = \frac{\tau_0}{I},$$

Eq. 1 can be rewitten as

$$\frac{d^2\theta}{dt^2} + 2\beta\frac{d\theta}{dt} + \omega_0^2\theta = \mu cos\omega t. \tag{2}$$

Thw solution to Eq 2 is

$$\theta(t) = \theta_{tr}(t) + \theta_{st}cos(\omega t + \varphi),$$

where the former term θ_{tr} denotes the transient solution that vanished exponentially as $t \to \infty$. The steady-state oscillation is with the amplitude

$$\theta_{st} = \frac{\mu}{\sqrt{(\omega_0^2 - \omega^2)^2 + 4\beta^2\omega^2}}$$

For small values of the damping coefficient β, the resonance angular frequency is close to the the natural angular frequency, and the amplitude of steady-state oscillations becomes large. The dependence of both the amplitude and the phase shift on the driving angular frequency are shown in the left and right Figure 1, respectively, for different values of the damping coefficient.

2 Experiment Setup

The BG-2 Pohl resonator consists of two main parts: a vibrometer and a control box. The vibrometer is shown in Figure 2. A copper balance wheel is mounted on a supporting frame, and the axis of the balance wheel is attached to the supporting frame with a scroll spring. The spring provides an elastic restoring torque to the wheel, which makes the balance wheel rotating about an equilibrium position.

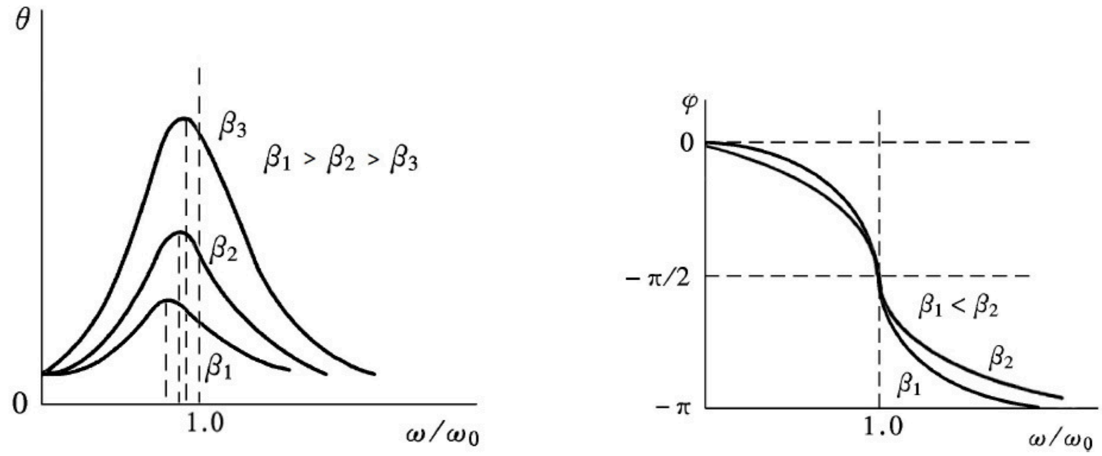


Figure 1: The dependence of the amplitude (left) and phase shift (right) of steady-state driven oscillations

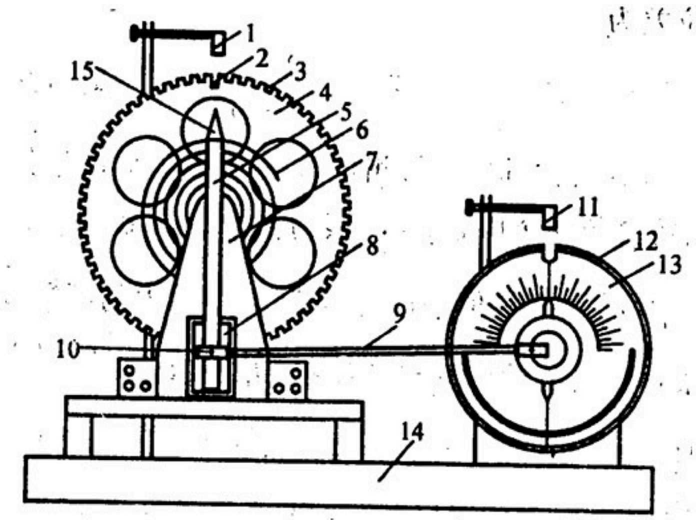
There are many notches on the edge of the balance wheel with one notch being much deeper than the others. A photoelectric detector is set above the deep notch. The detector is used to measure the amplitude and the period of oscillations, and it is connected to the electronic control box.

A pair of coils is placed at the bottom of the supporting frame, with the balance wheel fitting exactly into the gap between the two coils. Due to electromagnetic induction, the wheel will be acted upon an electromagnetic damping force when the coils are carrying current, and the magnitude of the damping force can be controlled by changing the current.

The device is equipped with a motor with an electric wheel and a rod used to drive the wheel. There is a Period Selection switch and a Period of Driving Force knob on the electric control box, which allow to control the speed of the motor precisely. Another photoelectric detector is set above the turntable and connected to the control box to measure the period of driving force.

The phase shift can be measured using the glass turntable with an angle scale and a strobe light. The strobe is controlled by the photoelectric detector above the wheel. When the deep notch passes the equilibrium position, the detector sends a signal and the strobe flashes. In a steady state, a line on the angle scale will be highlighted by the flash of the strobe and the phase difference can be read from the angle scale directly.

The amplitude of oscillations is measured by counting the notches on the wheel, and this measurement is performed by a photoelectric detector with the result displayed on the electronic control box.



1. Photoelectric detector (光电门) 2. Deep notch (深凹槽) 3. Shallow notch (浅凹槽)
4. Copper balance wheel (铜质摆轮) 5. Swinging rod (摇杆) 6. Scroll spring (蜗卷弹簧)
7. Supporting frame (支撑架) 8. Damping coil (阻尼线圈) 9. Connecting rod (连杆)
10. Adjustment screw (调节螺钉) 11. Photoelectric detector 12. Angle scale (角度盘)
13. Glass turntable (玻璃转盘) 14. Pedestal (底座) 15. Locking screw (夹持螺钉)

Figure 2: Vibrometer

The function Amplitude Display shows the oscillation amplitude of the balance wheel and Period Display shows the oscillation period in two modes. When the Period Selection switch is at position “1” , a single oscillation period will be displayed; when the Period Selection switch is at “10”, the time of 10 oscillation periods will be displayed. The reset button works only when the Period Selection button is at “10”.

The period of the driving force can be changed precisely by using the Period of the driving force knob, but please pay attention that the scale on the knob is not very accurate.

The Damping Selection knob changes the damping force by adjusting the electric current through the coils at the bottom of the wheel. There are six options, ranging from “0” (no current) to “5” (current of about 0.6 A). Here we use “2”, “3” or “4”.

The strobe generates a flash that allows you to read the phase difference from the angle scale directly. To protect the strobe, you should turn on the Strobe switch only when measuring the phase difference.

The Motor Switch is used to control the motor. You should turn the motor off when measuring the damping coefficient and the natural angular frequency.

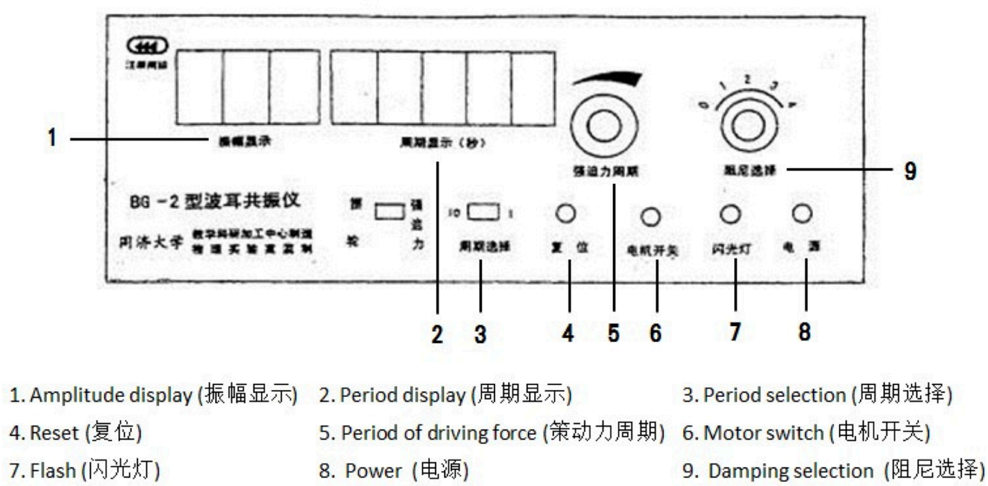


Figure 3: The front panel of the control box

2.1 Devices precision

The precisions of the devices are shown in Table 1.

Devices	Precision	Unit
Timer on BG-2 Pohl resonator	0.001	[s]
Angle on BG-2 Pohl resonator	1	[°]

Table 1: Devices precision

3 Measurement Procedure

3.1 Natural angular frequency

1. Turn the Damping Selection knob to ”0”.
2. Rotate the balance wheel to the initial angular position $\theta_0 \approx 150°$ and release it. Record the time of 10 periods.
3. REpeat for four times and calculate the natural angular frequency ω_0.

3.2 Damping coefficient

1. Turn the Damping Selection knob to ”2”, and the selection should not be changed during this part.

2. Rotate the balance wheel to the initial amplitude of approximately 150° and release it. Record the amplitude of each period and the time of 10 periods.

3. The solution to the homogeneous equation of motion, with the corresponding initial conditions, is $\theta_t = \theta_0 e^{-\beta t} cos(\omega_f t + \alpha)$. Hence $\theta_1 = \theta_0 e^{-\beta T}, \quad \theta_2 = \theta_0 e^{-\beta(2T)}, \ldots, \theta_n = \theta_0 e^{-\beta(nT)}$. The damping coefficient β can then be calculated as

$$ln\frac{\theta_i}{\theta_j} = ln\frac{\theta_0 e^{-\beta(iT)}}{\theta_0 e^{-\beta(jT)}} = (j-i)\beta T.$$

4. The value of T should be the average period, and $ln\frac{\theta_i}{\theta_{i+5}}$ should be obtained by the successive difference method as

$$\beta = \frac{1}{5T} ln\frac{\theta_i}{\theta_{i+5}}.$$

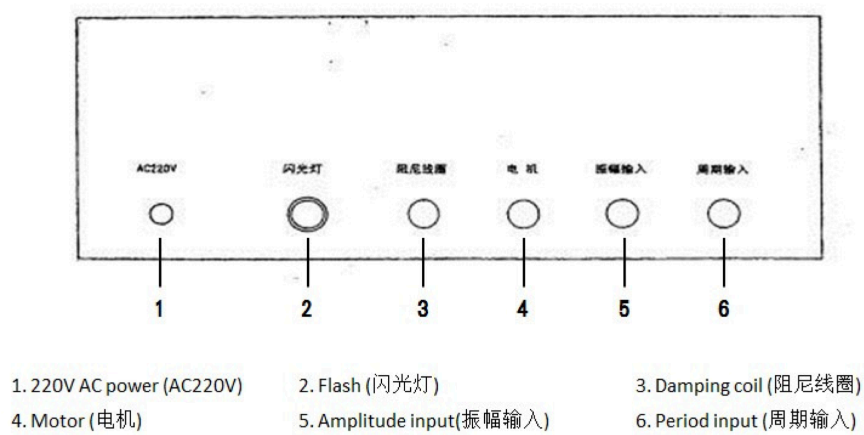


Figure 4: The rear panel of the control box

3.3 θ_{st} vs. ω and φ vs. ω Characteristics of forced oscillations

1. Keep the Damping Selection ar ”2”, and set the speed of the motor. Record the amplitude θ_{st}, the period T, and the phase shift φ when the oscillation reaches a

steady state.

2. Repeat the steps above by changing the speed of the motor. It will result in a change of the phase shift φ (referred to as $\Delta\varphi$). More data should be collected when φ and θ_{st} change rapidly (e.g. near to the resonance point). At least 15 data should be collected for plotting.

3. Choose Damping Selection "1" or "3". Repeat the above steps.

4. Plot the $\theta_{st}(\omega)$ characteristics, with ω/ω_0 on the horizontal axis and θ_{st} on the vertical axis. Two sets of data should be plotted on the same graph. Plot the $\varphi(\omega)$ characteristics, with ω/ω_0 on the horizontal axis and φ on the vertical axis. Two sets of data should be plotted on the same graph.

4 Result

5 Results

5.1 Measurement for natural angular frequency

We calculate the angular frequency from Table 2 by the formula

$$\omega_0 = \frac{2\pi}{T}.$$

	$10T[s] \pm 0.001[s]$
1	15.790
2	15.801
3	15.778
4	15.800

Table 2: Measurement of ten periods for the natural frequency

Hence the average value of $10T$ should be calculated as

$$\overline{10T} = \frac{1}{4}\sum_{i=1}^{4}(10T)_i = 15.79225 \pm 0.0107\ s, \quad u_{10T,r} = 0.07\%.$$

The value of ω_0 is

$$\overline{\omega_0} = \frac{20\pi}{10T} = \frac{20 \times 3.1416}{15.79225} = 3.9787 \pm 0.006\ rad/s, \quad u_{\omega_0,r} = 0.15\%.$$

5.2 Measurement for damping coefficient

The damping coefficient can be calculated by the following formula.

$$\beta = \frac{1}{5T} \ln \frac{\theta_i}{\theta_{i+5}}.$$

In the experiment, we choose Damping Selection 2.

Amplitude[°]±1[°]		Amplitude[°]±1[°]		$\ln(\theta_i/\theta_{i+5})$
θ_0	149	θ_5	95	0.4501
θ_1	136	θ_6	86	0.4583
θ_2	124	θ_7	79	0.4508
θ_3	114	θ_8	72	0.4595
θ_4	103	θ_9	65	0.4603
The average value of $\ln(\theta_i/\theta_{i+5})$				0.4558

Table 3: Measurement of the damping coefficient for Damping Selection 2

The experimental value of $\ln(\theta_i/\theta_{i+5})$ is shown below

$$\ln(\theta_i/\theta_{i+5}) = 0.4558 \pm 0.0050, \quad u_r = 1.09\%$$

Here, $T = 15.79225/10 = 1.5792 \pm 0.0001s$. Then we can easily obtain β as well,

$$\beta = \frac{1}{5 \times 1.5792} \times 0.4558 = 0.0577 \pm 0.003s^{-1}, \quad u_{\beta,r} = 5.2\%$$

5.3 Measurement for θ_{st} vs. ω and φ vs. ω

To study the relation between φ and ω/ω_0,

first we get the raw data of 10T, ϕ and θ, we then process the raw data and list them in Table 6 and Table 7.

	$10T[s] \pm 0.001[s]$	$\varphi[^\circ] \pm 1[^\circ]$	$\theta[^\circ] \pm 1[^\circ]$
1	15.098	-162	38
2	15.123	-163	39
3	15.542	-143	87
4	15.672	-118	130
5	15.707	-109	138
6	15.724	-103	141
7	15.736	-100	143
8	15.755	-94	145
9	15.763	-92	146
10	15.774	-89	144
11	15.788	-86	144
12	15.797	-84	144
13	15.810	-82	144
14	15.820	-79	142
15	15.841	-75	140
16	15.907	-64	130
17	15.946	-57	124
18	16.050	-46	105
19	16.131	-40	92
20	16.237	-33	78
21	16.472	-22	54
22	16.603	-18	46

Table 4: θ vs. $10T$ and φ vs. $10T$ raw data for Damping selection 2

	$10T[s] \pm 0.001[s]$	$\varphi[^\circ] \pm 1[^\circ]$	$\theta[^\circ] \pm 1[^\circ]$
1	15.057	-163	35
2	15.302	-155	50
3	15.526	-142	79
4	15.657	-123	108
5	15.708	-111	120
6	15.745	-104	125
7	15.760	-97	127
8	15.794	-93	128
9	15.789	-92	128
10	15.803	-89	128
11	15.814	-86	128
12	15.830	-83	128
13	15.849	-80	126
14	15.883	-74	124
15	15.903	-69	120
16	15.933	-62	114
17	16.013	-55	106
18	16.102	-47	93
19	16.168	-41	83
20	16.290	-33	70
21	16.452	-25	55
22	16.585	-18	46

Table 5: θ vs. $10T$ and φ vs. $10T$ raw data for Damping selection 3

Then We know that $\omega/\omega_0 = T_0/T$, thus we can have the following processed data.

	ω/ω_0	$\varphi \pm 1[°]$
1	1.0460	-162
2	1.0443	-163
3	1.0161	-143
4	1.0077	-118
5	1.0054	-109
6	1.0043	-103
7	1.0036	-100
8	1.0024	-94
9	1.0019	-92
10	1.0012	-89
11	1.0003	-86
12	0.9997	-84
13	0.9989	-82
14	0.9982	-79
15	0.9969	-75
16	0.9928	-64
17	0.9904	-57
18	0.9839	-46
19	0.9790	-40
20	0.9726	-33
21	0.9587	-22
22	0.9512	-18

Table 6: φ vs. ω/ω_0
Damping selection 2

	ω/ω_0	$\varphi \pm 1[°]$
1	1.0488	-163
2	1.0320	-155
3	1.0171	-142
4	1.0086	-123
5	1.0054	-111
6	1.0030	-104
7	1.0020	-97
8	0.9999	-93
9	1.0002	-92
10	0.9993	-89
11	0.9986	-86
12	0.9976	-83
13	0.9964	-80
14	0.9943	-74
15	0.9930	-69
16	0.9912	-62
17	0.9862	-55
18	0.9808	-47
19	0.9768	-41
20	0.9694	-33
21	0.9599	-25
22	0.9522	-18

Table 7: φ vs. ω/ω_0
Damping selection 3

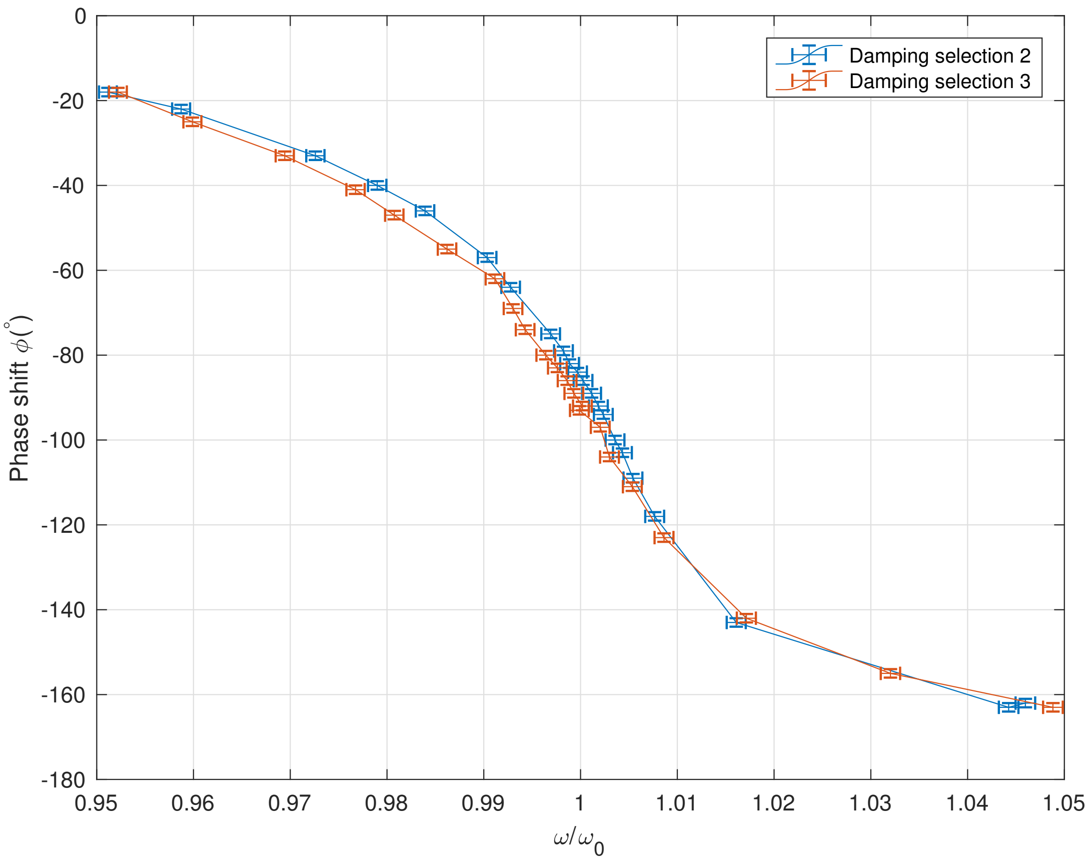


Figure 5: Phase shift φ vs. ω/ω_0

To study the relation between θ_{st} and ω/ω_0, we process the raw data and list them in Table 8 and Table 9.

	ω/ω_0	$\theta \pm 1[°]$
1	1.0460	38
2	1.0443	39
3	1.0161	87
4	1.0077	130
5	1.0054	138
6	1.0043	141
7	1.0036	143
8	1.0024	145
9	1.0019	146
10	1.0012	144
11	1.0003	144
12	0.9997	144
13	0.9989	144
14	0.9982	142
15	0.9969	140
16	0.9928	130
17	0.9904	124
18	0.9839	105
19	0.9790	92
20	0.9726	78
21	0.9587	54
22	0.9512	46

Table 8: θ vs. ω/ω_0
Damping selection 2

	ω/ω_0	$\theta \pm 1[°]$
1	1.0488	35
2	1.0320	50
3	1.0171	79
4	1.0086	108
5	1.0054	120
6	1.0030	125
7	1.0020	127
8	0.9999	128
9	1.0002	128
10	0.9993	128
11	0.9986	128
12	0.9976	128
13	0.9964	126
14	0.9943	124
15	0.9930	120
16	0.9912	114
17	0.9862	106
18	0.9808	93
19	0.9768	83
20	0.9694	70
21	0.9599	55
22	0.9522	46

Table 9: θ vs. ω/ω_0
Damping selection 3

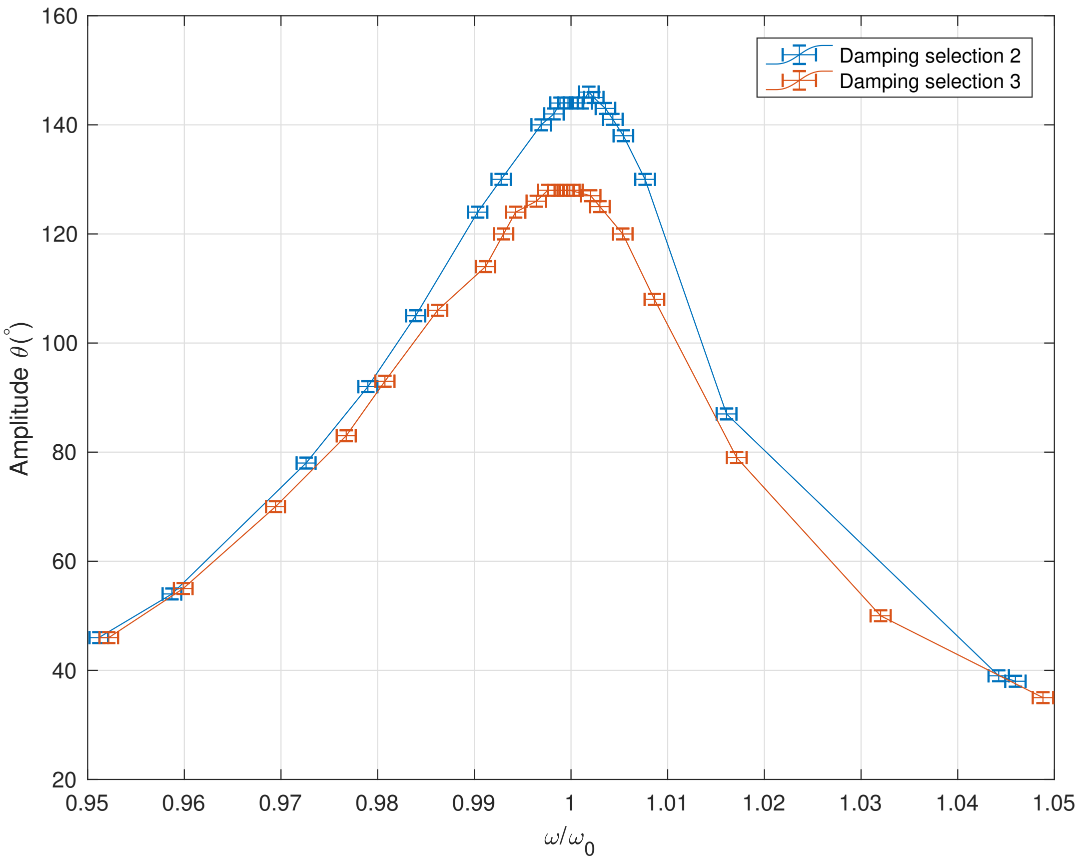


Figure 6: Amplitude θ_{st} vs. ω/ω_0

6 Measurement Uncertainty Analysis

6.1 Uncertainty for natural angular frequency

To estimate type-A uncertainty of $(10T)$, the standard deviation of the average value can be calculated as

$$t_{0.95} = 3.18, \quad n = 4,$$

$$s_{\overline{10T}} = \sqrt{\frac{1}{n(n-1)} \sum_{i=1}^{n} ((10T)_i - \overline{10T})^2}.$$

$$\Delta_{10T,A} = \frac{3.18}{\sqrt{4}} \times 0.0076811 \approx 0.017s.$$

The type-B uncertainty of $(10T)$ is 0.001s. Hence, the uncertainty of $(10T)$ is

$$u_{10T} = \sqrt{\Delta_A^2 + \Delta_B^2} = \sqrt{0.017^2 + 0.001^2} \approx 0.017s$$

For the uncertainty of ω_0,

$$\omega_0 = \frac{20\pi}{10T}, \quad \frac{\partial \omega_0}{\partial (10T)} = -\frac{20\pi}{(10T)^2}$$

$$u_{\omega_0} = \sqrt{\frac{\partial \omega_0}{\partial (10T)}^2 (u_{10T})^2} = \frac{20\pi}{(10T)^2} u_{(10T)}$$
$$= \frac{20 \times 3.1416}{15.485^2} \times 0.017 = 0.004 rad/s,$$

$$u_{\omega_0,r} = \frac{u_{\omega_0}}{\omega_0} \times 100\% = \frac{0.004}{4.058} = 0.10\%$$

6.2 Uncertainty of damping coefficient

The type-A uncertainty of $\ln(\theta_i/\theta_{i+5})$ can be determined by calculating $s \times t_{0.95}/\sqrt{n}$. Let $k = s \times t_{0.95}/\sqrt{n}$.

$$t_{0.95} = 2.78, \quad n = 5$$

$$\Delta_{A,k} = t_{0.95}/\sqrt{5} \times \sqrt{\frac{1}{5-1} \sum_{i=1}^{5} (k_i - \bar{k})^2} = 2.78/\sqrt{5} \times 0.00444 = 0.005$$

Then, for the type-B uncertainty,

$$\frac{\partial k}{\partial \theta_i} = \frac{1}{\theta_i}$$

$$\frac{\partial k}{\partial \theta_{i+5}} = -\frac{1}{\theta_{i+5}}$$

$$\Delta_{B,k} = \sqrt{\frac{\partial k}{\partial \theta_i}^2 (u_{\theta_i})^2 + \frac{\partial k}{\partial \theta_{i+5}}^2 (u_{\theta_{i+5}})^2} = \sqrt{(\frac{u_{\theta_i}}{\theta_i})^2 + (\frac{u_{\theta_{i+5}}}{\theta_{i+5}})^2}$$

When $i = 0$, $\theta_0 = 89°$, $u_{\theta_2} = 1°$, $\theta_5 = 54°$, $u_{\theta_5} = 1°$.

$$\Delta_{B,k} = \sqrt{(\frac{1}{89})^2 + (\frac{1}{54})^2} = 0.022$$

Considering the type-B uncertainty of $\ln(\theta_i/\theta_{i+5})$, the combined uncertainty is

$$u_k = \sqrt{(0.005)^2 + (0.022)^2} = 0.022,$$
$$u_{k,r} = \frac{u_k}{\bar{k}} \times 100\% = \frac{0.022}{0.511} \times 100\% = 4\%$$

Hence

$$\ln(\theta_i/\theta_{i+5}) = 0.456 \pm 0.005, \quad u_r = 1.09\%$$

For $10T = 15.792 \pm 0.001s$, we know that

$$T = 1.5792 \pm 0.0001s, \quad u_{r,T} = 0.006\%.$$

Then calculate the uncertainty of $\beta = \ln(\theta_i/\theta_{i+5})/(5T)$.

$$\frac{\partial \beta}{\partial T} = -\frac{k}{5T^2}$$

$$\frac{\partial \beta}{\partial k} = \frac{1}{5T}$$

$$
\begin{aligned}
u_\beta &= \sqrt{(\frac{\partial \beta}{\partial T})^2 (u_T)^2 + (\frac{\partial \beta}{\partial k})^2 (u_k)^2} = \sqrt{(\frac{k^2}{25T^4})(u_T)^2 + (\frac{1}{25T^2})(u_k)^2} \\
&= \sqrt{(\frac{0.511^2}{25 \times 1.5792^4})(0.0001^2) + (\frac{0.022^2}{25 \times 1.5792^2})} \\
&= 0.003s^{-1}
\end{aligned}
$$

where $k = \ln(\theta_i/\theta_{i+5})$.

Thus,

$$\beta = 0.0648 \pm 0.003s^{-1}, \quad u_{\beta,r} = 5\%$$

6.3 Uncertainty of θ_{st} vs. ω and φ vs. ω

We denote that $r = \omega/\omega_0$. To determine the uncertainty of ratio, we know that

$$
\begin{aligned}
r &= \frac{20\pi}{10T\omega_0} \\
\frac{\partial r}{\partial (10T)} &= -\frac{20\pi}{(10T)^2\omega_0} \\
\frac{\partial r}{\partial \omega_0} &= -\frac{20\pi}{(10T)\omega_0^2}
\end{aligned}
$$

$$
\begin{aligned}
u_r &= \sqrt{(\frac{\partial r}{\partial (10T)})^2 (u_{10T})^2 + (\frac{\partial r}{\partial \omega_0})^2 (u_{\omega_0})^2} \\
&= \sqrt{\frac{400\pi^2}{(10T)^4\omega_0^2}(u_{10T})^2 + \frac{400\pi^2}{(10T)^2\omega_0^4}(u_{\omega_0})^2}.
\end{aligned}
$$

As is calculated before, $\omega_0 = 3.9787 \pm 0.006\ rad/s$.

For instance, when $10T = 15.79225 \pm 0.001s$, the uncertainty of r is

$$u_r = \sqrt{\frac{400 \times 3.1416^2}{15.79225^4 \times 3.9787^2}(0.001)^2 + \frac{400 \times 3.1416^2}{15.79225^2 \times 3.9787^4}(0.004)^2} = 0.0009$$

For the uncertainty of φ and θ, these two data only obtain type-B uncertainty, so the combined uncertainty of them are both equal to $\Delta_B = 1°$, thus

$$u_\varphi = 1°$$

$$u_\theta = 1°.$$

7 Conclusion and Discussion

7.1 Natural angular frequency

Although by theory amplitude doesn't affect the period, as the amplitude gets larger, the negative work done by the frictional force isn't negligible. Thus, we need to keep the amplitude the same for every test to reduce this influence factor.

The third measurement of ten periods is obviously smaller than the other three, which might result from a severely different amplitude.

On the other hand, by measuring 10 times of T, the experimental value of ω_0 is quite precise (with small deviation). Still, $u_{10T,r} = 0.07\%$, that means our experiment setup works pretty well.

For recommendation, I think introducing a control system for the amplitude in addition to bare hand can bring more precision.

7.2 Damping coefficient

The damping coefficient is calculated from dividing the decreasing amplitude and log it. When calculating the damping coefficient, successive difference method is applied to reduce the deviation.

For more precision than successive difference method, we could use curve fitting, thus the whole list can be used to determine the damping coefficient β from $\theta_n = \theta_0 e^{-\beta(nT)}$, instead of only using 5 groups of isolated points.

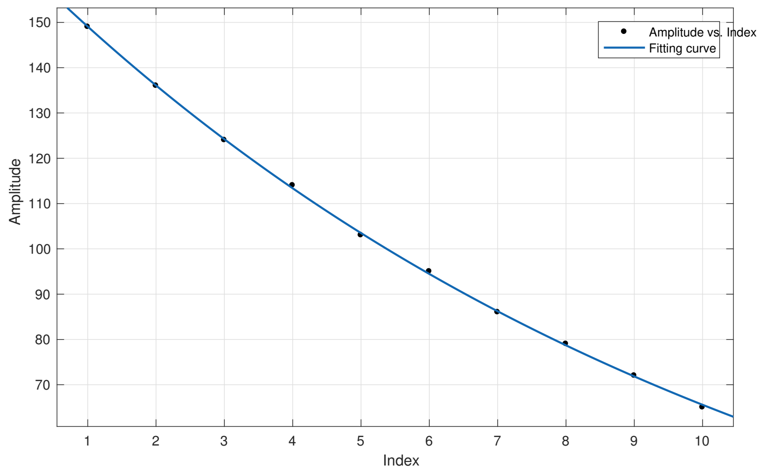


Figure 7: Curve fitting example

And we can find the b of the curve, which is -0.09125 Comparing $-5 \times b = 0.4562$ with $\beta = 0.4558$

$$u_{\beta,r} = 0.0878\%$$

We can reach a conclusion that although curve fitting can introduce more precise process, the present method is quite accurate.

7.3 φ vs. ω

Both of the φ vs. ω curve are in the shape of arc cotangent (see Figure 5). In theory we know that for the smaller damping coefficient, the left side curve of $\omega/\omega_0 = 1.0$ is higher than the other and the right side curve is lower than that.

My fit curve quit matches what a arc cotangent line looks like, one curve higher before the intersection point and lower after the intersection point. The time duration will be longer to reach the stable situation. The second half of phase points number lesser than the first one because it's hard to predict how the φ goes.

For recommendation, it hurts for us to bear the flashlight for reading the phase data. Electronic devices such as a camera can be introduced to record the data, instead of directly reading the data with human eyes.

7.4 θ_{st} vs. ω

The shape of both curve meets our expectation (in Figure 6).

In the figure, the curve's shape and position meet the expectation, that is damping 3's damping coefficient is greater than damping 2's. And the curve is same as the theory.

From the figure, it's obvious whether the peaks occur on the left or right hand side of the straight line $\omega/\omega_0 = 1.0$ for Damping Selection 2, but hard for Damping Selection 3. It is because that the deviation is smaller for Damping Selection 3. In theory, the peaks should occur on the slightly left of the straight line.

8 Reference

References

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9 Data Sheet