Tutorial exercises for week 2 (Basics, and Natural Deduction for Propositional Logic, part I)

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This tutorial deals with the content of weeks 1 and 2.

From week 1:

1. You should be able to explain:
    - What is a truth value?
    - What's an atom?
    - What is a predicate? (Was mentioned just vaguely and will become clear in week 5!)
    - What is a connective?
    - What is a truth table?
    - What is an interpretation?
2. If somebody gives you a logical formula and asks you whether it is *satisfiable*, *unsatisfiable*, or a *tautology*, can you answer that question? If so, do it! 🙂 I.e., make up a few formulae (stick to two or at most three variables!) and answer that question for each of these properties by constructing a truth table.
3. Follow-up on the last question: Do you know why we recommend to use at most 3 propositional symbols?
4. Based on what you learned in week 1, is the only way of proving that a formula is satisfiable, unsatisfiable, or a tautology to construct a complete truth table? Are there other ways based on what you know so far? (At least on some cases?)
5. Do you agree on the definition of the truth table for the implication? Can you come up with some arguments or an example why it might be problematic if it were defined differently?
6. In the lecture you saw that <-> (i.e., double-implication) can be represented using an AND and two implication symbols. Figure out how you can present the formula even without implications, i.e., just by using the connevtives AND, OR, and NEGATION.
7. Follow-up on the last question: Given an *arbitrary* propositional formula (i.e., you do not know in advance how it looks like), can you give a recipe how it can be presented as a large disjunction, where each disjunct (i.e., each part of the disjunction) uses only conjunctions and negations?

Steps:

1. **Identify True Rows:**
    - Identify the rows in the truth table where the formula evaluates to true.
2. **Construct Disjunctions:**
    - For each true row, construct a disjunction (logical OR) of conjunctions that represent the literals in that row.
3. **Negate False Rows:**
    - For the rows where the formula evaluates to false, negate the conjunction of literals in that row.
4. **Combine All Disjunctions:**
    - Combine all the disjunctions from steps 2 and 3 using conjunctions.

From week 2:

Basics:

- What is a *sequent*? Why do we need it if we already have truth tables?
- What does it mean to say that a sequent is *valid*? (Hint: it uses truth tables again!)
- Can you explain the difference between $\vDash$ and $\vdash$?

Natural Deduction:

Prove at least some of the following sequents in the natural deduction system.

1. $p \wedge q, r \wedge s \vdash (p \wedge r) \wedge (q \wedge s)$
2. $p \to (q \wedge r) \;\vdash\; (p \to q) \wedge (p \to r)$
3. $(p \to q) \wedge (p \to r) \;\vdash\; p \to (q \wedge r)$
4. $(p \wedge q) \to r \;\vdash\; (q \to p) \to (q \to r)$
5. $p \to (q \to r) \;\vdash\; (p \to q) \to (p \to r)$
6. $p \;\vdash\; (q \to r) \to ((p \to q) \to r)$

Chapter 2 of the Course Notes describes how to prove sequents like these.