Wave Optics

λ path difference $= 2\pi$ phase difference.

$$\boxed{\Delta_L = \frac{2\pi}{\lambda} L}$$

, refractive index is μ.

Optical path $= \mu \times$ physical path

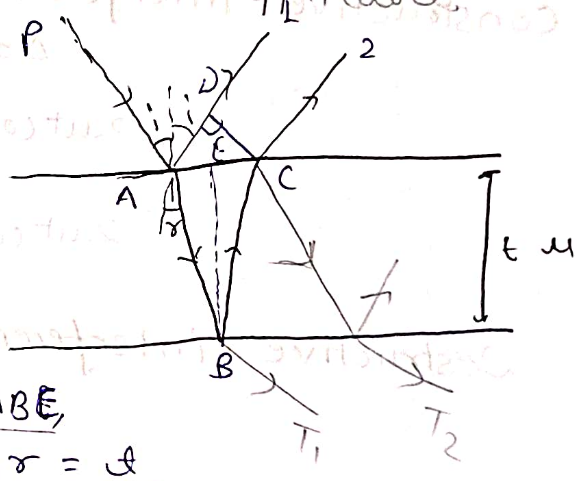
$= \mu \times L$

$$\boxed{\Delta_L = \frac{2\pi \mu L}{\lambda}}$$

Coherent waves :- phase difference is either 0 or constant.

If T_1 & T_2 ka path diff. nikale using same process, but cos r aaega, Stokes law not applicable.

Division of Amplitude :-



Path difference between 1 & 2.

$$= \mu(AB + BC) - AD \quad \text{—} \; (1)$$

in ΔABE,

$\cos r = \frac{t}{AB}$

$AB = \frac{t}{\cos r} = BC \quad \text{—} \; (2)$

in ΔABE,

$\tan r = \frac{AE}{t}$

$AE = t \tan r$

$AC = 2AE = 2t \tan r \quad \text{—} \; (3)$

in ΔADC,

$\sin i = \frac{AD}{AC}$

$AD = 2t \tan r . \sin i \quad \text{—} \; (4)$

$$\boxed{AD = 2t \frac{\sin r}{\cos r} \mu \sin r}$$

Path difference between ray 1 and 2,

$\mu(AB + BC) - AD$

$$= \mu\left(\frac{2t}{\cos r} - \frac{2t \sin^2 r}{\cos r}\right)$$

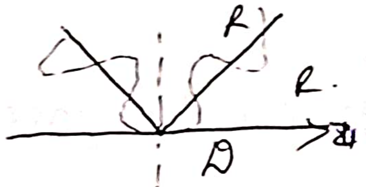
$$\Rightarrow \frac{2\mu t}{\cos r}(1-\sin^2 r) \Rightarrow \frac{2\mu t \cos^2 r}{\cos r}$$

$$\Rightarrow \boxed{\text{Path difference} = 2\mu t \cos r}$$

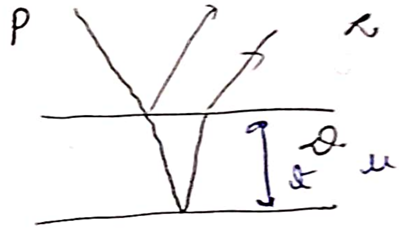
* If path difference $= m\lambda \rightarrow$ constructive interference $(m = 1, 2, 3, --)$

$(2n+1)\frac{\lambda}{2} \rightarrow$ Destructive interference

-Stokes law :-



$\Rightarrow$ (π) Phase difference occurs at this point.



$\rightarrow$ Path difference, $\Delta = 2\mu t \cos r \pm \frac{\lambda}{2}$

Constructive interference,

$$\Delta = m\lambda, \quad m = 0, 1, 2, ---$$

$$2\mu t \cos r - \frac{\lambda}{2} = m\lambda, \quad m = 1, 2, 3, --$$

$$\boxed{2\mu t \cos r = m\lambda + \frac{\lambda}{2}}$$

Destructive interference,

$$\Delta = (2m+1)\frac{\lambda}{2}, \quad m = 0, 1, 2, ---$$

$$2\mu t \cos r - \frac{\lambda}{2} = \frac{(2m+1)\lambda}{2}$$

$$\boxed{2\mu t \cos r = m\lambda}, \quad m = 1, 2, 3, --$$

Maximum thickness of slab :-

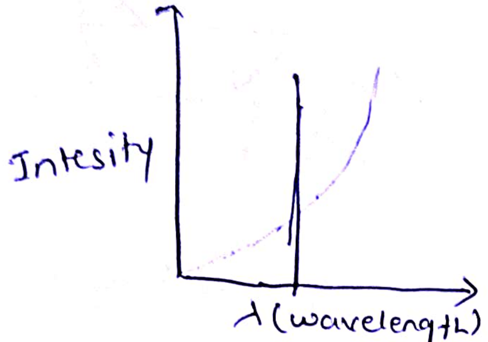
$$\Delta = 2\mu t \cos r - \frac{\lambda}{2}$$

$\Delta \leq$ coherence length of source

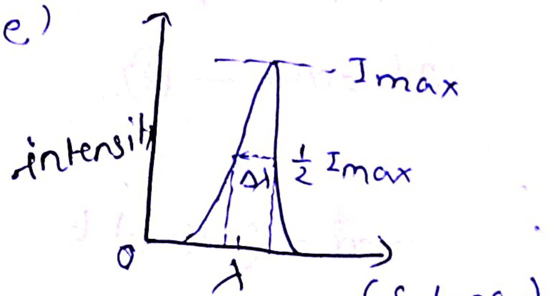
$$2\mu t \cos r - \frac{\lambda}{2} \leq \frac{\lambda^2}{\Delta\lambda}$$

$$2\mu t \cos r \leq \left(\frac{\lambda^2}{\Delta\lambda} + \frac{\lambda}{2}\right)$$

monochromatic light
(single wavelength)



(for ideal case)



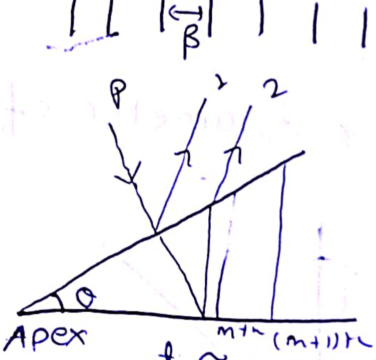
(δ func.)
(in natural)

Case - I in normal incidence,

$$2\mu t \cos r \le \frac{\lambda^2}{\Delta\lambda}$$

$$2\mu t \le \frac{\lambda^2}{\Delta\lambda}$$

$$\boxed{t \le \frac{\lambda^2}{2\mu\Delta\lambda}}$$



$t \approx \mu m$

θ = fraction of degree

- Wedge shaped thin film:-

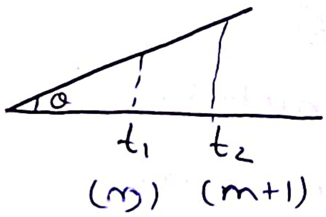
$2\mu t \cos r = m\lambda$, $m = 1, 2, 3, \ldots$

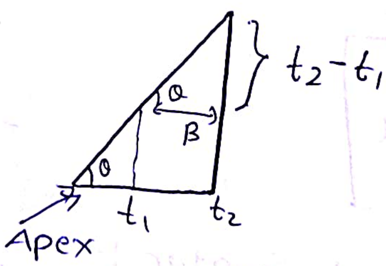
"dark pattern"

β = fringe width

$2\mu t_1 \cos r = m\lambda$

$2\mu t_2 \cos r = (m+1)\lambda$ (dark patterns).





$\theta \approx \tan\theta \approx \sin\theta$

$$\boxed{\tan\theta \approx \frac{t_2 - t_1}{\beta}}$$

$2\mu t_1 = m\lambda$ — ①

$2\mu t_2 = (m+1)\lambda$ — ②

$2\mu (t_2 - t_1) = \lambda$

$2\mu\beta \tan\theta = \lambda$

$$\boxed{\beta = \frac{\lambda}{2\mu\theta}}$$

$\Delta = 2\mu t \cos r - \frac{\lambda}{2}$

$\Delta = \frac{\lambda}{2}$

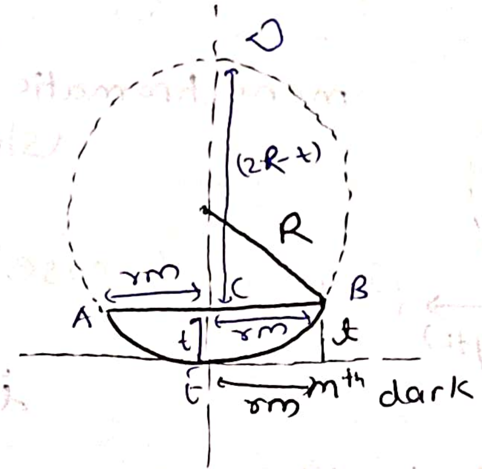
"fringes of equal thickness"
→ corresponding to a thickness either bright or dark pattern is obtained.

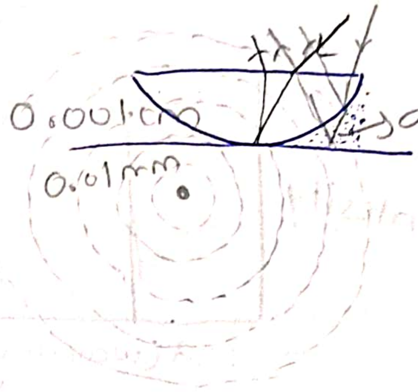
- Newton's Ring:-

$2\mu t = m\lambda$ — ①.

in air medium,

$2t = m\lambda$ — ②.





$r_m^2 = (2R-t)\,t$

$r_m^2 = 2Rt - t^2$

$r_m^2 = 2Rt$ — ③.

$r_m^2 = m\lambda R$

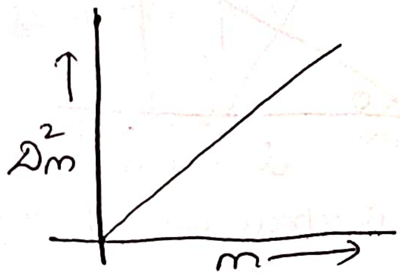
r_m^2 $\boxed{r_m = \sqrt{m\lambda R}}$

0.1mm<

not visible

* Diameter of m^{th} ring, $\boxed{D_m = 2\sqrt{m\lambda R}}$

$\boxed{D_{m_{air}}^2 = 4\lambda m R}$



slope= $4\lambda R$

$\cancel{D_m^2 + \beta =}$

$D_{m+\beta}^2 = 4(m+\beta)\lambda R$ — ②

$$\boxed{R = \frac{D_{m+\beta}^2 - D_{m_{avg}}^2}{4\lambda\beta}}$$ — ③.

To find R → spherometer is used.

$$\boxed{D_{m_{(\mu)}}^2 = \frac{4m\lambda R}{\mu}}$$ — ②

for destructive,

$D_m \propto \sqrt{m}$,

\# $$\boxed{\frac{D_{m\,(air)}^2}{(D_m^2)_{(\mu)}} = \mu}$$

- Michelson Interferometer :-

for constructive,

$D_m \propto \sqrt{2m+1}$

In transmitted case, for destructive, $D_m \propto \sqrt{2m+1}$

& constructive, $D_m \propto \sqrt{m}$

(coz $\pm \lambda/2$ ka factor x in path difference)

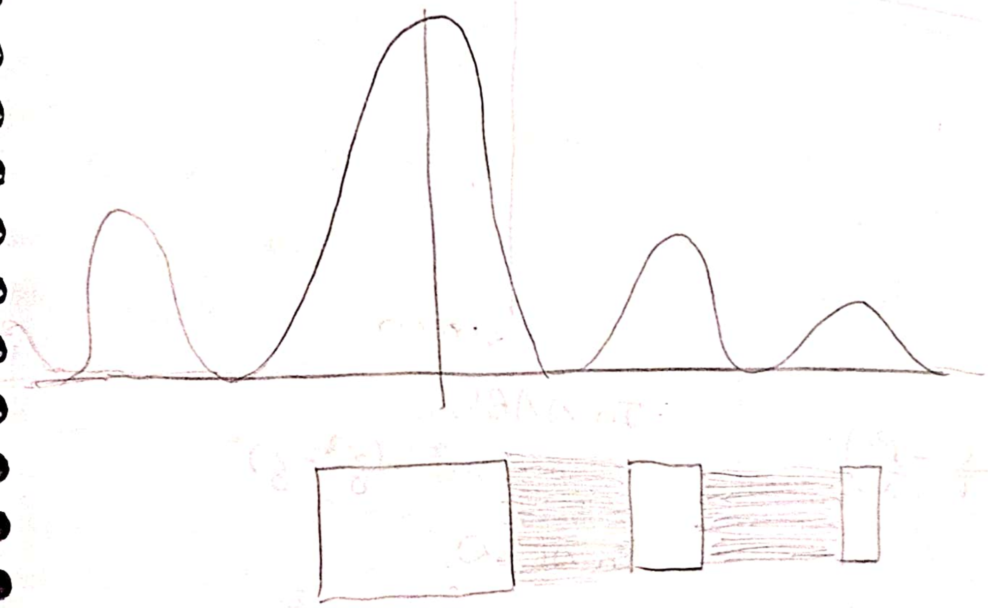
- **Diffraction:-** Fresnel diffraction → Source & screen at finite distance

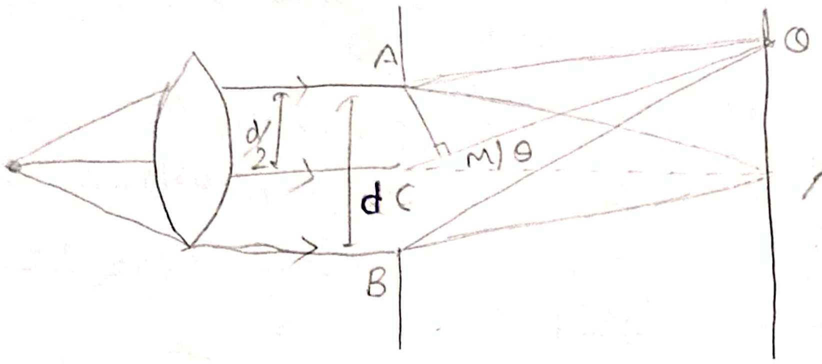
Fraunhofer diffraction → source & screen at infinite distance.

Diffraction occurs when,

$d > \lambda$ $d = \lambda$ $d << \lambda$

- **Single slit diffraction pattern:-**





$\Rightarrow d\sin\theta = \lambda$

Path difference is $\frac{d}{2}\sin\theta$

if $\frac{d}{2}\sin\theta = \frac{\lambda}{2}$ — (1)

$\frac{d}{4}\sin\theta = \frac{\lambda}{2}$

$d\sin\theta = 2\lambda$ — (2).

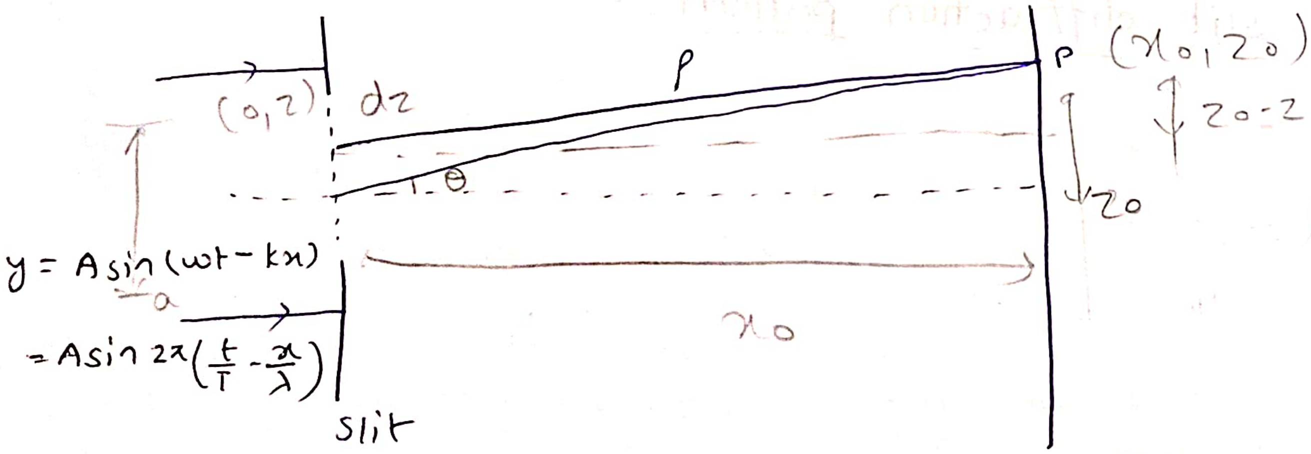
$\frac{d}{6}\sin\theta = \frac{\lambda}{2}$

$d\sin\theta = 3\lambda$ — (3).

$\frac{3}{2}\lambda, \frac{5}{2}\lambda, \frac{7}{2}\lambda, \ldots$

* $d\sin\theta = m\lambda$, $m = 1, 2, 3, \ldots$
(minima) ⇒ got dark patterns

* $d\sin\theta = \frac{(2m+1)\lambda}{2}$, $m = 1, 2, 3, \ldots$
(maxima)



$\rightarrow \quad dy = k\,dz \sin 2\pi\left(\frac{t}{T} - \frac{\rho}{\lambda}\right)$

$$y = \int_{-a/2}^{a/2} dy$$

$$y = \int_{-a/2}^{a/2} \sin 2\pi\left[\frac{t}{T} - \frac{(r - z\sin\theta)}{\lambda}\right] dz$$

In $\triangle ABC$,

$\rho^2 = x_0^2 + (z_0 - z)^2$

In $\triangle OBD$,

$r^2 = x_0^2 + z_0^2$

$\rho^2 = r^2 - z_0^2 + z_0^2 + z^2 - 2zz_0$

$\rho^2 = r^2 + z^2 - 2zz_0$

$$\Rightarrow \rho^2 = r^2\left[1+\frac{z^2}{r^2}-\frac{2zz_0}{r}\right]$$

$$\rho^2 = r^2\left[1-\frac{2zz_0}{r^2}\right]$$

$$\rho = r\left[1-\frac{2zz_0}{r^2}\right]^{1/2}$$ (using binomial theorem)

$$\Rightarrow \boxed{\rho = r\left[1-\frac{zz_0}{r^2}\right]}$$

$$\rho = r - z\left(\frac{z_0}{r}\right)$$

$$\boxed{\rho = r - z\sin\theta}$$

$$\Rightarrow \boxed{y = ka\frac{\sin\alpha}{\alpha}\sin 2\pi\left(\frac{t}{T}-\frac{r}{\lambda}\right)}$$

$$\alpha = \frac{\pi a\sin\theta}{\lambda}$$

→ Case-I :- Central maxima:-

$$\theta = 0$$
$$\alpha = 0$$

$$I = k^2a^2\frac{\sin\alpha}{\alpha} \qquad \alpha = \frac{\pi a\sin\theta}{\lambda}$$

$$\alpha \to 0\left(\frac{\sin\alpha}{\alpha}\right) = 1$$

$$\Rightarrow \boxed{I = k^2a^2}$$

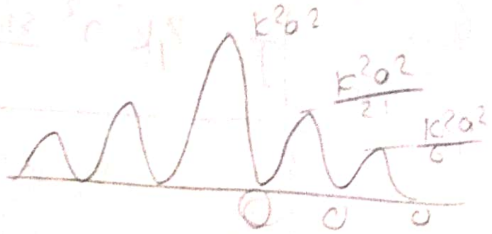
→ Case-II :- Secondary maxima:-

$$a\sin\theta = \frac{(2m+1)\lambda}{2}, \quad m = 1,2,3,\ldots$$

$$\alpha = \frac{\pi}{\cancel{\lambda}}\frac{(2m+1)}{2}\cancel{\lambda}$$

$$\boxed{\alpha = \frac{\pi}{2}(2m+1)} \quad m = 1,2,3,\ldots$$

$$\alpha = \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \ldots$$



$$\Rightarrow I = k^2a^2\frac{\sin^2\alpha}{\alpha^2}$$

→ Case-III :- Minima:-

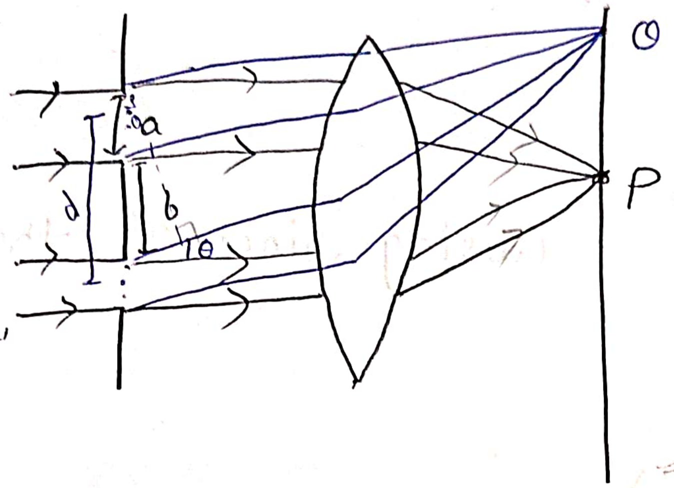
$$a\sin\theta = m\lambda, \quad m = 1,2,3,\ldots \qquad \alpha = \frac{\pi}{\lambda}m\lambda = m\pi$$

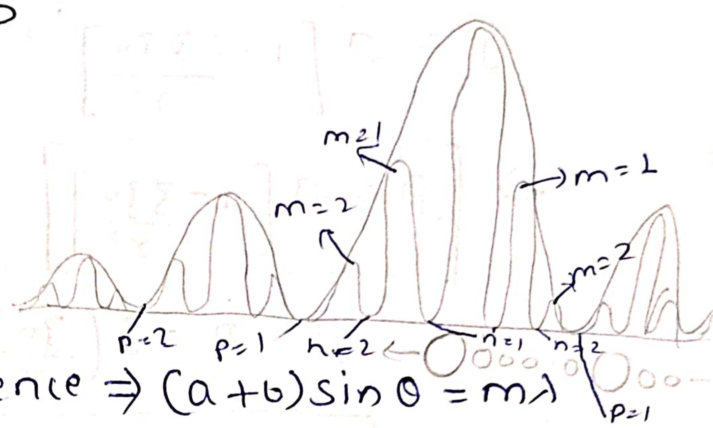
$$m = 1,2,3,\ldots$$

$$\Rightarrow \boxed{I = 0} \qquad \alpha = \pi, 2\pi, 3\pi, \ldots$$

- Double - Slit Diffraction Pattern :-

d = a+b
'Grating element'





① Interference :-

Maxima → Path difference ⇒ $(a+b)\sin\theta = m\lambda$

$m = 1, 2, 3,$

Minima → Path difference ⇒ $(a+b)\sin\theta = \frac{(2n+1)\lambda}{2}$,

$n = 1, 2, 3,$

② Diffraction :- $a\sin\phi = \beta\lambda$

$\beta = 1, 2, 3, \ldots$

(Interference ke liye 2 slit chahiye but diffraction ke liye only 1)

$$\Rightarrow y = \int_{-a/2}^{a/2} dy + \int_{d-a/2}^{d+a/2} dy \quad \text{—①}$$

$dy = k\,dz \sin 2\pi\left(\frac{t}{T} - \frac{\rho}{\lambda}\right)$

After solving this,

$$y = 2ka \frac{\sin\alpha}{\alpha} \cos\beta \sin 2\pi\left(\frac{t}{T} - \frac{\rho}{\lambda}\right)$$

$$\boxed{I = 4k^2a^2 \frac{\sin^2\alpha}{\alpha^2} \cos^2\beta}$$

$\alpha = \frac{\pi a \sin\theta}{\lambda}$

$\beta = \frac{\pi d \sin\theta}{\lambda}$

→ Central Maxima → $\boxed{I = 4k^2a^2}$

Q. Calculate the angles at which the first dark band & the next bright band are formed in the fraunhofer diffraction pattern from slit 0.3 mm wide and $\lambda = 5890\,Å$.

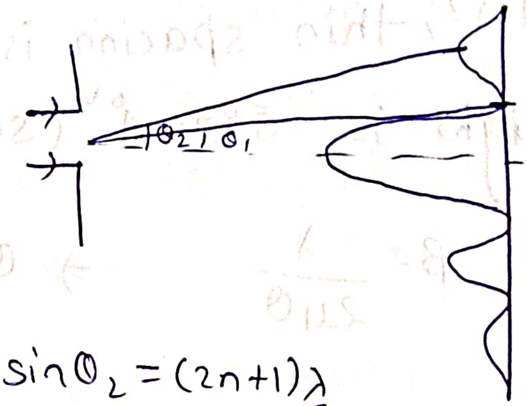
$\rightarrow$ $a = 0.3\,mm$

$\lambda = 5890\,A^{\circ}$

$a \sin\theta_1 = \lambda$, $n = 1$

radian

$\left(\times \frac{180}{\pi}\right)^{\circ}$



$a \sin\theta_2 = (2n+1)\frac{\lambda}{2}$

$\theta \rightarrow$ in radion

$a \sin\theta_2 = \frac{3\lambda}{2}$

So $\Rightarrow$ change in degree $\left(\times \frac{180^{\circ}}{\pi}\right)^{\circ}$

Q.- Find the wavelength of monochromatic source light in micholson interferometer experiment if 200 frindges cross the field of ~~view~~ view when path differences is created 0.0589 mm.

$\rightarrow$ $\Delta = 0.0589\,mm$ 200 fringes cross

$\boxed{d = \frac{N\lambda}{2}}$ $d = \frac{N\lambda}{2}$ $\Rightarrow$ $\lambda = \frac{d \times 2}{N}$ $\lambda = \frac{2 \times 0.0589 \times 10^{-3}}{200}$

$\boxed{\lambda = 5890 \times Å}$

Q. In a Newton's ring Exp., diameter of 10^{th} ray modifies from 1.40 cm to 1.27 cm when the air film is replaced by a drop of liquid. Find the refractive index of lens.

$\rightarrow$ $(D_{10})_{air} = 1.40\,cm$ $(D_{10})_{l} = 1.27\,cm$

$\frac{(D_m^2)_{air}}{(D_m^2)_{l}} = \mu$ $\Rightarrow$ $\mu = 1.509$

Q. Find the wedge angle for a thin glass wedge of $\mu = 1.52$, thin spacing is 0.1mm (β) and wavelength of light is 5890 Å (sodium light source.)

$\rightarrow$ $\beta = \dfrac{\lambda}{2\mu\theta}$ $\Rightarrow \theta = \dfrac{\lambda}{2\mu\beta} = \dfrac{5890\times10^{-10}}{3.04\times10^{-4}}$

$$\theta = \left(1,937.5\times10^{-6}\times\frac{180}{\pi}\right)^{\circ}$$

$\theta = 1.94\times10^{-3}$ rad

$$\boxed{\theta = 0.11^{\circ}}$$

~~If for the same value of θ~~

- Double slit :-

$\rightarrow$ Missing Order :- For same value of θ, Interference maxima is coincided over the,

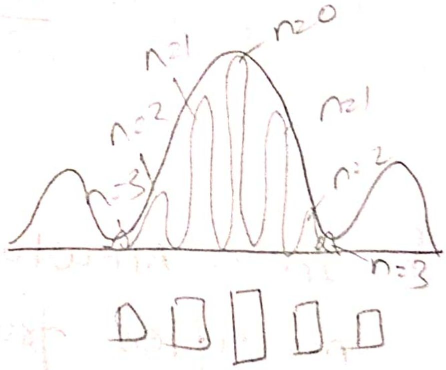
$d = a+b$

Interference maxima,

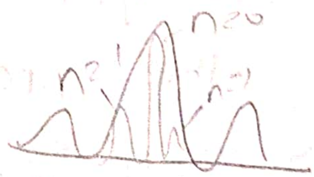
$(a+b)\sin\theta = n\lambda, \quad n = 1,2,3,\ldots$ —— (1)

Diffraction minima,

$a\sin\theta = p\lambda, \quad p = 1,2,3,$ —— (2)



$\dfrac{eq.(1)}{eq.(II)} = \dfrac{a+b}{a} = \dfrac{n}{p}$



Case-I, $a = b$

$n = 2p$

$\boxed{n=2}$ ($p=1$)

2, 4, 6, 8, 10 — —

$\Rightarrow$ missing order in Double slit.

$\Rightarrow$ IInd interference maxima coincided over Ist diffraction minima.

Case - II, $b = 2a$ — missing order $= 3, 6, 9, \ldots$

$\Rightarrow n = 3p$ — 5 $\Rightarrow$ interference maxima are observed.

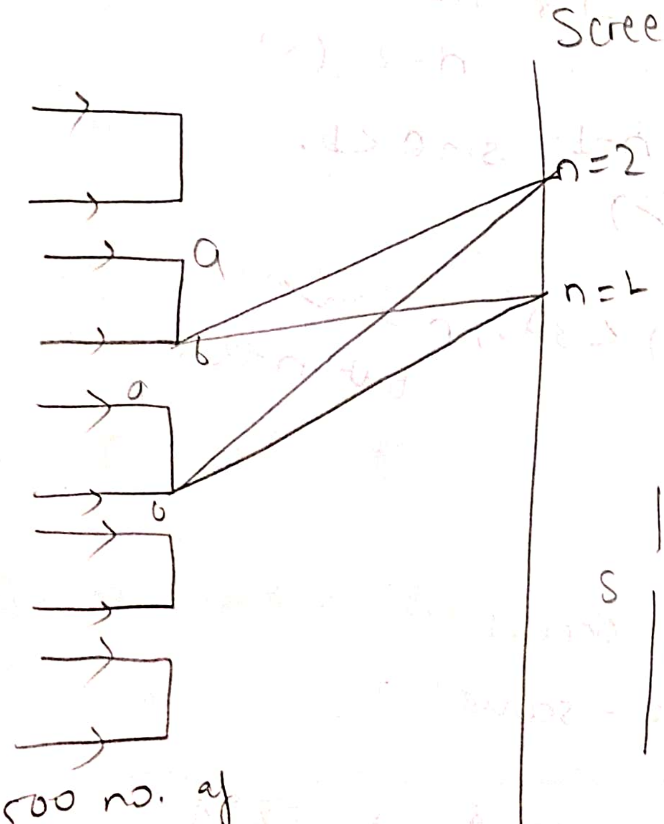
for $p = 1$, $\boxed{n = 3}$

- Transmmission Grating :- | # 3rd Interference maxima is coincided over 1st diff. minim

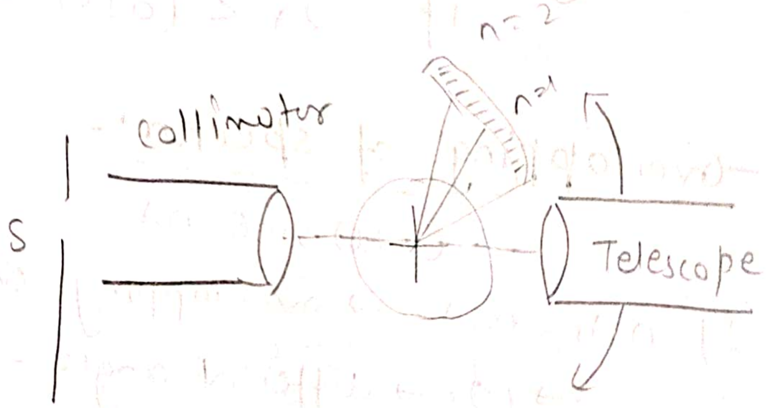
→ N-slit diffraction :-

$$y = \int_{-a/2}^{a/2} dy + \int_{d-a/2}^{d+a/2} dy + \int_{2d-a/2}^{2d+a/2} dy + \text{——}$$

$y =$ HW $\Rightarrow$ book se likh lena.



1500 no. of lines per inch



$$\underbrace{(a+b)}_{\text{Grating element}}\sin\theta = n\lambda$$

λ order

$n = 1, 2, 3, \ldots$

$$n = \frac{(a+b)\sin\theta}{\lambda}$$

$$\boxed{n_{max} = \frac{a+b}{\lambda}}$$

- Absent spectra:- (in transmission grating)

$(a+b)\sin\theta = n\lambda$

a ↓ slit width, b ↓ opaque separation

if $(a+b) < \lambda$, for $n=1$

$\sin\theta > 1$ (×) first order spectrum will not be observed

$\Rightarrow$ $n=1$ (×)

if $\lambda < (a+b) < 2\lambda$,

?

if $(a+b) < 2\lambda$, for $n=2$, $\sin\theta > 1$,

$n=2$ (×)

if $\lambda < (a+b)$ for $n=1$, $\sin\theta < 1$.,

$n=1$ (✓)

if $2\lambda < (a+b) < 3\lambda$,, $n=2$ ✓

but $n=3$ ×

- Overlapping of spectra:-

$(a+b)\sin\theta = n\lambda$

If $n_1\lambda_1 = n_2\lambda_2$ → overlapping occurs.

→ (θ) → different angle - same

Ex. - $\lambda_1 = 4000$ Å, $\lambda_2 = 8$ kÅ

$n=2$ $n=1$