Maulana Azad National Institute of Technology Bhopal
End-Term Examination (March 2022)

Course: B.Tech. Semester-I **Section: All**

Subject: Mathematics-I **Subject Code: MTH-101**

Time:3:00 Hours **Max. Marks: 50**

Note: All questions are compulsory.

S.No.	Question	Marks
1	Expand *cos x* using Taylor series upto order 4 about the point x = π/6. Can we calculate the value of *cos 32°* using Taylor Series? If yes, calculate the value of cos 32°.	05
2(a)	If $z = f(u,v), u = e^x \cos y, v = e^x \sin y$, show that $$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = (u^2 + v^2)\left(\frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2} \right)$$	05
2(b)	Using the properties of beta and gamma functions, show that $\int_0^{\pi/2} \sqrt{\sin\theta}\ d\theta \times \int_0^{\pi/2} \frac{1}{\sqrt{\sin\theta}}\ d\theta = \pi$	05
3(a)	Change the order of integration and hence evaluate $$\int_0^a \int_{\sqrt{ax}}^a \frac{y^2\ dydx}{\sqrt{y^4 - a^2 x^2}}$$	05
3(b)	Find the area of a loop of the curve $a^2 x^2 = 4y^2(a^2 - y^2)$	05
4(a)	Verify the Green's theorem for $f(x,y) = e^{-x}\sin y, g(x,y) = e^{-x}\cos y$, where C is the square with vertices at (0,0), (π/2,0), (π/2, π/2) and (0, π/2).	05
4(b)	Calculate $\iint_S \vec{F}.\vec{n}\ dS$, where $\vec{F} = 4xz\,\vec{\imath} - y^2\vec{\jmath} + yz\,\vec{k}$ and S is the surface of the cube bounded by the planes x = 0, x = 2, y = 0, y = 2, z = 0, z = 2.	05
5(a)	Solve the system of differential equations $$\frac{d^2 x}{dt^2} - \frac{dy}{dt} = 2x + 2t$$ $$\frac{dx}{dt} + 4\frac{dy}{dt} = 3y$$	08
5(b)	Solve the following differential equation by using the method of variation of parameter $$(3x+2)^2 \frac{d^2 y}{dx^2} + 3(3x+2)\frac{dy}{dx} - 36y = 3x^2 + 4x + 1$$	07