Physics Laboratory I

VP141

Exercise V

Damped and Driven Oscillations.
Mechanical Resonance

Name:
Tianyi Ge

Student Number:
516370910168

Teammate:
Yukang Tian

Student Number:
516370910093

Group:
17

Teaching Assistant:
Haojie Ye

Instructor:
Prof. Mateusz Krzyzosiak

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1 Introduction

The objective of this exercise is to study damped and driven oscillations in mechanical systems using the Pohl resonator. For driven oscillations, we will also observe and quantify the mechanical resonance phenomenon.

If a periodically varying external force is applied to a damped harmonic oscillator, the resulting motion is called forced (or driven) oscillations, and the external force is called the driving force. Assuming that the driving force is of the form

$$F = F_0(sin\omega t + \delta),$$

with the amplitude F 0 and angular frequency , the resulting steady-state forced oscillations will be simple harmonic with the angular frequency equal to that of the driving force. The amplitude of these steady-state oscillations turns out to depend on the angular frequency of the the driving force, in particular on how far it is from the natural angular frequency, and the damping coefficient. The amplitude may become quite large, and this phenomenon is known as the mechanical resonance.

Another interesting property of driven steady-state oscillations is the fact that there is a phase lag between the driving force and the displacement from the equilibrium position of the oscillating particle. This phase lag reaches $\pi/2$ (a quarter of the cycle) when the system is driven at the natural angular frequency.

In this experiment, forced oscillation of a balance wheel will be studied. The corresponding quantities (such as the force and the position) will be replaced by their angular counterparts.

The driving torque $\tau_{dr} = \tau_0 cos\omega t$ and a damping torque $\tau_f = -b\frac{d\theta}{dt}$, Also, we know the restoring torque $\tau = -k\theta$, its equation od motion is of the form

$$I\frac{d^2\theta}{dt^2} = -k\theta - b\frac{d\theta}{dt} + \tau_0 cos\omega t, \tag{1}$$

where I is the moment of inertia of the balance wheel, τ_0 is the amplitude of the driving torque, and ω is the angular frequency of the driving torque. Introducing the symbols

$$\omega_0 = \frac{k}{I}, \quad 2\beta = \frac{b}{I}, \quad \mu = \frac{\tau_0}{I},$$

Eq. 1 can be rewitten as

$$\frac{d^2\theta}{dt^2} + 2\beta\frac{d\theta}{dt} + \omega_0^2\theta = \mu cos\omega t. \tag{2}$$

Thw solution to Eq 2 is

$$\theta(t) = \theta_{tr}(t) + \theta_{st}cos(\omega t + \varphi),$$

where the former term θ_{tr} denotes the transient solution that vanished exponentially as $t \to \infty$. The steady-state oscillation is with the amplitude

$$\theta_{st} = \frac{\mu}{\sqrt{(\omega_0^2 - \omega^2)^2 + 4\beta^2\omega^2}}$$

For small values of the damping coefficient , the resonance angular frequency is close to the the natural angular frequency, and the amplitude of steady-state oscillations becomes large. The dependence of both the amplitude and the phase shift on the driving angular frequency are shown in the left and right Figure 1, respectively, for different values of the damping coefficient.

2 Experiment Setup

The BG-2 Pohl resonator consists of two main parts: a vibrometer and a control box. The vibrometer is shown in Figure 2. A copper balance wheel is mounted on a supporting frame, and the axis of the balance wheel is attached to the supporting frame with a scroll spring. The spring provides an elastic restoring torque to the wheel, which makes the balance wheel rotating about an equilibrium position.

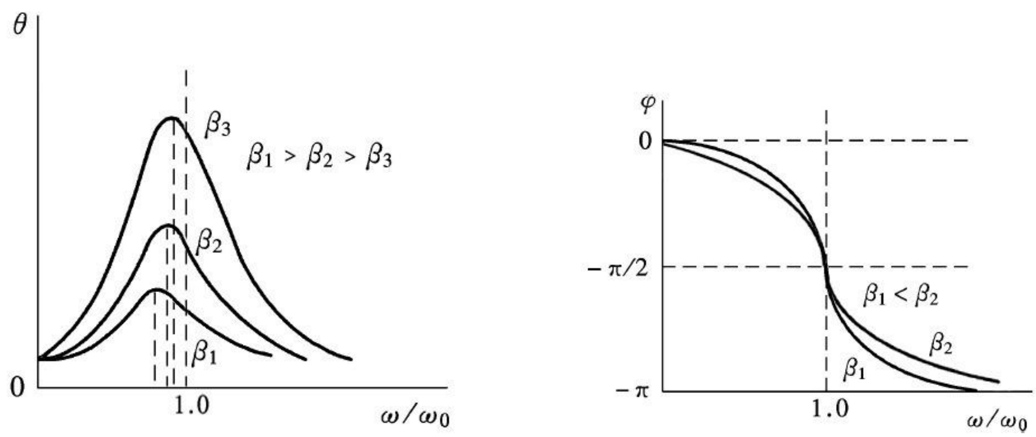

Figure 1: The dependence of the amplitude (left) and phase shift (right) of steady-state driven oscillations

There are many notches on the edge of the balance wheel with one notch being much deeper than the others. A photoelectric detector is set above the deep notch. The detector is used to measure the amplitude and the period of oscillations, and it is connected to the electronic control box.

A pair of coils is placed at the bottom of the supporting frame, with the balance wheel fitting exactly into the gap between the two coils. Due to electromagnetic induction, the wheel will be acted upon an electromagnetic damping force when the coils are carrying current, and the magnitude of the damping force can be controlled by changing the current.

The device is equipped with a motor with an electric wheel and a rod used to drive the wheel. There is a Period Selection switch and a Period of Driving Force knob on the electric control box, which allow to control the speed of the motor precisely. Another photoelectric detector is set above the turntable and connected to the control box to measure the period of driving force.

The phase shift can be measured using the glass turntable with an angle scale and a strobe light. The strobe is controlled by the photoelectric detector above the wheel. When the deep notch passes the equilibrium position, the detector sends a signal and the strobe flashes. In a steady state, a line on the angle scale will be highlighted by the flash of the strobe and the phase difference can be read from the angle scale directly.

The amplitude of oscillations is measured by counting the notches on the wheel, and this measurement is performed by a photoelectric detector with the result displayed on the electronic control box.

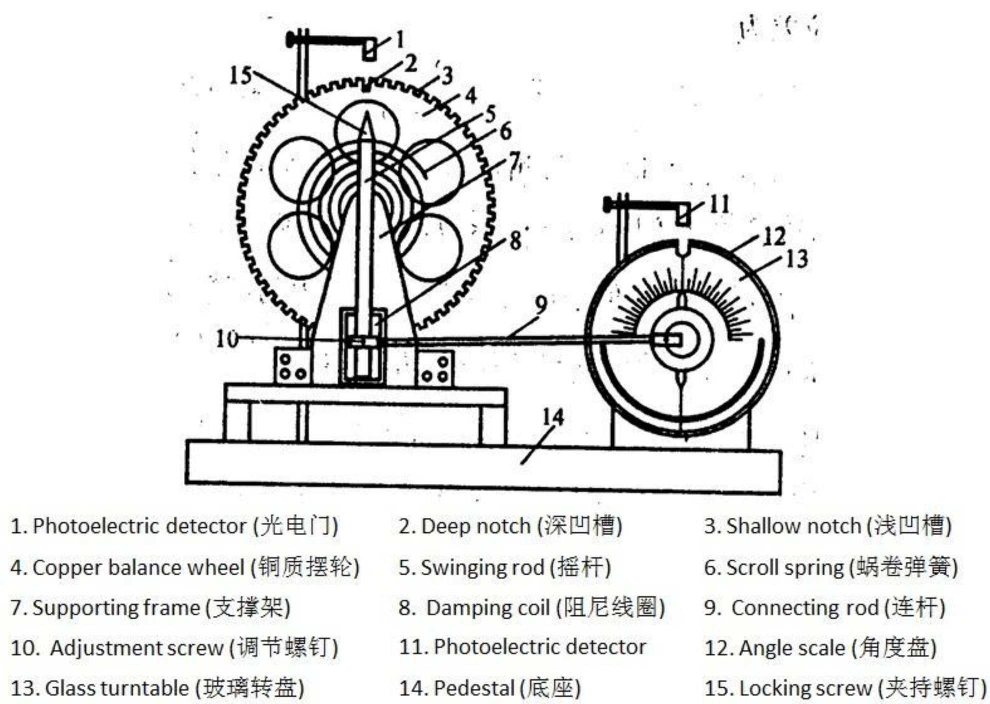

Figure 2: Vibrometer

The function Amplitude Display shows the oscillation amplitude of the balance wheel and Period Display shows the oscillation period in two modes. When the Period Selection switch is at position ”1”, a single oscillation period will be displayed; when the Period Selection switch is at ”10”, the time of 10 oscillation periods will be displayed. The reset button works only when the Period Selection button is at ”10”.

The period of the driving force can be changed precisely by using the Period of the driving force knob, but please pay attention that the scale on the knob is not very accurate.

The Damping Selection knob changes the damping force by adjusting the electric current through the coils at the bottom of the wheel. There are six options, ranging from ”0” (no current) to ”5” (current of about 0.6 A). Here we use ”2”, ”3” or ”4”.

The strobe generates a flash that allows you to read the phase difference from the angle scale directly. To protect the strobe, you should turn on the Strobe switch only when measuring the phase difference.

The Motor Switch is used to control the motor. You should turn the motor off when measuring the damping coefficient and the natural angular frequency.

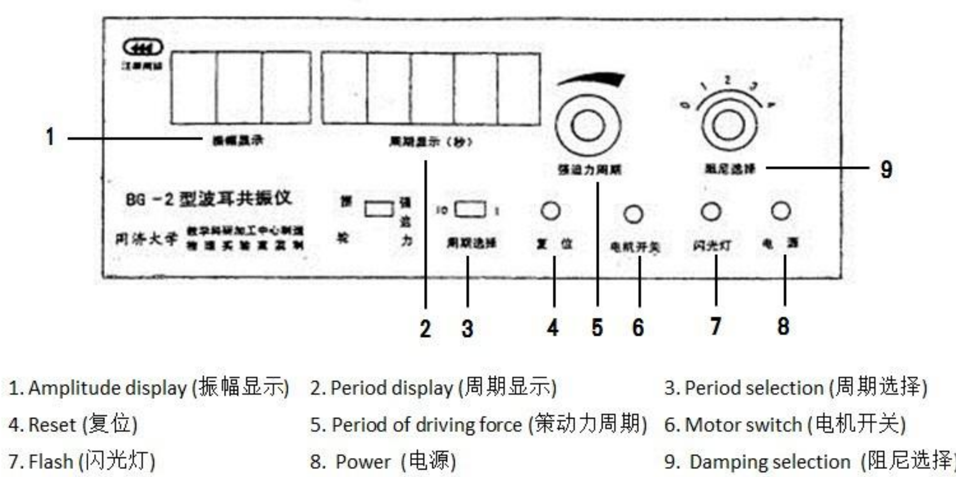

Figure 3: The front panel of the control box

2.1 Devices precision

The precisions of the devices are shown in Table 1.

Devices	Precision	Unit
Timer on BG-2 Pohl resonator	0.001	[s]
Angle on BG-2 Pohl resonator	1	[°]

Table 1: Devices precision

3 Measurement Procedure

3.1 Natural angular frequency

1. Turn the Damping Selection knob to "0".

2. Rotate the balance wheel to the initial angular position $\theta_0 \approx 150°$ and release it. Record the time of 10 periods.

3. REpeat for four times and calculate the natural angular frequency ω_0.

3.2 Damping coefficient

1. Turn the Damping Selection knob to "2", and the selection should not be changed during this part.

2. Rotate the balance wheel to the initial amplitude of approximately 150° and release it. Record the amplitude of each period and the time of 10 periods.

3. The solution to the homogeneous equation of motion, with the corresponding initial conditions, is $\theta_t = \theta_0 e^{-\beta t} cos(\omega_f t + \alpha)$. Hence $\theta_1 = \theta_0 e^{-\beta T}, \quad \theta_2 = \theta_0 e^{-\beta(2T)}, \ldots, \theta_n = \theta_0 e^{-\beta(nT)}$. The damping coefficient β can then be calculated as

$$ln\frac{\theta_i}{\theta_j} = ln\frac{\theta_0 e^{-\beta(iT)}}{\theta_0 e^{-\beta(jT)}} = (j-i)\beta T.$$

4. The value of T should be the average period, and $ln\frac{\theta_i}{\theta_{i+5}}$ should be obtained by the successive difference method as

$$\beta = \frac{1}{5T} ln\frac{\theta_i}{\theta_{i+5}}.$$

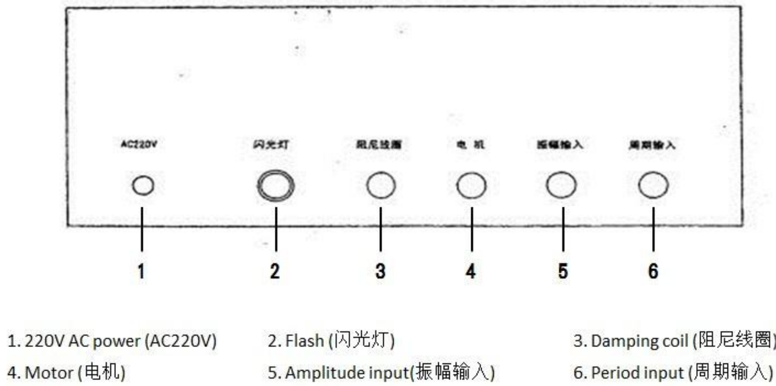

Figure 4: The rear panel of the control box

3.3 θ_{st} vs. ω and φ vs. ω Characteristics of forced oscillations

1. Keep the Damping Selection ar "2", and set the speed of the motor. Record the amplitude θ_{st}, the period T, and the phase shift φ when the oscillation reaches a steady state.

2. Repeat the steps above by changing the speed of the motor. It will result in a change of the phase shift φ (referred to as $\Delta\varphi$). More data should be collected when φ and θ_{st} change rapidly (e.g. near to the resonance point). At least 15 data should be collected for plotting.

3. Choose Damping Selection "1" or "3". Repeat the above steps.

4. Plot the $\theta_{st}(\omega)$ characteristics, with ω/ω_0 on the horizontal axis and θ_{st} on the vertical axis. Two sets of data should be plotted on the same graph. Plot the $\varphi(\omega)$ characteristics, with ω/ω_0 on the horizontal axis and φ on the vertical axis. Two sets of data should be plotted on the same graph.

4 Results

4.1 Measurement for natural angular frequency

We calculate the angular frequency from Table 2 by the fomula

$$\omega_0 = \frac{2\pi}{T}.$$

	$10T[s] \pm 0.001[s]$
1	15.481
2	15.480
3	15.496
4	15.481

Table 2: Measurement of ten periods

Hence the average value of $10T$ should be calculated as

$$\overline{10T} = \frac{1}{4}\sum_{i=1}^{4}(10T)_i = 15.485 \pm 0.017s, \quad u_{10T,r} = 0.11\%.$$

THe value of ω_0 is

$$\overline{\omega_0} = \frac{20\pi}{10T} = \frac{20 \times 3.1416}{15.485} = 4.058 \pm 0.004rad/s, \quad u_{\omega_0,r} = 0.10\%.$$

4.2 Measurement for damping coefficient

The damping coeffiecient can be calculated by the formula below.

$$\beta = \frac{1}{5T}\ln\frac{\theta_i}{\theta_{i+5}}.$$

Amplitude[°]±1[°]		Amplitude[°]±1[°]		$\ln(\theta_i/\theta_{i+5})$
θ_0	89	θ_5	54	0.500
θ_1	81	θ_6	49	0.503
θ_2	73	θ_7	44	0.506
θ_3	66	θ_8	40	0.501
θ_4	60	θ_9	36	0.511
The average value of $\ln(\theta_i/\theta_{i+5})$				0.504

Table 3: Measurement of the damping coefficient

The expermental value of $\ln(\theta_i/\theta_{i+5})$ is shown below

$$\ln(\theta_i/\theta_{i+5}) = 0.511 \pm 0.022[no\ unit], \quad u_r = 4\%$$

Here, $T = 15.561/10 = 1.5561 \pm 0.0001s$. Then we can easily obtain β as well,

$$\beta = \frac{1}{5 \times 1.5561} \times 0.511 = 0.0657 \pm 0.003s^{-1}, \quad u_{\beta,r} = 4\%$$

4.3 Measurement for θ_{st} vs. ω and φ vs. ω

To study the relation between φ and ω/ω_0, we process the raw data and list them in Table 4 and 5.

No.	ω/ω_0[ ]	u_{ω/ω_0}[ ]	φ[°]	u_φ[°]
1	1.0416	0.0010	-162	1
2	1.0296	0.0010	-153	1
3	1.0187	0.0010	-143	1
4	1.0110	0.0010	-131	1
5	1.0075	0.0010	-121	1
6	1.0050	0.0010	-110	1
7	1.0032	0.0010	-102	1
8	1.0018	0.0010	-95	1
9	1.0011	0.0010	-93	1
10	1.0006	0.0010	-90	1
11	0.9998	0.0010	-86	1
12	0.9980	0.0010	-82	1
13	0.9965	0.0010	-76	1
14	0.9943	0.0010	-70	1
15	0.9915	0.0010	-63	1
16	0.9892	0.0010	-57	1
17	0.9870	0.0010	-52	1
18	0.9860	0.0010	-50	1
19	0.9843	0.0010	-47	1
20	0.9812	0.0010	-43	1
21	0.9772	0.0010	-37	1
22	0.9712	0.0010	-30	1
23	0.9640	0.0010	-25	1

Table 4: φ vs. ω/ω_0 Damping selection 2

No.	ω/ω_0[ ]	u_{ω/ω_0}[ ]	φ[°]	u_φ[°]
1	0.9624	0.0010	-26	1
2	0.9669	0.0010	-30	1
3	0.9715	0.0010	-34	1
4	0.9771	0.0010	-41	1
5	0.9818	0.0010	-48	1
6	0.9868	0.0010	-56	1
7	0.9925	0.0010	-69	1
8	0.9954	0.0010	-77	1
9	0.9971	0.0010	-81	1
10	0.9976	0.0010	-85	1
11	1.0004	0.0010	-93	1
12	1.0022	0.0010	-99	1
13	1.0039	0.0010	-106	1
14	1.0056	0.0010	-112	1
15	1.0068	0.0010	-117	1
16	1.0083	0.0010	-122	1
17	1.0107	0.0010	-129	1
18	1.0132	0.0010	-135	1
19	1.0158	0.0010	-140	1
20	1.0191	0.0010	-146	1
21	1.0221	0.0010	-150	1
22	1.0290	0.0010	-155	1
23	1.0367	0.0010	-158	1

Table 5: φ vs. ω/ω_0 Damping selection 3

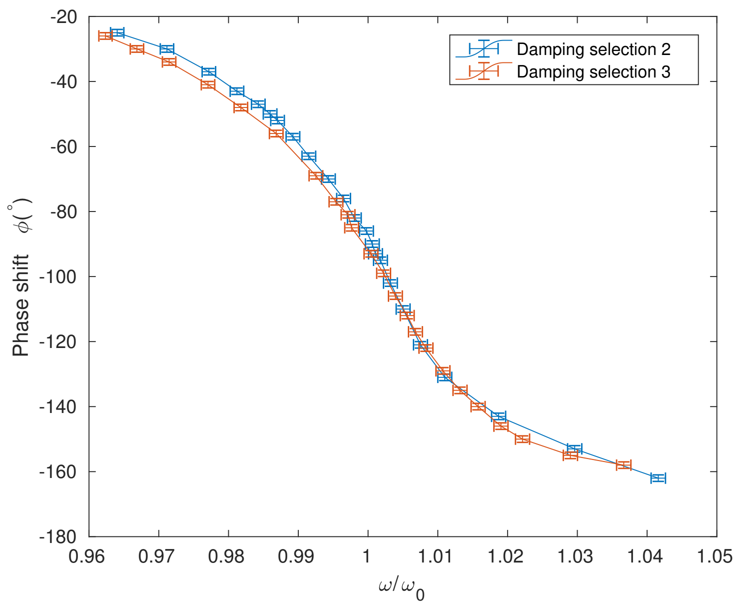

Figure 5: Phase shift φ vs. ω/ω_0

To study the relation between θ_{st} and ω/ω_0, we process the raw data and list them in Table 6 and 7.

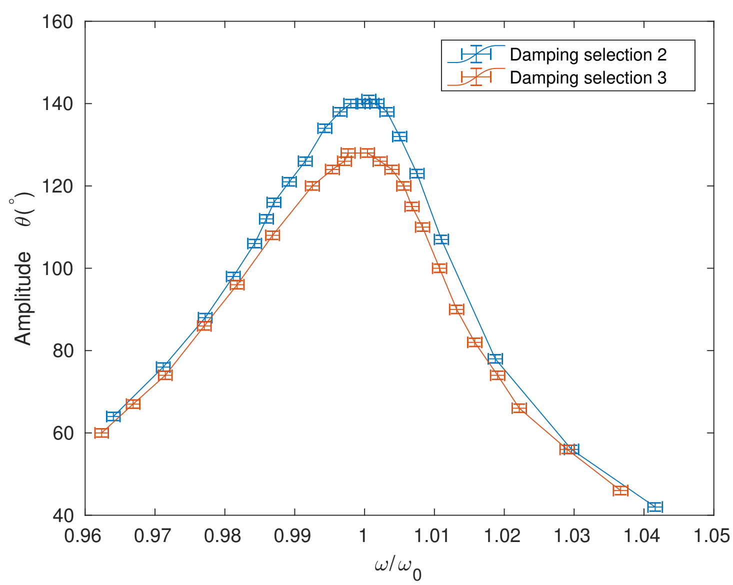

Figure 6: Amplitude θ_{st} vs. ω/ω_0

No.	$\omega/\omega_0[\,]$	$u_{\omega/\omega_0}[\,]$	$\theta_{st}[°]$	$u_{\theta_{st}}[°]$
1	1.0416	0.0010	42	1
2	1.0296	0.0010	56	1
3	1.0187	0.0010	78	1
4	1.0110	0.0010	107	1
5	1.0075	0.0010	123	1
6	1.0050	0.0010	132	1
7	1.0032	0.0010	138	1
8	1.0018	0.0010	140	1
9	1.0011	0.0010	140	1
10	1.0006	0.0010	141	1
11	0.9998	0.0010	140	1
12	0.9980	0.0010	140	1
13	0.9965	0.0010	138	1
14	0.9943	0.0010	134	1
15	0.9915	0.0010	126	1
16	0.9892	0.0010	121	1
17	0.9870	0.0010	116	1
18	0.9860	0.0010	112	1
19	0.9843	0.0010	106	1
20	0.9812	0.0010	98	1
21	0.9772	0.0010	88	1
22	0.9712	0.0010	76	1
23	0.9640	0.0010	64	1

Table 6: θ_{st} and ω/ω_0 Damping selection 2

No.	$\omega/\omega_0[\,]$	$u_{\omega/\omega_0}[\,]$	$\theta_{st}[°]$	$u_{\theta_{st}}[°]$
1	0.9624	0.0010	60	1
2	0.9669	0.0010	67	1
3	0.9715	0.0010	74	1
4	0.9771	0.0010	86	1
5	0.9818	0.0010	96	1
6	0.9868	0.0010	108	1
7	0.9925	0.0010	120	1
8	0.9954	0.0010	124	1
9	0.9971	0.0010	126	1
10	0.9976	0.0010	128	1
11	1.0004	0.0010	128	1
12	1.0022	0.0010	126	1
13	1.0039	0.0010	124	1
14	1.0056	0.0010	120	1
15	1.0068	0.0010	115	1
16	1.0083	0.0010	110	1
17	1.0107	0.0010	100	1
18	1.0132	0.0010	90	1
19	1.0158	0.0010	82	1
20	1.0191	0.0010	74	1
21	1.0221	0.0010	66	1
22	1.0290	0.0010	56	1
23	1.0367	0.0010	46	1

Table 7: θ_{st} and ω/ω_0 Damping selection 3

5 Measurement Uncetainty Analysis

5.1 Uncertainty for natural angular frequency

First, we will get the uncertainty of $(10T)$.

To estimate type-A uncertainty of $(10T)$, the standard deviation of the average value is caluculated as

$$t_{0.95} = 3.18, \quad n = 4,$$
$$s_{\overline{10T}} = \sqrt{\frac{1}{n(n-1)}\sum_{i=1}^{n}((10T)_i - \overline{10T})^2}.$$
$$\Delta_{10T,A} = \frac{3.18}{\sqrt{4}} \times 0.0076811 \approx 0.017s.$$

The type-B uncertainty of $(10T)$ is 0.001s. Hence, the uncertainty of $(10T)$ is

$$u_{10T} = \sqrt{\Delta_A^2 + \Delta_B^2} = \sqrt{0.017^2 + 0.001^2} \approx 0.017s$$

Hence, considering the uncertainty of ω_0,

$$\omega_0 = \frac{20\pi}{10T}, \quad \frac{\partial\omega_0}{\partial(10T)} = -\frac{20\pi}{(10T)^2}$$
$$u_{\omega_0} = \sqrt{\frac{\partial\omega_0}{\partial(10T)}^2 (u_{10T})^2} = \frac{20\pi}{(10T)^2} u_{(10T)} = \frac{20 \times 3.1416}{15.485^2} \times 0.017 \approx 0.004 rad/s,$$
$$u_{\omega_0,r} = \frac{u_{\omega_0}}{\omega_0} \times 100\% = \frac{0.004}{4.058} = 0.10\%$$

5.2 Uncertainty of damping coefficient

The type-A uncertainty of $\ln(\theta_i/\theta_{i+5})$ can be determined by calculating $s \times t_{0.95}/\sqrt{n}$. Here we denote it as k. Hence

$$t_{0.95} = 2.78, \quad n = 5$$
$$\Delta_{A,k} = t_{0.95}/\sqrt{5} \times \sqrt{\frac{1}{5-1}\sum_{i=1}^{5}(k_i - \bar{k})^2} = 2.78/\sqrt{5} \times 0.00444 \approx 0.005[no\ unit]$$

Then, for the type-B uncertainty, we calculate the propagated uncertainty.

$$\frac{\partial k}{\partial\theta_i} = \frac{1}{\theta_i},$$
$$\frac{\partial k}{\partial\theta_{i+5}} = -\frac{1}{\theta_{i+5}},$$
$$\Delta_{B,k} = \sqrt{\frac{\partial k}{\partial\theta_i}^2 (u_{\theta_i})^2 + \frac{\partial k}{\partial\theta_{i+5}}^2 (u_{\theta_{i+5}})^2} = \sqrt{(\frac{u_{\theta_i}}{\theta_i})^2 + (\frac{u_{\theta_{i+5}}}{\theta_{i+5}})^2}$$

For example, when $i = 0$, $\theta_0 = 89°$, $u_{\theta_2} = 1°$, $\theta_5 = 54°$, $u_{\theta_5} = 1°$.

$$\Delta_{B,k} = \sqrt{(\frac{1}{89})^2 + (\frac{1}{54})^2} = 0.022[no\ unit]$$

Choose the example propagated uncertainty as the type-B uncertainty of $\ln(\theta_i/\theta_{i+5})$. Thus the combined uncertainty is

$$u_k = \sqrt{(0.005)^2 + (0.022)^2} \approx 0.022[no\ unit],$$
$$u_{k,r} = \frac{u_k}{\bar{k}} \times 100\% = \frac{0.022}{0.511} \times 100\% = 4\%$$

Hence

$$\ln(\theta_i/\theta_{i+5}) = 0.511 \pm 0.022, \quad u_r = 4\%$$

For $10T = 15.561 \pm 0.001s$, we know that

$$T = 1.5561 \pm 0.0001s, \quad u_{r,T} = 0.006\%.$$

Then we will calculate the uncertainty of $\beta = \ln(\theta_i/\theta_{i+5})/(5T)$.

$$\begin{aligned}
\frac{\partial\beta}{\partial T} &= -\frac{k}{5T^2}, \\
\frac{\partial\beta}{\partial k} &= \frac{1}{5T}, \\
u_\beta &= \sqrt{(\frac{\partial\beta}{\partial T})^2(u_T)^2 + (\frac{\partial\beta}{\partial k})^2(u_k)^2} = \sqrt{(\frac{k^2}{25T^4})(u_T)^2 + (\frac{1}{25T^2})(u_k)^2} \\
&= \sqrt{(\frac{0.511^2}{25 \times 1.5561^4})(0.0001^2) + (\frac{0.022^2}{25 \times 1.5561^2})} \\
&\approx 0.003s^{-1}
\end{aligned}$$

where $k = \ln(\theta_i/\theta_{i+5})$.

Thus,

$$\beta = 0.0657 \pm 0.003s^{-1}, \quad u_{\beta,r} = 4\%$$

5.3 Uncertainty of θ_{st} vs. ω and φ vs. ω

We denote that $ratio = \omega/\omega_0$. To determine the uncertainty of ratio, we know that

$$\begin{aligned}
ratio &= \frac{20\pi}{10T\omega_0}, \\
\frac{\partial ratio}{\partial(10T)} &= -\frac{20\pi}{(10T)^2\omega_0}, \\
\frac{\partial ratio}{\partial\omega_0} &= -\frac{20\pi}{(10T)\omega_0^2}, \\
u_{ratio} &= \sqrt{(\frac{\partial ratio}{\partial(10T)})^2(u_{10T})^2 + (\frac{\partial ratio}{\partial\omega_0})^2(u_{\omega_0})^2} \\
&= \sqrt{\frac{400\pi^2}{(10T)^4\omega_0^2}(u_{10T})^2 + \frac{400\pi^2}{(10T)^2\omega_0^4}(u_{\omega_0})^2}.
\end{aligned}$$

As is calculated before, $\omega_0 = 4.058 \pm 0.004 rad/s$.

For example, when $10T = 14.865 \pm 0.001 s$, the uncertainty of $ratio$ is

$$u_{ratio} = \sqrt{\frac{400 \times 3.1416^2}{14.865^4 \times 4.058^2}(0.001)^2 + \frac{400 \times 3.1416^2}{14.865^2 \times 4.058^4}(0.004)^2} = 0.0010[no\ unit]$$

To avoid the similar calculation, the results of uncertainty are listed in Table 4.

Then we consider the uncertainty of φ and θ. Since these two data only obtain type-B uncertainty, the combined uncertainty of them are both equal to $\Delta_B = 1^\circ$, namely

$$u_\varphi = 1^\circ, \quad u_\theta = 1^\circ.$$

6 Conclusion and Discussion

6.1 Natural angular frequency

6.1.1 Discussions

Although we know that the amplitude does not affect the oscillation period, we still try to control the initial position to be the same. It's because little frictional force is unavoidable, which will make the period longer than it should be. As the amplitude gets larger, the negative work done by the frictional force gets more obvious. The third measurement of ten periods is obviously larger tha nthe other three, which might result from the larger amplitude(see Table 2). However, the experimental value of ω_0 is reliable since its relative uncertainty of is 0.10%.

6.1.2 Improvements and recommendations

As a device with high precision, the measurement result is quite precise although we only have four measuremnts. Despite that, I suppose controlling the amplitude is necessary to avoid the tiny friction.

6.2 Damping coefficient

6.2.1 Discussions

The damping coefficient is calculated from the exponentially decreasing amplitude. In the calculation of damping coefficient, we apply successive difference method to eliminate the deviation. However, the precision is not satisefying because the relative uncertainty reaches 4%. To be more specific, we want some other data processing methods.

6.2.2 Improvements and recommendations

Besides successive difference method, we can also apply curve fitting in this case. By exponential cauve fitting, we can also obtain the damping coefficient β from $\theta_i = \theta_0 e^{-\beta T}$.

```
General model Exp1:
   f(x) = a*exp(b*x)
Coefficients (with 95% confidence bounds):
     a =       98.7  (98.29, 99.1)
     b =   -0.06454  (-0.0651, -0.06398)

Goodness of fit:
  SSE: 0.2947
  R-square: 0.9999
  Adjusted R-square: 0.9999
  RMSE: 0.1919
```

Figure 7: Fitting curve information for β

From the information of the exponential fitting cure, we find that its R-square value is quite close to 1, which means it's very likely to be an exponential function. We notice that the b value here, which presents $-\beta$, is $-0.06454 \pm 0.0006 s^{-1}$. Compared with our result by successive difference method $\beta = 0.0657 \pm 0.003 s^{-1}$, the fitting value is more preicise. It shows that different methods to obtain some values are different in precision and reliability.

6.3 φ vs. ω

6.3.1 Discussions

Both of the φ vs. ω curve are decreasing in the shape of arc cotangent (see Figure 5). In theory we know that for the smaller damping coefficient, the left side curve of $\omega/\omega_0 = 1.0$ is higher than the other and the right side curve is lower than that. However, our results show that although the right half of the damping selection 3 curve is higher than damping selection 2 near the $\omega/\omega_0 = 1.0$, its gets lower quickly and beneath the damping selection 2, which is abnoraml. We recall the measurement procedure and conclude that it's blamed on the backslash error. During the second half of the experiment we thought the ω gap we chose is too small so we relatively quickly rotated the knob but made the gap too big. Therefore, we rotated the knob inversely to measure the middle ω again. Also, the time duration will be longer to reach the stable situation. This causes the second half of phase shifts to be smaller.

6.3.2 Improvements and recommendations

During the experiment we find it hard to predict the phase shift. We can only see the strobe flashes and read the phase shift. There are two methods to this problem.

1. Rotate the knob very carefully and be patient enough to read the phase shift for more times until it's stable. Patience can eliminate the deviation.

2. Redesign the machine. If there's an easier way to directly change the phase shift to the expected value, the entire experiment will be easy.

Whichever improvement we make, we have to be patient to ensure the oscillation is stable.

6.4 θ_{st} vs. ω

6.4.1 Discussions

The shape of both curve are within our expectation (see Figure 6). Obviously, the damping coeffiecient 2 is smaller than damping coeffiecient 3 beacuse the entire curve of damping selection 2 is above the curve of damping selection 3. we find that their beginning are roughly the same. However, it's hard to tell whether the peaks occur on the left or right hand side of the straight line $\omega/\omega_0 = 1.0$. In our expectation, the peaks should occur on the slightly left of the straight line. We suppose it's because the oscillation has not been stable yet. The value of $10T$ is smaller so that the calculated ω is larger than the real value, which makes the entire curve move rightwards. Therefore, the peak does not occur on the slightly left of $\omega/\omega_0 = 1.0$.

6.4.2 Improvements and recommendations

Sometimes we found the amplitude changes right after we recorded the periods. In order to make the result accurate, we still need patience.

References

[1] Qin Tian, Wang Yin, Mateusz Krzyzosiak Physics Laboratory Vp141 Exercise 5 Damped and driven oscillations, mechanical resonance.

[2] Qin Tian, Zeng Ming, Cao Jianjun, Han Xugen, Feng Yaming, Mateusz Krzyzosiak, Physics Laboratory (Vp141/Vp241) Student Handbook Introduction to Measurement Data Analysis.