Maulana Azad National Institute of Technology, Bhopal

Academic Year 2024-25

Assignment-3

Programme	B.Tech	Semester	Semester II
Course code	MTH24102	Section	All
Course Title	Mathematics-II	Department	MBC

Answer all questions.

Q. No.	Question	CO	BT
1	Consider the function $$f(x) = e^{-\|x\|}, \quad x \in \mathbb{R}.$$ Show that the function $f(x)$ satisfies the Dirichlet conditions for the Fourier Series by proving: • $f(x)$ is absolutely integrable: $\int_{-\infty}^{\infty} \|f(x)\|\, dx < \infty$. • $f(x)$ is piecewise continuous over any finite interval. Discuss the advantages of verifying that a function satisfies the Dirichlet conditions before computing its Fourier series.	4	2
2	Let the function $f(x)$ be defined on the interval $[-\pi, \pi]$ by $$f(x) = \begin{cases} \pi + x, & -\pi \le x < 0, \\ \pi - x, & 0 \le x \le \pi, \end{cases}$$ and extended periodically with period 2π. Check whether $f(x)$ satisfies the Dirichlet conditions or not. If yes, Compute the Fourier coefficients a_0, a_n, and b_n for $f(x)$ and write down the full Fourier series expansion for $f(x)$.	4	2
3.	Let $f(x) = x, \quad -\pi < x < \pi$, with $f(x + 2\pi) = f(x)$. (1) Derive the Fourier series of $f(x)$. (2) Using the series, evaluate $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$.	4	3
4.	Let $f(x) = \|x\|, \quad -\pi \le x \le \pi$, with periodic extension of period 2π. (1) Determine the Fourier series of $f(x)$. (2) Using the Fourier expansion, show that $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}$.	4	3

5.	Let $f(x) = x, \quad 0 < x < \pi$, and extend $f(x)$ as an odd function on $(-\pi, \pi)$. (a) Derive the Fourier sine series representation of $f(x)$. (b) By evaluating Fourier series at $x = \pi/2$, deduce that $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n-1} = \frac{\pi}{4}$.	4	5
6.	Let $f(x) = x^2, \quad 0 \le x \le \pi$, and extend $f(x)$ as an even function on $(-\pi, \pi)$. Derive the Fourier cosine series representation of $f(x)$. Then, compute $\sum_{n=1}^{\infty} \frac{1}{n^2}$.	4	5
7.	Let $f(x) = x(\pi - x), \quad 0 \le x \le \pi$, and extend $f(x)$ as an odd function on $(-\pi, \pi)$. Derive the Fourier sine series of $f(x)$. Then, evaluate the series $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}$.	4	5
8.	For $f(x) = e^{-3\|x\|}$, evaluate the Fourier transform and show that you are able to recover $f(x)$ using the inverse Fourier transform.	4	1
9.	Evaluate the Fourier transform of $f(x) = \begin{cases} 1, & \|x\| \le 1, \\ 0, & \|x\| > 1. \end{cases}$ Can we use Fourier transform of $f(x)$ to evaluate improper integral $\int_{-\infty}^{\infty} \frac{\sin^2 x}{x^2}\, dx$. If yes, then do it.	4	4
10.	Employ the Fourier transform technique to calculate the integral $I = \int_0^{\infty} \frac{\cos(2x)}{1+x^2}\, dx$.	4	5

Course Outcome (CO)

CO1: Demonstrate the ability to solve linear systems and perform matrix operations, including determining the rank, eigenvalues, and eigenvectors.
CO2: Apply the Cayley-Hamilton theorem to solve matrix-related problems.
CO3: Solve ordinary differential equations using Laplace transforms and interpret inverse Laplace transforms for engineering applications.
CO4: Develop proficiency in Fourier series and Fourier transforms and their application in signal analysis.
CO5: Analyze and solve partial differential equations (PDEs), including boundary value problems for heat and wave equations.

Bloom Taxonomy (BT)

1-Remember; 2-Understand; 3-Apply; 4-Analyze; 5-Evaluate; 6-Create