

Logic (PHIL 2080, COMP 2620, COMP 6262)  
*Chapter: Cheat Sheet for all Rules*  
 (Weeks 1 to 6)

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Natural Deduction

Propositional Logic: Conjunction

## AND / Conjunction / $\wedge$

**Conjunction Elimination:**

$$\frac{X \vdash A \wedge B}{X \vdash A} \wedge E$$

$$\frac{X \vdash A \wedge B}{X \vdash B} \wedge E$$

**Conjunction Introduction:**

$$\frac{X \vdash A \quad Y \vdash B}{X, Y \vdash A \wedge B} \wedge I$$

Propositional Logic: Implication

## Implication / $\rightarrow$

**Implication Elimination:**

$$\frac{X \vdash A \rightarrow B \quad Y \vdash A}{X, Y \vdash B} \rightarrow E$$

**Implication Introduction:**

$$\frac{X, A \vdash B}{X \vdash A \rightarrow B} \rightarrow I$$

disgard assumption A

Propositional Logic: Negation

## Double Negation

**Double-Negation Elimination and Introduction:**

$$\frac{X \vdash \neg\neg A}{X \vdash A} \neg\neg E$$

$$\frac{X \vdash A}{X \vdash \neg\neg A} \neg\neg I$$

## Single Negation / $\neg$

**(Single) Negation Elimination and Introduction:**

$$\frac{X \vdash A \quad Y \vdash \neg A}{X, Y \vdash \perp} \neg E$$

$$\frac{X, A \vdash \perp}{X \vdash \neg A} \neg I$$

discard assumption A

**Reductio ad Absurdum (RAA):** 2-step rule / RAA

$$\frac{X, B \vdash A \quad Y, B \vdash \neg A}{X, Y \vdash \neg B} RAA$$

discard assumption B

Propositional Logic: Disjunction

## OR / Disjunction / $\vee$

**Disjunction Introduction and Elimination:**

$$\frac{X \vdash A}{X \vdash A \vee B} \vee I$$

$$\frac{X \vdash B}{X \vdash A \vee B} \vee I$$

$$\frac{X \vdash A \vee B \quad Y, A \vdash C \quad Z, B \vdash C}{X, Y, Z \vdash C} \vee E$$

discard assumptions A & B

Predicate Logic: Existential Quantifier

## EXIST / Existential / $\exists$

**Existential Introduction Rule:**

$$\frac{X \vdash A_x^t}{X \vdash \exists x A} \exists I$$

Provided **t is not bound in  $A_x^t$**

typical case

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existential rule

**Existential Elimination Rule:**

$$\frac{X \vdash \exists x A_t^x \quad Y, A \vdash B}{X, Y \vdash B} \exists E$$

Provided **t** does not occur  
in  $B$  or any formula in  $Y$

existential rule + typical case

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typical conclusion

Predicate Logic: Universal Quantifier

## ALL / Universal / $\forall$

**Universal Introduction Rule:**

$$\frac{X \vdash A}{X \vdash \forall x A_x^x} \forall I$$

Provided **v does not occur in  $X$**

general case

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all special cases

**Universal Elimination Rule:**

$$\frac{X \vdash \forall x A}{X \vdash A_x^t} \forall E$$

Provided **t is not bound in  $A_x^t$**

all cases

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general case

## Semantic Tableau

## Propositional Logic: Conjunction, Disjunction, and Negation Elimination

### And Elimination:

$$\frac{T: A \wedge B}{T: A, T: B}$$

$$\frac{F: A \wedge B}{F: A \mid F: B}$$

### Or Elimination:

$$\frac{T: A \vee B}{T: A \mid T: B}$$

$$\frac{F: A \vee B}{F: A, F: B}$$

### Negation Elimination:

$$\frac{T: \neg A}{F: A}$$

$$\frac{F: \neg A}{T: A}$$

### Implication Elimination:

$$\frac{T: A \rightarrow B}{F: A \mid T: B}$$

$$\frac{F: A \rightarrow B}{T: A, F: B}$$

## Predicate Logic: Existential and Universal Quantifiers

$$\frac{F: \exists x Fx}{F: Fa, F: Fb, \dots}$$

for all  $a, b, \dots$   
in the branch –  
present and future!

$$\frac{T: \exists x Fx}{T: Fa}$$

if  $a$  is new to  
the branch

$$\frac{T: \forall x Fx}{T: Fa, T: Fb, \dots}$$

for all  $a, b, \dots$   
in the branch –  
present and future!

$$\frac{F: \forall x Fx}{F: Fa}$$

if  $a$  is new to  
the branch

≡

≡

≡

≡

$$\frac{X, F: \exists x A}{X, F: \exists x A, F: A_x^a}$$

for  $a$  in  $X$  or  $A$

$$\frac{X, T: \exists x A}{X, T: A_x^a}$$

for  $a$  not in  $X$  or  $A$

$$\frac{X, T: \forall x A}{X, T: \forall x A, T: A_x^a}$$

for  $a$  in  $X$  or  $A$

$$\frac{X, F: \forall x A}{X, F: A_x^a}$$

for  $a$  not in  $X$  or  $A$

## Predicate Logic: Existential and Universal Quantifier for Invalid Sequents

$$\frac{\text{inv} \quad T: \exists x Fx}{T: Fa \mid T: Fb \mid \dots \mid T: Fn}$$

for all  $a, b, \dots$  in the branch  
or  $n$  new to the branch

≡

$$\frac{\text{inv} \quad F: \forall x Fx}{F: Fa \mid F: Fb \mid \dots \mid F: Fn}$$

for all  $a, b, \dots$  in the branch  
or  $n$  new to the branch

≡

$$\frac{\text{inv} \quad X, T: \exists x A}{X, T: A_x^a \mid X, T: A_x^b \mid \dots \mid X, T: A_x^n}$$

for any/all  $a, b, \dots$  in  $X$  or  $A$ ,  
or  $n$  not in  $X$  or  $A$

$$\frac{\text{inv} \quad X, F: \forall x A}{X, F: A_x^a \mid X, F: A_x^b \mid \dots \mid X, F: A_x^n}$$

for any/all  $a, b, \dots$  in  $X$  or  $A$ ,  
or  $n$  not in  $X$  or  $A$