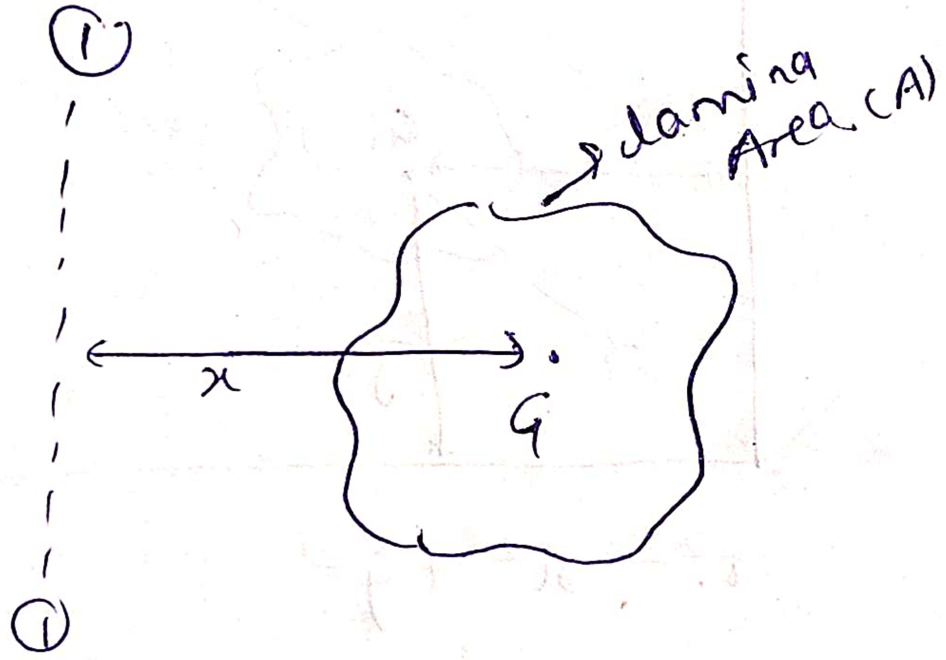
Moment of Inertia /

MOI / MI :-

moment of an Area → A × x

Area ↵ ↳ centroid distance



If moment of this area again multiplied by centroidal distance → $(A \times x) \times x$

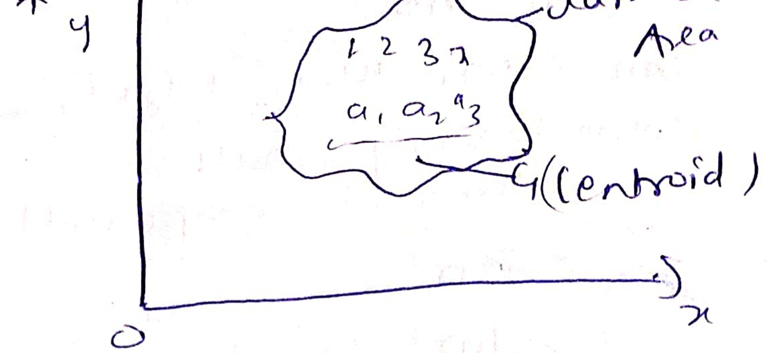
→ Ax^2

Moment of moment of an area → $A x^2$

Second moment of an area → Ax^2

Moment of inertia (MOI/MI)

$= Ax^2 = (mm)^4, (cm)^4, (m^4)$



Moment of inertia of whole lamina about Oy-axis
$= I_{oy}$.

$I_{oy} = a_1 x_1^2 + a_2 x_2^2 + a_3 x_3^2 + \dots$

$(x_1, x_2, x_3 \dots$ will be centroidal distance due to very small elemental component.

$$I_{oy} = \Sigma a x^2$$

$$I_{oy} = \Sigma da\, x^2 = \Sigma dA\, x^2$$

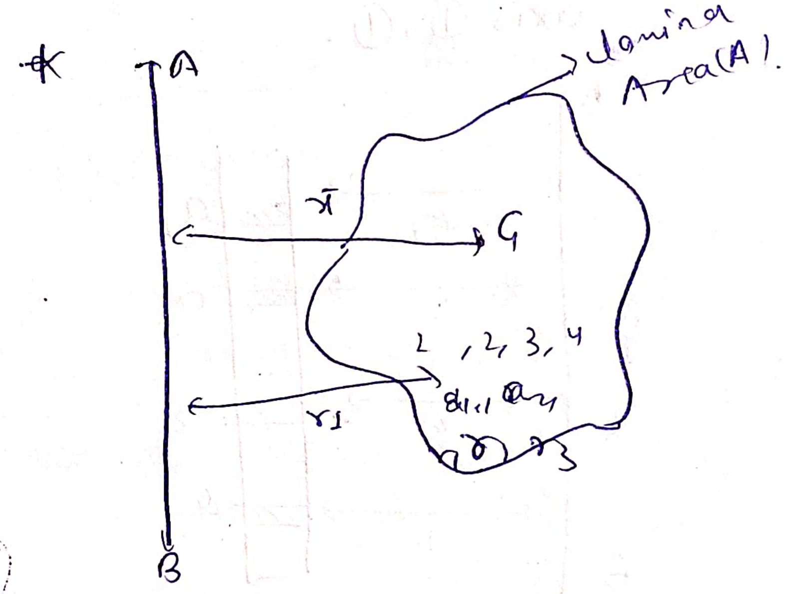
$$I_{oy} = \Sigma x^2 dA$$

Similarly, $I_{ox} = \Sigma a y^2$

$$I_{ox} = \Sigma a y^2 = \Sigma dA y^2$$

$$I_{ox} = \int y^2 dA$$

*



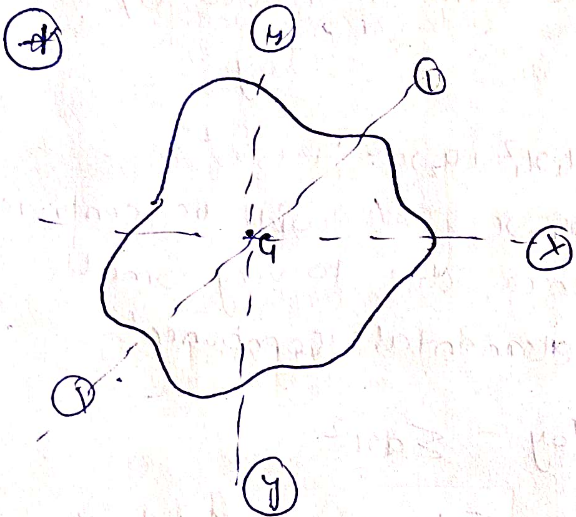
$$\boxed{I_{AB} = A \times \bar{x}^2}$$

∴ Moment of inertia about axis AB =

$I_{AB} = a_1 r_1^2 + a_2 r_2^2 + a_3 r_3^2 + \dots$ (due to very small elemental component)

$= \Sigma a r^2$

$= \Sigma d a r^2$ (uniform lamina)

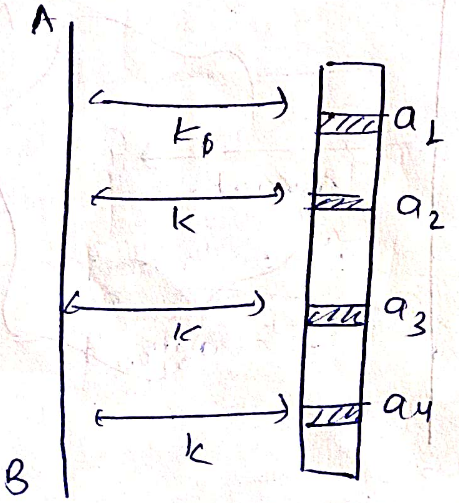


Moment of Inertia;

I_{xx} = moment about horizontal centroidal axis.

I_{yy} = moment about vertical centroidal axis.

I_{11} = moment about centroidal axis 1, 1.



$I_{AB} = \Sigma a k^2$ ($a_1 = a_2 = a_3 = a_4 = a$)

$I_{AB} = k^2 \Sigma a$ (where $\Sigma a = A$) whole area

$I_{AB} = k^2 A$

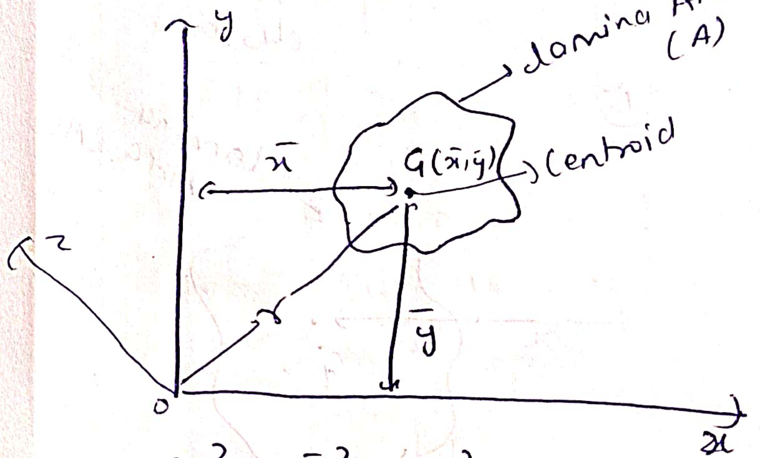
$\boxed{k = \sqrt{\frac{I_{AB}}{A}}}$ → radius of gyration.

$k_{xx} = \sqrt{\frac{I_{xx}}{A}}$;

$k_{yy} = \sqrt{\frac{I_{yy}}{A}}$

$k_{11} = \sqrt{\frac{I_{11}}{A}}$.

- Perpendicular axis theorem



$r^2 = \bar{x}^2 + \bar{y}^2$

$I_{ox} = A\bar{y}^2$; $I_{oy} = A\bar{x}^2$

$I_{oz} = A \times r^2$

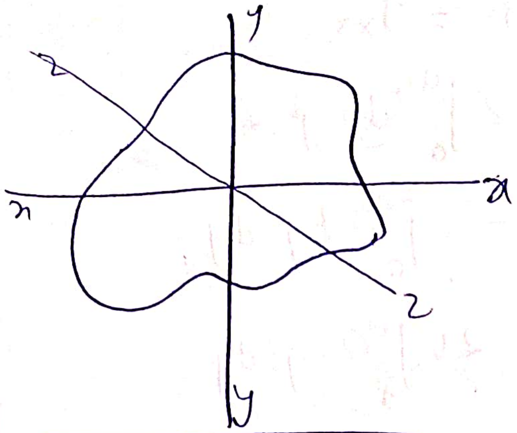
$= A(\bar{x}^2 + \bar{y}^2)$

$= A\bar{x}^2 + A\bar{y}^2$

$= I_{oy} + I_{ox}$

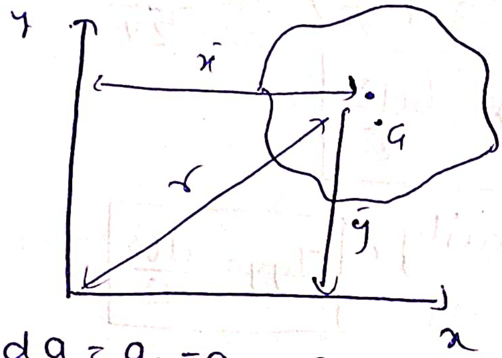
$\boxed{I_{oz} = I_{ox} + I_{oy}}$

- Pol ar ~~pore~~ moment of ~~fate~~ inertia.



$$\boxed{J_{zz} = J_{xx} + J_{yy}}$$

- Perpendicular axis theorem:-



$da = a_1 = a_2 = a_3 = - - ?$

If I_{ox} and I_{oy} be moment of Inertia of lamina about mutually perpendicular axis ox & oy in the plane of lamina and I_{oz} be the MOI of the lamina about an axis normal to the lamina and passing through the point of intersection of the axis Ox & oy. then,

$$I_{oz} = I_{ox} + I_{oy}$$

MoI of elemental component, about oz axis $= dar^2$

MoI of whole lamina about oz-axis :-

$$I_{oz} = \Sigma dar^2 \Rightarrow \because r^2 = x^2+y^2$$

$$I_{oz} = \Sigma da(x^2+y^2)$$

$$= \Sigma da x^2 + \Sigma da y^2$$

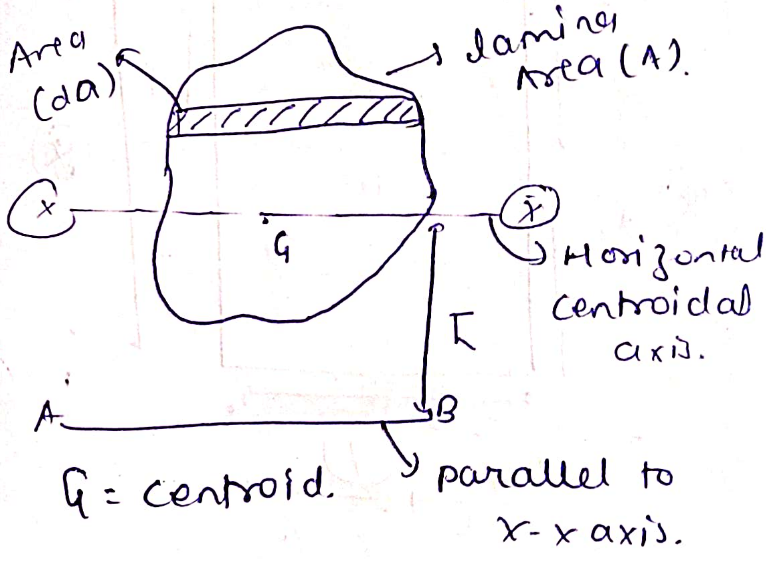
$$\boxed{I_{oz} = I_{oy} + I_{ox}}$$

↳ polar MOI

- Parallel Axis theorem/ Transfer formulae:-

$$\boxed{I_{AB} = I_{xx} + A\bar{h}^2}$$

The MOI of lamina about any axis in the plane of lamina is equal to the sum of MOI about a parallel centroidal axis and the product of the area and square of the distance b/w two axes.



G = centroid.

MOI of elemental component about axis AB $= da(\bar{h}+y)^2$

MOI of whole lamina about AB $= I_{AB}$

$$\boxed{I_{AB} = \Sigma da(\bar{h} \pm y)^2}$$

$I_{AB} = \Sigma da\bar{h}^2 + \Sigma day^2 + 2\Sigma da\bar{h}y$

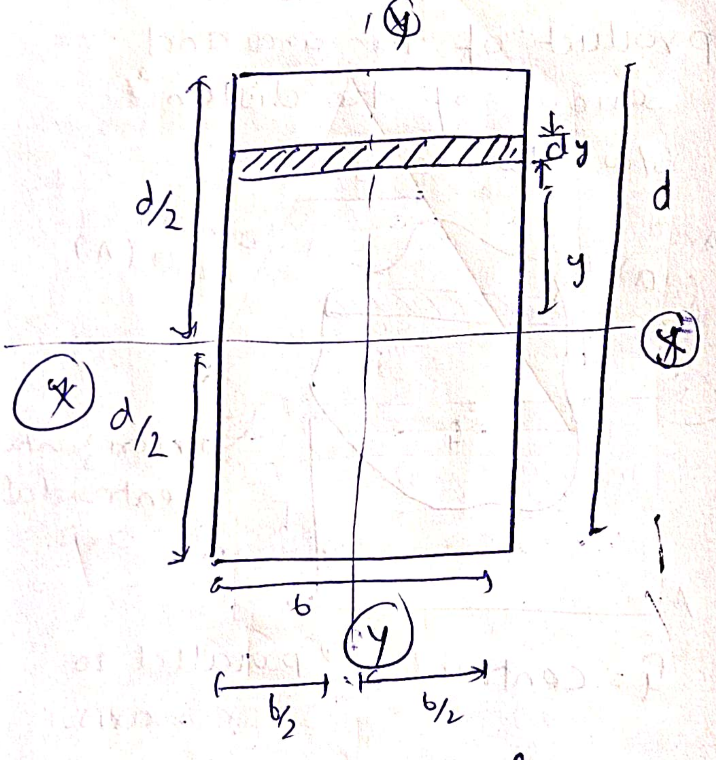
$= \Sigma da\bar{h}^2 + \Sigma day^2 + \Sigma day 2\bar{h}$

$\left[\because \Sigma da\bar{h}^2 = \bar{h}^2 A \atop \Sigma day^2 = I_{xx} \right]$

$\Sigma da.y$ = moment of area about horizontal centroidal axis and which is equals to zero.

$$\boxed{I_{AB} = I_{xx} + A\bar{h}^2}$$

(1). Rectangular lamina:-



Area of elemental component $da = \cancel{Ady}$ bdy

MOI of elemental component about x-x axis $= day^2$

[due to very small thickness of elemental component]

MOI of whole lamina about x-x axis $= I_{xx}$

$$I_{xx} = 2\int_0^{d/2} da.y^2$$

$$= 2\int_0^{d/2} by^2\,dy$$

$$= 2b\int_0^{d/2} y^2\,dy$$

$$= 2b\left(\frac{y^3}{3}\right)_0^{d/2}$$

$$= \frac{2b}{3}\left(\frac{d^3}{8}\right) = \frac{2bd^3}{24}$$

$$\boxed{I_{xx} = \frac{bd^3}{12}}$$

Similarily, $\boxed{I_{yy} = \frac{db^3}{12}}$

MOI about base $I_{①①}$:-

from parallel axes theorem,

$I_{AB} = I_{xx} + A\bar{h}^2$

where $I_{AB} = I_{①①}$;

$I_{xx} = \frac{bd^3}{12}$,

$A = b \times d$; $\bar{h} = \frac{d}{2}$

$\therefore I_{①①} = \frac{bd^3}{12} + (b\times d)\left(\frac{d}{2}\right)^2$

$= \frac{bd^3}{12} + \frac{bd^3}{4} = \frac{bd^3}{3}$

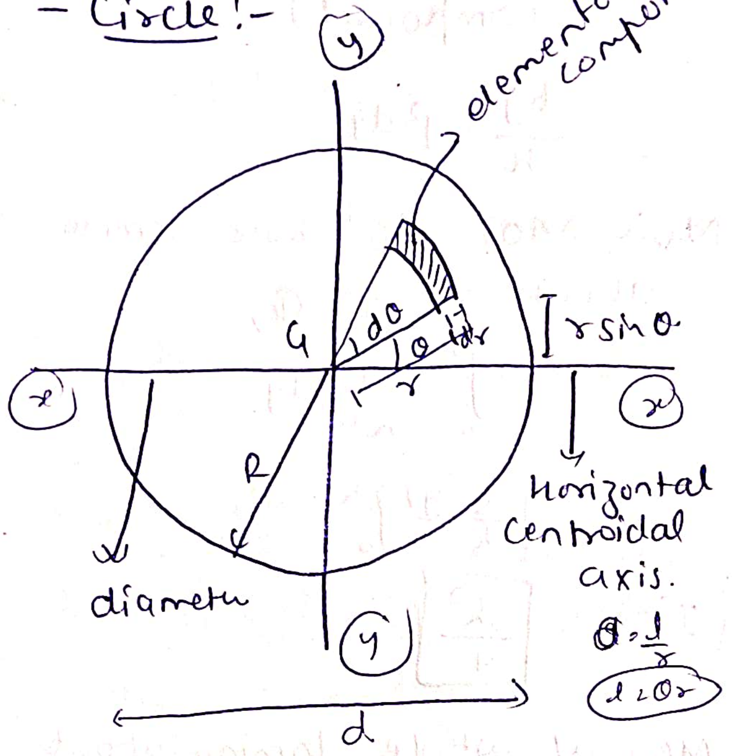
$\boxed{I_{①①} = \frac{bd^3}{3}}$ base.

$\boxed{I_{②②} = \frac{bd^3}{3}}$ Top.

$$\boxed{I_{33} = \frac{db^3}{3}}$$

$$\boxed{I_{44} = \frac{db^3}{3}}$$

- Circle :-



Area of elemental component $= da = r\,d\theta\,dr$

MoI of this elemental component about x-x axis,

$= da \times (r\sin\theta)^2$ [due to small elemental component]

$= r\,d\theta\,dr\,[r^2\sin^2\theta]$

$$\boxed{= r^3\sin^2\theta\,dr\,d\theta}$$

MoI of whole lamina about axis x-x $= I_{xx}$

$$I_{xx} = \int_0^R\int_0^{2\pi} r^3\sin^2\theta\,d\theta\,dr$$

$$= \int_0^R\int_0^{2\pi} \frac{r^3}{2}(1-\cos 2\theta)\,d\theta\,dr$$

$$I_{xx} = \int_0^{2\pi}\left(\frac{r^4}{8}\right)_0^R (1-\cos 2\theta)\,d\theta$$

$$= \frac{R^4}{8}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{2\pi}$$

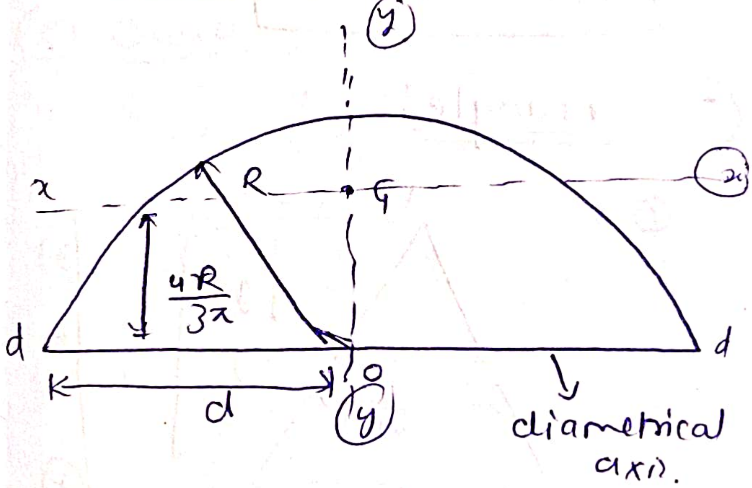
$$= \frac{R^4}{8}[2\pi]$$

$$\boxed{I_{xx} = \frac{\pi R^4}{4}}$$

MoI about vertical centroidal axis y-y $= I_{yy} = \frac{\pi R^4}{4}$

$\therefore$ MoI about diameter axis d-d $= I_{dd} = \frac{\pi R^4}{4}$

3. Semicircle :-



MoI of semicircle about diametrical axis dd ⇒

$I_{dd} = \frac{1}{2}$ MoI of circle about diametrical axis d-d.

$$I_{dd} = \frac{1}{2} \times \frac{\pi R^4}{4} = \boxed{\frac{\pi R^4}{8}}$$

$$\boxed{I_{dd} = \frac{\pi R^4}{8}}$$

MoI of semicircle about horizontal centroidal axis xx $= I_{xx} = ?$

From parallel axis theorem—

$I_{AB} = I_{xx} + A\bar{h}^2$

where,

$I_{AA} = I_{dd}$

$I_{xx} = I_{xx}$

$A = \frac{1}{2}\pi R^2$

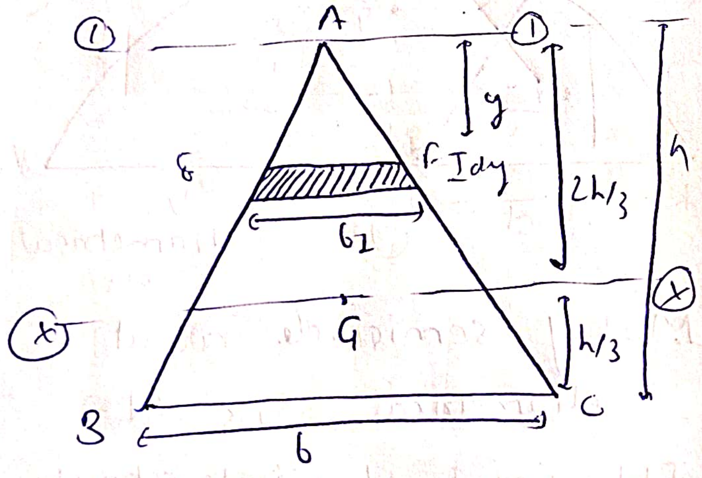
$\bar{h} = \frac{4R}{3\pi}$

$\frac{\pi R^4}{8} = I_{xx} + \frac{1}{2}\pi R^2 \left(\frac{4R}{3\pi}\right)^2$

$I_{xx} = 0.10976\, R^4$

$\boxed{I_{xx} = 0.11 R^4}$

(3) Triangle :-



Area of elemental component $= da$.

$da = b_1 \times dy$ [due to very small elemental component].

From $\Delta ABC \sim \Delta AEF$

$\frac{b_1}{b} = \frac{y}{h} \Rightarrow \boxed{b_1 = \frac{by}{h}}$

Now, $\boxed{da = \frac{by}{h} dy}$

MOI of this elemental component about 1-1.

$= da \times y^2$ [due to small elemental component]

$= \frac{by}{h} \times y^2 dy$

Now, MOI of whole lamina about axis 1-1,

$I_{11} = \int_0^h \frac{by^3}{h} dy$

$= \left[\frac{b}{h}\frac{y^4}{4}\right]_0^h$

$I_{11} = \boxed{\frac{bh^3}{4}}$

MOI of whole lamina about horizontal centroidal axis $x-x = I_{xx} = ?$

From parallel axis theorem

$I_{AB} = I_{xx} + A\bar{h}^2$

Here, $I_{AB} = I_{11} = \frac{bh^3}{4}$

$A = \frac{1}{2} \times b \times h \quad \bar{h} = \frac{2h}{3}$

$\frac{bh^3}{4} = I_{xx} + \frac{1}{2} bh \left(\frac{2h}{3}\right)^2$

$\boxed{I_{xx} = \frac{bh^3}{36}}$

MOI of whole lamina about axis $BC = I_{BC} = ?$

From parallel axis theorem

$\boxed{I_{AB} = I_{xx} + A\bar{h}^2}$

where, $I_{AB} = I_{BC}$;

$$I_{xx} = \frac{bh^3}{36}$$

$$A = \frac{1}{2} bh \; ; \; \bar{h} = \frac{h}{3}$$

$$I_{BC} = \frac{bh^3}{36} + \frac{1}{2} bh \left(\frac{h}{3}\right)^2$$

$$= \frac{bh^3}{36} + \frac{bh^3}{18} = \boxed{\frac{bh^3}{12}}$$

$$\boxed{I_{BC} = \frac{bh^3}{12}}$$