Quantum Mechanics :-

D' Broglie , $\lambda = \frac{h}{p}$

→ **Schrodinger Equation** :-

$p = \hbar k \qquad E = \hbar \omega$

$p = \frac{h}{2\pi} \frac{2\pi}{\lambda} \quad , \quad E = \frac{h}{2\pi} 2\pi\nu$

$$y = Ae^{i(kx - \omega t)}$$

$$\boxed{\psi = Ae^{i/\hbar\,(px - Et)}} \quad \text{—— (i).}$$

$$\frac{\partial \psi}{\partial x} = A\frac{i}{\hbar} p e^{i/\hbar\,(px - Et)}$$

$$\boxed{\frac{\partial \psi}{\partial x} = \frac{i}{\hbar} p \psi}$$

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{ip}{\hbar} \frac{\partial \psi}{\partial x}$$

$$= \left(\frac{ip^2}{\hbar}\right)\psi$$

$$\boxed{\frac{\partial^2 \psi}{\partial x^2} = \frac{-p^2}{\hbar^2} \psi} \quad \text{---(11)}$$

$$i\frac{\partial \psi}{\partial t} = \frac{iE}{\hbar}\psi$$

$$i\hbar \frac{\partial \psi}{dt} = E\psi$$

$$TE = KE + PE$$

$$\boxed{E = \frac{p^2}{2m} + V}$$

multiply by ψ,

$$\frac{p^2}{2m}\psi + V\psi = E\psi$$

$$\boxed{\frac{-\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + V\psi = E\psi}$$ ⇒ Time independent schrodinger equation

$$\boxed{\frac{-\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + V\psi = i\hbar\frac{\partial \psi}{\partial t}}$$ ⇒ Time dependent schrodinger equation.

- Properties of Equation :-

$\lambda = \frac{h}{p}$

$E = h\nu$, $\lambda = \frac{h}{p}$

$\cancel{v_p = \nu k}$

 $v_p = \frac{\omega}{k}$

(phase velocity)

$\boxed{v_p = \nu\lambda}$

$v_p = \frac{E}{h} \cdot \frac{h}{p}$

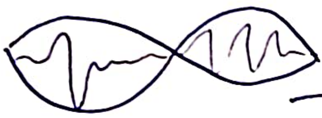
$= \frac{E}{p}$

$= \frac{mc^2}{mv}$

$\boxed{v_p = \frac{c^2}{v}}$

$\Rightarrow v_p > c$ (×) $\boxed{v_g < c}$ ✓

- Schrodinger :-

 $\longrightarrow v_g$

(group velocity)

$\boxed{v_g = \frac{d\omega}{dk}}$

$v_g < c.$

$v_g = \frac{d(k v_p)}{dk}$

$\boxed{k = \frac{2\pi}{\lambda}}$

$v_g = v_p + k\frac{dv_p}{dk}$

$\Rightarrow dk = -\frac{2\pi}{\lambda^2} d\lambda$

$v_g = v_p + \left(\frac{2\pi}{\lambda}\right) \frac{dv_p}{(-2\pi/\lambda^2)\, d\lambda}$

$\Rightarrow \boxed{v_g = v_p - \lambda\frac{dv_p}{d\lambda}}$

$$\boxed{v_g = v_p - \frac{\lambda\, d v_p}{d\lambda}}$$

$$\psi = A e^{i(kx-\omega t)}$$

$$\boxed{\frac{-h^2}{2m}\nabla^2\psi + V\psi = E\psi}$$

$$\psi = A + iB$$

$$\psi^* = A - iB$$

Probability density $= \psi^*\psi$

probability $= \int_{-\infty}^{\infty} \psi^*\psi\, dx$

- Properties of wave function ψ:-

1) ψ must be finite.
2). ψ must be continous.
3). ψ must be single valued.

- Postulates:- of wave function/Quantum mechanics:-

1). Concept of matter waves
2). Experimental value
3). Concept of operator.

③. operator ($\hat{O}$) :-

$$\hat{O}\psi = o\psi$$

$$\equiv$$

$$\hat{A}\psi = a\psi$$

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

$$\frac{p^2}{2m} = K.E.$$

$$\boxed{p = \sqrt{2m(KE)}}$$

$$\boxed{= \sqrt{3mKT}}$$

$eV = KE$

$p^2 = 2mk$ $\quad KE = \frac{3}{2}KT$

$$\lambda = \frac{h}{\sqrt{2meV}}$$

$eV = 200\,keV$

$= a e^{ikx}$

$eV = K.E. = 200\,keV$

$\sim 5\,keV$

$\psi = ax$

$$\frac{\langle x \rangle}{\langle p \rangle} = \frac{\int_{-\infty}^{\infty} \psi^* x \psi\, dx}{\int_{-\infty}^{\infty}}$$

$$eV = 200 \times 10^3 \times 1.6 \times 10^{-19}\, J$$

$$\Rightarrow \boxed{KE = \frac{3}{2}KT}$$

Eigen value:- equation:-

$$\hat{P}\psi = p\psi$$

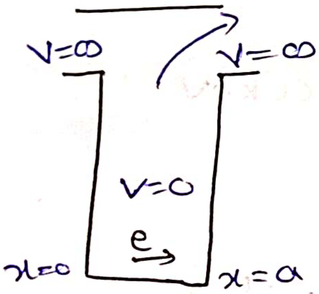
observation	operator.
x	$\hat{x}$
v	$\hat{v}$
p	$i\hbar\frac{\partial}{\partial x}$
KE	$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}$
TE	$-\frac{\hbar^2}{2m}\cdot\frac{\partial^2}{\partial x^2} = V$

TE | $i\hbar \frac{\partial}{\partial t}$

(4). Normalisation constant :-

$$\int_{-\infty}^{\infty} \psi^* \psi \, dx = 1$$

- Particle in a box :-



free particle
$\vec{F} = 0$

$$F = -\frac{dV}{d\vec{r}}$$

$$V = \int \vec{F} \cdot d\vec{r}$$

$$\boxed{V = 0}$$

only true in case of conservative force field.

Schrodinger

$$-\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V\psi = E\psi$$

$$\frac{d^2\psi}{dx^2} + \frac{2mE}{\hbar^2}\psi = 0$$

$$\frac{\partial^2 \psi}{\partial x^2} + K^2 \psi = 0 \quad ; \quad K = \frac{\sqrt{2mE}}{\hbar}$$

$$\Rightarrow \boxed{\psi = A \sin kx + B \cos kx}$$

where A, B are constant

Using boundary conditions,
$\psi(x) = 0$ at $x = 0$,

Probability of finding the particles at $x=0$ and at $x=a$ is zero.

Hence, wave function is zero. $\psi(x) = 0$

$$B = 0$$

$$\psi = A \sin kx$$

at $x = a$, $\psi(x) = 0$.

$$A \sin ka = 0$$

$$\Rightarrow A \neq 0$$

$$\sin ka = 0$$

$$ka = n\pi \quad , n = 0, 1, 2, \ldots$$

$$\Rightarrow k = \left(\frac{n\pi}{a}\right)$$

$$\Rightarrow \boxed{\psi = A \sin\left(\frac{n\pi x}{a}\right)}$$

$$\int_0^a \psi^*(x)\, \psi(x)\, dx = 1$$

$$\Rightarrow \int_0^a |\psi(x)|^2 dx$$

$$\Rightarrow A^2 \int_0^a \sin^2\left(\frac{n\pi x}{a}\right) dx = 1$$

$$\Rightarrow \boxed{A = \sqrt{\frac{2}{a}}}$$

$$\boxed{\psi = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right)}$$

$\rightarrow$ T.E. (E) = ? , KE = ? $(U=0)$

$$\hat{K.E.} = \frac{-\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2}$$

Eigen value:-

$$\hat{O}\psi = o\psi$$

position wave f^n. eigen values

$$\hat{KE} = \frac{\hbar^2}{2m}\sqrt{\frac{2}{a}} \frac{n^2\pi^2}{a^2} \sin\left(\frac{n\pi x}{a}\right)$$

$$\boxed{\hat{K.E.} = \frac{\hbar^2 n^2 \pi^2}{\sqrt{2}\, m\, a^{5/2}} \sin\left(\frac{n\pi x}{a}\right)}$$

$$\psi = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right)$$

$$\boxed{\hat{K.E.} = \frac{n^2\hbar^2\pi^2}{a^2\, 2m}\psi}$$

$$\frac{\partial \psi}{dx} = \sqrt{\frac{2}{a}} \frac{n\pi}{a} \cos\left(\frac{n\pi x}{a}\right)$$

$$\boxed{\frac{\partial^2 \psi}{dx^2} = -\sqrt{\frac{2}{a}} \left(\frac{n\pi}{a}\right)^2 \sin\left(\frac{n\pi x}{a}\right)}$$

$$\Rightarrow \frac{-\hbar^2 \partial^2}{2m\, dx^2}\psi = \frac{n^2\hbar^2\pi^2}{a^2\, 2m}\psi$$

$$\boxed{\frac{\partial^2 \psi}{\partial x^2} = -\left(\frac{n\pi}{a}\right)^2 \psi}$$

$$T.E. = \frac{n^2\hbar^2\pi^2}{2ma^2}, \quad n = \cancel{0}, 1, 2 \ldots$$

$$\psi = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right), \quad n = \cancel{0}, 1, 2 \ldots$$

$n=0$ (not allowed)

$n = 1, 2, 3, \ldots\ldots$

$$\boxed{E_0 = \frac{\pi^2\hbar^2}{2ma^2}} \rightarrow \text{Zero point energy}$$

Liquid He

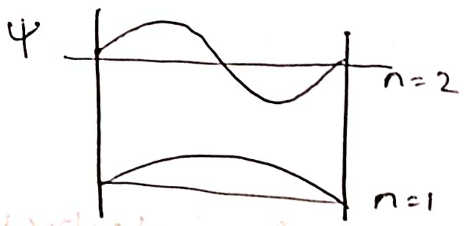
$T = 0K$

—— $n=3$ (2nd E.S.)
—— $n=2$ (1st E.S.)
—— $n=1$ (G.S.)

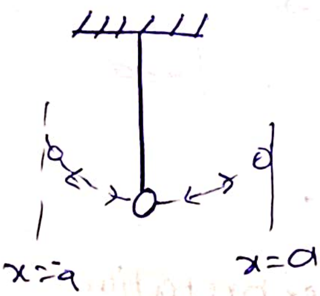
- Simple Harmonic Oscillator :- (SHO).

$$E_n = \frac{n^2\hbar^2\pi^2}{2ma^2}$$

$$\psi = \sqrt{\frac{2}{a}} \sin\left(\frac{\pi n x}{a}\right)$$



→ simple harmonic motion :-



$x(t)$

→ periodic / to and fro.

$\Rightarrow F \propto x$

$$\boxed{\vec{F} = -k\vec{x}}$$

→ 2nd law of 'Newtons' :-

$$m\frac{d^2x}{dt^2} = \sum F_{ext}$$

$$U = -\int \vec{F}\, d\vec{x}$$

$$U = \int kx\, dx$$

$$\boxed{U = \frac{1}{2}kx^2} \quad \text{—(1)}$$

P.E. $(U) = \frac{1}{2}kx^2$

K.E. $= \frac{1}{2}k(a^2 - x^2)$

T.E. $= \frac{1}{2}ka^2$

Schrodinger eqn :-

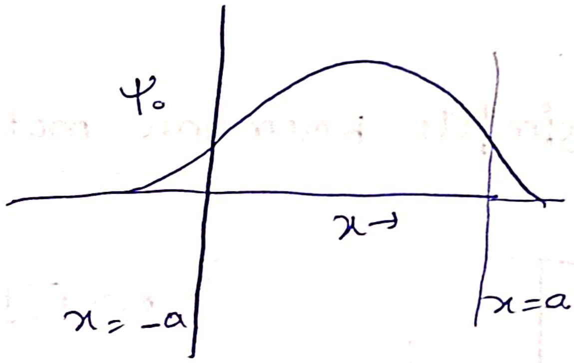
$$\boxed{-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V\psi = E\psi}$$

$V = \frac{1}{2} k x^2$ after solving equation.

$\Rightarrow \Psi = H_n e$

(book se)

$\boxed{E = (n+\frac{1}{2}) h\nu}$; $n = 0, 1, 2, \ldots\ldots$



Classical Interpretation

→ Amplitute can have any value ~~hence~~ hence energy is continous in

→ If amplitude is zero, energy can have zero value.

→ since velocity of the classical particle is max. at equi^m position and mini. at extremities hence, probability of finding the particle at equi^m is minimum at eq^m & max. at extremities.

Quantum Interpretation

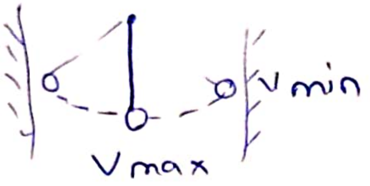
→ Energy value of quantum particle is quantised

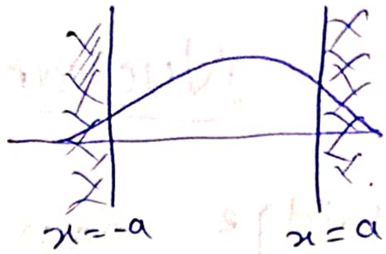
→ minimum energy,

$\boxed{E = \frac{1}{2} h\nu}$

⇒ zero point value.

→ Probability of finding the Quantum particle is maximum at equilibrium and minimum at extremities.





→ In ~~ca~~ this case, particle cannot exist if $x > a$ or $x < -a$.

→ There may be ~~fin~~ a finite probability of finding a particle in the region $x < -a$ & $x > +a$ (classically forbidden region)

- Tunnel Effect :-