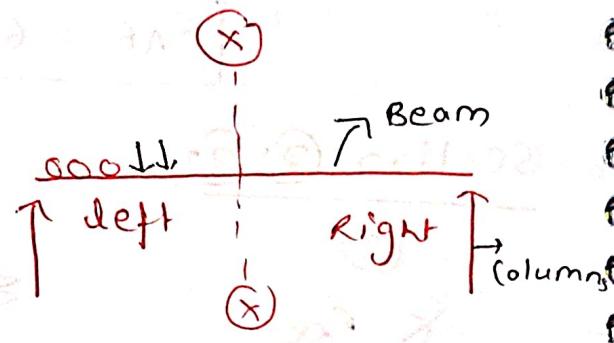
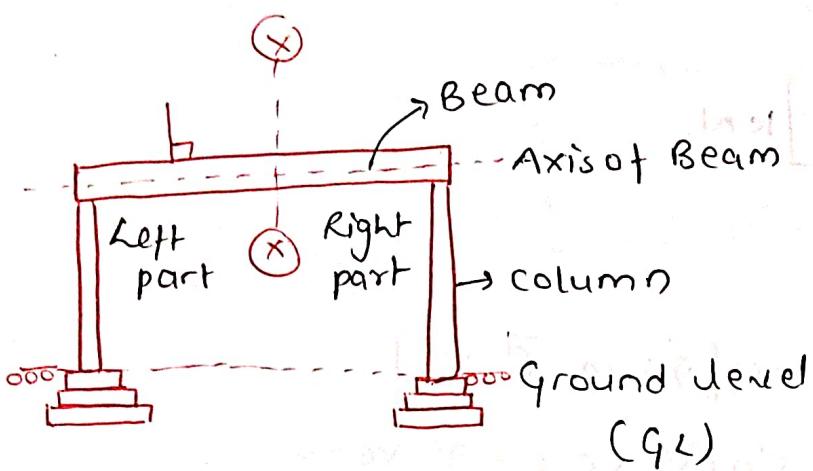


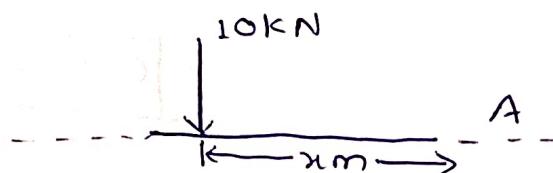
(SFD &
BMD)

SHEAR FORCE DIAGRAMS & BENDING MOMENT DIAGRAMS



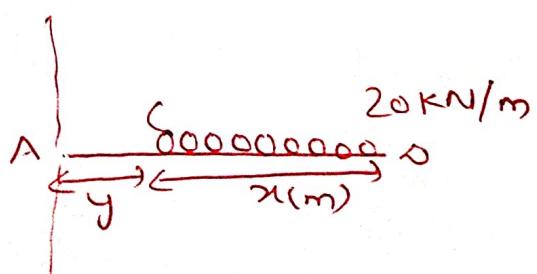
Beam :- Beam is a structural member which is acted upon by a system of external loads at right angles to the axis of beam.

Loads :- ①. Point load/concentrated load :-



$$\rightarrow \text{moment at } A = 10x \text{ kNm}$$

②. Uniformly distributed load (UDL) :-



\rightarrow moment at A,

$$= \text{load} \times \text{span} \times \left(\frac{1}{2} \text{span} + \text{remaining distance} \right)$$

$$= 20 \times x \times \left(\frac{x}{2} + y \right)$$

\rightarrow moment at C,

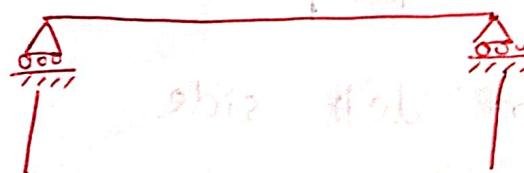
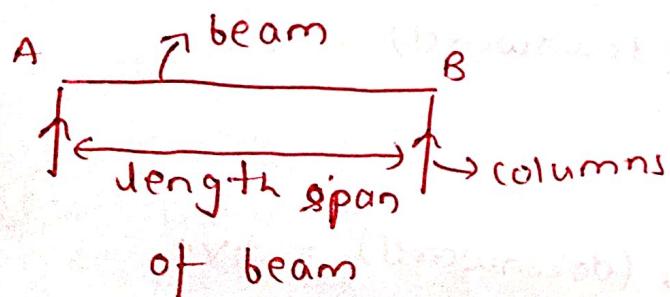
$$= 20 \times x \times \left(\frac{x}{2} + 0 \right) = 20x \left(\frac{x}{2} \right)$$

\rightarrow moment at D,

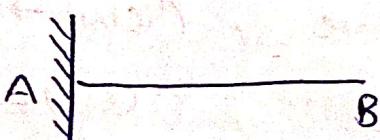
$$= 20 \times x \times \left(\frac{x}{2} \right)$$

- Shear force:- The algebraic sum of all vertical forces at any section of a beam to the right or left of the section is known as shear force.
- Bending moments :- The algebraic sum of the moments of all the forces acting to the right or left of the section is known as bending moment.
- Shear force diagrams:- (SFD) A SFD is one which showing the variation of shear force along the length of the beam.
- Bending moment Diagram (BMD):- A BMD is one which showing variation of bending moment along the length of the beam.
- Classification / Types of beams:-

- ①. Simply supported / Roller supported beam:- When a beam is supported with simply supported support or roller supported support at both the ends . It is known S.S. / R.S. beam.



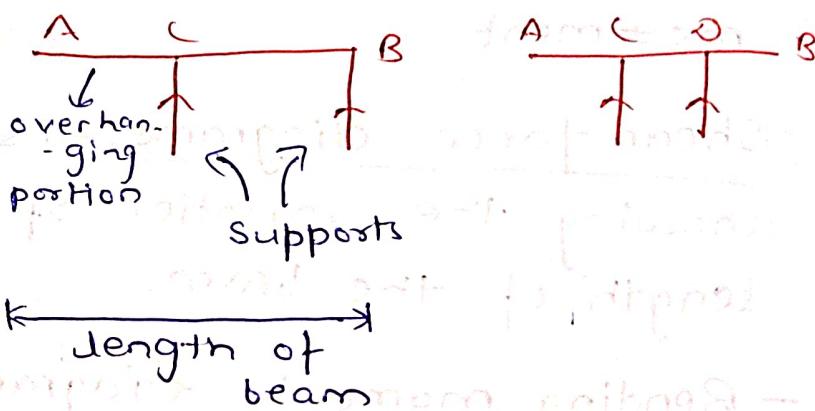
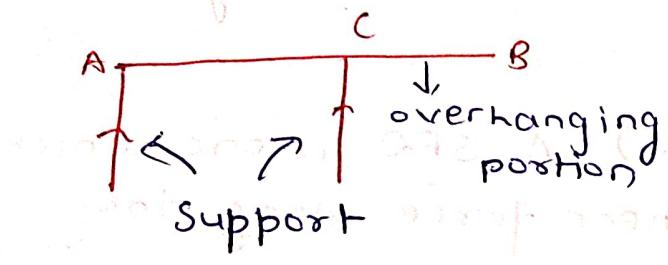
- ②. Cantilever Beam:- (one end fixed another in air)



③ Fixed beam :- (beam supported by fixed supporting support at both the ends)



④ overhanging beam :-



⑤ Continuous Beam :- (Beam having multiple supports more than 2).



Sign conventions :-

for shear force,

from right side,

$$\Rightarrow \uparrow (\text{upward}) = +\text{ve}, \downarrow (\text{downward}) = -\text{ve}$$

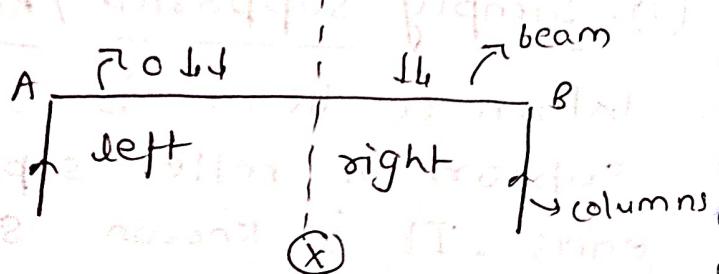
from left side,

$$\Rightarrow \uparrow (\text{upward}) = -\text{ve}, \downarrow (\text{downward}) = +\text{ve}$$

for bending moments,

from right side $\rightarrow \sigma(A(CW)) \rightarrow +\text{ve}, \tau(CW) \rightarrow -\text{ve}$

from left side $\rightarrow \sigma(A(CW)) = -\text{ve}, \tau(CW) = +\text{ve}$



\rightarrow = \rightarrow sagging moment, = +ve values

\rightarrow = \rightarrow (concavity)

\rightarrow = \rightarrow hogging moment, = -ve

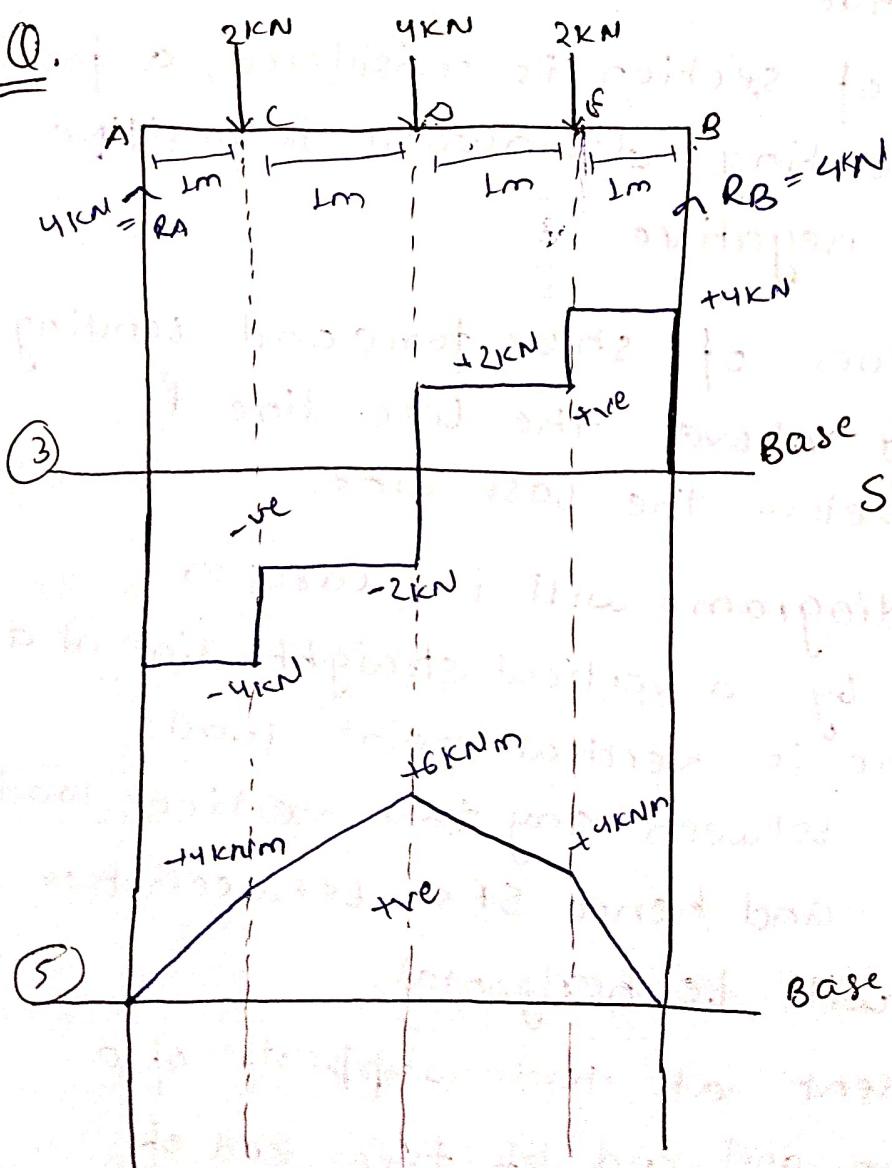
\rightarrow = \rightarrow (convexity)

- Point for drawing the SFD and BMD :-

- ① Consider the left or right portion of the section.
- ② Add the forces including reaction normal to beam on one of the portions.
- * If right portion is considered, a force in right portion acting upward is positive and downward is negative.
- * If left portion of section is considered, a force in left portion acting downward is positive and upward is negative.
- ③ The positive values of shear force and bending moment are plotted above the base line & negative values below the base line.
- ④ The shear force diagram will increase or decrease suddenly by a vertical straight line at a section where there is vertical point load.
- ⑤ The shear force between any two vertical loads will be constant and hence SFD between two vertical loads will be horizontal.
- ⑥ The bending moment at two supports of a simply supported beam and end of free end of

cantilever beam will be zero.

- ⑦. The SFD for uniformly distributed load will be straight inclined line.
- ⑧. The BMO for uniformly distributed load will be a parabolic curve.
- ⑨. The BM will be maximum where there is zero shearing forces and changing its sign from +ve to -ve and -ve to +ve.
- ⑩. The point of contraflexure will occur when there is zero bending moment and changing its sign from +ve to -ve and -ve to +ve.



→ ①. Reactions:-

Let the R_A and R_B be reactions at A and B respectively.

Taking moment about A,

$$\sum M_A = 0 \quad [\text{+ve, } \text{? -ve}]$$

$$-2x_1 - 4x_2 - 2x_3 + R_B \times 4 = 0$$

$$R_B = 4 \text{ kN}$$

Consider vertical equilibrium,

$$\sum V = 0 \quad [\text{T +ve, J -ve}]$$

$$+R_A + R_B - 2 - 4 - 2 = 0$$

$$R_A = 8 - 4$$

$$R_A = 4 \text{ kN}$$

②. Shear force calculation :-

[from right side - T +ve]

$$SF_B = +4 \text{ kN}$$

$$SF_E = +4 - 2 = +2 \text{ kN}$$

$$SF_O = +4 - 2 - 4 = -2 \text{ kN}$$

$$SF_C = +4 - 2 - 4 - 2 = -4 \text{ kN}$$

(Since, SF $\neq 0$, so we will calculate upto last load only).

④. Bending moment calculation:- [from right side]

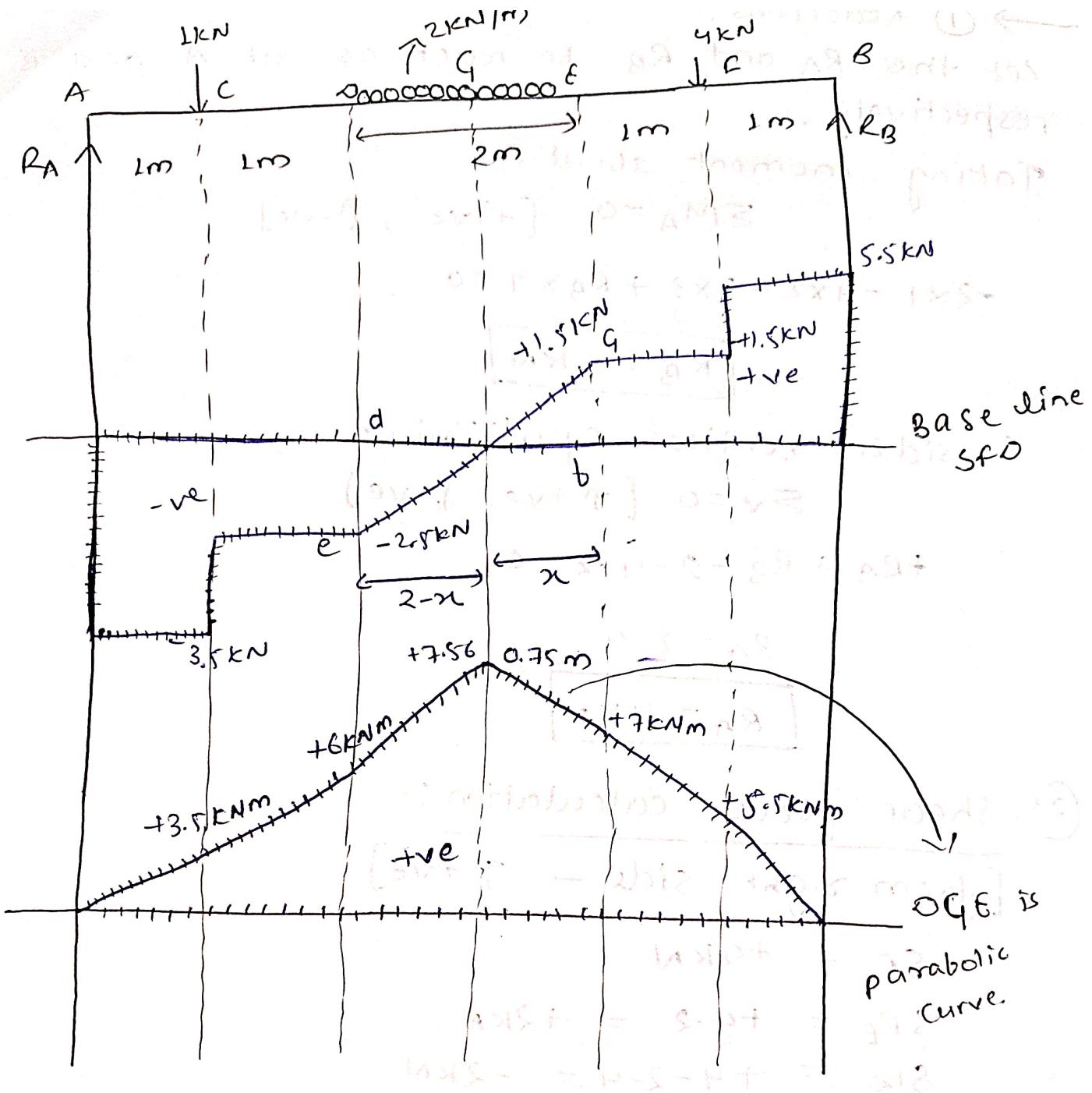
$$BM_B = 0 \quad [\text{+ve, } \text{? -ve}] = \text{? +ve}$$

$$BM_E = 4 \times 1 = +4 \text{ kNm}$$

$$BM_O = (4 \times 2) + (-2 \times 1) = 6 \text{ kNm}$$

$$BM_C = (4 \times 3) + (-2 \times 2) + (-4 \times 1) = 4 \text{ kN}$$

$$BM_A = @ (4 \times 4) - (2 \times 3) - (4 \times 2) - (2 \times 1) = 0$$



→ Reactions:-

Let R_A and R_B be reactions at A and B.

Now, taking moment about A:

$$\sum M_A = 0 \quad [↑+ve, \vec{r}-ve]$$

$$R_B \times 6 - 4 \times 5 - (2 \times 2 * (1+2)) - 1 \times 1 = 0$$

$$R_B = 5.5 \text{ kN}$$

Consider vertical equilibrium,

$$\sum V = 0 \quad [T+ve, J,-ve]$$

$$R_A + R_B - 4 - 1 - 2 \times 2 = 0$$

$$R_A = 9 - 5.5$$

$$R_A = 3.5 \text{ kN}$$

SF calculations :-

$$SF_B = +5.5 \text{ kN}$$

$$SF_F = +5.5 - 4 = +1.5 \text{ kN}$$

$$SF_E = +5.5 - 4 - 2 \times 0 = +1.5 \text{ kN}$$

$$SF_O = +5.5 - 4 - 2 \times 2 = -2.5 \text{ kN}$$

$$SF_C = +5.5 - 4 - 2 \times 2 - 1 = -3.5 \text{ kN}$$

Reaction of zero S.F. :-

From Δabc and Δade

$$\frac{ab}{bc} = \frac{ad}{de} \quad [\text{similarity}]$$

$$\frac{x}{1.5} = \frac{2-x}{2.5}$$

$$2.5x = 3 - 1.5x$$

$$4x = 3$$

$$x = 0.75 \text{ m}$$

Bending moment calculation

[from right side \rightarrow $J+ve$]

$$BM_B = 0$$

$$BM_F = +5.5 \times 1 = 5.5 \text{ kNm}$$

$$BM_E = (+5.5 \times 2) - (4 \times 1) = +7 \text{ kN}$$

$$BM_D = (+5.5 \times 4) - (4 \times 3)$$

$$- (2 \times 2 \times (1+0)) = +6 \text{ kN}$$

$$BM_C = + (5.5 \times 5) - (4 \times 4)$$

$$- (2 \times 2 \times (1+1)) = +3.5 \text{ kN}$$

$$BM_A = 0.$$

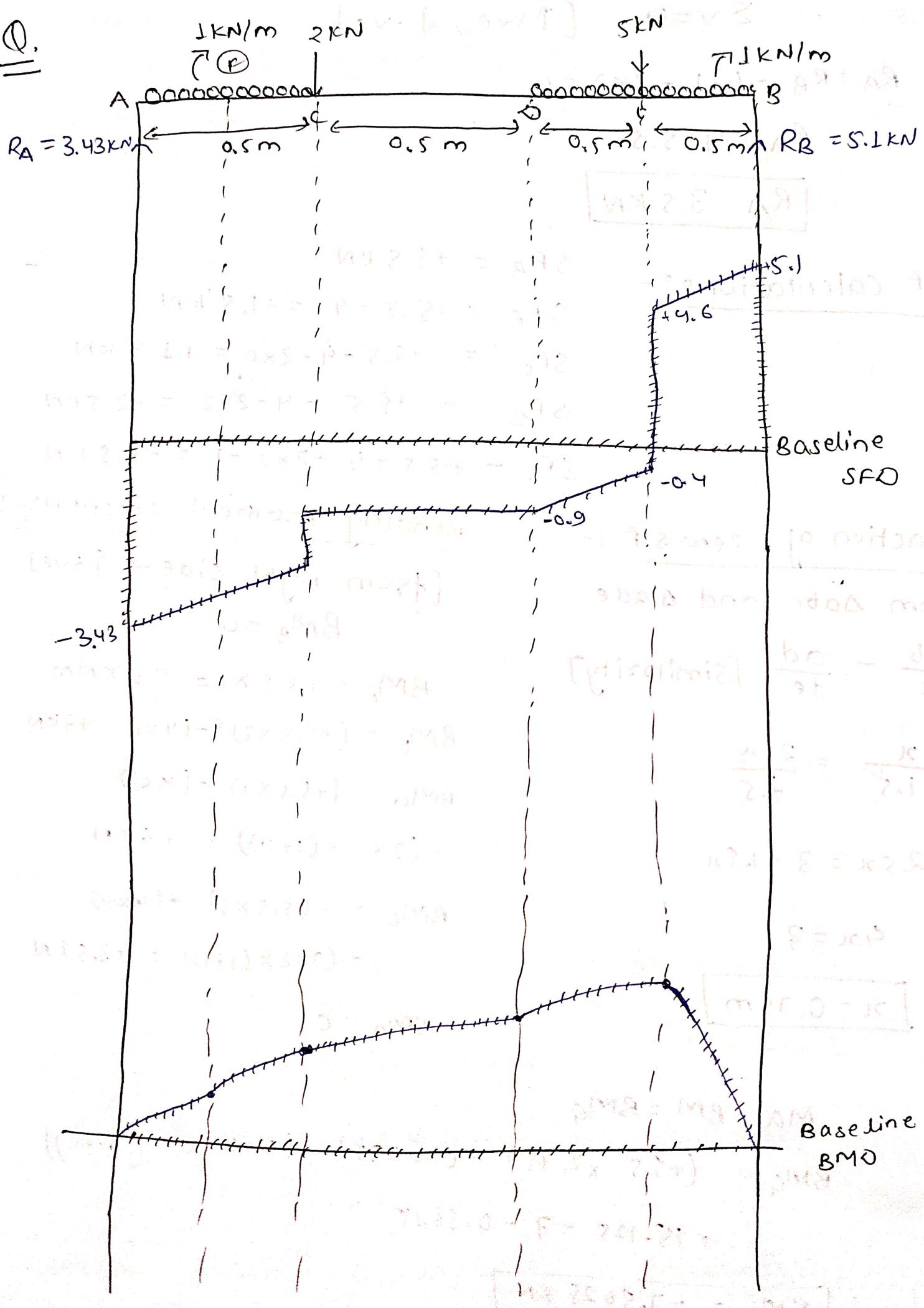
$$\text{Max } BM = BM_G$$

$$BM_G = (+5.5 \times 2.75) - (4 \times 1.75) - (2 \times 0.75 \times \left(\frac{0.75}{2}\right))$$

$$= 15.125 - 7 - 0.5625$$

$$BM_G = +7.5625 \text{ kN}$$

Q.



→ Reactions :- Let R_A and R_B be reactions at A and B respectively.

$$\sum M_A = 0 [S+ve, \vec{I}-ve]$$

$$R_B \times 2 - (1 \times 1 \times (\frac{1}{2} + 1.5)) - (5 \times 1.5) - (3 \times 0.5) - (1 \times 0.5 \times \frac{1}{2}) = 0$$

$$2R_B = 7 + 0.125$$

Consider vertical eqⁿ,

$$\sum V = 0 [T+ve, J, -ve]$$

SF calculations :- [from right side $\rightarrow \uparrow +ve$]

$$SF_B = 5.1 \text{ kN}$$

$$SF_E = +5.1 - (1 \times 0.5) = +4.6 \text{ kN}$$

without point load

$$SF_E = 5.1 - (1 \times 0.5) - (5 \times 1) = -0.4 \text{ kN}$$

with point load,

$$SF_O = +5.1 - 5 - (1 \times 1 \times \frac{1}{2}) =$$

$$SF_E =$$

BM calculations :- [from right side $\rightarrow S+ve$]

$$BM_B = 0$$

$$BM_E = -(1 \times 0.5 \times \frac{0.5}{2}) + (5 \times 0.5) = 4.25 \text{ kNm}$$

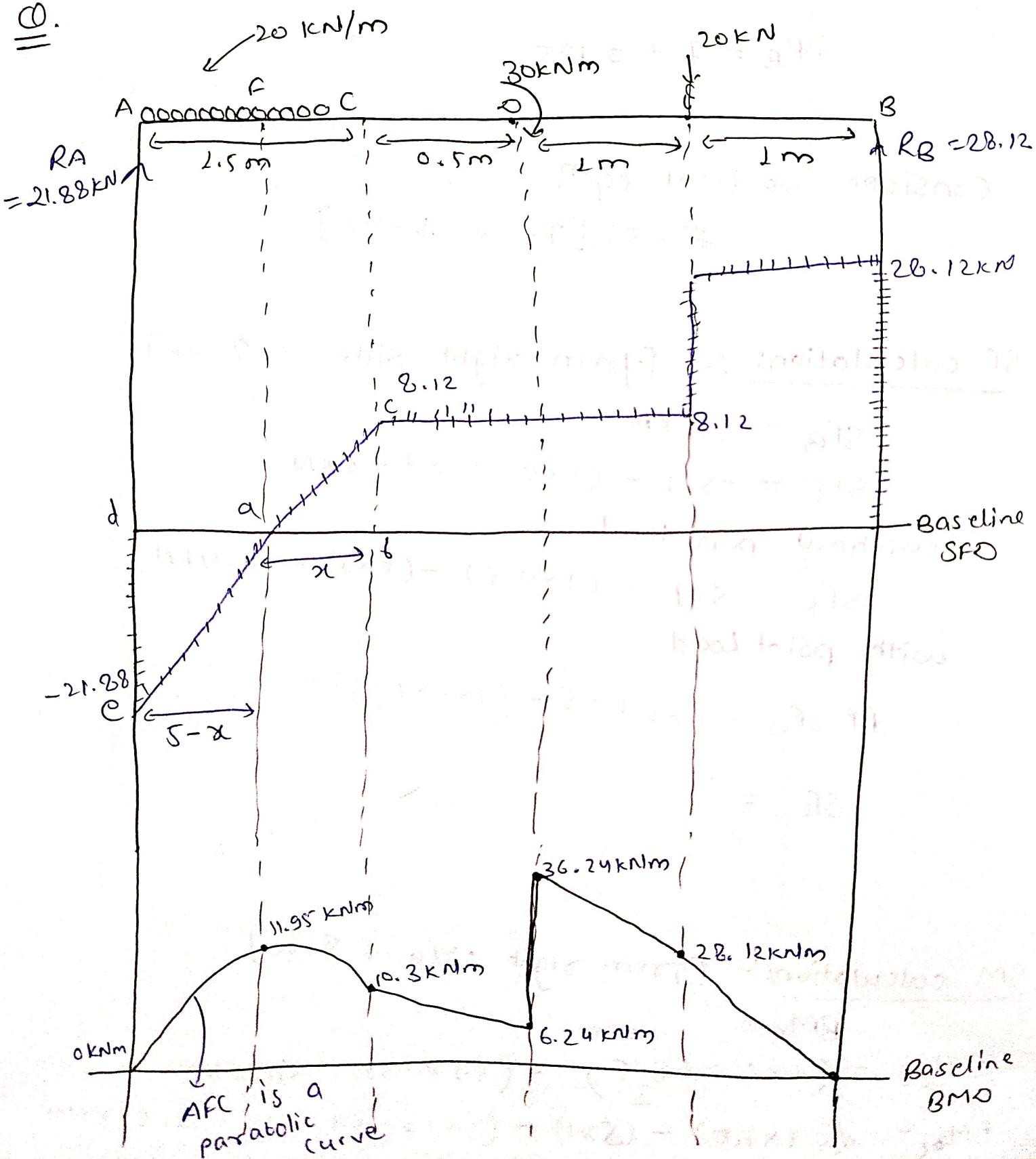
$$BM_{O_C} = (5 \times 1 \times 1.5) - (5 \times 1) - (1 \times 1 \times (\frac{1}{2} + 0.5)) = 3.65 \text{ kNm}$$

$$BMA = (5.1 \times 1) - (5 \times 0.5) - (1 \times 1 \times \frac{1}{2}) = 2.1 \text{ kNm}$$

$$BMA = 0$$

$$BMR = (5.1 \times 1.75) - (5 \times 1.25) - (1 \times 1 \times (\frac{1}{2} + 0.75)) \\ - (2 \times 0.25) - (1 \times 0.5 \times (\frac{0.5}{2}))$$

\therefore



→ Let R_A and R_B be reactions at A and B resp.

Taking moment at A,

$$\sum MA = 0 \quad [\text{F+ve, } \uparrow\text{-ve}]$$

$$R_B \times 4 - 20 \times 3 - 30 - (20 \times 1.5 \times \left(\frac{1.5}{2} + 0\right)) = 0$$

$$R_B = \frac{22.5 + 30 + 60}{4}$$

$$R_B = 28.125 \text{ KN}$$

Considering vertical equilibrium, $\sum V = 0$

$$\sum V = 0 \quad [\uparrow\text{+ve, } \downarrow\text{-ve}]$$

$$+ 28.125 - 20 - (20 \times 1.5) + R_A = 0$$

$$R_A = 21.88 \text{ KN}$$

SF calculations :- [from right side - $\uparrow\text{+ve}$]

$$SF_B = 28.12 \text{ KN}$$

$$SF_E = +28.12 - 20 = 8.12 \text{ KN}$$

$$SF_O = +28.12 - 20 = 8.12 \text{ KN}$$

$$SF_C = +28.12 - 20 = 8.12 \text{ KN}$$

$$SF_A = +28.12 - 20 - 20(1.5) = -21.88 \text{ KN}$$

from $\triangle abc$ & $\triangle ade$,

$$\frac{ab}{bc} = \frac{ad}{de}$$

$$\frac{x}{8.12} = \frac{5-x}{21.88}$$

$$21.88x = 8.12 \times 5 - 8.12x$$

$$x = 0.406 \text{ m}$$

BM & calculations :- [from right side \rightarrow +ve]

$$BM_B = 0$$

$$BM_F = 28.12 \times 1 = +28.12 \text{ kNm}$$

$$BM_o = 28.12 \times 2 - 20 \times 1 = +36.24 \text{ kNm}$$

without moment

$$BM_o = +28.12 \times 2 - 20 \times 1 - 30 = 6.24 \text{ kNm}$$

with moment

$$BM_c = +28.12 \times 2.5 - 20 \times 1.5 - 30 = 10.3 \text{ kNm}$$

Maximum BM will be at BM_F ,

$$BM_F = +28.12 \times 2.906 - 20 \times 1.906 - 30 - (20 \times 0.406 \times \frac{0.406}{2})$$

$$\boxed{BM_F = +11.95 \text{ kNm}}$$

Q. \rightarrow Reactions :-

Let R_A and R_B be the reaction of A & B resp.

Taking moments about A,

$$\sum M_A = 0 \quad [S+ve, R-ve]$$

$$-2 \times 7 + R_B \times 5 - (2 \times 5 \times \frac{5}{2}) - 5 \times 5 \times 2 = 0$$

$$5R_B = 14 + 11 + 25$$

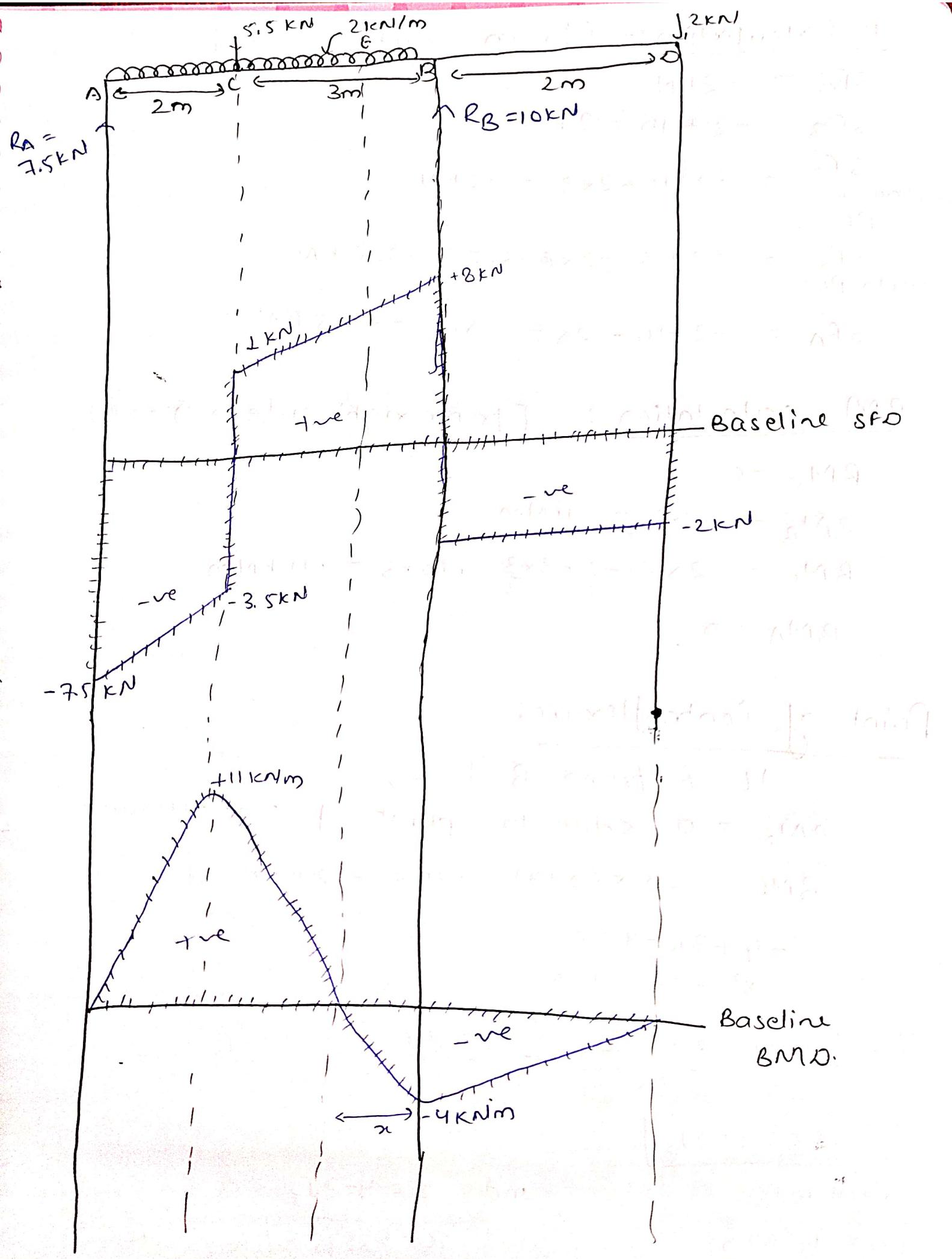
$$\boxed{R_B = 20 \text{ KN}}$$

Consider vertical equilibrium

$$\sum V = 0$$

$$+R_A + R_B - 2 - 2.5 - 5.5 = 0$$

$$\boxed{R_A = 7.5 \text{ KN}}$$



SF Calculation :- [from right side $\rightarrow \uparrow +ve$]

$$SF_0 = -2 \text{ kN}$$

$$SF_B = -2 + 10 = +8 \text{ kN}$$

$$SF_c = -2 + 10 - 2 \times 3 = +2 \text{ kN}$$

without PL

$$SF_c = -2 + 10 - 2 \times 3 - 5.5 = -3.5 \text{ kN}$$

with PL

$$SF_A = -2 + 10 - 2 \times 5 - 5.5 = -7.5 \text{ kN}$$

BM Calculation :- [from right side $\rightarrow \uparrow +ve$]

$$BM_0 = 0$$

$$BM_B = -2 \times 2 = -4 \text{ kNm}$$

$$BM_c = -2 \times 5 - 2 \times 3 \times \frac{3}{2} + 10 \times 3 = +11 \text{ kNm}$$

$$BM_A = 0.$$

Point of Contraflexure :-

At E from B to C,

$BM_E = 0$ (due to point of contraflexure)

$$BM_E = -2 \times (2+x) + 10x - 2x \times \frac{x}{2} = 0$$

$$-4 + 8x - x^2 = 0$$

$$x^2 - 8x + 4 = 0$$

$$x = \frac{8 \pm \sqrt{64-16}}{2} = \frac{8 \pm \sqrt{48}}{2} = \frac{8 \pm 4\sqrt{3}}{2}$$

$$\boxed{x = 4 \pm 2\sqrt{3}}$$

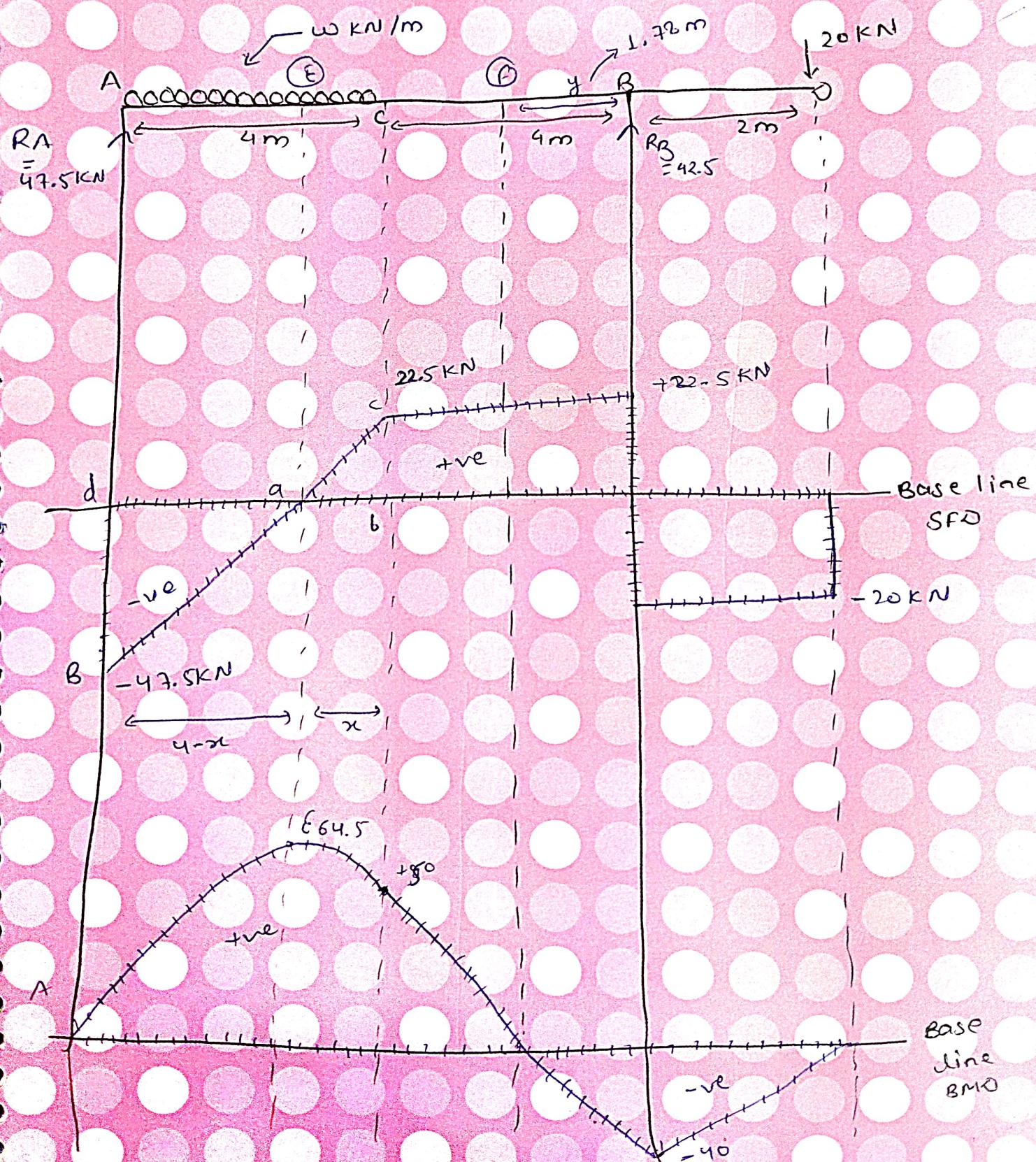
$$x = 4 + 2\sqrt{3}$$

$$\boxed{x = 7.964 \text{ m}}$$

$$\text{and } x = 4 - 2\sqrt{3}$$

$$\boxed{x = 0.535 \text{ m}}$$

* Q. Determine the value of UDL so that the B.M. at section C is 50 kN-m for the beam loaded as shown in fig. Draw the SFD and BMD. For this beam, for the calculated value of UDL, also noted the point of contraflexure if any.



→ Reactions:- Let R_A and R_B be the reactions at A and B respectively.

Let $\omega \text{ KN/m}$ be the intensity of UDL.

Taking moment about A,

$$\sum M_A = 0 \quad [\Sigma +ve, \bar{C} - ve]$$

$$-20 \times 10 + R_B \times 8 - (\omega \times 4 \times 4) = 0$$

$$8R_B - 8\omega = 200$$

$$R_B - \omega = 25 \quad \text{--- (1)}$$

Considering vertical equilibrium,

$$\sum V = 0 \quad [\uparrow +ve, \downarrow -ve]$$

$$-20 + R_B - \omega \times 4 + R_A = 0$$

$$R_B + R_A - 4\omega = 20 \quad \text{--- (2)}$$

∴ BM. at C is 50 KN.m.

- BM calculation at C:- [from right side, $\Sigma +ve$]

$$BM_C = -20 \times 6 + R_B \times 4 = 50$$

$$4R_B = 170$$

$$R_B = 42.5 \text{ KN}$$

from (1),

$$42.5 - \omega = 25$$

$$\omega = 17.5 \text{ KN}$$

from (2),

$$42.5 + R_A - 4 \times 17.5 = 20$$

$$R_A = 20 + 70 - 42.5 -$$

$$R_A = 90 - 42.5$$

$$R_A = 47.5 \text{ KN}$$

From Δabc & Δade , - SF (calculation):-

$$\frac{ab}{ad} = \frac{bc}{de}$$

[from right side - T+ve]

$$\Rightarrow \frac{x}{4-x} = \frac{22.5}{47.5}$$

A truss, $SF_D = -20 \text{ kN}$

$$SF_B = -20 + 42.5 = 22.5 \text{ kN}$$

$$47.5x = 90 - 22.5x$$

$$SF_c = -20 + 42.5$$

$$70x = 90$$

$$= 22.5 \text{ kN}$$

$$x = \frac{9}{7} = 1.28$$

$$SF_A = -20 + 42.5 - 17.5 \times 4$$

$$x = 1.28 \text{ m}$$

$$[1.28 + 20 = 22.5]$$

$$SF_A = -47.5 \text{ kN}$$

- BM Calculation:-

$$BM_D = 0$$

$$BM_B = -20 \times 2 = -40 \text{ kNm}$$

$$BM_C = -20 \times 6 + 42.5 \times 4$$

$$BM_C = 50 \text{ kNm}$$

$$BM_A = 0$$

$$At F, BM = 0$$

$$BM_F = -20(2+y) + 42.5y$$

$$\Rightarrow -40 - 20y + 42.5y = 0$$

$$\Rightarrow 22.5y = 40$$

$$y = 1.78 \text{ m}$$

Maximum BM will be

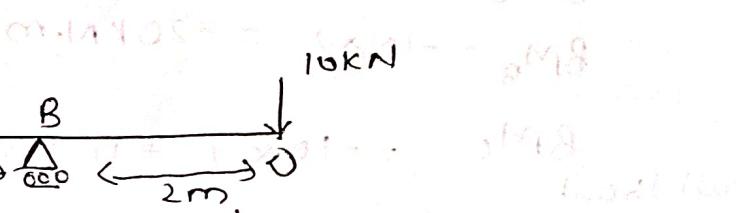
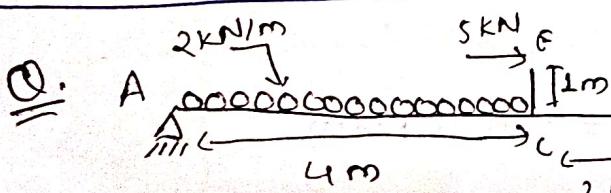
BM at F,

$$BM_F = -20 \times 7.28 + 42.5 \times 5.28$$

$$= 17.5 \times 1.28 \times \left(\frac{1.28}{2}\right) = 17.8 + 8.20 + 0.1 = 26.12$$

$$= 145.6 + 224.4 - 14.3$$

$$BM_F = 64.5 \text{ kNm}$$



→ Let R_{AV} and R_B be the reactions of A and B respectively.

Taking moment about A,

$$\sum MA = 0 \quad [\text{↑+ve, } \text{C-ve}]$$

$$-10 \times 8 + R_B \times 6 - 5 - 4 \times 2 \left(\frac{y}{2} \right) = 0$$

$$-80 + 6R_B - 5 - 16 = 0$$

$$R_B = \frac{101}{6}$$

$$R_B = 16.8 \text{ kN}$$

Considering vertical equilibrium,

$$\sum V = 0 \quad [\text{↑+ve, } \downarrow-\text{ve}]$$

$$-10 + R_B - 2 \times 4 + R_{AV} = 0$$

$$R_{AV} = 18 - 16.8$$

$$R_{AV} = 1.2 \text{ kN}$$

- SF calculation :- [from right side $\rightarrow \uparrow\text{+ve}$]

$$SF_D = -10 \text{ kN}$$

$$SF_B = -10 + 16.8 = 6.8 \text{ kN}$$

$$SF_C = -10 + 16.8 = 6.8 \text{ kN}$$

$$SF_A = -10 + 16.8 - 8 = -1.2 \text{ kN}$$

- BM calculation :- [from right side $\rightarrow \text{↑+ve}$]

$$BM_D = 0$$

$$BM_B = -10 \times 2 = -20 \text{ kNm}$$

$$BM_C = -10 \times 4 + 16.8 \times 2 = -6.4 \text{ kNm}$$

without

moment

$$BM_A = -10 \times 8 + 16.8 \times 6 - 5 - (2 \times 4 \times \frac{4}{2}) = -39.2$$

$R_A = 0$ because no shear force at A due to pinned support

$$BM_C = -6.4 - 5 = -11.4 \text{ kNm}$$

with moment = 0 at C

$$BM_F = -10 \times 7.4 + 16.8 \times 5.4 - 5 - 2 \times 3.4 \times \frac{3.4}{2} = 0.16 \text{ KN}$$

$$BM_F = -7.4 + 90.072 - 5 - 11.56 = 0.16 \text{ KN}$$

$$\boxed{BM_F = 0.16 \text{ KN}}$$

from Δabc & Δade

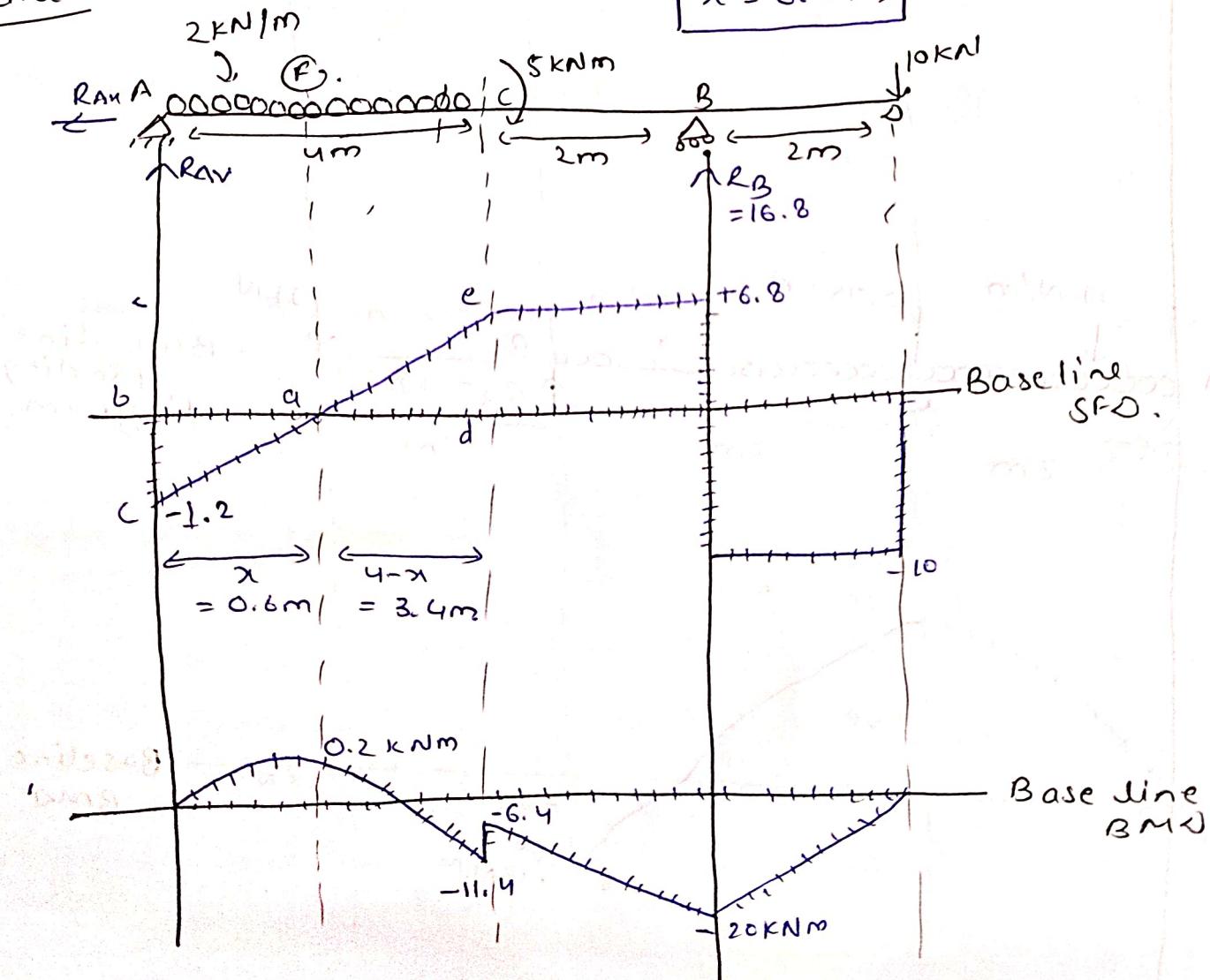
$$\frac{ab}{ad} = \frac{bc}{de}$$

$$\Rightarrow \frac{x}{4-x} = \frac{1.2}{6.8} \Rightarrow 6.8x = 4.8 - 1.2x$$

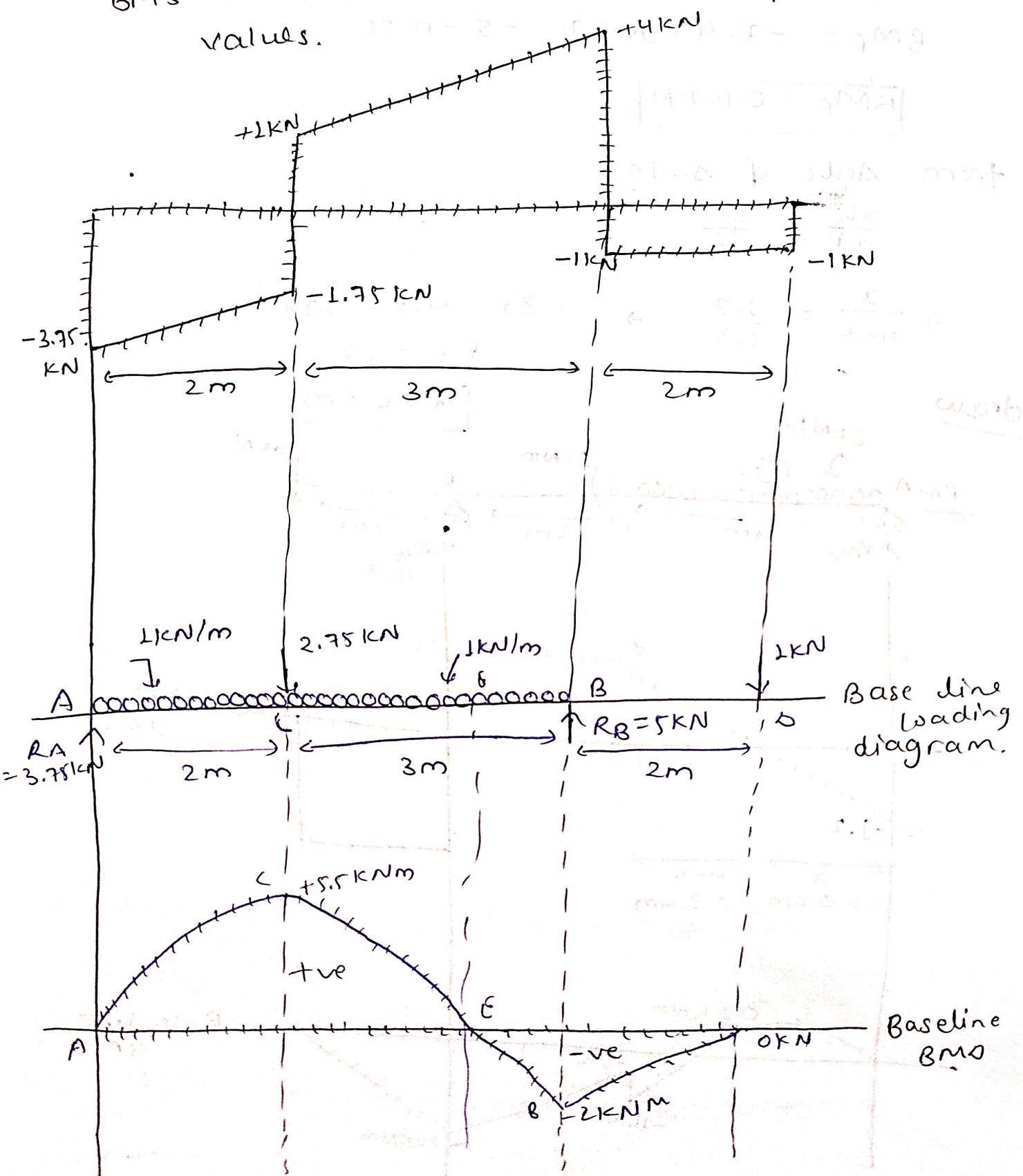
$$8x = 4.8$$

$$\boxed{x = 0.6 \text{ m}}$$

Redraw



- Q. The SFD for the beam rest on 2 supports one being at the left hand side as shown in fig.
- * Determine the values of loads & draw the loading diagram. Also calculate the values of BMs and draw the BMD with principle values.



- By the inspection of diagram. The other support will be at B whose SF increasing suddenly in upward direction.
- As given in fig. (problem) - one support is at RHS of fig.
- The SF is constant 0 to B and it is going down at 0 so there is point load of 1KN in downward direction.
- The value of reaction of B. R_B will be
- $$R_B = 4 + 1 = 5 \text{ KN}$$
- \because SF decreasing from 4KN to 1KN, from B to C, linearly, there should be a UDL with intensity
- $$= \frac{+4 - (+1)}{3} = \boxed{+1 \text{ KN/m}}$$
- There is a drop of SF from 1KN to -1.75KN at C there is point load of $(1 + 1.75)$
- $$\boxed{= 2.75 \text{ KN}}$$
- \because The SF again varies from -1.75 to -3.75 KN linearly from C to A, there should be a UDL of intensity $= \frac{(-1.75) - (-3.75)}{2} = \boxed{1 \text{ KN/m}}$
- BM calculation :-
- $$BM_A = 0$$
- $$BM_B = -1 \times 2 = -2 \text{ KN-m}$$
- $$BM_C = (- \times 5) - (1 \times 3 \times \frac{3}{2}) = +5.5 \text{ KN}$$
- $$+ 5 \times 3$$
- $$BM_A = 0 \text{ KN.}$$

- Point of contraflexure :- At E from B to C.

$$BM_E = 0$$

$$BM_E = -1x(2+x) + 5x - 1x \frac{2x}{2} = 0$$

$$\Rightarrow -2 + 4x - \frac{x^2}{2} = 0$$

$$-4 + 8x - x^2 = 0$$

$$x^2 - 8x + 4 = 0$$

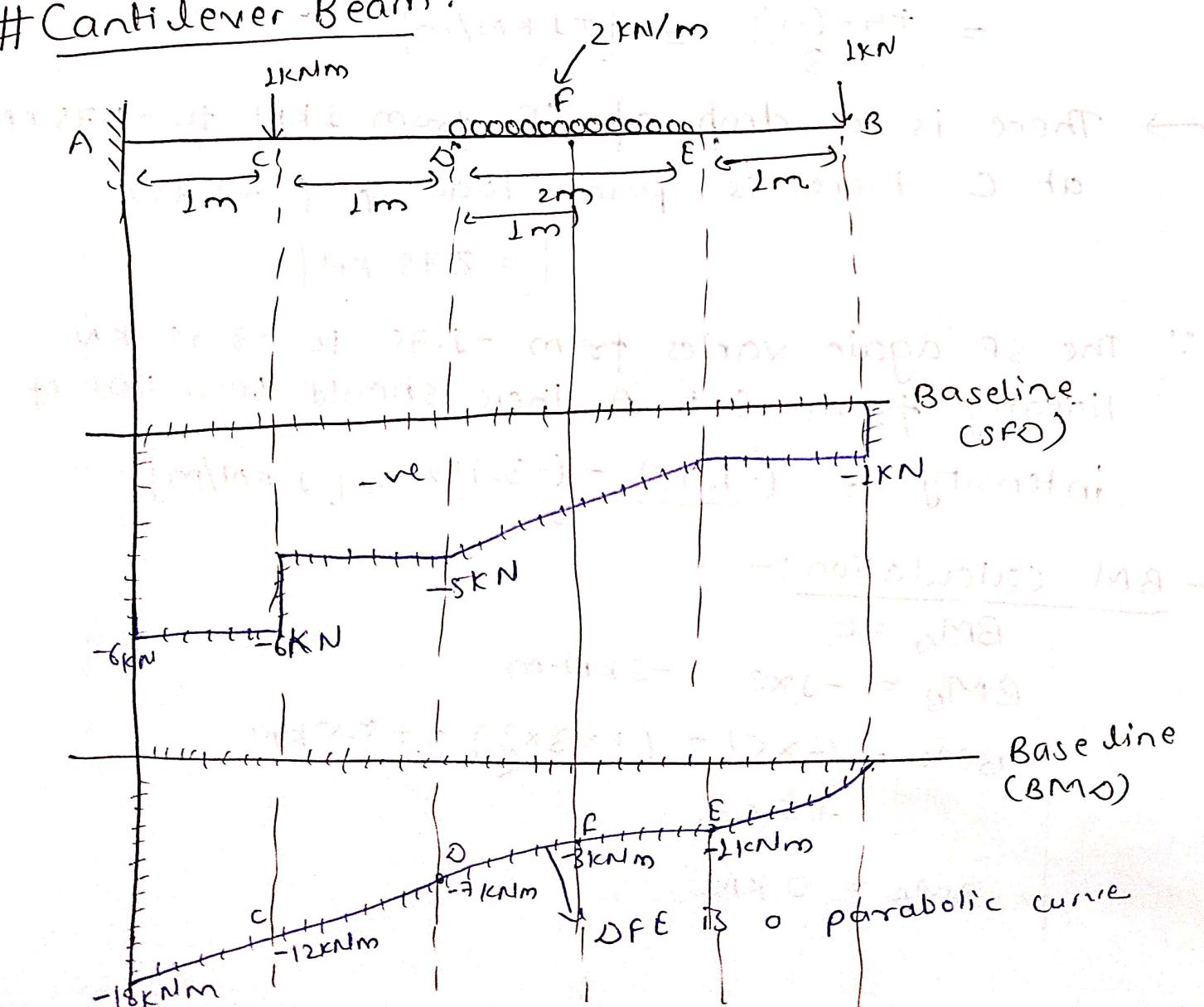
$$x = \frac{8 \pm \sqrt{64-16}}{2} = \frac{8 \pm 4\sqrt{3}}{2}$$

$$x = 4 - 2\sqrt{3}$$

$$x = 4 \pm 2\sqrt{3}$$

$$x = 0, 8.4m$$

Cantilever Beam :-



→ SF calculation :- [from right side → ↑ +ve]

$$SF_B = -1 \text{ kN}$$

$$SF_E = -1 \text{ kN}$$

$$SF_D = -1 - 2 \times 2 = -5 \text{ kN}$$

$$SF_C = -1 - 2 \times 2 - 1 = -6 \text{ kN}$$

$$SF_A = -6 \text{ kN}$$

BM Calculation :- [from right side → ↗ +ve]

$$BM_B = 0$$

$$BM_E = -1 \times 1 = -1 \text{ kNm}$$

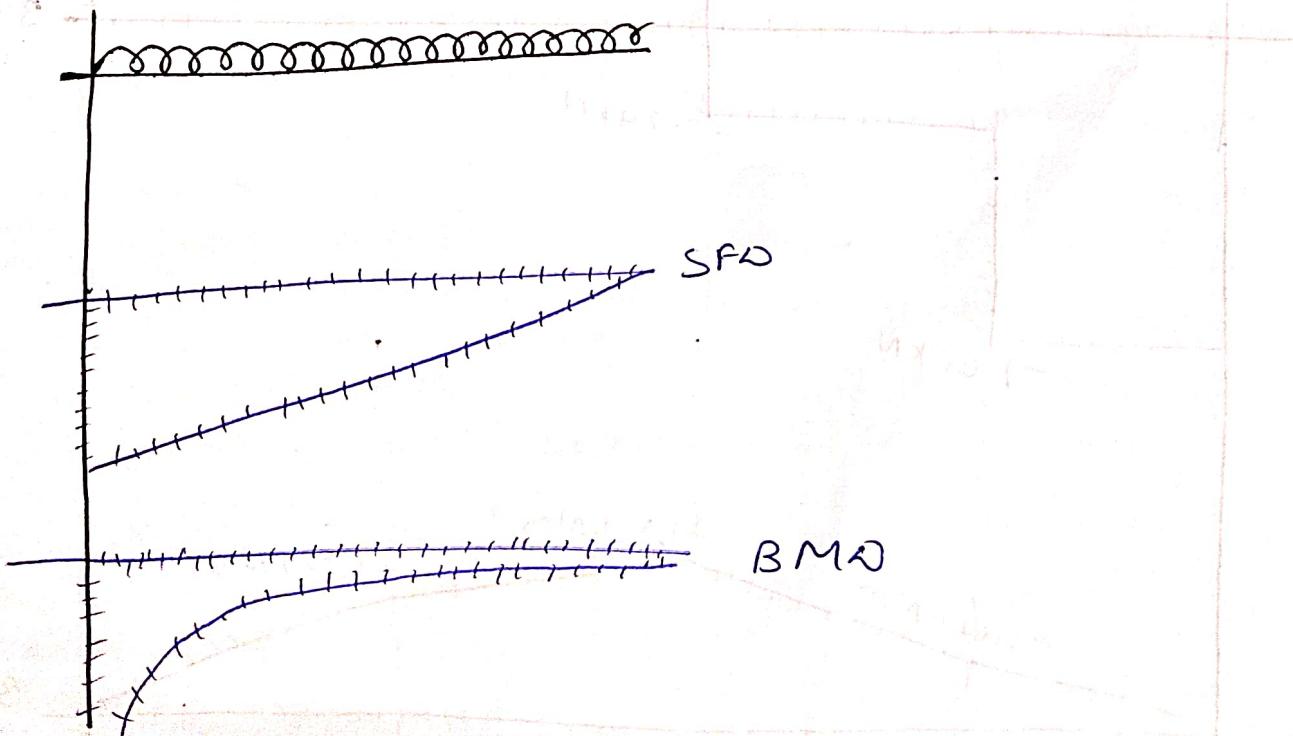
$$BM_D = -1 \times 3 - 2 \times 2 \times \frac{2}{2} = -7 \text{ kNm}$$

$$BM_C = -1 \times 4 - 2 \times 2 \times \left(1 + \frac{2}{2}\right) = -12 \text{ kNm}$$

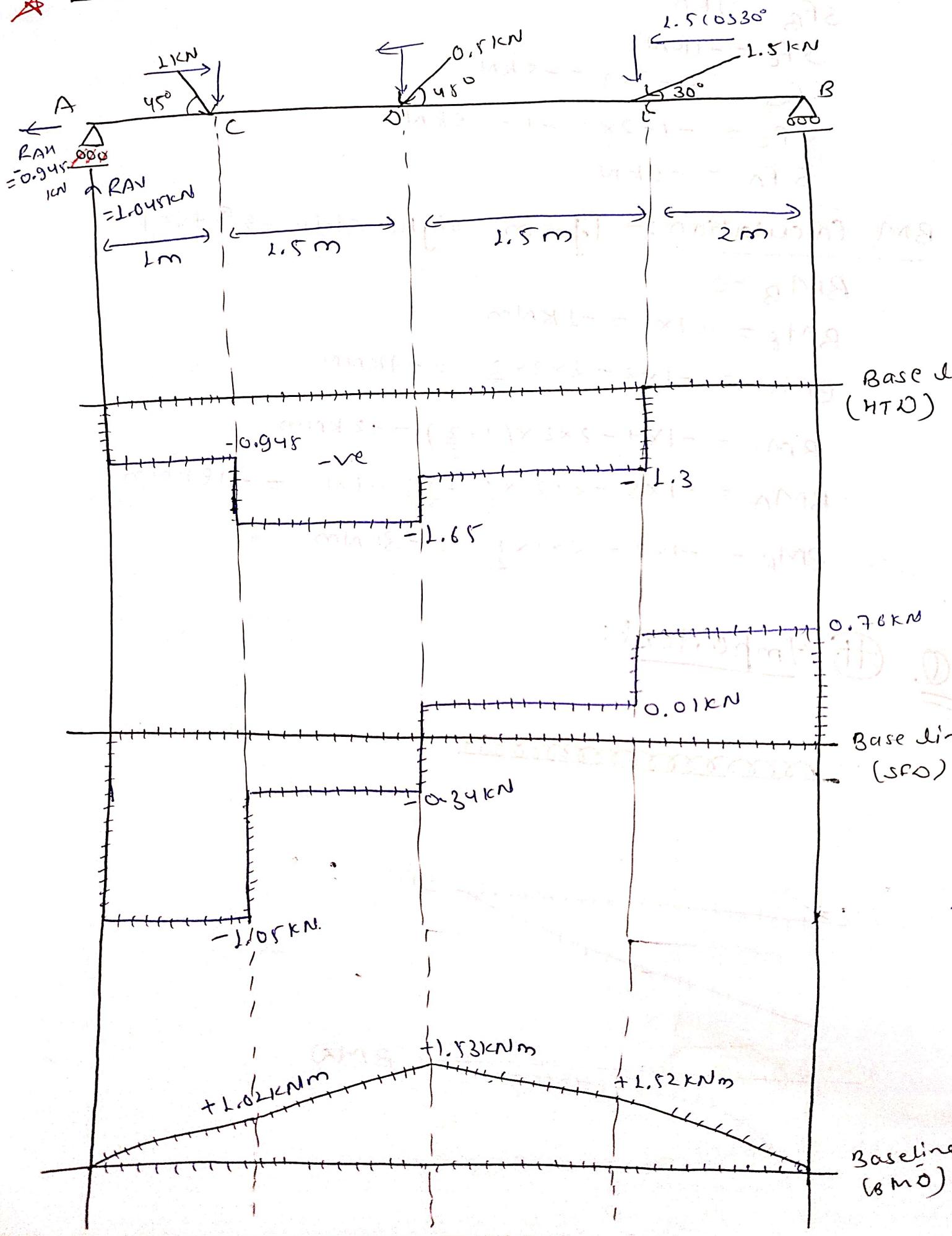
$$BM_A = -1 \times 5 - 2 \times 2 \times \left(2 + \frac{2}{2}\right) - 1 \times 1 = -18 \text{ kNm}$$

$$BM_F = -1 \times 2 - 2 \times 1 \times \frac{1}{2} = -3 \text{ kNm}$$

Q # Important :-



~~#~~ HTD : Horizontal Thrust Diagram



Let R_{AV} & R_{AH} be reaction at A. Let R_B be the reaction at B.

Taking moment about A,

$$\sum M_A = 0 \quad [+ve, -ve]$$

$$\Rightarrow R_B \times 6 - 1.5 \sin 30^\circ \times 4 - 0.5 \sin 45^\circ \times 2.5 - 1 \cos 45^\circ \times 1 = 0$$

$$\Rightarrow 6R_B - 3 - 0.88 - 0.70 = 0$$

$$R_B = 0.76 \text{ kN}$$

Consider vertical equilibrium,

$$\sum V = 0 \quad [\uparrow +ve, \downarrow -ve]$$

$$+ R_A + R_B - 1.5 \sin 30^\circ - 0.5 \sin 45^\circ - 1 \sin 45^\circ = 0$$

$$R_A = 0.353 + 0.707 + 0.75 - 0.76$$

$$R_A = 1.045 \text{ kN}$$

Consider horizontal equilibrium,

$$\sum H = 0 \quad [\rightarrow (+ve), \leftarrow (-ve)]$$

$$- 1.5 \cos 30^\circ - 0.5 \cos 45^\circ + 1 \cos 45^\circ - R_{AH} = 0$$

$$R_{AH} = -0.945 \text{ kN}$$

- HTD Calculation:- SF calculation:-

$$[\text{from right side} \rightarrow (+ve)] \quad [2 \text{ st } 2] \quad [\text{from right side} \rightarrow \uparrow +ve]$$

$$HT_B = 0 \text{ kN} \quad SF_B = +0.76 \text{ kN}$$

$$HT_E = -1.5 \cos 30^\circ = -1.3 \text{ kN} \quad SF_E = +0.76 - 1.5 \sin 30^\circ$$

$$HT_D = -1.5 \cos 30^\circ - 0.5 \cos 45^\circ = -1.65 \text{ kN} \quad = 0.61 \text{ kN}$$

$$HT_C = -1.5 \cos 30^\circ - 0.5 \cos 45^\circ + 1 \cos 45^\circ \quad SF_D = 0.76 - 0.75 - 0.5 \sin 45^\circ \\ = -0.945 \text{ kN} \quad = -0.34 \text{ kN}$$

$$HT_A = -0.945 \text{ kN}$$

$$SF_C = 0.76 - 0.75 - 0.5 \sin 45^\circ - 1 \sin 45^\circ = -1.05 \text{ kN}$$

- BM calculation :- (from sign convention)

$$BM_B = 0$$

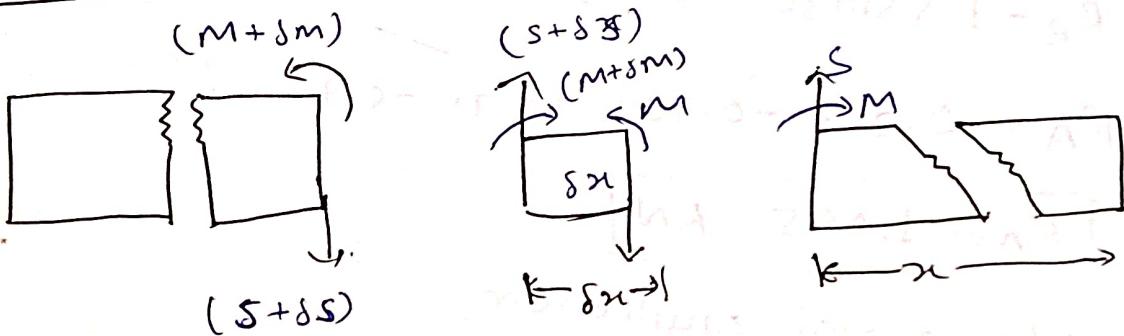
$$BM_E = R_B \times 2 = 0.76 \times 2 = +1.52 \text{ kNm}$$

$$BM_D = 0.76 \times 3.5 - 1.5 \sin 30^\circ \times 1.5 = 2.66 - 1.225 \\ = +1.535 \text{ kNm}$$

$$BM_C = 0.76 \times 5 - 1.5 \sin 30^\circ \times 3 - 0.5 \sin 45^\circ \times 1.5 \\ = 3.8 - 2.25 - 0.535 = 1.02 \text{ kN}$$

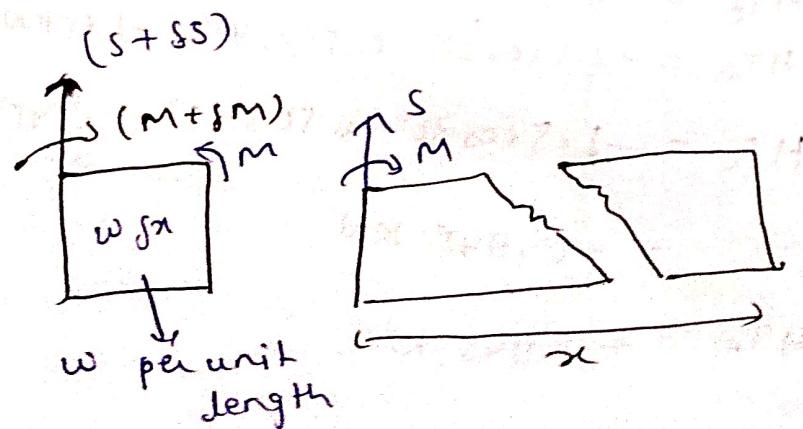
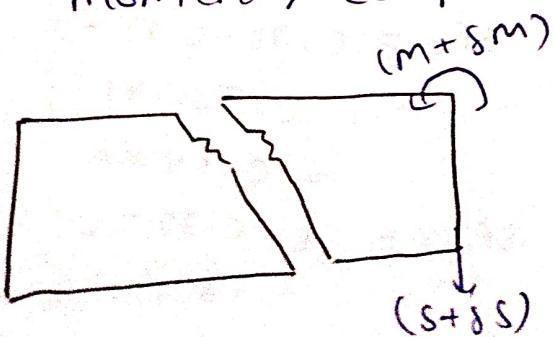
$$BM_A = 0.$$

- Relation between shear force & Bending moment :-



Consider a short length δx of a beam at a distance x from some origin. Let the load over the short length be w per unit length acting vertically downward.

Then, the shearing forces over this short length will increase from S to $S + \delta S$ while the BM increases from M to $M + \delta M$. This short length is in equilibrium under both vertical forces &



Vertical forces :- $(S + \delta S) - S = \omega \delta x$

$$\frac{\delta S}{\delta x} = \omega \quad \text{or} \quad \boxed{\frac{dS}{dx} = \omega} \quad \rightarrow ①$$

Moments/Couples :- $(M - (M + \delta M)) = -S \delta x - \omega \delta x \left(\frac{\delta x}{2}\right)$

$$\Rightarrow -\delta M = -S \delta x - \frac{\omega}{2} (\delta x)^2$$

$\because (\delta x)^2$ is very small quantity of 2nd order,
so it can be taken as zero.

$$\Rightarrow -\delta M = -S \delta x$$

$$\boxed{\frac{\delta M}{\delta x} = S = \frac{dM}{dx}} \quad \rightarrow ②$$

In the integral form, we get,

$$S = \int \omega dm \quad \rightarrow ③$$

$$M = \int S dm \quad \rightarrow ④$$

Conclusion:-

①. from eq. ①. It is concluded that rate of change of shear force at any section represents the ~~rate of~~ loading at that section.

②. from eq. ②. we can conclude that rate of change of bending moment at any section represents the shear force at that section.