Convolution theorum

2\. Laplace

Let $f(t)$ be a function defined by then the integral ;

$$f(p) = \int_0^\infty e^{-pt} (f(t)) dt$$ is called Laplace transf.

of $f(t)$ provided the integral exisist dt is generally denoted by

$$L(f(t)] = f(P)$$

$$L^{-1}\left[f(P]\right] = f(t)$$

Let $f(t) = 1$

$$\Rightarrow L f(t) = L[1] = \int_0^\infty e^{-pt} dt.$$

$$= \boxed{\frac{1}{p}} \quad p > 0$$

(i) If $f(t) = t$

$$L(t) = \int_0^\infty e^{-pt} t dt \qquad e^{-pt} = v$$

$$= \frac{1}{p^2}, \ p > 0. \qquad e^{-pt} dt = dv$$

$$e^{-pt} dt = \frac{dv}{p}.$$

(ii) If $f(t) = e^{at}$

$$L[e^{at}] = \int_0^\infty e^{-pt} e^{at} dt = \frac{1}{p-a} \quad p > a$$

4) $f(t) = \sin at$

$$L[\sin at] = \int_0^\infty e^{-pt} \sin at . dt$$

$$= \frac{a}{p^2 + a^2}$$

5) $f(t) = \cos at$

$$L[\cos at] = \int_0^\infty e^{-pt} \cos at$$

$$= \frac{p}{a^2 + p^2}$$

6) $f(t) = \sinh at$.

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$L[\sinh at] = \int_0^\infty e^{-pt} \left[\frac{e^{+at} - e^{-at}}{2} \right] dt$$

$$L\left[\frac{e^{at} - e^{-at}}{2} \right] = \frac{a}{p^2 - a^2}$$

7) $f(t) = \cosh at$

$$\cosh x = \frac{e^{-x} + e^x}{2}.$$

$$L[\cosh at] = \frac{p}{p^2 - a^2}$$

Convolution theorum

$$\frac{e^{ax}}{a^2+b^2}\left[a\cos b - b\sin x\right]$$

$$\frac{e^{ax}}{a^2+b^2}\left[a\sin ax - b\cos ax\right]$$

8) $L[t^n]$, n (+) integer

$$L[t^n] = \int_0^\infty e^{-pt}\, t^n = \frac{\lfloor n}{p^{n+1}}$$

LINEARITY OF LAPLACE THEORUM

Let $f(t)$ and $g(t)$ be any two function whose LT exist then for any two constant

$$L[\alpha f(t) + \beta g(t)] = \alpha L[f(t)] + \beta L[g(t)]$$

$$L[\alpha f(t) + \beta g(t)] = \int_0^\infty e^{-pt}[\alpha f(t) + \beta g(t)]\, dt$$

$$L[\alpha f(t) + \beta g(t)] =$$

$$\alpha F(p) + \beta G(p)$$

Questions

#1. $L\{(t^2+1)^2\}$

$L(t^4+1+2t^2)$

$L(t^4 + L[1] + 2L[t^2]$

$\frac{4!}{p^5} + \frac{1}{p} + 2\times\frac{3!}{p^3} = \frac{24+4p^2+p^4}{p^5}$

#2 $L[(\sin t + \cos t)^2]$

$L[1+\sin 2t]$

$L[1] + L[\sin 2t]$

$\frac{1}{p} + \frac{2}{p^2+4}$

$= \frac{p^2+4+2p}{p(p^2+4)}$

#3 $L[3t^4] - 2L[t^3] + 4L[e^{-3t}] - 2L[\sin 5t] + 3L[\cos 2t]$

$\frac{3\,4!}{p^5} - \frac{2\,3!}{p^4} + \frac{4}{p+3} + \frac{-2\times5}{p^2+25} + \frac{3p}{p^2+4}$

Convolution theorum

#4.

$L[f(t)]$

$f(t) = \begin{cases} (t-1)^2 & t > 1 \\ 0 & 0 < t < 1 \end{cases}$

$L(t^2+1-2t) + L[f(t)] \Rightarrow \int_0^\infty f\,dt$

$L(t^2) + L(t) - 2L(t)$

$\frac{3!}{p^4} + \frac{t}{p^2} - \frac{2}{p^2}$

$\frac{6}{p^4} - \frac{1}{p^2}$

$\boxed{\frac{e^{-p} 2}{p^3}}$

Ans.

$L[f(t)] = \int_0^\infty e^{-pt} f(t)\,dt$

$\int_0^1 e^{-pt}(f(t))\,dt + \int_1^\infty e^{-pt} f(t)\,dt \Rightarrow \int_1^\infty e^{-pt}(t-1)^2 dt + \int_0^1 e^{-pt} \cdot 0\,dt$

$\Rightarrow \int_1^\infty e^{-pt}(t-1)^2 dt$ let $t-1 = x$
$dt = dx$

$\int_0^{+\infty} e^{-p} e^{-px} x^2\,dx$

$\Rightarrow e^{-p} \int_0^\infty e^{-px} x^2\,dx$

$\Rightarrow \boxed{e^{-p} L[x^2] = e^{-p} \frac{2!}{p^3}}$

1. First shifting theorem
(first trans.

If $L[f(t)] = f(p)$

then $L[e^{at} f(t)] = f(p-a)$

proof :- Let $f(p) = \int_0^\infty e^{-pt} f(t)\,dt$

$f(p-a) = \int_0^\infty e^{-(p-a)t} f(t)\,dt$

$\Rightarrow \int_0^{\infty} e^{-pt} e^{at} (f(t)\,dt$

Ans. $\Rightarrow L[e^{at} f(t)]$

$L(\cos 2t\, e^{-3t})$

$L(\cos 2t) = \frac{s}{s^2+4}$

$L(e^{-3t}\cos 2t) = \frac{s+3}{(s+3)^2+4}$.

for eg.

$L(\sin 3t) = \frac{3}{s^2+9}$.

$L(e^{2t}\sin 3t) = \frac{3}{(s-2)^2+9}$

$L^{-1}(f(s)) = f(t)$

$L^{-1}(f(s-a)) = e^{at} f(t)$

$L^{-1} f(s-a) = e^{at} L^{-1}(f(s))$

$L(e^{2t}\sin 3t) = \frac{3}{(s-2)^2+9}$ ⟶

$3L^{-1}\left(\frac{1}{(s-2)^2+9}\right)$

$3e^{2t}\left(\frac{1}{s^2+9^2}\right)$.

$3e^{2t}\frac{\sin 3t}{3}$

$\boxed{e^{2t}\sin 3t}$

2. Second shifting theorem.

Let $L[f(t)] = F(p)$ & $G(t)$ = $\left\{ \begin{matrix} f(t-a) & t>a \\ 0 & t<a \end{matrix} \right\}$.

$L(f(t)) = f(s)$.

$L f(t-a) H(t-a) = e^{-as} f(s)$.

$\left\{ \begin{matrix} \sin(t-\frac{\pi}{3}) & t>\frac{\pi}{3} \\ 0 & t<\pi/3 \end{matrix} \right\}$

$\sin(t-\frac{\pi}{3}) \cdot \left\{ \begin{matrix} 1 & t>\pi/3 \\ 0 & t<\pi/3 \end{matrix} \right\}$.

$\boxed{\sin(t-\frac{\pi}{3}) H(t-\frac{\pi}{3})}$

$L(\sin t) = \frac{1}{s^2+1}$

$L(\sin(t-\frac{\pi}{3}) H(t-\frac{\pi}{3})) = e^{-\frac{\pi}{3}s}\frac{1}{s^2+1}$

then $L[G(t)] = e^{-ap} f(p)$

doubt.

Question :-

Find $L[f(t)]$, $f(t) = \left\{ \begin{matrix} (t-1)^2 & t>1 \\ 0 & 0<t<1 \end{matrix} \right\}$

$G(t)$.

Let $f(t) = t^2$

** $\boxed{L[G(t)] = e^{-p}\frac{2!}{p^3}}$

using first shifting theorem

$\left\{ \begin{matrix} e^{t-a} & t>a \\ 0 & t<a \end{matrix} \right\}$.

$e^{t-a} \left\{ \begin{matrix} 1 & t>a \\ 0 & t<a \end{matrix} \right\}$.

$e^{t-a} H(t-a)$

$L(e^{t}) = \frac{1}{s-a}$.

$L(e^{t-a} H(t-a)) = e^{-as}\frac{1}{s-1}$

3. Theorem : change of scale property.

→ If $L[f(t)] = F(p)$ then

$L[f(at)] = \frac{1}{a} F(\frac{p}{a})$

multiplication

$L(f(t)) \times t =$

Definition

→ **Function of exponential order (α)**

A function $f(t)$ is said to be of exponential order α if there exist constant α and $M > 0$ such that

$$|f(t)| \leq M e^{\alpha t}, \quad t \geq 0.$$

$$|f(t)| \leq M e^{\alpha t}, \quad t \geq 0$$

$$|t| \leq e^{t}$$

$$|e^{-2t}| \leq e^{t}$$

$$|\cos t| \leq e^{t}, \quad t \geq 0$$

LT of Derivatives

Theorem..

theorum: Let $f(t)$ be a continuous function for all $t \geq 0$ and of exponential order α. then LT of the derivative $f(t)$ exist and defined by $L[f'(t)] = pL[f(t)] - f(0)$.

theorum: Let $f(t)$ and its derivatives f' f'' f''' $f^{n}(t)$ are continuous for all $t \geq 0$ and of exponential order α. $L[f^{n}(t)] = p^{n}L[f(t)] - p^{n-1}f(0) - f'(0)p^{n-2} - p^{n-3}f''(0) \dots$

$p^{n-3} f''(0) \dots f^{n-1}(0)$.

$p^{n-2} f'(0) \cdot p^{n-3} f''(0) + \dots$

Question :- Find $L[e^{-t}\sin^2 t]$

Let $f(t) = \sin^2 t = \frac{1-\cos 2t}{2}$.

$L[f(t)] = L\left[\frac{1-\cos 2t}{2}\right]$

$\frac{L[1]}{2} - \frac{1}{2}L(\cos 2t)]$

$\rightarrow \frac{1}{2}\left[\frac{1}{p}\right] - \frac{1}{2}\frac{p}{p^2+4}$.

$\rightarrow \frac{2}{p(p^2+4)} = F(p)$

e^{+at} e^{-t}

$(a=-1)$ $(p-a)$

$p-(-1)$

$(p+1)$

#

$$F(p+1) = \frac{2}{(p+1)\left([(p+1)^2+4]\right)}$$

$$f(p-2) = \frac{2+4(p-6)^2+4(p-6)}{(p-6)^3}$$

$$\frac{4p^2-44p+122}{(p-6)^3}$$

Question 1:

$$L[e^{6t}(t+2)^2] = \left(\frac{4p^2-44p+122}{(p-6)^3}\right)$$

$$\boxed{\frac{4}{p-6} + \frac{2}{(p-6)^2} + \frac{2}{(p-6)^3}}$$ Ans.

Question 2: Find

$$L[f(t)] = L[t^2+4+4t]$$

$$= 4\frac{1}{p} + 4\cdot\frac{1}{p^2} + 1\times\frac{2}{p^3} = \frac{(2p+1)^2+1}{p^3} = \frac{2+4p^2+4p}{p^3}$$

$$f(t) = \begin{cases} \cos\left(t-\frac{2\pi}{3}\right) & t > \frac{2\pi}{3} \\ 0 & t < \frac{2\pi}{3} \end{cases}$$

$g(t) = \cos t$

$$L[g(t)] = L[\cos t] = \frac{p}{p^2+1} = G(p)$$

then $L[f(t)]$

second shifting theorem.

Answer.

$$L[f(t)] = e^{-\frac{2}{3}\pi p}\, G(p) = \boxed{e^{-\frac{2\pi}{3}p} \times p/p^2+1}$$

L.T. of integrals

Theorum: If $f(t)$ is a function and satisfies $|f(t)| < Me^{at}$ for all $t \geq 0$ (continuous)

$$L\left[\int_0^t f(x)\,dx\right] = \frac{1}{p} L[f(t)] = \frac{F(p)}{p}$$

Theorum: If $L[f(t)] = F(p)$ then all ~~...~~.

$$L[t f(t)] = -F'(p)$$

similarly $L[t^n f(t)] = (-1)^n \frac{d^n}{dp^n}[F(p)]$

Theorum:

If $L[f(t)] = F(p)$ then

then $L\left[\frac{f(t)}{t}\right] = \int_p^{\infty} F(x)\,dx.$

provided $\lim_{t\to 0}\left[\frac{f(t)}{t}\right]$ exists

Question $f(t) = L[\sin^2 t]$

$f(0) = 0$ $\quad L[f'(t)] = pL[f(t)] - f(0).$

$f'(t) = 2\sin t \cos t$
$= \sin 2t$

$L[\sin 2t] = \boxed{pL[f(t)] - 0.}$

$L[f(t)] = \frac{1}{p} L[\sin 2t] = \frac{2}{p(p^2+4)}$

Question

$\Rightarrow L[f'(t)] = p[Lf(t)] - 0.$ $\quad L(f(t))$

$L[t] = \frac{1}{p}L[1]$

$\Rightarrow \frac{1}{p^2}$

$\Rightarrow L[t^3]$

$f(t) = t^3$

$f'(t) = 3t^2$

$f''(t) = 6t$

$f'''(t) = 6.$ $\quad \left(L[1] = \frac{1}{p}\right)$

$f(0)$ & $f'(0)$ & $f''(0) = 0$

$L[f'''(t)] = p^3 L(f(t)) - p^2 f(0) - p f'(0) - f''(0)$

$L[f(t)] = \frac{1}{p^3} L[f'''(t)] = \frac{6}{p^3} L[1] = \frac{6}{p^4}.$

Question :- Find $L[t^2 \cos at]$

$L[t^n f(t)] = (-1)^n \frac{d^n}{dp^n} F(p)$

$n=2$

Let $f(t) = \cos at = f(p) = \frac{p}{p^2+a^2}$

$L(t^2 \cos at) = (-1)^2 \frac{d^2}{dp^2}\left(\frac{p}{p^2+a^2}\right)$

$= \frac{2p(p^2-3a^2)}{(p^2+a^2)^3}.$

Question : $L[(t^2-3t+2)\sin 3t]$

$\frac{6p^4 - 18p^3 + 12p^2 - 162p + 432}{(p+9)^2}.$

$L[t^2 \sin 3t] - 3L[t \sin 3t] + 2L[\sin 3t]$

$d^2p\left(\frac{3}{p^2+9}\right) + 3\,dp\left(\frac{3}{p^2+9}\right) + 2\left(\frac{3}{p^2+9}\right)$

Question:

Find $L\left[\frac{\sin t}{t}\right]$ also find the value of $L\left[\frac{\sin at}{t}\right]$ Does the LT of $\frac{\cos \alpha t}{t}$ exist ?

Let $f(t) = \sin t$

$L(f(t)) = L[\sin t]$

$= \frac{1}{p^2+1} = f(p)$.

then $L\left[\frac{f(t)}{t}\right] = \int_p^\infty f(x)\,dx$

provided.

$\lim_{t \to 0} \frac{f(t)}{t} \to$ (exist)

$\Rightarrow$. $\lim_{t\to 0} \frac{f(t)}{t} = \lim_{t\to 0} \frac{\sin t}{t} \to \cos t =$ (existing)

$\cos 0 = 1$.

then $L\left[\frac{a \sin at}{at}\right] = aL\left[\frac{\sin at}{at}\right]$

$\to a \times \frac{1}{a} \tan^{-1}\left(\frac{1}{p/a}\right)$.

$\to \tan^{-1}\left(\frac{a}{p}\right)$.

$L\left(\frac{P(t)}{t}\right) = \frac{\sin t}{t}$

$= \int_p^\infty \frac{1}{x^2+1} = \tan^{-1}(x)\Big|_0^\infty$

$= \frac{\pi}{2} - \tan^{-1} p$

or $\cot^{-1} p$ or $\boxed{\tan^{-1}\frac{1}{p}}$

$\int \frac{a}{x^2+a^2} = \tan^{-1}\left(\frac{x}{a}\right)$

$= \frac{\pi}{2} - \tan^{-1}\frac{p}{a}$

$\boxed{\tan^{-1}\frac{a}{p}}$

$\to L\left[\frac{f(t)}{t}\right]$

$\to L\left[\frac{\sin t}{t}\right] = \int_p^\infty \frac{1}{x^2+1}\,dx$.

$\left[\tan^{-1} x\right]_p^\infty$

$\frac{\pi}{2} - \tan^{-1} p$

$\cot^{-1} p$

ans: $\boxed{\tan^{-1}\left(\frac{1}{p}\right)}$

Question: Does the LT of $\frac{\cos at}{t}$ exist ?

$f(t) = \cos at$.

$L[f(t)] = \frac{p}{p^2+a^2} = F(p)$.

$\left[\frac{\cos at}{t}\right] = \int_p^\infty F(x)\,dx$.

$\boxed{\lim_{t\to 0} \frac{\cos t}{t} = \infty \quad \frac{1}{0} \text{ does not ex}}$

$x^2+a^2 = t$

$2x\,dx = dt$

$x\,dx = \frac{dt}{2}$

$\int \frac{1}{t}\,dx$

$= \int_p^\infty \frac{x}{x^2+a^2}\,dx$

$\frac{1}{2}\left[\log(x^2+a^2)\right]_p^\infty$

$\frac{1}{2}\lim_{x\to\infty} \log(x^2+a^2) - \frac{1}{2}\log(p^2+a^2)$

non exist

Question given $L[\sin\sqrt{t}] = \frac{\sqrt{\pi}}{2p^{3/2}} e^{-1/4p}$.

Show that; $L\left[\frac{\cos\sqrt{t}}{\sqrt{t}}\right] = \sqrt{\frac{\pi}{p}}\, e^{-1/4p}$.

Question

$$\boxed{\text{given evaluate. } \int_0^\infty \frac{e^{-at} - e^{-bt}}{t}\, dt}$$

$f(t) = e^{-at} - e^{-bt}$.

$L[f(t)] = L[e^{-at} - e^{-bt}] =$

$\frac{1}{p+a} - \frac{1}{p+b} = F(b)$.

$L\left[\frac{f(t)}{t}\right] = \int_p^\infty f(x)\,dx$.

$= \int_p^\infty \left(\frac{1}{x+a} - \frac{1}{x+b}\right) dx$

$= \left[\log\frac{(x+a)}{(x-a)}\right]_p^\infty$

$\log\left(\frac{x+a}{x+b}\right) - \log\left(\frac{p+a}{p-b}\right)$

$\lim_{x\to\infty} \boxed{\log\left(\frac{1+a/x}{1+b/x}\right) - \log\left(\frac{p+a}{p+b}\right)}$

$\lim_{x\to\infty} \boxed{\log\left(\frac{p+b}{p+a}\right)}$ → answer.

final ans.

$p=0$

$\log\left(\frac{b}{a}\right)$

Question :-

Prove that

$\int_0^\infty t^3 e^{-t} \sin t\, dt = 0$

Let $f(t) = t^3 \sin t$

$L[f(t)] = L[t^3 \sin t]$

$L[f(t)] = (-1)^3 \frac{d^3}{dp^3}\left(\frac{1}{p^2+1}\right)$.

$$\boxed{L[f(t)] = \frac{24(p^2+\,-1)p}{(p^2+1)^4}}$$

→ $\int_0^\infty e^{-pt} t^3 \sin t\, dt = \frac{6(p^2-1)p}{(p^2+1)}$ taking $\boxed{p=1.}$

→ $\int_0^\infty e^{-t} t^3 \sin t\, dt = 0$

$L\left(\frac{1-\cos t}{t}\right) \to \frac{1}{s} - \frac{s}{s^2+1}$.

$\int_{s}^\infty \frac{1}{s} - \frac{s}{s^2+1}$

$\left(\log s - \frac{1}{2}\log(s^2+1)\right]_s^\infty$.

$\frac{1}{2}\left[2\log s - \log(s^2+1)\right]_s^\infty$.

$\frac{1}{2}\left\{\left[\log\frac{s^2}{s^2+1}\right]_\infty - \left[\log\frac{s^2}{s^2+1}\right]_s\right\}$.

$\frac{1}{2}\left[\log 1 - \log\frac{s^2}{s^2+1}\right]$

ans. $\boxed{\frac{-1}{2}\log\frac{s^2}{s^2+1}}$

Inverse Laplace transform (ILT)

If $F(p)$ is LT of $f(t)$ is $L[f(t)] = f(p)$ then $f(p)$ is called inverse Laplace transform of $f(p)$. Its written as

$$L^{-1}[F(p)] = f(t)$$

it is unique;

Linearity :- If $F_1(p)$ & $F_2(p)$ be the LT. of $F_1(t)$ then for any constant α and β.

$$L^{-1}[\alpha F_1(p) + \beta f_2F(p)] = \alpha^{-1}L[f_1(p)] + \beta L^{-1}[F_2 p)$$
$$= \alpha F_1(t) + \beta f_2(t).$$

(1) ILT of $\frac{1}{p}$ ie $L^{-1}(1/p) = 1$.

(2) ILT of $\frac{1}{p^{n+1}}$ ie $L^{-1}\left[\frac{1}{p^{n+1}}\right] = \frac{t^n}{n!}$ or $\frac{t^n}{\sqrt{n+1}}$.

(3) ILT. of $\left[\frac{1}{p-a}\right]$ ie $L^{-1}\left[\frac{1}{p-a}\right] = e^{at}$.

(4) ILT. of $\left[\frac{1}{p^2+a^2}\right] = \frac{1}{a}\sin at$.

(5) ILT. of $\left[\frac{p}{p^2+a^2}\right] = \cos at$

(6) ILT. of $\left[\frac{1}{p^2-a^2}\right] = \frac{1}{a}\sinh at$,

(7) ILT of $\left[\frac{p}{p^2-a^2}\right] = \cosh at$,

Linearity.
+ Laplace →
transformation

Question :-

1. $L^{-1}\left[\frac{1}{p^4}\right] = L^{-1}\left[\frac{1}{p^{3+1}}\right] = \frac{t^3}{3!}$

2. $L^{-1}\left[\frac{1}{p^2+4}\right] = \frac{1}{2}\sin 2t$

3. $L^{-1}\left[\frac{5}{p^2} + \left(\frac{\sqrt{p}-1}{p}\right)^2 - \frac{7}{3p+2}\right]$

$\frac{p-1}{p^2}$

$5\frac{t}{1!} + L^{-1}\left[\frac{1}{p}\right] - L^{-1}\left[\frac{1}{p^2}\right] - L^{-1}\left[\frac{7}{3p+2}\right]$

$5t + 1 - t - L^{-1}\left[\frac{7}{3p+2}\right]$

$4t + 1 - L^{-1}\left[\frac{7/3}{p+2/3}\right]$

$$\boxed{\left[4t + 1 - \frac{7}{3}e^{-2/3t}\right]}$$

4. $L^{-1}\left[\frac{1}{p}\sin\frac{1}{p}\right]$

$L^{-1}\frac{1}{p}\left[\frac{1}{p} - \frac{1}{3!\,p^3} + \frac{1}{5!\,p^5}\right]$

$L^{-1}\cdot\frac{1}{p^2} - \frac{1}{3!\,p^4} + \frac{1}{5!\,p^6} \ldots$

$L^{-1}\left[\frac{1}{p^2}\right] + \frac{-1}{3}\left[\frac{1}{p^4} + \frac{1}{5!}L^{-1}\left(\frac{1}{p^6}\right)\right]$

$$\boxed{\frac{t}{1!} + -\frac{1}{3!}\frac{t^3}{3!} + \frac{1}{5!}\frac{t^5}{5!}}$$

FIRST SHIFTING THEORUM

If $L^{-1}[f(p)] = f(t)$

then

$L^{-1}[f(p-a)] = e^{at} f(t)$.

$= e^{at} L^{-1}[F(p)]$.

SECOND. SHIFTING THEORUM.

If $L^{-1}[F(p)] = f(t)$.

then

$L^{-1}[e^{ab} f(b)]$
$= G(t)$.

$G(t) \Rightarrow \begin{cases} f(t-a), & t > a \\ 0, & t < a. \end{cases}$

theorum : change of scale property.

$L^{-1}[f(p)] = f(t)$

then

$L^{-1}[f(ab)] = \frac{1}{a} f\left(\frac{t}{a}\right)$.

$L^{-1}\left[\frac{3p-2}{p^2-4p+20}\right] : L^{-1}\left[\frac{3p-2}{(p-2)^2+16}\right]$

$L^{-1}\left[\frac{3(p-2)+4}{(p-2)^2+16}\right]$

$3L^{-1}\frac{(p-2)}{(p-2)^2+4^2} + 4L^{-1}\left[\frac{1}{(p-2)^2+4^2}\right]$

answer: $\boxed{3\cos(4t)e^{2t} + L^{-1}\sin(4t)e^{2t}}$

Evaluate :-

$L^{-1}\left[\frac{3p+7}{p^2-2p+3}\right]$

$L^{-1}\left[\frac{3p+7}{(p-1)^2+2}\right] = L^{-1}\left[\frac{3(p-1)+4}{(p-1)^2+2}\right]$

$3L^{-1}\left[\frac{p-1}{(p-1)^2+2}\right] + 2L^{-1}\left[\frac{2}{(p-1)^2+2}\right]$

$3\cos(\sqrt{2}t)e^t + 2\sin(\sqrt{2}t)e^t$.

---

if $L^{-1}[f(p)] = f(t)$ then $L^{-1}[F^n(p)] = L^{-1}\left[\frac{d^n}{dp^n}F(p)\right]$

$\rightarrow (-1)^n t^n f(t)$

$\rightarrow (-1)^n t^n L^{-1}[f(p)]$

#. $L^{-1}[f(p)] = f(t)$

then $L\left[\frac{f(t)}{t}\right] = \int_0^\infty f(x)dx$.

$L^{-1}\left[\int_0^\infty f(x)dx\right] = \frac{f(t)}{t}$

---

if $L[f(p)] = f(t)$

$f(0) = 0$.

then $L^{-1}(pf(p)] = F'(t)$.

---

theorum of LT.- If $f(t)$ is a piecewise continuous and satisfies $f(t) \leq Me^{\alpha t}$.

then $\boxed{L\left[\int_0^t f(t)\,dt\right] = \frac{1}{p} f(p).}$

$$\frac{s}{s^2-4s+13}$$

$$\Rightarrow \frac{s}{s^2-4s+4+9}$$

$$\Rightarrow \frac{s}{(s-2)^2+3^2} - \frac{2}{(s-2)^2+3^2} + \frac{2}{(s-2)^2+3^2}$$

$$\Rightarrow \frac{s-2}{(s-2)^2+3^2} + \frac{2}{(s-2)^2+3^2}$$

$$\Rightarrow e^{2t}\cos(3t) + \frac{2}{3}\sin 3t\, e^{2t}$$

$$L\left\{ e^{t}\frac{\sin t}{t} \right\}$$

$$\frac{\sin t}{t} = \int_s^\infty \frac{1}{s^2+1} = \left[\tan^{-1} s\right]_s^\infty$$

$$= \frac{\pi}{2} - \tan^{-1} s.$$

$$\frac{\sin t}{t} e^{t} = \frac{\pi}{2} - \tan^{-1}(s-1)$$

Answer

$e^{at}\cos bt = \frac{s-a}{(s-a)^2+b^2}$

$$\cos 3t = \frac{s}{s^2+3^2}$$

$$e^{2t}\cos 3t = \frac{s-2}{(s-2)^2+3^2}$$

$t\sin at = \frac{2as}{(s^2+a^2)^2}$

$t\cos at = \frac{s^2-a^2}{(s^2+a^2)^2}$

$\frac{1}{p}$ wali property

$$L\left\{ \int_0^t\int_0^t\int_0^t t\sin t\, dt\,dt\,dt \right\} = \frac{1}{s^3} f(s)$$

$$= \frac{1}{s^3} \times \frac{2s}{(s^2+1)^2}$$

$$\left\{ e^{-2t} t\cos t \right\}$$

$$\cos t = \frac{s}{s^2+1}$$

$$t\cos t = \frac{s^2-1}{(s^2+1)^2} \quad -\frac{d}{ds}(\,)$$

$$e^{-2t} t\cos t \longrightarrow \frac{(s-2)^2-1}{((s-2)^2+1)^2}$$

INITIAL

Inverse

1. $L^{-1}\left\{\frac{1}{(s+2)^3}\right\}$

$L\{e^{-2t}t^2\} = \frac{2}{(s+2)^3}$.

$\Rightarrow \boxed{\frac{e^{-2t}t^2}{2}}$ answer.

2. $L^{-1}\left\{\frac{1}{(s+2)(s-3)}\right\}$

$\frac{s+2}{s(s+2)(s-3)} - \frac{(s-3)}{s(s+2)(s-3)}$.

$\frac{L^{-1}}{s}\left\{\frac{1}{s+2}\right\} - \frac{L^{-1}}{s}\left\{\frac{1}{s-3}\right\}$.

$\boxed{\frac{e^{-2t}}{5} - \frac{e^{3t}}{5}}$ Answer.

3. $L^{-1}\left\{\log\left(\frac{s+1}{s-1}\right)\right\}$

$f(s) = L^{-1}\{\log s+1\} - L^{-1}\{\log(s-1)\}$

$\frac{-d}{ds}f(s) = L^{-1}\left\{\frac{-d}{ds}\log(s+1)\right\} - L^{-1}\left\{\frac{-d}{ds}\log(s-1)\right\}$

$+t f(t) = \frac{1}{s+1} - \frac{1}{s-1}$

$+t f(t) = e^{-t} - e^{t}$

$\boxed{f(t) = \frac{e^{-t} - e^{t}}{t}.}$

4. $L^{-1}\left\{\log\left(\frac{s^2+1}{s(s+1)}\right)\right\}$

$\log(s^2+1) - \log s - \log(s+1)$

$\frac{d}{ds}f(s) \Rightarrow$

$L^{-1}\left\{\frac{d}{ds}f(s)\right\} = L^{-1}\left\{\frac{2s}{s^2+1}\right\} - L^{-1}\left(\frac{1}{s}\right) - L^{-1}\left(\frac{1}{s+1}\right)$

$-t f(t) = 2\cos t - 1 - e^{-t}$

$\boxed{f(t) = \frac{2\cos t - 1 - e^{-t}}{-t}}$

\# $\dfrac{1}{s^3(s^2+4)}$

$L^{-1}\left(\dfrac{1}{s^2+4}\right) = \dfrac{\sin 2t}{2}$

$\dfrac{1}{s(s^2+4)} = \int_0^t \dfrac{\sin 2t}{2} dt = \dfrac{1}{2}\left(-\dfrac{\cos 2t}{2}\right)_0^t = \dfrac{1}{4}(1-\cos t$

$\dfrac{1}{s^2(s^2+4)} = \dfrac{1}{4}\int_0^t (1-\cos t) = \dfrac{1}{4}\left(t - \dfrac{\sin 2t}{2}\right)$

$\dfrac{1}{s^3(s^2+4)} = \dfrac{1}{8}\int_0^t (2t - \sin 2t) dt$

\# $\dfrac{1}{s^2(s+1)}$

$L^{-1}\left(\dfrac{1}{s+1}\right) = e^{-t}$

$L^{-1}\left(\dfrac{1}{s(s+1)}\right) = \int_0^t e^{-t} dt = \left(\dfrac{e^{-t}}{-1}\right)_0^t$

$= 1 - e^{-t}$.

$\dfrac{1}{s^2(s+1)} = \int_0^t 1 - e^{-t} \, dt = (t + e^{-t})_0^t = t + e^{-t} - 1$

$L^{-1}\left(\dfrac{s}{(s^2+a^2)^2}\right)$

$L^{-1}\left(\dfrac{s}{s^2+a^2} * \dfrac{s}{s^2+a^2}\right) =$

$\downarrow \qquad \downarrow$

$\cos at \qquad \cos at$

$\int_0^t \cos au \cos a(t-u) du$.

$\dfrac{1}{2}\int_0^t \cos((a-a)u + at) +$

$\cos((a+a)u - at) du$.

$\dfrac{1}{2}\int_0^t \cos at + \cos(2au - at) du$

$\dfrac{1}{2}\left(u \cos at + \dfrac{\sin(2au - dt)}{2a}\right)$

$\dfrac{1}{2}\left(t\cos at + \dfrac{\sin at}{2a}\right) - \left(\dfrac{\sin at}{2a}\right)$

$\dfrac{1}{2}\left(t\cos at + \dfrac{\sin at}{a}\right)$.

$\dfrac{at\cos at + \sin at}{2a}$.

W.C.R.

#

$L^{-1}\left[\frac{p}{p^2+1} * \frac{1}{p^2+4}\right]$

$\rightarrow \int_0^t \frac{1}{2}\sin 2u \cos(t-u)\,du$

$\rightarrow \frac{1}{4}\int_0^t [\sin(u+t) + \sin(3u-t)\,du$

$\boxed{\frac{1}{3}(\cos t - \cos 2t)}$ answer.

#

$L^{-1}\ \frac{1}{p}\left(\frac{1}{p^2-a^2}\right)$

$f(t) = \frac{1}{a}\sinh bt$

$g(t) = 1.$

$L^{-1}\left(\frac{1}{p(p^2-a^2)}\right) = \int_0^t \frac{1}{a}\sinh au \cdot du$

$\boxed{\frac{1}{a^2}(\cosh at - 1)}$

answer.

INITIAL VALUE THEORUM

$L(f t) = \bar{f}(s)$

$\lim_{t\to 0} f(t) = \lim_{s\to\infty} \bar{f}(s)\, s.$

$L(f'(t)) = s\bar{f}(s) - f(0)$

$\int_0^\infty e^{-st} f'(t)\,dt = s\bar{f}(s) - f(0)$

$\lim_{s\to\infty}$ both side.

$\int_0^\infty \left(\lim_{s\to\infty} e^{-st}\right) f'(t)\,dt = \lim_{s\to\infty} s\bar{f}(s) - f(0)$

$0 \cdot f'(t)\,dt = \lim_{s\to\infty} s\bar{f}(s) - f(0)$

$\boxed{f(0) = \lim_{s\to\infty} s\bar{f}(s) \quad \lim_{t\to 0} f(t)}$

final value theorum:

$L[f(t)] = \bar{f}(s)$ then

$\lim_{t \to \infty} f(t) = \lim_{s \to 0} s\bar{f}(s)$

we know that;

$L(f'(t)) = s\bar{f}(s) - f(0)$

$\int_0^\infty e^{-st} f'(t)\,dt = s\bar{f}(s) - f(0)$.

$\lim s \to 0$ both sides

$\lim_{s \to 0} \int_0^\infty e^{-st} f'(t)\,dt = \lim_{s \to 0} [s\bar{f}(s) - f(0)]$

$\int_0^\infty (\lim_{s \to 0} e^{-st}) f'(t)\,dt = \lim_{s \to 0} (s\bar{f}(s) - f(0))$

$\int_0^\infty f'(t)\,dt = \lim_{s \to 0} s\bar{f}(s) - f(0)$

$\lim_{t \to \infty} f(t) - f(0) + f(0) = \lim_{s \to 0} s\bar{f}(s)$

$$\boxed{\lim_{t \to \infty} f(t) = \lim_{s \to 0} s\bar{f}(s).}$$

(s-1)

2.

2

$\frac{2}{3}$.

Convolution theorum

$L^{-1}(f(s)) = f(t)$

$L^{-1}(g(s)) = g(t)$

$L^{-1}(f(s) * g(s)) = \int_0^t f(u)\, g(t-u)\, du.$

example : $\frac{1}{s^2(s^2+4)}$

$L^{-1}\left(\frac{1}{s^2}\right) = t$

$L^{-1}\left(\frac{1}{s^2+4}\right) = \frac{\sin 2t}{2}$

$L^{-1}\left(\frac{1}{s^2(s^2+4)}\right) = \int_0^t \frac{\sin 2u}{2}(t-u)\, du$

$\frac{1}{2}\left[(t-u)\int \sin 2u\, du - \int \left(\frac{d(t-u)}{du}\int \sin 2u\, du\right) du\right)_0^t$

$\frac{1}{2}\left\{(t-u) - \frac{\cos 2u}{2} + \frac{\sin 2u}{4}\right\}_0^t$

$\frac{1}{2}\left[\left(0 + \frac{\sin 2t}{4}\right) - \left(\frac{1}{2}t\right)\right]$

$\frac{1}{2}\frac{\sin 2t}{4} + \frac{t}{4}$

example : $L^{-1}\left(\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right.$.

$L^{-1}\frac{s}{s^2+a^2} = \cos at$ $\qquad \cos A \cos B = \frac{1}{2}(\cos(A+B) + \cos(A-B))$

$L^{-1}\frac{s}{s^2+b^2} = \cos bt$.

$\int_0^t \cos au * \cos b(t-u)\, du.$

$\frac{1}{2}\int_0^t \cos[(a-b)u + bt + \cos(a+b)u + bt)]\, du$

$\frac{1}{2}\left[\frac{\sin(a-b)u+bt}{a-b.} + \frac{\sin(a+b)u\ bt}{a+b.}\right]_0^t$

$\frac{1}{2}\left(\frac{\sin at}{a-b} + \frac{\sin at}{a+b}\right) - \left(\frac{\sin bt}{a-b} - \frac{\sin bt}{a+b}\right]$

$\frac{1}{2}\left(\left(\frac{1}{a-b} + \frac{1}{a+b}\right)\sin at - \left(\frac{1}{a-b} - \frac{1}{a+b}\right)\sin bt\right)$

$\frac{1}{2}\left(\frac{2a}{a^2-b^2}\sin at - 2b\frac{\sin bt}{a^2-b^2}\right) = \boxed{\frac{a\sin at - b\sin bt}{a^2-b^2}}$

L^{-1} $\frac{s}{(s^2+a^2)} \times \frac{1}{(s^2+b^2)}$

$\cos A \sin B = \frac{1}{2}(\sin(A+B) - \sin(A-B))$

W.C.R.

$\cos at \qquad \frac{\sin bt}{b}$

$$L^{-1}\left[\frac{s}{s^2+a^2} * \frac{1}{b^2+s^2}\right] = \int_0^t \cos au \frac{\sin b(t-u)}{b}\,du$$

$$\frac{1}{2b}\int_0^t \sin(a-b)u + bt - \sin((a+b)u - bt)\,du$$

$$\frac{1}{2b}\left[-\frac{\cos(a-b)u+bt}{a-b} + \frac{\cos(a+b)u-bt}{a+b}\right]_0^t$$

$$\frac{1}{2b}\left[\left(\frac{-\cos at}{a-b} + \frac{\cos at}{a+b}\right) - \left(-\frac{\cos bt}{a-b} + \frac{\cos bt}{a+b}\right)\right]$$

$$\frac{1}{2b}\left[\left(\frac{1}{a+b} - \frac{1}{a-b}\right)\cos at - \left(\frac{1}{a+b} - \frac{1}{a-b}\right)\cos bt\right]$$

$$\frac{1}{2b}\left[\frac{1}{a+b} - \frac{1}{a-b}(\cos at - \cos bt)\right]$$

$$\frac{1}{2b}\left[\frac{a-b-a-b}{a^2-b^2}(\cos at - \cos bt)\right] = \boxed{\frac{\cos at - \cos bt}{b^2-a^2}}$$

$$L^{-1}\left(\frac{s}{s^2+a^2} \times \frac{1}{s^2+a^2}\right)$$

L^{-1} $\cos at \qquad \frac{\sin at}{a}$

$$L^{-1}\left[\frac{s}{s^2+a^2} * \frac{1}{s^2+a^2}\right] = \int_0^t \cos au \frac{\sin a(t-u)}{a}\,du$$

$$\frac{1}{2a}\int_0^t \sin((a-a)u + at) - \sin((a+a)u - at)\,du$$

$$\frac{1}{2a}\int_0^t \sin at - \sin(2au - at)\,du$$

$$\frac{1}{2a}\left(u\sin at + \frac{\cos(2au - at)}{2a}\right]_0^t$$

$$\frac{1}{2a}\left(t\sin at + \frac{\cos at}{2a}\right) - \left(0 + \frac{\cos at}{2a}\right) = \boxed{\frac{t\sin at}{2a}}$$

$$\frac{u\cos at + \sin at}{2a}.$$

Application of Laplace transformation

(i) Solution of ordinary dif. eq^n

DE

- ordinary → no. different dependants
  - simple DE → one indep + one dependent.
    $\boxed{\frac{dy}{dx} + x = x^2}$
  - system DE. → one independ. + more than one dependant
    + $\frac{dy}{dx} + x = 1.$
    ↳ $\frac{dy}{dx} + x^2 = 0.$
- partial differential equations → different dependants
  - simple PDE → one or more independent variable of single dependent
  - system PDE. → one or move independ. & more than one depend

Let us consider or n^{th} order DE with constant coeff.

$$\frac{d^ny}{dx^n} + a_1 \frac{d^{n-1}y}{dx^{n-1}} + a_2 \frac{d^{n-2}y}{dx^{n-2}} \cdots a_n y = f(x).$$

with condition (initial / boundary)

$$y(0) = A_0 \quad (y)'(0) = A_1$$

$$y''(0) = A_2 \quad \cdots\cdots \quad y^{n-1}(0) = A_{n-1}.$$

Take LT of (1) in both sides:

$$L\left[\frac{d^ny}{dx^n} + a_1 \frac{d^{n-1}y}{dx^{n-1}} + a_2 \frac{d^{n-2}}{dx^{n-2}} + \cdots a_n y\right] = L(f x).$$

$L[y] =$ expression

$y = L^{-1}[\text{expression}]$.

$L[y] = y(b)$.

#1.
Question

Find the solⁿ. of $(D^2 + k^2)y = 0$
#1

$y(0) = A$

$y'(0) = B$.

taking LT of pt. (1)

$L[D^2y + k^2y) = L(0)$

$L[y'' + k^2y] = 0$

$y[y''] + k^2[y] = 0$

$p^2 L[y] . py(0) - y'(0) + k^2 L(y) = 0$

$(p^2 + k^2) L(y) - p(A + B) = 0$.

$L[y] = y[p] = \frac{Ap + B}{p^2 + k^2}$.

$\overbrace{p^2 L(y) - p\,y(0) - y'(0)} + \overbrace{k^2 L(y)}$

$(p^2 + k^2) Ly - Ap - B = 0$

$$y = L^{-1}\left[\frac{Ap+B}{p^2+k^2}\right] = AL^{-1}\left[\frac{p}{p^2+k^2}\right] + BL^{-1}\left[\frac{1}{p^2+k^2}\right].$$

$$= A\cos kx + \frac{B}{k}\sin kx$$

$$= A\cos kx + c\sin kx$$

$$\Rightarrow y = L^{-1}\left[\frac{Ap+B}{p^2+k^2}\right] = AL^{-1}\left[\frac{p}{p^2+k^2}\right] + BL^{-1}\left[\frac{1}{p^2+k^2}\right]$$

$$= A\cos kx + \frac{B}{k}\sin kx.$$

$$= A\cos kx + c\sin x.$$

#2

$$L[y''] + L[y] = (t)$$

$$p^2\bar{y}(p) - py(0) - y'(0) + \bar{y}(p) = \frac{1}{p^2}$$

$$(p^2+1)\bar{y}(p) = Ap + 1 + \frac{1}{p^2}$$

Solve $y''(t) + y(t) = t$

$y'(0) = 1$

$y(0) = A$ $\quad y(\pi) \Rightarrow 0, \; y(0) = A$

$$p^2 y(p) + py(0) = \boxed{y'(0) + y(0) + \frac{1}{p^2}.}$$

$$(p^2+1)\, y(p) = Ap + 1 + \frac{1}{p^2}.$$

$$s^2 L(y) + sL(y) = s\,y(0) + y(0) + y'(0) + t$$

$SA \quad 0 \quad 1 + \quad \frac{1}{p^2}$

$6 \times 4 \quad 2$

$\frac{10}{5}$

$\frac{6}{5} + 2$

$$y(p) = \left(Ap + \frac{p^2+1}{p^2}\right) = \frac{Ap}{p^2+1} + \frac{1}{p^2}.$$

$$y = L^{-1}\left[\frac{p}{p^2+1}\right] + L^{-1}\left[\frac{1}{p^2}\right] = A\cos t + t$$

using (II) condution

$$y(\pi) = A\cos\pi + \pi = 0$$

$$-A = -\pi$$

$$A = \pi$$

$$y = \pi\cos t + t.$$

3

Solution of ODE with variable coefficient ÷

solve $t[y''] + [y'] + 4t[y] = 0$

if $y(0) = 3.$

$y'(0) = 0.$

$$L[ty''] + L[y'] + 4L[ty] = [0]$$

$$-\frac{d}{dp}L[y''] + L[y'] - 4\frac{d}{dp}\{L(y)\} = 0$$

$$-\frac{d}{dp}\left[p^2 y - p y(0) y'(0)\right] + \left[p y - y(0)\right] - 4\frac{d}{dp} y = 0.$$

$$-\frac{d}{dp}\left[p^2 y - 3p\right] + \left[p y - 3\right] - 4\frac{dy}{dp} = 0.$$

$$-(p^2+4)\frac{dy}{dp} + y = 0.$$

$$\frac{dy}{dp} + \frac{p}{p^2+4}\, dp = 0.$$

$$\boxed{\log y + \frac{1}{2}\log(p^2+4) = \log c.}$$

$$y = \frac{c}{\sqrt{p^2+4}}.$$

$$y = L^{-1}\left[\frac{\cancel{2}\, c}{p^2+4}\right].$$

$$c = L^{-1}\left[\frac{1}{\sqrt{p^2+0}}\right].$$

Ordinary

$L(D^2 y) = s^2 L(y) - s y(0) - y'(0)$

$L(Dy) = sLy - y(0)$.

$L(D^3 y) = s^3 L(y) - s^2 y(0) - s y'(0) - y''(0)$.

① $L^{-1}\left(\frac{1}{s^2+1}\right) = \sin t$

$\frac{1}{s}\left(\frac{1}{s^2+1}\right) = \int_0^t \sin t \, dt = 1 - \cos t$

$\frac{1}{s^2(s^2+1)} = \int_0^t 1 - \cos t = t - \sin t$

Quest 1) $(D^2+1)y = t$ $y(0) = 1$
$y'(0) = -2$

$L(D^2 y) + L(y) = L(t)$.

$s^2 L(y) - s y(0) - y'(0) + L(y) = \frac{1}{s^2}$

$(s^2+1) L(y) - s + 2 = \frac{1}{s^2}$

$(s^2+1) L(y) = \frac{1}{s^2} + s - 2$

$L(y) = \frac{1}{s^2(s^2+1)} + \frac{s}{s^2+1} - \frac{2}{s^2+1}$.

$\boxed{y = t - \sin t + \cos t - 2\sin t}$.

(2) $(D^2+1)y = t$ then $y(0) = 1$
$y'(0) = k$.

$L(D^2y) + L(y) = L(t)$

$s^2 L(y) - sy(0) - y'(0) + L(y) = \frac{1}{s^2}$

$(s^2+1) L(y) - s - k = \frac{1}{s^2}$

$L(y) = \frac{1}{s^2(s^2+1)} + \frac{s}{s^2+1} + \frac{k}{s^2+1}$.

$y = L^{-1}\left(\frac{1}{s^2(s^2+1)}\right) + L^{-1}\left(\frac{s}{s^2+1}\right) + kL^{-1}\left(\frac{1}{s^2+1}\right)$

$y = t - \sin t + \cos t + k\sin t$

$y = t + \cos t + (k-1)\sin t$.

$y\left(\frac{\pi}{2}\right) = -1$

$y = -1$
at $t = \pi/2$

$-1 = \frac{\pi}{2} + \cos\frac{\pi}{2} + (k-1)\sin\frac{\pi}{2}$

$k = -\pi/2$.

$$\boxed{y = t + \cos t + \left(\frac{\pi}{2} - 1\right)\sin t}$$

final ans.

Question ③ $(D^2+9)y = \cos 2t$ $\quad y(0)=1$, $y(\frac{\pi}{2}) = -1$

Let $y'(0)=k$.

$$L(D^2y) + 9L(y) = L(\cos 2t)$$

$$s^2L(y) - sy(0) - y'(0) + 9L(y) = \frac{s}{s^2+4}.$$

$$(s^2+9)L(y) - s - k = \frac{s}{s^2+4}$$

$$(s^2+9)L(y) = \frac{s}{s^2+4} + s + k$$

$$L(y) = \frac{s}{(s^2+9)(s^2+4)} + \frac{s}{s^2+9} + \frac{k}{s^2+9}$$

$$y = L^{-1}(\ ) + (\ ) + (\ )$$

$$y = \frac{\cos 3t - \cos 2t}{4-9} + \cos 3t + \frac{k}{3}\operatorname{sen} 3t$$

$$\frac{s}{(s^2+a^2)(s^2+b^2)} = \frac{\cos at - \cos bt}{b^2-a^2}.$$

$$y = \frac{\cos 3t - \cos 2t}{-5} + \cos 3t + \frac{k}{3}\sin 3t$$

$$y\left(\frac{\pi}{2}\right) = 1$$

$y = 1$
at $t = \pi/2$

$$1 = \frac{\cancel{\cos \frac{3\pi}{2}}^{\,0} - \cos\pi}{-5} + \cancel{\cos \frac{3\pi}{2}}^{\,0} + \frac{k}{3}\sin\frac{3\pi}{2}.$$

$$1 = \frac{-1}{5} - \frac{k}{3} \Rightarrow$$

$$k = 12/5$$

$$\boxed{y = \frac{\cos 3t - \cos 2t}{-5} + \cos 3t + \frac{4}{5}\sin 3t}$$

(4)

$$(D^2+3D+2)x=1 \qquad x(0)=0=x'(0).$$

$$L[D^2(x)+3L(Dx)+2L(x)=L(1).$$

$$[s^2L(x)-sx(0)-x'(0)]+3[sL(x)-x(0)]+2L(x)=\frac{1}{s}$$

$$(s^2+3s+2)L(x)=\frac{1}{s}$$

$$L(x)=\frac{1}{s(s^2+3s+2)}=\frac{1}{s(s+1)(s+2)}.$$

$$x=L^{-1}\left[\frac{1}{2s}-\frac{1}{s+1}+\frac{1}{2(s+2)}\right]$$

$$\boxed{x=\frac{1}{2}-e^{-t}+\frac{1}{2}e^{-2t}}$$

at 0

$$x=\frac{1}{2}-1+\frac{1}{2}$$

$$x=0.$$

$$\frac{A}{s}+\frac{B}{s+1}+\frac{C}{s+2}$$

$A=1/2$

$s=0$

$s+1=0 \quad B=\frac{1}{-1(-1+2)}=-1$

$s=-1$

$s+2=0$

$s=-2. \quad C=\frac{1}{-2(-2+1)}$

$=\frac{1}{2}$

Partial

$$L\left(\frac{du}{dt}\right) = s\bar{u} - u(x,0)$$

$$L\left(\frac{d^2u}{dt^2}\right) = s^2\bar{u} - su(x,0) - u_t(x,0)$$

$\frac{du}{dt}$ at $t=0$

$$L\left(\frac{du}{dx}\right) = \frac{d\bar{v}}{dx}$$

$$L\left(\frac{d^2u}{dx^2}\right) = \frac{d^2\bar{u}}{dx^2}$$

$$L(u x,t) = \bar{u}(x,s)$$

↓

Laplace.

1. $\frac{du}{dt} = 2\frac{\partial^2 u}{\partial x^2}$

given $u(0,t) = 0 = u(5,t)$
, $u(x,0) = 10 \sin 4\pi x$

$L(\ ) = L(\ )$

$s\bar{u} - u(x,0) = 2\frac{d^2\bar{u}}{dx^2}$

$2\frac{\partial^2 \bar{u}}{\partial x^2} - s\bar{u} = -u(x,0)$

$\frac{d^2\bar{u}}{dx^2} - \frac{s}{2}\bar{u} = \frac{-u(x,0)}{2}$

$\left(D^2 - \frac{s}{2}\right) u = -\frac{u(x,0)}{2}$

$\left(D^2 - \frac{s}{2}\right)\bar{u} = -5 \sin 4\pi x$

$m^2 - \frac{s}{2} = 0$

$m = \pm\sqrt{\frac{s}{2}}$

$y = c_1 e^{\sqrt{s/2}\,x} + c_2 e^{-\sqrt{s/2}\,x}$

for PI

$$\bar{u} = \frac{-5\sin 4\pi x}{D^2 - 5/2}$$

$$\bar{u} = \frac{-5\sin 4\pi x}{-16\pi^2 - \frac{5}{2}} = \boxed{\frac{10\sin 4\pi x}{5+32\pi^2}}$$

$\bar{u} = $ cf + PI

$$= c_1 e^{\sqrt{5/2}\,x} + c_2 e^{-\sqrt{5/2}\,x} + \frac{10\sin 4\pi x}{5+32\pi^2}$$

$u(0,t) = 0 = u(5,t)$

$c_1 = c_2 = 0$

$$L(u(x,t)) = \frac{10\sin 4\pi x}{5+32\pi^2}$$

$$u(x,t) = 10\sin 4\pi * L^{-1}\left(\frac{1}{s+32\pi^2}\right)$$

$$\boxed{u(x,t) = 10\sin 4\pi * e^{-32\pi^2 t}}$$

check ans $t = 0$

$u(x,0) = 10\sin 4\pi x$

2. $\frac{du}{dx} - \frac{du}{dt} = 1 - e^{-t}$

$0 < x < 1$

$t > 0$

$u(x,0) = x$

$$L\left(\frac{du}{dx}\right) - L\left(\frac{du}{dt}\right) = L(1) - L(e^{-t})$$

$$\frac{d\bar{u}}{dx} - \{s\bar{u} - u(x,0)\} = \frac{1}{s} - \frac{1}{s+1}$$

$$\frac{d\bar{u}}{dx} - s\bar{u} + x = \frac{1}{s(s+1)}$$

$$(D-s)\bar{u} = \frac{1}{s(s+1)} - x$$

$m - s = 0$

$m = s$

$$\boxed{q = ce^{sx}}$$

PI:

$$\bar{u} = \frac{\frac{1}{s(s+1)} - x}{D - s}$$

$$\bar{u} = -\frac{1}{s}\left(\frac{\frac{1}{s(s+1)} - x}{1 - D/s} \right)$$

$$\bar{u} = \left(1 - \frac{D}{s}\right)^{-1} \left[-\frac{1}{s^2(s+1)} + \frac{x}{s} \right)$$

$$\bar{u} = \left(1 + \frac{D}{s}\right)\left(-\frac{1}{s^2(s+1)} + \frac{x}{s} \right)$$

o dif

$$\bar{u} = \frac{1}{s^2(s+1)} + \frac{x}{s} - \frac{1}{s^2}$$

$$L^{-1}\left(\frac{1}{s(s+1)}\right) = \int_0^t e^{-t}\,dt = 1 - e^{-t}$$

$$\frac{1}{s^2(s+1)} = \int_0^t 1 - e^{-t}\,dx$$

$$t + e^{-t} - 1$$

$$\bar{u} = c e^{5x} + \left(\frac{1}{s^2} - \frac{1}{s^2(s+1)} + \frac{x}{s} \right)$$

Apply $0 < x < 1$ $t > 0$.

$\rightarrow c = 0$.

$$u(x,t) = L^{-1}\left(\frac{1}{s^2}\right) - L^{-1}\left(\frac{1}{s^2(s+1)}\right) + x L^{-1}\left(\frac{1}{s}\right)$$

$$u(x,t) = t - (t + e^{-t} - 1) + x$$

$$\boxed{u(x,t) = -e^{-t} + 1 + x}$$

3. $\frac{du}{dx} = 2\frac{du}{dt} + u$

$x > 0$
$t > 0$

$L\left(\frac{du}{dx}\right) = 2L\left(\frac{du}{dt}\right) + L(u)$ $\quad u(x,0) = 6e^{-3x}$.

$\frac{d\bar{u}}{dx} = 2(s\bar{u} - u(x,0)) + \bar{u}$

$\frac{d\bar{u}}{dx} - (2s+1)\bar{u} = -2u(x,0)$.

$(D-(2s+1))\bar{u} = -12e^{-3x}$.

$m - (2s+1) = 0$

$m = 2m+1$

$cf \Rightarrow ce^{(2s+1)x}$

$PI \Rightarrow$

$\bar{u} = \frac{-12e^{-3x}}{D-(2s+1)}$.

$\bar{u} = \frac{-12e^{-3x}}{-3-2s-1} = \boxed{\frac{6e^{-3x}}{s+2}}$

$$L(u_{x,t}) = c e^{(2s+1)x} + \frac{6e^{-3x}}{s+2}$$

$x>0$

$t>0 \quad c=0$

$$L\, u_{x,t} = \frac{6e^{-3x}}{s+2} = 6e^{-3x} L^{-1}\left(\frac{1}{s+2}\right)$$

$$\boxed{6e^{-3x}e^{-2t}}$$

answer.

$$[tD^2 + (1-2t)D - 2]y = 0$$

$$y(0) = 1$$
$$y'(0) = 2$$

$$tD^2y + y - 2tDy - 2y = 0$$ taking Laplace inverse

$$-\frac{d}{dp}L[y''] + y - 2\left[-\frac{d}{dp}L[y]\right] - 2y = 0.$$

$$-\frac{d}{dp}[p^2\bar{y} - py(0) - y'(0)] - L[y] + \frac{2d}{dp}[p\bar{y} - y(0)]$$

$$-\frac{d}{dp}(p^2L[y] - py(0) - y'(0)) - L[y] + 2\frac{d}{dp}(pL[y] - y(0))$$

$$-\frac{d}{dp}[p^2L[y] - p - 2] - L[y] + 2pL[y] - 2$$

$$-\frac{d}{dp}p^2y + 1 + \cancel{\frac{d}{dp}(2)} - y + 2p\frac{dy}{dp} + \cancel{2\frac{d}{dp}(1)}$$

$$\frac{dy}{dp}[2p - p^2] = y - 1$$

$$\frac{dy}{dp} = \frac{y-1}{p(-p+2)}$$

Question : $[tD^2+(1-2t)D-2]y=0$

$L(tD^2)+L(1-2t)D-2L(y)$ taking laplace.

$\Rightarrow -\frac{d}{ds}\left[s^2L(y)-sy(0)-y'(0)\right]+$

$[sL(y)-y(0)]+2\frac{d}{ds}[sL(y)-y(0)]-2L(y)$

$\Rightarrow -2sL(y)-s^2L'(y)\cancel{+1}+sL(y)\cancel{-1}+2L(y)+2sL'y-2L(y)$

$\Rightarrow L'(y)[-s^2+2s]+L[y](-2s+s)=0$

$\Rightarrow L'(y)[-s(s-2))]+L(y)(-s)=0$

$\Rightarrow L(y)\cancel{s}=L'(y)(-\cancel{s}(s-2))$

$\Rightarrow \int\frac{L'(y)}{y}=-\int\frac{1}{s-2}$

$\Rightarrow \log L(y)=-\log(s-2)+\log c.$

$\Rightarrow \boxed{\log L(y)=\log\frac{c}{s-2}}$

$\Rightarrow y=\frac{c}{s-2}$

$\Rightarrow \boxed{y=ce^{2t}}$

Answer

$y=0$

$y(0)=ce^0$

$c=1$

$\boxed{y=e^{2t}}$

Answer

Solve] Solution of system of simultaneous diff equation

$x(0) = y(0) = D(y(0)) = 0.$

and $D(x(0)) = 2$?

Taking LT of both eq^n;

$$L[x'' - 3x] - 4L[y] = 0$$
$$L[x] + L[y'' + y] = 0$$
$\to$ set A.

~~#~~ using boundary condition;

$$p^2\bar{x} - p x(0) - x'(0) - 3\bar{x} - 4\bar{y} = 0$$
$$\bar{x} + p^2\bar{y} - p(y(0) - y'(0)) + \bar{y} = 0.$$

$$p^2\bar{x} - 3\bar{x} - 4\bar{y} = 2$$
$$\bar{x} + p^2\bar{y} + \bar{y} = 0 \to \left\{\begin{array}{l}(p^2-3)\bar{x} - 4\bar{y} = 2\\ \bar{x} + (p^2+1)\bar{y} = 0\end{array}\right\}$$

set B

$\bar{x} = -(p^2+1)\bar{y}$

$-(p^2-3)(p^2+1)\bar{y} - 4\bar{y} = 2$

$2 = [4 + [(p^2-3)(p^2+1)]\bar{y}$

$[-4 - (p^2-3)(p^2+1)]\bar{y} = 2$

$$\cancel{(p^2-3)\bar{x} - 4\bar{y} = 2}$$
$$\cancel{(p^2-3)\bar{x} + (p^2+1)(p^2-3)\bar{y} = 0.}$$

after solving,

$p^4 - 3p^2 + p^2 - 3 + 4$

$\boxed{p^4 + 1 - 2p^2}$

$$\boxed{\bar{y} = \frac{-2}{4 + (p^2-3)(p^2+1)}}$$

$$\bar{x} = \frac{2(p^2+1)}{(p^2-1)^2} = \frac{1}{(p-1)^2} + \frac{1}{(p+1)^2}$$

$$\boxed{\bar{x} = \frac{+2(p^2+1)}{(p^2-3)(p^2+1)+4}}$$

$$\bar{y} = \frac{-2}{(p+1)^2(p-1)^2} \quad \text{——— (3)}$$

$$\Rightarrow \frac{1}{2}\left[\frac{-1}{p+1} + \frac{1}{p-1} + \frac{-1}{(p+1)^2} - \frac{1}{(p\ldots}\right.$$

$$y = \frac{1}{2}\left[\frac{-1}{(p+1)} + \frac{1}{p+1} - \frac{1}{(p+1)^2} - \frac{1}{(p-1)^2}\right] \quad \text{——}(4)$$

1 # $x(t) = L^{-1}\left[\frac{1}{(p-1)^2} + \frac{1}{(p+1)^2}\right] = e^{t}t + e^{-t}t = \boxed{t(e^{t} + e^{-t})}$ → Answer

2 # $y(t) = L^{-1} \frac{1}{2}\left[\frac{-1}{p+1} + \frac{1}{p-1} - \frac{1}{(p+1)^2} - \frac{1}{(p-1)^2}\right]$

first shifting theorem

$x(t) = te^{t} - te^{-t}$

$\cancel{y(t) = \frac{1}{2}\left[-[e^{-t}]e^{t}\right.}$

$-\frac{1}{2} L^{-1} \frac{1}{p+1} + \frac{1}{2} L^{-1} \frac{1}{p-1} - \frac{1}{2} L^{-1} \frac{1}{(p+1)^2} - \frac{1}{2} L^{-1} \frac{1}{(p-1)^2}$

$\frac{1}{2}[-e^{-t} + e^{t} - te^{-t} - te^{t}]$