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Assignment #1 Mathematics - I (MTH 110)

1. If
$$V = \ln \sin \frac{\pi(2x^2 + y^2 + xz)^{1/2}}{2(x^2 + xy + 2yz + z^2)^{1/3}},$$
find the value of $x\frac{\partial V}{\partial x} + y\frac{\partial V}{\partial y} + z\frac{\partial V}{\partial z}$ when $x = 0$, $y = 1$ and $z = 2$.

2. Find the value of the parameter n so that $V = r^n(3\cos^2\theta - 1)$ satisfies
$$\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right) = 0.$$

3. If $u(x, y) = \csc^{-1}\left(\sqrt{x + \sqrt{y}}\right)$, discuss the degree of the function u, and also prove that
$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \tan u\left(\frac{13}{12} + \frac{\tan^2 u}{12}\right).$$

4. Find three positive numbers whose sum is 48 and such that their product is as large as possible.

5. An international airline has a regulation that each passenger can carry a suitcase having the sum of its width, length, and height less than or equal to 129 cm. Find the dimensions of the suitcase of maximum volume that a passenger can carry under this regulation.

6. Discuss the maxima and minima of the function $u(x, y, z) = \sin x \sin y \sin z$, where x, y, and z are the angles of a triangle.

7. Locate the stationary points of $x^4 + y^4 - 2x^2 + 4xy - 2y^2$ and also determine their nature.

8. Find all the stationary points of the function $x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$. Also, examine whether the function is maximum or minimum at these points.

9. Locate the points of the surface $x^2 - yz = 5$ that are closest to the origin.

10. Find the extreme value of $x^2 + y^2 + z^2 + xy + xz + yz$ subject to the constraints $x + y + z = 1$ and $x + 2y + 3z = 3$.

11. Suppose that a function $f(x, y)$ is differentiable at the point $(1, 1)$ with $f_x(1, 1) = 2$ and $f(1, 1) = 3$. Let $L(x, y)$ denote the local linear approximation of f at $(1, 1)$. If $L(1.1, 0.9) = 3.15$, find the value of $f_y(1, 1)$.

12. Obtain the linear approximation to the function $f(x, y) = e^y \log(x + y)$ using Taylor's series about the point $(1, 0)$. Also, estimate the maximum absolute error over the rectangle $|x - 1| < 0.1$, $|y - 1| < 0.1$.

13. Expand $f(x, y) = \sin(x + 2y)$ in a Taylor series up to third-order terms about the point $(0, 0)$. Find the maximum error over the rectangle $|x| < 0.1$, $|y| < 0.1$.