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Assignment #4 Mathematics - I

1. Harsh and Raj plan to evaluate the line integral $I = \int \vec{F} \cdot \vec{dr}$ along a path in xy-plane from $(0, 0)$ to $(1, 1)$. The force field is $\vec{F}(x, y) = (x+2y)\hat{i} + (-x+y^2)\hat{j}$. Harsh chooses the path that runs along the x-axis from $(0, 0)$ to $(1, 0)$ and then runs along the vertical line $x = 1$ from $(1, 0)$ to $(1, 1)$. Raj chooses the direct path along the diagonal line $y = x$ from $(0, 0)$ to $(1, 1)$. Which one of them has larger value of I?

2. A torus is generated by rotating a circle C about a straight line L in space so that C does not intersect or touch L. If L is z-axis and C has radius b and its center has distance $(a > b)$ from L, then compute the surface area of the torus where surface of the torus is given by $\vec{r}(u, v) = (a + b \cos v) \cos u\hat{i} + (a + b \cos v) \sin u\hat{j} + b \sin v\hat{k}$. Here u is the angle of rotation and v is the angle used in describing the circle.

3. Evaluate the flux of the vector field $\vec{F}(\vec{r}) = \frac{\vec{r}}{a^3}$ through the sphere with the center at origin and radius a. Comment on the flux so obtained.

4. For the function $f(x, y) = \frac{y}{(x^2+y^2)}$, find the value of directional derivative in the direction making an angle $30°$ with the positive x-axis at the point $(0, 1)$.

5. The velocity vector field of a fluid is $\vec{F} = x\hat{i} + y\hat{j} + z\hat{j}$. Find the flow along the helix represented by $\vec{r}(t) = \cos t\hat{i} + \sin t\hat{j} + t\hat{k}$, where t lies in $[0, 2\pi]$.

6. Check whether $\vec{f} \times \vec{g}$ are irrotational or solenoidal if $\vec{f}$ and $\vec{g}$ are irrotaional vectors.

7. Verify Greens theorem for the following vector field $\vec{F} = e^{-x} \sin y\hat{i} + e^x \cos y\hat{j}$, where C is the square with the vertices $(0, 0), (0, \pi/2), (\pi/2, 0), (\pi/2, \pi/2)$.

8. Find the values of a, b and c for which the vector field $\vec{F} = (x + 2y + az)\hat{i} + (bx - 3y - z)\hat{j} + (4x + cy + 2z)\hat{k}$ is irrotational.