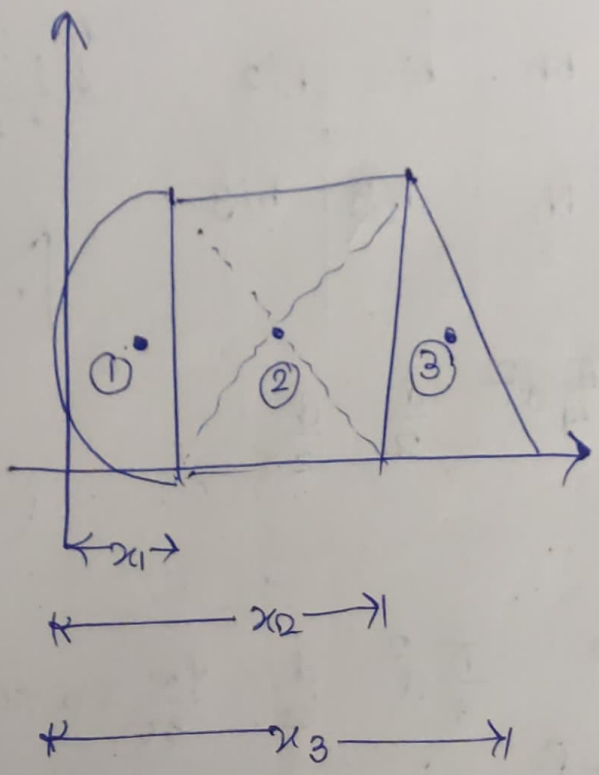
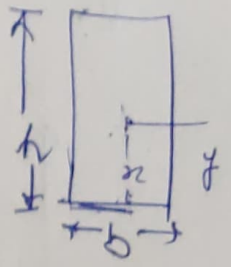
2- II. Centroid & CG.

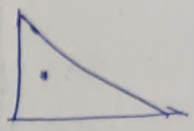


$$\bar{x} = \frac{a_1x_1 + a_2x_2 + a_3x_3}{a_1 + a_2 + a_3} = \frac{\Sigma ax}{\Sigma a}$$

$$\bar{y} = \frac{a_1y_1 + a_2y_2 + a_3y_3}{a_1 + a_2 + a_3} = \frac{\Sigma ay}{\Sigma a}$$

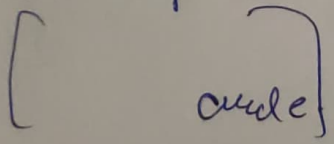
Shape	area	$\bar{x}$	$\bar{y}$
Rectangle	bh	$b/2$	$h/2$
Triangle	$\frac{bh}{2}$	$b/3$	$h/3$
circle	$\frac{\pi}{4}d^2$	$\frac{d}{2}$	$\frac{d}{2}$
semicircle	$\frac{\pi}{8}d^2$	$\frac{d}{2}$	$\frac{4r}{3\pi} = 0.424r$
Quadrant	$\frac{\pi}{16}d^2$	$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$
Sector of αr^2		$\bar{x} = \frac{2r\sin\alpha}{3\alpha}$	$\bar{y} = 0$



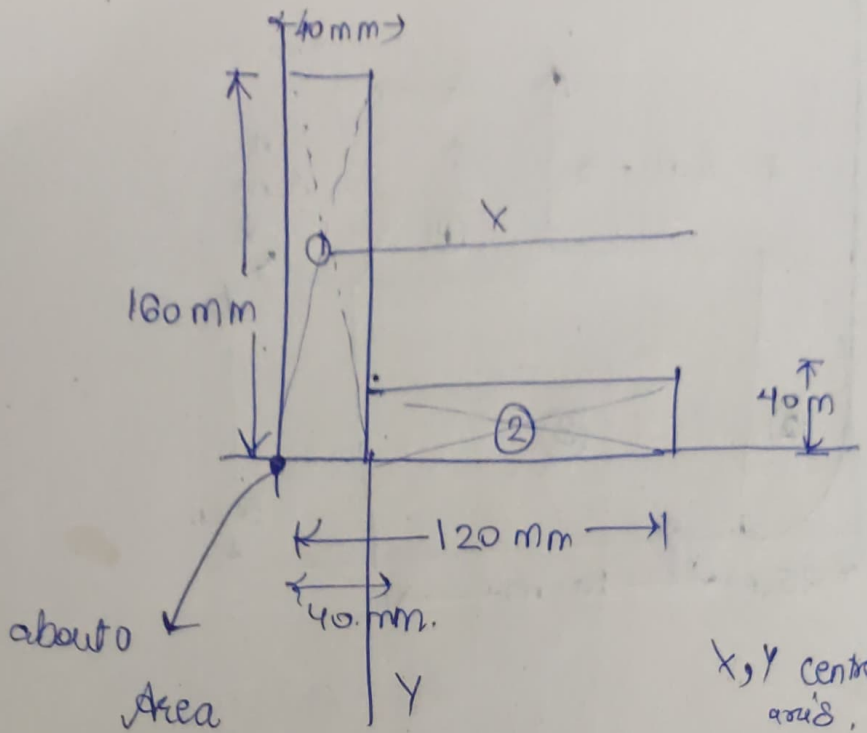


$\frac{\pi r^2}{4}$ $\frac{\pi(\frac{d}{2})^2}{4}$ $\frac{\pi d^2}{16}$.

diameter.



α in radians.



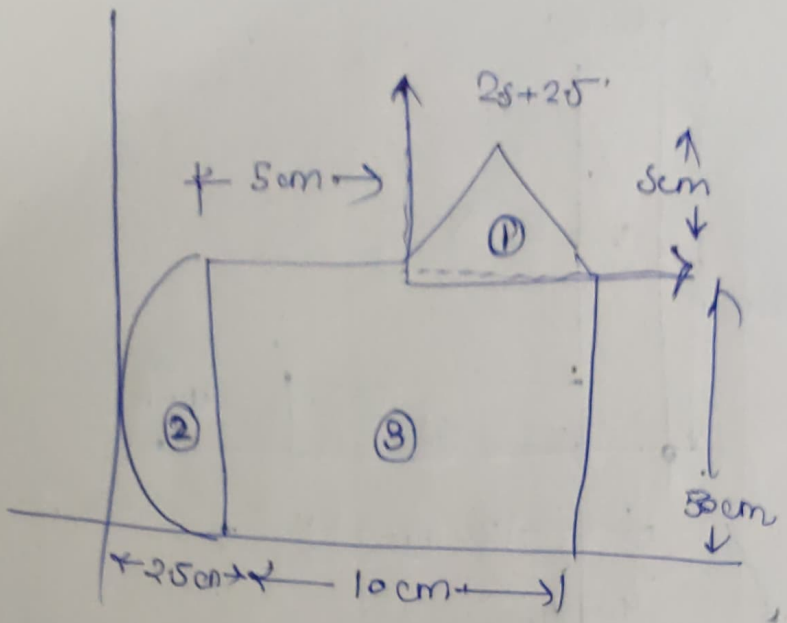
x, y centroidal axis.

Area mm^2 (a) $\bar{x}$ $\bar{y}$ ax ay.

rect 1)	160 x 40	20	80	12800	512000
rect 2)	80 x 40	80	20	25600	6400
	Σa.			Σax	Σay
				384000	576000

$$\frac{\Sigma ax}{\Sigma a} = 40$$

$$\frac{\Sigma ay}{\Sigma a} = 80$$



area.

		x	y	ax	ay.
triangle:	$\frac{5 \times 5}{2}$	$5+2.5+2.5$	$5+\frac{5}{3}=6.67$.		
semicircle	$\frac{\pi}{2}(2.5)^2$	$2.5-\frac{4r}{3\pi}=1.44$	2.5.		
rect.	10×5.	$2.5+5$.	2.5.		

$\bar{x}$ = 7.11 cm

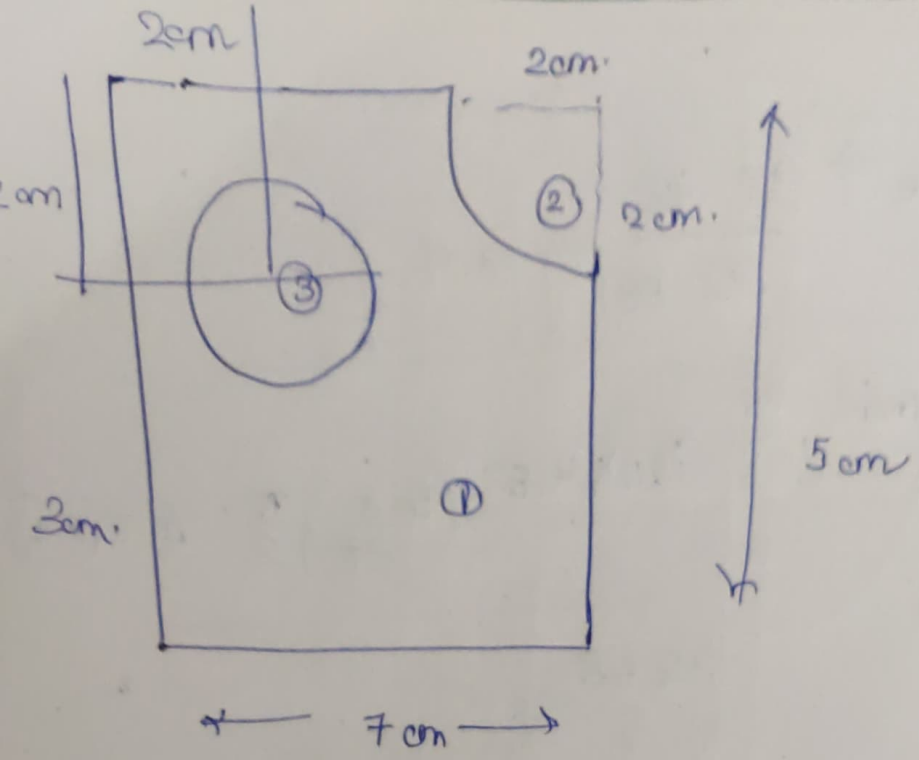
$\bar{y}$ = 3.32 cm

ans.

πr^2

$\frac{1}{2} \pi (2.5)^2$

$\frac{\pi r^2}{4}$



1.	Rectangle.	7×5	3.5	2.5
		area.	x	y
2.	Quadrant.	$\frac{\pi}{8}(2)^2$	$7 - \frac{4(2)}{3\pi}$	$5 - \frac{8}{3\pi}$
3.	circle.	$\pi(1.5)^2$	2	3.

x centre $\rightarrow$

$$\frac{7 \times 5 \times 3.5 - \left(7 - \frac{8}{3\pi}\right) \times \frac{\pi}{8}(4)^2 - \pi(1.5)^2(2)}{-\frac{\pi}{8}(4) - \left(7 - \frac{4(2)}{3}\right) + 35}$$

$$\frac{122.5 - 9.65 - 14.13}{-1.57 - 0.849 + 35}$$

$x_{centre} \Rightarrow \frac{98.72}{32.581} = \boxed{3.7.}$

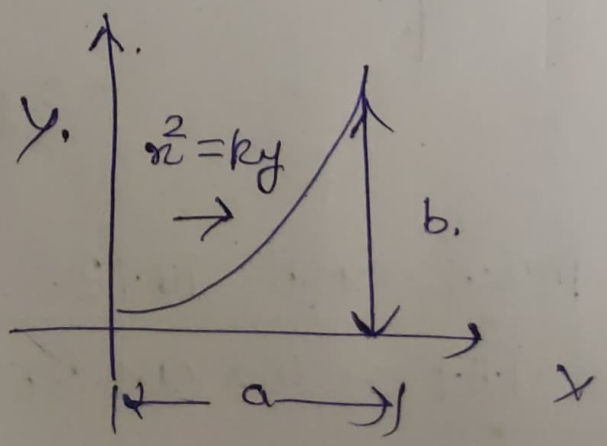
$y_{centroid} \Rightarrow$

$$\frac{35 \times 2.5 - \left(5 - \frac{8}{3\pi}\right)\frac{\pi}{2} - 8 \times (1.5)^2}{32.58}$$

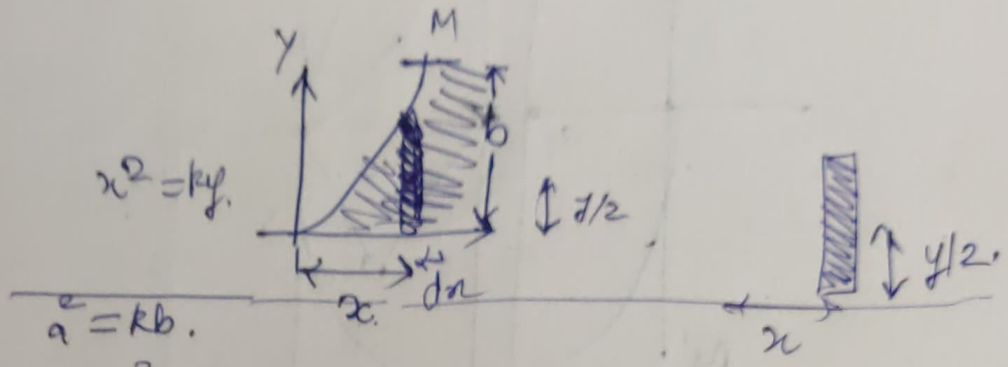
y centroid.

$$\frac{87.5 - 6.51 - 21.145}{32.581} = \boxed{1.63.}$$

Ques Determine position of centre of gravity of shaded position the curve OM being a parabola with axes



Area of cross sectional S/p



$k = \frac{a^2}{b}$. → $x^2 = ky$

$$\boxed{x^2 = \frac{a^2}{b} y}$$ (eq^n of parabola)

Area of elementary strip /Section $= y\,dx$.

$$\bar{x} = \frac{\int_0^a y\,dx \cdot x}{\int_0^a y\,dx} = \frac{3a}{4}$$

$$= \frac{\frac{b}{a^2}\int_0^a x^3\,dx}{\frac{b}{a^2}\int_0^a x^2\,dx}$$

$\frac{a^4}{4} / \frac{a^3}{3}$.

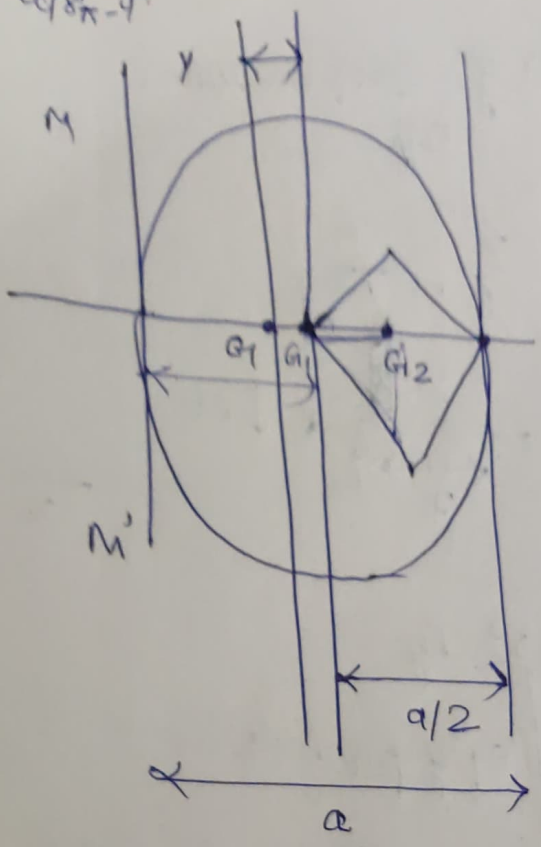
$$\bar{y} = \frac{\int_0^a y\,dx \cdot \frac{y}{2}}{\int_0^a y\,dx} = \frac{3b}{10}$$

$\frac{y\,dx}{2} \int \frac{b^2}{a^4} x^4\,dx$

$\int \frac{b}{a^2} x^2$

a square hole is punched out circular dia 'a' with one diagonal of the square coincide with radius of the circle show that the centre of remainder is at a distance of $\frac{a}{8\pi - 4}$ from the centre of circle

Q/8π-4.



	(area 'A')	(x. from MM')	(Ax)
circular ~~comma~~ :	$\frac{\pi a^2}{4}$	$\frac{a}{2}$	$\frac{\pi a^3}{8}$
square hole(2) :	$-\frac{a^2}{8}$	$l^2+l^2=\frac{a^2}{8}$ $\frac{a^2}{4}$ $l^2=\frac{a^2}{8}$	$\frac{3a^3}{4}$ $-\frac{3a^3}{32}$

total area of comma shape

$$\# \Sigma a = \frac{\pi a^2}{4} - \frac{a^2}{8} = a^2\left(\frac{\pi}{4} - \frac{1}{8}\right)$$

Σax

$$\vec{x} = \frac{\Sigma Ax}{\Sigma A} = \frac{\frac{\pi a^3}{8} - \frac{3a^3}{32}}{a^2\left(\frac{\pi}{4} - \frac{1}{8}\right)}$$

$$= \frac{\frac{a^3}{8}\left(\pi - \frac{3}{4}\right)}{a^2\left(\frac{\pi}{4} - \frac{1}{8}\right)}$$

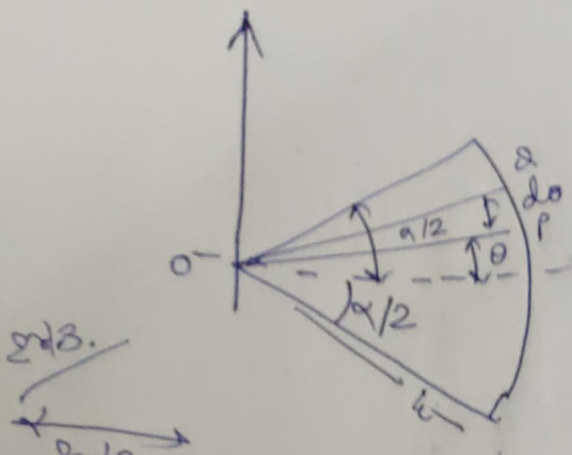
$$= \frac{\frac{a^3}{8}\left(\frac{4\pi - 3}{4}\right)}{a^2\left(\frac{2\pi - 1}{8}\right)}$$

$$= \frac{a}{4}\left(\frac{4\pi - 3}{2\pi - 1}\right)$$

Centre of remainder from the centre of circle

$$\frac{a}{2} - \frac{a}{4}\left(\frac{4\pi - 3}{2\pi - 1}\right) =$$

$$\frac{a}{8\pi - 4}.$$



Area of $\Delta = \frac{1}{2} r^2 d\theta$.
its c.g. on OP at a distance $\frac{2r}{3}$ from O.
$\frac{2}{3} r \cos\theta$.

$$\bar{x} = \frac{\int_0^{\alpha/2} \frac{2r}{3}\cos\theta \cdot \frac{r^2}{2} d\theta}{\int_0^{\alpha/2} \frac{r^2}{2} d\theta} = \frac{\frac{2r}{3}\int_0^{\alpha/2} \cos\theta \, d\theta}{\int_0^{\alpha/2} d\theta}$$

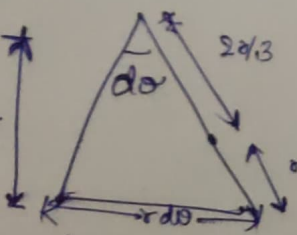
$$= \frac{2r}{3} \frac{\sin\alpha/2}{\alpha/2}$$

$$\Rightarrow \frac{4r}{3a} \sin\frac{\alpha}{2}$$

$$\frac{2r}{3} \frac{\sin\alpha/2}{\alpha/2} \quad \Sigma.$$

ΣAx

Question

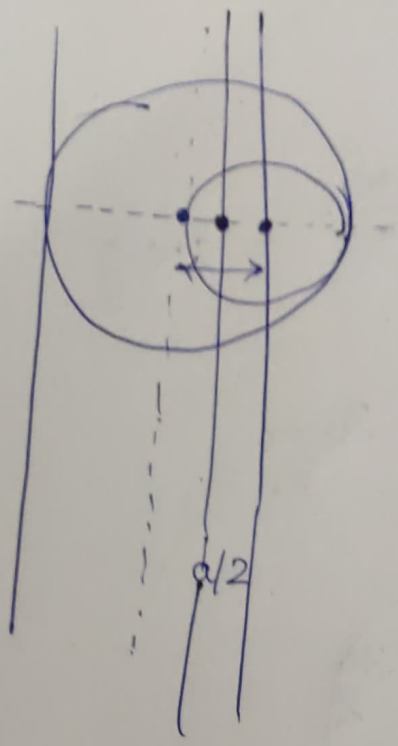


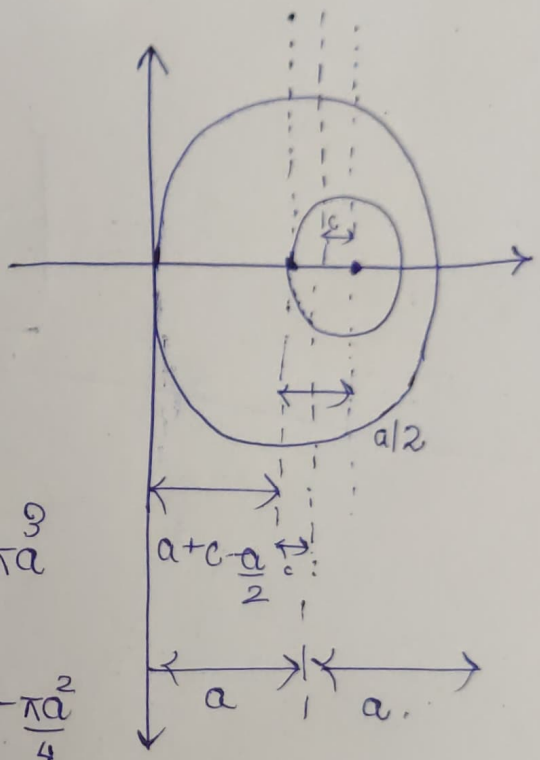
$\frac{1}{2} \times r \times r\, d\theta$

$\boxed{y = 0}$.

Question :- From a uniform circular disc of radius a is cut off a circular portion of radius a/2 find the dist b/w the centre of disc and hole.

If centre of mass of the remaining it on circumference of hole.





	Area	$\bar{x}$	$A\bar{x}$
circular disc	πa^2	a	πa^3
circular hole.	$-\frac{\pi a^2}{4}$	$a+c$	$-\frac{\pi a^2}{4}$

$$\Sigma A = 3\frac{\pi a^2}{4}$$

$$A\bar{x} = \pi a^3 - \frac{\pi a^2}{4}(a+c)$$

$$\bar{x} = \frac{\Sigma A\bar{x}}{\Sigma A} = \frac{\pi a^3 - ( )}{3\pi a^2/4}$$

$$\frac{\pi a^3 - \frac{\pi a^2}{4}(a+c)}{\frac{3\pi a^2}{4}}$$

$$\frac{A\bar{x}}{A}$$

$$\bar{x} = a + c - \frac{a}{2}$$

$$\frac{\pi a^2\left(a-\frac{a+c}{4}\right)}{3\pi a^2/4} = a+c-\frac{a}{2}.$$

$$\frac{a}{2}+c = \frac{3a+c}{3}.$$

$$c = \frac{3a}{8}$$