

Maulana Azad National Institute of Technology Bhopal
Department of Mathematics, Bioinformatics and Computer Applications
 Assignment-2

Programme	B.Tech	Semester	Semester II (2024-25)
Course code	MTH24110	Section	All
Course Title	Mathematics-II	Department	MBC

Q. No.	Question Text	CO	BT
1.	Find the Laplace transform of a. $f(t) = \sin(mt) \sin(nt)$ where $m, n \in \mathbb{R}$ b. $f(t) = e^{-2t} \int_0^t \frac{\sin(\tau)}{\tau} d\tau$	3	1,2,3,5
2.	Using Convolution Theorem for Laplace Transform, find the inverse Laplace transform of the following function $\frac{s^2}{(s^2 + a^2)(s^2 + b^2)}$	3	1,2,3,5
3.	Discuss the application of Laplace transform to solve the variable coefficient initial value problem $ty'' + 2y' + ty = \cos(t)$ with initial conditions $y(0) = 1, y'(0) = 2.$	3	3,4,5
4.	Using the application of Laplace transform solve the following initial value problem. $\frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} + 3\frac{dy}{dx} - y = x^2e^x$ with initial conditions $y(0) = 1, y'(0) = 0$ and $y''(0) = 2.$	3	3,4,5
5.	Solve the following integral equation using the method of Laplace transform. $t^2 = \int_0^t e^\tau x(\tau) d\tau$	3	3,4,5

Q. No.	Question Text	CO	BT
6.	<p>Suppose we have a mass spring system $x''(t) + x(t) = f(t)$, $x(0) = 0$, $x'(0) = 0$ where</p> $f(t) = \begin{cases} 0 & , 0 < t < 1 \\ 1 & , 1 \leq t < 5 \\ 0 & , t \geq 5 \end{cases}$ <p>Find the displacement of the spring as a function of time and hence evaluate $x(5)$.</p>	3	3,4,5
7.	<p>Given the initial value problem $y'' + 4y = 3 \sin(t)$ with initial conditions $y(0) = 1$ and $y'(0) = -1$. Find the solution of the system using the method of Laplace transform.</p>	3	3,4,5
8.	<p>Given that $f : (0, \infty) \rightarrow \mathbb{R}$ is a continuous function and $c > 0$ is a fixed real number such that $-c \leq f(t) \leq c$ for every $t > 0$. Does f satisfy the sufficient conditions for the existence of Laplace transform? Give explanation in support of your answer.</p>	3	2,3,4
9.	<p>Check whether the following functions satisfy the sufficient conditions for existence of Laplace transform.</p> <p>a. $f(t) = e^{\cos(t)}$</p> <p>b. $f(t) = e^{7t^3 - 5t^2 + 1}$</p> <p>c. $f(t) = \begin{cases} e^{9t^2 - 5} & , 0 < t < 20 \\ e^{9t} & , t \geq 20 \end{cases}$</p> <p>d. $f(t) = \begin{cases} e^{9t} & , 0 < t < 20 \\ e^{9t^2 - 5} & , t \geq 20 \end{cases}$</p>	3	2,3,4
10.	<p>For a function $f : (0, \infty) \rightarrow \mathbb{R}$, we are given its Laplace Transform $\mathcal{L}(f(t))(s) = F(s)$. If $a > 0$ is a fixed real number, what will be the Laplace transform of</p> <p>a. $g(t) = \begin{cases} f(t) & , 0 < t < a \\ 0 & , t \geq a \end{cases}$</p> <p>b. $g(t) = \begin{cases} 0 & , 0 < t < a \\ f(t) & , t \geq a \end{cases}$</p> <p>c. $g(t) = \begin{cases} f(t) & , 0 < t < a \\ e^{5t} f(t) & , t \geq a \end{cases}$</p>	3	2,4,6

Course Outcome (CO)

CO1: Demonstrate the ability to solve linear systems and perform matrix operations, including determining the rank, eigenvalues, and eigenvectors.

CO2: Apply the Cayley-Hamilton theorem to solve matrix-related problems.

CO3: Solve ordinary differential equations using Laplace transforms and interpret inverse Laplace transforms for engineering applications.

CO4: Develop proficiency in Fourier series and Fourier transforms and their application in signal analysis.

CO5: Analyze and solve partial differential equations (PDEs), including boundary value problems for heat and wave equations.

Bloom Taxonomy (BT)

1-Remember; 2-Understand; 3-Apply; 4-Analyze; 5-Evaluate; 6-Create