Analytic function →

→ regular or harmonic

$w = u + iv$

$\frac{du}{dx} = \frac{dv}{dy}$

$\frac{du}{dy} = \frac{-dv}{dx}$

Q1. $f(z) = z^2$

$= x^2 + 2ixy + i^2y^2$

$f(z) = x^2 - y^2 + 2xyi$

$$\boxed{\begin{aligned} \frac{du}{dx} &= 2x \\ \frac{du}{dy} &= 2x \\ \frac{dv}{dy} &= -2y \\ \frac{du}{dx} &= 2y \end{aligned}}$$

CR eq satisfy → harmonic

(2) $e^{-x}\cos y + ie^{-x}\sin y$

$u + iv$

$\frac{du}{dx} = -e^{-x}\cos y$

$\frac{du}{dy} = -e^{-x}\sin y$

$\frac{dv}{dy} = e^{-x}\cos y$

$\frac{dv}{dx} = -e^{-x}\sin y.$

$\left[\begin{matrix} \frac{du}{dx} \neq \frac{dv}{dy} \\ \frac{du}{dy} \neq \frac{dv}{-dx} \end{matrix} \right]$ → not satisfy.

$$f(z) = \boxed{\sin z} = \sin(x+iy)$$
$$= \sin x \cos iy + \cos x \sin iy$$
$$= \sin x \cosh y + i \cos x \sinh y.$$

$u = \sin x \cosh y$
$v = \cos x \sinh y$

$\cos iy = \cosh y$
$\sin iy = i \sinh y.$

$$\frac{du}{dx} = \cos x \cosh y$$

$$\frac{du}{dy} = \sin x \sinh y$$

$$\frac{dv}{dy} = \cos x \cosh y$$

$$\frac{dv}{dx} = -\sin x \cosh y.$$

CR eq^n $\rightarrow$ ✗

$$f'(z) = \frac{du}{dx} + i\frac{dv}{dx}.$$

$$f'(z) = \frac{du}{dx} + i\frac{dv}{dx}.$$
$$\cos x \cosh y - i(\sin x \cosh y)$$
$$\cos x \cos iy - \sin x \sin iy$$
$$\cos(x+iy)$$
$$\boxed{\cos z}$$

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analytic ✓ ✗

z^n $\sin z$

at all point

e^z

$\cos z$.

⇤ Harmonic eqⁿ ⇥

$f(z) = u + iv \rightarrow u = x^2 - y^2$

$\frac{d^2u}{dx^2} = 2 \quad \frac{d^2u}{dy^2} = -2$

$\downarrow$

0

$\frac{du}{dx} = \frac{dv}{dy}. \qquad \frac{du}{dy} = -\frac{dv}{dx}$

$\frac{d^2u}{dx^2} = \frac{d^2v}{dx\,dy}. \qquad \frac{d^2u}{dy^2} = -\frac{d^2v}{dx\,dy}$

$+$

$\frac{d^2u}{dx^2} + \frac{du^2}{dy^2} = 0$

$\frac{d^2v}{dx^2} + \frac{dv^2}{dy} = 0$

③ $u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 2x + 1.$

$\frac{d^2u}{dx^2} + \frac{d^2u}{dy^2} = 0$

$dv = \frac{dv}{dx} \times dx + \frac{dv}{dy} \times dy$

$= -\frac{du}{dy} dx + \frac{du}{dx} dy$

$= 2y\,dx + 2x\,dy$

$= 2(y\,dx + x\,dy)$

$\int dv = \int 2\,d(xy) + c$

$\boxed{2xy + c = v}$

$dv = \frac{dv}{dx} \times dx + \frac{dv}{dy} \times dy.$

$= -\frac{du}{dy} \times dx + \frac{du}{dx} dy$

$= 6xy\,dx + 3x^2 dy + 6(y\,dx + x\,dy) - \int 3y^2 + 2\,dy$

$\int v = \int (3x^2y)\,dxy + 6\int d(xy) - y^3 + 2y + c$

$\boxed{v = 3x^2y + 6xy - y^3 + 2y + c}$

②

$3x^2y - y^3 = v(x, y)$

1. $\frac{d^2v}{dx^2} + \frac{\partial^2 v}{\partial y^2} = 0$ ✓

$du = \frac{du}{dx} dx + \frac{du}{dy} dy$

$= \frac{dv}{dy} dx - \frac{dv}{dx} dy$

$= (3x^2 - 3y^2)dx - 6xy\,dy = 3x^2 dx - 3(y^2 dx + 2xy\,dy)$

$\int du = 3\int x^2 dx - 3\int d(xy^2) + c$

$\boxed{u = x^3 + 3xy^2 + c}$

Milne Thomson method →

→ $v = x^3 - 3xy^2$

if p real part of analytical funct $f(z)$ find $f(z)$

$\frac{du}{dx} = 3x^2 - 3y^2$

$\frac{du}{dy} = -6xy$

$\phi_1(z,0) = 3z^2 - 0$

$\phi_2(z,0) = 0$

$f(z) = \int (3z^2 - i0)\, dz + c$

$\boxed{f(z) = z^3 + c}$

→

$3x^2y - y^3$ imp. part of analytical function

$\frac{dv}{dy} = 3x^2$ $\quad \frac{dv}{dx} = 6xy$

$\phi_1(z,0) = 3z^2$ $\quad \phi_2(z,0) = 0$

$f(z) = \int \phi_1(z,0) + i\phi_2(z,0)\, dz + c$

$= \int 3z^2\, dz + c$

$f(z) = z^3 + c$

$(u-v) = (x-y)(x^2+4xy+y^2)$

analytic fund $f(z) = u+iv$

$f(z) = u+iv$

$if(z) = iu-v$

$(1+i)f(z) = (u-v)+i(u+v)$

$\boxed{\begin{array}{l} f(z) = U+iV \\ F = V+iV \end{array}}$

$V = (x-y)(x^2+4xy+y^2)$

$\frac{dv}{dx} = (x^2+4xy+y^2) + (x-y)(2x+4y)$

$\phi_1(z,0) = z^2+(z-0)(2z+0) = 3z^2$

$\frac{dV}{dy} = -(x^2+4xy+y^2) + (x-y)(4x+2y)$

$\phi_2(z,0) = -(z^2)+(z-0)(4z+0)$

Real part

$f(z) = \int(3z^2 - 3z^2 i)\,dz = 0$

$(1+i)f(z) = 3(1-i)\int z^2 dz = 0.$

$f(z) = \frac{(1-i)}{(1+i)} z^3 + \frac{c}{1+i}$

$\boxed{f(z) = \frac{(1-i)(1-i)}{1-i^2}(z^3) + \frac{c}{1+i}}$

$$\# \quad u+v = \frac{2\sin 2x}{e^{2y}+e^{-2y}-2\cos 2x}.$$

$$u+v = \frac{2\sin 2x}{2\cosh 2y - 2\cos 2x}. \qquad (1+i)f(z) = (u-v)+i(u+v)$$

imaginary

$$v = \frac{\sin 2x}{\cosh 2y - \cos 2x}$$

$$\frac{dv}{dy} = \frac{-\sin 2x * 2\sinh 2y}{(\cosh 2y - \cos 2x)^2}$$

$$\phi(z,0) = 0.$$

$$\frac{dv}{dx} = \frac{(\cosh 2y - \cos 2x)(2\cos 2x) - \sin 2x\,(0+2\sin 2x)}{(\cosh 2y - \cos 2x)^2}$$

$$\phi_2(z,0) = \frac{2}{1-\cos 2z}.$$

$$f(z) = \int \phi_1(z,0) + i\phi_2(z,0)\,dz + C$$

$$(1+i)f(z) = \int \left(0 + \frac{2i}{1-\cos 2z}\right) dz + C.$$

$$f(z) = \frac{1}{1+i} \cdot 2i \int \frac{1}{2\sin^2 z}\ \cancel{1-\cos}\ \operatorname{cosec}^2 dz$$

$$\boxed{f(z) = \frac{-i}{1+i} \cot z + C}$$

Q. $\dfrac{2xy^2}{x^3+3y^3}$ $\neq f(0,0)$

$= f(0,0)$.

$y = mx$.

$(mx)^2 x 2$ $\quad \frac{0}{0}$ form

$\dfrac{2m^2x^3}{x^3+3m^3x^3}$

$\dfrac{2m^2\cancel{x^3}}{\cancel{x^3}(1+3m^3)} = \dfrac{2m^2}{1+3m^3}$ does'nt exist.

X

Q. $\lim_{x \to 0} \dfrac{xy^3}{x^2+y^6}$.

$y = x$

$\dfrac{x^4}{x^2+x^6} = \dfrac{x^2}{1+x^4} = 0$ exist

$y^2 = x$. $= \frac{1}{2}$

$y \to 0$.

Limit is diff

X exist

Q. x^2+4y

$x \to 1$

$y \to 2$ $\quad 9 \neq 0$

X continue

$$\begin{cases} \frac{y^2-x^2}{x^2+y^2} \\ 0 \end{cases}$$

$y = mx$.

$\lim_{x \to 0} (x, mx) = 0$

limit exist

$f(x,y) = f(0,0)$

continuous.

$\frac{3x-2y}{2x-3y}$ $\lim_{x \to 0} \frac{3-2m}{2-3m}$.

dys. $f(x) =$ at $x=0$

$\lim \frac{f(x)-f(0)}{x-a}$ exist