Q.1 If z is a function of x and y, given by the equation $x^x y^y z^z = c$, where c is a constant, Show that at $x = y = z$, $\frac{\partial^2 z}{\partial x \partial y} = -[x \log_e x]^{-1}$

Q.2 If $u = \tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$, Prove that

(i) $x\left(\frac{\partial u}{\partial x}\right) + y\left(\frac{\partial u}{\partial y}\right) = \sin 2u$.

(ii) $x^2\left(\frac{\partial^2 u}{\partial x^2}\right) + 2xy\left(\frac{\partial^2 u}{\partial x \partial y}\right) + y^2\left(\frac{\partial^2 u}{\partial y^2}\right) = \sin 2u\,(1 - 4\sin^2 u)$

Q.3 Examine the maximum and minimum values of the function $\sin x + \sin y + \sin(x+y)$. In interval (0, pi)

Q.4 Find the minimum value of the function $u^2+v^2+w^2$ subject to the condition $au+bv+cw = a+b+c$.

Q.5 Find the maximum and minimum distance of the point $(3, 4, 12)$ from the sphere $x^2+y^2+z^2 = 1$ using Lagrange's method of multipliers.

Q.6 Examine the function $f(x,y) = x^3+y^3-63(x+y)+12xy$ for maxima and minima.

Q.7 Find the Maclaurin series for $\ln(1+x)$ and hence that for $\ln\left(\frac{1+x}{1-x}\right)$.

Q.8 Find first 3 terms in the Maclaurin series for $\cos(\sin x)$. Hence find $\lim_{x \to 0} \frac{1-\cos(\sin x)}{x^2}$.

Q.9 (a) Evaluate first 3 non-zero terms of the Maclaurin series for $f(x) = e^{-x^2} \sin x$.

(b) Find $\lim\limits_{x \to 0} \frac{(f(x) - x)}{x^3}$.

Q.10 The expansion of function $f(x) = (1-x)^{5/2}$ is

$$f(0) + x f'(0) + \frac{x^2}{2} f''(\theta x)$$

Find the value of θ as $x \to 1$.