✓MATRICES✓

1. square
   - row matrix
   - column
   $[6\ 8\ 5]_{1\times3}$
   Row column

$\boxed{\text{Types}}$ 2x2
3x3

if diagonal matric — except diag everything is 0

$\begin{bmatrix}1&0\\0&1\end{bmatrix} \rightarrow$ Scalar — if diagon matric dia elements same

$\begin{bmatrix}-1&0\\0&1\end{bmatrix}$

null $\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}$

upper $\begin{bmatrix}2&3&4\\0&1&6\\0&0&2\end{bmatrix}$ lower $\begin{bmatrix}2&0&0\\3&4&0\\-2&6&4\end{bmatrix}$

2. $\boxed{\text{transpose}}$

$A^T = A$ sym. $A^T = -A$ skew

$\begin{bmatrix}2&3\\6&8\\4&2\end{bmatrix}_{3\times2} \rightarrow \begin{bmatrix}2&6&4\\3&8&2\end{bmatrix}_{2\times3}$

3. $\boxed{\text{operators}}$

- scalar multiplication
- add or subs

$I = \begin{bmatrix}1&0\\0&1\end{bmatrix}$

$A = \begin{bmatrix}1&1\\1&1\end{bmatrix}$

$8A = \begin{bmatrix}8&8\\8&8\end{bmatrix}$

A+B or A−B order same

a. $A+B = B+A$
b. $3(A+B) = 3A+3B$
c. $(A+B)+C = A+(B+C)$

4. $\boxed{\text{Adjoint}}$

2 order, 3 order

$A = \begin{bmatrix}3&4\\2&6\end{bmatrix}$

cofactor of $A = \begin{bmatrix}6&-2\\4&3\end{bmatrix}$

Adj $A = \begin{bmatrix}6&-2\\4&3\end{bmatrix}^T = \begin{bmatrix}6&4\\-2&3\end{bmatrix}$

$A = \begin{bmatrix}3&1&2\\4&2&1\\6&2&5\end{bmatrix}$

$10-2=8$

cofactor $= \begin{bmatrix}8&+14&-4\\+1&3&0\\-3&-5&2\end{bmatrix}$

sign $\begin{bmatrix}8&-14&-4\\-1&3&0\\-3&5&2\end{bmatrix}$

adj = transpose

$\begin{bmatrix}8&-1&-3\\-14&3&5\\-4&0&2\end{bmatrix}$

5. $\boxed{\text{singular \& non}}$

$A = \begin{bmatrix}7&0&2\\1&2&6\\4&5&3\end{bmatrix}$

$|A| = 7(6-30) - 0(\ ) + 2(5-8)$

$|A| \neq 0 \rightarrow$ non singular.

$A = \begin{bmatrix}1&4\\6&9\end{bmatrix}$ = non sing $\Rightarrow 9-24$

6. inverse

$$A^{-1} = \frac{\text{adj } A}{|A|}$$

trace: sum of all diagonal elements in square matrix

$tr(kA) = tr(A)k$

$tr(A+B) = tr(A) + tr(B)$

simultaneous

$x+y+z=3$

$x+2y+3z=4$

$x+4y+9z=6$

$$\underbrace{\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} 3 \\ 4 \\ 6 \end{bmatrix}}_{B}$$

$AX = B$

$X = A^{-1}B$

$$A^{-1} = \frac{\text{adj } A}{|A|}$$

$$A^{-1} \rightarrow \frac{\text{adj } A}{|A|}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 6 & -5 & 1 \\ -6 & 8 & -2 \\ 2 & -3 & 1 \end{bmatrix}\begin{bmatrix} 3 \\ 4 \\ 6 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 4 \\ 2 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 18-20+6 \\ -18+32-12 \\ 6-12+6 \end{bmatrix}$$

$$= \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

7. $\boxed{\text{Rank}}$ of a matrix

$$A = \begin{bmatrix} 1 & 3 & 5 \\ 2 & -1 & 4 \\ -2 & 8 & 2 \end{bmatrix}$$

$R_2 \rightarrow R_2 - 2R_1$

$R_3 \rightarrow R_3 + R_1 2$

$$A = \begin{bmatrix} 1 & 3 & 5 \\ 0 & -7 & -6 \\ 0 & 14 & 12 \end{bmatrix}$$

$R_3 = R_3 + 2R_2$

$$\begin{bmatrix} 1 & 3 & 5 \\ 0 & -7 & -6 \\ 0 & 0 & 0 \end{bmatrix}$$

Rank → 2

$R(A) = 2$

$\boxed{\text{Non zero row.}}$

8. inverse

by partition of method

$$A = \left[\begin{array}{cc|cc} 1 & 2 & 3 & 1 \\ 1 & 3 & 3 & 2 \\ \hline 2 & 4 & 3 & 3 \\ 1 & 1 & 1 & 1 \end{array}\right]$$

$$P = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} \quad R = \begin{bmatrix} 2 & 4 \\ 1 & 1 \end{bmatrix}$$

$$Q \begin{bmatrix} 3 & 1 \\ 3 & 2 \end{bmatrix} \quad S = \begin{bmatrix} 3 & 3 \\ 1 & 1 \end{bmatrix}$$

\# $A = \left[\begin{array}{c|c} P & Q \\ \hline R & S \end{array}\right]$

if $|P| \neq 0$

$$A^{-1} = \begin{bmatrix} X & Y \\ Z & W \end{bmatrix}$$

1) P^{-1}
2) $T = R \cdot P^{-1}$
3) $W = [S - TQ]^{-1}$
4) $Z = -WT$
5) $Y = -P^{-1}QW$
6) $X = P^{-1} - YT$

Q1.

$$\left[\begin{array}{c|cc} 1 & 2 & 3 \\ \hline 2 & 4 & 5 \\ 3 & 5 & 6 \end{array}\right]$$

$P = [1]$

$P^{-1} = 1$

$T = RP^{-1}$

1. $T = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

2. $W = (S - TQ)^{-1}$

$$\left\{ \begin{bmatrix} 4 & 5 \\ 5 & 6 \end{bmatrix} - \underset{2\times1}{\begin{bmatrix} 2 \\ 3 \end{bmatrix}} \underset{1\times2}{\begin{bmatrix} 2 & 3 \end{bmatrix}} \right\}^{-1}$$

$$\begin{bmatrix} 4 & 5 \\ 5 & 6 \end{bmatrix} - \begin{bmatrix} 4 & 6 \\ 6 & 9 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ -1 & -3 \end{bmatrix}^{-1} = \frac{\begin{bmatrix} -3 & 1 \\ 1 & 0 \end{bmatrix}}{-1} = \begin{matrix} 3 & -1 \\ -1 & 0 \end{matrix}$$

3. $Z = -WT$

$$= -\begin{bmatrix} 3 & -1 \\ -1 & 0 \end{bmatrix} \begin{matrix} 2 \\ 3 \end{matrix} = -\begin{bmatrix} 3 \\ -2 \end{bmatrix} = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$$

4. $Y = -P^{-1}QW$

$$= -1 \begin{bmatrix} 2 & 3 \end{bmatrix} \begin{bmatrix} 3 & -1 \\ -1 & 0 \end{bmatrix} = -\begin{bmatrix} 3 & -2 \end{bmatrix} = \begin{bmatrix} -3 & 2 \end{bmatrix}$$

$$X = P^{-1} - YT$$
$$= 1 - \begin{bmatrix} -3 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$
$$= 1 - [-6 + 6]$$
$$= 1 - 0$$
$$X = 1$$

$$A^{-1} = \begin{bmatrix} X & Y \\ Z & W \end{bmatrix}$$
$$= \begin{bmatrix} 1 & -3 & 2 \\ -3 & 3 & -1 \\ 2 & -1 & 0 \end{bmatrix}$$

Simultaneous

Rank.

1. $$\begin{bmatrix} 1 & 4 & 3 & 2 \\ 1 & 2 & 3 & 4 \\ 2 & 6 & 7 & 5 \end{bmatrix}$$

↳ 0 baneye → non zero rows = rank

2. $$\begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 3 & 4 & 3 \\ 0 & 0 & 0 & -6 \\ 0 & 0 & 0 & -2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 3 & 4 & 3 \\ 0 & 0 & 0 & -6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

help se

$R_2 \rightarrow R_2 - 3R_1$
$R_3 \rightarrow R_3 - R_1$

[rank = 2]

↳ normal form banayegen

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

NORMAL.

3. $$\begin{bmatrix} C_2 \rightarrow C_2 - 3C_1 \\ C_3 \rightarrow C_3 - 4C_1 \\ C_4 \rightarrow C_4 - 3C_1 \end{bmatrix}$$

↳

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_2 \rightleftharpoons C_4$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -6 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_2 \rightarrow \frac{C_2}{-6}$

↳

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow \text{Rank } 2.$$

Rank 3 → $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

normal form

unit matrix

#3.

$$A = \begin{bmatrix} 6 & 1 & 3 & 8 \\ 4 & 2 & 6 & 7 \\ 10 & 3 & 9 & 7 \\ 16 & 4 & 12 & 15 \end{bmatrix}$$

$C_1 \rightleftharpoons C_2$

$$A = \begin{bmatrix} 1 & 6 & 3 & 8 \\ 2 & 4 & 6 & 7 \\ 3 & 10 & 9 & 7 \\ 4 & 16 & 12 & 15 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 6 & 3 & 8 \\ 0 & -8 & 0 & -17 \\ 0 & -8 & 0 & -17 \\ 0 & -8 & 0 & 17 \end{bmatrix}$$

$\boxed{\begin{matrix} R2 = R_2 - 2R_1 \\ R3 = R_3 - 3R_1 \\ R4 = R_4 - 4R_1 \end{matrix}}$

$$A = \begin{bmatrix} 1 & 6 & 3 & 8 \\ 0 & -8 & 0 & -17 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\boxed{\begin{matrix} R_3 \to R_3 - R_2 \\ R_4 \to R_4 - R_2 \end{matrix}}$

Rank = 2

1. $C_2 \to C_2 - 6C_1$
$C_3 \to C_3 - 3C_1$
$C_4 \to C_4 - 8C_1$

2. $A = C_2 \to \dfrac{C_2}{-8}$

$C_4 \to \dfrac{C_4}{-17}$

3. $C_4 \to C_4 - C_2$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \to C_4 \to C_4 - C_2 \quad \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} I_2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\rho(A) = 2$

Matrix

4×4 diff same $x \to 4$

2 row same $\to 2$

3 row same. $\to 3$

4 same $\to 1$.

→ Eigen value & Eigen Vector.

* Characterstic matrix of A. →

$A - \lambda I \Rightarrow$ matrix

* Charactrix polynomial of A →

$|A - \lambda I|$

* characterstic equation of A →

$|A - \lambda I| = 0$.

4 roots → eigen values or latent values of (matrix A)

Eigen vector →

$\lambda = \lambda$ is eigen value of $n \times n$ matrix then $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ of eq^n.

$[A - \lambda I] X = 0$

eigen vector of A.

$$A = \begin{bmatrix} -2 & 1 & 1 \\ +1 & 4 & 5 \\ -1 & 1 & 0 \end{bmatrix}$$

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A - \lambda I = \begin{bmatrix} -2-\lambda & 1 & 1 \\ -11 & 4-\lambda & 5 \\ -1 & 1 & 0-\lambda \end{bmatrix}$$

$|A - \lambda I| \rightarrow$

$$-2-\lambda[4-\lambda(-\lambda)-5] - 1(1\cdot\lambda+5] + 1[-1+4-\lambda]$$

$|A - \lambda I| \rightarrow$

$$-\lambda^3 + 2\lambda^2 + \lambda - 2 = 0$$

$$\lambda^3 - 2\lambda^2 - \lambda + 2 = 0$$

$$\lambda^2(\lambda-2) - 1(\lambda-2) = 0$$

$$(\lambda-2)(\lambda^2-1) = 0$$

$$\boxed{\lambda = 2, 1, -1}$$

eigen values.

$\lambda_1 = 1$; $|A - I| = \begin{bmatrix} -3 & 1 & 1 \\ -11 & 3 & 5 \\ -1 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\lambda_2 = 2$; $|A - 2I| =$

$\lambda_3 = -1$; $|A + I| =$

$$\frac{x_1}{\begin{vmatrix} 3 & 5 \\ 1 & -1 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} -11 & 5 \\ -1 & -1 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} -11 & 3 \\ -1 & 1 \end{vmatrix}}$$

$$\frac{x_1}{-8} = \frac{-x_2}{16} = \frac{x_3}{-8}$$

$\boxed{1 \quad 2 \quad 1}$ $\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$

way to find.

$$\begin{bmatrix} -2 & 1 & 1 \\ -11 & 4 & 5 \\ -1 & 1 & 0 \end{bmatrix} \xrightarrow{\text{values is}} \begin{bmatrix} -2 & 1 & 1 \\ -11 & 4 & 5 \\ -1 & 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = 1\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

1 value eigen.

Hermitian matrix

$$\boxed{\bar{A}^T = A^\theta}$$

θ = Transpose conjugate.

or

conjugate transpose.

A^θ conjugate.

$\boxed{i \to -i}$

* v important

$(i \to -i)$
- $\to A^T$: transpose
- $\to \bar{A}$: conjugate
- $\to (\bar{A})^T = A^\theta$

$$A^\theta \Rightarrow \begin{bmatrix} 2 & 3i & 5 \\ 7-i & 11+i & -i \end{bmatrix}$$

$$\begin{bmatrix} 2 & 7+i \\ -3i & 11-i \\ 5 & i \end{bmatrix}$$

diagonal elements eigen values real.

Skew hermitian

$$\boxed{A^\theta = A}$$

diag imag or zero. & eigen values

Hermitian.

$$\boxed{A^\theta = A}$$

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orthogonal matrix.

$$AA^T = I = A^TA$$

$$A^{-1} = A^T.$$

$$\left[\begin{array}{cc|cc|c} 2 & 3 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ \hline 0 & 0 & 4 & 1 & 0 \\ 0 & 0 & 3 & 2 & 0 \\ \hline 0 & 0 & 0 & 0 & 5 \end{array}\right]$$

$A_1 : 4$
$A_2 : 5$
$A_3 : 5$
×

$$\left|\begin{array}{cc|cc} 1 & 6 & 2 & 8 \\ 3 & 2 & 4 & 1 \\ \hline 0 & 0 & 2 & 4 \\ 0 & 0 & 6 & 8 \end{array}\right|$$

upper triangle.

diagonal block

$$\begin{vmatrix} A & B \\ 0 & D \end{vmatrix} \to |A||D|$$

$|A||D|$

---

unitary matrix.

$$AA^\theta = I$$

Block matrix.

$$\begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix} \to A \text{ so } \boxed{A^{100} \to 2}$$

#2.

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

Scalar matrix →

∴ diagonal matrix $\begin{bmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{bmatrix}$

$$A-\lambda I = \begin{bmatrix} 2-\lambda & -1 & 1 \\ -1 & 2-\lambda & -1 \\ 1 & -1 & 2-\lambda \end{bmatrix}$$

$$|A-\lambda I| = 0$$

$$-\lambda^3 + 6\lambda^2 - 9\lambda + 4 = 0$$

$$-\lambda^2(\lambda-1) + 5\lambda(\lambda-1) - 4(\lambda-1) = 0$$

$$(\lambda-1)(\lambda^2-5\lambda+4) = 0$$

$$(\lambda-1)(\lambda-1)(\lambda-4) = 0.$$

$$(\lambda = 1, 1, 4)$$

Eigenvalues → $\lambda_1 = \lambda_2 = 1$

$$|A-I| = \begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

rank < 2
[rank is 1]

$$\boxed{x_1 - x_2 + x_3 = 0}$$

$x_2 = k$
$x_3 = 0$
$x_1 = k.$

$x_3 = k$
$x_2 = 0$
$x_1 = -k.$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} k.$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = k \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$$

1. Matrix → 3 eigen values ⇒ non zero. → Rank 3.

1 eigValue → put rank 2

(eigen vector) → ∞ solution

CALEY-HAMILTAN THEORUM

trace = sum of eigen value

1. Verify

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

digonal = trace = 5.

$$|A - \lambda I| = \begin{bmatrix} 2-\lambda & 1 & 1 \\ 0 & 1-\lambda & 1 \\ 1 & 1 & 2-\lambda \end{bmatrix} = -\lambda^3 + 5\lambda^2 - 7\lambda + 3 = 0$$

coeff trace same.
check point

$$-A^3 + 5A^2 - 7A + 3I_3 = 0.$$

$$\boxed{A^3 - 5A^2 + 7A - 3I_3 = 0}$$

1. A^2 [ ][ ] =
2. $A^2 \cdot A = A^3$

put.

$$A^3 - 5A^2 + 7A - 3I_3 = 0$$

$$\underbrace{\begin{bmatrix} 14 & 13 & 13 \\ 0 & 1 & 0 \\ 13 & 13 & 14 \end{bmatrix} - 5\begin{bmatrix} 5 & 4 & 4 \\ 0 & 1 & 0 \\ 4 & 4 & 5 \end{bmatrix} + 7\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{0}$$

finding inverse

(4) 16 → 20
mid → (4)
end → (5) (7)
man (7)

$$A^{-1}(A^3-5A^2+7A-3I)=0$$

$$A^2-5A+7I-3A^{-1}=0$$

$$3A^{-1}=(\quad)$$

$$A^{-1}=\frac{1}{3}(A^2-5A+7I)$$

$$A^{-1}=\frac{1}{3}\left(\begin{bmatrix}5&4&4\\0&1&0\\4&4&5\end{bmatrix}-5\begin{bmatrix}2&1&1\\0&1&0\\1&1&2\end{bmatrix}+7\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\right)$$

#3. $\begin{vmatrix}2&1&1\\0&1&0\\1&1&2\end{vmatrix}$ and hence

$$A^8-5A^7+7A^6-3A^5+A^4-5A^3+8A^2-2A+I_3.$$

$$|A-\lambda I| = \lambda^3+5\lambda^2-7\lambda+3=0$$

$$\boxed{A^3-5A^2+7A-3I=0}$$

Q. $A^8-5A^7+7A^6-3A^5+A^4-5A^3+8A^2-2A+I_3$.

$$A^5(A^3-5A^2+7A-3I)+A^4-5A^3+7A^2+A^2-2A+I_3.$$

$$A^5(0)+A(3I)+A^2-2A+I$$

$$3A+A^2-2A+I$$

$$A^2+A+I$$

$$[\quad]+[\quad]+[\quad]$$

$$\Rightarrow \begin{bmatrix}8&5&5\\0&3&0\\5&5&8\end{bmatrix}.$$

Consistency & inconsistency

(1)

$$A = \begin{bmatrix} 1 & 1 & 1 & 6 \\ 2 & 1 & 3 & 13 \\ 5 & 2 & 1 & 12 \\ 2 & -3 & -2 & 10 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$
$R_3 \to R_3 - 5R_1$
$R_4 \to R_4 - 2R_1$

$$A = \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & -1 & 1 & 1 \\ 0 & -3 & -4 & -21 \\ 0 & -5 & -4 & -27 \end{bmatrix}$$

$R_3 \to R_3 - 3R_2$
$R_4 \to R_4 - 5R_2$

$$A = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & -1 & 1 & 1 \\ 0 & 0 & -7 & -21 \\ 0 & 0 & -9 & -27 \end{array}\right]$$

× $\rho(A) = 3$

$A \cdot B = 3$

$R_4 \to 7R_4 - 9R_3$

$$A = \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & -1 & 1 & 1 \\ 0 & 0 & \cancel{-7}\ 0 & \cancel{-21}\ 0 \end{bmatrix}$$

✓

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & -1 & 1 & 1 \\ 0 & 0 & -7 & -21 \\ 0 & 0 & 0 & 0 \end{array}\right] \to \rho(AB) = 3$$

Rank.

Rank(A) = 3

$x+y+z=6$

$2x+y+3z=13$

$5x+2y+z=12$

$2x-3y-2z=-10$

→

consistent $= \rho(AB)=\rho(A)$ & $=n$ (no. of unknown)

→ unique solⁿ $= \rho(AB)=\rho(A)$ is $<n$

inconsistent $= \rho(AB) \neq \rho(A)$

$x+y+z=6$

$-y+z=1$

$-7z=-21$

$x=3$

$y=2$

$z=1$

UNIQUE SOLUTION

Idempotent matrix or projection matrix

$A=[a_{ij}]$ if $A^2=A$

$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ $A^2=A \rightarrow$ indempotent

1. If A is idempotent matrix $I-A$ is also idempotent

2. If A and B are matrices of same order $AB=A$, $BA=B$ then A and B are itempotent

3. A and $B \rightarrow$ Indempotent of same order then $A+B$ is id :- $AB=BA=0$

4. A is idemp then $k \in N$ $\boxed{A^k=A}$

5. k and l are 2 no's A is indempotent

$(kI+lA)^n =$

$k^nI + [(k+l)^n - k^n] A$

$x+y-3z=1$

Question 2 $4x-2y+6z=8$

$15x-3y+9z=21$

$$\begin{bmatrix} 1 & 1 & -3 \\ 4 & -2 & 6 \\ 15 & -3 & 9 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 8 \\ 21 \end{bmatrix}$$

$AX = B$.

$$AB = \begin{bmatrix} 1 & 1 & -3 & 1 \\ 4 & -2 & 6 & 8 \\ 15 & -3 & 9 & 21 \end{bmatrix}$$

$R_2 \to R_2 - 4R_1$

$R_3 \to R_2 - 15R_1$

$$A.B = \begin{bmatrix} 1 & 1 & -3 & 1 \\ 0 & -6 & 18 & 12 \\ 0 & -18 & 54 & 36 \end{bmatrix}$$

$R_3 = R_3 - 3R_3$

$$AB = \begin{bmatrix} 1 & 1 & -3 & 1 \\ 0 & -6 & 18 & 12 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Infinite solution

$\rho(AB = 2$

$\rho(B) = 2$

unknown $= 3$

$x + y - 3z = 7$

$-6y + 18z = 12$

$\boxed{z = k}$

$y = \frac{-18k + 12}{-6} = \boxed{-2 + 3k}$

$x + y - 3(k) = 7$

$\boxed{x = 3}$

Question 3

$x - 4y + 7z = 14$

$3x + 8y - 2z = 13$

$7x - 8y + 26z = 5$

$$\begin{bmatrix} 1 & -4 & 7 \\ 3 & 8 & -2 \\ 7 & -8 & 26 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 14 \\ 13 \\ 5 \end{bmatrix}$$

$$(A|B) = \left[\begin{array}{ccc|c} 1 & -4 & 7 & 14 \\ 3 & 8 & -2 & 13 \\ 7 & -8 & 26 & 5 \end{array}\right]$$

$R_3 \to R_3 - 7R_1$
$R_2 \to R_2 - 3R_1$

$$(A|B) = \left[\begin{array}{ccc|c} 1 & -4 & 7 & 14 \\ 0 & 20 & -23 & -29 \\ 0 & 20 & -33 & -91 \end{array}\right]$$

$R_3 \to R_3 - R_2$

$$AB = \left[\begin{array}{cccc} 1 & -4 & 7 & 14 \\ 0 & 20 & -23 & -29 \\ 0 & 0 & 0 & 64 \end{array}\right]$$

$\rho(AB) = 3$

$\rho(A) = 2$

inconsistent

$\boxed{\text{NO SOLUTION}}$

LU decomposition / Crout's method
← method →

$x+5y+z=14$
$2x+y+3z=13$
$3x+y+4z=17$

$\rightarrow \begin{bmatrix} 1 & 5 & 1 \\ 2 & 1 & 3 \\ 3 & 1 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 14 \\ 13 \\ 17 \end{bmatrix}$

$$A = \begin{bmatrix} 1 & 5 & 1 \\ 2 & 1 & 3 \\ 3 & 1 & 4 \end{bmatrix} = LU$$

$$\begin{bmatrix} 1 & 5 & 1 \\ 2 & 1 & 3 \\ 3 & 1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

$$\begin{bmatrix} u_{11} & u_{12} & u_{13} \\ l_{21}u_{11} & l_{21}u_{12}+u_{22} & l_{21}u_{13}+u_{23} \\ l_{31}u_{11} & l_{31}u_{12}+l_{32}u_{22} & l_{31}u_{13}+l_{32}u_{23}+u_{33} \end{bmatrix}$$

Row (1) →

$u_{11}=1$

$u_{12}=5$

$u_{13}=1$

Column (1) → $l_{21}u_{11}=2 \rightarrow l_{21}=2$

$l_{31}u_{11}=3 \rightarrow l_{31}=3$

row 2 $\rightarrow$

$l_{21} u_{12} + u_{22} = 1$

$2(5) + u_{22} = 1$

$\boxed{u_{22} = -9}$

$l_{21} u_{13} + u_{23} = 3$

$2(1) + u_{23} = 3$

$\boxed{u_{23} = 1}$

colum 2 $\rightarrow$

$l_{31} u_{12} + l_{32} u_{22} = 1$

$3(5) + l_{32}(-9) = 1$

$l_{32}(-9) = +4$

$\boxed{l_{32} = 14/9}$

$l_{31} u_{13} + l_{32} u_{23} + u_{33} = 4$

$\boxed{u_{33} = -5/9}$

$$A = LU = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & \frac{14}{9} & 1 \end{bmatrix} \begin{bmatrix} 1 & 5 & 1 \\ 0 & -9 & 1 \\ 0 & 0 & -5/9 \end{bmatrix}$$

L — U

$AX = B$

$LUX = B$

$LY = B$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 14/9 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 14 \\ 13 \\ 17 \end{bmatrix}$$

1. $\boxed{y_1 = 14}$

2. $2y_1 + y_2 = 13$

$2y_1 + y_2 = 13$

$\boxed{y_2 = -15}$

3. $3y_1 + \frac{14}{5} y_2 + y_3 = 17$

$\boxed{y_3 = -5/3}$

$$\begin{bmatrix} 1 & 5 & 1 \\ 0 & -9 & 1 \\ 0 & 0 & -5/9 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 14 \\ -15 \\ -5/3 \end{bmatrix}$$

Let $UX = Y$

$x + 5y + z = 14 \longrightarrow y = 1$

$-9y + z = -15 \longrightarrow y = 2$

$-\frac{5}{9} z = -5/3$

$z = 3$

Involutary matrix of reflection matrix

1: A square matrix $A = [a_{ij}]_{n \times n}$ is said to be involutary matrix if $A^2 = I$

If A and B are involutary which commute with each other AB is involutary

$$(AB)^2 = ABAB = AABB = A^2B^2 = I$$

2: If A is involutary then

$$A^n = \begin{cases} A & n \text{ odd} \\ I & n \text{ even} \end{cases}$$

n even $\quad n = 2m$

$$A^n = A^{2m} = (I)^m = I$$

n odd $\quad n = 1 \quad A^n = A$

$n = 2m+1 \quad A^n = 2^{2m+1} = A^{2m}A$

$= IA = A$

$$A = \begin{bmatrix} 1 & 2 & 5 \\ 11 & 13 & 19 \\ 17 & 15 & 9 \end{bmatrix}$$

diagonal entities match with (A)

$[1], [9], \begin{bmatrix} 13 & 19 \\ 15 & 9 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 11 & 13 \end{bmatrix}$: principal submatrix of square matrix

---

Echelon form of matrix

A matrix A is in Echelon form if

1. Zero rows of A should be in the botton
2. Leading non-zero element of non zerows is 1
3. Number of zeros before leading non zero element in any non zero row is more than the number of zero before leading non-zero element in previous row.

**

$$\begin{bmatrix} 1 & 0 & 2 & 3 & 4 \\ 0 & 1 & 5 & 6 & 7 \\ 0 & 0 & 0 & 1 & 8 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow \boxed{\text{Echelon form}}$$

Rank = 3

Non zero elements (rows) => Rank.

Elementary →

#1.

identity matrix → ① operation

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \times 5 \rightarrow \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$ or $\begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}$ elementary m

→ trace (ABC) ⇒ ~~Trace (ACB)~~

→ tr (ACB) ≠ BAC = CBA   trac (BCA) = (CAB)

Class : Rank :- of a matrix.

1. $\begin{bmatrix} 3 & 1 & 7 \\ 1 & 2 & 4 \\ 4 & -1 & 7 \\ 2 & 1 & 5 \end{bmatrix}$

$\boxed{\text{RANK} = 2}$

$R_2 \to 3R_2 - 8R_1$

rank $\Rightarrow$ 2.

$C_1 \rightleftharpoons C_2$

$\begin{bmatrix} 1 & 3 & 7 \\ 2 & 1 & 4 \\ -1 & 4 & 7 \\ 1 & 2 & 5 \end{bmatrix}$

$R_2 \to R_2 - 2R_1$
$R_3 \to R_3 + R_1$
$R_4 \to R_4 - R_1$

$\begin{bmatrix} 1 & 3 & 7 \\ 0 & -5 & -10 \\ 0 & 7 & 14 \\ 0 & -1 & -2 \end{bmatrix}$

$R_3 \to R_3 + \frac{7R_2}{5}$
$R_4 \to R_4 + \frac{R_2}{-5}$

$\begin{bmatrix} 1 & 3 & 7 \\ 0 & -5 & -10 \\ 0 & 0 & \cancel{0}24 \\ 0 & 0 & 0 \end{bmatrix}$

$14 + \frac{7}{5} \times -10$

$-2 \times + \frac{-10}{-5}$

$\boxed{\text{Rank} = 2}$

2. $\begin{bmatrix} 1 & 0 & 0 & 29 \\ 0 & 1 & 0 & 16 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 1 & 3 \end{bmatrix}$

Rank = 3

row

$\begin{matrix} 1 & 0 & 0 & 29 \\ 0 & 1 & 0 & 16 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{matrix}$

row

$\begin{bmatrix} 1 & 0 & 0 & 29 \\ 0 & 1 & 0 & 16 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

column.

$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$\begin{bmatrix} I_3 & 0 \\ 0 & 0 \end{bmatrix}$

3. $\begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & 1 & -3 & -3 \\ 1 & 0 & 1 & 2 \\ 1 & -1 & 3 & 3 \end{bmatrix}$ 4x4

* $\boxed{\text{rank}(A) = 3}$

so basically.

row

$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & -2 & -2 \\ 0 & -1 & 0 & 1 \\ 0 & -2 & 2 & 2 \end{bmatrix}$

$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & -2 & -2 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & -2 & -2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$\begin{matrix} 1 & & & \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{matrix}$

$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & -2 & -2 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \div 2$

$\begin{matrix} 1 & 1 & 1 & 1 \\ 0 & 1 & -1 & -1 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{matrix}$

$\boxed{R_4 \to R_4 + R_2}$

$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & -2 & -2 \\ 0 & -1 & 0 & 1 \\ 0 & -2 & 2 & 2 \end{bmatrix}$

→ there are 2 ways to find rank:-

determinant way → checking out the sub matrices determinant → square matrices.

non-determinant way. → echelon way. → normal way form.

Question:

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & 1 & -3 & -3 \\ 1 & 0 & 1 & 2 \\ 2 & 1 & 3 & 3 \end{bmatrix} \quad 4\times4$$

$A_{11} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = 2$

$A_{12} = \begin{bmatrix} 1 & 1 \\ 1 & -3 \end{bmatrix} = -4$

$A_{13} = \begin{bmatrix} 1 & 1 \\ -3 & -3 \end{bmatrix} = 0$

$A_{21} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

$A_{22} = \begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix}$

A_{23} $\begin{bmatrix} -3 & -3 \\ 1 & 2 \end{bmatrix}$

A_{31} $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

A_{32} $\begin{bmatrix} 0 & 1 \\ 13 & \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

rank = 3.

det A ≠ 0

$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \end{bmatrix}$

Y = 2X

rank ≠ 2

r, or 1

230 → 30

30 +

78 → 6 P!

32

Quest 1) $A = \begin{bmatrix} 2 & 1 & 1 \\ 4 & 2 & 2 \\ 1 & 2 & 2 \end{bmatrix}$

$\det(A) = 0$

rank < 3

may be 2, 1, 0.

$A_{11} = \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix} \to 0$

$A_{12} = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \to 0$

$A_{21} = \begin{bmatrix} 4 & 2 \\ 1 & 2 \end{bmatrix} \to 6$

$A_{22} = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} \to 0$

Question 2)

$A = \begin{bmatrix} 3 & 1 & 7 \\ 1 & 2 & 4 \\ 4 & -1 & 7 \\ 2 & 1 & 5 \end{bmatrix}_{4\times 3}$

rank = 2.

$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = 5$

$A_{11} = \begin{bmatrix} 1 & 7 \\ 2 & 4 \end{bmatrix} = -10$

$A_{12} = \begin{bmatrix} 1 & 2 \\ 4 & -1 \end{bmatrix} = -9$

$A_{21} = \begin{bmatrix} 2 & 4 \\ -1 & 7 \end{bmatrix} = 18$

$A_{22} =$

$A_{31} = \begin{bmatrix} 4 & -1 \\ 2 & 1 \end{bmatrix} = 6$

$M_{31} = \begin{bmatrix} -1 & 7 \\ 1 & 5 \end{bmatrix} = -12$.

Question 3)

mxn matrix of A : then

1. $0 \leq \varrho(A) \leq \min\{m, n\}$
   $\varrho(A) = 0 \iff A = 0$
2. $\varrho(I_n) = n$
3. $\varrho(KA) = \varrho(A)$ if $k \neq 0$
4. $\varrho(A^T) = \varrho(A)$

rank of matrix $\geq$ submatrix.

5. Let $AX = 0$ then for non zero solution ie $(x = 0)$ $\varrho(A) \leq n-1$
6. rank of a diag matrix equal the no. of non zero diagonal element it posses
7. $\varrho(A) = \varrho(A^T) = \varrho(AA^T) = \varrho(A^TA)$
8. $\varrho(A) < n$ for square matrix $|A| = 0$
9. rank of every non sing matrix of order n is 'n'.
10. rank of every non 0 matrix is ≥ 1

Rank of a matrix

1. $\gamma(A+B) = \gamma(A) + \gamma(B)$

2. $\gamma(A-B) \geq |\gamma(A) - \gamma(B)|$

3. $\gamma(AB) \leq \min\{\gamma(A), \gamma(B)\}$

4. $\gamma(AB) \neq \gamma(BA)$ in general

5. $\gamma\begin{bmatrix} A & B \\ 0 & D \end{bmatrix} \geq \gamma(A) + \gamma(D)$

6. $\gamma\begin{bmatrix} A & 0 \\ C & D \end{bmatrix} \geq \gamma(A) + \gamma(D)$

rank : non singular
zero in the echelon form of matrix

$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \end{bmatrix}$

rank = 2

(i) system solution $AX = B$ ↳ $X = A^{-1}B$.

(ii) coplanar or collinear

(iii) one-one and onto transformation

$(2,4,6) = 2(1,2,3)$

$A = d\,B$.

(iv) to find the determinant

(v) Stability of computation system

$$A = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 1 & 4 \\ 2 & 8 & 2 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$
$R_3 \to R_3 + R_1$

$$\begin{bmatrix} 1 & 3 & 5 \\ 0 & -7 & -6 \\ 0 & 14 & 12 \end{bmatrix}$$

$-1-6 = -7$
$4-10 = -6$

$R_3 \to R_3 - R_2$

Rank: 2

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

2

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 5 \\ 1 & 5 & 5 & 7 \\ 8 & 1 & 14 & 17 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$
$R_3 \to R_3 - R_1$
$R_4 \to R_4 - 8R_1$

2

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -3 & -2 & -3 \\ 0 & 3 & 2 & 3 \\ 0 & -15 & -10 & -15 \end{bmatrix}$$

$R_3 \to R_3 + R_2$
$R_4 \to R_4 - R_2 \times 5$

14-24
10
17-32

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -3 & -2 & -3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

rank of matrix remains unchanged under elementary row opertions

normal form:
elemantary transformation
any non zero matrix A. can be reduced to the one the following form.

1. $[I_r]$

2. $[I_r \; 0]$ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ $[I_2 \; 0]$

rank = 2

3. $\begin{bmatrix} I_r \\ 0 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ rank=3

$\begin{bmatrix} I_3 \\ 0_{2\times3} \end{bmatrix}$

4. $\begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ rank = 2.

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 3 & 0 & -1 & 0 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$

$R_3 \to R_3 - 3R_1$

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -3 & -2 & -5 \\ 0 & -6 & -10 & -12 \end{bmatrix}$$

$R_3 \to R_3 \pm 2R_2$

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -3 & -2 & -5 \\ 0 & 0 & 0 & -2 \end{bmatrix}$$

$C_2 \to C_2 - 2C_1$

$C_3 \to C_3 - 3C_1$

$C_4 \to C_4 - 4C_1$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -3 & -2 & -5 \\ 0 & 0 & 0 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -3 & -2 & -5 \\ 0 & 0 & 0 & -2 \end{bmatrix}$$

1 0 0 0 $\boxed{\text{rank} = 3}$

1.

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 3 & 0 & 5 & -10 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -3 & -2 & -5 \\ 0 & -6 & -4 & -22 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -3 & -2 & -5 \\ 0 & 0 & 0 & -12 \end{bmatrix} \quad R_3 \to R_3 - 2R_2$$

$-22+10$

$C_2 \to C_2 - 2C_1$

$C_3 \to C_3 - 3C_1$

$C_4 \to C_4 - 4C_1$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -3 & -2 & -5 \\ 0 & 0 & 0 & -12 \end{bmatrix}$$

$C_2 \to \frac{C_2}{-3}$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & \frac{2}{3} & \frac{5}{3} \\ 0 & 0 & 0 & -12 \end{bmatrix}$$

$C_3 \to C_3 - \frac{2}{3}C_2$

$C_4 \to C_4 - \frac{5}{3}C_2$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -12 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & +1 \end{bmatrix}$$

$R_3 \to \frac{R_3}{-12}$

2. $\begin{bmatrix} 1 & 2 & -1 & 3 \\ 4 & 1 & 2 & 1 \\ 3 & -1 & 1 & 2 \\ 1 & 2 & 0 & 1 \end{bmatrix}$

$\sim \begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & -7 & 6 & -11 \\ 0 & -7 & 4 & -7 \\ 0 & 0 & 1 & -2 \end{bmatrix}$

$R_2 \to R_2 - 4R_1$
$R_3 \quad R_3 - 3R_1$
$R_4 \to R_4 - R_1$

$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -7 & 6 & -11 \\ 0 & -7 & 4 & -7 \\ 0 & 0 & 1 & -2 \end{bmatrix}$

$C_2 \to C_2 - 2C_1$
$C_3 \to C_3 + C_1$
$C_4 \to C_4 - 3C_1$

$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -7 & 6 & -11 \\ 0 & 0 & -2 & 4 \\ 0 & 0 & 1 & -2 \end{bmatrix}$

$R_3 \to R_3 + -R_2$

$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 6 & -11 \\ 0 & 0 & -2 & 4 \\ 0 & 0 & 1 & -2 \end{bmatrix}$

$\boxed{\begin{matrix} C_3 \to C_3 - 6C_2 \\ C_4 \to C_4 - 11C_2 \end{matrix}}$

$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -2 & 4 \\ 0 & 0 & 1 & -2 \end{bmatrix}$

$R_4 \to R_4 + \frac{R_3}{2}$

$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -2 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$\begin{bmatrix} I_3 & 0 \\ 0 & 0 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & 0 & \\ 0 & 1 & 0 & \\ 0 & 0 & -2 & \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$C_4 \to C_4 + 2C_3$

1. unique no sol^n $\rightarrow$ Infinite sol^n.

2. system of eq^m $\nearrow$ homogeneous $AX=0$ ie $b=0$ $\rightarrow$ always consistent
$\searrow$ non homogen' $AX=b$
ie $b \neq 0$
$X = A^{-1}b'$

3. $AX=b$
inconsis consist
no sol^n unique infinite
$\underbrace{\text{unique infinite}}_{\text{non hom.}}$
hom $\rightarrow$ consist.

4. case of $soln$.

$a_1x + b_1y = c.$
$a_2x + b_2y = c_2.$

1) unique $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$
2) no sol^n
3) infinite sol^n $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_1}$

all equal.

5. method for sol^n.

$AX=b.$

$[A:b] \rightarrow$ argumented matrix

check (1) if rank $[A:b]$ = rank $[A]$ $\rightarrow$ consistent
if rank $[A:b] \neq$ rank $[A]$ $\rightarrow$ inconsistent

$$x + y + 2z = 3$$
$$x + 2y + z = 4$$
$$2x + y + 5z = \lambda.$$

1. ~~incons~~	$\rho(AB) = \rho(A) = n$
2. ~~∞ sol^n~~	$\rho(AB) = \rho(A) < n$
3. ~~cons~~	$\rho(AB) \neq \rho(A)$
inconsis. no sol^n.	

$$\begin{bmatrix} 1 & 1 & 2 & 3 \\ 1 & 2 & 1 & 4 \\ 2 & 1 & 5 & \lambda \end{bmatrix} =$$

$$\begin{bmatrix} 1 & 1 & 2 & 3 \\ 0 & 1 & -1 & 1 \\ 0 & -1 & 1 & \lambda-6 \end{bmatrix}$$

$$\Rightarrow \left[\begin{array}{ccc:c} 1 & 1 & 2 & 3 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & \lambda-5 \end{array}\right]$$

\# $\rho(A) \approx \rho(AB) \approx n$ (unknown)

$$\downarrow \qquad \downarrow$$

$\lambda - 5 = 0$: 2

$\lambda = 5$

but then;

$\rho(AB) =$ non zero.

or

** $\boxed{\text{Rank becomes } 3}$

NO UNIQUE SOL^n.

non-homogeneous.

(i) $rank[A:b] = rank[A] = n$

(no. of variables)

$\hookrightarrow$ unique solⁿ.

$rank[A:b] = rank[A] < n$

infinite

homog $\rightarrow$ consistent.

\# $\begin{cases} x+y+z=3 \\ x+2y+cz=4 \\ 2x+3y+2cz=k. \end{cases}$

$$[AB] \Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 1 & 2 & c & 4 \\ 2 & 3 & 2c & k \end{array}\right]$$

$$[AB] \Rightarrow \left[\begin{array}{cccc} 1 & 1 & 1 & 3 \\ 0 & 1 & c-1 & 1 \\ 0 & 1 & 2c-2 & k-6 \end{array}\right]$$

$\boxed{\begin{array}{l} R_3 \to R_3 - 2R_1 \\ R_2 \to R_2 - R_1 \end{array}}$

Important

$$[AB] \Rightarrow \left[\begin{array}{cccc} 1 & 1 & 1 & 3 \\ 0 & 1 & c-1 & 1 \\ 0 & 0 & c-1 & k-7 \end{array}\right]$$

$R_3 \to R_3 - R_1$.

$\boxed{\text{unique}}$: $r(A) = r(AB) = 3$ (no. of unknown)

$3=3=3$ possible

$c-1 \neq 0$, $c \neq 1$ | $k-7=0$, $k=7$ | $k-7 \neq 0$, $k \neq 0$

(~~no sol^n^~~)

$r(A) \neq r(A:b)$

$\Downarrow$

$2 \neq 3$

$c=1$, $k=1$ (∞)

$c-1=0$ $k-7 \neq 0$

$\boxed{c=1 \ \& \ k \neq 7}$

$$x_1 + 2x_2 + x_3 = 0$$

$$\# \qquad -x_1 - x_2 + x_3 = 0$$

$$3x_1 + 4x_2 + ax_3 = 0$$

$$\begin{bmatrix} 1 & 2 & 1 & 0 \\ -1 & -1 & 1 & 0 \\ 3 & 4 & a & 0 \end{bmatrix} = 0$$

$$\begin{array}{l} R_2 \to R_2 + R_1 \\ R_3 \to R_3 - 3R_1 \end{array} \begin{pmatrix} 1 & 2 & 1 & 0 \\ 0 & +1 & 2 & 0 \\ 0 & -2 & a-3 & 0 \end{pmatrix}$$

$$R_3 \to R_3 + 2R_1 \begin{pmatrix} 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & a+1 & 0 \end{pmatrix}$$

$a - 3 + 4$

$a = -1$

$\text{rank} = \rho(AB) = 2$

$\rho(A) = 2$

$(n) = 3$

∞ soln

* Eigen values & eigen vector
imp.

$\lambda_1 + \lambda_2 + \lambda_3 = \text{trace}$

$|d(A)| = \lambda_1 \cdot \lambda_2 \cdot \lambda_3$

eigen space : set of all eigen values

$$A - \lambda I = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

$$\begin{bmatrix} 1-\lambda & 2 \\ 4 & 3-\lambda \end{bmatrix}$$

$(3-\lambda)(1-\lambda) - 8$

$3 - \lambda - 3\lambda + \lambda^2 - 8$

$\lambda^2 - 4\lambda - 5$

$\lambda^2 - 5\lambda + \lambda - 5$

$\lambda(\lambda-5) + 1)(\lambda-5)$

$(\lambda+1)(\lambda-5)$

$\boxed{\lambda = -1, 5.}$

$$\begin{bmatrix} 1 & 2 & 0 \\ 2 & 2 & 2 \\ 0 & 2 & 3 \end{bmatrix} - \lambda I.$$

$$\rightarrow \begin{bmatrix} 1-\lambda & 2 & 0 \\ 2 & 2-\lambda & 2 \\ 0 & 2 & 3-\lambda \end{bmatrix}$$

$$1-\lambda\left((2-\lambda)(3-\lambda)-4\right) - 2(0) + 0$$

$$6-3\lambda+\lambda^2-2\lambda$$

$$1-\lambda(\lambda^2-5\lambda+6)$$

$$\boxed{\cancel{-\lambda^3+5\lambda^2-6\lambda+1}} \qquad \boxed{\lambda^3-6\lambda^2+3\lambda+10}$$

$\lambda = 2, -1, 5$

~~1-~~

Question:

$$A = \begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix}$$

$$X = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$$X_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$AX = \lambda X$?

$(A - \lambda I) X = 0$

$$AX_1 = \begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ -2 \end{bmatrix} = \lambda_1 X_1 = \begin{bmatrix} 3 \\ -6 \end{bmatrix} = 3\begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$$AX_2 = \begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = 1\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \lambda_2 X_2$$

$\lambda_1 = 3$

$\lambda_2 = 1$

$$A = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$$

$$\lambda(A) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

$$(A - \lambda I) = 0$$

characterstic equation

$$\begin{vmatrix} 1-\lambda & 4 \\ 3 & 2-\lambda \end{vmatrix} = \boxed{\lambda = -2, 5}$$

** ans.

for. $\lambda = -2$

$$\begin{bmatrix} 3 & 4 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Rank < 2

$$\begin{bmatrix} 3x_1 + 4x_2 = 0 \end{bmatrix}$$

Let $x_2 = k$.

$3x_1 = -4k$

$$\boxed{x_1 = -\frac{4k}{3}}$$

$$\boxed{\begin{aligned} x_1 &= -\frac{4}{3} \\ x_2 &= 1 \end{aligned}}$$

$$X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -\frac{4k}{3} \\ k \end{bmatrix} = k\begin{bmatrix} -4/3 \\ 1 \end{bmatrix}$$

for $= 5$.

$$\begin{bmatrix} -4 & 4 \\ 3 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 \\ 3 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-x_1 + x_2 = 0$$

Let $x_1 = k$.

$$\boxed{x_2 = k.}$$

☆

if you have dist eigen values

↓

linearly independent

$$X_2 = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} k \\ k \end{bmatrix} = k\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

$$A - \lambda I = \begin{bmatrix} 1-\lambda & 0 & 0 \\ 0 & 2-\lambda & 1 \\ 2 & 0 & 3-\lambda \end{bmatrix}$$

$$(1-\lambda)[(2-\lambda)(3-\lambda)$$

$$6 - 3\lambda + \lambda^2 - 2\lambda.$$

$$(1-\lambda)(\lambda^2 - 5\lambda + 6)$$

This is the answer $\boxed{-\lambda^3 + 5\lambda^2 - 6\lambda + 1 = 0}$

$$+\lambda^2 - 5\lambda + 6. = -\lambda^3 + 6\lambda^2 - 11\lambda + 7$$

$\lambda = 1, 2, 3$ $\qquad$ $-1 + 6 - 11 + 7$

$\lambda_1 = 1$

$$X_1 = k\begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$$

$$X_2 = k\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

X_3 $\lambda = 3$

$$= k\begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$$

#

$$\begin{bmatrix} 2 & 0 & 0 \\ 4 & 2 & 0 \\ 6 & 0 & 2 \end{bmatrix}$$

$(\lambda-2)^3$

$\boxed{\lambda = 2,2,2}$

Answer.

$|A-2I|$

$$\Rightarrow \begin{bmatrix} 0 & 0 & 0 \\ 4 & 0 & 0 \\ 6 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

1. $R_3 \to R_3 - \frac{6}{4} R_2$

$\cancel{6 - \frac{6}{4}}$

2. $\frac{R_3}{4} + R_1$

3. $R_3 \leftrightarrow R_1$

Ans. $\begin{pmatrix} x_1 = 0 \\ x_2 = k \\ x_3 = M \end{pmatrix}$

$x_1 + 0 + 0 = 0$

unused to the same el.

$$X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{bmatrix} 0 \\ k \\ M \end{bmatrix} = k\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + M\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

x_1 x_2

eigen ~~value~~ Vector = |Rank − no. of variables|

$|1-3| = 2$ eigen value

#

$$\begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$|A - \lambda I| = 0$$

$$\lambda = 3, 3, 5$$

---

for $\lambda = 5$

$$\begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & -1 \\ 0 & 2 & -4 \\ 0 & 2 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & -1 \\ 0 & 2 & -4 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$-x_1 + x_2 - x_3 = 0.$$

$$2x_2 - 4x_3 = 0.$$

$x_3 = k$

$x_2 = 2x_3 = 2k.$

$$\boxed{\begin{aligned} x_1 - x_2 - x_3 &= 2k - k \\ x_1 &= k. \end{aligned}}$$

$$X_1 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} k \\ 2k \\ k \end{bmatrix} = k\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

$2 = |1-3|$

agen ~~value~~ vector

$$\begin{pmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{pmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{pmatrix} 1 & 1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{matrix} 0 \\ 0 \\ 0 \end{matrix}$$

$x_1 + x_2 - x_3 = 0$

$x_2 = k$

$x_3 = \Delta.$

$x_1 = x_3 - x_2 = \Delta - k.$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$X = \Delta - K.$

$$\begin{bmatrix} K + 0\Delta \\ \Delta + 0K \end{bmatrix} = y\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + K\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$$

x_2 x_3.