Maulana Azad National Institute of Technology Bhopal-462003

Mini Test (April-2023)

Course: B.Tech. **Semester-II** **Section: B**

Subject: Mathematics-II **Subject Code:MTH-110**

Time: 55 Minutes **Max. Marks: 10**

Note: All questions are compulsory.

S.No.	Question	Marks
1	Solve the following system of equations using Crout's method $2x+3y+z=9$ $x+2y+3z=6$ $3x+y+2z=8$	3
2	Solve the equation $(D^2+9)y=\cos 2t, \textit{if } y(0)=1 \;\&\; y\left(\frac{\pi}{2}\right)=-1$ by using the Laplace transform.	4
3	Find the eigenvalues and eigenvectors of the matrix $A=\begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$.	3

Maulana Azad National Institute of Technology, Bhopal

Mid Term Examination (May, 2023)

B.Tech. | Semester II | Group: A & B
Max. Marks: 20 | Duration: 90 minutes | Date: 29/05/2023
Sub: Mathematics - II | Sub. Code: MTH 110 | Roll No.:

Note: Attempt all Five Questions. Standard notations are used. Assume missing data if any.

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1. Calculate all the eigenvalues and corresponding eigenvector of Matrix A. Moreover, using the concept of eigenvalues, find the determinant of A^7 if

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

[Marks 4]

2. Test the consistency and solve if the following system of equations is consistent:

$$\begin{aligned} x + 2y + z &= 3 \\ 2x + 3y + 2z &= 5 \\ 3x - 5y + 5z &= 2 \\ 3x + 9y - z &= 4 \end{aligned}$$

[Marks 4]

3. Find $\mathcal{L}^{-1}\left\{\dfrac{1}{(s-2)^3(s^2-4s+5)}\right\}$ and use it to solve the following

$$(D^3 + 3D^2 - 3D + 1)y = e^t \sin t, \quad y(0) = y'(0) = y''(0) = 0, \text{ where } D = \frac{d}{dt}.$$

[Marks 4]

4. Find the fourier series expansion of the following periodic function with period 2π

$$f(x) = \begin{cases} \pi + x, & -\pi < x < 0, \\ 0, & 0 \le x < \pi. \end{cases}$$

Using the result, prove $\dfrac{1}{1^2} + \dfrac{1}{3^2} + \dfrac{1}{5^2} + \cdots = \dfrac{\pi^2}{8}$. [Marks 4]

5. Using the method of separation of variables, solve the following heat equation

$$u_t = u_{xx}, \quad 0 < x < 1, \quad t > 0$$

subject to $u(0,t) = u(1,t) = 0$, and $u(x,0) = x - x^2$.

[Marks 4]

Maulana Azad National Institute of Technology, Bhopal

End Term Examination (June 2023)

B.Tech.
Max. Marks: 50
Sub: Mathematics - II

Semester II
Duration: 180 minutes
Sub. Code: MTH 110

Group: A & B
Date: 26/06/2023
Roll No.:

Note: Total five questions. Attempt all Questions. Standard notations are used. Assume missing data if any. Use of Calculators is **not permitted**.

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1. (a) Using LU Decomposition Method, calculate the inverse of the matrix $A = \begin{bmatrix} 4 & 1 & 1 \\ 1 & 4 & -2 \\ 3 & 2 & -4 \end{bmatrix}$. Then, using inverse of the matrix A, solve the following system of linear equations

$$\begin{aligned} 4x + y + z &= 4 \\ x + 4y - 2z &= 4 \\ 3x + 2y - 4z &= 6 \end{aligned}$$

[Mark 5]

(b) Using Cayley-Hamilton theorem, evaluate the value of A^4 without evaluating A^3 and Matrix multiplication A^2A^2, where $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$. Moreover, calculate A^{-1} without evaluating the Determinant of A. **[Mark 5]**

2. (a) Investigate whether $f(t) = t^2e^{2t} + e^t\cos t + \sin\sqrt{t}$ is of exponential order α. If yes, then find Laplace transform of $f(t)$. **[Mark 5]**

(b) Calculate the sum of infinite series $1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots$ using the Fourier series enpension of $f(x) = \begin{cases} \pi + x & \text{if } -\pi \le x \le 0 \\ \pi - x & \text{if } 0 < x \le \pi \end{cases}$ **[Mark 5]**

3. (a) Solve $\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial x \partial y} + \frac{\partial u}{\partial y} - u = 2\cos(x+2y) + e^x + e^y - e^{2x+y}$ **[Mark 5]**

(b) A tightly stretched string with fixed end points $x = 0$ and $x = L$ is initially in a position given by $u = u_0\sin^3\frac{\pi x}{L}$. If it is released at a time $t = 0$ from the position $u = u_0\sin^3\frac{\pi x}{L}$, find the formula for the displacement at a distance x from one end and at a time t, i.e., $u(x,t)$. **[Mark 5]**

4. (a) Present the Classification of $f(z) = e^{\frac{1}{(z-2)^2}} + \frac{\sin z}{z}$ Singularities using Laurent series and calculate residue at each isolated singularity. **[Mark 5]**

(b) Using Residue Theorem, solve the Definite integral $\int_0^{2\pi} \frac{\sqrt{2}\, d\theta}{(3+\cos\theta)^2}$ **[Mark 5]**

5. (a) Find the first five non-vanishing terms in the powers series solution about $x = 0$ of the initial value problem: $\frac{d^2y}{dx^2} - \frac{dy}{dx} + \frac{y}{4} = 0, \quad y(2) = 0, \quad \left.\frac{dy}{dx}\right|_2 = 1.$ **[Mark 5]**

(b) Identify all singularities and obtain the series solution of $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - \frac{1}{9})y = 0$ about a regular singular point. **[Mark 5]**