L4F Workshop 2 (week 4)

Using a SAT solver to Check for Logical Implications

Theoretical Background – Discussion in Class

This week's puzzle will be less about the specifics of L4F (you should continue to train this via the Guide and by trying so solve some of the easy puzzles by yourself), and instead more about how a SAT solver can be used to check whether some formula follows from some others. (While also practicing the formalization of natural language into Logic.)

How to state that some formula A follows logically from some set of formulae X? That's $X \models A$! But SAT solvers don't prove sequents!

1. What do SAT solvers do? They get a formula (or a set of formulae) and find one (or more) interpretations that make it true. So, since we can't ask for $X \models A$, can we ask for $(A_1 \wedge \ldots \wedge A_n) \rightarrow A$ instead? (Assuming $X = \{A_1, \ldots, A_n\}$.)

   No! Consider two examples: $a, a \rightarrow b \models b$ will turn into $(a \wedge (a \rightarrow b)) \rightarrow b$. What will L4F tell? And what does this tell you? Now consider another sequent: $a \vee b \models a \wedge b$. Is it valid? This turns into $(a \vee b) \rightarrow (a \wedge b)$. What will L4F tell? And what does it tell you? In particular when you compare the two sequents? **Discuss in class!**

2. Now, consider the deduction equivalence, i.e., $X, A \vdash B$ iff $X \vdash A \rightarrow B$. By using it (and some other intermediate steps), we can turn a sequent $A_1, \ldots, A_n \vdash A$ into $\vdash (A_1 \wedge \ldots \wedge A_n) \rightarrow A$. Recall what that means! $(A_1 \wedge \ldots \wedge A_n) \rightarrow A$ is a tautology. SAT solvers (like L4F) can however not find out that a formula is a tautology, they can only check whether there's a satisfying interpretation. So, how can a SAT-solver be used to find out whether a formula is tautology? **Discuss in class!**

3. Finally, putting it all together, how to use a SAT solver to to prove $A_1, \ldots, A_n \models A$ with a SAT solver? **Discuss in class!**

4. Although we now already know how we could check $X \models A$, there's also another way. Consider the following:

   - With the negation introduction rule, we know that we can derive $X \vdash \neg A$ from $X, A \vdash \bot$. (And note that this is equivalent to deriving $X \vdash A$ from $X, \neg A \vdash \bot$.)
   - By realizing that A is equivalent to $(\neg A) \rightarrow \bot$, we can exploit the deduction equivalence to derive $X, \neg A \vdash \bot$ from $X \vdash A$.

   $\rightarrow$ So, in total we just showed that $X \vdash A$ holds iff $X, \neg A \vdash \bot$.

   So, what's a second way to use a SAT solver to show $X \models A$? **Discuss in class!**

A Logic Riddle: The Winds and the Windows

This is a problem adapted from Lewis Carroll, Symbolic Logic. We are given the following statements:

1. There is always sunshine when the wind is in the East.
2. When it is cold and foggy, my neighbor practices the flute.
3. When my fire smokes, I set the door open.
4. When it is cold and I feel rheumatic, I light my fire.
5. When the wind is in the East and comes in gusts, my fire smokes.
6. When I keep the door open, I am free from headache.
7. Even when the sun is shining and it is not cold, I keep my window shut if it is foggy.
8. When the wind does not come in gusts, and when I have a fire and keep the door shut, I do not feel rheumatic.
9. Sunshine always brings on fog.
10. When my neighbor practices the flute, I shut the door, even if I have no headache.
11. When there is a fog and the wind is in the East, I feel rheumatic

One of the following statements is a logical consequence of the list of statements above:

- **A** I feel rheumatic whenever the wind comes in gusts.
- **B** When the wind is in the east, I keep my window shut.

Can you tell which using L4F?

Homework for after the tutorial

This tutorial focused on showing validity of sequents in propositional Logic – so more advanced features of L4F were not used yet.

At home, you should read through the section *Next Things Next*, the content of which will be helpful for the next tutorial.