

**Maulana Azad National Institute of Technology Bhopal-462003**

**End-Term Examination (Session: March-June 2021)**

**Course: B.Tech.**

**Semester-II**

**Section: All**

**Subject: Mathematics-II**

**Subject Code: MTH-121**

**Max. Marks: 50**

**Time: 3 Hours**

**Note: All questions are compulsory.**

S.No	Question	Marks
1(a)	Find the series solution of the following differential equation about $x=0$ $\frac{d^2 y}{dx^2} + x \frac{dy}{dx} + x^2 y = 0$	5
1(b)	Solve the following differential equation using Laplace transform $t \frac{d^2 y}{dt^2} + (1-2t) \frac{dy}{dt} - 2y = 0, \text{ if } y(0) = 1 \text{ \& } \frac{dy(0)}{dt} = 2$	5
2(a)	Solve the equation $\frac{dy}{dt} + 2y + \int_0^t y(t-u) du = \sin t, y(0) = 1$ by using the Laplace transform.	5
2(b)	If $v(x, y) = y + 3x^2 y - y^3$ , then show that $v(x, y)$ is a harmonic function. Also, find the corresponding conjugate harmonic function $u(x, y)$ and construct the analytic function $f(z) = u + iv$	5
3(a)	Evaluate $\oint_C \frac{e^{z^2} dz}{z^2(z-i)^2}$ , where C: the square with vertices at $\pm 3 \pm 3i$	4
3(b)	Use Charpit's method to find the complete integral of $p = (qy + z)^2$	4
4(a)	Solve $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} - 2 \frac{\partial^2 z}{\partial y^2} + 2 \frac{\partial z}{\partial x} + 2 \frac{\partial z}{\partial y} = e^{2x+3y} + \sin(2x+y)$	4
4(b)	Solve the heat equation $u_t = u_{xx}$ in the range $0 \leq x \leq 2\pi$ , subject to the boundary conditions $u(x, 0) = \sin^3 x$ for $0 \leq x \leq 2\pi$ and $u(0, t) = u(2\pi, t) = 0$ for $t \geq 0$	5
5	Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 2 & 0 \\ -1 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$ . Also, show that eigenvalues of $A^3$ are cube of those of A.	5
6(a)	Find the Fourier transform of $e^{-at^2}$ , where $a > 0$ .	4
6(b)	Find the Fourier cosine series of the function $f(x) = \begin{cases} kx, & 0 \leq x \leq 1 \\ k(2-x), & 1 \leq x \leq 2 \end{cases}$	4