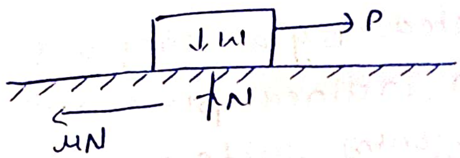
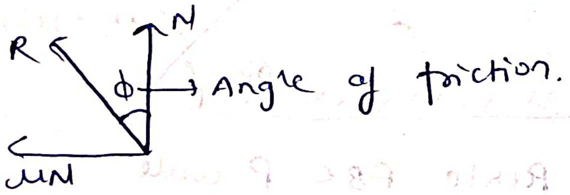
Friction [MKST]



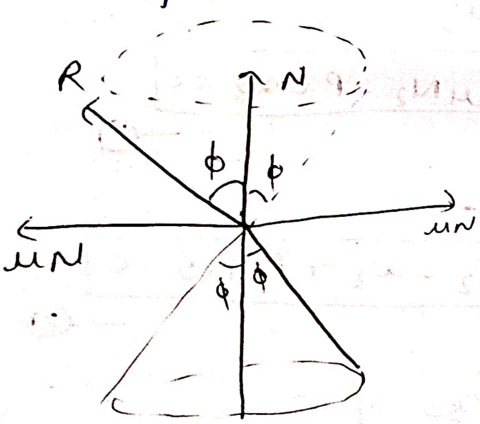
- Limiting friction force :- It is the maximum frictional force ~~exte~~ exerted at the time of impending motion i.e. when the motion is about to begin.

- Angle of friction :- (ϕ)

It is an angle between normal reaction force and resultant of normal reaction force & friction force.

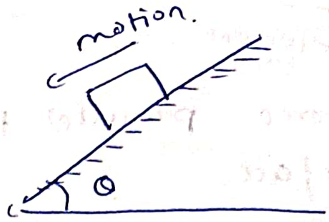


- Cone of friction :- If the resultant reaction is rotated about normal reaction force, it will form a cone which is known as cone of friction.



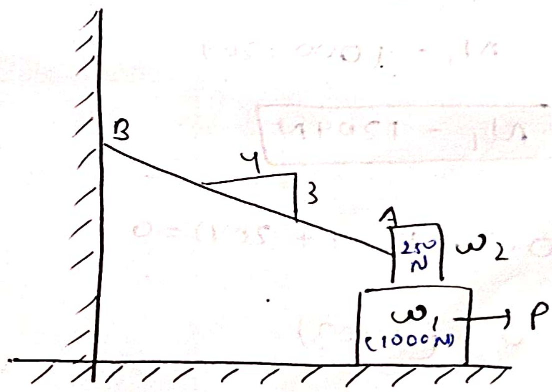
cone of friction.

- Angle of Repose (θ) :-



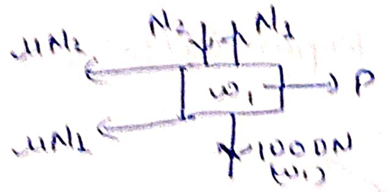
when a body is about to slide down on an inclined plane ~~wt~~ due to its own weight, then the angle made by the plane with ~~respect to~~ the horizontal is known as angle of repose.

Q.1 A block of weight $W_1 = 1000N$ rests on a horizontal surface and supports on its top another block of weight $W_2 = 250N$. The block of weight W_2 is attached by an inclined string AB to the vertical wall. Find the magnitude of the horiz. force P applied to the lower block to cause slipping to impend. The co-efficient of friction for all contacting surfaces may be assumed to be $\mu = 0.3$.



→ FBD of W_1,

⊛ Jispe P laga ho,



(*) friction force parallel to the surface.

$\Sigma F_x = 0$

$P - \mu N_2 - \mu N_1 = 0$

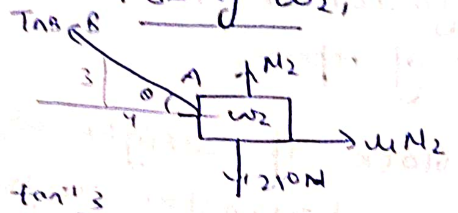
$\boxed{P - \mu(N_1 + N_2) = 0}$ —(1)

$\Sigma F_y = 0$

$N_1 - N_2 - W_1 = 0$

$\boxed{N_1 - N_2 = 1000}$ —(2)

Now, FBD of W_2,



$\theta = \tan^{-1}\frac{3}{4}$

$\boxed{\theta = 37^\circ} = \boxed{36.87^\circ}$

$\Sigma F_x = 0$

$\mu N_2 - T_{AB}\cos(36.87^\circ) = 0$ —(3)

$\Sigma F_y = 0$

$N_2 - 210 + AB\sin(36.87^\circ) = 0$ —(4).

by eq. (3) & (4) :-

⇒N_2

$\boxed{N_2 = 204\ N}$

$\boxed{T_{AB} = 76.5\ N}$

bye eq. (2) :-

$N_1 = 1000 + 204$

$\boxed{N_1 = 1204 N}$

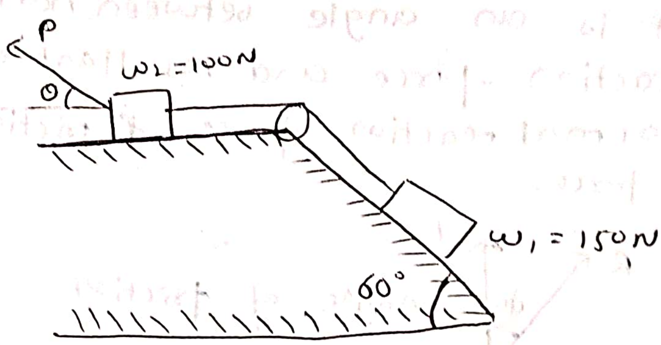
$P - 0.3(1204 + 204) = 0$

$P = 0.3(2408)$

$\boxed{P = 722.4\ N}$

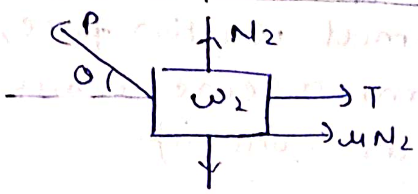
$\boxed{P = 722.4 N}$

Q.2 Two rectangular blocks of weight $W_1 = 150N$ & $W_2 = 100N$ are connected by a string and rest on an inclined plane and on a horizontal surface as shown in fig. The co-efficient of friction for all contiguous surfaces is $\mu = 0.2$. Find the magnitude and direction of the least force P at which the motion of the blocks will impact.



(*) Pahle FBD P wale block ka banayenge

→ FBD of Block W_2,



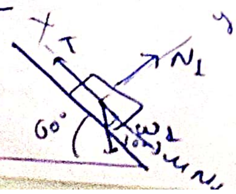
$\Sigma F_x = 0$,

$\boxed{T + \mu N_2 - P\cos\theta = 0}$ —(1)

$\Sigma F_y = 0$,

$\boxed{N_2 - W_2 + P\sin\theta = 0}$ —(2)

Now, FBD of W_1 :-



$\Sigma F_y = 0$:-

$N_1 - w_1 \sin 30° = 0$

$N_1 = \frac{180}{2} \Rightarrow \boxed{N_1 = 75\ N}$

$\Sigma F_x = 0$:-

$w_1 \cos 30° + \mu N_1 - T = 0$

$\frac{180 \times \sqrt{3}}{2} + 0.2 \times 75 = T$

$T = 75\sqrt{3} + 0.2 \times 75$

$T = 75(0.2 + \sqrt{3})$

$\boxed{T = 144.9 N}$

by eq. 1 & 2 :-

~~144.9 +~~

$0.2 N_2 + 144.9 - P\cos\theta = 0$ —— (3)

$N_2 - 100 + P\sin\theta = 0$ —— (4)

$\Rightarrow \boxed{N_2 = 100 - P\sin\theta}$

Now in eq. (3) :-

$0.2(100 - P\sin\theta) + 144.9 - P\cos\theta = 0$

$0.2(100 - P\sin\theta) = -144.9 + P\cos\theta$

$\Rightarrow 20 - 0.2P\sin\theta = P\cos\theta - 144.9$

$P\cos\theta + (0.2)\sin\theta(P) = 164.9$

$$\boxed{P = \frac{164.9}{\cos\theta + 0.2\sin\theta}}$$

for $P = P_{min}$,

$\frac{d}{d\theta}(\cos\theta + 0.2\sin\theta) = 0$

$-\sin\theta + 0.2\cos\theta = 0$

$0.2\cos\theta = \sin\theta$

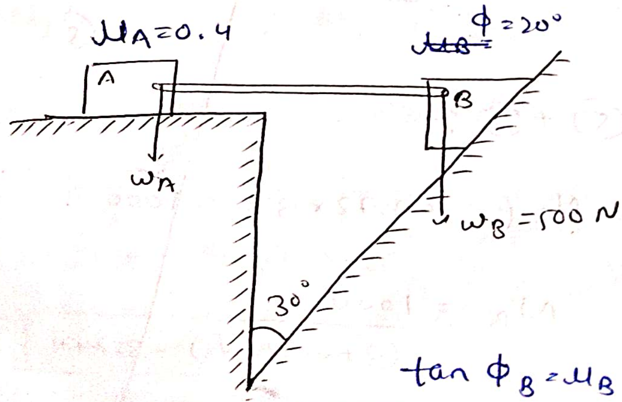
$\tan\theta = 0.2$

$\boxed{\theta = 11.3°}$

then, $P_{min} = \frac{164.9}{\cos(11.3°) + 0.2\sin(11.3°)}$

$\boxed{P_{min} = 161.7 N}$

Q.3 Two blocks are connected by a horizontal link AB and rest on two planes as shown in fig. what is the smallest weight W_A of the block A for which the equillibrium can exist? Assume the coefficient of friction for the block A and the horizontal surface to be 0.4 and the angle of friction for the block B on the inclined plane is $\phi = 20°$

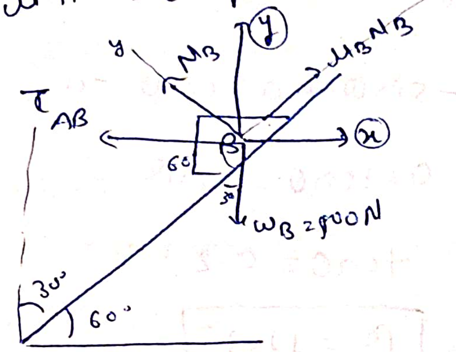


$\tan\phi_B = \mu_B$

$\boxed{\mu_B = 0.36}$

→ FBD of Block B,

* link → compression.



$\Sigma F_y = 0$:-

$N_B - w_B \sin 30° + T_{AB} \sin 60° = 0$ —(1)

$\Sigma F_x = 0$:-

$\mu_B N_B - w_B \cos 30° - T_{AB} \cos 60° = 0$ —(2).

$0.36 N_B - 250\sqrt{3} - \frac{T_{AB}}{2} = 0$

$\boxed{0.72 N_B - 500\sqrt{3} - T_{AB} = 0}$ —(3)

by (1) :-

$N_B - 250 + T_{AB} \frac{\sqrt{3}}{2} = 0$

$\boxed{2N_B - 500 + \sqrt{3}\, T_{AB} = 0}$ —(4)

eqn. (3) $\times \sqrt{3}$:-

$0.72 \times \sqrt{3} N_B - 1500 - \sqrt{3} T_{AB} = 0$ —(5)

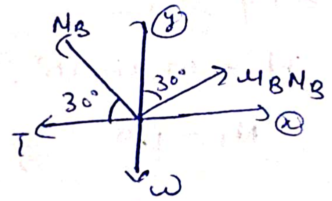
(5) + (4) :-

$N_B (2 + 0.72 \times \sqrt{3}) = 1000$

$N_B = \frac{1000}{(2 + 0.72 \times \sqrt{3})} = 3.247$

$N_B =$

$\Sigma F_x = 0$,



$-T + \mu_B N_B \sin 30°$

$- N_B \cos 30° = 0$

$N_B (\mu_B \sin 30° - \cos 30°) = T$

$N_B \left(0.36 \times \frac{1}{2} - \frac{\sqrt{3}}{2}\right) - T = 0$

$N_B (0.18 - 0.866) - T = 0.$

$(-0.686) N_B - T = 0$

$\boxed{T + 0.686 N_B = 0}$ —(1).

$\Sigma F_y = 0$,

$-w_B + \mu_B N_B \cos 30° + N_B \sin 30° = 0$

$\Rightarrow N_B (\mu_B \cos 30° + \sin 30°) = 500$

$N_B \left(0.36 \times \frac{\sqrt{3}}{2} + \frac{1}{2}\right) = 500$

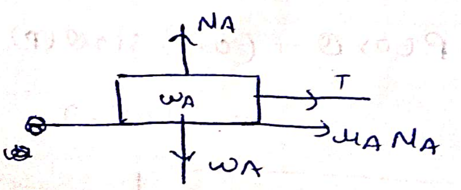
$N_B (1.623) = 1000$

$\boxed{N_B = 616.1 \text{ N}}$

$T = -0.686 (616.1)$

$\boxed{T = -422.6 \text{ N}}$

Now, FBD for A :-



$\Sigma F_x = 0$,

$\mu_A N_A + T = 0$

$N_A = \frac{422.5}{0.4}$

$N_A = 1056.25\,N$

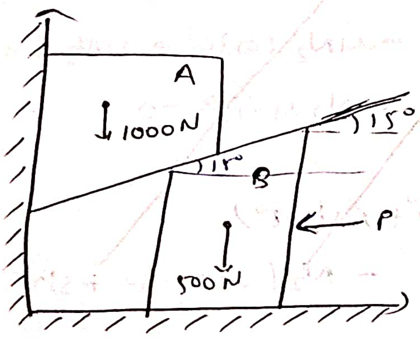
$\Sigma F_y = 0,$

$N_A - W_A = 0$

$W_A = N_A$

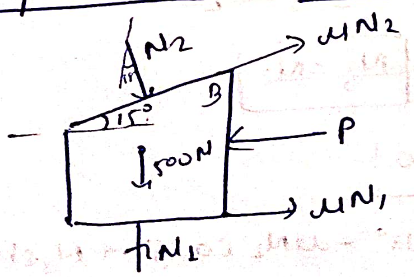
$\boxed{W_A = 1056.25\,N}$

Q.4 Block A weighing 1000 N is to be raised by means of 15° wedge B weighing 500 N. Assuming the coefficient of friction between all contact surfaces be 0.2, find what minimum horizontal force P should be applied to raised block.

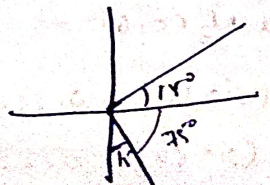


→ $\mu = 0.2$

FBD of ~~block B~~ : wedge B :-



$\Sigma F_x = 0$:-



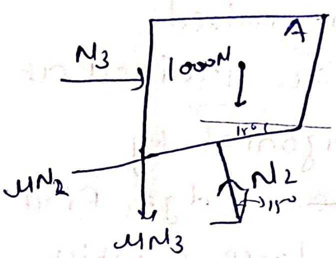
$\mu N_1 - P + \mu N_2 \cos 15° + N_2 \sin 15° = 0$ —— (1)

$\Sigma F_y = 0$:-

$N_1 - 500 + \mu N_2 \sin 15° - N_2 \cos 15° = 0$

$\boxed{N_1 + N_2(\mu \sin 15° - \cos 15°) = 500}$ —— (2)

Now, FBD of block A :-



$\Sigma F_x = 0$:-

$N_3 - \mu N_2 \cos 15° - N_2 \sin 15° = 0$

$\boxed{N_3 - N_2(\mu \cos 15° + \sin 15°) = 0}$ —— (3)

$\boxed{N_3 - N_2(0.45) = 0}$ —— (4)

$\Sigma y = 0$:-

$N_2 \cos 15° - 1000 - \mu N_3 - \mu N_2 \sin 15° = 0$

$N_2(\cos 15° - \mu \sin 15°) - \mu N_3 = 1000$

$\boxed{N_2(0.91) - 0.2 N_3 = 1000}$ —— (5)

$N_3 = 548.78\,N$ $N_2 = 1219.5\,N$

from eq. (1),

$N_1 = 500 - 1219.5(0.2\sin 15° - \cos 15°)$

$= 500 + 1114.8$

$\boxed{N_1 = 1614.8\,N}$

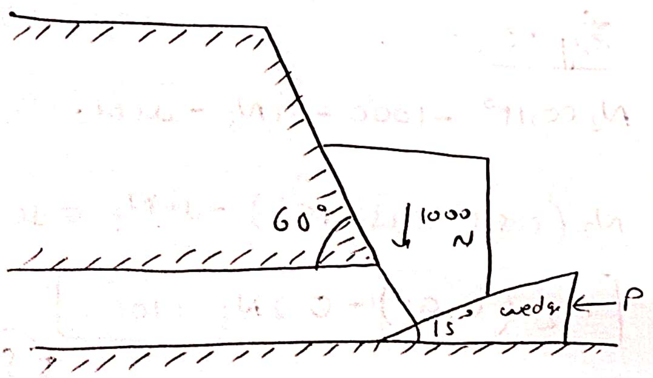
from eq. (1) :-

$(0.2)(1619.8) + (0.2)(1219.5)\cos 15^\circ + (1219.5)\sin 15^\circ = P$

$P = 322.96 + 235.59 + 315.63$

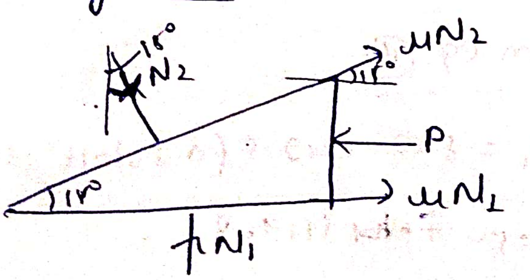
$\boxed{P = 874.18\ N}$

Q.5 A ~~surface~~ block weighing 1000 N is to be raised against a surface inclined at 60° to the horizontal by means of a 15° ~~me~~ wedge. Find the horizontal force P which will just start the block to move if the coefficient of friction between all the surfaces of contact be 0.2. Assume the wedge to be of negligible weight.



$\mu = 0.2$

FBD of wedge:-



$\Sigma F_x = 0$:-

$\mu N_1 - P + \mu N_2\cos 15^\circ + N_2\sin 15^\circ = 0$

$\mu N_1 - P + N_2(\mu\cos 15^\circ + \sin 15^\circ) = 0$

$\boxed{\mu N_1 - P + N_2(0.45) = 0}$ —(1)

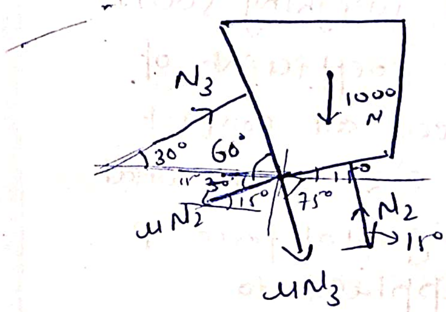
$\Sigma F_y = 0$:-

$N_1 + \mu N_2\sin 15^\circ - N_2\cos 15^\circ = 0$

$N_1 + N_2(\mu\sin 15^\circ - \cos 15^\circ) = 0$

$\boxed{N_1 - 0.91\, N_2 = 0}$

Now, FBD of block:-



$\Sigma F_x = 0$:-

$N_3\cos 30^\circ - \mu N_2\cos 30^\circ + \mu N_3\sin 15^\circ - N_2\sin 15^\circ = 0$

$N_3(\cos 30^\circ + \mu\sin 15^\circ) - N_2(\mu\cos 30^\circ + \sin 15^\circ) = 0$

$\cancel{(N_3 - N_2)(\cos 30^\circ + \mu\sin 15^\circ) = 0}$

$\cancel{0.91(N_3 - N_2) = 0}$

$\boxed{\cancel{N_3 = N_2}}$

$\Sigma F_x = 0$:-

$-N_2\sin 15^\circ - \mu N_2\cos 15^\circ + N_3\sin 60^\circ + \mu N_3\cos 60^\circ = 0$

$\Rightarrow -N_2(\sin 15^\circ + 0.2\cos 15^\circ) + N_3(\sin 60^\circ + 0.2\cos 60^\circ) = 0$

$N_2(0.45) - N_3(0.966) = 0$ —(III).

$\Sigma F_y = 0$:-

$N_2\cos15^\circ - N_2\sin15^\circ + N_3\cos60^\circ - \mu N_3\sin60^\circ$
$= 1000$

$\Rightarrow N_2(\cos15^\circ - 0.2\sin15^\circ) +$
$N_3(\cos60^\circ - 0.2\sin60^\circ) = 0$

$\Rightarrow (0.91)N_2 + (0.327)N_3 = 1000$ —(IV).

on solving, we get :-

$\boxed{N_2 = 941.3\,N}$
$\boxed{N_3 = 438.5\,N}$

by (2) :-

$N_1 = 0.91(N_2)$

$N_1 = 0.91 \times 941.3\,N$

$\boxed{N_1 = 856.58\,N}$

by eq. (1) :-

$0.2N_1 - P + 0.2N_2\cos15^\circ$
$+ N_2\sin15^\circ = 0$

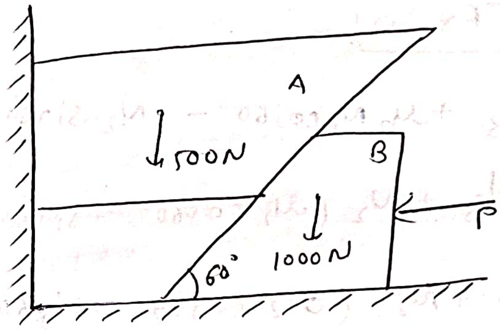
$P = 0.2(856.58) + N_2(0.45)$

$P = 171.32 + 423.58$

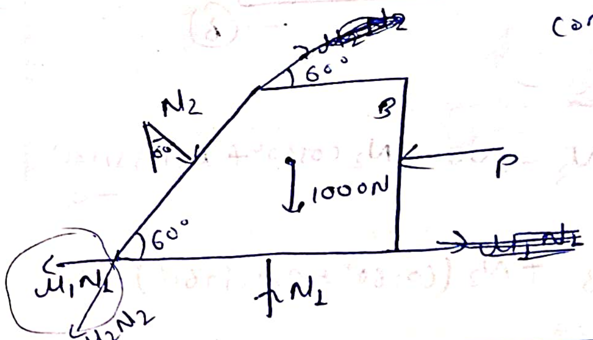
$\boxed{P = 594.89\,N}$

$\Rightarrow$ 596.76

Q.6 Two blocks A & B are resting against a wall and the floor as shown in fig. Find the value of the horizontal force P applied to the lower block that will hold the system in equilibrium, coefficients of friction are 0.25 at the floor; 0.3 at the wall; 0.2 between the blocks.



→ FBD of ~~wedge~~ block B :-



condition of eq.

$\boxed{\mu_1 = 0.25,\ \mu_2 = 0.2,\ \mu_3 = 0.3}$

(*) agar P nahi lagayenge to friction ander ki taraf dagega

$\Sigma F_x = 0$:-

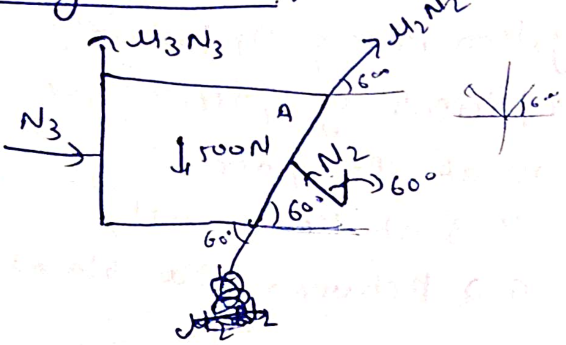
$\boxed{N_2\sin60^\circ + \mu_2N_2\cos60^\circ - P - \mu_1N_1 = 0}$ —(1)

$\Sigma F_y = 0$:-

$\boxed{N_1 - 1000 + \mu_2N_2\sin60^\circ - N_2\cos60^\circ = 0}$

eq —(2).

FBD of block A :-



$\Sigma F_x = 0$:-

$N_3 + \mu_2 N_2 \cos 60° - N_2 \sin 60° = 0$

$N_3 + N_2(\mu_2 \cos 60° - \sin 60°) = 0$

$N_3 + N_2(0.2 \cos 60° - \sin 60°) = 0$

$\boxed{N_3 - 0.766 N_2 = 0}$ —(3)

$\Sigma F_y = 0$:-

$\mu_3 N_3 - 500 + N_2 \cos 60° + \mu_2 N_2 \sin 60° = 0$

$0.3 N_3 + N_2((\cos 60° + 0.2 \sin 60°) = 500$

$\boxed{0.3 N_3 + (0.67) N_2 = 500}$ —(4)

on solving eq. (3) & (4), we get,

$\boxed{N_3 = 425.65 \text{ N}}$

$\boxed{N_2 = 555.68 \text{ N}}$

by eq. (2) :-

$N_1 = 1000 + N_2(\mu_2 \sin 60° + \cos 60°)$

$N_1 = 1000 + (0.67)(555.68)$

$\boxed{N_1 = 1372.3 \text{ N}}$

from eq. (1) :-

$P = -(0.25)(1372.3) - (0.2)(555.68)\cos 60° + (555.68)\sin 60°.$

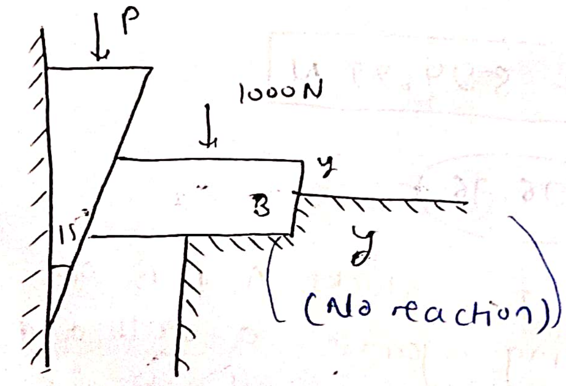
$P = -343 - 55.57 + 481.23$

$\boxed{P = 82.66 \text{ N}}$

(*). Isme FBD mein friction ki direction ko bina P force ke consider kiya hai.

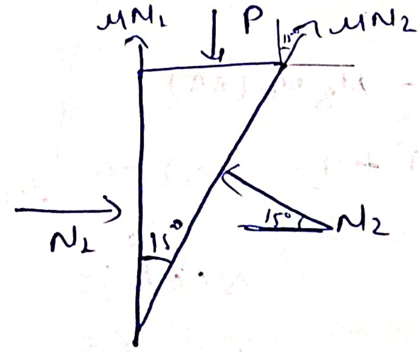
Q.7 A 15° wedge of negligible weight is to be driven to tighten a body B which is supporting a vertical load of 1000 N. If the coefficient of friction for all contacting surfaces be 0.25. Find the minimum force P required to drive the wedge.

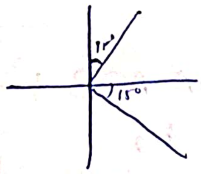
(Assume the reaction of surface y-y as zero).



→ $\boxed{P = ? \text{ (minimum)}}$

FBD of wedge:-





$\Sigma F_x = 0$:-

$N_1 - N_2 \cos 15° + \mu N_2 \sin 15° = 0$

$N_1 + N_2 (0.25 \sin 15° - \cos 15°) = 0$

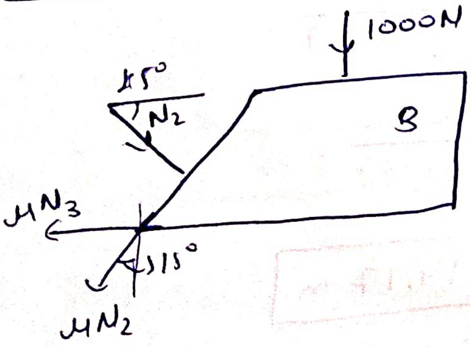
$\boxed{N_1 - 0.9 N_2 = 0}$ — (1)

$\Sigma F_y = 0$:-

$\mu N_1 - P + N_2 \sin 15° + \mu N_2 \cos 15° = 0$

$\boxed{0.25 N_1 - P + N_2 (\sin 15° + (0.25)\cos 15°) = 0}$ — (2).

FBD of Block B :-



$\Sigma F_x = 0$:-

$-\mu N_3 - \mu N_2 \sin 15° + N_2 \cos 15° = 0$

$\rightarrow -0.25 N_3 + N_2 (\cos 15° - 0.25 \sin 15°) = 0$

$\boxed{-0.25 N_3 + 0.9 N_2 = 0}$ — (3)

$\Sigma F_y = 0$:-

$N_3 - 1000 - \mu N_2 \cos 15° - N_2 \sin 15° = 0$

$\rightarrow N_3 - N_2 (\mu \cos 15° + \sin 15°) = 1000$

$\boxed{N_3 - N_2 (0.5) = 1000}$ — (4).

by solving eq (3) & (4) :-

$\boxed{N_3 = 1161.29 \text{ N}}$

$\boxed{N_2 = 322.58 \text{ N}}$

from eq: (1) :-

$N_1 = 0.9 \times 322.58 \text{ N}$

$\boxed{N_1 = 290.32 \text{ N}}$

from eq. (2) :-

$P = 0.25 (290.32) + (322.58)(0.5)$

$\boxed{P = 233.96}$ Ans

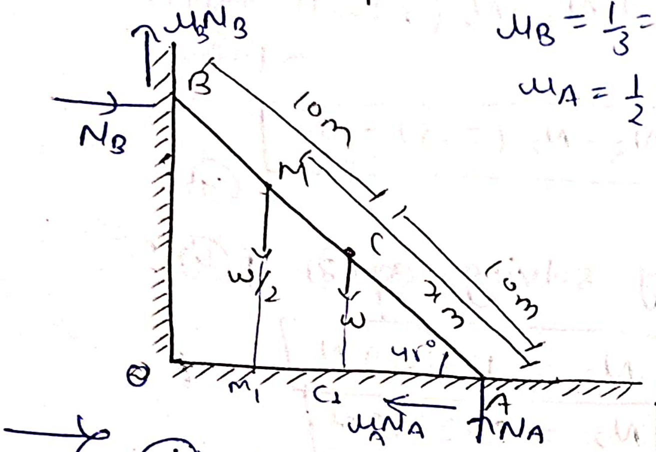
(*) Isme sirf reaction wali cheez par dhyan dena tha.

Q.-8 A uniform ladder AB of length $l = 20$ m and weight W is supported by the horizontal floor at A. and by a vertical wall at B. It makes an angle 45° with the horizontal. If a man whose weight is one-half that of the ladder, ascends the ladder, how much length x of the ladder he shall climb before the ladder slips.

Given:-

$\mu_B = \frac{1}{3} = 0.33$

$\mu_A = \frac{1}{2} = 0.5$



→ (*) Moment apply karna padega

$\Sigma F_x = 0$:-

$N_B - \mu N_A = 0$

$\boxed{N_B = \mu_A N_A}$

$N_B = 0.5 N_A \Rightarrow \boxed{\frac{1}{2} N_A = N_B}$ —(1)

$\Sigma F_y = 0$:-

$N_A + \mu_B N_B - \omega - \frac{\omega}{2} = 0$

$\Rightarrow N_A + \frac{1}{3}\left(\frac{1}{2} N_A\right) = \frac{3\omega}{2}$

$\Rightarrow N_A + \frac{1}{6} N_A = \frac{7}{6} N_A = \frac{3W}{2}$

$\boxed{N_A = \frac{9}{7} W}$ —(2).

in eq. (1) :-

$N_B = \frac{1}{2} N_A$

$\boxed{N_B = \frac{9}{14} \omega}$ —(3).

Moment about A = 0 :-

$\Sigma M_A = 0$..

$$\boxed{-N_B (OB) - \mu_B N_B (OA) + \omega (C_1A) + \frac{\omega}{2} (AM_1) = 0}$$ —(4).

(#) ΔAOB :-

$\sin 45° = \frac{1}{\sqrt{2}}$

$\frac{OB}{20} = \frac{1}{\sqrt{2}}$

$\boxed{OB = 10\sqrt{2}}$

$\boxed{OB = 14.14 m}$

$\boxed{OA = 14.14 m}$

ΔCC_1A :-

$C_1A = 5\sqrt{2}$

$\boxed{C_1A = 7.07 m}$

ΔM_1AM :-

$\boxed{M_1A = \frac{x}{\sqrt{2}} m}$

$\boxed{M_1A = 0.707x\, m}$

In eq. (4) :-

$-\frac{9}{14}(14.14)\cancel{\omega} - 0.33\left(\frac{9}{14}\cancel{\omega}\right)(14.14)$

$+\cancel{\omega}(7.07) + \frac{\cancel{\omega}}{2}(0.707)x = 0$

$-12.08 + 7.07 + \frac{0.707x}{2} = 0$

$x = \frac{5.01 \times 2}{0.707}$

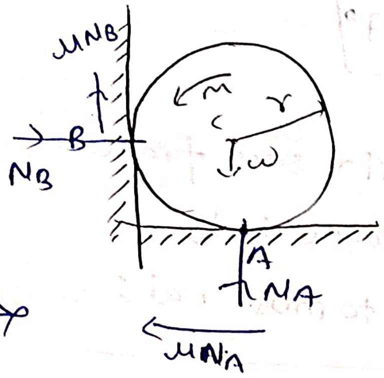
$x = \frac{10.02}{0.707}$

$\Rightarrow \boxed{x = 14.17 m}$

$\Rightarrow$ 14.437

Q.-9 A homogeneous cylinder of weight W rests on a horizontal floor in contact with a wall. If the co-efficient of friction for all contact surfaces be μ, determine the couple M acting on the cylinder which will start counter-clockwise rotation.

$M = ?$



→

$\Sigma F_x = 0$:-

$N_B - \mu N_A = 0$

$$\boxed{N_B = \mu N_A} \quad -(1)$$

$\Sigma F_y = 0$:-

$\mu N_B + N_A - w = 0$

$\mu^2 N_A + N_A - w = 0$

$N_A(\mu^2 + 1) = w$

$$\boxed{N_A = \frac{w}{1+\mu^2}} \quad -(2)$$

from eq. (1) :-

$$\boxed{N_B = \frac{\mu w}{1+\mu^2}} \quad -(3)$$

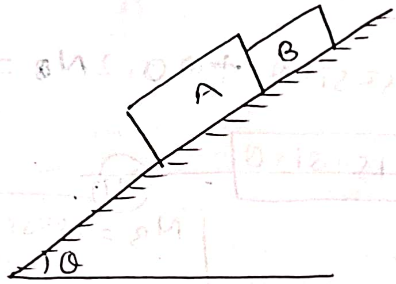
$\Sigma M_A = 0$ → moment =

$M - \mu N_B r - N_B r = 0$

$M = \mu r\left(\frac{\mu w}{1+\mu^2}\right) + \frac{\mu w r}{1+\mu^2}$

$$\boxed{M = \frac{\mu^2 w r + \mu w r}{1+\mu^2}} \quad \text{Ans}$$

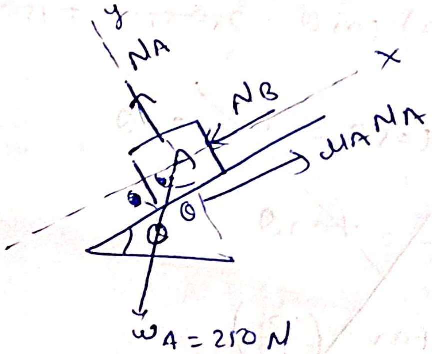
Q-10 Two blocks A & B of weights 250N and 150N rest on a plane which is slowly raised from the horizontal position to an angle θ. Find the maximum angle that can be reached before bodies slip down the incline. Assume :- μ between block B and the plane = 0.2 and μ between block A & the plane = 0.3.



→ Given :- $\mu_A = 0.3$

$\mu_B = 0.2$

FBD of block A :-



$\Sigma F_x = 0$:-

$\mu_A N_A - N_B - w_A \sin\theta = 0$

$$\boxed{0.3 N_A - N_B - 250 \sin\theta = 0} \quad -(1)$$

$\Sigma F_y = 0$:-

$N_A - w_A \cos\theta = 0$

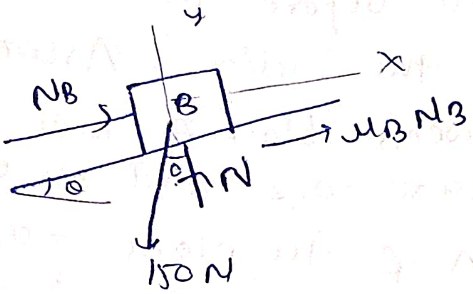
$$\boxed{N_A = 250 \cos\theta} \quad -(2)$$

in eq. (1) :-

$(0.3)(210)\cos\theta - 210\sin\theta = N_B$

— (11).

FBD of Block B :-



$\Sigma F_x = 0$:-

$N_B - 150\sin\theta + 0.2 N_B = 0$

$\boxed{\cancel{N_B = 150\sin\theta}}$ — ~~(111)~~

$\boxed{N_B = 150\sin\theta - 150(0.2)\cos\theta}$

$\Sigma F_y = 0$:-

$N - 180\cos\theta = 0$

$\boxed{N = 180\cos\theta}$

in eq. (11) :-

$\cancel{(0.3)(250)\cos\theta - 250\sin\theta = 150\sin\theta}$

$\cancel{75\cos\theta = 400\sin\theta}$

$\cancel{\frac{75}{400} = \tan\theta}$

$\cancel{\theta = \tan^{-1}\left(\frac{3}{16}\right)}$

$\cancel{\boxed{\theta = 10.61^\circ}}$

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$\boxed{N_B = 180\sin\theta - 30\cos\theta}$

in eq (11) :-

$75\cos\theta - 250\sin\theta = 180\sin\theta - 30\cos\theta$

$105\cos\theta = 400\sin\theta$

$$\tan\theta = \cancel{\frac{105}{400}}\ \frac{21}{\cancel{80}}$$

$$\theta = \tan^{-1}\left(\frac{21}{80}\right)$$

$$\boxed{\theta = 14.7^{\circ}}$$