VV186: HW 2

Due on October 13, 2016 at 8:00am

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Exercise 2.2

1. Proof 1 : $m,n \in \mathbb{N} \wedge (\frac{m^2}{n^2} < 2) \Rightarrow 2n^2 > m^2 \Rightarrow m^2 + 4mn + 4n^2 > 2m^2 + 4mn + 2n^2 \Rightarrow m^2 + 2 \times m \times 2n + 4n^2 > 2(m^2 + 2mn + n^2) \Rightarrow (m+2n)^2 > 2(m+n)^2 \Rightarrow \frac{(m+2n)^2}{(m+n)^2} > 2$

   Proof 2 : $\frac{m^2}{n^2} < 2 \Rightarrow 2n^2 > m^2 \wedge m,n \in \mathbb{N} \Rightarrow m^3(m+2n) < 2mn^2(m+2n) \Rightarrow m^2n^2 + 4mn^3 + 4n^4 < 4m^2n^2 + 8mn^3 + 4n^4 - m^4 - 2m^3n - m^2n^2 \wedge n \neq 0 \Rightarrow m^2 + 4mn + 4n^2 < \frac{\left(m^2+2mn+n^2\right)\left(4n^2-m^2\right)}{n^2} \wedge (m,n \in \mathbb{N}) \Rightarrow \frac{(m+2n)^2}{(m+n)^2} < 4 - \frac{m^2}{n^2} \Rightarrow \frac{(m+2n)^2}{(m+n)^2} - 2 < 2 - \frac{m^2}{n^2}$

2. Proof 1 : $m,n \in \mathbb{N} \wedge (\frac{m^2}{n^2} > 2) \Rightarrow 2n^2 < m^2 \Rightarrow m^2 + 4mn + 4n^2 < 2m^2 + 4mn + 2n^2 \Rightarrow m^2 + 2 \times m \times 2n + 4n^2 < 2(m^2 + 2mn + n^2) \Rightarrow (m+2n)^2 < 2(m+n)^2 \Rightarrow \frac{(m+2n)^2}{(m+n)^2} < 2$

   Proof 2 : $\frac{m^2}{n^2} > 2 \Rightarrow 2n^2 < m^2 \wedge m,n \in \mathbb{N} \Rightarrow m^3(m+2n) > 2mn^2(m+2n) \Rightarrow m^2n^2 + 4mn^3 + 4n^4 > 4m^2n^2 + 8mn^3 + 4n^4 - m^4 - 2m^3n - m^2n^2 \wedge n \neq 0 \Rightarrow m^2 + 4mn + 4n^2 > \frac{\left(m^2+2mn+n^2\right)\left(4n^2-m^2\right)}{n^2} \wedge (m,n \in \mathbb{N}) \Rightarrow \frac{(m+2n)^2}{(m+n)^2} > 4 - \frac{m^2}{n^2} \Rightarrow \frac{(m+2n)^2}{(m+n)^2} - 2 > 2 - \frac{m^2}{n^2}$