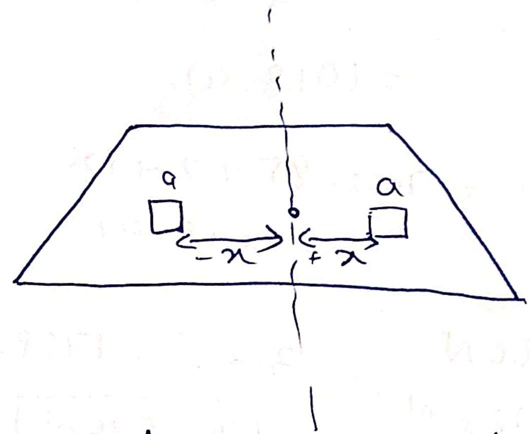
Centroid

- Moment of whole lamina Area about centroidal axis = zero.



sum of moment of all elemental components about axis of symmetry

$= ax - ax + - -$

$=$ zero.

- difference between centre of gravity & centroid :-

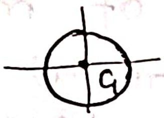
(I). The term C.G. apply to bodies having mass and weight and centroid apply to plane lamina only.

(II). C.G. of a body is a point through which resultant gravitational act for any orientation of body whereas centroid is point in a plane Area

Such that the moment of area about any axis through that point is zero.

- Centroid :-

① Circle :-

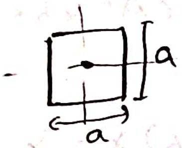


= centre

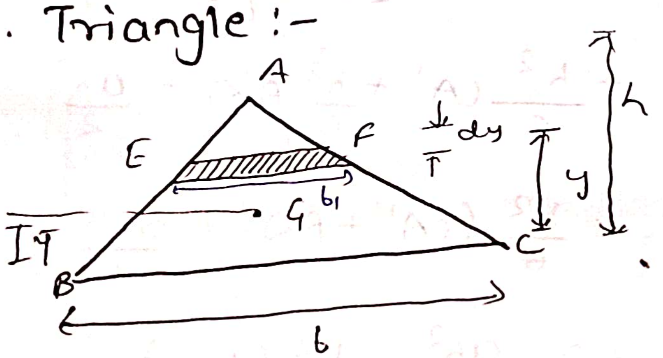
② Rectangle :-

$\left(\frac{b}{2}, \frac{d}{2}\right)$

③. square :-



④. Triangle :-



ΔABC & ΔAEF → similar Δs.

$$\frac{b_1}{b} = \frac{h-y}{h}$$

$$b_1 = \frac{b(h-y)}{h} = b\left(1-\frac{y}{h}\right)$$

Area of whole lamina :-

$$A = \frac{1}{2} bh$$

Area of elemental component :- $dA = b_1 \times dy$

(due to very small elemental component)

$$dA = b\left(1-\frac{y}{h}\right)dy$$

$$\text{Centroid} = \frac{\text{moment of area}}{\text{total area}}$$

$$= \frac{\int y\, dA}{A}$$

$$= \frac{\int y \cdot b\left(1-\frac{y}{h}\right)dy}{A}$$

Centroid =

$$b\left[\frac{y^2}{2} - \frac{y^3}{3h}\right]_0^h$$

$$= \frac{bh^2}{6A}$$

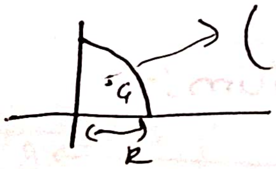
$$= \frac{h}{3}$$

$$\boxed{\bar{y} = \frac{h}{3}}$$

⑤. semicircle :-

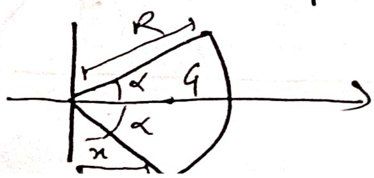
$$\bar{y} = \frac{4R}{3\pi}$$

⑥. Quarter circle :-



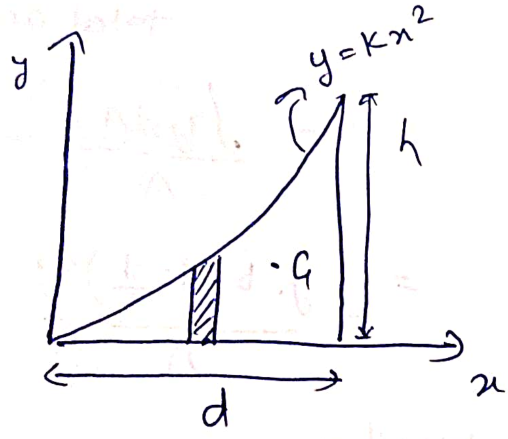
$\left(\frac{4R}{3\pi}, \frac{4R}{3\pi}\right)$

⑦. Section of a circle :-



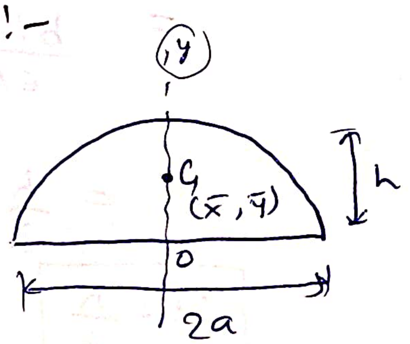
$$\boxed{\bar{x} = \frac{2R}{3\alpha}\sin\alpha}$$

(8). Parabolic spendrel :-



$$\bar{Y} = \frac{3h}{10}$$

(9). Parabola :-

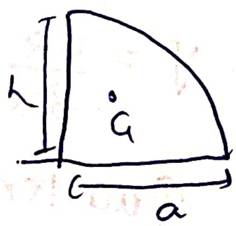


$$\boxed{\bar{y} = \frac{3h}{5}}$$

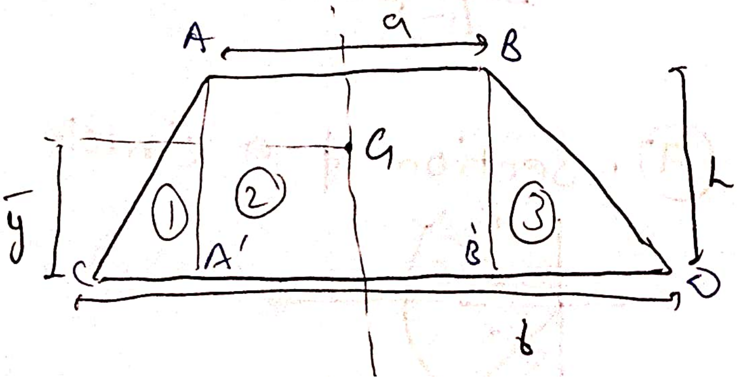
(10). semi-parabola :-

$$\bar{x} = \frac{3a}{8}$$

$$\bar{y} = \frac{3h}{5}$$



(11). Trapezium :-



elemental component,

(i) $\Delta CAA'$

(ii) $\square ABA'B'$

(iii). $\Delta BB'D$

Sum of moment of all elemental component about base (CD)
= moment of whole lamina about base (CD)

$$= \left\{\frac{1}{2} \times CA' \times h \times \frac{h}{3}\right\} + \left\{(a \times h) \times \frac{h}{2}\right\} + \left[\frac{1}{2} \times B'D \times h \times \frac{h}{3}\right]$$

$$= \frac{h^2}{6} CA' + \frac{h^2}{6} B'D + \frac{ah^2}{2}$$

$$= \frac{h^2}{6}(CA' + B'D) + \frac{ah^2}{2}$$

$$= \frac{ah^2}{2} + \frac{h^2}{6}(b-a)$$

$$\boxed{= \frac{h^2}{6}[2a+b]}$$

Total area of lamina :-

$$A = \frac{1}{2}(a+b) \times h$$

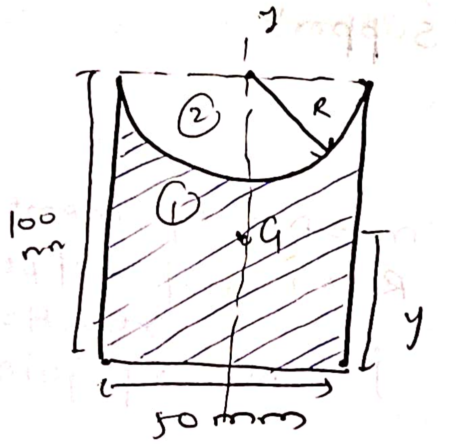
$$\bar{y} = \frac{\text{moment of whole lamina}}{\text{total area}}$$

$$= \frac{\frac{h^2}{6}(2a+h)}{\frac{1}{2}(a+b)h}$$

$$\boxed{\bar{y} = \frac{h}{3}\left(\frac{2a+b}{a+b}\right) \text{ from base}}$$

$$\boxed{\bar{y} = \frac{h}{3}\left(\frac{2b+a}{b+a}\right) \text{ from top}}$$

* Shaded lamina:-



Let $\bar{y}$ be the position of centroid from bottom of lamina.

To determine $\bar{y}$, divide the whole lamina into no. of elemental components.

elemental component →

① rectangle $(b \times d)$

$= 50\text{ mm} \times 100\text{ mm}$

② semicircle $(R) = \frac{50}{2} = 25\text{ mm}$

Now area,

$a_1 = 50 \times 100 = 5000\text{ mm}^2$

$a_2 = \frac{1}{2}\pi R^2 = \frac{1}{2}\pi(25)^2$

$= 981.25\text{ mm}^2$

Centroidal distance of components from bottom of lamina,

elemental component ①

$\rightarrow y_1 = \frac{100}{2} = 50\text{ mm}$ (due to rectangle d/2)

elemental component ②

$\rightarrow y_2 = 100 - \frac{4R}{3\pi}$

[due to semicircle from diameter]

$y_2 = 100 - \frac{4(25)}{3\pi}$

$$\boxed{y_2 = 89.38\text{ mm}}$$

Now, $\bar{y} = \frac{a_1 y_1 - a_2 y_2}{a_1 - a_2}$

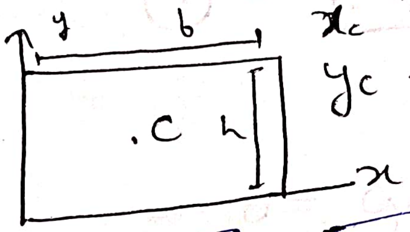
$= \frac{5000 \times 50 - 981.25 \times 89.38}{5000 - 981.25}$

~~$\bar{y} = 40.9$~~

$$\boxed{\bar{y} = 40.38\text{ mm}}$$

Centroid of various sections

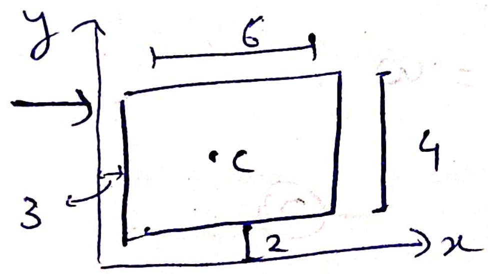
1. Rectangle :-



$x_c = b/2$

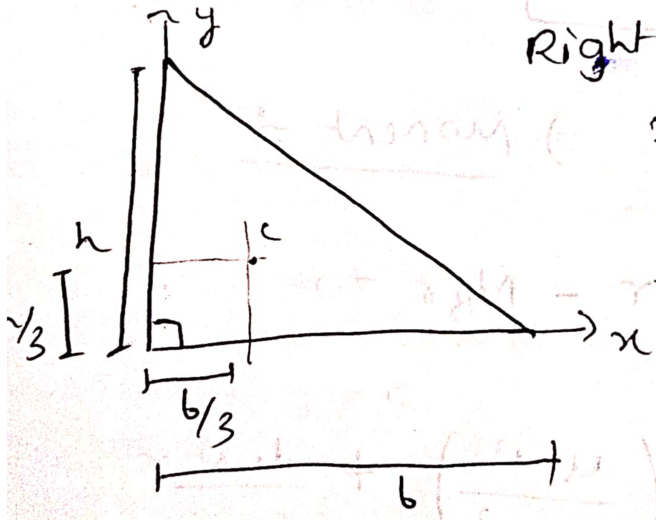
$y_c = h/2$

$\boxed{x_c = b/2}$, $\boxed{y_c = \frac{h}{2}}$



$\boxed{x_c = 6}$, $\boxed{y_c = 4}$

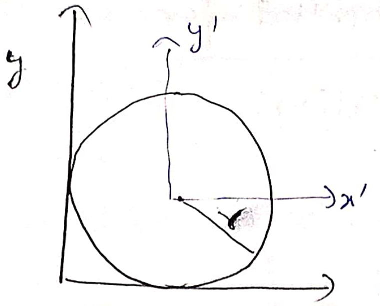
2. Triangle :-



Right angled triangle

$\boxed{x_c = b/3}$, $\boxed{y_c = h/3}$

3. circle :-

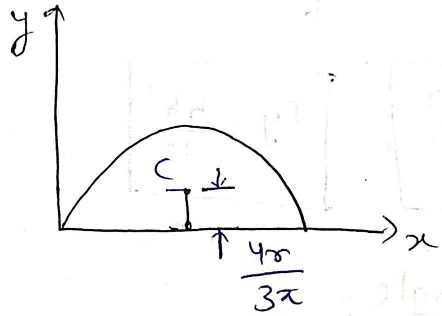


$\boxed{x_c = r}$

$\boxed{y_c = r}$

$\boxed{x'_c = 0}$, $\boxed{y'_c = 0}$

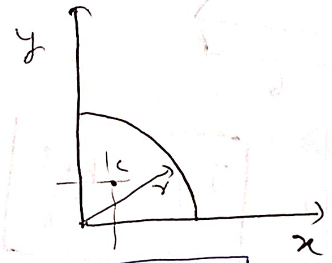
4). Semi-circle:-



$\boxed{x_c = r}$

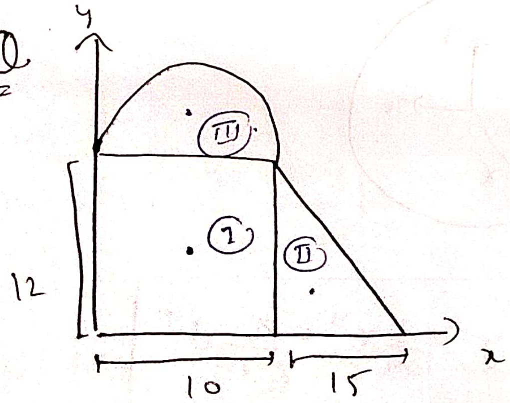
$\boxed{y_c = \frac{4r}{3\pi}}$

5). Quarter of a circle:-



$\boxed{x_c = \frac{4r}{3\pi}}$, $\boxed{y_c = \frac{4r}{3\pi}}$

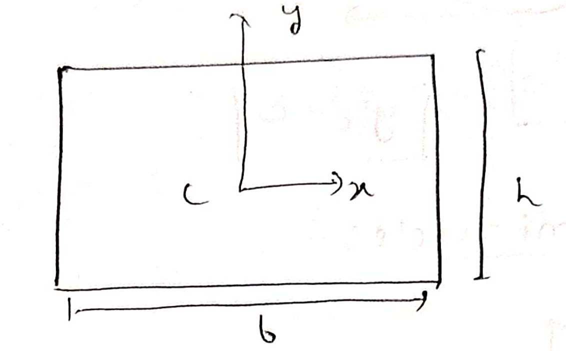
Q



Section ①:- $x_{c_1} = 5$, $y_{c_1} = 6$.

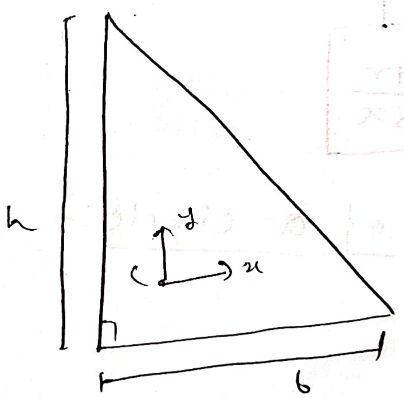
- Moment of Inertia for Various Section :-

①. Rectangle :-



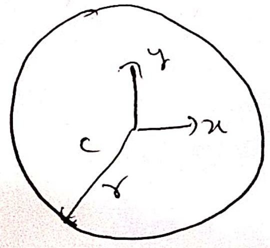
$$\boxed{I_x = \frac{bh^3}{12}}, \quad \boxed{I_y = \frac{hb^3}{12}}$$

②. Triangle :-



$$\boxed{I_x = \frac{bh^3}{36}}, \quad \boxed{I_y = \frac{hb^3}{36}}$$

③. Circle :-



$$\boxed{I_x = I_y = \frac{\pi}{4} r^4 = \frac{\pi}{64} d^4}$$