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Department of Mathematics, Bioinformatics and Computer Applications

Assignment-I

Programme	B.Tech	Semester	Semester II (2024-25)
Course code	MTH24110	Section	G
Course Title	Mathematics-II	Department	MBC

Q. No.	Question Text	CO	BT
1.	Using the application of Gaussian elimination, solve the following system of equations $$\begin{aligned} x_1 - 3x_2 + 2x_3 - x_4 + 2x_5 &= 2, \\ 3x_1 - 9x_2 + 7x_3 - x_4 + 3x_5 &= 7, \\ 2x_1 - 6x_2 + 7x_3 + 4x_4 - 5x_5 &= 7. \end{aligned}$$	1	5
2.	Discuss the solutions of the given system of linear equations for given constant b $$\begin{aligned} x_1 + 2x_2 - 3x_3 - 2x_4 + 4x_5 &= 1, \\ 2x_1 + 5x_2 - 8x_3 - x_4 + 6x_5 &= 4, \\ x_1 + 4x_2 - 7x_3 + 5x_4 + 2x_5 &= 8, \\ 2x_1 - x_2 + 5x_3 - 7x_4 + x_5 &= 0, \\ -x_1 - 11x_2 + 22x_3 - 14x_4 - 5x_5 &= b. \end{aligned}$$	1	4,5
3.	Determine the inverse of the following matrices using the application of Gauss Jordan method. (a) $\begin{bmatrix} 1 & 2 & -4 \\ -1 & -1 & 5 \\ 2 & 7 & -3 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 4 & -4 \\ 1 & 5 & -1 \\ 3 & 13 & -6 \end{bmatrix}$	1	5
4.	Solve the given system of linear equations using LU decomposition. $$\begin{aligned} x_1 + 2x_2 + x_3 &= 5 \\ 2x_1 + 3x_2 + 3x_3 &= -3 \\ -3x_1 - 10x_2 + 2x_3 &= 21 \end{aligned}$$	1	5
5.	The characteristic equation of a matrix A is given as $\lambda^3 - 7 = 0$. What can you say about A^{-1}? Give reasons in support of your answer.	1	1,2,4

6.	For the given matrix (1) Find the eigenvalues and the eigenvectors. (2) Verify that $det(A)$ = product of all eigenvalues (3) Verify Cayley-Hamilton Theorem and find A^{-1} and A^6. $\begin{bmatrix} 5 & 2 & 2 \\ 3 & 6 & 3 \\ 6 & 6 & 9 \end{bmatrix}$	1,2	3,4,5
7.	Find all eigenvalues and eigenvectors and verify Cayley-Hamilton Theorem for each of the given matrices. (a) $\begin{bmatrix} 4 & 3 & 0 & 0 \\ -2 & -1 & 0 & 0 \\ 0 & 0 & 4 & 3 \\ 0 & 0 & -3 & -2 \end{bmatrix}$ (b) $\begin{bmatrix} -2 & 2 & 3 \\ 2 & 1 & 6 \\ 3 & 6 & 6 \end{bmatrix}$ (c) $\begin{bmatrix} 6 & 2 & -2 \\ 2 & 6 & -2 \\ -2 & -2 & 10 \end{bmatrix}$	1,2	4,5
8.	If the characteristic equation of a matrix has no term of degree 0 (i.e. has no constant term), then the matrix is singular. True or False? Support your answer with reasons.	1	1,2,4
9.	Given the characteristic equation of a matrix is $\lambda^6 - 7\lambda^4 + 25\lambda - 125 = 0$. Using the fact that $det(A)$ = product of all eigenvalues, what is $det(A)$?	1	1,2,4
10.	Given a system of linear equations $AX = B$. Let $[A : B]$ denote the augmented matrix of the system. Which of the following cases are possible and if possible, what happens to the solution set of the given system in that case. (1) $\text{rank}(A) < \text{rank}([A : B])$ (2) $\text{rank}(A) = \text{rank}([A : B])$ (3) $\text{rank}(A) > \text{rank}([A : B])$	1	2,3,4,5,6
11.	Find all eigenvalues and eigenvectors for the given matrix and its transpose. $A = \begin{bmatrix} 2 & 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 & 2 \end{bmatrix}$ What can you say about the eigenvalues and eigenvectors of $7A^{-1} + 6I$ where I denotes the identity matrix.	1	2,4,5

Course Outcome (CO)

CO1: Demonstrate the ability to solve linear systems and perform matrix operations, including determining the rank, eigenvalues, and eigenvectors.
CO2: Apply the Cayley-Hamilton theorem to solve matrix-related problems.
CO3: Solve ordinary differential equations using Laplace transforms and interpret inverse Laplace transforms for engineering applications.
CO4: Develop proficiency in Fourier series and Fourier transforms and their application in signal analysis.
CO5: Analyze and solve partial differential equations (PDEs), including boundary value problems for heat and wave equations.

Bloom Taxonomy (BT)

1-Remember; 2-Understand; 3-Apply; 4-Analyze; 5-Evaluate; 6-Create