Arbitrarily Sampled Signal Reconstruction Using Relative Difference Features

1st Chin-Yun Yu
School of Electronic Engineering and Computer Science
Queen Mary University of London
London, UK
chin-yun.yu@qmul.ac.uk

2nd Johan Pauwels
School of Electronic Engineering and Computer Science
Queen Mary University of London
London, UK
j.pauwels@qmul.ac.uk

3rd György Fazekas
School of Electronic Engineering and Computer Science
Queen Mary University of London
London, UK
george.fazekas@qmul.ac.uk

***Abstract*—Signal reconstruction on arbitrarily sampled positions can be solved by graph-based methods if the pair-wise differences of target quantity between samples can be estimated. We demonstrate this idea on time-of-arrival estimation of head-related transfer functions by viewing it as a phase unwrapping problem. This shows the possibility of generalising various techniques for tasks that fall into this problem definition.**

***Index Terms*—signal reconstruction, linear programming, phase estimation, time-of-arrival estimation**

I. Introduction

Reconstructing signals without measurements of their absolute values is intrinsic to phase unwrapping (PU). In PU, given a set of sampled points, their target is to estimate the absolute phase at each point as the measured phase is wrapped inside $[-\pi, \pi)$. Assuming the density of measurement points is enough to capture the phase changes, one can utilise the wrapped phase differences between points to "unwrap" the phase. We can view the points and connections between points as graph nodes and edges. PU is simply integrating the computed differences on edges along a given path that traverses all the nodes. Various techniques have been proposed to solve PU on a graph, such as minimum-cost network flow (MCF) [1] and graph cuts [2].

The above paradigm can be applied to other signal reconstruction tasks as long as 1) the measurement points can be connected into a graph and 2) estimations of the target quantity differences between points are feasible. Once the requirement is satisfied, various methods developed by the PU community can also be used with minor changes on these tasks. We explored this idea in estimating the time-of-arrival (TOA) of head-related transfer functions (HRTFs).

II. Problem Definition

Assuming the target quantity $f(\theta)$ varies as a function of θ and $\theta \in \mathcal{M}$, $\mathcal{M}$ is the chosen coordinate system. We sampled $\mathcal{M}$ at N different positions $\Theta = \{\theta_0, \dots, \theta_{N-1}\}$. These positions are connected by a graph G with vertices $V(G) = \{0, \dots, N-1\}$ and edges $E(G) = \{e_0, e_1, \dots\}$. We have $X_{u,v} \approx f(\theta_v) - f(\theta_u) : (u, v) \in E(G)$, representing a close estimation of the quantity difference between θ_u and θ_v. The target is to estimate $f(\theta_i) : \theta_i \in \Theta$ given G and $X_e : e \in E(G)$.

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III. Methodology

A. Solution with Linear Programming

Let us represent the true quantity difference as:

$$f_v - f_u = X_{u,v} - K_{u,v}, \tag{1}$$

where $f_i = f(\theta_i)$. The residual $K_{u,v}$ should be small for most edges, assuming robust difference estimations exist. We can formulate the task as finding a set of f_i that the sum of resulting residuals' magnitude $\sum_{e \in E(G)} |K_e|$ is at the minimum. We can represent this and (1) as a linear programming (LP) problem:

$$\begin{gathered} \min_{\mathbf{k},\mathbf{y}} \ \mathbf{w}^T |\mathbf{k}| \\ s.t. \ \begin{bmatrix} \mathbf{A} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{y} \\ \mathbf{k} \end{bmatrix} = \mathbf{x} \\ A_{ij} = \left. \begin{cases} -1, & j = u \\ 1, & j = v \\ 0, & otherwise \end{cases} \right\} : (u, v) = e_i, \end{gathered} \tag{2}$$

where $\mathbf{y} = [f_0, \dots, f_{N-1}]^T, \mathbf{k} = [K_{e_0}, K_{e_1}, \dots]^T, \mathbf{x} = [X_{e_0}, X_{e_1}, \dots]^T$. $\mathbf{w}$ is a user-defined weighting vector.

B. Connection to Phase Unwrapping

Eq. 2 is called the edgelist method [3] in the PU literature and equals to MCF [1] under some conditions. In the PU context, $f(\theta)$ is the scaled phase $\frac{\phi(\theta)}{2\pi}$ and $X_{u,v}$ is the scaled wrapped phase difference $\frac{\mathcal{W}(\psi_v - \psi_u)}{2\pi} \in [-\pi, \pi)$, where ψ_i is the wrapped phase and $\mathcal{W}$ the wrapping function. Dividing by 2π restricts the solutions of $K_{u,v}$ to be only integers. Thus, the unwrapped phase is $2\pi\mathbf{y}$.

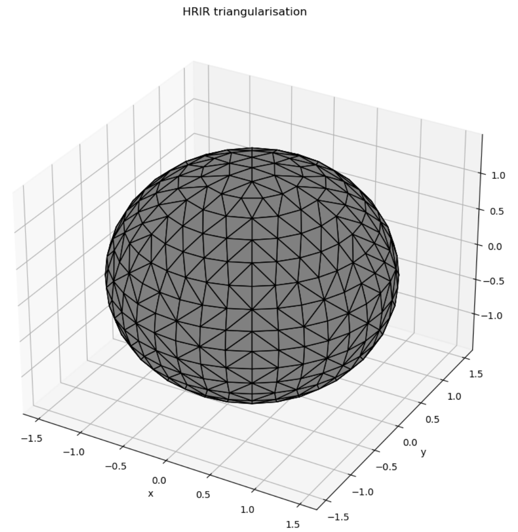

Fig. 1. The resulting graph on the measurement points of HUTUBS HRTFs.

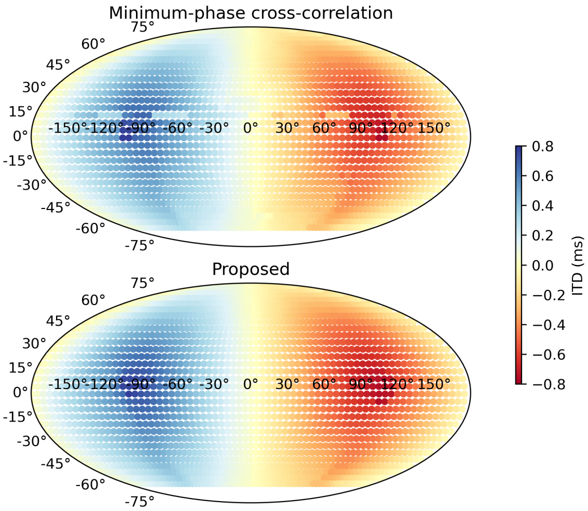

Fig. 2. The ITDs of MRT01.sofa computed from the baseline (top) and the proposed one (bottom). We use Mollweide projection to plot the hemisphere; each dot is a sampled direction.

IV. Application: Time-of-Arrival Estimation in Spatial Audio

The time domain representation of HRTFs is a set of impulse responses (HRIRs) that encode the acoustic scattering patterns from the sound source to human ears in different directions. The time each impulse arrives in the ears is different between directions. Aligning HRIRs based on their time-of-arrival (TOA) could facilitate further processing of HRTFs, like data compression and spatial interpolation.

A. *Configurations*

In TOA estimation, $\mathcal{M} \equiv (\mathbb{S}^2, \{L, R\})$. The HRIRs $h[t]$ were measured in N distinct directions on a 2-sphere with a subject wearing in-ear microphones in the middle. Both ears were measured, so the total number of data points is 2N. We connect the points by running the convex hull algorithm on the spherical points first, which results in a graph illustrated in Fig. 1; then, we connect the left and right ears' data by duplicating the graph and connect the two with N edges $(i, L) \leftrightarrow (i, R)$. We take the maximum cross-correlation index between HRIRs $X_{u,v} = \arg\max_t h_u[t] \star h_v[t]$ as a noise-robust estimation of the time difference, either between spatial directions [4] or ears [5].

B. *Experiment on AACHEN HRTFs*

We tested the proposed method on the subject MRT01 from the AACHEN HRTFs database. We used uniform weights for convenience. We compared the performance to the naive minimum-phase cross-correlation method [6], which does not utilise time differences information. We evaluated the performance on interaural time difference (ITD), which is $f((*, L)) - f((*, R))$.

Fig. 2 shows that the baseline method [6] is less accurate at directions near the horizontal plane, creating non-smooth ITDs. In contrast, the proposed LP-based method predicts smoother ITDs, proving the effectiveness of relative time difference features and considering TOA estimation as a joint optimisation problem for all directions.

V. Conclusion and Future Works

In this paper, we propose to formulate any sampled signal reconstruction task using a graph data structure as long as the difference in the target quantity between sample points can be estimated. We demonstrate this approach on TOA estimation of HRTFs using the edgelist method [3], initially proposed for PU. The estimation results vary more smoothly and are more accurate than the baseline, which does not consider relative differences between sample points.

More work is needed to test the proposed method for other TOA estimation problems in spatial audio and similar tasks, such as HRTFs phase unwrapping. Other optimisation methods from the PU literature could be explored as well.

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