KIRCHHOFF'S LAWS -

For complex circuit computations, the following two laws first stated by Kirchhoff are indispensable.

(1) ***Kirchhoff's Point Law or Current Law*** **(KCL). It states as follows :**

The sum of the currents entering a junction is equal to the sum of the currents leaving the junction.

Refer Fig. 27.

If the currents *towards a junction are considered positive and those away from the same junction negative,* then this law states that the *algebraic sum of all currents*

meeting at a common junction is zero.

i.e., $$\Sigma \text{ Currents entering} = \Sigma \text{ Currents leaving}$$

$$I_1 + I_3 = I_2 + I_4 + I_5$$

or $$I_1 + I_3 - I_2 - I_4 - I_5 = 0$$

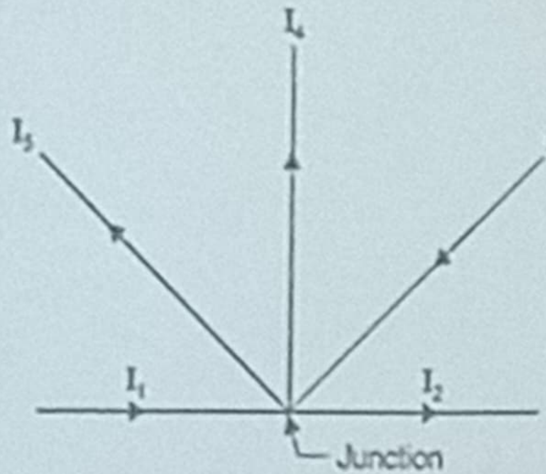

Fig. 27

(2) *Kirchhoff's Mesh Law or Voltage Law (KVL). It states as follows :*

The sum of the e.m.fs (rises of potential) around any closed loop of a circuit equals the sum of the potential drops in that loop.

Considering a rise of potential as positive (+) and a drop of potential as negative (−), the *algebraic sum of potential differences (voltages) around a closed loop of a circuit is zero :*

$\Sigma E - \Sigma IR$ *drops* $= 0$ *(around closed loop)*

i.e., $\Sigma E = \Sigma IR$

or Σ Potential rises $= \Sigma$ Potential drops

To apply this law in practice, assume an arbitrary current direction for each branch current. The end of the resistor through which the current enters, is then positive, with respect to the other end.

*** If the solution for the current being solved turns out negative, then the direction of that current is opposite to the direction assumed.*

Rule (1)

A voltage drop exists when tracing through a resistance with or in the same direction as the current, or through a battery or generator against their voltage, that is from positive (+) to negative (–). As shown in Fig. 28.

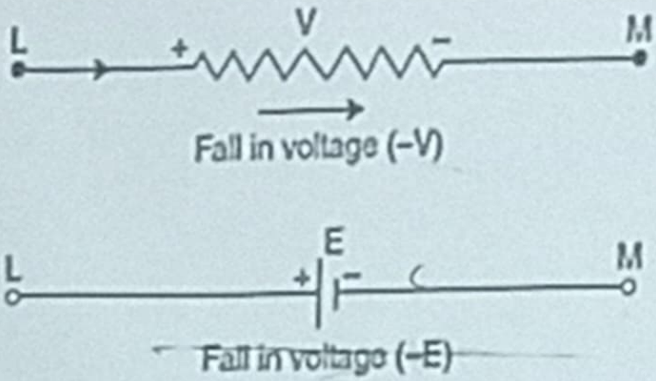

Fig. 28

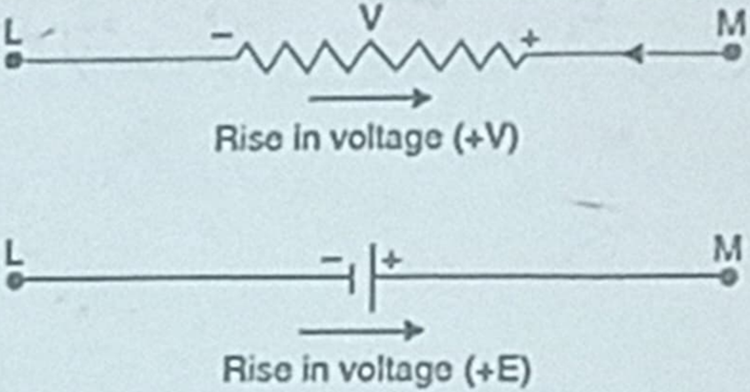

Fig. 29

Rule (2)

A voltage rise exists when tracing through a resistance against or in opposite direction to the current or through a battery or a generator with their voltage that is from negative (–) to positive (+). As shown in Fig. 29.

APPLICATIONS OF KIRCHHOFF'S LAWS- It is employed in following methods for solving networks.

1. Branch-current method
2. Maxwell's loop (or mesh) current method
3. Nodal voltage method.

(1) Branch-Current Method

For a multi-loop circuit the following *procedure is adopted for writing equations :*

1. Assume currents in different branch of the network.
2. Write down the smallest number of voltage drop loop equations so as to include all circuit elements ; these loop equations are independent.

If there are *n nodes of three or more elements in a circuit, then write (n – 1) equations as per* current law.

3. Solve the above equations simultaneously.

The assumption made about the directions of the currents initially is arbitrary. In case the actual direction is *opposite to the assumed one, it will be reflected as a negative value for that current* in the answer.

The branch-current method (oldest method) involves more labour and is not used *except for very simple circuits.*

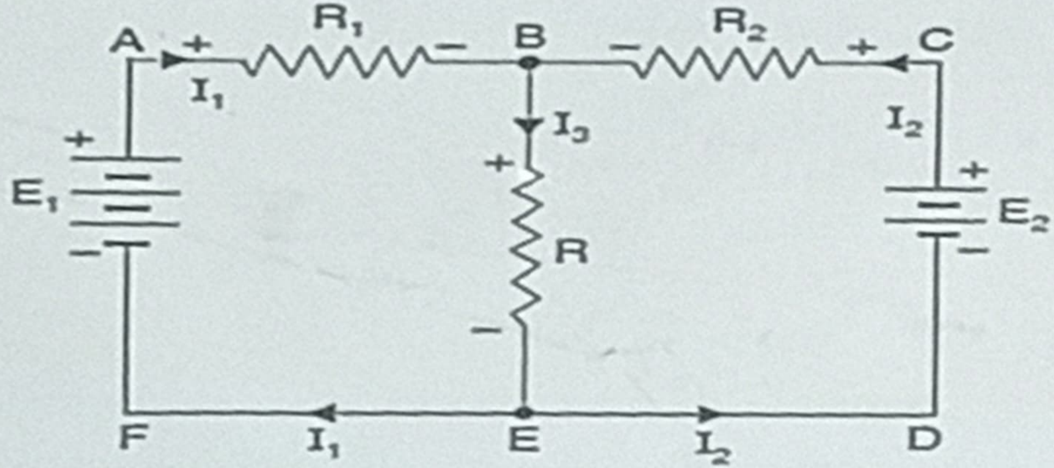

Fig. 30

Illustration. Consider a circuit shown in Fig. 30.

Considering the loop ABEFA, we get

$$-I_1R_1 - I_3R_3 + E_1 = 0$$

or $$E_1 = I_1R_1 + I_3R_3 \quad \text{(where } I_3 = I_1 + I_2\text{)} \quad ...(i)$$

Considering the loop BCDEB, we have

$$I_2R_2 - E_2 + I_3R_3 = 0 \quad ...(ii)$$

or $$E_2 = I_2R_2 + I_3R_3$$

If E_1, E_2, R_1, R_2 and R_3 are known, then I_1, I_2 and I_3 can be calculated from eqns. (*i*) and (*ii*).

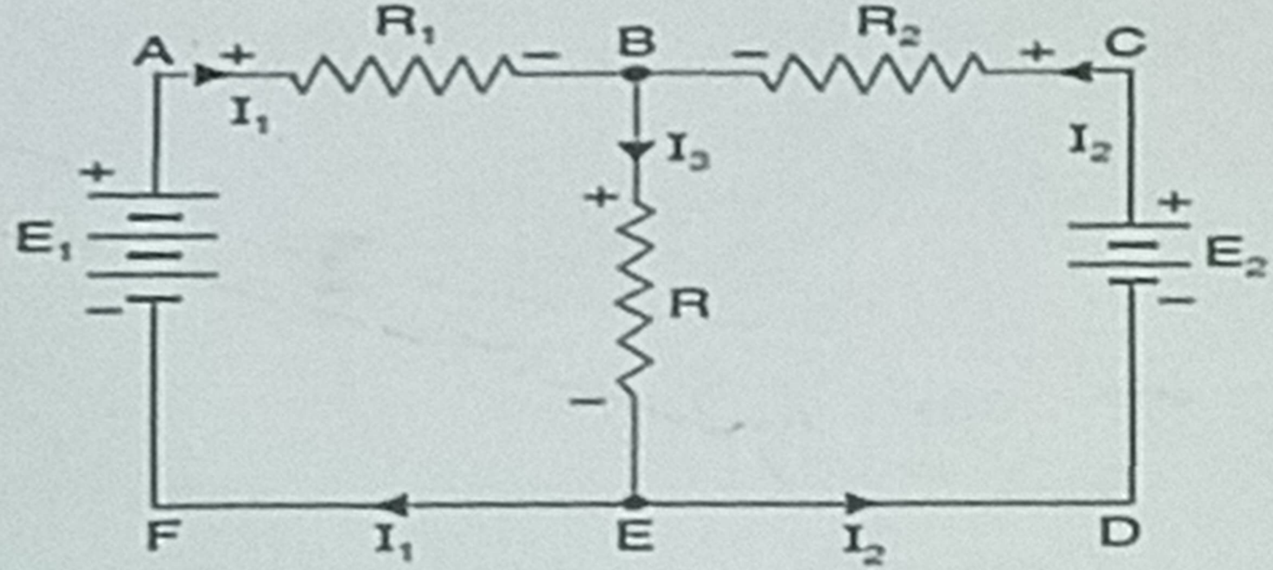

Fig. 30

Illustration. Consider a circuit shown in Fig. 30.

Considering the loop **ABEFA**, we get

$$-I_1R_1 - I_3R_3 + E_1 = 0$$

or $\quad E_1 = I_1R_1 + I_3R_3 \quad$ (where $I_3 = I_1 + I_2$) ...(*i*)

Considering the loop **BCDEB**, we have

$$I_2R_2 - E_2 + I_3R_3 = 0 \qquad ...(ii)$$

or $\quad E_2 = I_2R_2 + I_3R_3$

If E_1, E_2, R_1, R_2 and R_3 are known, then I_1, I_2 and I_3 can be calculated from eqns. (*i*) and (*ii*).

Example (1). ***In the circuit of Fig. 31-***

Find the current through each resistor and voltage drop across each resistor.

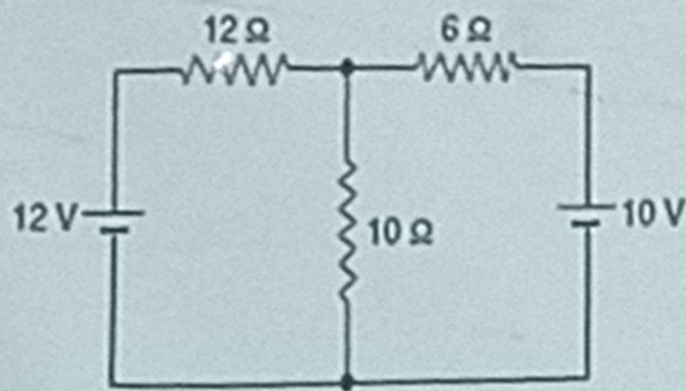

Fig. 31

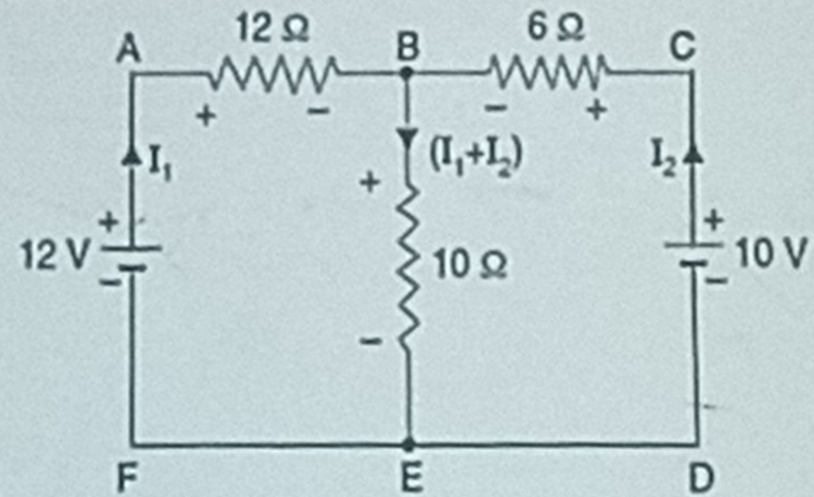

Fig. 32

Solution. Let the currents be as shown in Fig. 32.

Applying Kirchhoff's voltage law to the circuit ABEFA, we get

$$-12I_1 - 10(I_1 + I_2) + 12 = 0$$

$$-22I_1 - 10I_2 + 12 = 0$$

$$+11I_1 + 5I_2 - 6 = 0$$

Example(2). *Find the magnitude and direction of currents in each of the batte shown in Fig. 33.*

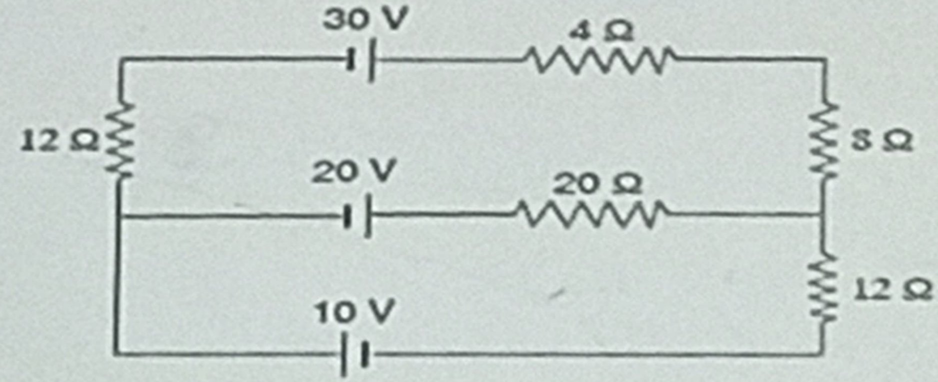

Fig. 33

Solution. Let the directions of currents I_1, I_2 and I_3 in the batteries be as shown in Fig.

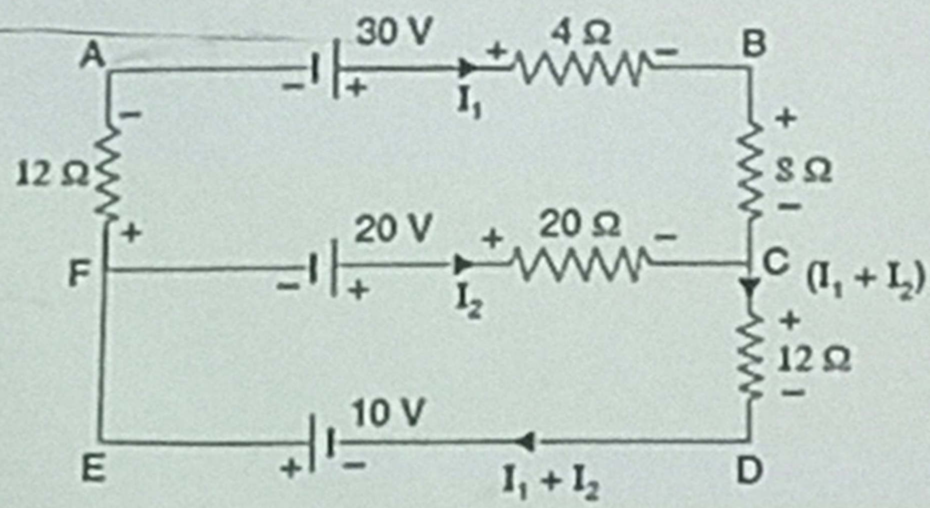

Fig. 34

Applying Kirchhoff's voltage law to the circuit ABCFA, we get

$$30 - 4I_1 - 8I_1 + 20I_2 - 20 - 12I_1 = 0$$

$$-24I_1 + 20I_2 + 10 = 0$$

or $$12I_1 - 10I_2 - 5 = 0$$

Circuit ECDEF gives,

$$20 - 20I_2 - 12(I_1 + I_2) + 10 = 0$$

$$20 - 20I_2 - 12I_1 - 12I_2 + 10 = 0$$

$$-12I_1 - 32I_2 + 30 = 0$$

or $$6I_1 + 16I_2 - 15 = 0$$

Multiplying eqn. (*ii*) by 2 and subtracting it from (*i*), we get

$$-42I_2 + 25 = 0$$

i.e., $$I_2 = 0.595 \text{ A}$$

Substituting this value of I_2 in eqn. (*i*), we get

$$12I_1 - 10 \times 0.595 - 5 = 0$$

or $$I_1 = 0.912 \text{ A}$$

Hence current through,

30 V battery, $I_1 = 0.912$ A. (Ans.)

20 V battery, $I_2 = 0.595$ A. (Ans.)

10 V battery, $(I_1 + I_2) = 1.507$ A. (Ans.)

Applying Kirchhoff's voltage law to the circuit ABCFA, we get

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20 V battery, $I_2 = 0.595$ A. (Ans.)

10 V battery. $(I_1 + I_2) = 1.507$ A. (Ans.)

(2) Maxwell's Loop (or Mesh) Current Method-

- This method is *used in solving networks having some* degree of complexity
- It *eliminates a great deal of tedious work involved in branch-current method*
- It is *best suited when energy sources are voltage sources rather than current sources*
- This method can be used only for *planar circuits* (No wires crossing each other)

The **procedure for writing the equations is as follows-**

(1) Assume the smallest number of mesh currents so that at least one mesh current links every element. As a matter of convenience, all mesh currents are assumed to have a *clockwise direction.* The *number of mesh currents is equal to the number of meshes in the circuit.*

(2) For each mesh write down the Kirchhoff's voltage law equation. Where more than one mesh current flows through an element, the algebraic sum of currents should be used. The algebraic sum of mesh currents may be sum or the difference of the currents flowing through the element depending on the direction of mesh currents.

(3) Solve the above equations and from the mesh currents find the branch currents

Example . ***Determine the currents through various resistors of the circuit show in Fig. 47 using the mesh current method.***

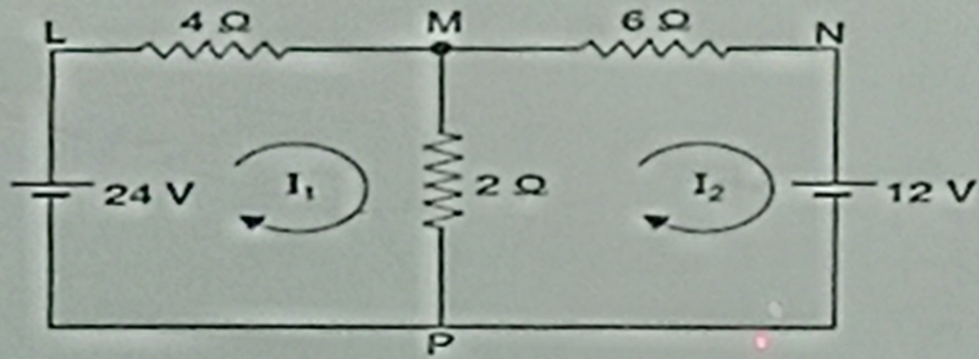


Fig. 47

Since there are two meshes, let the loop currents be as shown.

Applying Kirchhoff's law to *loop 1*, we get

$$24 - 4I_1 - 2(I_1 - I_2) = 0$$

$$-6I_1 + 2I_2 + 24 = 0$$

or $$3I_1 - I_2 = 12 \qquad \ldots(i)$$

For *loop 2*, we have

$$-2(I_2 - I_1) - 6I_2 - 12 = 0$$

$$2I_1 - 8I_2 - 12 = 0$$

$$I_1 - 4I_2 = 6 \qquad \ldots(ii)$$

Solving (*i*) and (*ii*), we get $I_1 = \frac{42}{11}$ A

and $$I_2 = -\frac{6}{11}\text{ A}$$

Hence Current through 4 Ω resistor = $\frac{42}{11}$ A (from L to M). (Ans.)

Current through 6 Ω resistor = $\frac{6}{11}$ A (from N to M). (Ans.)

Current through 2 Ω resistor = $\frac{42}{11} - \left(-\frac{6}{11}\right) = \frac{48}{11}$ A (from M to P). (Ans.)

Example . ***Determine the current supplied by each battery in the circuit shown in Fig. 48.***

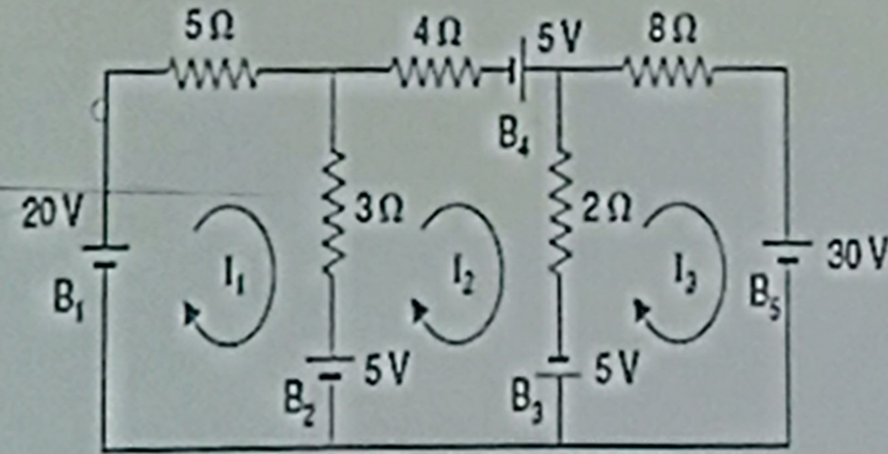


Fig. 48

As there are three meshes, let the three loop currents be as shown in figure 48

Applying Kirchhoff's law to *loop 1*, we get

$$20 - 5I_1 - 3(I_1 - I_2) - 5 = 0$$

or
$$8I_1 - 3I_2 = 15 \qquad ...(i)$$

For *loop 2*, we have

$$-4I_2 + 5 - 2(I_2 - I_3) + 5 + 5 - 3(I_2 - I_1) = 0$$

$$3I_1 - 9I_2 + 2I_3 = -15 \qquad ...(ii)$$

For *loop 3*, we have

$$-8I_3 - 30 - 5 - 2(I_3 - I_2) = 0$$

$$2I_2 - 10I_3 = 35 \qquad ...(iii)$$

Eliminating I_1 from (*i*) and (*ii*), we get

$$63I_2 - 16I_3 = 165 \qquad ...(iv)$$

Solving (*iii*) and (*iv*), we get

$$I_2 = 1.82 \text{ A} \quad \text{and } I_3 = -3.15 \text{ A}$$

(– ve sign means direction of current is *counter-clockwise*)

Substituting the value of I_2 in (*i*), we get

$$I_1 = 2.56 \text{ A}$$

Current through battery B_1 (discharging current) $= I_1 = 2.56$ A. (Ans.)

Current through battery B_2 (charging current) $= I_1 - I_2 = 2.56 - 1.82 = 0.74$ A. (Ans.)

Current through battery B_3 (discharging current) $= I_2 + I_3 = 1.82 + 3.15 = 4.97$ A. (Ans.)

Current through battery B_4 (discharging current) $= I_2 = 1.82$ A. (Ans.)

Current through battery B_5 (discharging current) $= I_3 = 3.15$ A. (Ans.)

Example . *Determine the currents through the different branches of the bridge circuit shown in Fig. 49.*

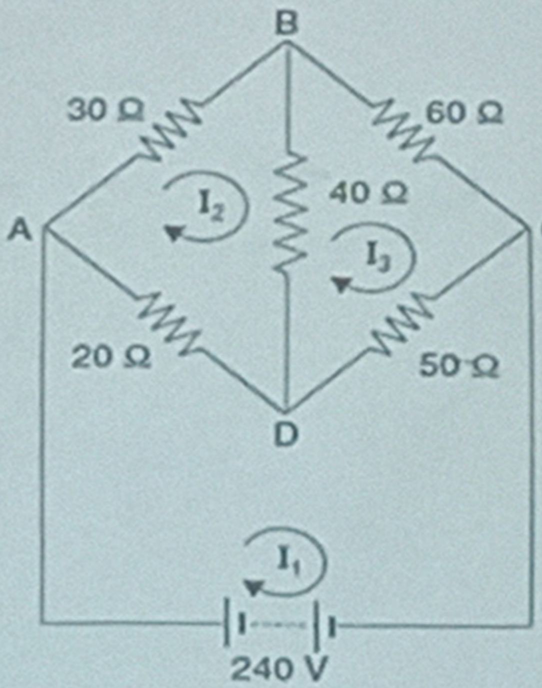


Fig. 49

The equations for the three meshes are :

For loop 1 : $240 - 20(I_1 - I_2) - 50(I_1 - I_3) = 0$

$$-70I_1 + 20I_2 + 50I_3 = -240$$

$$70I_1 - 20I_2 - 50I_3 = 240 \quad ...(i)$$

For loop 2 : $-30I_2 - 40(I_2 - I_3) - 20(I_2 - I_1) = 0$

$$20I_1 - 90I_2 + 40I_3 = 0$$

$$2I_1 - 9I_2 + 4I_3 = 0 \quad ...(ii)$$

For loop 3 $-60I_3 - 50(I_3 - I_1) - 40(I_3 - I_2) = 0$

$$50I_1 + 40I_2 - 150I_3 = 0$$

$$5I_1 + 4I_2 - 15I_3 = 0 \quad ...(iii)$$

Solving these equations, we get

$$I_1 = 6.10\text{ A}, I_2 = 2.56\text{ A}, I_3 = 2.72\text{ A}$$

Current through 30 Ω resistor $= I_2$

$= 2.56$ A (A to B). (Ans.)

Current through 60 Ω resistor $= I_3 = 2.72$ A (B to C). (Ans.)

Current through 20 Ω resistor $= I_1 - I_2 = 6.10 - 2.56 = 3.54$ A (A to D).

Current through 50 Ω resistor $= I_1 - I_3 = 6.10 - 2.72 = 3.38$ A (D to C).

Current through 40 Ω resistor $= I_3 - I_2 = 2.72 - 2.56 = 0.16$ A (D to B).

3. Nodal Voltage Method –

Under this method the following *procedure is adopted :*

1. Assume the voltages of the different independent nodes.
2. Write the equations for each node as per Kirchhoff's current law.
3. Solve the above equations to get the node voltages.
4. Calculate the branch currents from the values of node voltages.

Let us consider the circuit shown in the Fig. 52. *L and M are the two independent nodes* ; *M* can be taken as the reference node. Let the voltage of node *L (with respect to M) be VL.*

Using Kirchhoff's law, we get

$$I_1 + I_2 = I_3$$

Ohm's law gives $\quad I_1 = \dfrac{V_1}{R_1} = \dfrac{(E_1 - V_L)}{R_1}$

$$I_2 = \frac{V_2}{R_2} = \frac{(E_2 - V_L)}{R_2}$$

$$I_3 = \frac{V_3}{R_3} = \frac{V_L}{R_3}$$

$$\frac{E_1 - V_L}{R_1} + \frac{E_2 - V_L}{R_2} = \frac{V_L}{R_3}$$

Rearranging the terms, we get

$$V_L\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right] - \frac{E_1}{R_1} - \frac{E_2}{R_2} = 0$$

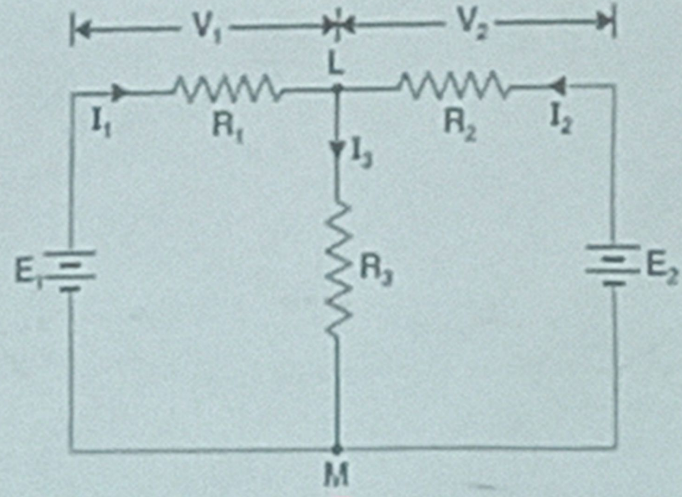


Fig. 52

It should be noted that the above nodal equation contains the following terms :

(i) *The node voltage multiplied by the sum of all conductance's* connected to that node. This* term is positive.

(ii) *The node voltage at the other end of each branch (connected to this node) multiplied by* the conductance of branch. These terms are negative.

— In this method of solving a network the *number of equations required for the solution is* ~~one less than the~~ number of independent nodes *in the network.*

#In general the nodal analysis *yields similar solutions.*

* Conductance (G) is the reciprocal of resistance.

Example.

For the circuit shown in Fig. 53, find the currents through the resistances R3 and R4.

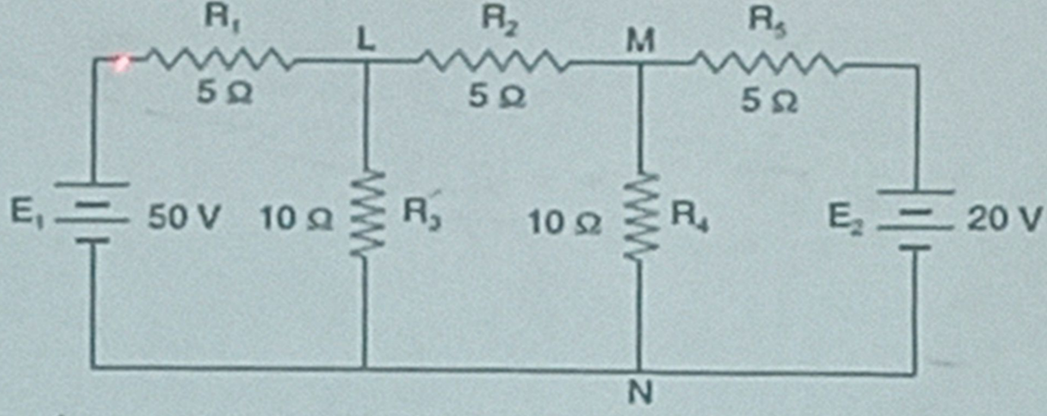


Fig. 53

Let *L, M and N are Independent nodes, and VL and VM are Voltages of nodes L and M with respect to reference node N*

The nodal equations for the nodes *L and M can be written as* :

$$V_L\left[\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right]-\frac{E_1}{R_1}-\frac{V_M}{R_2}=0 \qquad \text{...(i)}$$

$$V_M\left[\frac{1}{R_2}+\frac{1}{R_4}+\frac{1}{R_5}\right]-\frac{E_2}{R_5}-\frac{V_L}{R_2}=0 \qquad \text{...(ii)}$$

Substituting the values in (*i*) and (*ii*) and simplifying, we get

$$V_L\left(\frac{1}{5}+\frac{1}{5}+\frac{1}{10}\right)-\frac{50}{5}-\frac{V_M}{5}=0$$

$$2.5V_L-V_M-50=0$$

$$V_M\left(\frac{1}{5}+\frac{1}{10}+\frac{1}{5}\right)-\frac{20}{5}-\frac{V_L}{5}=0$$

$$2.5V_M-V_L-20=0$$

$$-V_L+2.5V_M-20=0$$

Solving (*iii*) and (*iv*), we get

$$V_L=27.6\text{ V},\ V_M=19.05\text{ V}$$

Current through R_3 $=\dfrac{V_L}{R_3}=\dfrac{27.8}{10}=2.76$ **A. (Ans.)**

Current through R_4 $=\dfrac{V_M}{R_4}=\dfrac{19.05}{10}=1.905$ **A. (Ans.)**

Example .

Find the branch currents in the circuit of Fig. 55 by nodal analysis.

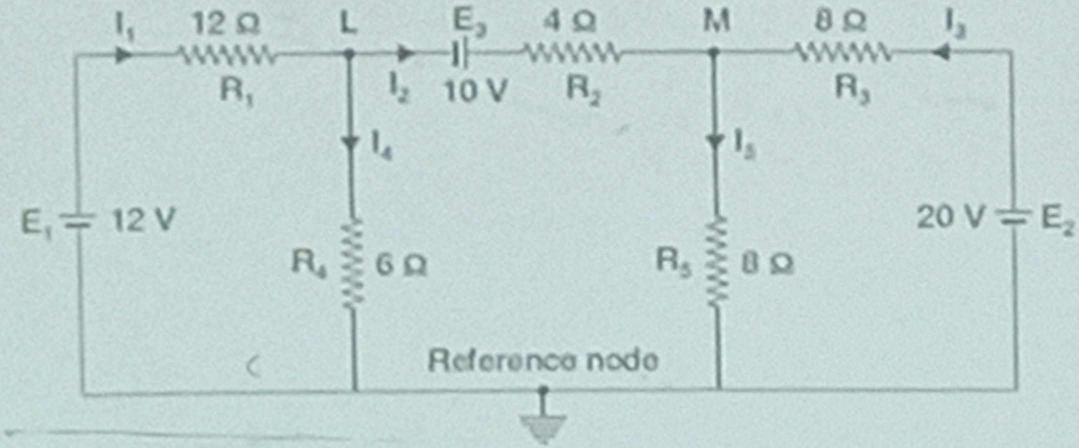


Fig. 55

Equation for Node L :

$$V_L\left[\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_4}\right]-\frac{E_1}{R_1}-\frac{V_M}{R_2}+\frac{E_3}{R_2}=0$$

Substituting the given data, we get

$$V_L\left[\frac{1}{12}+\frac{1}{4}+\frac{1}{6}\right]-\frac{12}{12}-\frac{V_M}{4}+\frac{10}{4}=0$$

$$V_L\left[\frac{1+3+2}{12}\right]-1-\frac{V_M}{4}+\frac{10}{4}=0$$

$$0.5V_L-1-0.25V_M+2.5=0$$

$$2V_L-V_M=-6 \qquad \text{...(i)}$$

Equation for Node M:

$$V_M\left[\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_5}\right]-\frac{E_2}{R_3}-\frac{V_L}{R_2}-\frac{E_1}{R_2}=0$$

$$V_M\left[\frac{1}{4}+\frac{1}{8}+\frac{1}{8}\right]-\frac{20}{8}-\frac{V_L}{4}-\frac{10}{4}=0$$

$$V_M\left[\frac{2+1+1}{8}\right]-\frac{20}{8}-\frac{V_L}{4}-\frac{10}{4}=0$$

$$\frac{V_M}{2}-\frac{20}{8}-\frac{V_L}{4}-\frac{10}{4}=0$$

$$V_M-\frac{V_L}{2}=10 \qquad \text{...(ii)}$$

From eqn. (*i*) and (*ii*), we get

$$V_L = \frac{8}{3} V; \quad V_M = \frac{34}{3} V$$

$$I_1 = \frac{E_1 - V_L}{R_1} = \frac{12 - \frac{8}{3}}{12} = \frac{7}{9} \text{ A. (Ans.)}$$

$$I_2 = \frac{V_L + E_1 - V_M}{R_2} = \frac{\frac{8}{3} + 10 - \frac{34}{3}}{4} = \frac{1}{3} \text{ A. (Ans.)}$$

$$I_3 = \frac{E_2 - V_M}{R_3} = \frac{20 - \frac{34}{3}}{8} = \frac{13}{12} \text{ A. (Ans.)}$$

$$I_4 = \frac{V_L}{6} = \frac{\frac{8}{3}}{6} = \frac{4}{9} \text{ A. (Ans.)}$$

$$I_5 = \frac{V_M}{R_5} = \frac{\frac{34}{3}}{8} = \frac{17}{12} \text{ A. (Ans.)}$$

Example . Determine the current's in all branches of the network shown in Fig. E2.7 by nodal method.

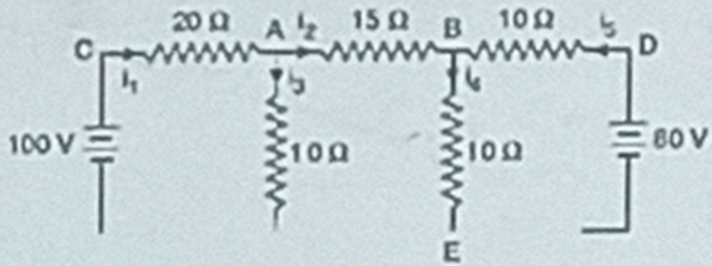


Fig. E2.7

Solution. Taking node *E* as reference and since the potentials of nodes *C* and *D* with respect to *E* are known hence only two independent nodes are *A* and *B* where the voltages be V_A and V_B respectively.

Writing nodal equation at node A

$$\frac{100-V_A}{20}-\frac{V_A-V_B}{15}-\frac{V_A}{10}=0$$

or

$$5=\frac{V_A}{20}+\frac{V_A}{15}-\frac{V_B}{15}+\frac{V_A}{10}$$

$$25=\frac{V_A}{4}+\frac{V_A}{3}-\frac{V_B}{3}+\frac{V_A}{2}$$

$$300=3\,V_A+4V_A-4V_B+6V_A$$

or

$$13V_A-4V_B=300 \qquad (1)$$

At node B

$$\frac{V_A - V_B}{15} + \frac{80 - V_B}{10} = \frac{V_B}{10}$$

$$\frac{V_A - V_B}{3} + \frac{80 - V_B}{2} = \frac{V_B}{2}$$

$$2V_A - 2V_B + 240 - 3V_B = 3V_B$$

$$2V_A - 8V_B = -240$$

$$-V_A + 4V_B = 120 \quad \text{...(2)}$$

Adding equation (1) and (2) we have

$$12V_A = 420$$

$$V_A = 35 \text{ volts}$$

Hence $\quad -35 + 4V_B = 120$

or $\quad 4V_B = 155$

or $\quad V_B = 38.75$ volts

Hence current $\quad i_1 = \frac{100 - 35}{20} = \frac{65}{20} = 3.25 \text{ A}$

$$i_2 = \frac{35 - 38.75}{15} = -\frac{3.75}{15} = -0.25$$

The value of i_2 is coming out to be negative which means in the actual circuit the direction of current should be opposite to what is shown in Fig. E2.7

Hence $\quad i_3 = i_1 + i_2 = 3.25 + 0.25 = 3.5$

$$i_4 = \frac{V_B}{10} = \frac{38.75}{10} = 3.875 \text{ A}$$

and hence $\quad i_5 = i_2 + i_4 = 0.25 + 3.875$

Source Conversion –

- A given voltage source with a series resistance can be converted into equivalent current source with a parallel resistance. How ?

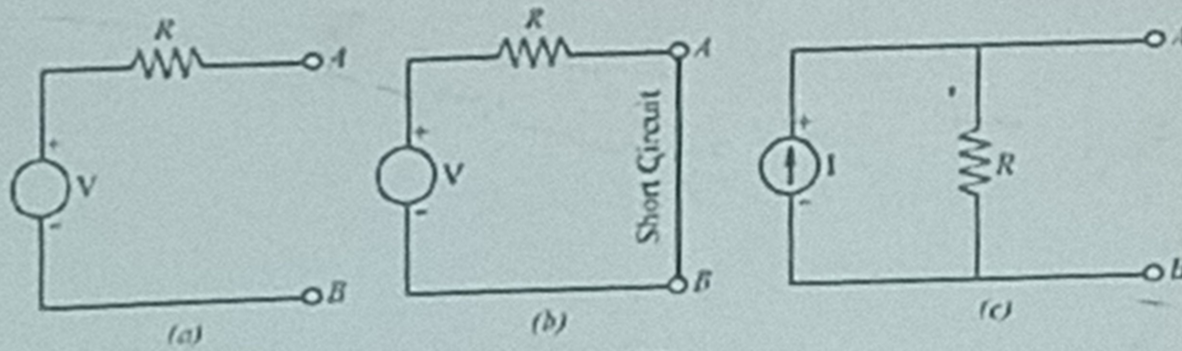


Fig. 2.75

Suppose, we want to convert the voltage source of Fig. 2.75 *(a) into an equivalent current source.*

(1) First, we will find the value of current supplied by the source when a 'short' is put across in terminals *A and B as shown in* Fig. 2.75 *(b). This current is* $I = V/R$

(2) A current source supplying this current *I and having the same resistance* R *connected in parallel* with it represents the equivalent source. It is shown in Fig.2.75(c)

(3) Conversely, a current source with a parallel resistance can be converted into a voltage source with a series resistance

Example . Convert the voltage source of Fig. 2.76 (a) into an equivalent current source.

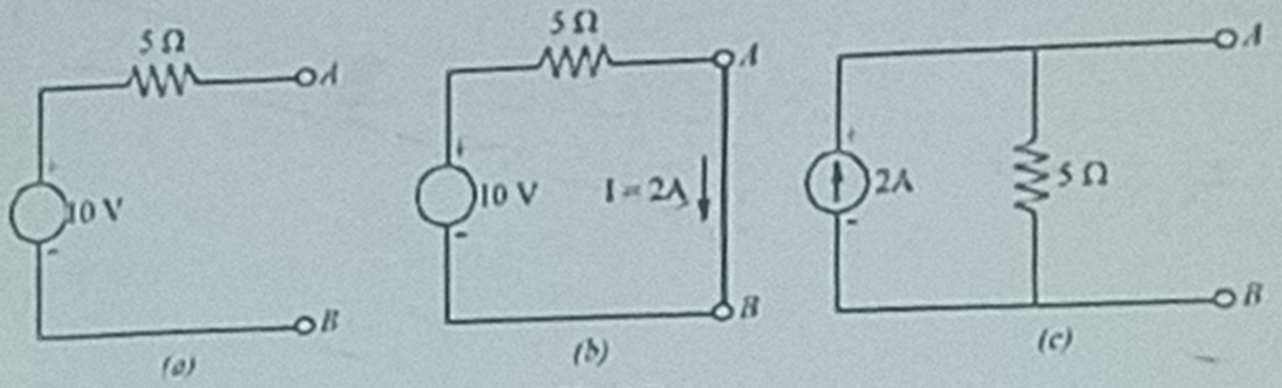


Fig. 2.76

As shown in Fig 2.76 (b), current obtained by putting a short across terminals A and B is 10/5 = 2 A.

Hence, the equivalent current source is as shown in Fig. 2.76 (c).

Example . Find the equivalent voltage source for current source in Fig 2.77(a)

The open-circuit voltage across terminals *A and B* *Voc = drop across R* = 5 x 2 =10 V

Hence, voltage source has a voltage of 10 V and the same resistance of 2 ohm connected in series.

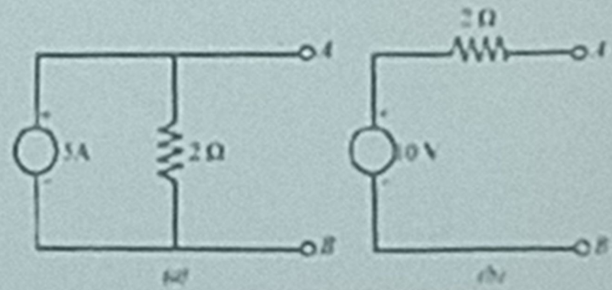


Fig. 2.77

Example Use Source Conversion technique - to find the load current in the circuit of Fig. 2.78 (a).

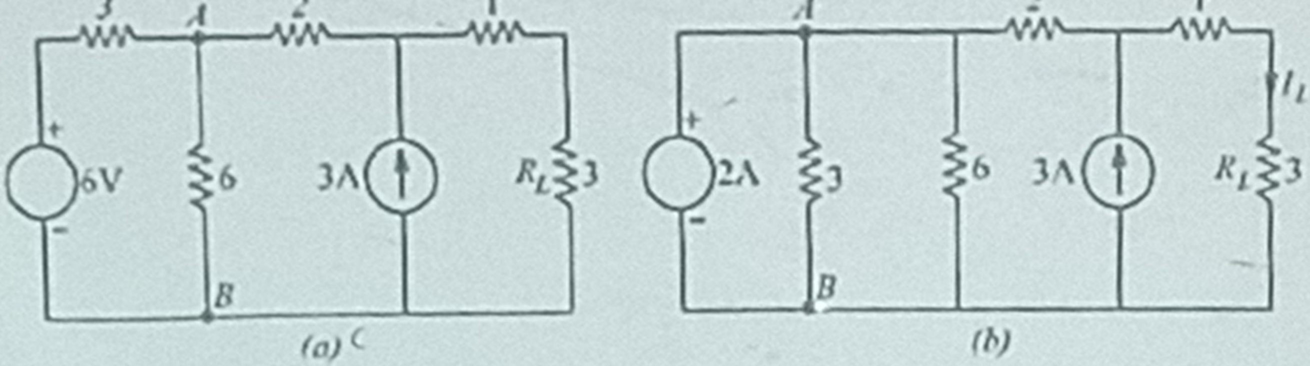


Fig. 2.78

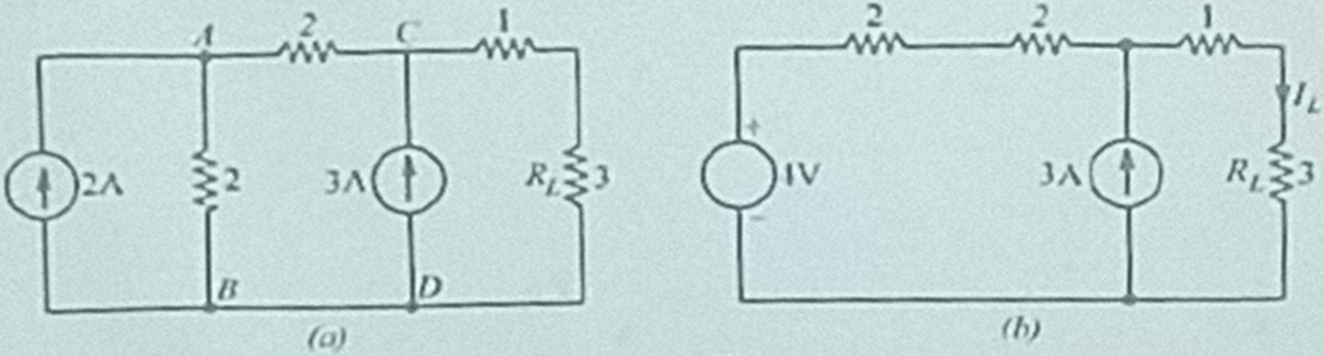


Fig. 2.79

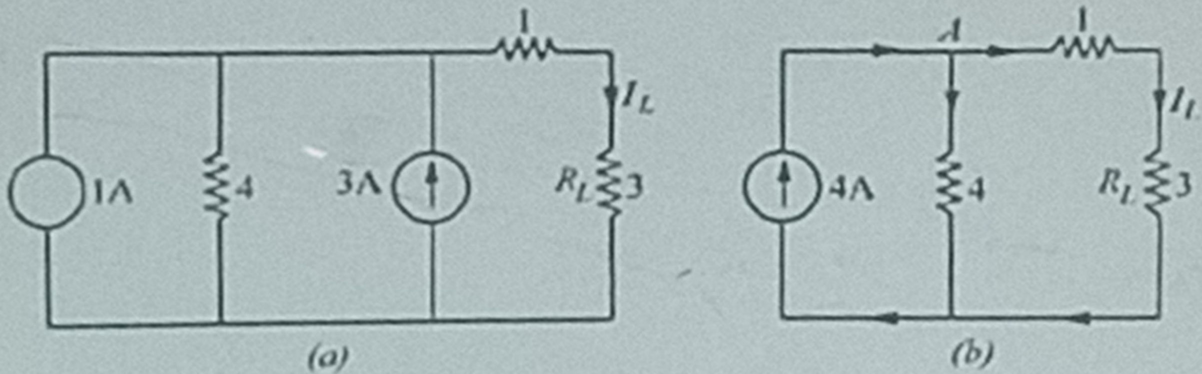


Fig. 2.80

Note That –

➢The two current sources cannot be combined together because of the 2 Ω resistance present between points *A and C*.

➢To remove this hurdle, we convert the 2 A current source into the equivalent 4 V voltage source as shown in Fig. 2.79

➢Two current sources can be combined into a single 4-A source as shown in Fig.2.80(b) because there is no resistance between them

➢ Finally At point A 4 amps current is divided in two equal part therefore,
Load current = 2 amps [Answer]

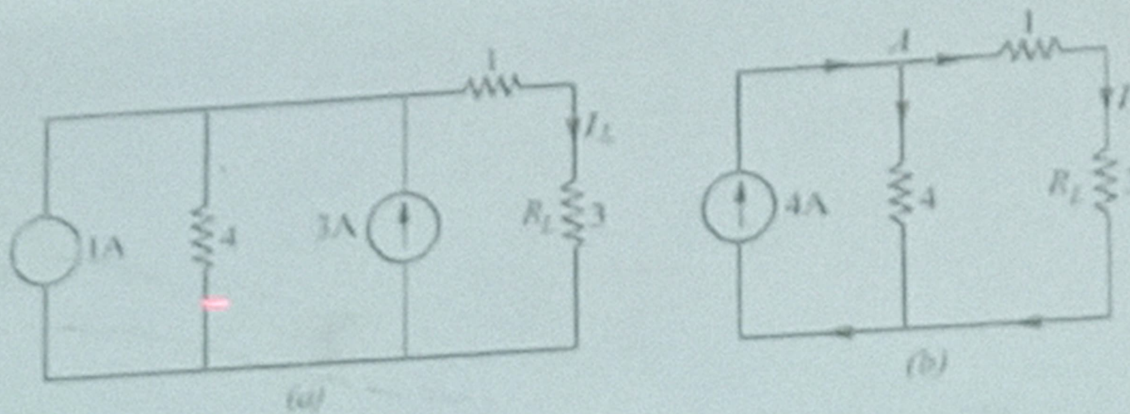


Fig. 2.80

Note That –

➢The two current sources cannot be combined together because of the 2 Ω resistance present between points *A and C*.

➢To remove this hurdle, we convert the 2 A current source into the equivalent 4 V voltage source as shown in Fig. 2.79

➢Two current sources can be combined into a single 4-A source as shown in Fig.2.80(b) because there is no resistance between them

➢ Finally At point A 4 amps current is divided in two equal part therefore,

Load current = 2 amps [Answer]

Example Calculate the direction and magnitude of the current through the 5 Ω resistor between points A and B of Fig. 2.81 (a) by using nodal voltage method. Mistake- consider 4 ohm in place of 3 ohm to get the solution

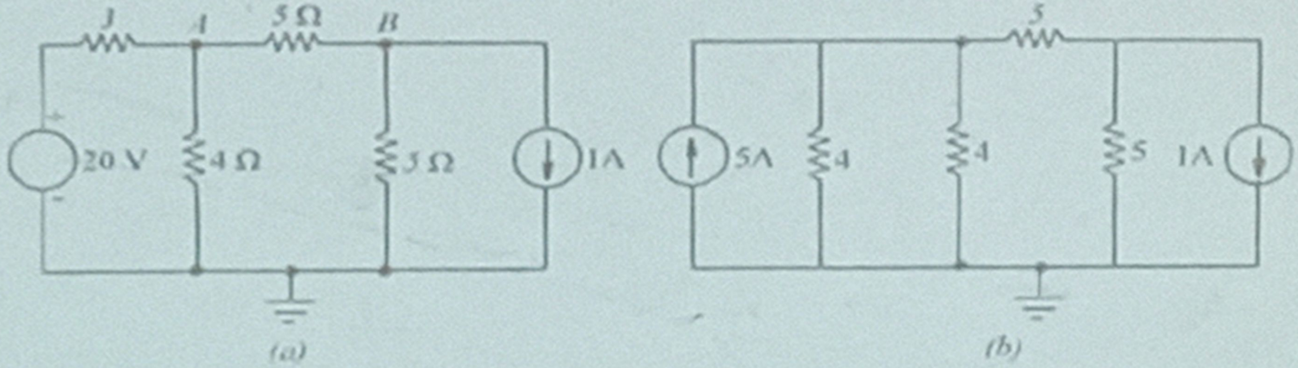


Fig. 2.81

- Convert the voltage source into the current sources as shown in Fig. 2.81 *(b)*.
- Combine two parallel resistances of 4 Ω each to get single resistance of 2 Ω [Fig. 2.82 *(a)*].

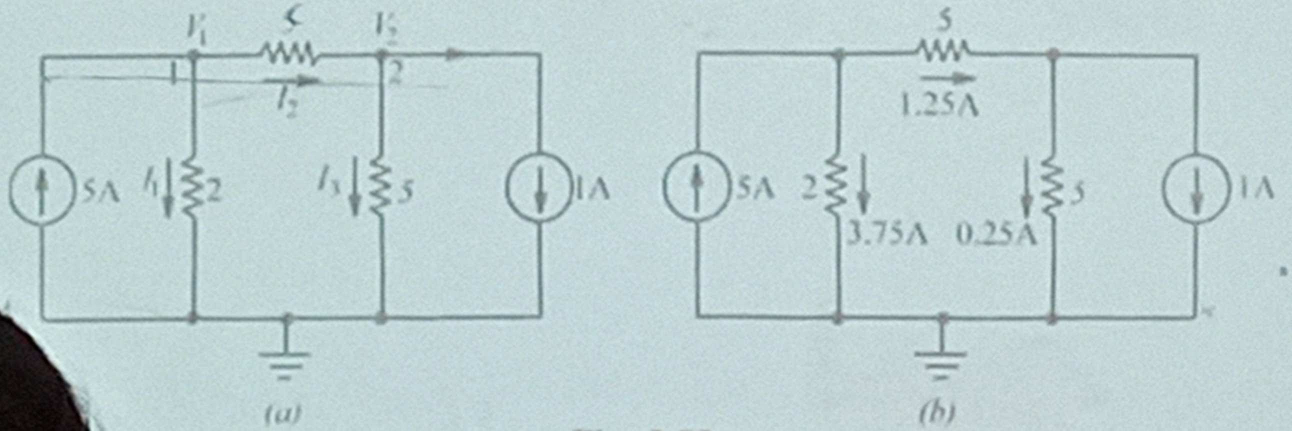


Fig. 2.82

➢ Let the current directions be as indicated in Fig 2.82

Applying the nodal rule to nodes 1 and 2, we get

Node 1

$$V_1\left(\frac{1}{2}+\frac{1}{5}\right)-\frac{V_2}{5} = 5 \quad \text{or} \quad 7V_1 - 2V_2 = 50$$

Node 2

$$V_2\left(\frac{1}{5}+\frac{1}{5}\right)-\frac{V_1}{5} = -1 \quad \text{or} \quad V_1 - 2V_2 = 5$$

Solving for V_1 and V_2, we get $V_1 = \dfrac{15}{2}$ V and $V_2 = \dfrac{5}{4}$ V.

$$\therefore \quad I_2 = \frac{V_1 - V_2}{5} = \frac{15/2 - 5/4}{5} = 1.25 \text{ A}$$

Similarly, $I_1 = V_1/2 = 15/4 = 3.75$ A; $I_3 = V_2/5 = 5/20 = 0.25$ A.

The actual current distribution remains as shown in Fig. 2.82 *(b)*.

****

Example . Calculate the effective resistance of the following combination of resistances and the voltage drop across each resistance when a P.D. of 60 V is applied between points A and B.

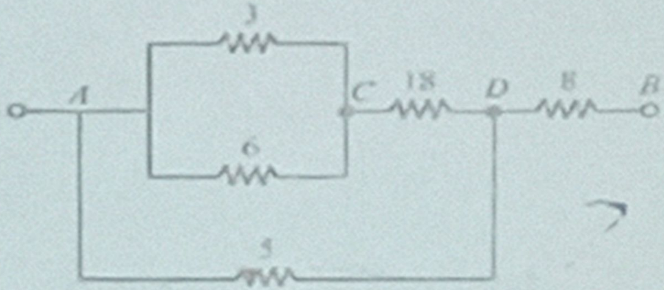


Fig. 1.17

Solution: Resistance between A and C (Fig. 1.17).

$$= 6 \| 3 = 2\,\Omega$$

Resistance of branch ACD $= 18 + 2 = 20\,\Omega$

Now, there are two parallel paths between points A and D of resistances $20\,\Omega$ and $5\,\Omega$.

Hence, resistance between A and $D = 20 \| 5 = 4\,\Omega$

$\therefore$ Resistance between A and $B = 4 + 8 = 12\,\Omega$

Total circuit current $= 60/12 = 5$ A

C

Current through $5\,\Omega$ resistance $= 5 \times \frac{20}{25} = 4$ A

Current in branch ACD $= 5 \times \frac{5}{25} = 1$ A

$\therefore$ P.D across $3\,\Omega$ and $6\,\Omega$ resistors $= 1 \times 2 = 2$ V

P.D across $18\,\Omega$ resistors $= 1 \times 18 = 18$ V

P.D across $5\,\Omega$ resistors $= 4 \times 5 = 20$ V

P.D across $8\,\Omega$ resistors $= 5 \times 8 = 40$ V

Example. *Find current through 4 resistance.*

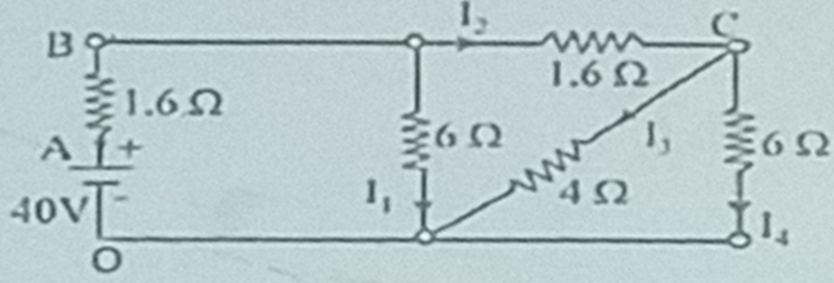


Fig. 1.26

Solution. Simplifying the series-parallel combinations, and solving the circuit, the source current is 10 amp. With respect to 0, $V_A = 40$, $V_B = 40 - 16 = 24$ volts.

$I_1 = 4$ amp, hence $I_2 = 6$ amp

$V_C = V_B - I_2 \times 1.6 = 24 - 9.6 = 14.4$ volts

$I_3 = 14.4/4 = 3.6$ amp, which is the required answer.

Further *current in 6 ohm resistor (I4) =2.4 amp.*

****

Superposition Theorem-

Statement –

In a linear network, containing more than one source of e.m.f., the current which flows at any point is the sum of all the currents which would flow at that point if each generator were considered separately and all the other generators replaced for the time being by resistances equal to their internal resistances.

Key point – (1) If internal resistances of sources are unknown then independent voltage sources must be replaced by short circuit while independent current sources should be replaced by open circuit

(2) Superposition theorem does not apply to power as power is proportional to the square of the current which is not a linear function

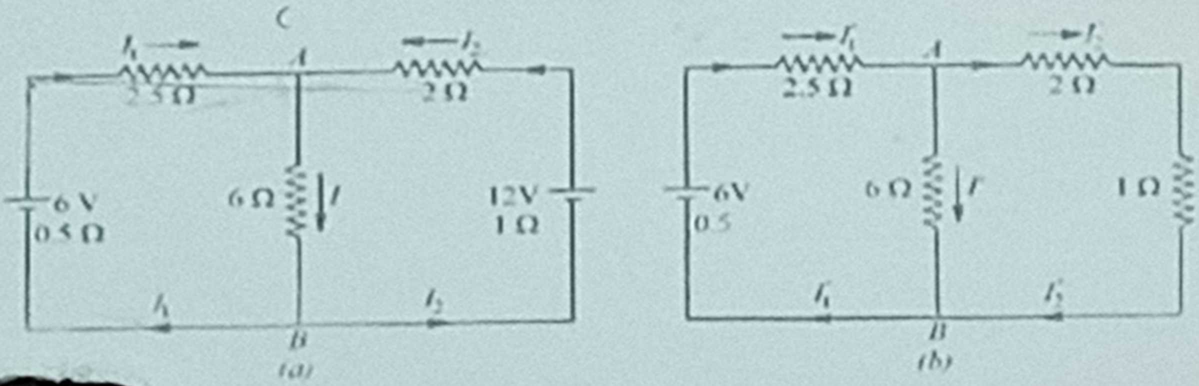
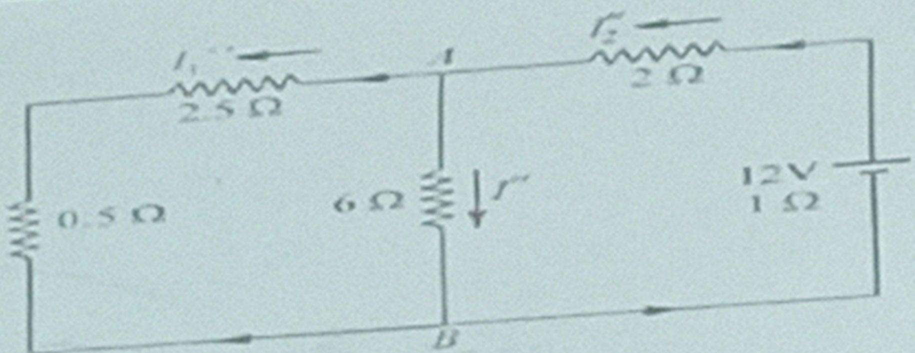


Fig. 2.95



Obviously, $I_1 = I_1' - I_1''$, $I_2 = I_2'' - I_2'$, $I = I' + I''$.

Steps To apply Superposition Theorem-

Step 1 : Select a single source acting alone. Short the other voltage sources and open the current sources, if internal resistances are not known. If known, replace them by their internal resistances.

Step 2 : Find the current through or the voltage across the required element, due to the source under consideration, using a suitable network simplification technique.

Step 3 : Repeat the above two steps for all the sources

Step 4 : Add the individual effects produced by individual sources, to obtain the total current in or voltage across the element.

Example- Use the superposition theorem to calculate the current in branch PQ of the circuit shown in Figure 2.64

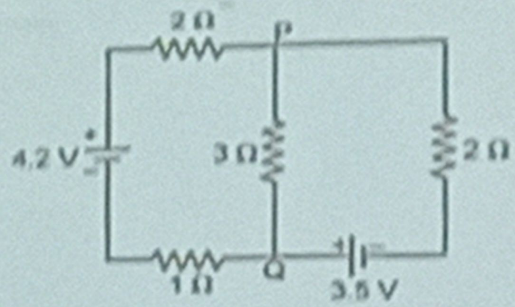


Fig. 2.64

Step 1 : Let us consider 4.2 V, replacing other by short circuit.

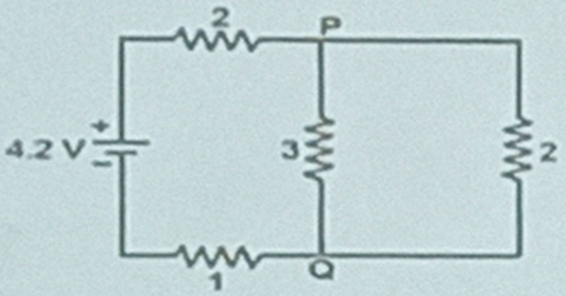


Fig. 2.64 (a)

The resistances 3 Ω and 2 Ω are in parallel

$$\therefore \quad 3 \parallel 2 = \frac{3\times 2}{3+2} = 1.2\ \Omega$$

$$\therefore \quad I = \frac{4.2}{(2+1.2+1)} = 1\ A$$

Now we want I_{PQ}, hence using current division formula,

$$I'_{PQ} = 1\ A \times \frac{2}{2+3} = 0.4\ A \downarrow$$

... due to 4.2 V alone

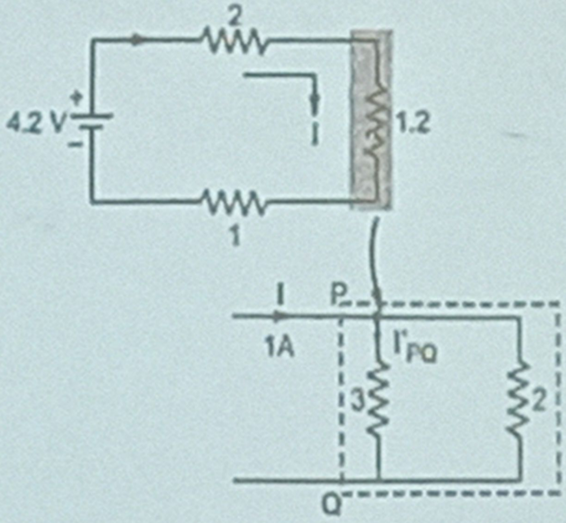


Fig. 2.64 (b)

Step 2 :

Now consider 3.5 V source, replacing other by a short circuit.

The resistances 2 and 1 are in series hence

$$2 \text{ series } 1 = 2 + 1 = 3\,\Omega$$

The resistances 3 and 3 are in parallel.

$$\therefore \quad 3 \parallel 3 = \frac{3\times 3}{3+3} = 1.5\,\Omega$$

$$I = \frac{3.5}{(2+1.5)} = 1\text{ A}$$

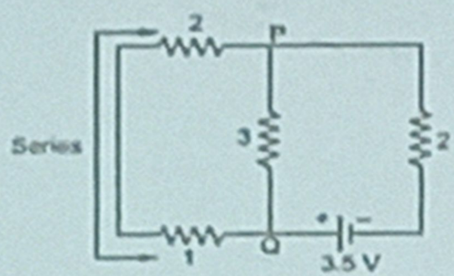


Fig. 2.64 (c)

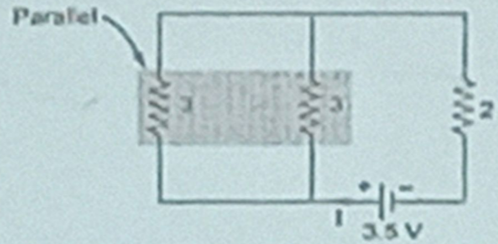


Fig. 2.64 (d)

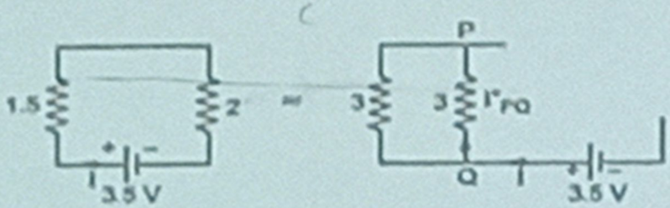


Fig. 2.64 (e)

But we want I_{PQ}, hence using current division formula we get,

$$I''_{PQ} = 1\times\frac{3}{(3+3)}$$

$$= 1\times\frac{3}{(3+3)}$$

$$= 0.5 \uparrow \text{A}$$

... due to 3.2 V alone

Hence total current through PQ branch

$$= 0.4\text{ A}\downarrow + 0.5\text{ A}\uparrow = 0.1\text{ A}\uparrow$$

ANSWER

Exercise- In the circuit shown Find the current through branch AB superposition theorem.

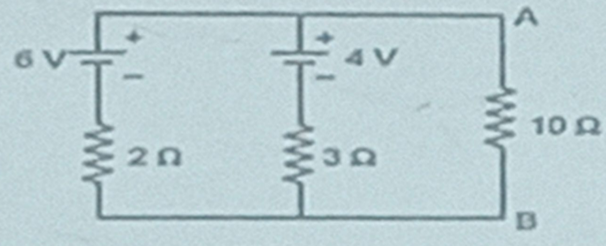


Fig. 2.67

Solution : Step 1 : Consider 6 V source alone

Now, resistances 10 Ω and 3 Ω are in parallel. Hence total current, I is

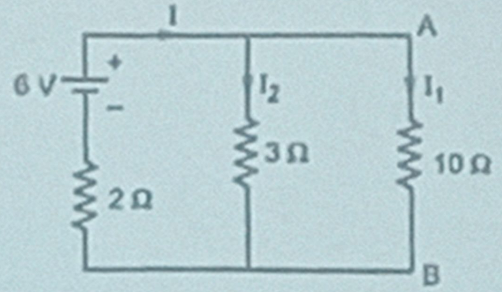


Fig. 2.67 (a)

$$I = \frac{6}{2+(3||10)} = \frac{6}{2+\left(\frac{3\times10}{3+10}\right)} = \frac{6}{2+2.307}$$

$$\therefore \qquad I = 1.3928 \text{ A}$$

As per current distribution in parallel branches,

$$I_1 = I\times\frac{3}{3+10} = \frac{1.3928\times3}{13} = 0.3214 \text{ A}\downarrow \quad \text{... (6 V alone)}$$

This is I_{AB} due to 6 V battery alone.

Step 2 : Consider 4 V battery alone.

Now, the resistances 2 Ω and 10 Ω are in parallel. Hence, current I can be obtained as,

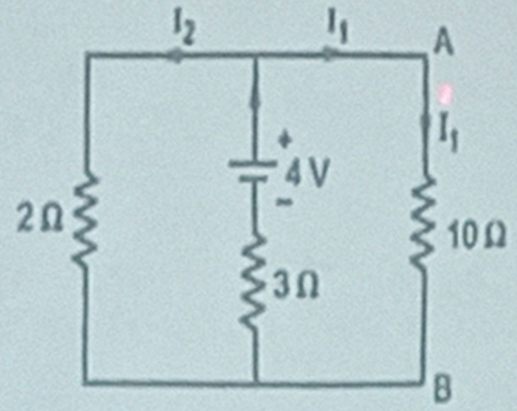


Fig. 2.67 (b)

$$I = \frac{4}{3+(2\|10)} = \frac{4}{3+\left(\frac{2\times10}{2+10}\right)}$$

$$= \frac{4}{3+1.67} = 0.8571\ \text{A}$$

According to current distribution in parallel branches,

$$I_1 = I\times\frac{2}{(2+10)} = 0.8571\times\frac{2}{12}$$

$$= 0.1428\ \text{A}\downarrow \qquad \text{... (4 V alone)}$$

This is I_{AB} due to 4 V battery alone.

According to Superposition theorem,

$$\text{Total } I_{AB} = 0.3214\ \text{A}\downarrow + 0.1428\ \text{A}\downarrow = 0.4642\ \text{A}\downarrow$$ Answer

Example- Calculate current through 15 ohm resistor using Kirchhoff's law and verify your answer using superposition theorem as well. The circuit is shown in Fig 2.65

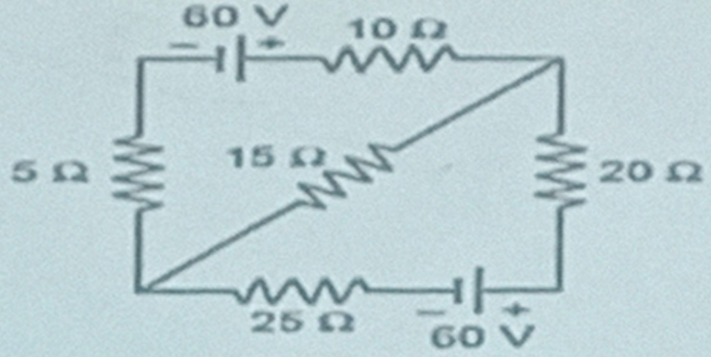


Fig. 2.65

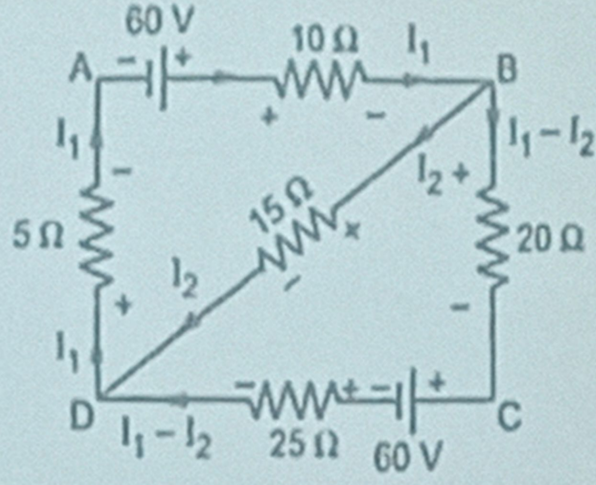
Solution : To use Kirchhoff's law, let us indicate various currents as shown in the Fig. 2.65 (a).

Consider loop ABDA,

$$+60 - 10\,I_1 - 15\,I_2 - 5\,I_1 = 0$$

$$\therefore \quad 15\,I_1 + 15\,I_2 = 60$$

$$\therefore \quad I_1 + I_2 = 4 \qquad \ldots (1)$$



Consider loop BDCB,

$$-15\, I_2 + 25\,(I_1 - I_2) + 60 + 20\,(I_1 - I_2) = 0$$

$$-45\, I_1 + 60\, I_2 = 60$$

$$\therefore \quad -3\, I_1 + 4 I_2 = 4$$

Add [3 × equation (1)] to (2),

$$-3\, I_1 + 4 I_2 = 4$$

$$+3\, I_1 + 3 I_2 = 12$$

$$\therefore \quad 7\, I_2 = 16$$

$$\therefore \quad I_2 = 2.2857 \text{ A}$$

So current through 15 Ω resistance is 2.2857 A from B to D.

Now let us use Superposition Theorem, consider upper 60 V battery alone the lower battery is replaced by short circuit as shown in the Fig. 2.65 (b).

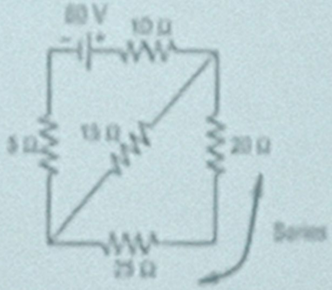
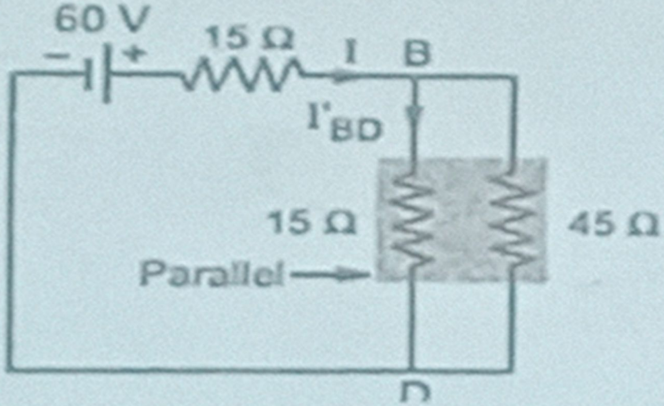


Fig. 2.65 (b)

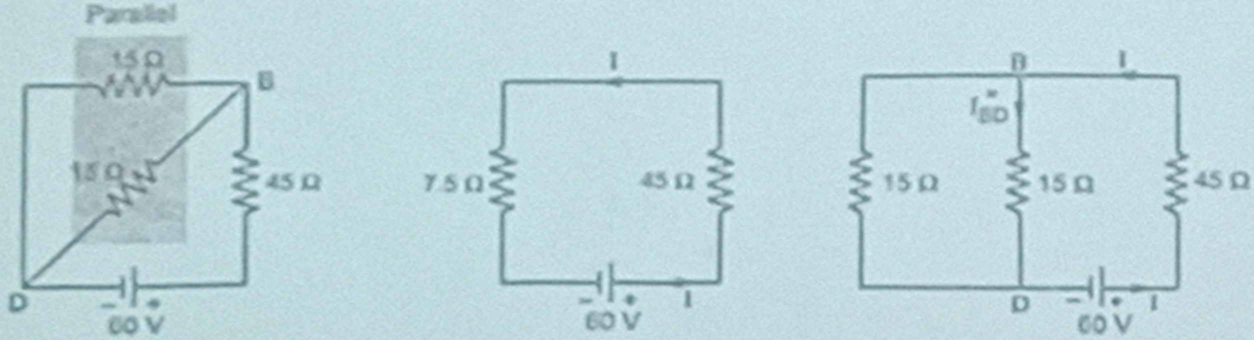


Hence the total current is,

$$I = \frac{60}{15+(15||45)} = \frac{60}{15+11.25} = 2.2857 \text{ A}$$

$$\therefore \quad I'_{BD} = I \times \frac{45}{(45+15)} = 2.2857 \times \frac{45}{60}$$

Consider other 60 V battery now, hence circuit reduces as,



$$\therefore \text{ Total current } \quad I = \frac{60}{45+(15||15)} = \frac{60}{45+7.5} = 1.1428 \text{ A}$$

$$\therefore \quad I''_{BD} = I \times \frac{15}{15+15} = 1.1428 \times \frac{1}{2} = 0.5714 \text{ A} \downarrow$$

$\therefore \quad I''_{BD} = 0.5714 \text{ A} \downarrow$ due to other 60 V battery.

Hence according to Superposition Theorem,

$$I_{15\Omega} = 1.7142 + 0.5714 = 2.2857 \text{ A} \downarrow$$

This is same as calculated by Kirchhoff's laws.

******Answer Verified using Superposition theorem *******

Example . *Using Superposition theorem, find the value of the output voltage Vo in the curcuit of Fig. 2.107*

(a) When 4 A and 6 V sources are killed*

As shown in Fig. 2.108 (a), 4 A source has been replaced by an open circuit and 6 V source by a short-circuit. Using the current-divider rule, we find current I_1 through the 2 Ω resistor = 6 × 1/(1 + 2 + 3) = 1 A ∴ V_{01} = 1 × 2 = 2 V.

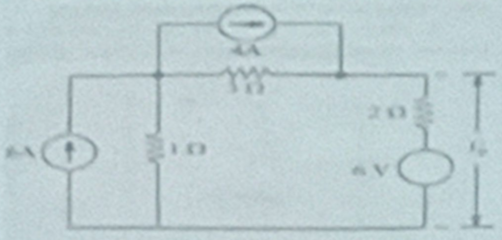
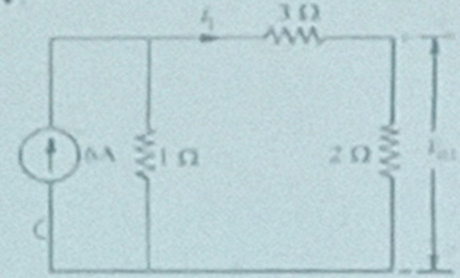
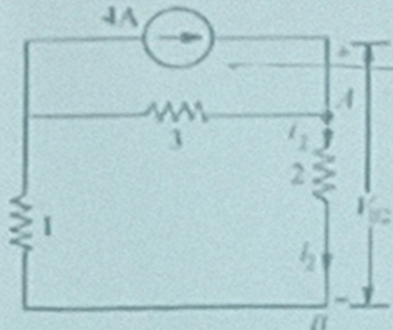


Fig. 2.107

(b) When 6 A and 6 V sources are killed



(a)

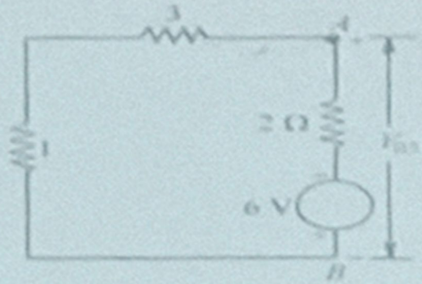


(b)

There are two parallel paths across current source of 4 amps. One has R=3 ohm and other has R=(2+1) ohm. Therefore Current divides equally therefore, V02 = 4V

(c) When 6 A and 4 A sources are killed

As shown in Fig. 2.108 (c), drop over 2 Ω resistor = 6 × 2/6 = 2 V. The potential of point B with respect to point A is = 6 − 2 = + 4 V. Hence, V_{AB} = − 4 V.



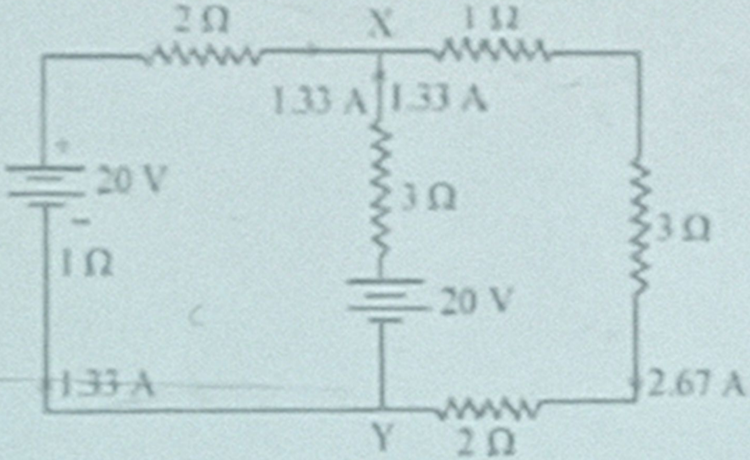
(c)

According to Superposition theorem, we have

[illegible]

Answer

Example . Find the current flowing in the branch XY of the circuit shown in Fig. by superposition theorem and verify whether marked values of currents are correct. ?



Thevenins Theorem -

It provides a mathematical technique for replacing a given network, as viewed from two output terminals, by *a single voltage source* with *a series resistance as shown in Fig 2.217*

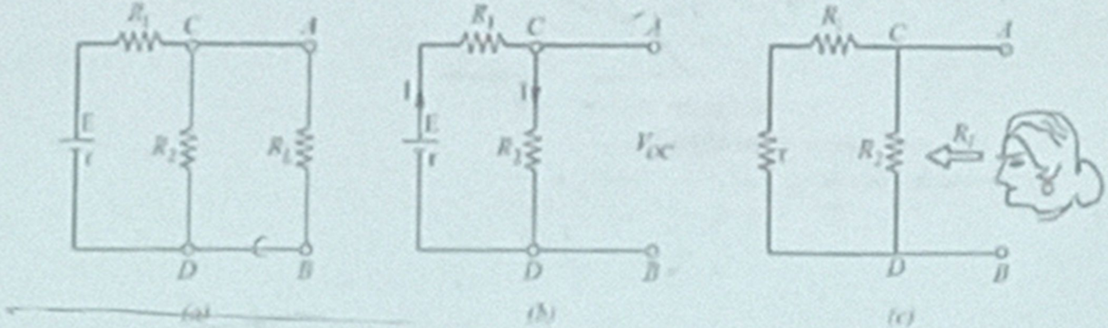


Fig. 2.127

Suppose, it is required to find current flowing through load resistance R_L, as shown in Fig. 2.127 (a). We will proceed as under :

1. Remove R_L from the circuit terminals A and B and redraw the circuit as shown in Fig. 2.127 (b). Obviously, the terminals have become open-circuited.
2. Calculate the open-circuit voltage V_{oc} which appears across terminals A and B when they are open i.e. when R_L is removed.

   As seen, V_{oc} = drop across $R_2 = IR_2$ where I is the circuit current when A and B are open.

   $I = \dfrac{E}{R_1 + R_2 + r}$ $\quad\therefore\quad$ $V_{oc} = IR_2 = \dfrac{ER_2}{R_1 + R_2 + r}$ [r is the internal resistance of battery]

   It is also called 'Thevenin voltage' V_{th}.

3. Now, imagine the battery to be removed from the circuit, leaving its internal resistance r behind and redraw the circuit, as shown in Fig. 2.127 (c). When viewed *inwards* from terminals A and B, the circuit consists of two parallel paths : one containing R_2 and the other containing $(R_1 + r)$. The equivalent resistance of the network, as viewed from these terminals is given as

$$R = R_2 \| (R_1 + r) = \frac{R_2(R_1 + r)}{R_2 + (R_1 + r)}$$

This resistance is also called, *Thevenin resistance R_{th} (though, it is also sometimes written as R_i or R_o).

Consequently, as viewed from terminals A and B, the whole network (excluding R_1) can be reduced to a single source (called Thevenin's source) whose e.m.f. equals V_{th} (or V_{AB}) and whose internal resistance equals R_{th} (or R_i) as shown in Fig. 2.128.

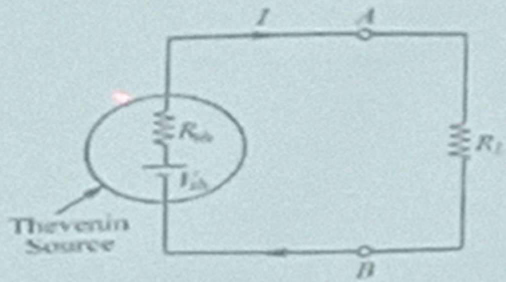


Fig. 2.128

4. R_L is now connected back across terminals A and B from where it was temporarily removed earlier. Current flowing through R_L is given by

$$I = \frac{V_{th}}{R_{th} + R_L}$$

Hence, Thevenin's theorem, as applied to d.c. circuits, may be stated as under :

The current flowing through a load resistance R_L connected across any two terminals A and B of a linear, active bilateral network is given by V_{oc} || $(R_i + R_L)$ where V_{oc} is the open-circuit voltage (i.e. voltage across the two terminals when R_L is removed) and R_i is the internal resistance of the network as viewed back into the open-circuited network from terminals A and B with all voltage sources replaced by their internal resistance (if any) and current sources by infinite resistance.

How to Thevenize a Given Circuit?

1. Temporarily remove the resistance (called load resistance R_L) whose current is required.
2. Find the open-circuit voltage V_{oc} which appears across the two terminals from where resistance has been removed. It is also called Thevenin voltage V_{th}.
3. Compute the resistance of the whose network as looked into from these two terminals after all voltage sources have been removed leaving behind their internal resistances (if any) and current sources have been replaced by open-circuit *i.e.* infinite resistance. It is also called Thevenin resistance R_{th} or T_i.
4. Replace the entire network by a single Thevenin source, whose voltage is V_{th} or V_{oc} and whose internal resistance is R_{th} or R_i.
5. Connect R_L back to its terminals from where it was previously removed.
6. Finally, calculate the current flowing through R_L by using the equation.

$$I = V_{th}/(R_{th} + R_L) \quad \text{or} \quad I = V_{oc}/(R_i + R_L)$$

Example- By applying Thevenin's theorem find the current in the load resistance of 15 ohms in the network of Fig 2.132 (a)

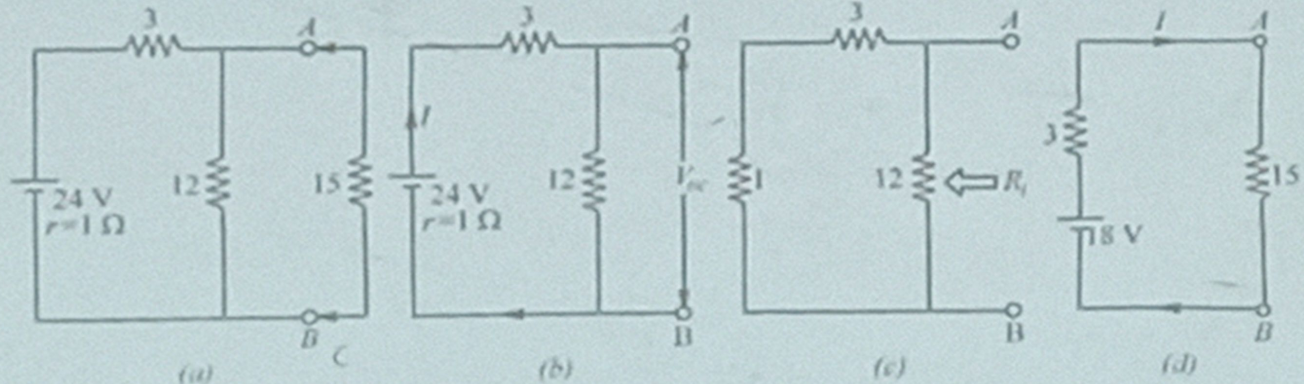


Fig. 2.132

Solution. (i) Current in the network before load resistance is connected [Fig. 2.132 (a)]

$$= 24/(12 + 3 + 1) = 1.5\ \text{A}$$

$\therefore$ voltage across terminals $AB = V_{oc} = V_{th} = 12 \times 1.5 = 1.8\ \text{V}$

Hence, so far as terminals A and B are concerned, the network has an e.m.f. of 18 volt (and not 24 V).

(ii) There are two parallel paths between points A and B. Imagine that battery of 24 V is removed but not its internal resistance. Then, resistance of the circuit as looked into from point A and B is [Fig. 2.132 (c)]

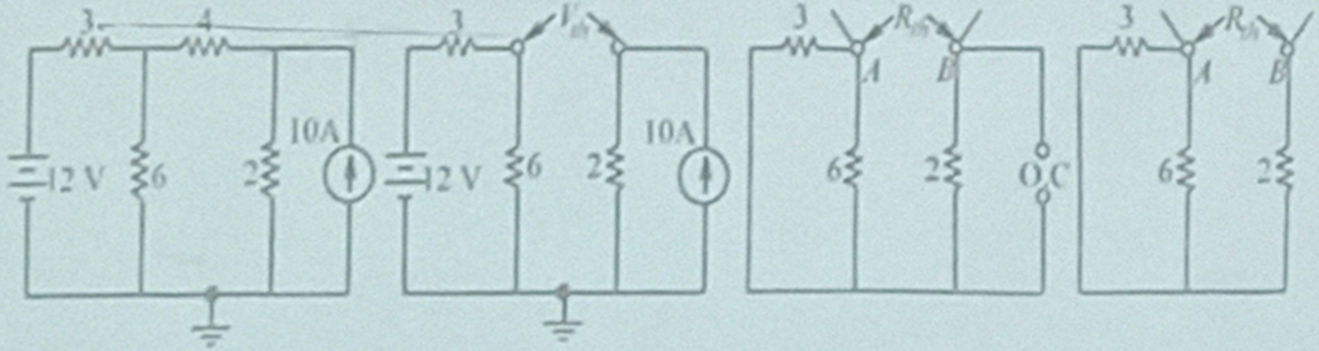
$$R_i = R_{ab} = 12 \times 4/(12 + 4) = 3\ \Omega$$

(*iii*) When load resistance of 15 Ω is connected across the terminals, the network is reduced to the structure shown in Fig. 2.132 (*d*).

$$I = V_{oc}/(R_{th} + R_L) = 18/(15 + 3) = 1\ \text{A}$$

Example-Using Thevenin theorem, calculate the current flowing through the 4 Ω-resistor of Fig. 2.133(a).

c



(a) (b) (c) (d)

Fig. 2.133

(i) Finding Vth-

Hence, drop across 6 Ω resistor = 12 × 6/(3 + 6) = 8 V.

V_A = 8 V with respect to the common ground

$V_{th} = V_{BA} = V_B - V_A = 20 - 8 = 12$ V—with B at a higher potential

(ii) Finding Rth-

Loking from A &B termial 6and 3 are in parallel therefore 6x3 / 9 =2 ohm. Now this is in series with 2 ohm resistor Therefore Thevinins equivelent resistance =4 ohm as shown below. In Fig 2.134

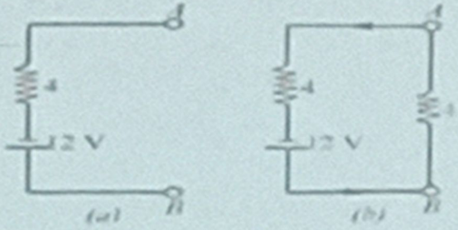


Fig. 2.134

$I = 12/(4 + 4) = 1.5$ A—from B to A Answer

Example *For the circuit shown in Fig. 2.135 (a), calculate the current in the 10 ohm resistance. Use Thevenin's theorem only.*

Solution. When the 10 Ω resistance is removed, the circuit becomes as shown in Fig. 2.135 (b).

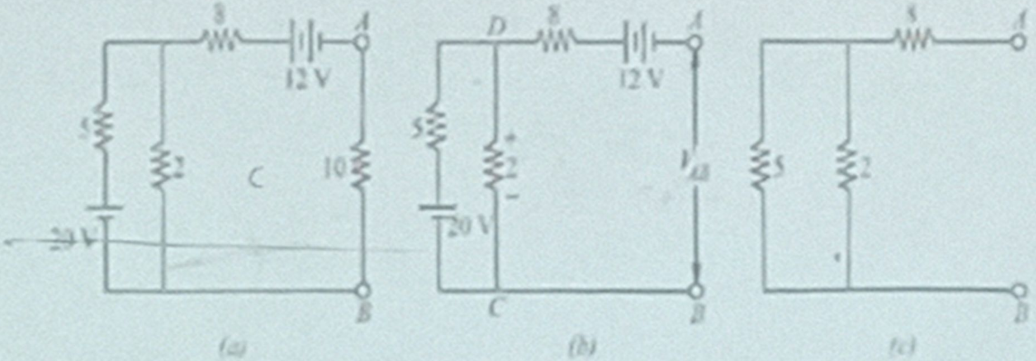


Fig. 2.135

Drop in 2 ohm resistance =20 x 2/(2 + 5) =5.71 V with the polarity as shown in figure.2.135 (b)

$$V_{AB} = V_{th} = +5.71 - 12 = -6.29 \text{ V}$$

The negative sign shows that point A is negative with respect to point B or which is the same thing, point B is positive with respect to point A.

For finding $R_{AB} = R_{th}$, we replace the batteries by short-circuits as shown in Fig. 2.128 (c).

$$\therefore \quad R_{AB} = R_{th} = 8 + 2 \parallel 5 = 9.43 \ \Omega$$

Hence, the equivalent Thevenin's source with respect to terminals A and B is an shown in Fig. 2.136. When 10 Ω resistance is reconnected across A and B, current through it is $I = 6.24/9.43 + 10)$ $= 0.32$ A.

Example - *Using Thevenin's theorem, calculate the p.d. across terminals A and B in Fig. 2.137 (a)*

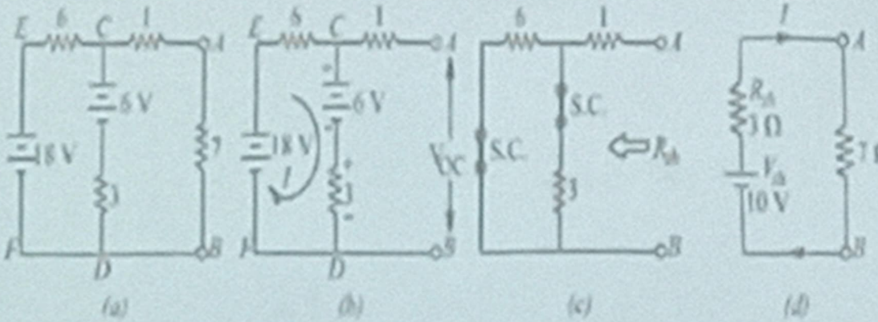


Fig. 2.137

Answer = next slide

Steps to apply maximum power transfer theorem-

Step 1 : Calculate Thevenin's voltage V_{TH} or Norton's current I_N.

Step 2 : Calculate R_{eq} as viewed through the load terminals.

Step 3 : Draw Thevenin's equivalent or Norton's equivalent.

Step 4 : $R_L = R_{eq}$ gives the condition for maximum power transfer to load.

Step 5 : And maximum power is given by,

$$P_{max} = \frac{V_{TH}^2}{4R_{eq}}$$

Key Point : *If in the problem only the value R_L for maximum power transfer is asked then only R_{eq} is to be calculated. In such case there is no need to calculate V_{TH} or I_N. These values are required only if magnitude of P_{max} is required.*

Steps to apply maximum power transfer theorem-

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Example 2.30 : *Find the value of R_L for maximum power transfer and the magnitude of maximum power dissipated in the resistor R_L in the circuit shown in the Fig. 2.78.*

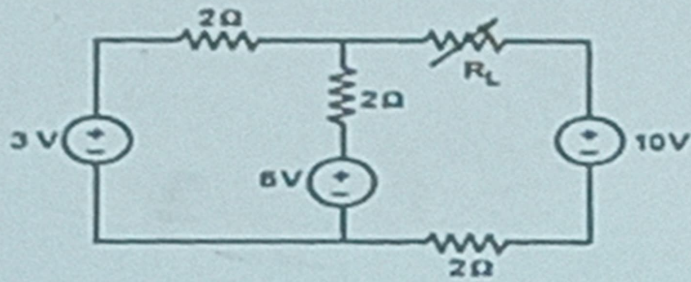


Fig. 2.78

Solution : Step 1 : Remove the load resistance R_L.

Step 2 : Find the Thevenin's voltage V_{TH} across the open terminals as magnitude of maximum power is asked.

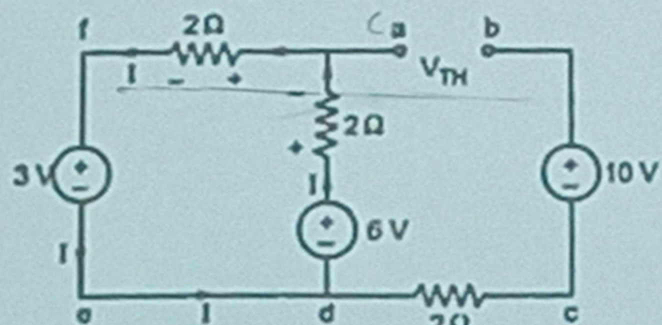


Fig. 2.78 (a)

Applying KVL to the loop,

$$-2I - 2I - 3 + 6 = 0$$

$$\therefore \quad 4I = 3$$

$$\therefore \quad I = 0.75 \text{ A}$$

$\therefore$ Drop across 2 Ω = 2×0.75 = 1.5 V

The 2 Ω resistance in branch cd is not carrying any current hence drop across it is zero volts. Trace the path from a to b and show the various voltage drops as shown in the Fig. 2.78 (b).

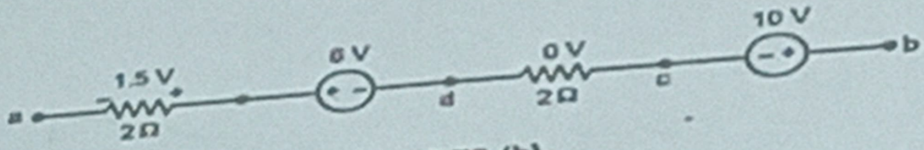


Fig. 2.78 (b)

$V_{TH} = 11.5 - 6 = 5.5$ V with b positive with respect to a.

Step 3 : Calculate R_{eq} replacing all voltage sources by short circuits.

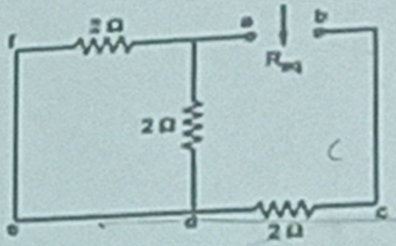


Fig. 2.78 (c)

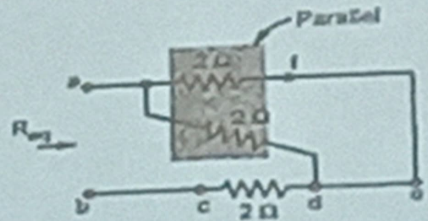


Fig. 2.78 (d)

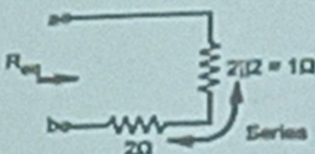


Fig. 2.78 (e)

As seen from the Fig. 2.78 (d) and 2.78 (e) we can write,

$$R_{eq} = [2 \parallel 2] + 2 = 1 + 2 = 3\ \Omega$$

Thus for maximum power transfer to load,

$$R_L = R_{eq} = 3\ \Omega$$

Step 4 : The maximum power transfer is given by,

$$P_{max} = \frac{V_{TH}^2}{4 R_{eq}}$$

$$= \frac{(5.5)^2}{4 \times 3} = 2.5208 \text{ W}$$

Answer

Example 2.31 : *Find the value of R_{AD} for maximum power transfer, in the circuit shown in the Fig. 2.79.*

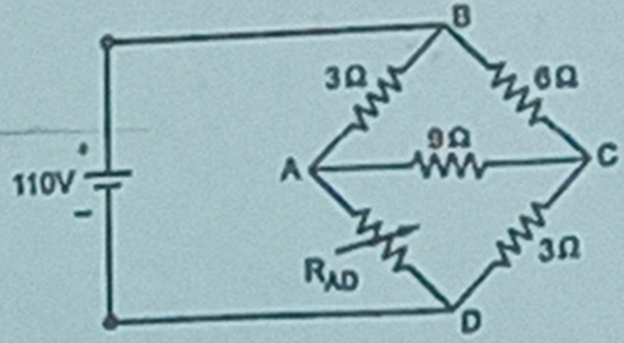


Fig. 2.79

Solution in next slide

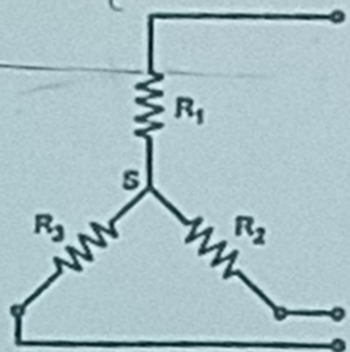
Star- delta Transformation-

What is the need ?

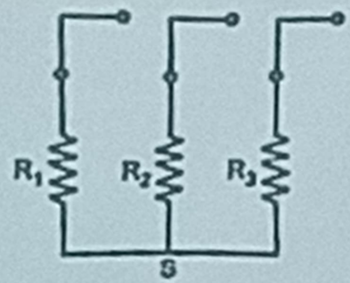
- Some times It is time consuming to solve complicated network involving large no of Unknown.
- We use star –delta transformation to make such complicated network simple before Applying KCL/KVL or any network theorem.

What is Star connection?

If the three resistances are connected in such a manner that one end of each is connected together to form a junction point called Star point, the resistances are said to be connected in Star.



(a)



(b)

Example 2.10 : *Find equivalent resistance between points A-B.*

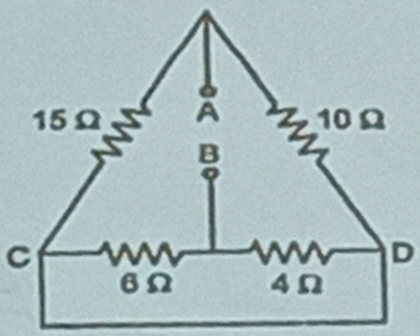


Fig. 2.50

Solution : Redraw the circuit.

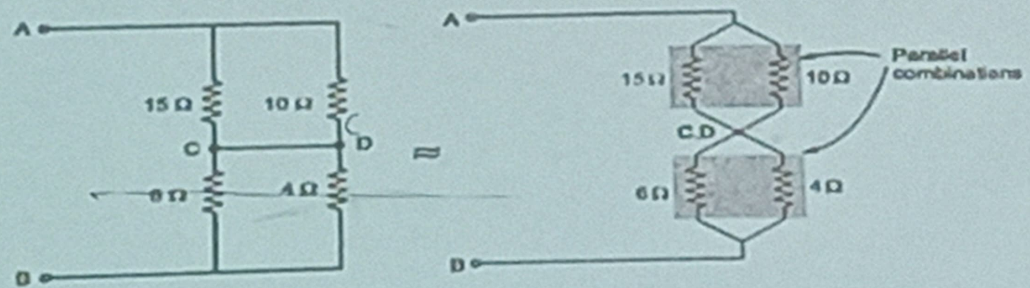
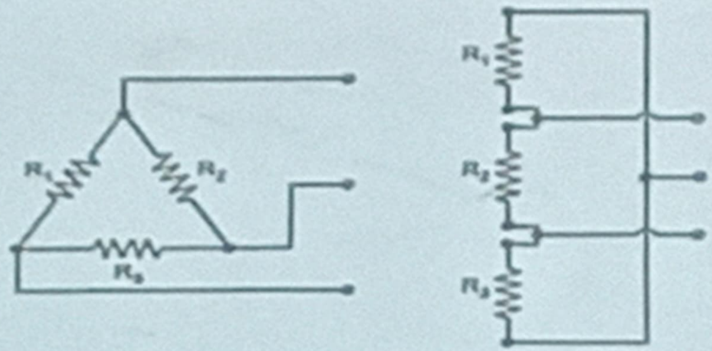


Fig. 2.50 (a)

Answer- Rab = 8.4 ohms

What is delta connection-

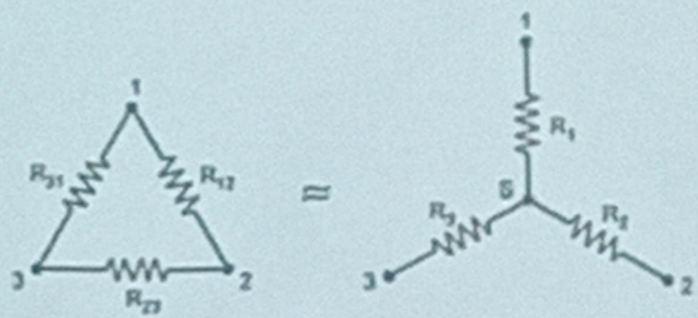
If the three resistances are connected in such a manner that one end of the first is connected to first end of second, the second end of second to first end of third and so on to complete a loop then the resistances are said to be connected in Delta.



(a) (b)

Key Point- Delta connection always forms a close loop

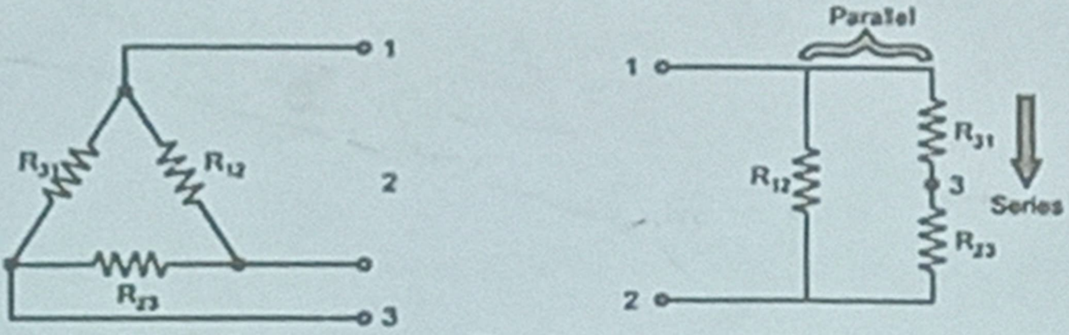
Delta to star Transformation-



Given Delta Fig. 2.40 Equivalent Star

Key Point- To call these two arrangements as equivalents, resistance between any two terminals must be same in both types of connections

Let us analyse Delta connection first, shown in the Fig. 2.40 (a).



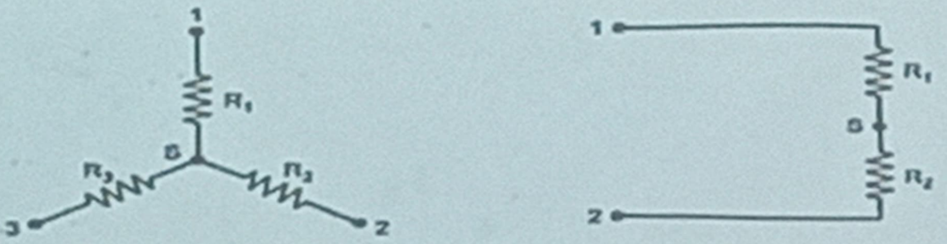
(a) Given Delta Fig. 2.40 (b) Equivalent between 1 and 2

R_{12} parallel with $(R_{31} + R_{23})$ as R_{31} and R_{23} are in series.

$\therefore$ Between (1) and (2) the resistance is,

$$= \frac{R_{12}(R_{31}+R_{23})}{R_{12}+(R_{31}+R_{23})} \quad \text{...(a)}$$

Now consider the same two terminals of equivalent Star connection



$\therefore$ Between (1) and (2) the resistance is $= R_1 + R_2$ ... (b)

Now to call this Star as equivalent of given Delta it is necessary that the resistances calculated between terminals (1) and (2) in both the cases should be equal and hence

$$\frac{R_{12}(R_{31}+R_{23})}{R_{12}+(R_{23}+R_{31})} = R_1 + R_2 \qquad \text{...(c)}$$

Similarly if we find the equivalent resistance as viewed through terminals (2) and (3) in both the cases and equating, we get,

$$\frac{R_{23}(R_{31}+R_{12})}{R_{12}+(R_{23}+R_{31})} = R_2 + R_3 \qquad \text{...(d)}$$

Similarly if we find the equivalent resistance as viewed through terminals (3) and (1) in both the cases and equating, we get,

$$\frac{R_{31}(R_{12}+R_{23})}{R_{12}+(R_{23}+R_{31})} = R_3 + R_1 \qquad \text{...(e)}$$

Now we are interested in calculating what are the values of R_1, R_2, R_3 interms of known values R_{12}, R_{23}, and R_{31}.

Subtracting (d) from (c),

$$\frac{R_{12}(R_{31}+R_{23})-R_{23}(R_{31}+R_{12})}{(R_{12}+R_{23}+R_{31})} = R_1 + R_2 - R_2 - R_3$$

$$\therefore \quad R_1 - R_3 = \frac{R_{12} R_{31} - R_{23} R_{31}}{R_{12} + R_{23} + R_{31}} \qquad \ldots(f)$$

Adding (f) and (e),

$$\frac{R_{12} R_{31} - R_{23} R_{31} + R_{31}(R_{12} + R_{23})}{R_{12} + R_{23} + R_{31}} = R_1 + R_3 + R_1 - R_3$$

$$\therefore \frac{R_{12} R_{31} - R_{23} R_{31} + R_{31} R_{12} + R_{31} R_{23}}{R_{12} + R_{23} + R_{31}} = 2R_1$$

$$2R_1 = \frac{2 R_{12} R_{31}}{R_{12} + R_{23} + R_{31}}$$

$$\boxed{R_1 = \frac{R_{12} R_{31}}{R_{12} + R_{23} + R_{31}}}$$

Similarly by using another combinations of subtraction and addition with equations (c), (d) and (e) we can get,

$$\boxed{R_2 = \frac{R_{12} R_{23}}{R_{12} + R_{23} + R_{31}}}$$

and

$$\boxed{R_3 = \frac{R_{23} R_{31}}{R_{12} + R_{23} + R_{31}}}$$

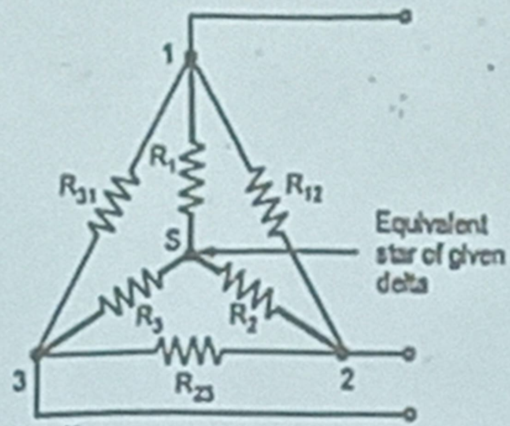
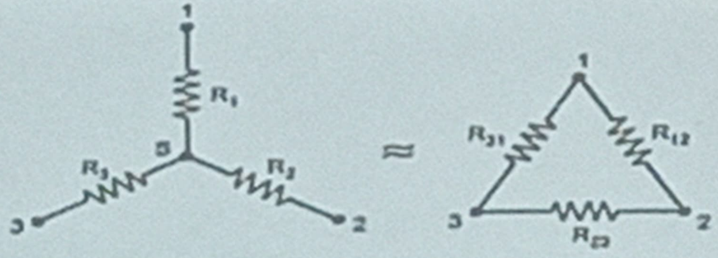


Fig. 2.43 Delta and equivalent Star

Star to delta Transformation-



Given Star Fig. 2.44 Equivalent Delta

For this we can use set of equations derived in previous article. From the result of Delta-Star transformation we know that,

$$R_1 = \frac{R_{12}R_{31}}{R_{12}+R_{23}+R_{31}} \quad ...(g)$$

$$R_2 = \frac{R_{12}R_{23}}{R_{12}+R_{23}+R_{31}} \quad ...(h)$$

$$R_3 = \frac{R_{23}R_{31}}{R_{12}+R_{23}+R_{31}} \quad ...(i)$$

Now multiply (g) and (h), (h) and (i), (i) and (g) to get following three equations.

$$R_1R_2 = \frac{R_{12}^2 R_{31} R_{23}}{(R_{12}+R_{23}+R_{31})^2} \quad ...(j)$$

$$\therefore \quad R_2R_3 = \frac{R_{23}^2 R_{12} R_{31}}{(R_{12}+R_{23}+R_{31})^2} \quad ...(k)$$

$$R_3R_1 = \frac{R_{31}^2 R_{12} R_{23}}{(R_{12}+R_{23}+R_{31})^2} \quad ...(l)$$

Now add (j) ,(k) and (l)

$$\therefore \quad R_1R_2+R_2R_3+R_3R_1 = \frac{R_{12}^2R_{31}R_{23}+R_{23}^2R_{12}R_{31}+R_{31}^2R_{12}R_{23}}{(R_{12}+R_{23}+R_{31})^2}$$

$$\therefore \quad R_1R_2+R_2R_3+R_3R_1 = \frac{R_{12}R_{31}R_{23}(R_{12}+R_{23}+R_{31})}{(R_{12}+R_{23}+R_{31})^2}$$

$$\therefore \quad R_1R_2+R_2R_3+R_3R_1 = \frac{R_{12}R_{31}R_{23}}{R_{12}+R_{23}+R_{31}}$$

But $\dfrac{R_{12}R_{31}}{R_{12}+R_{23}+R_{31}} = R_1$ From equation (g)

$\therefore$ Substituting in above in R.H.S. we get,

$$\therefore \quad R_1R_2+R_2R_3+R_3R_1 = R_1 R_{23}$$

$$\therefore \quad \boxed{R_{23} = R_2+R_3+\frac{R_2 R_3}{R_1}}$$

Similarly substituting in R.H.S., remaining values, we can write relations for remaining two resistances.

$$\boxed{R_{12} = R_1+R_2+\frac{R_1 R_2}{R_3}}$$

and

$$\boxed{R_{31} = R_3+R_1+\frac{R_1 R_3}{R_2}}$$

How to remember ?

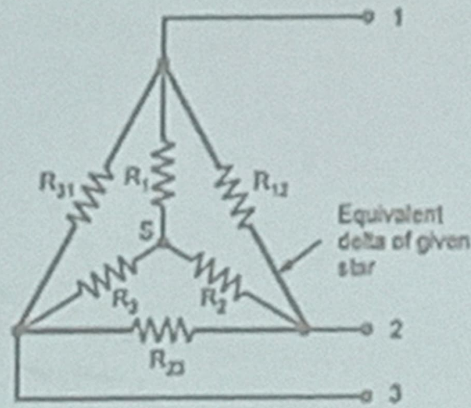


Fig. 2.45 Star and equivalent Delta

The equivalent delta connected resistance to be connected between any two terminals is sum of the two resistances connected between the same two terminals and star point respectively in star, plus the product of the same two star resistances divided by the third star resistance.

Q.1 – Find Equivalent resistances in star for all the resistance connected in delta having same resistance say R.

R1=R2=R3= ?

Q.2- Find Equivalent resistances in delta, for all the resistance connected in star having same resistance say R.

R12=R23=R31= ?

Key Point-

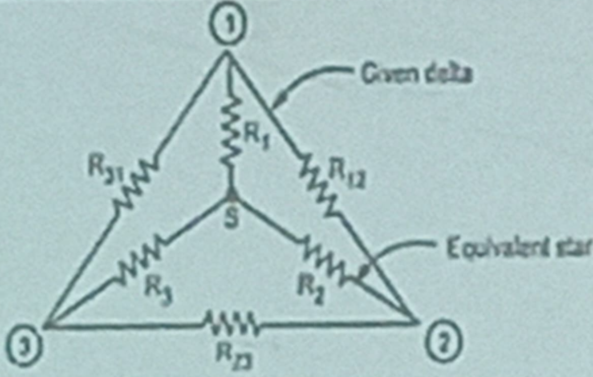
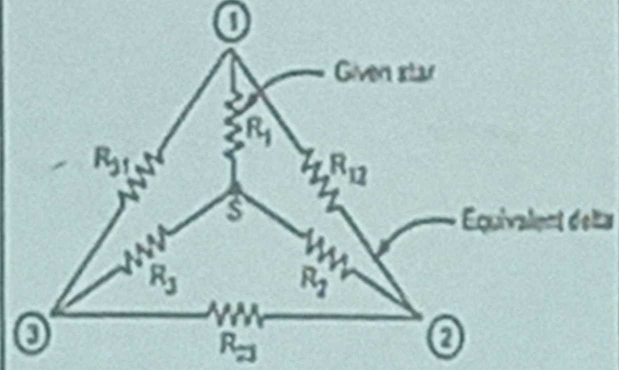
Delta-Star	Star-Delta
	
$R_1 = \frac{R_{12}R_{31}}{R_{12}+R_{23}+R_{31}}$	$R_{12} = R_1 + R_2 + \frac{R_1R_2}{R_3}$
$R_2 = \frac{R_{12}R_{23}}{R_{12}+R_{23}+R_{31}}$	$R_{23} = R_2 + R_3 + \frac{R_2R_3}{R_1}$
$R_3 = \frac{R_{23}R_{31}}{R_{12}+R_{23}+R_{31}}$	$R_{31} = R_3 + R_1 + \frac{R_1R_3}{R_2}$

Table 2.1 Star-Delta and Delta-Star Transformations

Example-

Convert the given Delta in the Fig. 2.46 into equivalent Star.

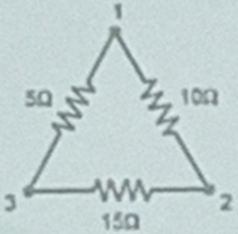


Fig. 2.46

Solution : Its equivalent star is as shown in the Fig. 2.47.

where

$$R_1 = \frac{10\times5}{5+10+15} = 1.67\ \Omega$$

$$R_2 = \frac{15\times10}{5+10+15} = 5\ \Omega$$

$$R_3 = \frac{5\times15}{5+10+15} = 2.5\ \Omega$$

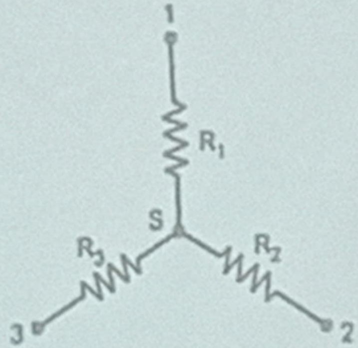


Fig. 2.47

Example-

Convert the given star in the Fig. 2.48 into an equivalent delta.

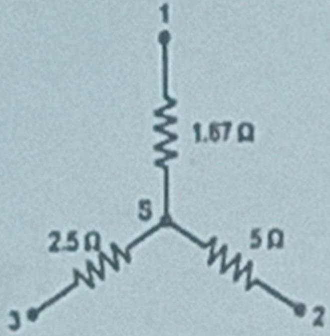


Fig. 2.48

Solution : Its equivalent delta is as shown in the Fig. 2.48 (a).

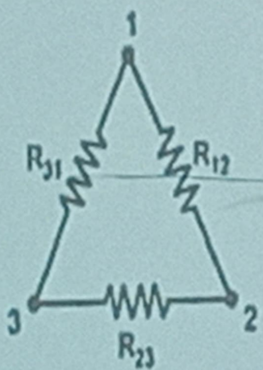


Fig. 2.48 (a)

$$R_{12} = 1.67 + 5 + \frac{1.67 \times 5}{2.5} = 1.67 + 5 + 3.33 = 10\ \Omega$$

$$R_{23} = 5 + 2.5 + \frac{5 \times 2.5}{1.67} = 5 + 2.5 + 7.5 = 15\ \Omega$$

$$R_{31} = 2.5 + 1.67 + \frac{2.5 \times 1.67}{5} = 2.5 + 1.67 + 0.833 = 5\ \Omega$$

Solution : As magnitude of P_{max} is not required, only R_{eq}, as seen through terminals is to be obtained with voltage source shorted.

As points B and D are directly connected, the circuit can be redrawn as shown in Fig. 2.79 (b), showing B and D as a single point.

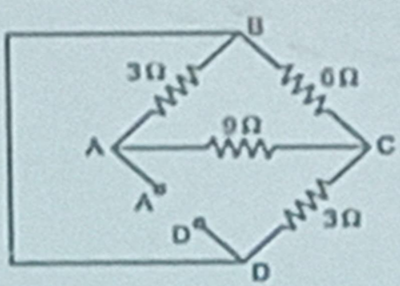


Fig. 2.79 (a)

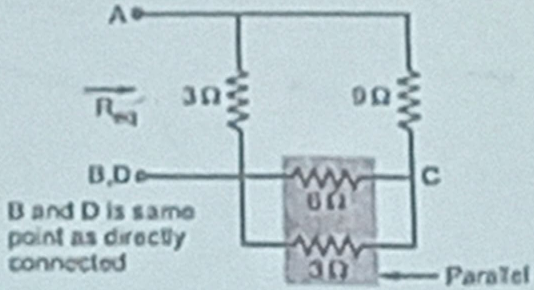


Fig. 2.79 (b)

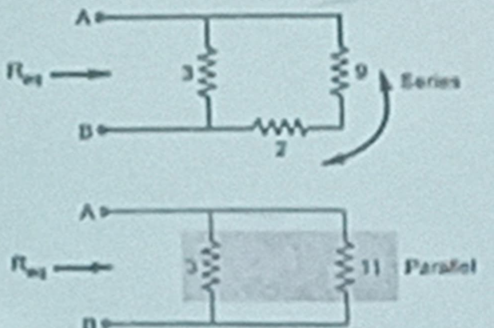


Fig. 2.79 (c)

$$\therefore \quad R_{eq} = (3) \parallel [9 + (6 \parallel 3)]$$
$$= (3) \parallel [9 + 2]$$
$$= (3) \parallel (11)$$
$$= 2.3571\ \Omega$$

For P_{max}, $R_{AD} = R_{eq} = 2.3571\ \Omega$

Answer

Example -Find the current through 10 ohm branch in Figure 2.188

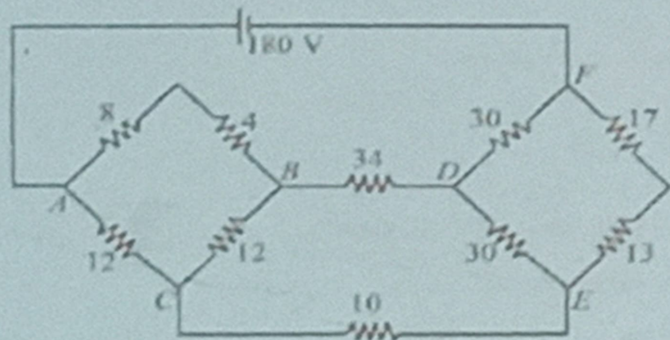
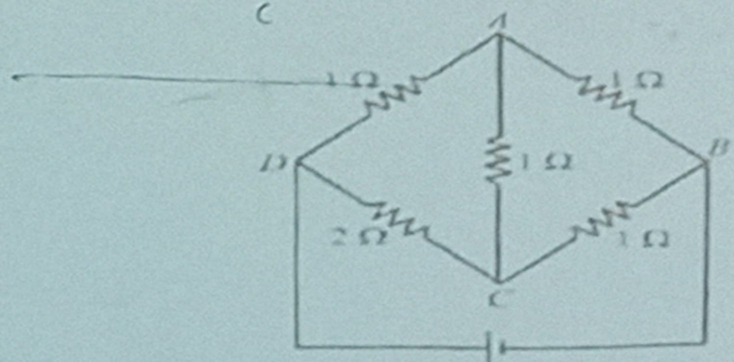


Fig. 2.188

Example 2.91. *A bridge network ABCD has arms AB, BC, CD and DA of resistances 1, 1, 2 and 1 ohm respectively. If the detector AC has a resistance of 1 ohm, determine by star/delta transformation, the network resistance as viewed from the battery terminals.*

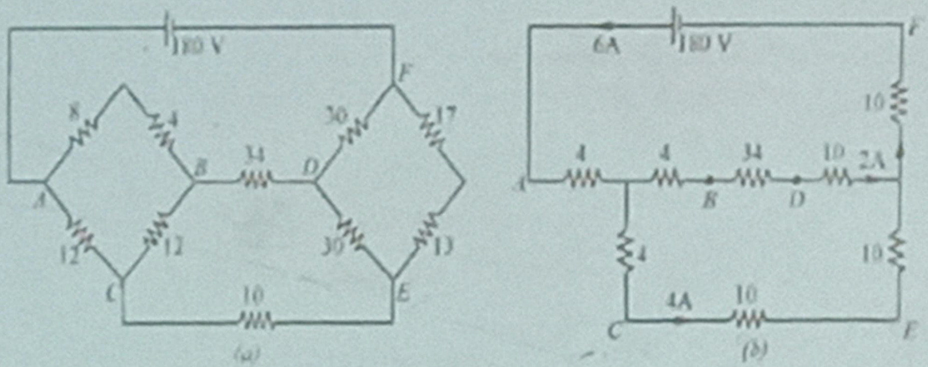


Answer in next slide

Solution 1

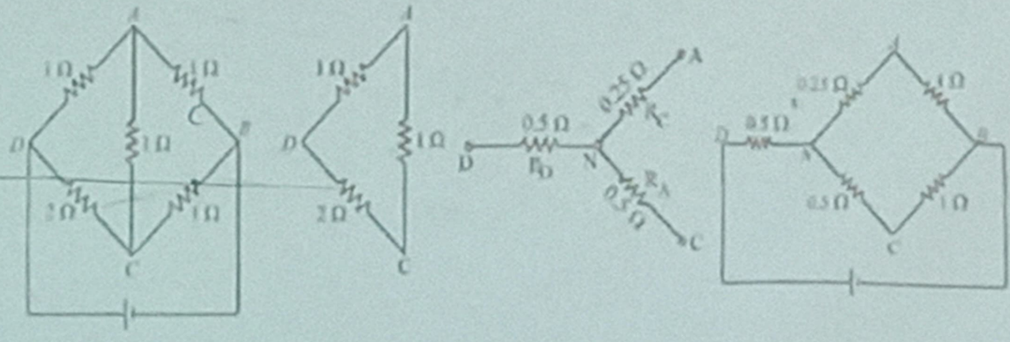
The total circuit resistance between A and $F = 4 + 48 \parallel 24 + 10 = 30\ \Omega$. Hence $I = 180/30 = 6$ A.

Current through $10\ \Omega$ resistor as given by current-divider rule $= 6 \times 48/(48 + 24) = 4$ A.



(a) (b)

Solution -2



(a) (b) (c) (d)

Total resistance of the network between points D and B is

$$= 0.5 + \frac{15}{22} = \frac{13}{11}\ \Omega$$

Solution 1 The total circuit resistance between A and $F = 4 + 48 \parallel 24 + 10 = 30\ \Omega$. Hence $I = 180/30 = 6$ A.
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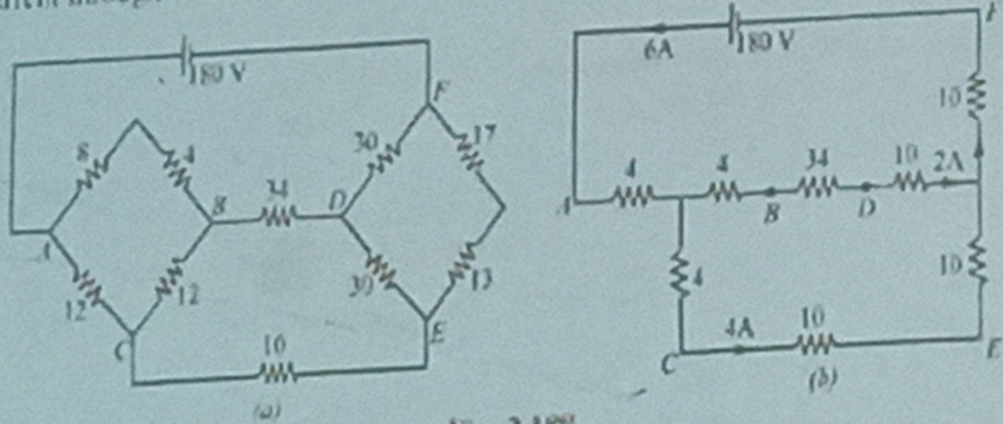
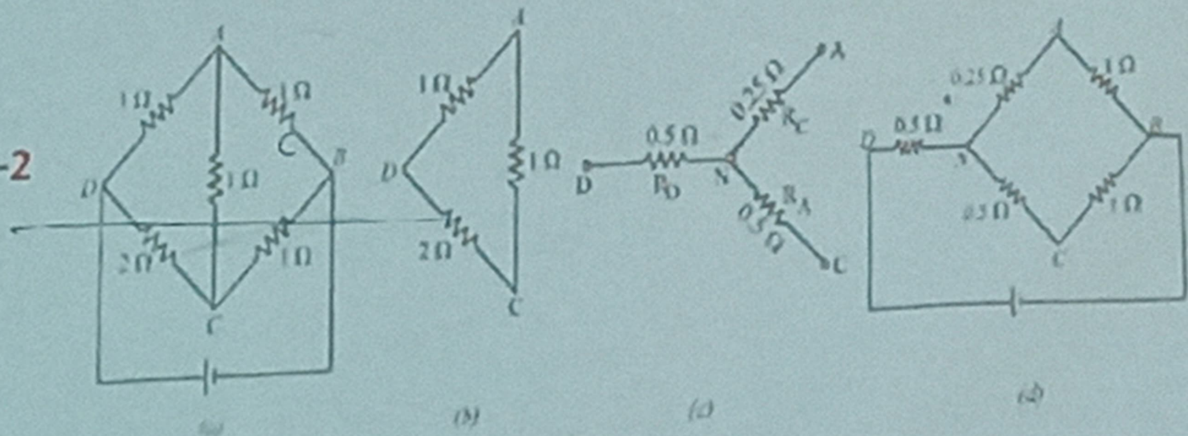


Solution -2



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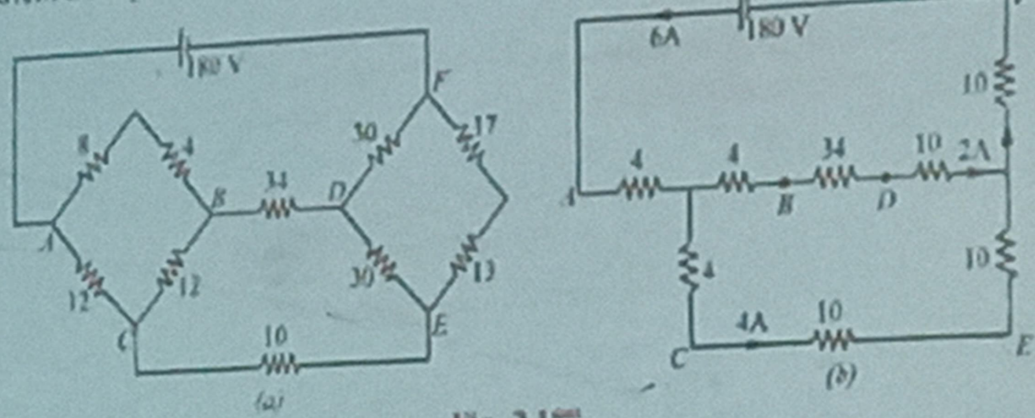
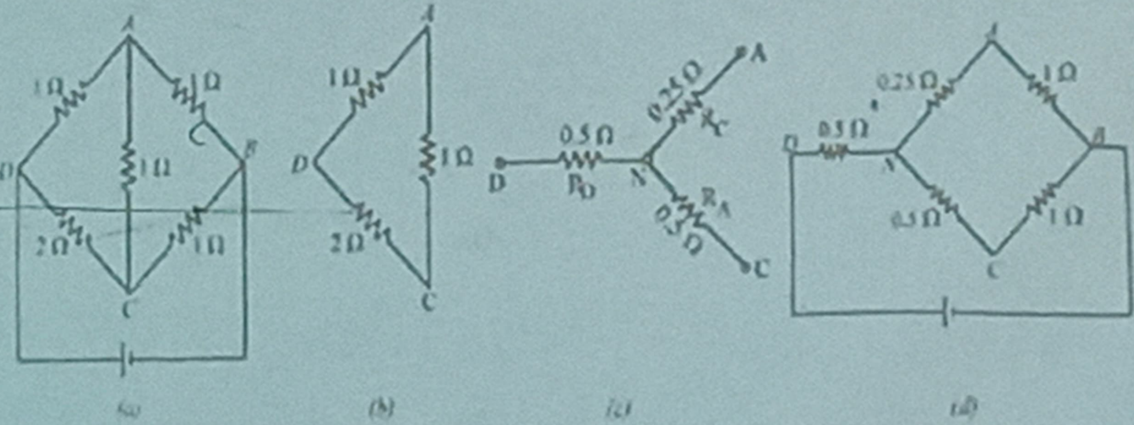


Solution -2



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