Maulana Azad National Institute of Technology, Bhopal

Mid Term Examination (November 18, 2024)

Session: July to December 2024

Course: B.Tech. (Sem I) **Section:** All

Subject: Mathematics - I **Subject Code:** MTH 24101

Duration: 60 Minutes **Maximum Marks:** 20

Instructions:
- **All questions are compulsory and carry equal marks.**
- **Calculator is not allowed.**
- **Proper justifications should be given for each answer.**

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1. If $x = r\cos\theta$, $y = r\sin\theta$, prove that $\dfrac{\partial^2 r}{\partial x^2} + \dfrac{\partial^2 r}{\partial y^2} = \dfrac{1}{r}\left\{\left(\dfrac{\partial r}{\partial x}\right)^2 + \left(\dfrac{\partial r}{\partial y}\right)^2\right\}$.

2. Find the extreme values of $f(x, y, z) = x^2 + y^2 + z^2 + xy + xz + yz$ subject to the constraints $x + y + z = 1$ and $x + 2y + 3z = 3$.

3. Trace the curve $y^2(1 - x^2) = x^2(1 + x^2)$ by describing the appropriate properties.

4. Find the area bounded by the curves in the first quadrant $y = x^2$, $8x = y^2$ and $y = (\sqrt{3} - 1)(2x + \sqrt{3})$.

** End of the question paper **