Q.1 Find $\lim\limits_{x \to \infty} \dfrac{\sqrt{x} - x}{\sqrt{x} + x}$.

Q.2 Find the value of $\lim\limits_{x \to 0} \left[\dfrac{8 \text{ in } [x-3]}{[x-3]} \right]$.

where $[\cdot]$ represents the greatest integer function.

Q.3 Discuss the continuity of the function :

$$f(x, y) = \begin{cases} \dfrac{x^2 y^2}{x^4 + y^4} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$$

Q.4 Show that the function given by :

$$f(x, y) = \begin{cases} \dfrac{xy(x^2 - y^2)}{x^2 + y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$$

is differentiable at the origin.

Q.5 Examine the differentiability of :

$$f(x, y) = \begin{cases} \dfrac{xy}{\sqrt{x^2 + y^2}} & \text{if } x^2 + y^2 \neq 0 \\ 0 & \text{if } x = y = 0 \end{cases}$$

Q.6 If $u = \log(x^3 + y^3 + z^3 - 3xyz)$ show that

$$\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y} + \dfrac{\partial u}{\partial z} = \dfrac{3}{x + y + z}$$

Q.7 If $\theta = t^n e^{-r^2/4t}$, find the value of n for which

$$\dfrac{1}{r^2} \left[\dfrac{\partial}{\partial r} \left(r^2 \dfrac{\partial \theta}{\partial r} \right) \right] = \dfrac{\partial \theta}{\partial t}$$

Q.8 If $u = f(x + ay) + g(x - ay)$, show that $4 \dfrac{\partial^2 u}{\partial x^2} = \dfrac{\partial^2 u}{\partial y^2}$.