☆ 1Φ Transformer ☆

☆ Need of a transformer:

- The a.c voltages can be raised or lowered as per requirements in different stages of electrical network (generation, Transmission, Distribution & utilisation) with the help of static device called as "Transformer"

- Transformer works on the principle of "Mutual Induction"

- Transfer an electrical energy from one circuit to other when there is no electrical connection b/w two circuits.

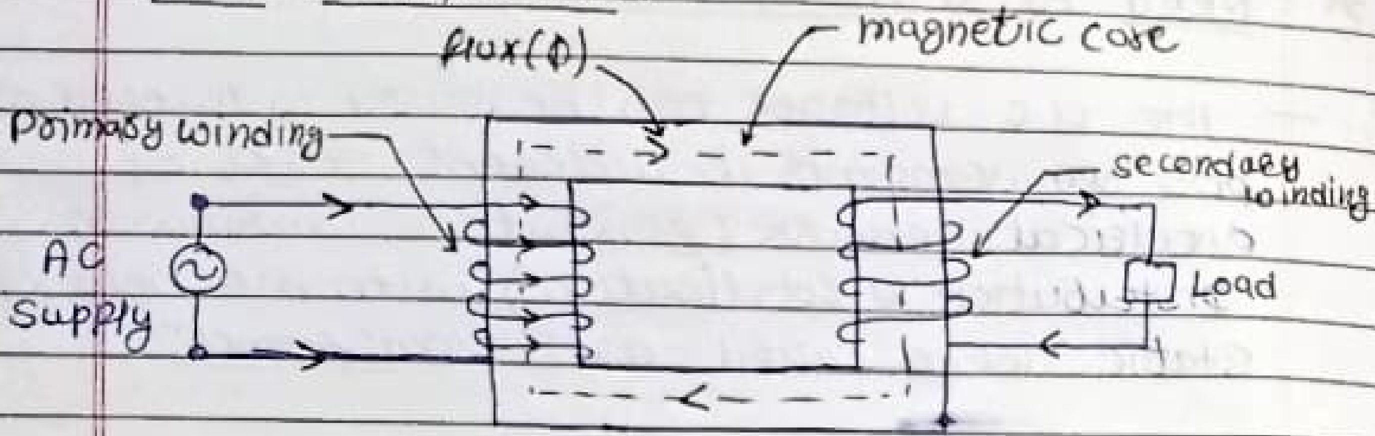
Definition of Transformer: The transformer is a static device by means of which an electrical ~~device~~ power is transferred from one ac circuit to another with the desired change in voltage & current without any change in the frequency.

Working principle of transformer:

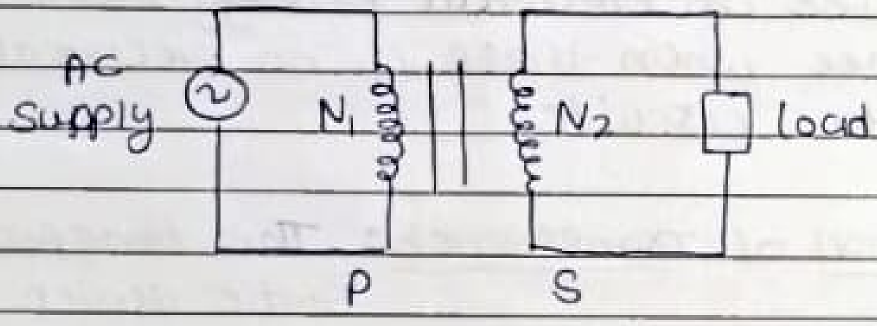
* Work on the principle of "Mutual Induction"

Mutual Induction: The principle of mutual induction states that when two coils are inductively coupled & if current in one coil changed uniformly then an emf gets in the other coil.

Basic transformer:



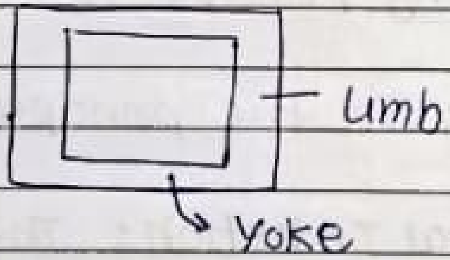
Symbolic representation:



★ Construction of transformer:

Two basic parts
(I) magnetic core (II) windings or coils

(I) magnetic core:

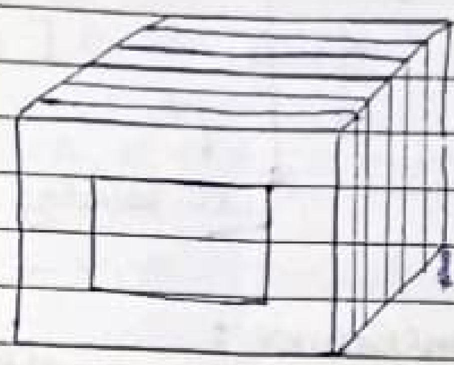


- Core provides path for magnetic flux (ϕ)
- two loses occur in the core.

Hysterisis loss & Eddy current (magnetic losses or ~~Ios~~ Iron losses)

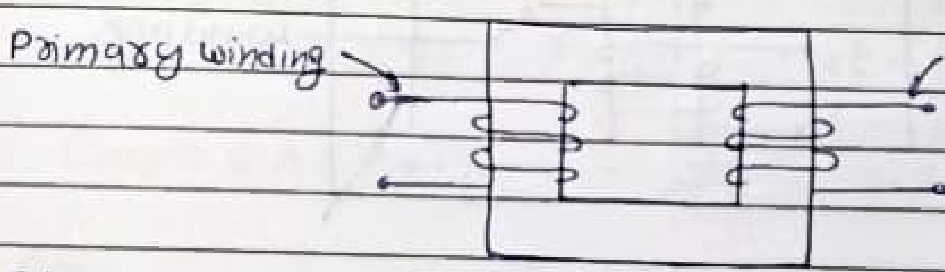
- generally core is made up of high grade silicon steel (magnetic material with high permeability{ earlier core is made up of Iron) in order to minimise Hysteresis losses.

- Core is made up of thin laminations, to minimise eddy current losses.



insulated from each other by varnish.

(II) <u>windings</u> or <u>coils</u>:-

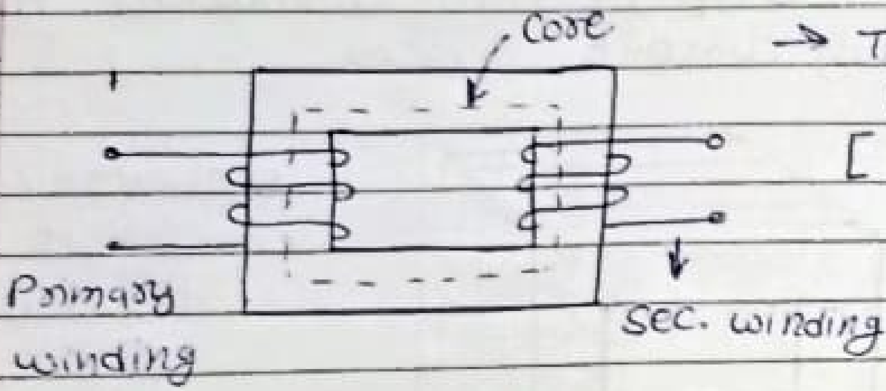


- electric current passes through windings.
- windings are made up of copper & are wound on the limbs of magnetic core. windings are insulated from each other.
- windings are subjected to I^2R losses also called as copper losses.

Based on the arrangement of core & windings:

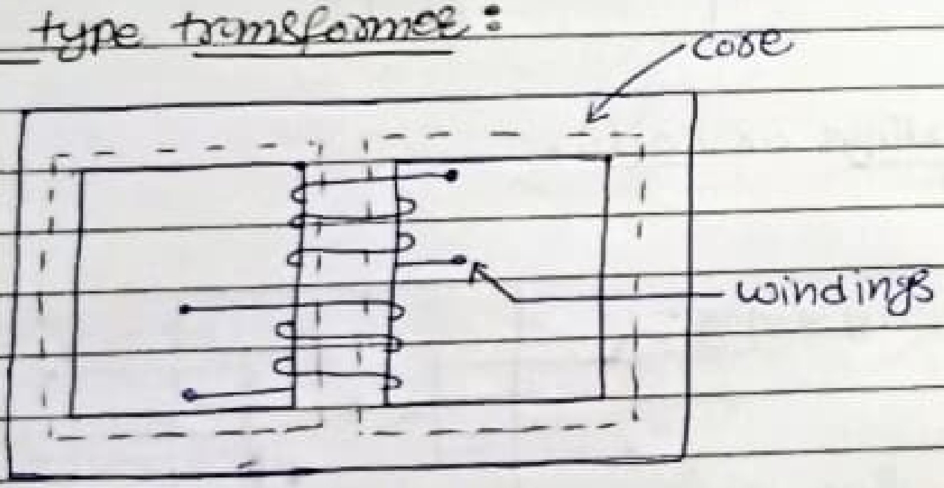
Two type of Transformer:

① Core type transformer:



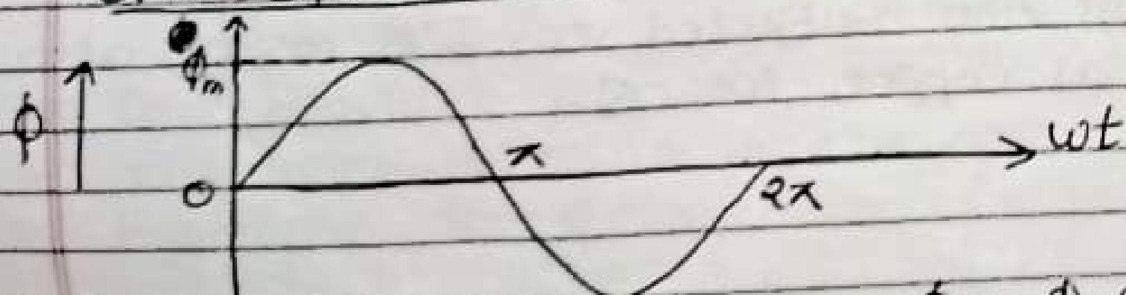
→ The winding encircles the Core
[core is surrounded by windings]

Ⓘ Shell type transformer:



→ core encircles the windings.
[windings are surrounded by core]

☆ EMF equation of a transformer:-



$\phi = \phi_m \sin\omega t$

By Faraday's law of electromagnetic induction.
emf induced.

$$e = \frac{N d\phi}{dt}$$

by lenz law.

$$e = -\frac{N d\phi}{dt}$$

emf induced at the primary side

$$e_1 = -\frac{N_1 d\phi}{dt}$$

$$e_1 = -N_1 \frac{d}{dt}[\phi_m \sin wt]$$

$$e_1 = -N_1 \phi_m \cdot \cos wt \cdot \omega$$

$$e_1 = N_1 \phi_m \omega [-\cos wt]$$

$$\therefore e_1 = \phi_m 2\pi f . N_1 \cdot \sin(\omega t - 90^\circ)$$

Comparing with

$$e_1 = E_{m_1} \cdot \sin(\omega t + \varphi)$$

$$E_{m_1} = 2\pi f . \phi_m \cdot N_1 \quad ; \quad \varphi = -90^\circ$$

Rms value of emf $= \frac{E_{m_1}}{\sqrt{2}}$

$$= \frac{2\pi f \cdot \phi_m \cdot N_1}{\sqrt{2}}$$

$$\boxed{E_1 = 4.44 \, \phi_m \cdot f \cdot N_1}$$

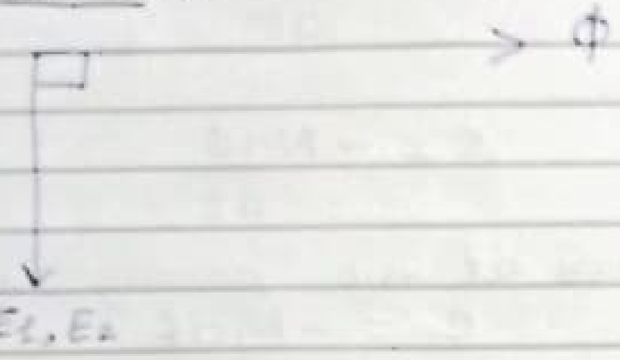
Similarly, secondary side emf

$$\boxed{E_2 = 4.44 \phi_m \, f \cdot N_2}$$

→ ~~[illegible]~~

E_1 & E_2 lag flux ϕ by $90°$ or $\pi/2$

Phasor diagram:



✱ Ratios in a Transformer:

$$E_1 = 4.44\,\phi_m\, f\, N_1$$
$$E_2 = 4.44\,\phi_m \cdot f \cdot N_1$$

$$\therefore \frac{E_2}{E_1} = \frac{N_2}{N_1} = \frac{\text{Secondary No. of turns}}{\text{Primary No. of turns}}$$

If we neglect Resistance & leakage reactance
$[\, R_1 \approx R_2 \approx 0 \;:\; X_1 \approx X_2 \approx 0 \,]$
$E_1 \approx V_1$ & $E_2 \approx V_2$

$$\Rightarrow \boxed{\frac{E_2}{E_1} = \frac{V_2}{V_1} = \frac{N_2}{N_1} = k \text{ (turns ratio)}}$$ voltage ratio

✱ Transformer rating is in VA (volt-Amp)

primary side VA = secondary side VA

$$V_1 I_1 = V_2 I_2$$

$$\frac{V_2}{V_1} = \frac{I_1}{I_2}$$

but $\frac{V_2}{V_1} = \frac{N_2}{N_1} = K$

$$\therefore \quad \frac{I_p}{I_s} \quad \boxed{\frac{I_1}{I_2} = \frac{N_2}{N_1} = K}$$ Current ratio

$$\boxed{\frac{E_2}{E_1} = \frac{V_2}{V_1} = \frac{I_1}{I_2} = \frac{N_2}{N_1} = K}$$

* <u>Based on turns Ratio (K)</u> :

(a) $K > 1 \Rightarrow \frac{N_2}{N_1} > 1 \Rightarrow N_2 > N_1$

$E_2 > E_1$

$\therefore$ Step up transformer [Voltage is raised from primary to secondary]

(b) $K < 1 \Rightarrow \frac{N_2}{N_1} < 1 \Rightarrow N_2 < N_1$

$E_2 < E_1$

$\therefore$ Step-down transformer [Voltage is lowered from primary to secondary]

(c) $K = 1 \Rightarrow \frac{N_2}{N_1} = 1 \Rightarrow N_2 = N_1$

$E_2 = E_1$

1:1 transformer / Isolation transformer

Q. A 50KVA, 2200/440V, 50HZ Single phase Transformer has primary turns of 200. Determine:
(i) flux in core (ii) secondary turns
(iii) Rated primary current (iv) Rated sec. current

Ans

Emf equation of transformer,

$$E_1 = 4.44 \Phi_m \cdot f \cdot N_1$$

$$2200 = 4.44 \Phi_m \times 50 \times 200$$

$$\boxed{\Phi_m = 0.0495 \text{ wb}}$$

$$\frac{E_1}{E_2} = \frac{N_1}{N_2} \Rightarrow \frac{2200}{440} = \frac{200}{N_2}$$

$$\boxed{N_2 = 40}$$

$V_1 I_{1fl}$ = VA of a transformer

$\because V_1 = E_1$

$\therefore E_1 I_{1fl} = VA$

$$2200 I_{1fl} = 50{,}000 \text{ VA}$$

$$\boxed{I_{1fl} = \frac{50000}{2200} = 22.7273 \text{ A}}$$

$E_2 I_{2fl}$ = VA of a transformer

$$440 I_{2fl} = 50000$$

$$\boxed{I_{2fl} = 113.6364 \text{ A}}$$

Q. A 40 KVA, 3300/240 V, 50Hz, 1 phase transformer has 660 turns on the primary. determine
(I) no. of turns on the secondary
(II) The maximum value of flux in the core
(III) The approximate value of primary and secondary full load current.

ans

$$\frac{E_1}{E_2} = \frac{N_1}{N_2} \Rightarrow \frac{3300}{240} = \frac{660}{N_2} \Rightarrow \boxed{N_2 = 48}$$

emf eq. of transformer

$$E_1 = 4.44\ \Phi_m \cdot f \cdot N_1$$

$$3300 = 4.44 \cdot \Phi_m \times 50 \times 660$$

$$\boxed{\Phi_m = 0.0225\ wb}$$

$$V_1 I_{1fl} = VA \Rightarrow E_1 I_{1fl} = VA$$

$$3300\ I_{1fl} = 40000 \Rightarrow \boxed{I_{1fl} = 12.1212A}$$

$$E_2 I_{2fl} = VA \Rightarrow 240\ I_{2fl} = 40000$$

$$\boxed{I_{2fl} = 166.66674\ A}$$

Q. A 5KVA, 240/2400 V, 50Hz Single phase transformer has maximum value of flux density as ~~1.2~~ 1.2 Tesla. If the emf per turn is 8V. Calculate the no. of primary and secondary turns and the primary and secondary ~~fault~~ current at full load.

given $B_m = 1.2T = 1.2\ wb/m^2$

Ans $\frac{E_1}{N_1} = 8 \Rightarrow \frac{240}{N_1} = 8 \Rightarrow \boxed{N_1 = 30}$

$\frac{E_2}{N_2} = 8 \Rightarrow \frac{2400}{N_2} = 8 \Rightarrow \boxed{N_2 = 300}$

$\Rightarrow \quad kVA = V_1 I_{1fl} = E_1 I_{1fl}$

$5000 = 240 \cdot I_{1fl}$

$\boxed{I_{1fl} = 20.833A}$

$\Rightarrow \quad kVA = E_2 I_{2fl} \Rightarrow 5000 = 2400\, I_{2fl}$

$\boxed{I_{2fl} = 2.0833A}$

Q. A 3000/200-V, 50Hz, single phase transformer has a cross-sectional area of 150 cm^2 for the core. If number of turns on the low voltage winding is 80, determine number of turns on the high voltage winding and maximum value of flux density in the core.

Ans given. $N_{LV} = 80 = N_2$

$\frac{E_1}{E_2} = \frac{N_1}{N_2} \Rightarrow \frac{3000}{200} = \frac{N_1}{80} \Rightarrow \boxed{N_1 = 1200}$

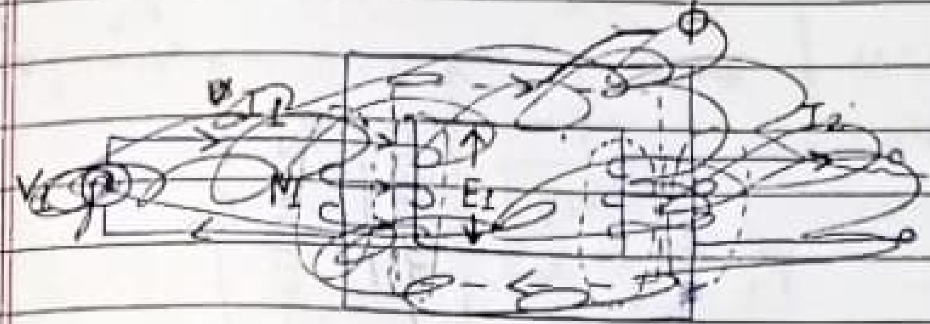
emf eq. of transformer

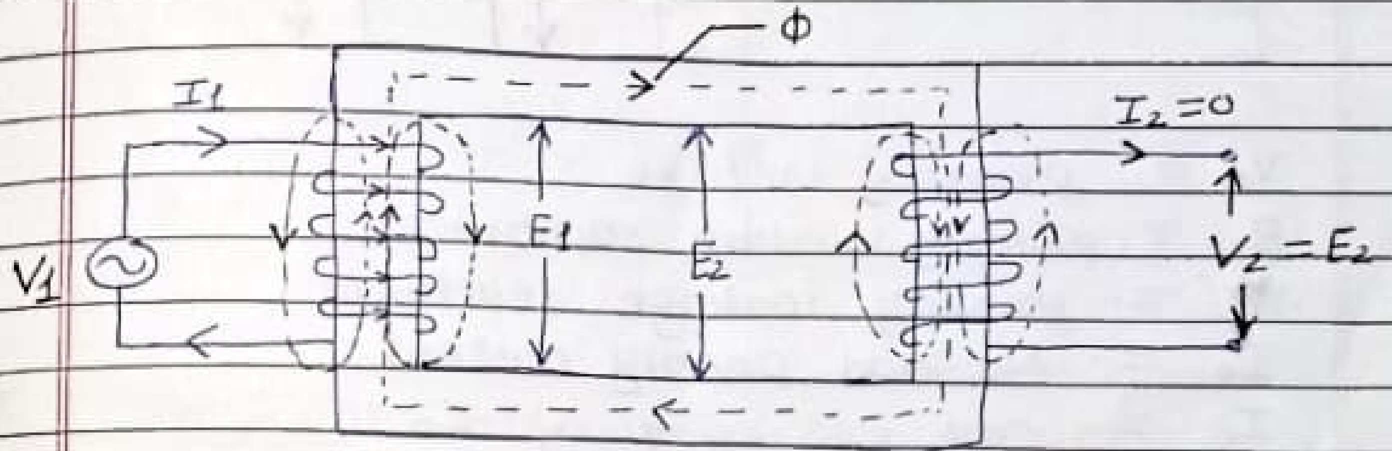
$E_2 = 4.44\, \phi_m \cdot f \cdot N_2$

$200 = 4.44\, \phi_m \times 50 \times 80 \Rightarrow \phi_m = 0.0113 wb$

$\boxed{B_m = \frac{\phi_m}{\text{Area of core}} \Rightarrow = \frac{0.0113}{150 \times 10^{-4}} = 0.7508\ wb/m^2}$

☆ Practical transformer on NO-LOAD:





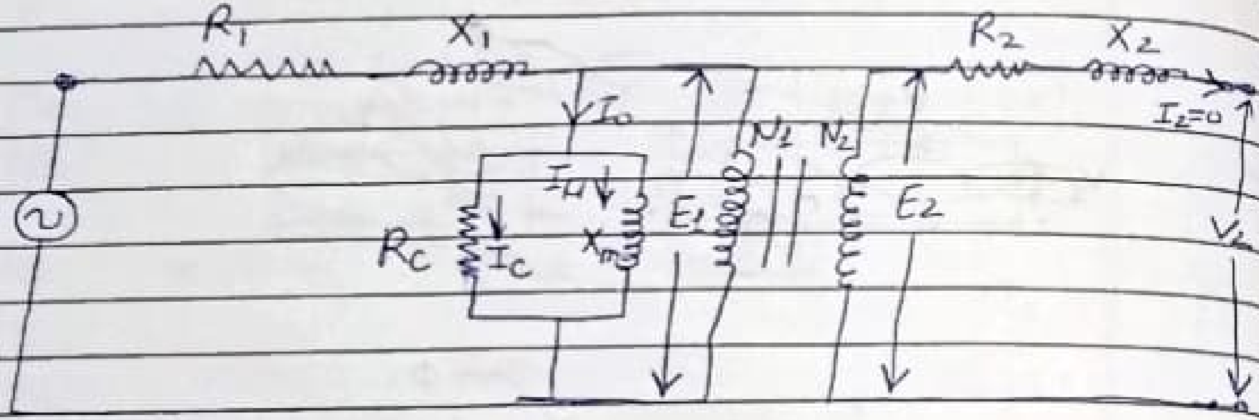
I_1 = No load primary current (I_0)

I_0 has two components

(i) generation of magnetic flux. (ϕ)
(magnetising component of I_0 (I_μ))

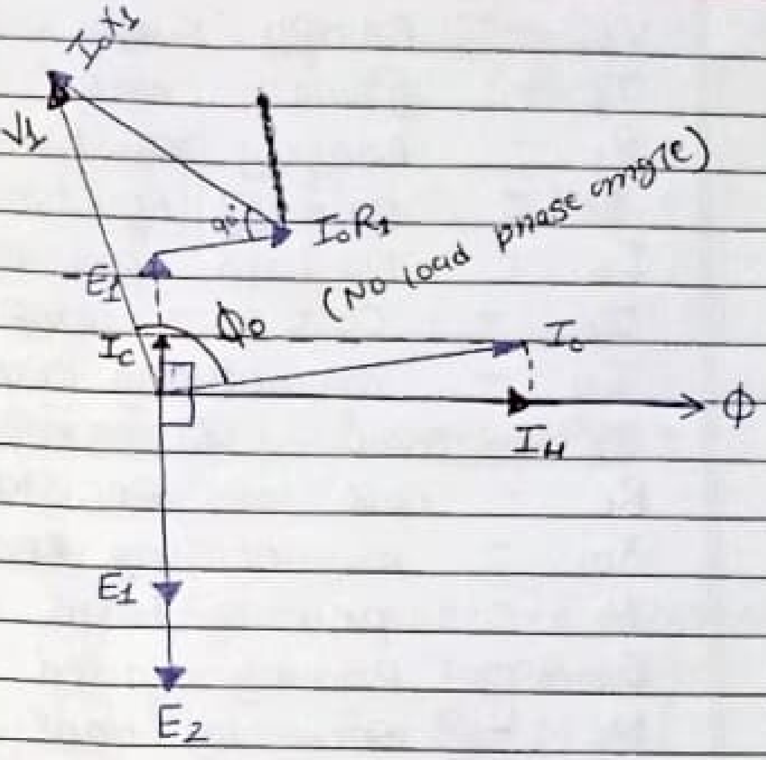
(ii) Component which is responsible for magnetic losses [Hysteresis & Eddy current] and primary winding I^2R losses is called as core loss component of I_0 (I_c)

Equivalent circuit diagram of transformer :-

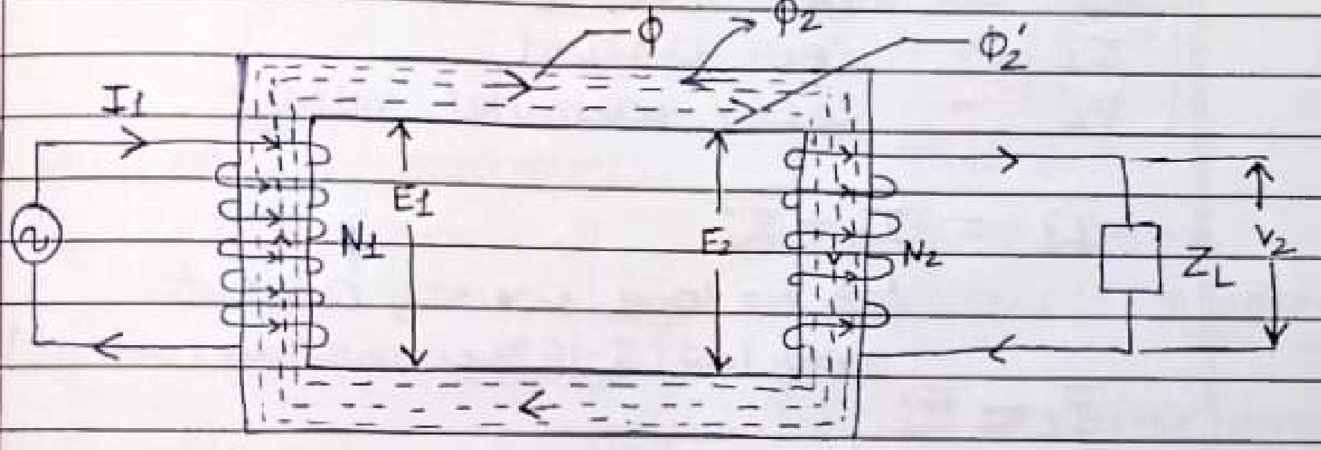


V_1 = primary voltage
R_1 = primary winding resistance
X_1 = primary leakage reactance
I_0 = No load primary current
I_c = core loss comp. of I_0
I_μ = magnetising comp. of I_0
R_c = core loss resistance
X_m = magnetising reactance
E_1 = primary induced emf
N_1 = No. of primary winding turns
R_2 = Secondary winding resistance.
X_2 = secondary leakage reactance
N_2 = No. of secondary winding turns.
E_2 = Secondary induced emf
V_2 = terminal (secondary) voltage.

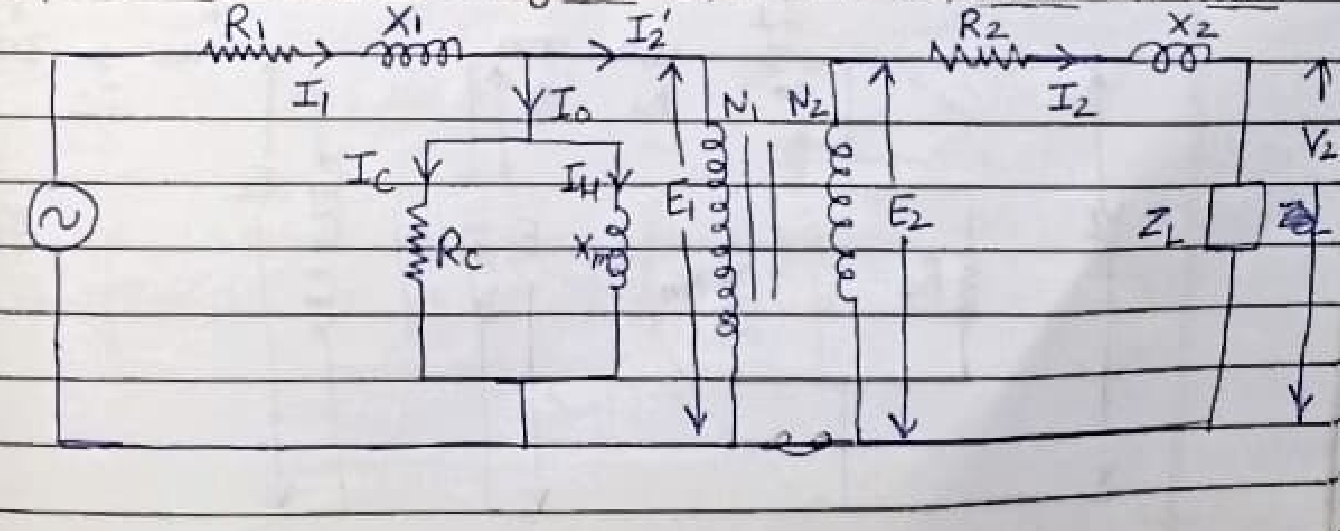
phasor diagram:



☆ Transformer ON LOAD:



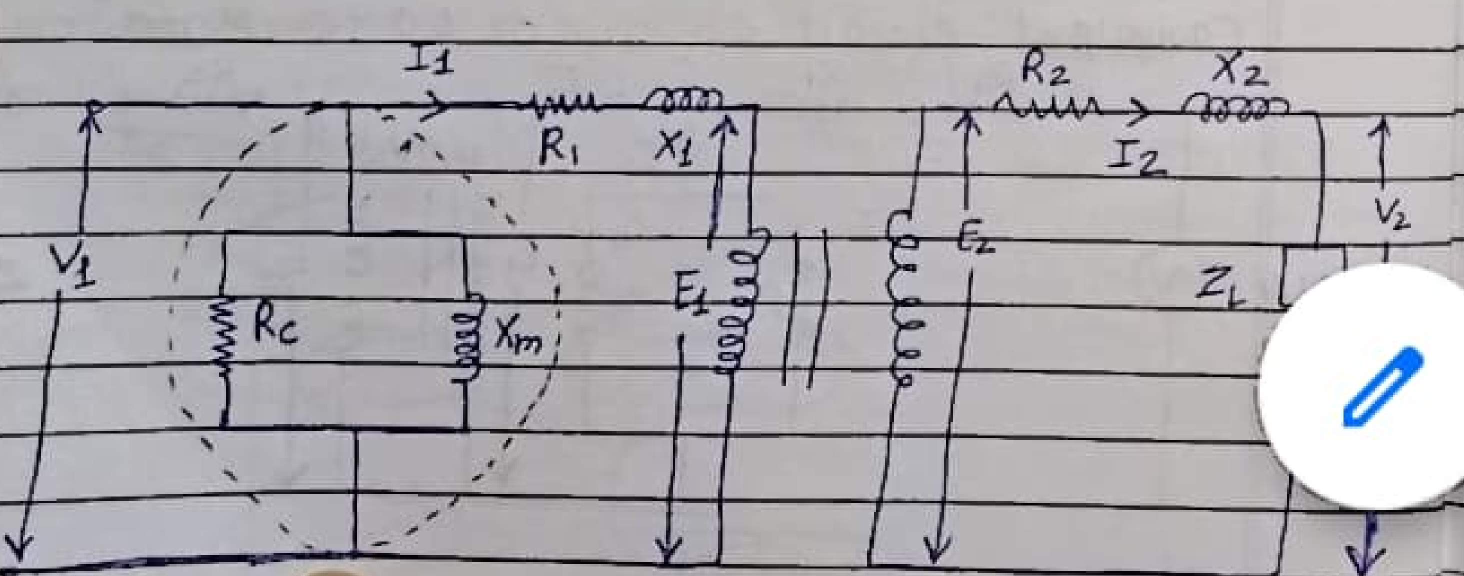
Equivalent circuit diagram of 1 ϕ transformer on LOAD:

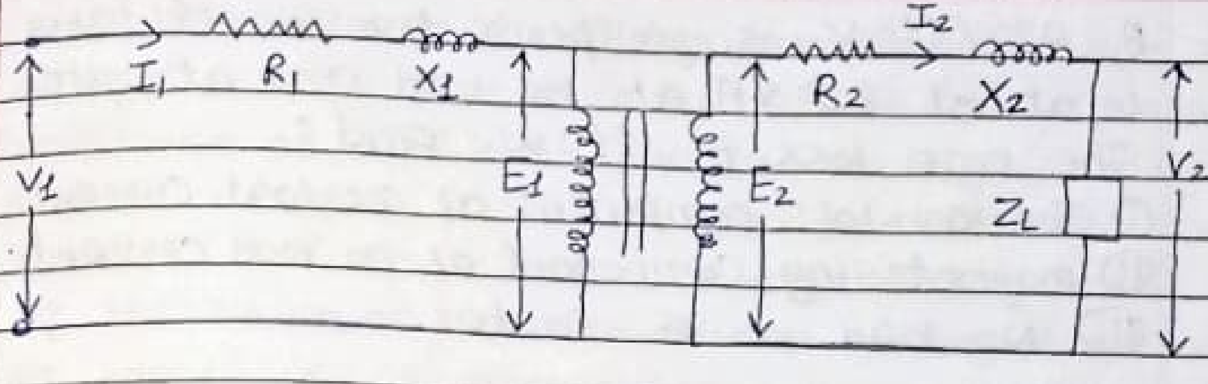


V_1 — primary / Supply Voltage.
I_1 — primary current
R_1 — primary resistance
X_1 — primary Reactance
I_0 — No load primary current
I_c — Core loss comp. of I_0
I_μ — magnetising comp. of I_0
I_2' — comp. of I_1 to nullify effect of ϕ_2
R_c — Core loss Resistance
X_m — magnetising Reactance
N_1 — primary turns
E_1 — Primary induced emf
N_2 — Secondary turns
E_2 — Secondary induced emf
R_2 — Sec. resistance
X_2 — Sec. leakage reactance
I_2 — Sec. current
V_2 — Sec. / terminal voltage.

$\overline{I_1} = I_0 + I_2'$
↳ No load primary current
very low (5–10% of full load current)

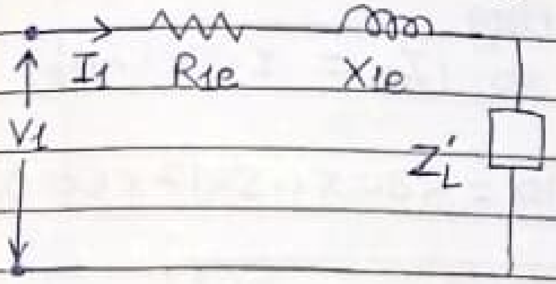
$\therefore \overline{I_1} \approx \overline{I_2'}$





→ Equivalent circuit diagram of 1ϕ transformer :

(a) Referred to primary



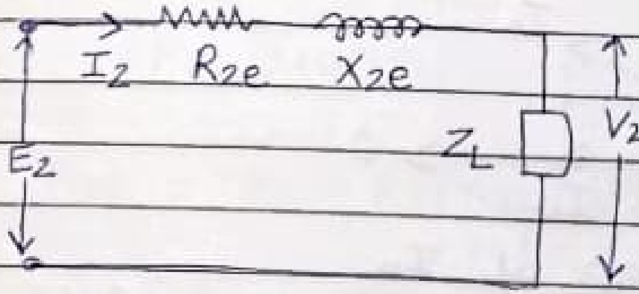
R_{1e} - eq. resistance referred to primary)

$$\boxed{R_{1e} = R_1 + R_2\left(\frac{N_1}{N_2}\right)^2}$$

X_{1e} - eq. reactance referred to primary)

$$\boxed{X_{1e} = X_1 + X_2\left(\frac{N_1}{N_2}\right)^2}$$

(b) Referred to secondary :-



R_{2e} - eq. resistance referred to ~~primary~~ secondary

X_{2e} - eq. reactance referred to secondary

$$\boxed{R_{2e} = R_2 + R_1\left(\frac{N_2}{N_1}\right)^2}$$

$$\boxed{X_{2e} = X_2 + X_1\left(\frac{N_2}{N_1}\right)^2}$$

Q. A 230/110 V. single phase transformer takes an input of 350 VA at no load and at rated voltage. The core loss is 110 W. Find :

(I) The iron loss component of no load current

(II) magnetising component of no load current

(III) No load power factor.

ans given

$$350VA, \quad \frac{E_1}{E_2} = \frac{230}{110}, \quad w_i = 110W$$

→ No load primary current.

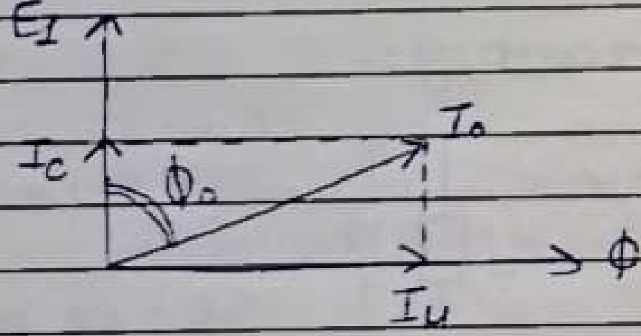
$$V_o I_o = VA \text{ rating}$$

$$230 I_o = 350 \Rightarrow \boxed{I_o = 1.5217A}$$

$$W_i = V_o I_o \cos\phi_o \Rightarrow 110 = 230 \times 1.5217 \times \cos\phi_o$$

$$\boxed{\cos\phi_o = 0.3143}$$

$$\phi_o = 71.68^\circ$$



I_c = iron loss comp. of I_o

$$I_c = I_o \cos\phi_o \Rightarrow I_c = 1.5217 \times 0.3143$$

$$\boxed{I_c = 0.4783A}$$

I_u = magnetising comp. of I_o

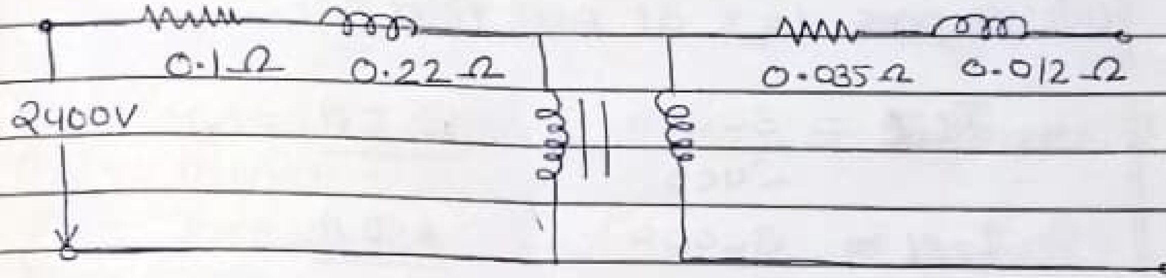
$$I_u = I_o \sin\phi \Rightarrow I_u = 1.5217 \times \sin 71.68^\circ$$

$$\boxed{I_u = 1.4446A}$$

Q. A 30KVA, 2400/120V, 50HZ, transformer a high-voltage winding resistance of 0.1 Ω and a leakage resistance of 0.22Ω. The low-voltage winding resistance ~~of~~ is 0.035 Ω and the leakage reactance is 0.012 Ω. calculate for the transformer-

(I) eq. resistance as referred to both primary & secondary.
(II) eq. reactance as referred to both primary & secondary.
(III) eq. impedance as referred to both primary & secondary
(IV) copper loss at full load.

Ans



$R_1 = 0.1\,\Omega$: $X_1 = 0.22\,\Omega$
$R_2 = 0.035$: $X_2 = 0.012\,\Omega$

(I)

$$R_{1e} = R_1 + R_2\left(\frac{N_1}{N_2}\right)^2 = 0.1 + 0.035\left(\frac{2400}{120}\right)^2$$

$$\boxed{R_{1e} = 14.1\,\Omega}$$

$$R_{2e} = R_2 + R_1\left(\frac{N_2}{N_1}\right)^2 = 0.035 + 0.1\left(\frac{120}{2400}\right)^2$$

$$\boxed{R_{2e} = 0.03525\,\Omega}$$

(II)

$$X_{1e} = X_1 + X_2\left(\frac{N_1}{N_2}\right)^2 = 0.22 + 0.012\left(\frac{2400}{120}\right)^2$$

$$\boxed{X_{1e} = 5.02\,\Omega}$$

$$X_{2e} = X_2 + X_1\left(\frac{N_2}{N_1}\right)^2 = 0.012 + 0.22\left(\frac{120}{2400}\right)^2$$

$$\boxed{X_{2e} = 0.01255\,\Omega}$$

(iii) $Z_{1e} = \sqrt{R_{1e}^2 + X_{1e}^2}$

$= \sqrt{(14.1)^2 + (5.02)^2}$

$\boxed{Z_{1e} = 14.967\,\Omega}$

$Z_{2e} = \sqrt{R_{2e}^2 + X_{2e}^2} = \sqrt{(0.03525)^2 + (0.01255)^2}$

$\boxed{Z_{2e} = 0.0374\,\Omega}$

(iv) copper loss at full load

$I_{1fl} = \frac{30000}{2400} = \underline{12.5\,A}$

$I_{2fl} = \frac{30000}{120} = \underline{250\,A}$

$P_{cu} = I_{1fl}^2 R_1 + I_{2fl}^2 R_2 = (12.5)^2 \times 0.1 + (250)^2 \times (0.035)$

$\boxed{P_{cu} = 2.2031\,kW}$

$P_{cu} = I_{1fl}^2 R_{1e} = (12.5)^2 \times 14.1 = 2.2031\,kW$

$P_{cu} = I_{2fl}^2 R_{2e} = (250)^2 \times 0.03525 = 2.2031\,kW$

★ <u>Losses in a transformer</u>

Core losses
- core get subjected to alternating flux causing core losses

Copper losses
- The winding carry currents when transformer is loaded causing copper losses

Core / Iron / Constant losses

① Hysteresis loss: due to AC flux set up in the magnetic core of the transformer. It undergoes ~~to~~ a cycle of magnetisation & demagnetisation. Due to Hysteresis effect there is a loss of energy in this process, which is called as hysteresis loss.

$$\boxed{P_h = K_h B_m^{1.67} \cdot f \cdot V}$$

K_h - Hysteresis constant depends on ~~its~~ material
B_m - maximum flux density
f - frequency
V - Volume of the core.

② Eddy current losses: The induced emf in the core tries to set up eddy current in the core & hence responsible for eddy current losses.

$$\boxed{P_e = K_e \cdot B_m^2 \cdot f^2 \cdot t^2}$$

K_e - Eddy current constant
t - thickness of the core.

- flux in the core is ~~ato~~ almost constant as supply voltage V_1 at rated frequency f is always constant.

∴ P_h & P_e are constant losses (Iron losses)

- high grade core material like silicon steel & with lamination of smallest thickness.

★ Copper losses / variable losses:

- The copper losses are due to the power wasted in the form of I^2R loss due to the resistances of the primary & secondary windings.

$$\boxed{P_{cu} = I_1^2 R_1 + I_2^2 R_2}$$

- copper losses depend upon the amount of load current which can be changed depends upon the load connected.
Hence copper losses are variable losses.

$$\text{Total losses} = P_i + P_{cu}$$
$$= (P_h + P_e) + (I_1^2 R_1 + I_2^2 R_2)$$

⇒ why rating of transformer is in VA/KVA in place of W/KW like other equipment.

ans - Rating of any device depends upon the losses occuring in that device.
- In transformer two types of losses present
(i) Core loss: depends on the supply voltage

(ii) Copper loss: depends on the current through windings.
- In order to know total losses in a transformer. by knowing VA of a transformer.
- at the time of design, nature ($\cos\phi$) of the load is not known

Efficiency of transformer:

$$\eta = \frac{\text{output power}}{\text{Input power}}$$

$$\eta = \frac{\text{output power}}{\text{output power} + \text{losses}}$$

For full Load:

$$\eta = \frac{V_2 I_2 \cos\phi_2}{V_2 I_2 \cos\phi_2 + W_i + W_{cu}}$$

$$\boxed{\eta = \frac{V_2 I_2 \cos\phi_2}{V_2 I_2 \cos\phi_2 + W_i + I_2^2 R_{2e}}}$$

For fractional load:

$$x = \frac{\text{actual load}}{\text{full load}} = \frac{I_2}{I_{2(FL)}}$$

$$\therefore I_2 = x I_{2(FL)}$$

$$\therefore \eta = \frac{x \cdot V_2 I_{2(FL)} \cos\phi_2}{x \cdot V_2 I_{2(FL)} \cos\phi_2 + W_i + x^2 I_{2(FL)}^2 R_{2e}}$$

$$\boxed{\eta = \frac{x \cdot (\text{VA rating}) \cdot PF}{x (\text{VA rating}) \cdot PF + W_i + x^2 W_{cu(FL)}}}$$

Maximum Efficiency

$$\eta = \frac{V_2 I_2 \cos\phi_2}{V_2 I_2 \cos\phi_2 + W_i + I_2^2 R_{2e}}$$

diff. η w.r.t I_2

$$\therefore \frac{d\eta}{dI_2} = \frac{(V_2 I_2 \cos\phi_2 + W_i + I_2^2 R_{2e}) \cdot V_2 \cos\phi_2 - V_2 I_2 \cos\phi_2 (V_2 \cos\phi_2 + 0 + 2 I_2 R_{2e})}{(V_2 I_2 \cos\phi_2 + W_i + I_2^2 R_{2e})^2}$$

To get η_{max},

$$\frac{d\eta}{dI_2} = 0$$

$$\therefore (V_2 I_2 \cos\phi_2 + W_i + I_2^2 R_{2e}) \cdot V_2 \cos\phi_2 - V_2 I_2 \cos\phi_2 (V_2 \cos\phi_2 + 2 I_2 R_{2e}] = 0$$

$$\Rightarrow \cancel{V_2 \cos\phi_2} [\cancel{V_2 I_2 \cos\phi_2} + W_i + I_2^2 R_{2e} - \cancel{V_2 I_2 \cos\phi_2} + 2 I_2^2 R_{2e}] = 0$$

$$\Rightarrow W_i - I_2^2 R_{2e} = 0 \Rightarrow$$

$$W_i = I_2^2 \cdot R_{2e}$$

$$\boxed{\text{Iron losses} = \text{Copper losses}}$$

This is the condition for η_{max}.

Load current at η_{max}:

$$I_2^2 R_{2e} = W_i$$

$$\cancel{I_2^2 R_{2e} = x \cdot I_{2(FL)}}$$

$$\therefore x^2 \cdot I_{2(FL)}^2 R_{2e} = W_i$$

$\therefore \quad x^2 \cdot W_{cu(FL)} = W_i$

$$\boxed{x = \sqrt{\frac{W_i}{W_{cu(FL)}}}}$$

KVA supplied at η_{max} :

$$\boxed{\text{KVA at } \eta_{max} = x \times (\text{KVA rating})}$$

maximum efficiency

$$\boxed{\eta_{max} = \frac{x\,(\text{VA rating}) \cdot PF}{x\,(\text{VA rating}) \cdot PF + 2W_i}}$$

Q. 40 KVA transformer have iron loss of 450W and full-load copper loss of 850W. If the power factor of the load of 0.8 lagging. calculate:
(I) full load efficiency
(II) The load at which max. efficiency occurs
(III) The max. efficiency.

ans

$$\eta = \frac{x\,(\text{KVA rating}) \times PF}{x\,(\text{KVA rating} \times PF + W_i + x^2 W_{cu(FL)}}$$

→ KVA rating = 40 ; W_i = 450W ; $W_{cu(FL)}$ = 850W
PF = 0.8 (lag)
x = fraction of full load

(I) $x = 1$:

$$\eta = \frac{1\,(40000) \times 0.8}{1(40000) \times 0.8 + 450 + 850 \times 1^2}$$

$$= 0.961$$

$\% \eta = 96.1\%$

(i) for η_{max}

$$x = \sqrt{\frac{W_i}{W_{cu(FL)}}} = \sqrt{\frac{450}{850}} = 0.7276$$

load KVA for η_{max} = x (KVA rating)
$= 0.7276 \times 40$
$= 29.1043$

(ii)

$$\eta_{max} = \frac{x_{(\eta_{max})} \cdot \text{KVA rating} \cdot PF}{x_{(\eta_{max})} \cdot \text{KVA rating} \cdot PF + W_i + x^2_{\eta_{max}} W_{cu(FL)}}$$

$$= \frac{0.7276 \times 40000 \times 0.8}{0.7276 \times 40000 \times 0.8 + 450 + (0.7276)^2 \times 850}$$

$\eta_{max} = 0.9628$

$$\boxed{\% \eta_{max} = 96.28\%}$$

Q. The efficiency of a 200 KVA, 1100/230 V transformer is a maximum of 98.0% at 50% of rated load. Calculate -

(i) core loss
(ii) Efficiency at rated load
(iii) Efficiency at a load of 75%

Ans given: $\eta_{max} = 0.98$ assume $PF = 1$
$x = 0.5$

(i)

$$\eta_{max} = \frac{x\,(\text{KVA rating})\, PF}{x\,(\text{KVA rating})\, PF + W_i + \underbrace{x^2 W_{cu(FL)}}_{W_i}}$$

for η_{max} ⇒ $W_i = W_{cu}$
$W_i = x^2 W_{cu(FL)}$

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$$0.98 = \frac{0.5\times200\times10^3\times1}{0.5\times200\times10^3\times1 + 2w_i}$$

$$\Rightarrow 0.5\times200\times10^3 + 2w_i = \frac{0.5\times200\times10^3}{0.98}$$

$$\boxed{w_i = 1020.4081\,W}$$

(ii) $$x_{(\eta_{max})} = \sqrt{\frac{w_i}{w_{cu(FL)}}}$$

$$0.5 = \sqrt{\frac{1020.4081}{w_{cu(FL)}}} \Rightarrow \boxed{w_{cu(FL)} = 4081.6327\,W}$$

efficiency at rated load, $x = 1$

$$\eta = \frac{x\,(\text{kVA rating})\times PF}{x(\text{kVA rating})\times PF + w_i + x^2 w_{cu(FL)}}$$

$$\eta = \frac{1(200\times10^3)\times1}{1\times200\times10^3\times1 + 1020.4081 + 1^2\times4081.6327}$$

$$\eta = 0.9751 \Rightarrow \boxed{\%\eta = 97.51}$$

(iii) efficiency at 75% of rated load, $x = 0.75$

$$\eta = \frac{0.75\times200\times10^3\times1}{0.75\times200\times10^3\times1 + 1020.4081 + (0.75)^2\times4081.6327}$$

$$= 0.9784$$

$$\boxed{\%\eta = 97.84\%}$$

HV (secondary)

Q. A 100 KVA, 1000V/10000V, 50Hz 1-Phase transformer iron losses of 1100 watts the copper loss with 5A in high voltage winding is 400 watts. Calculate the efficiency at 25% of full load at:

(i) UPF

(ii) 0.8 lagging pf

ans KVA rating = 100 ; $\frac{E_1}{E_2} = \frac{1000}{10000}$; $x = 0.25$

$W_i = 1100W$; $W_{cu} = 400W$ (5A current in HV winding)

$$I_{2(FL)} = \frac{100 \times 10^3}{10000} = 10A$$

$$W_{cu} = I^2 R$$

$$W_{cu} \propto I^2$$

$$\Rightarrow \frac{W_{cu}}{W_{cu(FL)}} = \left(\frac{I_2}{I_{2(FL)}}\right)^2$$

$$W_{cu(FL)} = W_{cu}\left(\frac{I_{2(FL)}}{I_2}\right)^2$$

$$= 400 \times \left(\frac{10}{5}\right)^2$$

$$\boxed{W_{cu(FL)} = 1600W}$$

(i) $$\eta = \frac{0.25 \times 100 \times 10^3 \times 1}{0.25 \times 100 \times 10^3 \times 1 + 1100 + (0.25)^2 \times 1600}$$

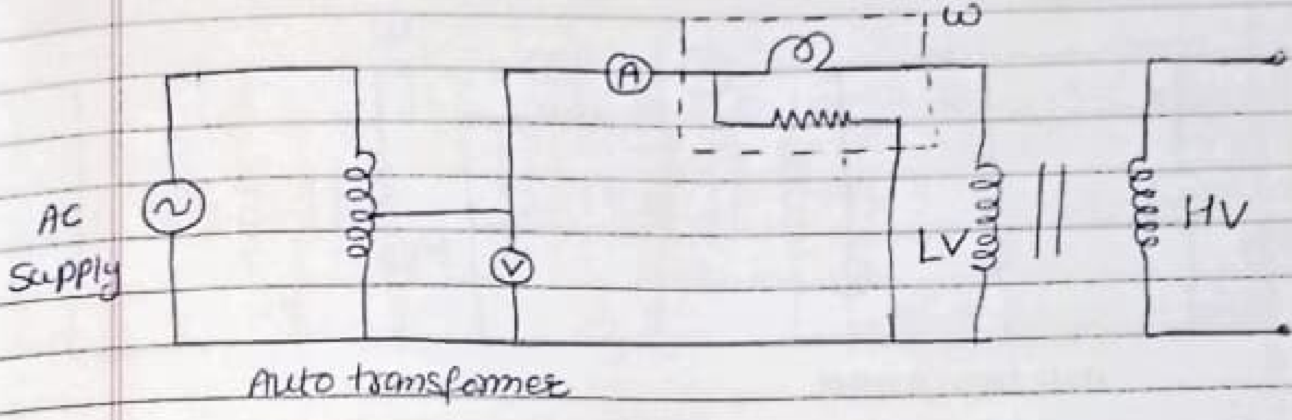
$$= 0.9542$$

(ii) $$\eta = \frac{0.25 \times 100 \times 10^3 \times 0.8}{0.25 \times 100 \times 10^3 \times 0.8 + 1100 + (0.25)^2 \times 1600}$$

$$= 0.9434$$

$$\boxed{\% \eta = 94.34\%}$$

☆ Open circuit test on transformer (O.C test):

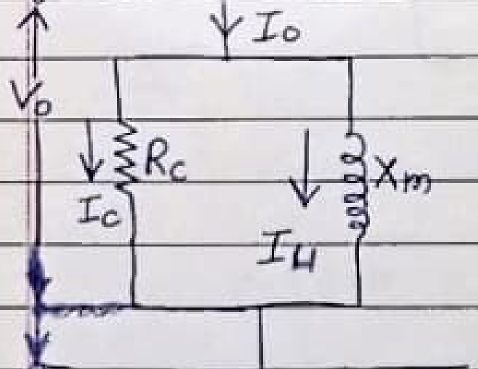


- OC test is performed on LV winding keeping HV winding open circuited
- V_0 (No load primary voltage / Rated voltage)
  I_0 (No load primary current) [very low value]
  W_0 (No load losses) [Core losses]
- Copper losses occuring in the winding will be very less because of very low value of I_0

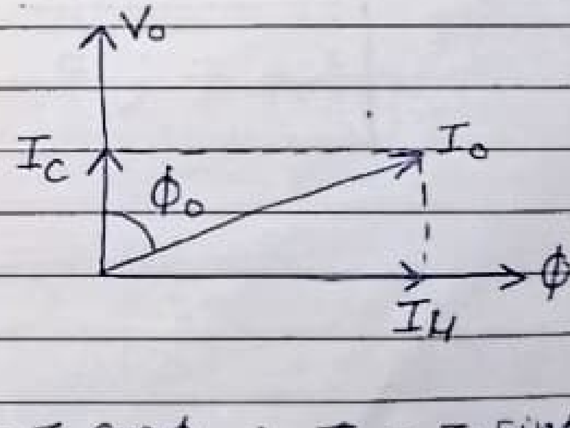
From O.C test: V_0, I_0, W_0

$$W_0 = V_0 I_0 \cos\phi_0$$

$$\cos\phi_0 = \frac{W_0}{V_0 I_0} \Rightarrow \phi_0 \text{ [No load PF angle]}$$



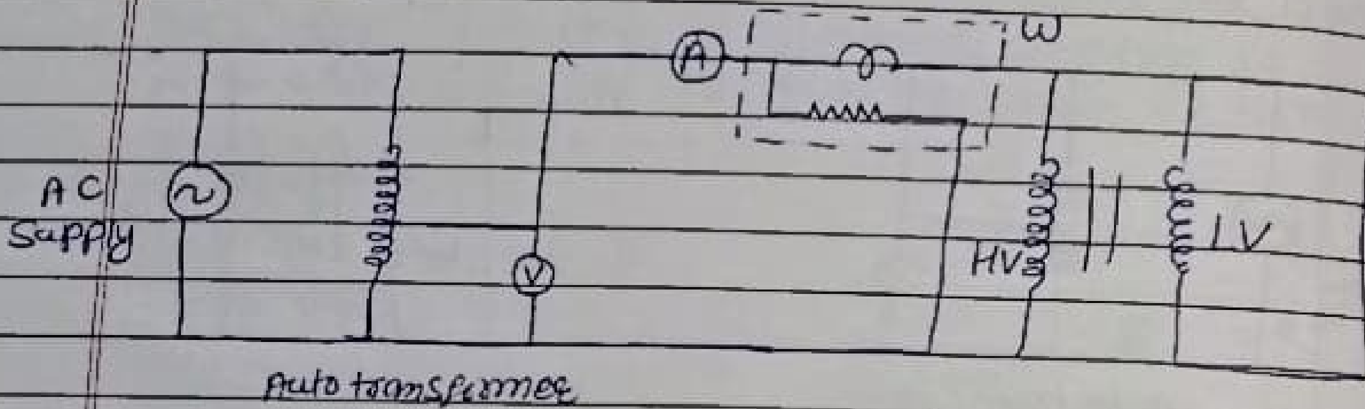
$\Rightarrow$



$$I_c = I_0 \cos\phi_0 \; ; \; I_\mu = I_0 \sin\phi_0$$

$$\boxed{R_c = R_0 = \frac{V_0}{I_c}} \; ; \; \boxed{X_m = X_0 = \frac{V_0}{I_\mu}}$$

* Short circuit test on transformer (SC test):



- Normally SC test is performed on HV winding short circuiting LV winding.
- V_{sc} (Short circuit voltage) [very low compared to Rated voltage]
  I_{sc} (Short circuit current) [comparable with full load current]
  W_{sc} (Short circuit power) [copper losses]

- wattmeter will read copper losses. because of high current passing through windings.

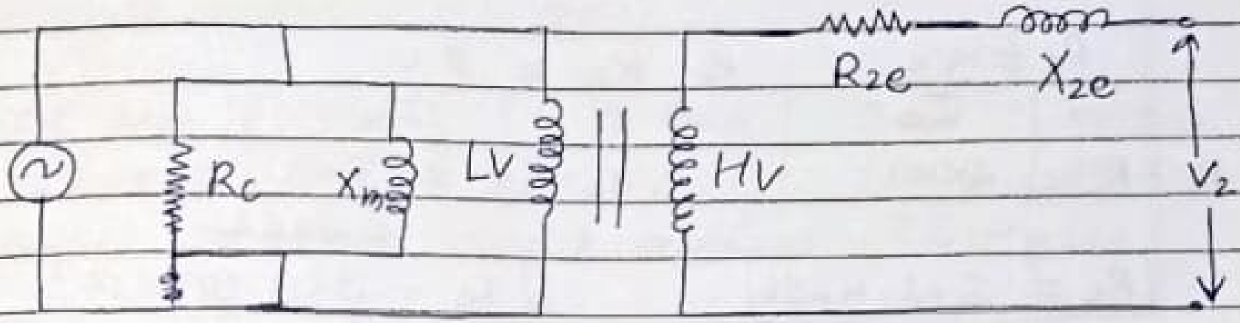
From SC test: V_{sc}, I_{sc}, W_{sc}

$$W_{sc} = I_{sc}^2 \cdot R_{2e}$$

$$\boxed{R_{2e} = \frac{W_{sc}}{I_{sc}^2}} \quad ; \quad Z_{2e} = \frac{V_{sc}}{I_{sc}}$$

$$\boxed{X_{2e} = \sqrt{Z_{2e}^2 - R_{2e}^2}}$$

Equivalent Circuit diagram of transformer :



- Core losses [OC test]
- Copper losses [SC test]
- * efficiency & voltage regulation

Q. Obtain the equivalent circuit referred to high voltage side of a 400/200 volts, 50 Hz single phase transformer from following tests.

O.C test	200V	0.7 A	70 W	on L.V side
S.C test	150 V	10A	80 W	on H.V side

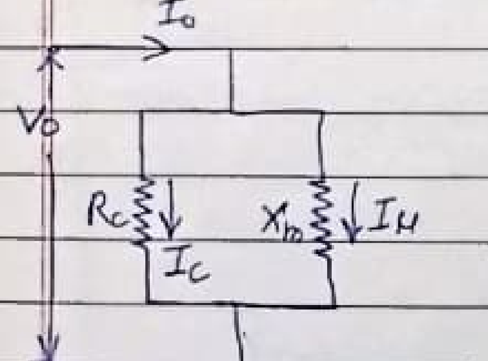
ans

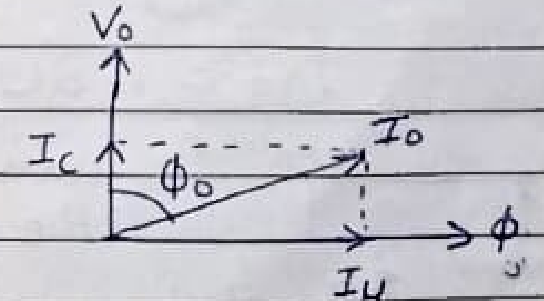
(a) O.C test : $V_0 = 200\,V$, $I_0 = 0.7A$. $W_i = 70W$

$$W_i = V_0 I_0 \cos\phi_0 \Rightarrow 70 = 200 \times 0.7 \times \cos\phi_0$$

$$\boxed{\cos\phi_0 = 0.5} \Rightarrow \boxed{\phi_0 = 60^\circ}$$

$$\boxed{\sin\phi_0 = 0.8667}$$





$$I_c = I_0 \cos\phi_0 = 0.7 \times 0.5 = 0.35A$$

$$I_\mu = I_0 \sin\phi_0 = 0.7 \times 0.8667 = 0.6062A$$

$$R_c = \frac{V_0}{I_c} \quad \& \quad X_m = \frac{V_0}{I_\mu}$$

$$R_0 = R_c = \frac{200}{0.35} \qquad X_0 = X_m = \frac{200}{0.6062}$$

$$\boxed{R_c = 571.4286} \qquad \boxed{X_0 = 329.9144\,\Omega}$$

ⓑ SC test: $V_{sc} = 15V$, $I_{sc} = 10A$, $W_{sc} = 80W$

$$W_{sc} = I_{sc}^2 \cdot R_{1e} \Rightarrow 80 = 10^2 R_{1e}$$

$$\boxed{R_{1e} = 0.8\,\Omega}$$

$$Z_{1e} = \frac{V_{sc}}{I_{sc}} = \frac{15}{10} \Rightarrow \boxed{Z_{1e} = 1.5\,\Omega}$$

$$Z_{1e} = \sqrt{R_{1e}^2 + X_{1e}^2} \Rightarrow 1.5 = \sqrt{(0.8)^2 + X_{1e}^2}$$

$$X_{1e} = 1.2689$$

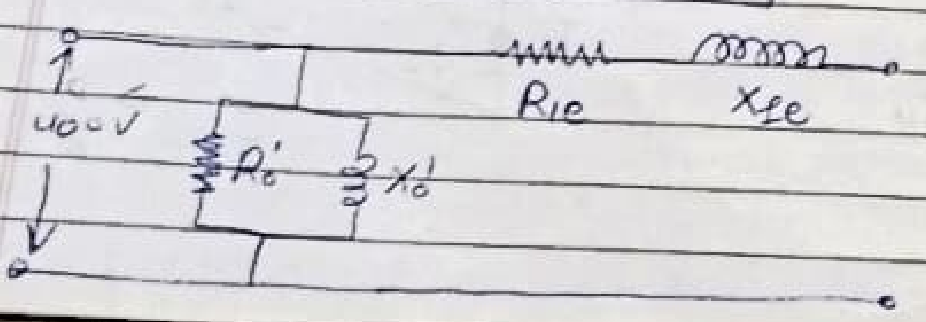
equivalent circuit ref. to HV (Primary)

$$R_0' = R_0\left(\frac{400}{200}\right)^2 = 571.4286 \times 4$$

$$\boxed{R_0' = 2.2857\,k\Omega}$$

$$X_0' = X_0\left(\frac{400}{200}\right)^2 = 329.9144 \times 4$$

$$\boxed{X_0' = 1.3197\,k\Omega}$$



Q. A 5KVA, 1000/200V, 50Hz, single phase transformer gave the following results during open circuit and short circuit tests.

OC test	1000V	0.24A	90W	HV side
SC test	50V	5A	110W	HV side

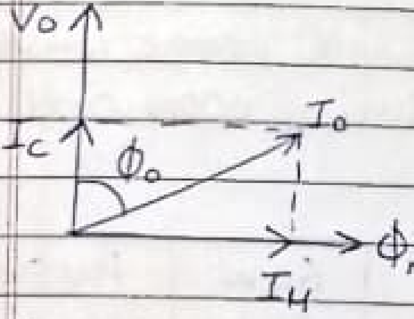
obtain equivalent circuit referred to HV side.

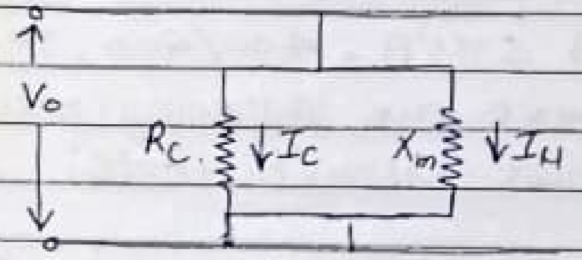
ans (a) OC test: $V_o = 1000V$, $I_o = 0.24A$, $W_i = 90W$

$$W_i = I_o V_o \cos\phi_o \Rightarrow 90 = 0.24 \times 1000 \cos\phi_o$$

$$\boxed{\cos\phi_o = 0.375}$$

$\phi_o = 67.98°$ $\sin\phi_o = 0.927$





$$I_c = I_o \cos\phi_o = 0.24 \times 0.375 \Rightarrow I_c = 0.09A$$
$$I_\mu = I_o \sin\phi_o = 0.24 \times 0.927 \Rightarrow I_\mu = 0.2225A$$

$$R_c = R_o = \cancel{V/I}\ \frac{V_o}{I_c} = \frac{1000}{0.09} \quad \& \quad X_c = X_o = \frac{V_o}{I_H} = \frac{1000}{0.2225}$$

$$\boxed{R_o = 1111.1111\ \Omega}$$

$$\boxed{X_o = 4494.382\ \Omega}$$

(b) SC test: $V_{sc} = 50V$; $I_{sc} = 5A$; $W_{sc} = 110W$

$$W_{sc} = I_{sc}^2 R_{1e} \Rightarrow R_{1e} = \frac{110}{(5)^2}$$

$$\boxed{R_{1e} = 4.4\ \Omega}$$

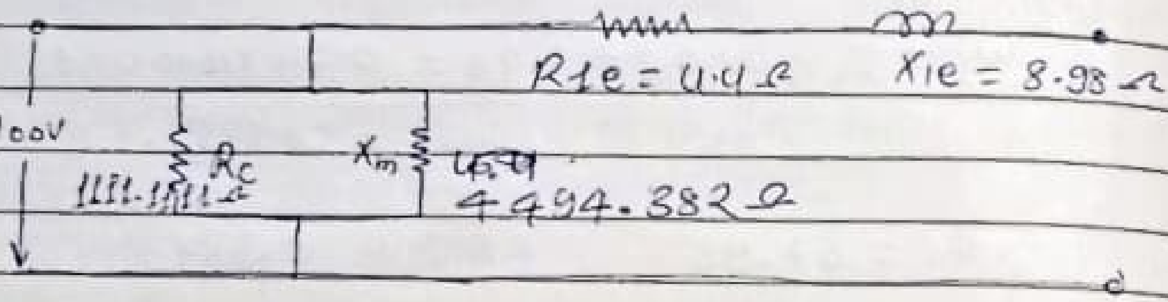
$$Z_{1e} = \frac{V_{sc}}{I_{sc}} = \frac{50}{5}$$

$$\boxed{Z_{1e} = 10\,\Omega}$$

X_{1e} – $Z_{1e} = \sqrt{R_{1e}^2 + X_{1e}^2} \Rightarrow 10 = \sqrt{(4.4)^2 + X_{1e}^2}$

$$\boxed{X_{1e} = 8.98\,\Omega}$$

eq. circuit referred to HV side



Q. A 5KVA, 400/200, 50Hz, single phase transformer gave the following results during open and short circuit tests.

OC test	400V	1A	60W	HV side
SC test	15V	12.5A	50W	HV side

Calculate:

(i) No load parameters R_0 & X_0

(ii) eq. resistance and reactance ref. to HV side

(iii) Iron & copper losses at full load

(iv) Efficiency at half load and 0.8 pf lagging.

Ans

(a) OC test: $V_0 = 400V$, $I_0 = 1A$, $W_0 = W_i = 60W$

$W_i = V_0 I_0 \cos\phi_0 \Rightarrow 60 = 400 \times 1 \times \cos\phi_0$

$\cos\phi_0 = 0.15$

$\phi_0 = 81.37°$

$\sin\phi_0 = 0.9887$

$I_c = I_0 \cos\phi_0 = 1 \times 0.15 = 0.15A$

$I_\mu = I_0 \sin\phi_0 = 1 \times 0.9887 = 0.9887\,A$

$$R_c = R_0 = \frac{V_0}{I_c} = \frac{400}{0.15} = 2666.6667\,\Omega$$

$$X_m = X_0 = \frac{V_0}{I_\mu} = \frac{400}{0.9887} = 404.5717\,\Omega$$

(b) SC test: $V_{sc} = 15V$, $I_c = 12.5A$; $W_{sc} = 50W$

$$W_{sc} = I_{sc}^2 \cdot R_{1e} \Rightarrow 50 = (12.5)^2 \cdot R_{1e}$$

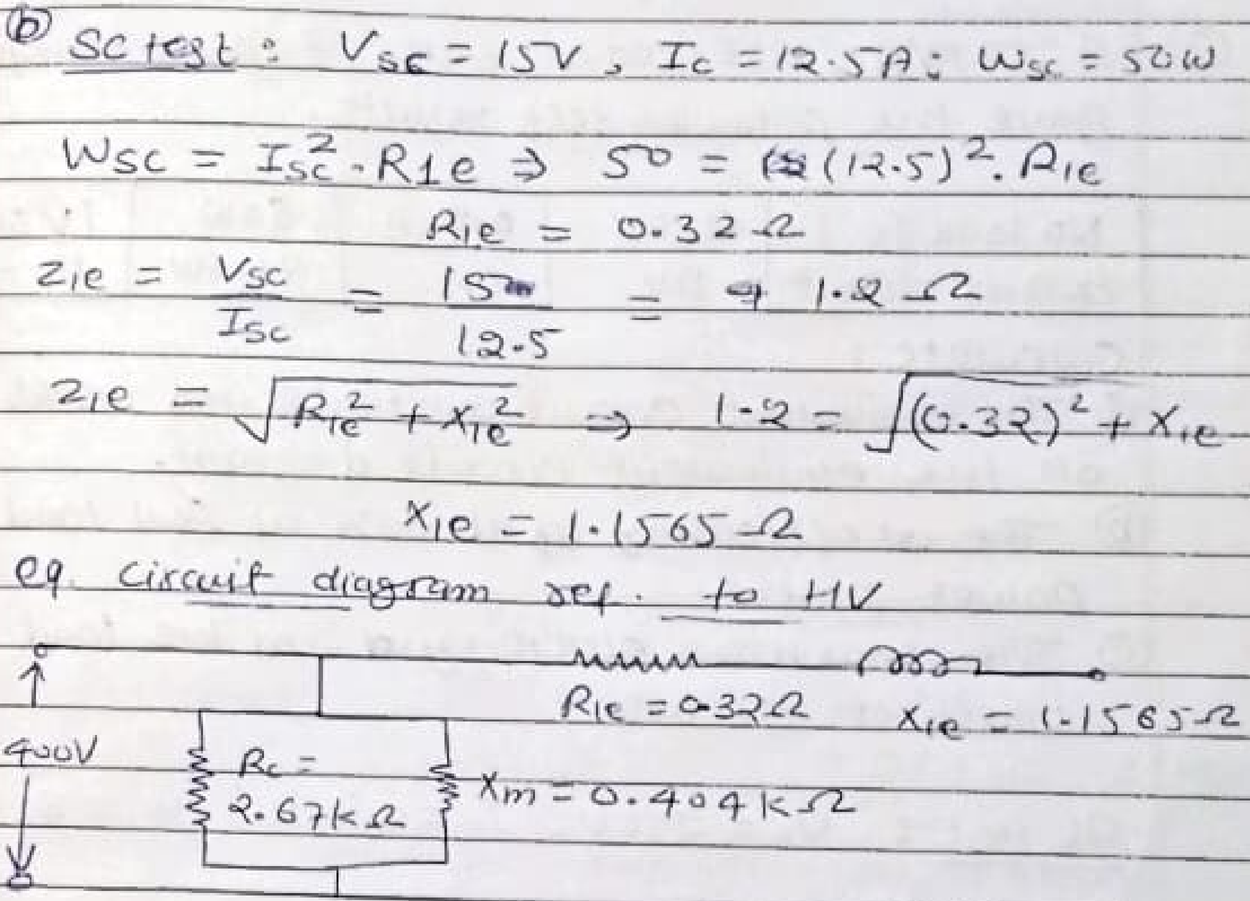
$$R_{1e} = 0.32\,\Omega$$

$$Z_{1e} = \frac{V_{sc}}{I_{sc}} = \frac{15}{12.5} = 1.2\,\Omega$$

$$Z_{1e} = \sqrt{R_{1e}^2 + X_{1e}^2} \Rightarrow 1.2 = \sqrt{(0.32)^2 + X_{1e}}$$

$$X_{1e} = 1.1565\,\Omega$$

eq. circuit diagram ref. to HV



Calculation of η:

$W_{sc} = 50W$; $I_{sc} = 12.5A\,(I_1)$

$$I_{1fl} = \frac{5000}{400} = 12.5A$$

$$W_{cu(fl)} = W_{sc} \times \left(\frac{I_{1fl}}{I_1}\right)^2 = 50\left(\frac{12.5}{12.5}\right)^2$$

$$= 50W$$

$W_{iron} = 60\,W$

$$\eta = \frac{x(\text{KVA rating}) \times pf}{~~x(\text{KVA} \text{ rating}) \times pf \times W_i \times x^2 \text{ cu } W_{cu(FL)}~~}$$

$$= \frac{(0.5) \times 5000 \times 0.8}{0.5 \times 5000 \times 0.8 + 60 + (0.5)^2 \times 50}$$

$$= 0.965$$

$$\boxed{\%\eta = 96.5}$$

Q. A 50 KVA, 250/500-V, 50Hz, 1-phase transformer gave the following test results.

No load (OC)	250V	0.75A	60W	LV side
~~SC~~ Short circuit	9V	6A	21.6W	HV side

Calculate:

(a) The equivalent circuit constants and insert these on the equivalent circuit diagram.

(b) The efficiency at 60% of full load unity power factor.

(c) The maximum efficiency and the load at which it occurs.

ans

OC test: $V_0 = 250V$; $I_0 = 0.75A$; $W_i = 60W$

$W_i = V_0 I_0 \cos\phi_0$ ~~$= 250 \times 0.75 \cos\phi_0$~~

$60 = 250 \times 0.75 \cos\phi_0 \Rightarrow \cos\phi_0 = 0.32$

$\phi_0 = 71.34°$

$\sin\phi_0 = 0.9474$

$I_c = I_0 \cos\phi_0 = 0.75 \times 0.32 = 0.24\,A$

$I_\mu = I_0 \sin\phi_0 = 0.75 \times 0.9474 = 0.7106A$

$R_c = \frac{V_0}{I_c} = \frac{250}{0.24}$ & $X_m = \frac{V_0}{I_\mu} = \frac{250}{0.7106}$

$= 1041.6667\,\Omega$ $= 351.8154\,\Omega$

(b) SC test: $V_{sc} = 9V$; $I_{sc} = 6A$; $W_{sc} = 21.6\,W$

$$W_{sc} = I_{sc}^2 R_{2e} \Rightarrow 21.6 = 6^2 \times R_{2e}$$

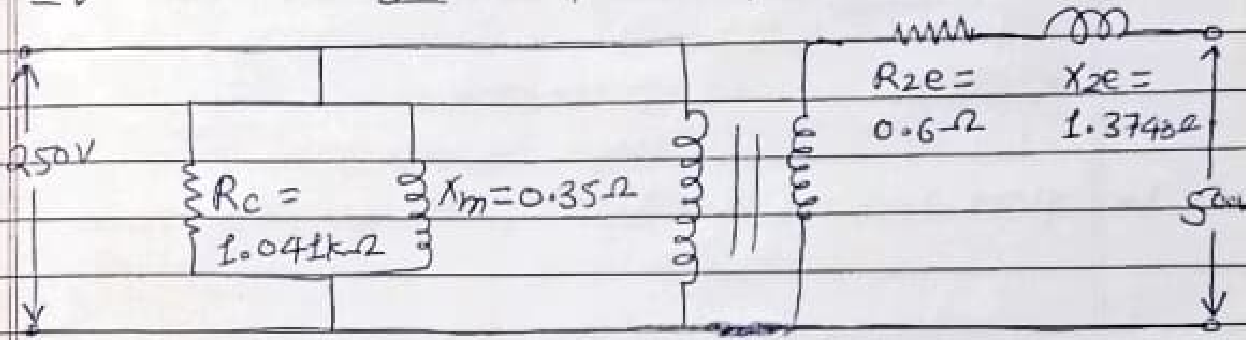
$$R_{2e} = 0.6\,\Omega$$

$$Z_{2e} = \frac{V_{sc}}{I_{sc}} = \frac{9}{6} = 1.5\,\Omega$$

$$Z_{2e} = \sqrt{R_{2e}^2 + X_{2e}^2} \Rightarrow 1.5 = \sqrt{0.6^2 + X_{2e}^2}$$

$$X_{2e} = 1.3748\,\Omega$$

eq. circuit diagram of transformer.



$$\text{efficiency } \eta = \frac{x\,(\text{kVA rating}) \times pf}{x\,(\text{KVA rating}) \times pf + W_i + x^2 W_{cu(fl)}}$$

$x = 0.6$, $pf = 1$; $KVA = 50$; $W_i = 60W$

$$I_{2fl} = \frac{50000}{500} = 100A$$

$$\frac{W_{cu(fl)}}{W_{sc}} = \left(\frac{I_{2fl}}{I_{sc}}\right)^2 \Rightarrow W_{cu(fl)} = 21.6 \times \left(\frac{100}{6}\right)^2$$

$$\boxed{W_{cu(fl)} = 6000W}$$

$$\eta = \frac{0.6 \times 50000 \times 1}{0.6 \times 50000 \times 1 + 60 + 0.6^2 \times 6000} = 0.9311$$

$$\%\eta = 93.11$$

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$$x(\text{for } \eta_{max}) = \sqrt{\frac{W_i}{W_{cu(fl)}}} = \sqrt{\frac{60}{6000}} = 0.1$$

Load KVA where η is max $= x \times (\text{KVA rating})$

$= 0.1 \times 50$

Load KVA $= 5$

$$\eta_{max} = \frac{0.1 \times 5000 \times 1}{0.1 \times 5000 \times 1 + 60 + (0.1)^2 \times 6000}$$

$= 0.9766$

$$\boxed{\% \ \eta_{max} = 97.66\%}$$