22B008

Maulana Azad National Institute of Technology Bhopal-462003

End Term Examination (Session: 2022-23)

Course: B.Tech. **Semester-I** **Section: All**

Subject: Mathematics-I **Subject Code: MTH-101** **Time: 3 Hours**

Max. Marks: 50

Note: All questions are compulsory.

S.No.	Question	Marks
1(a)	If $\theta = f\left(\frac{y-x}{xy}, \frac{z-x}{xz}\right)$, *show that* $x^2\frac{\partial\theta}{\partial x} + y^2\frac{\partial\theta}{\partial y} + z^2\frac{\partial\theta}{\partial z} = 0$	4
1(b)	Find an approximate value of f(1. , 0.8) using the Taylor series quadratic approximation, if $f(x, y) = \tan^{-1}(xy)$.	4
2(a)	Discuss the maxima and minima of the function $u = \cos A\cos B\cos C$ where A, B, C are the angles of a triangle.	4
2(b)	If $u = \sin^{-1}\frac{x+2y+3z}{x^8+y^8+z^8}$, find the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}$.	4
3(a)	Find the area lying inside the circle $r = \sin\theta$ and outside the cardioid $r = (1-\cos\theta)$.	5
3(b)	The loop of the curve $2ay^2 = x(x-a)^2$ revolves about the x-axis. Find the volume of the solid so generated.	5
4(a)	Verify the Stokes's theorem for the vector field $v = (3x-y)i - 2yz^2 j - 2y^2z k$, where S is the surface of the sphere $x^2 + y^2 + z^2 = 16$, $z > 0$	4
4(b)	Use the divergence theorem to evaluate $\iint_S (V.n)\,dA$, where $V = x^2zi + yj - xz^2k$ and S is the boundary of the region bounded by the paraboloid $x^2 + y^2 = z$ and the plane $z = 4y$.	4
4(c)	Evaluate $\oint_C (x^2 + y^2)dx + (y + 2x)dy$, where C is the boundary of the region in the first quadrant, which is bounded by the curves $x^2 = y$ & $y^2 = x$.	4
5(a)	Solve $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 8(x^2 + e^{2x} + \sin 2x)$	4
5(b)	Solve by using the method of variation of parameter $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = x^2e^x$	4
5(c)	Solve $(y + \frac{1}{3}y^3 + \frac{1}{2}x^2)dx + \frac{1}{4}(x + xy^2)dy = 0$	4