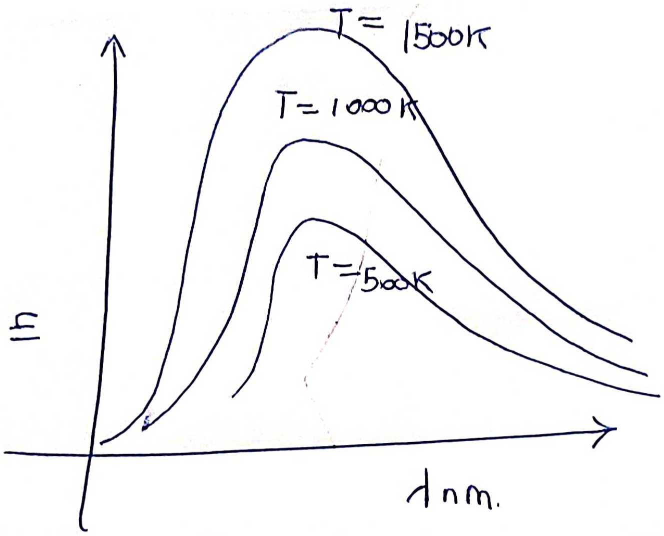
3 mechanics q: quantum

$\frac{1}{2}mv^2 = KE$

3 cases $\quad mv = \frac{p^2}{2m} = KE$

p diff. $\quad$ KE $\quad$ freedom.

$\underline{KE = eV}$ $\quad \underline{p = \sqrt{2mKE}}$ $\quad \underline{KE = \frac{3}{2}KT}$



1. Blackbody radiations

$$\boxed{E = \frac{h\nu^3}{e^3[e^{h\nu/KT} - 1]}}$$

2. debroglie hypothesis.

3. Phase velocity

$v_p = \lambda \nu$

$\lambda = \frac{2\pi}{K}$

$\nu = \frac{\omega}{2\pi}$

$p = \frac{E}{c}$ or mv

$E = h\nu$

[associated with moving particle]

$$\boxed{v_p = \frac{\omega}{K} = \frac{2\pi}{k} \times \frac{\omega}{2\pi}}$$

velocity $\quad$ wave

$$\boxed{v_p = \lambda \nu = \frac{\cancel{h}}{p} \times \frac{E}{\cancel{h}} = \frac{E}{p}}$$

$$E = mc^2$$

$$p = mv$$

non realistic

(i) $v_p = \frac{mc^2}{mv} = \frac{c^2}{v}$

$\boxed{c \Rightarrow v}$

less

realistic (ii) $E = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$

$\boxed{c \lesssim > \cancel{\gg} v}$

more

$$E = \sqrt{p^2c^2 + m_0^2c^4}$$

$$v_p = \frac{\sqrt{p^2c^2 + m_0^2c^4}}{p}$$

$$v_p = \sqrt{\frac{p^2c^2}{p^2} + \frac{m_0^2c^4}{p^2}}$$

$$\boxed{v_p = c\sqrt{1 + \left[\frac{m_0^2c^2}{p^2}\right]}}$$

$$v_p > c$$

Group Velocity

ω_1, ω_2

k_1, k_2

$$y_1 = A\sin(\omega_1 t - k_1 x)$$

$$y_2 = A\sin(\omega_2 t - k_2 x)$$

$$y = y_1 + y_2$$

$$y \Rightarrow 2A\sin\left[\left(\frac{\omega_1+\omega_2}{2}\right)t - \left(\frac{k_1+k_2}{2}\right)x\right\} * \cos\left\{\left(\frac{\omega_1-\omega_2}{2}\right)t - \left(\frac{k_1-k_2}{2}\right)x\right\}$$

$y \Rightarrow$

$$2A\sin(\omega t - Kx)\cos\left(\frac{\Delta\omega}{2}t - \frac{\Delta K}{2}x\right)$$

$$v_p = \omega/k.$$

$$v_g = \frac{\Delta\omega}{2} / \Delta k/2$$

$$\boxed{\frac{\Delta\omega}{\Delta K} = \frac{d\omega}{dk}.}$$

ans.

→ .

phase velocity ↔ group velocity

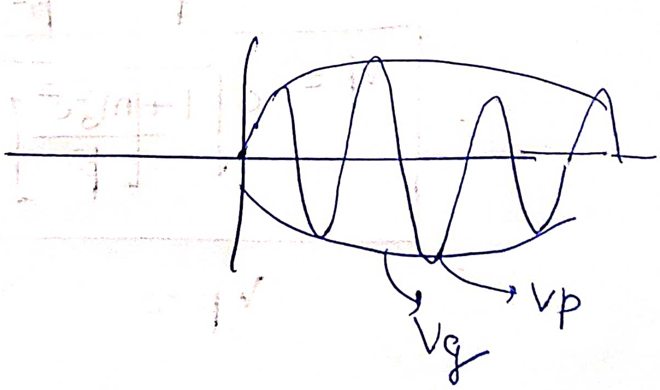
$$v_p = \frac{\omega}{K}$$

&

$$\omega = v_p K \quad \text{——} \quad (1)$$

$$v_g = \frac{d\omega}{dK}$$

$$v_g \to \frac{d(v_p K)}{dK}$$



$$v_g = v_p + k\frac{dv_p}{dK}$$

$$= v_p + \frac{2\pi}{\lambda}\left[\frac{dv_p}{-\frac{2\pi}{\lambda^2}d\lambda}\right]$$

$$= v_p - \lambda\frac{dv_p}{d\lambda}$$

$$dK = -\frac{2\pi}{\lambda^2}d\lambda$$

$\psi(r,t) \rightarrow$ Complex quantity.

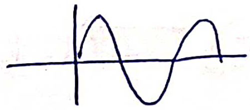
$|\psi(r,t)|^2 \rightarrow \psi(r,t) \cdot \psi^*(r,t) \rightarrow$ physical meaning (prob density)

r: position (x, y, z)
t: time

well behaved wave funcⁿ should be →

1. $\left.\begin{matrix} x \\ y \\ z \end{matrix}\right]$ ∞ to $-\infty$ of r

2. $\psi(r,t)$ should be single valued (one value at one point)
3. $\psi(r,t)$ should be continuous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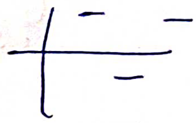
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4. normalised wave funcⁿs

$$|\psi\, r, t|^2 = \int_{-\infty}^{\infty} \psi(r,t) * \psi^*(r,t)\, d\tau = 1$$

tow

$$dv = dx\,dy\,dz$$
$$d\tau = dx\,dy\,dz\,dt$$

$$\int_{-\infty}^{\infty} |\psi_N(r,t)|^2\, d\tau = 1$$

normalised wave funcⁿ

$$|c|^2 \int_{-\infty}^{\infty} |\psi(r,t)|^2\, dt = 1$$

$$\boxed{|c|^2 = \frac{1}{\int_{-\infty}^{\infty} |\psi(r,t)|^2\, dt}}$$

Schrodinger's eqⁿ

$$\boxed{\psi(x,t) = Ae^{i(\omega t + kx)}}$$

$$\omega = \frac{2\pi}{\lambda}$$

$$\times \quad \lambda = \frac{h}{p} = \frac{c}{\nu p}$$

$$h\nu = E$$

$$E = h\nu$$
$$= \frac{h}{2\pi} \times 2\pi\nu$$

$$\boxed{E = \hbar \times \omega} \quad \text{and} \quad p = \frac{h}{\lambda} \times \frac{2\pi}{2\pi} \quad \boxed{p = \hbar k}$$

put these values,

$$\psi(x,t) \longrightarrow Ae^{i\left(-\frac{E}{\hbar}t + \frac{p}{\hbar}x\right)}$$

$$\longrightarrow Ae^{\frac{i}{\hbar}(px - Et)}$$

$$\longrightarrow$$

* diff wrt x;

$$\frac{d\psi}{dx} = Ae^{\frac{i}{\hbar}(px-Et)} \times \frac{pi}{\hbar} = \frac{1}{\hbar} p\psi i$$

$$\frac{d\psi}{dx^2} = \psi \frac{p^2 i^2}{\hbar^2} = \frac{\psi p^2}{\hbar^2} \times (-1)$$

$$(1) \quad \boxed{p^2\psi = -\hbar^2 \frac{d^2\psi}{dx^2}}$$

* diff wrt to t;

$$\frac{d\psi}{dt} = Ae^{\frac{i}{\hbar}(px-Et)} \left(-\frac{i}{\hbar}E\right)$$

$$\frac{d\psi}{dt} = -\frac{i}{\hbar}\psi E$$

$$\boxed{\frac{d\psi}{dt^2} = -\psi \frac{E^2}{\hbar^2}}$$

$$E\psi = -\frac{\hbar}{i} \times \frac{i}{i} \frac{d\psi}{dt}$$

$$E\psi = +\hbar i \frac{d\psi}{dt} \quad (2)$$

total energy of particle;

$$E\psi = \left(\frac{p^2}{2m} + V\right)\psi$$

P.E

$$E\psi = \frac{p^2}{2m}\psi + V\psi$$

(putting values in 1, 2)

$$i\hbar\frac{d\psi}{dt} = \frac{1}{2m}\left(-\hbar^2\frac{d^2\psi}{dx^2}\right) + V\psi$$

$$\boxed{-\frac{\hbar^2}{2m}\frac{d^2\psi}{\partial x^2} + V\psi = i\hbar\frac{d\psi}{dt}} \quad (3)$$

time dependent sch.- eq^n

TIME independent sch. wave func

$$\psi(r,t) = \psi_r * \phi(t)$$

putting eq^n $\psi(r,t)$ in (3)

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}[\psi\phi] + V\psi\phi = i\hbar\frac{\partial}{\partial t}[\psi\phi]$$

$$-\frac{\phi\hbar^2}{2m}\frac{\partial^2}{\partial x^2}[\psi] + V\psi\phi = \psi\, i\hbar\frac{\partial}{\partial t}[\phi]$$ constant taken out;

$\div$ by $\psi\phi$, $$-\frac{\hbar^2}{2m}\left(\frac{1}{\psi}\right)\frac{\partial^2\psi}{\partial x^2} + V = i\hbar\frac{1}{\phi}\frac{d\phi}{dt} = E.$$

$$-\frac{\hbar^2}{2m}\left(\frac{1}{\psi}\right)\frac{\partial^2\psi}{\partial x^2} + V = E$$

$$\boxed{E\phi = i\hbar\frac{\partial\phi}{\partial t}}$$

$$\boxed{E = i\hbar\frac{\partial}{\partial t}}$$

$$\boxed{-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V\psi = E\psi}$$

$$\boxed{\frac{\partial^2 \psi}{\partial x^2} + \frac{2m}{\hbar^2}(\epsilon - V)\psi = 0}$$

time indep schr eqⁿ

$$\Rightarrow \frac{-\hbar^2}{2m}\left[\frac{\partial^2 \psi}{\partial x^2}\right] + V\psi = \epsilon\psi$$

$$\Rightarrow \frac{-\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)$$

$$\Rightarrow \boxed{\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}}$$

$$\Rightarrow \boxed{-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = \epsilon\psi}$$

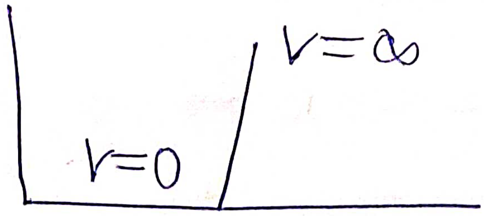
3D eqⁿ

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{2m}{\hbar^2}(\epsilon - V)\psi = 0$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\psi + \frac{2m}{\hbar^2}[\epsilon - V]\psi = 0$$

$$\boxed{\nabla^2\psi + \frac{2m}{\hbar^2}[\epsilon - V]\psi = 0}$$

Particle in a box



$$V = \begin{cases} 0 & 0 < x < L \\ \infty & x \geq L,\ x \leq 0 \end{cases} \quad \text{--- (1)}$$

Schrondinger time independent wave eq^n

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{2m}{\hbar^2}[E - V]\psi = 0 \quad \text{--- (2)}$$

for $0 < x < L$;

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{2m}{\hbar^2}[E - 0]\psi = 0$$

$$\frac{\partial^2 \psi}{\partial x^2} + \underbrace{\frac{2m}{\hbar^2}}_{K^2} E\psi = 0 \qquad K = \sqrt{\frac{2mE}{\hbar^2}}$$

$$\boxed{\frac{\partial^2 \psi}{\partial x^2} + K^2 \psi = 0}$$

The sol^n of eq^3 is

$$\psi = Ae^{ikx} + Be^{-ikx}$$

$$\psi = A\sin kx + B\cos kx.$$

Boundary condition;

$\psi = 0$ at $x = 0$

$\psi = 0$ at $x = L$

at $x=0$, $0 = 0 + B \Rightarrow B = 0$

at $x=L$, $0 = A\sin kL + 0$

$$A \sin kL = 0$$

$$kL = n\pi$$

Putting in eq 4,

$$\psi = A\sin kx + 0$$

$$kL = n\pi$$

$$k^2L^2 = n^2\pi^2$$

$\cancel{kx = \frac{n\pi x}{L}}$

$$\boxed{\frac{2mEL^2}{\hbar^2} = n^2\pi^2}$$

$$\underbrace{(a+b)}_{A} \sin kx + \underbrace{(a-b)}_{B} \cos kx$$

$$\frac{2mEL^2}{\hbar^2} = n^2\pi^2$$

** v imp

$$E \rightarrow \frac{n^2\pi^2\hbar^2}{2mL^2} = \frac{n^2\pi^2 h^2/4\pi^2}{2mL^2}$$

$$\boxed{E = \frac{n^2h^2}{8mL^2}}$$

from $kL = n\pi \Rightarrow k = \frac{n\pi}{L}$: $\boxed{\psi = A\sin\frac{n\pi x}{L}}$ normalised eqn

$$\Rightarrow \int_{-\infty}^{\infty} \psi\,\psi^* dx = \int_{-\infty}^{0} |\omega|^2 dx + \int_{0}^{L} |\omega|^2 dx + \int_{L}^{\infty} |\omega|^2 dx.$$

$$\Rightarrow \int_{-\infty}^{\infty} |\omega|^2 dx = 1$$

$$\Rightarrow \int_0^L A^2 \sin^2 \frac{n\pi x}{L}\, dx = 1$$

$$\Rightarrow \frac{A^2}{2}\int_0^L \left(1 - \cos\frac{2n\pi x}{L}\right) dx = 1$$

$$\Rightarrow \frac{A^2}{2}\left[x - \frac{\sin\frac{2n\pi x}{L}}{\left(\frac{2n\pi}{L}\right)}\right]_0^1 = 1$$

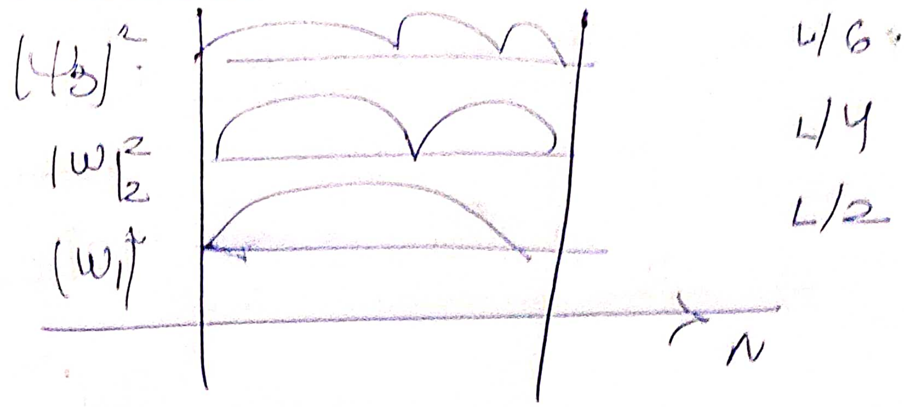
$$\Rightarrow \frac{A^2}{2}\left[L - 0 - 0 - 0\right] = 1.$$

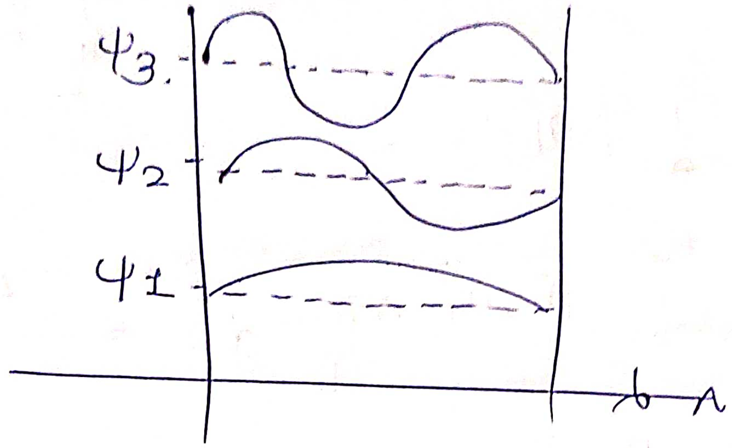
$$\frac{A^2}{2}L = 1$$

$$\boxed{A = \sqrt{\frac{2}{L}}}$$

$$\boxed{\psi_n = \sqrt{\frac{2}{L}}\sin\frac{n\pi x}{L}}$$

$$\psi_1 = \sqrt{\frac{2}{L}}\sin\frac{\pi x}{L}$$





$$\frac{A_2}{A_1} = \frac{K_1 - K_2}{K_1 + K_2}$$

$$\frac{A_3}{A_1} = \frac{K_1 \times 2}{K_1 + K_2}$$

$R = \frac{|A_2|^2}{|A_1|^2}$ reflected / incident $\neq 0$

$R = \frac{|A_3|^2}{|A_1|^2}$ transmitted / incident $\neq 1$

at $n=0$ $E = 0$

$n=1$ $E = \frac{h^2}{8mL^2}$

[zero potential energy]

→ particle has a minimal energy at L/2

at $n=1$. middle of box] ψ^2 probability of finding.

$n=2$ never

$n=1$:- 1st excited state.

Question: $n=0$
$E \neq 0$
why?

$n=0$:- particle ground state.

quantum part: $E \Rightarrow 0$
$p \Rightarrow 0$
if $\lambda \Rightarrow \infty$ ⟶ [it can be confined]
↳ wave nature

$$\boxed{E_1 = \frac{h^2}{8mL^2}}$$

$\longleftarrow$ zero point energy $\longrightarrow$

$$\boxed{\psi_x = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}}$$

Energy in 3D :-
& wave function

$$\sqrt{\frac{8}{L_x L_y L_z}} \sin \frac{n_x \pi x}{L_x} \sin \frac{n_y \pi y}{L_y} \sin \frac{n_z \pi z}{L_z}$$

$$E_{n_x n_y n_z} = \frac{n_x^2 h^2}{8mL_x^2} + \frac{n_y^2 h^2}{8mL_y^2} + \frac{n_z^2 h^2}{8mL_z^2}$$

$$E_{n_x n_y n_z} = \frac{h^2}{8m}\left[\frac{n_x^2}{L_x^2} + \frac{n_y^2}{L_y^2} + \frac{n_z^2}{L_z^2}\right]$$

HARMONIC OSCILLATOR

$$F = -kx$$

$$F = m\frac{d^2x}{dt^2}$$

spring
k: constant

Putting it in eq(1)

$$m\frac{d^2x}{dt^2} = -kx$$

$$\boxed{\frac{d^2x}{dt^2} + \frac{k}{m}x = 0}$$

$$\boxed{\frac{d^2x}{dt} + \omega^2 x = 0}$$

$$\boxed{x = A\cos(\omega t + \phi)}$$

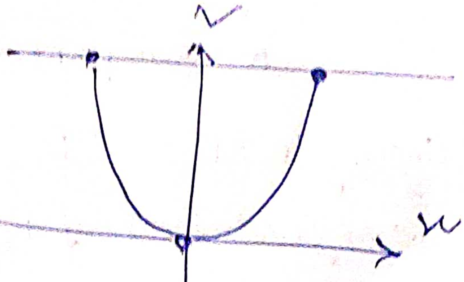
$$\omega^2 = \frac{k}{m}$$

$$\omega = \sqrt{\frac{k}{m}}$$

$$\boxed{\nu = \frac{1}{2\pi}\sqrt{\frac{k}{m}}}$$

* Potential energy →

$$V = -\int_0^x F\,dx$$



$V = \frac{1}{2}kx^2$ or $\int_0^x kx\,dx$ (1) energy spectrum can not be continuous it is discreat.

(2) The lowest energy can not be zero it should have same ~~diff differ~~ definite value of energy $E \neq E_0$.

(3) The is finite prob that particle is out side the potential wall.

Schrodinger's eq^n:

$$\frac{\partial^2\psi}{\partial x^2} + \frac{2m}{h^2}(E - V)\psi = 0$$

$$\frac{d^2\psi}{dx^2} + \frac{2m}{h^2}\left(E - \frac{1}{2}kx^2\right)\psi = 0$$

$$\frac{d^2\psi}{dx^2} + (\alpha - y^2)\psi = 0 \text{ ——— } (4)$$

where :→ $\alpha = \frac{2mE}{h^2}$

:→ $y^2 = \frac{\cancel{2}m}{h^2}\frac{1}{\cancel{2}}kx^2$

$y^2 = \frac{kx^2m}{\hbar^2}$

$$\left[\, y = \frac{x}{\hbar}\sqrt{km} \,\right]$$

After solving eqⁿ ④

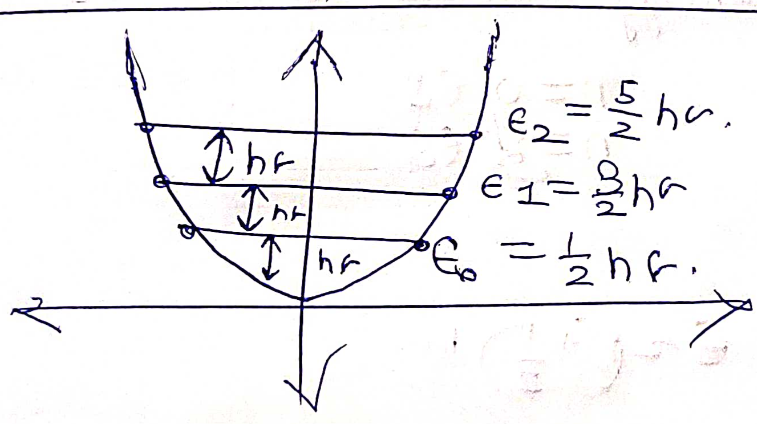
$$\psi_n \rightarrow \left[\frac{2m\omega}{\hbar}\right]^{1/4} \left[2^n n!\right]^{-1/2} H_n(y)\, e^{-y^2/2}$$

$H_n(y)$ is hermite polynomial;

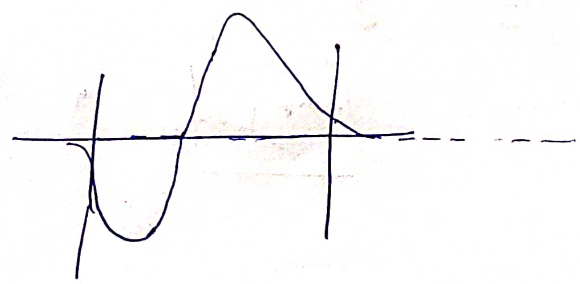
	n	$H_n(y)$	α_n	E_n
$\psi = 0$	0	1	1	$\frac{1}{2}\hbar\omega$
$\psi \Rightarrow$	1	$2y$	3	$\frac{3}{2}\hbar\omega$
	2	$4y^2 - 2$	5	$\frac{5}{2}\hbar\omega$
	3			

energy of H.O.

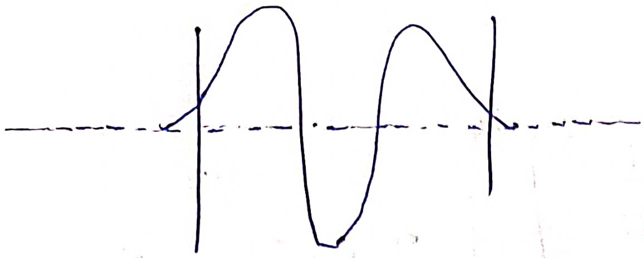
$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega$$



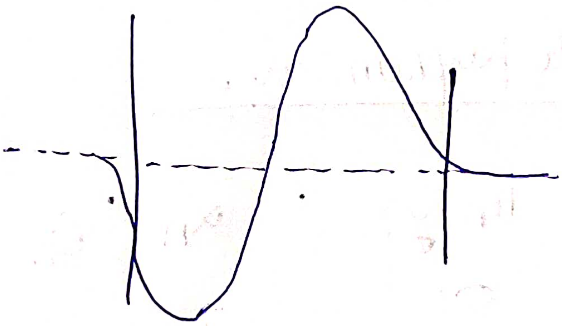
ψ_1:



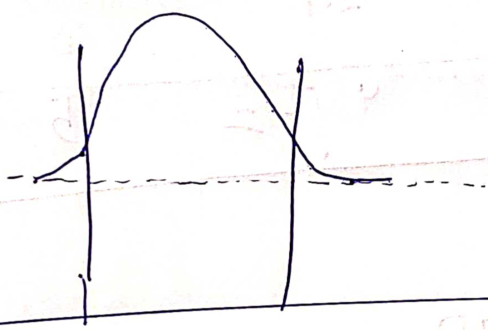
ψ_2



ψ_1



ψ_0



Question :→

Cal· energy 2^{nd} ES &

$n=0$ ES.

$n=2$ ES

$E = \left(n+\frac{1}{2}\right) h\nu$

$\frac{7}{2} - \frac{1}{2} h\nu$

$\boxed{\rightarrow 3h\nu}$

$\frac{9}{8}\,\frac{n^2h^2}{mL^2} - \frac{1}{8}$

$\boxed{\rightarrow \frac{n^2h^2}{mL^2}}$

Sol → Q. An e^- is confined to move b/w two rigid wall sep by 1 nm find de-B wavelength w/e ref the first 2 allowed energy states of e^- & corresp. energy

$$\lambda = \frac{h}{mv}$$

$$\lambda = \frac{h}{\sqrt{2mE}}$$

$$\boxed{E_{difference} = \frac{4h^2}{8mL^2} - \frac{h^2}{8mL^2}}$$

$$\frac{4-1}{8}\frac{h^2}{mL^2} = \frac{3h^2}{8m}$$

$$\frac{h}{\sqrt{2m\frac{3h^2}{8mL^2}}}$$

$$\cancel{\frac{2h}{3m}}$$

$$\boxed{\frac{2L}{\sqrt{3}}}$$

2 different value.

$$\frac{h^2}{2mL^2} \qquad \frac{h^2}{8mL^2}$$

$$\boxed{\frac{2}{\sqrt{3}}\times 10^{-9}\ nm}$$

$$\frac{h\sqrt{2mL^2}}{\sqrt{2m\times h^2}} \qquad \frac{h\sqrt{8mL^2}}{\sqrt{2m\times h^2}}$$

$$L \qquad 2L \qquad \frac{2\sqrt{2}}{\pi}$$