PYQ physics & assignments

← midsem 5 →

MATRICES

1) adjoint

$A=\begin{bmatrix}3&-4\\2&6\end{bmatrix} \to \begin{bmatrix}6&4\\-2&3\end{bmatrix}$

$adj A / |A| = A^{-1}$

$A=\begin{bmatrix}3&1&2\\4&2&1\\6&2&5\end{bmatrix}$

sign → ...

2) Rank

$\begin{pmatrix}6&1&3&8\\4&2&6&-1\\10&3&9&7\\16&4&12&15\end{pmatrix}$ $\begin{bmatrix}1&6&3&8\\2&4&6&-1\\3&10&9&7\\4&16&12&15\end{bmatrix}$

$C_1 \rightleftharpoons C_2$

$R_2 = R_2 - 2R_1$
$R_3 = R_3 - 3R_1$
$R_4 = R_4 - 4R_1$

$R_3 = R_3 - R_2$
$R_4 = R_4 - R_2$

imp

Rank = 2

normal eqn

$C_2 \to C_2 - 6C_1$, $C_3 \to C_3 - 3C_1$, $C_4 \to C_4 - 8C_1$

$C_4 \to C_4 - C_2$

normal, echelon form

3) eigen value

$|A - \lambda I| = 0$

exp. distinct

2 vector, 1 vector

$\lambda = 3,3,5$

$\lambda = 5$

$\lambda = 1, 1, 4$

$(A-\lambda I) = \begin{bmatrix}1&1&-1\\1&1&-1\\1&1&1\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$

$-x_1 + x_2 - 3 = 3$

$2x_2 - 4x_3$

$[2,2,2]$

$\lambda = 3$

$x_1 + x_2 - 3x_3 = 0$

$x_1 = 0$, $x_2 = k$, $x_3 = ...$

$x_1 - x_2 + x_3 = 0$

1) trace = sum eig.
2) det = multiply
3) row sum same → λ_1

4) Solution → homo.

$\rho(AB) = \rho(A) = n$ → unique consistent

$\rho(AB) = \rho(A) < n$ → infinite solⁿ

$\rho(AB) \neq \rho(A)$ → inconsistent, no solⁿ

non homogeneous

$x + y - 3z = -1$
$4x - 2y + 6z = 8$
$15x - 3y + 9z = 21$

$-y + 3 = 1$
$y + y + 3 = 6$

solution eqns.

$\begin{bmatrix}1&1&-3\\4&-2&6\\15&-3&9\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}-1\\8\\21\end{bmatrix} \to \begin{bmatrix}1&1&1&6\\0&1&1&1\\0&0&-7&-2\\0&0&0&0\end{bmatrix}$

A, AB rank.

LU decomp / crout's method

(5)

$x + 5y + z = 14$
$2x + y + 3z = 13$
$3x + y + 4z = 17$

$\begin{bmatrix}1&5&1\\2&1&3\\3&1&4\end{bmatrix} = \begin{bmatrix}1&0&0\\l_{21}&1&0\\l_{31}&l_{32}&1\end{bmatrix}\begin{bmatrix}u_{11}&u_{12}&u_{13}\\0&u_{22}&u_{23}\\0&0&u_{33}\end{bmatrix}$

→ B

L & U find

$LY = B$

$y_1 / y_2 / y_3$

$UX = Y$

$x / y / 3$.

$A\ X_1\ X_2$

$AX_1 = \begin{bmatrix}-1&-2\\0&3\end{bmatrix}\begin{bmatrix}1\\2\end{bmatrix} = \begin{bmatrix}3\\-6\end{bmatrix} = 3\begin{bmatrix}1\\2\end{bmatrix}$ eigen value 3

eigen variation

$\alpha A \to \alpha\lambda$
$A^m \to \lambda^m$
$A \pm kI \to \lambda \pm k$
$(A \pm kI)^{-1} \to \frac{1}{\lambda \pm k}$
$A^{-1} \to \frac{1}{\lambda}$
A & A^T same.

CAYLEY-HAMILTON

$A - \lambda I \to -\lambda^3 + 5\lambda^2 - 7\lambda + 3$.

$A^3 - 5A^2 + 7A - 3I_3 = 0$

inverse finding.

$A^2 - 5A + 7I - 3A^{-1} = 0$

$A^{-1} = \frac{(\ )}{3}$

row sum same

$adj = \frac{|A|}{\lambda}$

eigenvalue $A^{-1} \to \frac{1}{\lambda}$

$A^k \to \lambda^k$

3 eign value → 3 distinct, Rank = 3.

LAPLACE

1) $f(p) = \int_0^\infty e^{-pt} f(t)\,dt$

2) $L[1] = \frac{1}{p}$

3) $L[t] = \frac{1}{p^2}$

4) $L[e^{at}] = \frac{1}{p-a}$

5) $L[t^n] = \frac{\angle n}{p^{n+1}} = \frac{n!}{p^{n+1}}$

6) $L[\sin at] = \frac{a}{p^2+a^2}$

7) $L[\cos at] = \frac{p}{a^2+p^2}$

8) $L[\sinh at] = \frac{a}{p^2-a^2}$

9) $L[\cosh at] = \frac{p}{p^2-a^2}$

10) $\frac{e^{ax}}{a^2+b^2}[a\cos bx - b\sin bx] \to \int e^{ax}\cos bx$

11) $\frac{e^{ax}}{a^2+b^2}[a\sin bx - b\cos bx] \to \int e^{ax}\sin bx$

$\sinh x = \frac{e^x - e^{-x}}{2}$

Scalar

→ $L[f(t)] = f(p)$

→ $L[f(at)] = \frac{1}{a} f\left(\frac{p}{a}\right)$

$|\cos t| \le e^t$
$|e^{-2t}| \le e^t$

$L[f'(t)] = pL[f(t)] - f(0)$ →

$L[f^n(t)] = p^n L[f(t)] - p^{n-1} f(0) - f'(0)p^{n-2}$ →

$\int_0^t$

$L\left[\int_0^t f(x)\,dx\right] = \frac{1}{p} L[f(t)] = \frac{f(p)}{p}$

①

② $L[t^n f(t)] = (-1)^n \frac{d^n [f(p)]}{dp^n}$

③ $\int_p^\infty F(x)\,dx = L\left[\frac{f(t)}{t}\right]$

$\frac{\cos t}{t}$ ✗

exist ✓

$\frac{\sin t}{t} \to \frac{\pi}{2} - \tan^{-1} p$ by $\int_p^\infty \frac{1}{1+x^2}$

$\frac{\sin at}{t} = \frac{\pi}{2} - \tan^{-1}\frac{p}{a}$ $\int \frac{a}{x^2+a^2}$

$\int_p^\infty \frac{1}{x+a} - \frac{1}{x+b}$

$x \to \infty$, $p = \infty$

$\log \frac{x+a}{x+b}$

1st shifting

$e^{at} f(t) = \frac{1}{(p-a)}$

eg: $\cos 2t\, e^{-3t} = \frac{(s+3)}{(s+3)^2+4}$

$e^{2t}\sin 3t = \frac{3}{(s-2)^2+9}$

2nd shifting

1. $\cos(t - \pi/3) = e^{-\frac{\pi}{3}s} \frac{s}{s^2+1}$

2. $e^{t-a} = e^{-as}\frac{1}{s-1}$

3. $(t-1)^2 = \frac{2!}{p^3} e^{-p}$

★★

Q. $\int_1^\infty e^{-pt}(t-1)^2\,dt$.

$e^{-p} e^{-px} x^2\,dx$

$e^{-p}\int_0^\infty e^{-px} x^2\,dx$

$\boxed{e^{-p}\frac{2!}{p^3}}$

$\sin^2 t \to \left[\frac{1-\cos 2t}{2}\right] \to$

$\frac{2}{p(p^2+4)}$

$L[\sin 2t] = p\,L[\sin^2 t] - f(0)$

$\frac{L[\sin 2t]}{p} = L[\sin^2 t]$

eg $L[f'''(t)] =$

$p^3 L[f(t)] - p^2 f(0) - p f'(0) - f''(0)$

$\int\int\int_0^t t\sin t\,dt\,dt\,dt$

$\Rightarrow \frac{1}{p^3} \times \frac{2p}{(p^2+1)^2}$

$t\sin t$

INVERSE.

#

$\sin\frac{1}{p} = \frac{1}{p} - \frac{1}{3!}\frac{1}{p^3} + \frac{1}{5!}\frac{1}{p^5}$

$L\left[\frac{1}{p}\sin\frac{1}{p}\right] = \frac{1}{p^2} - \frac{1}{3!}\frac{1}{p^4} + \frac{1}{5!}\frac{1}{p^6}$

$= t - \frac{1}{3!}\frac{t^3}{3!}$

V. important

#

$t\sin at = \frac{2as}{(s^2+a^2)^2}$

$t\cos at = \frac{s^2-a^2}{(s^2+a^2)^2}$

eg. $\frac{7}{3p+2}$

$\frac{7/3}{p+2/3}$

$\frac{7}{3}e^{-2t/3}$

$\frac{\sin t}{t}e^{-at} = \frac{\pi}{2} - \tan^{-1}(s-a)$

$\frac{\sin t}{t} \to \frac{\pi}{2} - \tan^{-1}s$

$\frac{\sin at}{t} \to \frac{\pi}{2} - \sin^{-1}\frac{p}{a}$

Question

$L^{-1}\left[\frac{3(p-2)+4}{p^2-4p+20}\right]$ $3p-2$

$3\left[\frac{p-2}{(p-2)^2+4^2}\right] + 4L^{-1}\left[\frac{1}{(p-2)^2+4^2}\right]$

$\boxed{3\cos(4t)e^{2t} + \frac{1}{4}\sin(4t)e^{2t}}$

APPLICATIONS

ordinary — partial

1. $L[D^2y] = s^2L[y] - sy(0) - y'(0)$
2. $L[Dy] = sL[y] - y(0)$
3. $L[D^3y] = s^3L[y] - s^2[y(0)] - sy'[0] - y''[0]$

eg 1) $y''(t) + y(t) = t$

1 → Laplace $L[y''(t)] + L[y] = L[t]$

2 → value put $p^2\bar{y}(p) - py(0) - y'(0) + \bar{y}(p) = \frac{1}{p^2}$

$\boxed{(p^2+1)\bar{y}(p) = Ap + 1 + \frac{1}{p^2}}$

$y(t) = L^{-1}\left[\frac{p}{p^2+1}\right]\times A + L^{-1}\left[\frac{1}{p^2}\right]$

$\boxed{A\cos t + t}$

eg 3 →

$\boxed{L^{-1}\frac{1}{s^2(s^2+1)} = t - \sin t}$

eg 4:

$\boxed{\frac{s}{(s^2+9)(s^2+4)} = \frac{\cos at - \cos bt}{b^2-a^2}}$

convulation.

eg 5:

$\frac{1}{s(s+1)(s-2)} = \frac{-1}{2s} - \frac{1}{s+1} + \frac{1}{2(s+2)}$

$\boxed{\frac{1}{2} - e^{-t} + \frac{1}{2}e^{2t}}$

$A = \frac{1}{(s+1)(s-2)}$ $A = \frac{1}{2}$ $\boxed{s=0}$

Partial fraction.

eg 2) $L[ty''] + L[y'] + 4 + L[y]$

$-\frac{d}{dp}L[y''] + L[y'] + 4\frac{d}{dp}[L(y)]$

$-\frac{d}{dp}[p^2\bar{y} - py(0) - y'(0)] + \bar{y} + 4\frac{d\bar{y}}{dp}$

$-\frac{d\bar{y}}{dp}[p^2+4] - py = 0$

$\frac{dy}{y} + \frac{p}{p^2+4}dp = 0$

$\log y + \frac{1}{2}\log(p^2+4) = \log c$

type 1 type 2

$\boxed{\bar{y} = \frac{c}{\sqrt{p^2+4}}}$

ans $y = L^{-1}[\ ]$

eg 6.

$\boxed{\begin{array}{l}-\frac{d}{ds}[s^2L(y)] \quad \text{multiply.} \\ -2sL[y] - s^2L'(y)\end{array}}$

** $L^{-1}\left(\frac{1}{(p+1)^2}\right) \to e^{-t}xt$

$L^{-1}\left(\frac{1}{(p+1)^2}\right) \to e^{-t}xt$

eg 7.

$(p^2-3)\bar{x} - 4\bar{y} = 2$

$\bar{x} + (p+1)\bar{y} = 0.$

$\bar{x} = \frac{2(p+1)}{(p^2-1)^2} = \frac{1}{(p-1)^2} + \frac{1}{(p+1)^2}$

$\bar{y} = \frac{-2}{(p+1)^2(p-1)^2} = \frac{1}{2}\left[\frac{1}{p+1} + \frac{1}{p-1} - \frac{1}{(p+1)^2} - \frac{1}{(p-1)^2}\right]$

FOURIER.

$f(x) \to \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \cos\left(\frac{n\pi x}{l}\right) + b_n \sin\left(\frac{n\pi x}{l}\right)\right]$

$a_0 = \frac{1}{l} \int_{-l}^{l} f(x)\,dx$

① $a_n = \frac{1}{l} \int_{-l}^{l} f(x) \cos\left(\frac{n\pi x}{l}\right) dx$

$b_n = \frac{1}{l} \int_{-l}^{l} f(x) \sin\left(\frac{n\pi x}{l}\right) dx$

even.

$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{l}$

$a_0 = \frac{2}{l} \int_0^l f(x)dx$

$a_n = \frac{2}{l} \int_0^l f(x) \cos \frac{n\pi x}{l}$

by parts ✓ odd funct ≠ 0.

$\cos n\pi \to (-1)^n$

$\cos n\pi$

$\boxed{a_0 + a_n \cos\left(\frac{n\pi x}{l}\right) + b_n}$

② $a_0 = \frac{1}{2\pi} \int_0^{2\pi} f(x)dx$

$a_n = \frac{1}{\pi} \int_0^{2\pi} f(x) \cos nx\,dx$

$b_n = \frac{1}{\pi} \int_0^{2\pi} f(x) \sin nx\,dx$

③ $a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x)dx$

$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)dx \cos nx$

...tion of System of ODE/

DIFF & INTEGRATION BASIC FORMULAS

$\sin \to \cos$

$\cos \to -\sin$

$\tan \to \sec^2$

$\cot \to -\csc^2$

$\csc \to -\csc \cot$

$\sec \to \sec \tan$

$\ln x \to \frac{1}{x}$ $\boxed{\cancel{\log} a^x \to \frac{1}{(\log a) x}}$

$\int a^x \to a^x \log x$ $\log_a x \to \frac{1}{x \log a}$

$x^x \to x^x + \ln x$

$e^x \to e^x$

$\sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$

$\cos^{-1} x = -\frac{1}{\sqrt{1-x^2}}$

$\tan^{-1} x \to \frac{1}{1+x^2}$

$\cot^{-1} x \longrightarrow \frac{-1}{1+x^2}$

$\sec^{-1} x \longrightarrow \frac{1}{|x|\sqrt{x^2-1}}$

$\csc^{-1} x \longrightarrow \frac{-1}{|x|\sqrt{x^2-1}}$

$\frac{d}{dx}\frac{f(x)}{g(x)} = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$

$\int \tan x = \log|\sec x|$

$\int \cot x = \log|\sin x|$

$\int \sec x = \log|\sec + \tan|$

$\int \csc x = \log|\csc x - \cot x|$

$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \log\left|\frac{x-a}{x+a}\right| + c$

$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \log\left|\frac{a+x}{a-x}\right| + c$

$\int \frac{dx}{x^2+a^2} = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right)$

$\int \frac{dx}{\sqrt{x^2-a^2}} = \log|x + \sqrt{x^2-a^2}|$

$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right)$

$\int \frac{dx}{\sqrt{x^2+a^2}} = \log|x + \sqrt{x^2+a^2}| + c$

1. $\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2} \log|x + \sqrt{x^2-a^2}|$
2. $\int \sqrt{x^2+a^2}\,dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2} \log|x + \sqrt{\ }|$
3. $\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2} \sin^{-1}\left(\frac{x}{a}\right)$