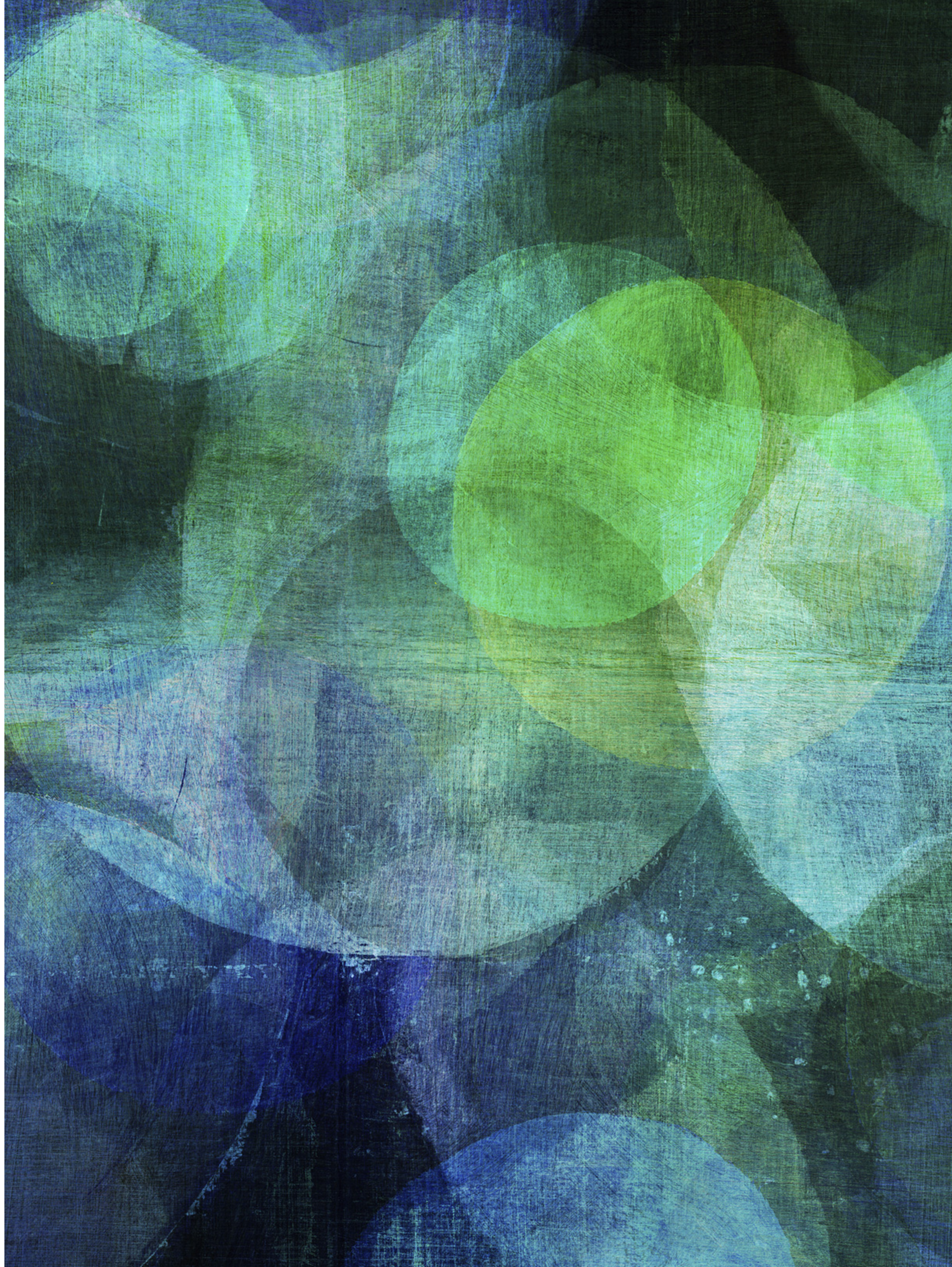
SEMANTICS OF FIRST ORDER LOGIC



GOAL, INFORMALLY

We aim to understand the correspondence between language and reality (can be counterfactual):

<u>*The Syntactic World (Language)*</u>

"Gödel is taller than Tarski"

"Pascal looks more formal than John"

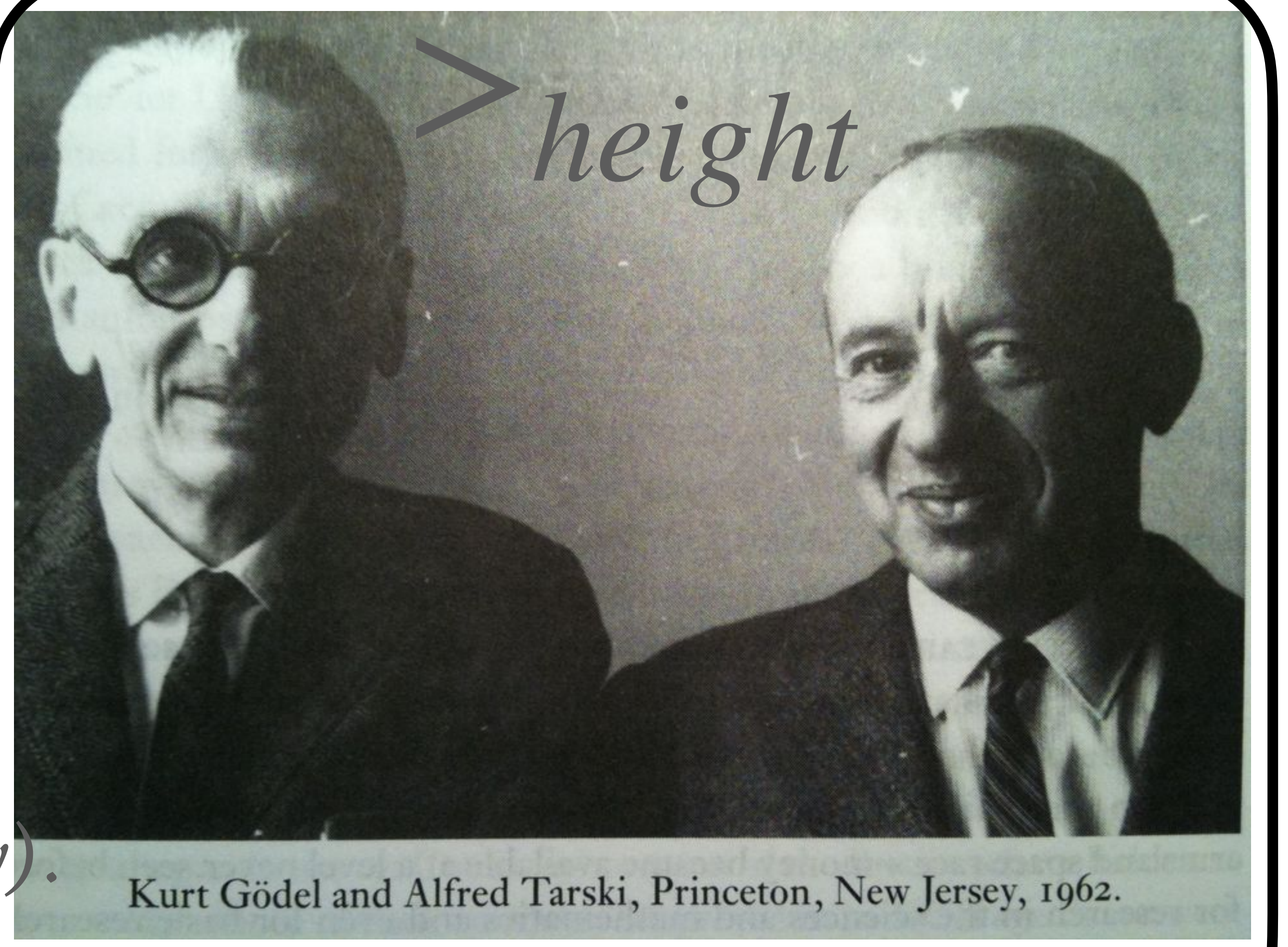
Interpretation ⇓ *Mapping from Language to Reality*

Alfred Tarski (1933):

"Setting out a mathematical

definition of truth

for formal languages"



https://www.facebook.com/alfredtarski/

$>_{LF}$

<u>*The Semantic World (Reality; does not have to be true in this world)*</u>

What is Language? A finitary system of symbols (proof theory).

What is Reality? A (infinitary) system of entities (e.g., sets) and their truths (model theory)

GOAL, FORMALLY

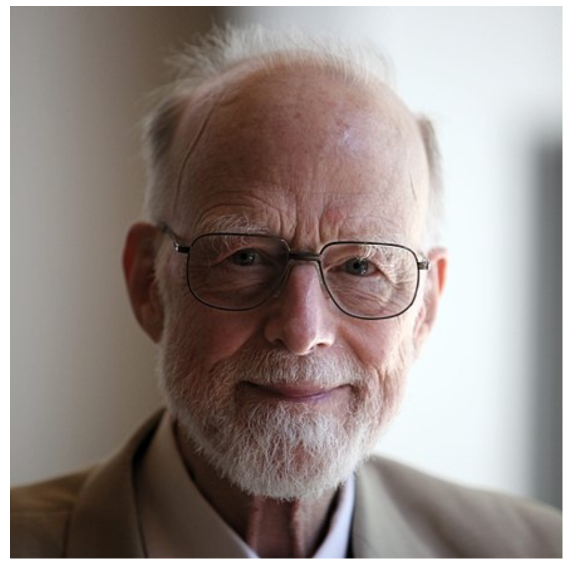
Formal Symbols in First-Order Logic (Syntax)	Interpretation/Denotations/Referents (Semantics; Model Theory)
Names (aka. constants): $a, b, \ldots$	Domain D (non-empty set): $I(a), I(b) \ldots \in D$
Function symbols: $f(x_1, \ldots, x_n)$	Functions on the domain: $I(f) : D^n \to D$
Predicate symbols: $R(x_1, \ldots, x_n)$	Relations: $I(R) : D^n \to \{1,0\}$ or $I(R) \subseteq D^n$
Formulae: $A(x_1, \ldots, x_n)$	Truth value functions: $I(A) : D^n \to \{1,0\}$
Formulae with all variables quantified: A	Truth values: $I(A) = 1$ or 0

Notation: $(x_1, \ldots, x_n)$ means $x_1, \ldots, x_n$ are free variables

SEMANTICS AND COMPOSITIONALITY

- Symbols in proof systems have no meaning (denotations); they just follow syntactic rules (cf. formalism). Semantics or interpretation gives meaning (denotations) to them.
- An interpretation of propositional logic is an assignment of truth values (*0* or *1*) to atoms (atomic propositions; propositional variables) $p, q, r \ldots$.
  - Once we assign truth values to atoms, all formulae are assigned with truth values, which is called compositionality, i.e., the denotation of a complex expression is determined on the basis of the denotations of its simpler parts (and the way they are connected).
  - Remark: Compositional semantics of programming languages is essential for formal verification of programs, especially for safety-critical systems such as self-driving cars.

Frege



Tony Hoare

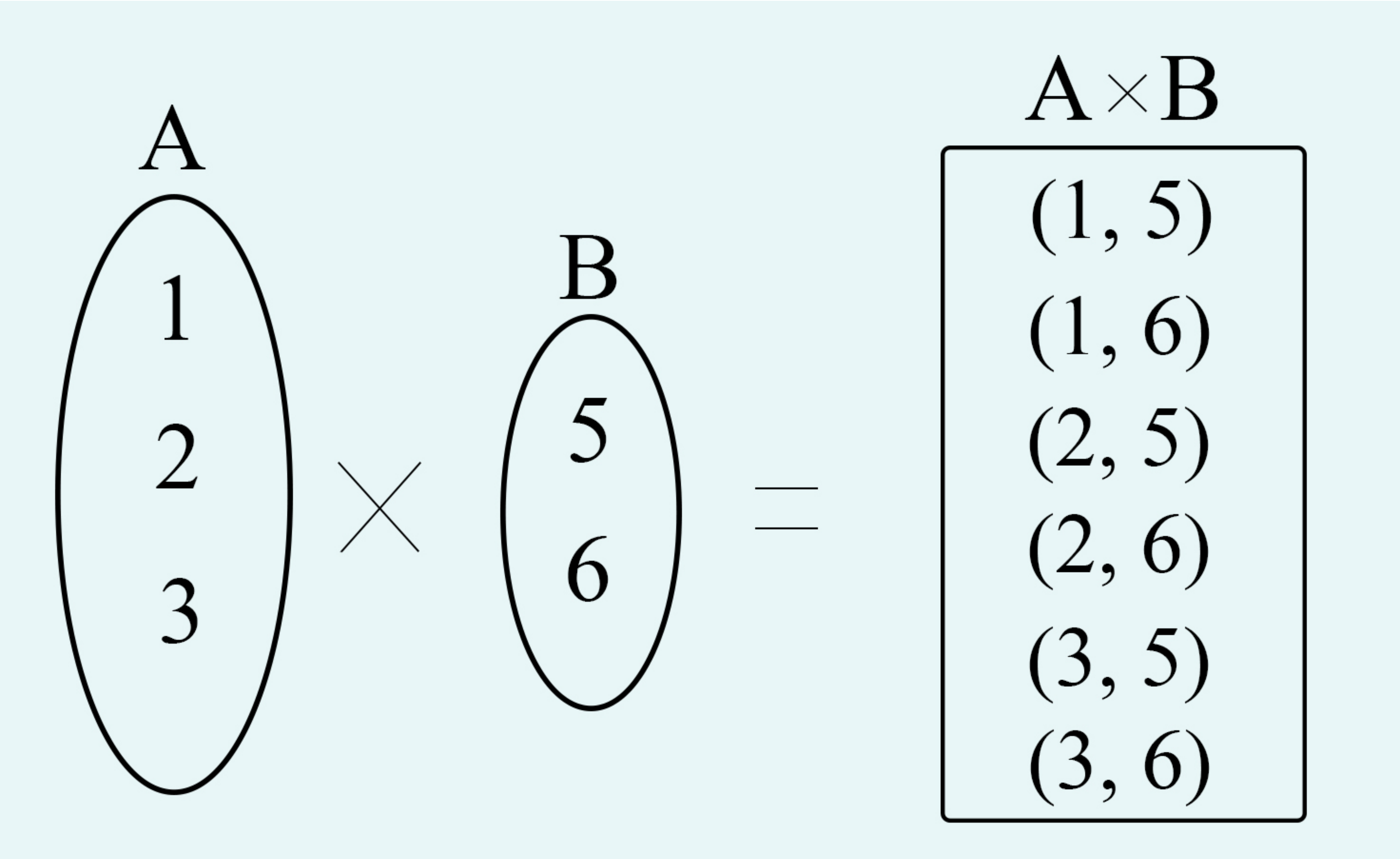
SEMANTICS AND COMPOSITIONALITY (CONT'D)

- In general, an interpretation of a logical system is an assignment of denotations (aka. referents) to symbols, especially non-logical ones.
  - It is fixed how to interpret logical symbols (including logical connectives).
- In first-order logic, the non-logical symbols are names, function symbols, and predicate symbols.
- Once we fix denotations (interpretations) of these symbols, we are able to determine the truth values of all formulae in first-order logic, from simple to complex ones.
  - The same kind of compositionality as in the semantics of propositional logic.
  - Remark: Semantics in natural language processing in AI is not necessarily compositional.

PRESUPPOSITIONS IN SYNTAX AND SEMANTICS

- In semantics (model theory), we rely on the language of set theory (sets, functions, relations, etc.); in syntax (or proof theory), we construct everything from scratch.

- Recall: $x \in D$ (i.e., x is an element of D) and $A \subset B$ (i.e., A is a subset of B). Product (aka. cartesian product) and functions are very important too:

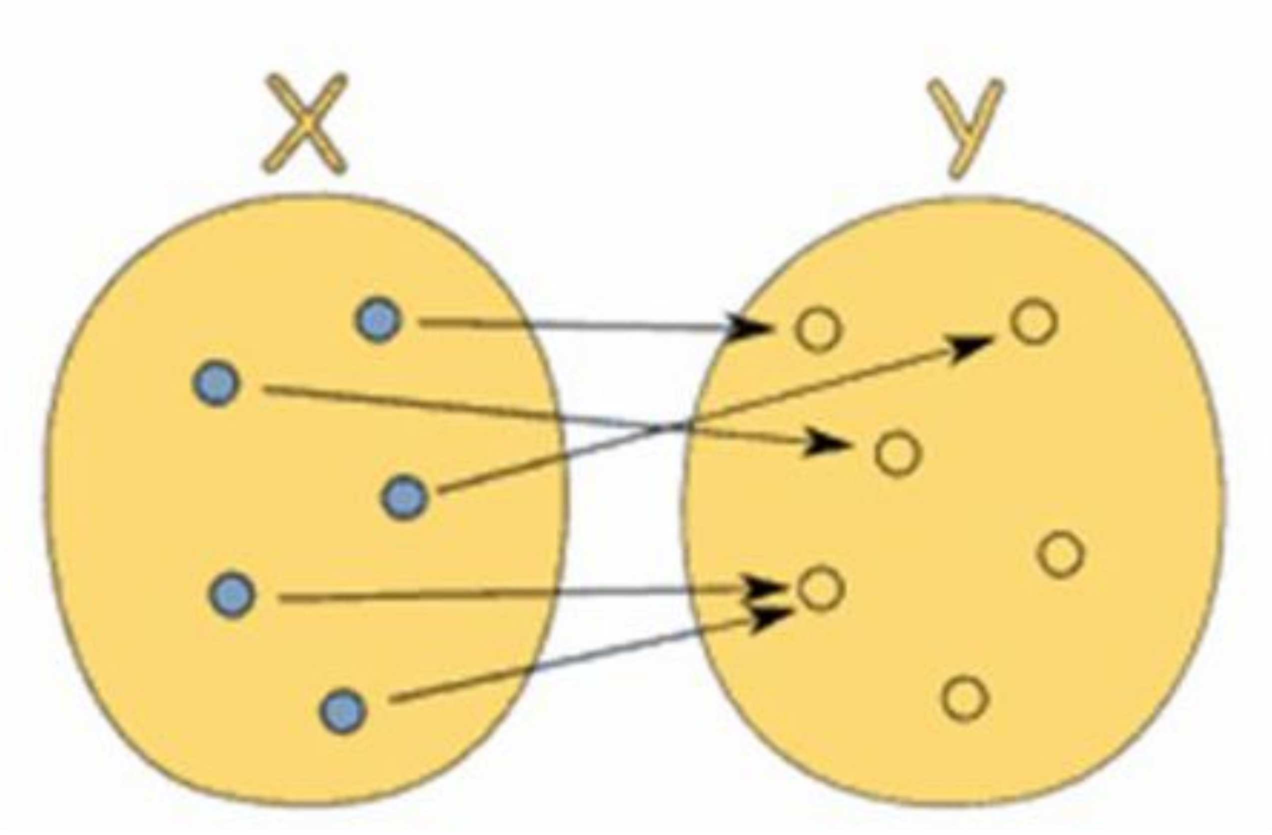


See also Pascal's set theory slides on Wattle

https://hyperskill.org/learn/step/8653

- A function relates **each element** of a set with **exactly one** element of another set (possibly the same set).

$f : X \to Y$



$f(x) \in Y$

https://slideplayer.com/slide/4158335/

RELATION AS SUBSET AND FUNCTION

- A binary relation R on a set X is a subset of $X \times X$, i.e., $R \subset X \times X$. An n-ary relation R is a subset of X^n, i.e., $R \subset X^n$.
- Equivalently, a binary relation is $R : X \times X \to \{0,1\}$; likewise, an n-ary relation is $R : X^n \to \{0,1\}$.
- Given a subset $R \subset X \times X$, we have $S : X \times X \to \{0,1\}$ defined by: $S(x_1, x_2) = 1$ iff $(x_1, x_2) \in R$ (and it's 0 otherwise).
- Given a function $S : X \times X \to \{0,1\}$, we have $R \subset X \times X$ defined by: $(x_1, x_2) \in R$ iff $S(x_1, x_2) = 1$. So they are equivalent.
- Note: relations are set-theoretical entities; relation symbols are syntactic symbols.

EXAMPLES OF RELATIONS

- The equality relation is $\{(x, x) \mid x \in X\} \subset X \times X$, or equivalently, $R_= : X \times X \to \{0,1\}$ such that $R_=(x, y) = 1$ if $x = y$, and $R_=(x, y) = 0$ otherwise.
- The inequality relation on $\mathbb{N}$ (set of natural numbers) is $\{(n, m) \mid n < m\}$, or $R_< : \mathbb{N} \times \mathbb{N} \to \{0,1\}$ such that $R_<(n, m) = 1$ if $n < m$ and $R_<(n, m) = 0$ otherwise.

DOMAIN AND INTERPRETATION

- An interpretation of first-order language consists of D and I such that:
- (i) D is a non-empty set, called a domain (of discourse).
  - Examples of domains: the set of natural numbers, a set of animals, etc.
- (ii) I gives denotations (interpretations) of symbols in the following manner.

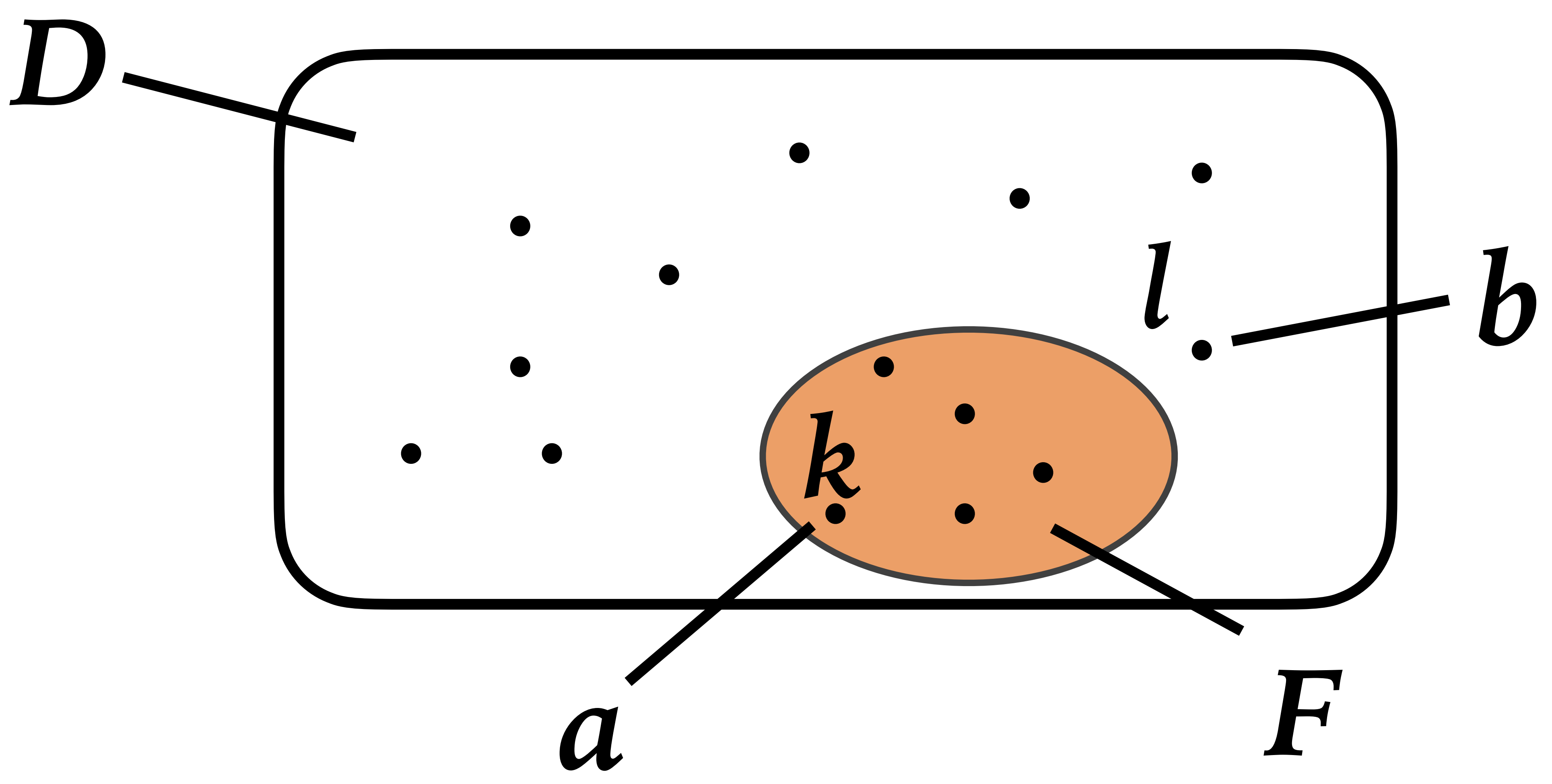
INTERPRETATION OF NAMES AND RELATIONS

- Names (aka. constants) are interpreted as elements of the domain D:
  - For a name m, I assigns an element $I(m) \in D$ to m.
- Predicate symbols are interpreted as relations, i.e., for an n-ary predicate symbol R, $I(R)$ is defined as:
  - A subset $I(R) \subseteq D^n$;
  - Equivalently, a function $I(R) : D^n \to \{1,0\}$.

THE UNDERLYING INTUITION

- The intuition behind two views on the interpretations of relations:
  - The relation as a function view: $I(R)(c_1, \ldots, c_n) = 1$ means the n-ary relation R holds for $c_1, \ldots, c_n$.
  - The relation as a subset view: $(c_1, \ldots, c_n) \in I(R)$ means the same thing.
- Put another way: $(c_1, \ldots, c_n) \in I(R)$ if and only if $I(R)(c_1, \ldots, c_n) = 1$ (true).

EXAMPLES



In this example, the domain is the set D. The names “a” and “b” are interpreted as the two elements k and l in D, which are their denotations.

The unary (i.e., one-variable) predicate symbol F is interpreted as the relation indicated by the orange subset.

Equivalently, the interpretation of F can be represented as a function:

$I(F) : D \to \{1,0\}$ where in particular, $I(F)(k) = 1$ and $I(F)(l) = 0$.

EXAMPLES (CONT'D)

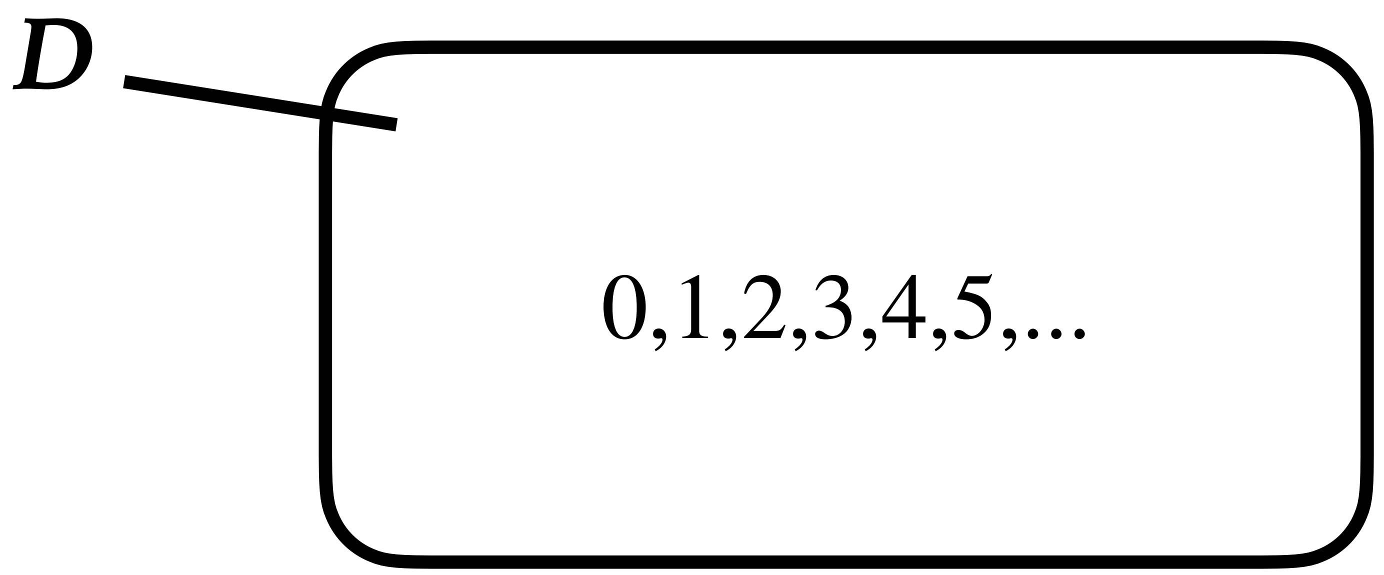
$D = \{2,3,13\}$ is the domain.

The binary predicate symbol "F" is interpreted as the binary relation "<": i.e., $I(F) = \{(2,3),(3,13),(2,13)\} \subset D \times D$; or equivalently, $I(F) : D \times D \rightarrow \{1,0\}$ such that $I(F)(2,3) = 1$, $I(F)(3,13) = 1$, $I(F)(2,13) = 1$, and gives 0 in all the other cases.

INTERPRETATION OF FUNCTION SYMBOLS

- Function symbols are interpreted as follows: for an n-ary function symbol f, its interpretation is a function $I(f) : D^n \to D$.
  - Suppose the interpretations of names $t_1, \ldots, t_n$ are elements $a_1, \ldots, a_n$ in D, respectively. The term $f(t_1, \ldots, t_n)$ behaves like a name. So its interpretation $I(f(t_1, \ldots, t_n))$ is defined as an element $I(f)(a_1, \ldots, a_n)$ in D.
  - More formally, if $I(t_1) = a_1, \ldots, I(t_n) = a_n$ then $I(f(t_1, \ldots, t_n))$ is defined as $I(f)(I(t_1), \ldots, I(t_n)),$ i.e., $I(f)(a_1, \ldots, a_n)$.
- Note that the above interpretation combines or composes the interpretations of names with the interpretation of a function. A general term (a name or of form $f(t_1, \ldots, t_n)$ where each t_i is a term) is interpreted in a similar, compositional manner.

EXAMPLES



In this example, the domain D is the set of all natural numbers. Let a, b be the names for the numbers 2,3, respectively. Let the binary function symbol f be interpreted as the function on the natural numbers $h : D^2 \to D$ such that:

$$h(x, y) = x^2 + y^2$$

Then we have:

$$I(f(a, b)) = I(f)(I(a), I(b)) = I(f)(2,3) = h(2,3) = 13$$

INTERPRETATION OF PROPOSITIONAL CONNECTIVES

We assign truth values to formulae with propositional connectives as follows:

- $I(A \wedge B) = 1$ iff $I(A) = 1$ and $I(B) = 1$. (It's 0 otherwise.)
  - A formula is called open if there is a free variable in it; called closed if not.
  - In general, it's an open formula: if the free variables of $A \wedge B$ are $x_1, \ldots, x_n$, $I(A \wedge B)(c_1, \ldots, c_n) = 1$ iff $I(A)(c_1, \ldots, c_n) = 1$ and $I(B)(c_1, \ldots, c_n) = 1$.
  - $I(\neg A) = 1$ iff $I(A) = 0$. The rest omitted (the same as in propositional logic).

Compositionality: the value of $A \wedge B$ is determined on the basis of values of A and B.

EXAMPLES (CONT'D)

Consider the following formula $A(x, y)$ with two variables x and y:

$$F(a, f(x)) \wedge F(b, y)$$

Suppose: the set of natural numbers is the domain; F is interpreted as inequality relation $<$; f is interpreted as $h(x) = x^2$; and a, b are interpreted as numbers 5,10 resp.

Then the interpretation $I(A)$ is a function $I(A) : D^2 \to \{1,0\}$ where, e.g., $I(A)(3,13)$ is true and $I(A)(2,13)$ is false. In general, it is interpreted as $5 < x^2 \wedge 10 < y$.

INTERPRETATION OF QUANTIFIERS

We assign truth values to quantified formulae in the following way:

- $I(\forall x A(x)) = 1$ iff $I(A)(a) = 1$ for any element a in the domain D.
- $I(\exists x A(x)) = 1$ iff $I(A)(a) = 1$ for some element a in the domain D.
    - In general, there may be additional free variables.

For example, consider the formula $\forall x(Fx \rightarrow Gx)$.

$I(\forall x(Fx \rightarrow Gx)) = 1$ iff $I(Fx \rightarrow Gx)(a) = 1$ for any a in D.

EXAMPLE

Consider the formula $\forall x \exists y\ Rxy$ in the following interpretation: the domain D is the set of natural numbers 0, 1, 2, …, and $I(R)$ is the binary relation "<".

The interpretation of the formula $\forall x \exists y\ Rxy$ is as follows. $I(\forall x \exists y\ Rxy) = 1$ (true) iff:

- $I(\exists y\ Rxy) : D \to \{1,0\}$ evaluates to 1 (true) for all natural numbers, i.e., for any natural number a, $I(\exists y\ Rxy)(a) = 1$, which means:
- Given any natural number a, there is a natural number b such that $I(Rxy)(a, b) = 1$.

The formula says that for any natural number there is a bigger natural number. What if the domain is the natural numbers plus ∞ bigger than any numbers? It's false then.

SUMMARY

Formal Symbols in First-Order Logic (Syntax)	Interpretation/Denotations/Referents (Semantics; Model Theory)
Names (aka. constants): $a, b, \ldots$	Elements of the domain: $I(a), I(b) \ldots \in D$
Function symbols: $f(x_1, \ldots, x_n)$	Functions on the domain: $I(f) : D^n \to D$
Predicate symbols: $R(x_1, \ldots, x_n)$	Relations: $I(R) : D^n \to \{1,0\}$ or $I(R) \subseteq D^n$
Open formulae: $A(x_1, \ldots, x_n)$	Truth value functions: $I(A) : D^n \to \{1,0\}$
Closed formulae (all variables quantified): A	Truth values: $I(A) = 1$ or $I(A) = 0$

NB. A formula is called open if there is a free variable in it; called closed if not.

INTERPRETATION OF SEQUENTS

Finally we define (semantic) validity:

- Closed formula case: $X \vdash A$ is valid iff $I(X) = 1$ implies $I(A) = 1$, for any interpretation I. Here note that $I(X) = 1$ is defined as: $I(B) = 1$ for any $B \in X$.
- Open formula case with n free variables: $X \vdash A$ is valid iff $I(X)(c_1, \ldots c_n) = 1$ implies $I(A)(c_1, \ldots, c_n) = 1$, for any interpretation I and any $c_1, \ldots, c_n \in D$.

We write $X \vDash A$ if $X \vdash A$ is (semantically) valid. It's called semantic entailment relation.

MODELS

An interpretation is a **model** of a set of (closed) formulae X iff any formula in X is true in that interpretation.

For example, consider the formula "$R_{taller}(g, t)$". If the domain is the set of logicians, $I(R_{taller})$ is the is-taller-than relation in the real world, and $I(g)$ and $I(t)$ are Gödel and Tarski, resp., then it is a model of $R_{taller}(g, t)$, assuming Gödel is taller than Tarski.

You can also think of a counterfactual world in which Tarski is taller than Gödel, and then it is not a model of $R_{taller}(g, t)$, assuming the interpretation of R_{taller} remains the same.

MODELS (CONT'D)

- An interpretation may be any possible interpretation, however strange it is, but a model (of X) must be an interpretation making formulae concerned (i.e. X) true.

- What are the possible models for the formula: $A \vee B$? (where A and B are formulae with no free variables.)
  - For $A \vee B$ to be true in an interpretation I, there are exactly three possibilities: In I, 1. A is true, B is false; 2. A is false, B is true; 3. A is true, B is true.

MODELS (CONT'D)

- Advanced example: The standard interpretation of 0, s (successor function symbol), $+$, $\cdot$, and $=$ with the domain of all natural numbers give a model of the seven axioms of arithmetic I gave in (the introductory part of) the last lecture.

Peano Axioms for natural numbers

PA1 $\forall x(\neg(s(x) = 0))$
PA2 $\forall x \forall y(s(x) = s(y) \to x = y)$
PA3 $\forall x(x + 0 = x)$
PA4 $\forall x \forall y(x + s(y) = s(x + y))$
PA5 $\forall x(x \cdot 0 = 0)$
PA6 $\forall x \forall y(x \cdot s(y) = x \cdot y + x)$
PA7 $[A(0) \wedge \forall x(A(x) \to A(s(x)))] \to \forall x A(x)$

https://www.chegg.com/homework-help/questions-and-answers/relation-u-greater-equal-v-natural-numbers-n-defined-formula-ez-u-z-v--give-formal-proof-u-q64888301

SOUNDNESS AND COMPLETENESS

SOUNDNESS AND COMPLETENESS

- Two ways of doing first-order logic.
- 1. Syntactically with a proof system.
- 2. Semantically with interpretations (models).
- Then:
- 1. The proof system defines provability.
- 2. The interpretations (models) define validity (truth).
- Question: do they coincide with each other?

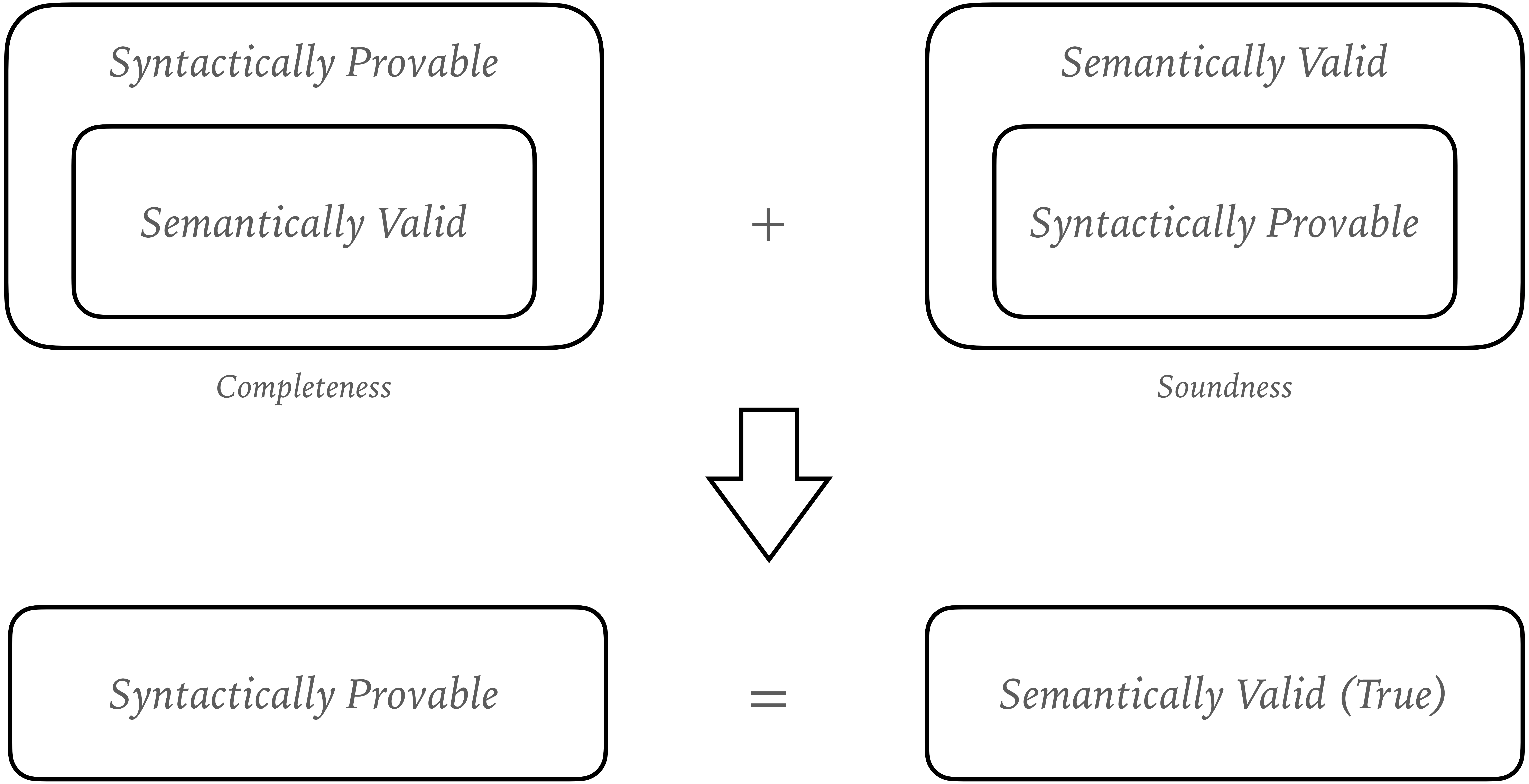
DEFINITIONS

- **Soundness** of a proof system means that every provable sequent is semantically valid.
  - We cannot prove anything that is false; if we prove something, it is true.

- **Completeness** of a proof system means that every valid sequent is provable.
  - We can prove everything that is true; there is no true, unprovable sequent.

- Soundness and completeness of semantics are defined in the same manner.
  - Some logicians think a proof system is prior to semantics; others the other way around. Related to the denotational vs. inferential semantics issue (coming soon).

SOUNDNESS AND COMPLETENESS, PICTORIALLY



NB. Completeness (theorems) sometimes mean both soundness and completeness in the above sense

THE NATURE OF PROOF SYSTEM

- Symbols in a proof system have no meaning; you just mechanically apply rules without semantically interpreting them.

- Leo Corry "Axiomatics between Hilbert and the New Math" (2007): "It is well known that Hilbert once explained his newly introduced approach by saying that in his system one might write "chairs," "tables" and "beer mugs," instead of "points," "lines" and "planes," and this would not affect the structure and the validity of the theory presented."

- In a proof system, one may write "chairs," "tables" and "beer-mugs", instead of "$\wedge$", "$\neg$", and "$\rightarrow$". It does not matter whey they are as long as they follow the rules specified; they have no meaning other than following the rules.

SYNTAX VS SEMANTICS

Syntax (Proof Theory)

- A proof system with symbols and deductive rules, such as $R, \forall, a$, and $\wedge$I, etc.
- The symbols have no intrinsic meaning and they only mechanically follow certain derivation rules to form proofs.
- Syntactic (or proof-theoretic) entailment is represented by the symbol $\vdash$. $X \vdash A$ means A is provable from X via the deductive rules (of natural deduction or sequent calculus which we learn soon).

Semantics (Model Theory)

- Gives meaning to the syntactic symbols.
- Semantic (or model theoretic) entailment is represented by the symbol $\vDash$. $X \vDash A$ means that formula A is true in all models of X.
- Soundness: if $X \vdash A$ then $X \vDash A$. It suffices to show that all rules preserve validity.
- Completeness: if $X \vDash A$ then $X \vdash A$. Shown by assuming $X \nvdash A$ and constructing a model to show $X \nvDash A$. The model consists of syntactic symbols (cf. symbolic construction of reality).

SOUNDNESS OF CONJUNCTION ELIMINATION RULE

- Consider the conjunction elimination rule: $$\frac{X \vdash A \wedge B}{X \vdash A}$$

- The soundness of the rule means that: if $X \vDash A \wedge B$ then $X \vDash A$, i.e., the rule preserves semantic validity.

- The soundness of this rule holds because: if any interpretation that makes X true also makes $A \wedge B$ true, then any interpretation that makes X true also makes A true.

- Proving the soundness of the entire system amounts to proving that all the deductive rules of natural deduction preserve semantic validity.

SOUNDNESS OF DISJUNCTION ELIMINATION

How about the disjunction elimination $\vee$**E**?

$$\frac{X \vdash A \vee B \quad Y, A \vdash C \quad Z, B \vdash C}{X, Y, Z \vdash C} \; \vee\mathrm{E}$$

We suppose the three input sequents are valid:

$$X \vDash A \vee B \quad Y, A \vDash C \quad Z, B \vDash C$$

We want to show that the output of the rule is valid:

$$X, Y, Z \vDash C$$

SOUNDNESS OF DISJUNCTION ELIMINATION

To prove a semantic entailment, we have to show that the conclusion is true in any interpretation that makes the assumptions true.

Consider an interpretation that makes the assumptions X, Y, Z true. Since we assumed the validity of the following sequents:

$$X \vDash A \vee B \quad Y, A \vDash C \quad Z, B \vDash C$$

we know that $A \vee B$ is true in I. Then either Y, A is true or Z, B is true in I. Either case makes C true in I (because of the above assumptions), and thus we have got:

$$X, Y, Z \vDash C$$

SOUNDNESS EXAMPLES: $\forall$I

Now consider the $\forall$ **I** rule:

$$\frac{X \vdash A}{X \vdash \forall x A} \; \forall \mathrm{I}$$

Assume $X \vDash A$, which means $I(X)(c) = 1$ implies $I(A)(c) = 1$, for any $c \in D$; for simplicity we assume there is only one free variable x (if not just add $c_1, \ldots, c_n$). By the side condition of the rule, X actually does not include any free variable x.

Then: "$I(X)(c) = 1$ implies $I(A)(c) = 1$" actually means "$I(X) = 1$ implies $I(A)(c) = 1$", which implies "$I(X) = 1$ implies $I(\forall x A) = 1$" since c is any element in D. Thus we have: $X \vDash \forall x A$. The side condition is essential in this proof.

Soundness proofs for the other rules are similar or simpler.

SOUNDNESS

- All rules of natural deduction preserve semantic validity.
  - Including the assumption rule.
- Thus: any sequent provable via natural deduction is semantically valid.
  - In other words: we cannot prove anything invalid using natural deduction.
- Formally: $X \vdash A$ implies $X \vDash A$ (in first-order logic).

COMPLETENESS PROOF

To prove $X \vDash A$ implies $X \vdash A$, we prove the contrapositive: if $X \nvdash A$ then $X \nvDash A$, which is shown by constructing a syntactic model of X that invalidates A (called Henkin model).

See the logic notes, which give some ideas and references for a completeness proof.

[NB. It is known that there is no finitary proof of completeness (it is known, for example, in Reverse Mathematics that Gödel's completeness theorem for countable languages is equivalent to Weak König's Lemma, an infinitary principle of proof; for general languages, a stronger principle is required, e.g. the existence of ultrafilters or Zorn's Lemma).]

You only need to understand the concept of completeness in this course (not its proof).

DUALITY

- Thanks to soundness and completeness, we can use either of a proof system and semantics to verify or refute sequents.
  - Proof theory (proof system) is basically useful for verifying a sequent $X \vdash A$.
  - Model theory (semantics) is basically useful for refuting a sequent $X \vdash A$ (i.e. verifying $X \nvdash A$).
- Remark: In general, it is not easy to prove $X \nvdash A$ using proof-theoretic methods only, but doing so has both mathematical and philosophical merits.

Appendix *(Just for Fun)*

ADVANCED ISSUES

- There are several philosophical/mathematical issues I would like to mention but you don't have to fully understand:
- 1. Presuppositions in proof theory and model theory.
- 2. Denotationalism vs. inferentialism in the theory of meaning. Do you really need the Tarskian model-theoretic semantics (the one you just learned) in order to confer meaning on logical symbols?
- 3. Completeness beyond logic (just to mention).

Prawitz developed natural deduction originally discovered by Gentzen.

He problematised the identity of proof.

Proofs in natural deduction correspond to programs in functional programming.

Origins of inferentialism and proof-theoretic semantics

Schock Prize Awarded to Prawitz and Martin-Löf

https://dailynous.com/2020/03/13/schock-prize-awarded-prawitz-martin-lof/

Year	Name(s)	Country
1993	Willard V. Quine	United States
1995	Michael Dummett	United Kingdom
1997	Dana S. Scott	United States
1999	John Rawls	United States
2001	Saul A. Kripke	United States
2003	Solomon Feferman	United States
2005	Jaakko Hintikka	Finland
2008	Thomas Nagel	Yugoslavia / United States
2011	Hilary Putnam	United States
2014	Derek Parfit	United Kingdom[3]
2017	Ruth Millikan	United States[4]
2018	Saharon Shelah	Israel[5]
2020	Dag Prawitz and Per Martin-Löf	Sweden Sweden [6]

https://en.wikipedia.org/wiki/Rolf_Schock_Prizes

PRESUPPOSITIONS IN PROOF AND MODEL THEORIES

- Proof theory (syntax) is finitary and independent of set theory, but model theory (semantics) is dependent upon set theory and in general infinitary.
  - Proof theory was originally conceived for proving the consistency of mathematics; formalise mathematics and prove its consistency, i.e., prove formal unprovability of a contradiction $\vdash A \wedge \neg A$.
  - To do this, proof theory must be finitary; otherwise, it only proves the consistency of infinitary math on the basis of infinitary math, which does not make much sense (if not no sense). It has to prove the consistency of infinitary math upon a finitary basis.
- How to prove $\nvdash A \wedge \neg A$ for the logic you learned? You can prove it semantically. How to prove it in a purely syntactic, finitary manner? It's an origin of sequent calculus.

PRESUPPOSITIONS (CONT'D)

- Related remarks on presupossitions: what do you presuppose when you use or define a proof system (or semantics)?

- Is logic defining logical connectives from scratch? No. We cannot define anything without presupposing anything at all.

  - The conj. intro. rule: if you have $X \vdash A$ and $X \vdash B$, then you may derive $X \vdash A \wedge B$. We are using conjunction and implication in our meta-language to talk about ND.

- Object-level language: natural deduction. Meta-language: natural language. But we do not need the full power of natural language to do natural deduction.

- What exactly do we need? It's a subtle question of both math. and phil. significance.

https://opinionator.blogs.nytimes.com/2012/01/04/remembering-michael-dummett/

Dummett

DENOTATIONALISM VS. INFERENTIALISM

- Some logicians actually argue that proof systems are sufficient for conferring meaning on syntactic symbols. Called inferentialism or proof-theoretic semantics.
  - Simplistically, saying, they argue deductive rules themselves give meaning. The meaning of logical symbols is how to use them in inferences (cf. Wittgenstein below).
- What we learned is denotational semantics: an expression denotes something, which is meaning. In inferential semantics, they argue inferential rules governing the expression give meaning (cf. operational semantics in programming language theory).

Brandom

https://www.thebritishacademy.ac.uk/fellows/robert-brandom-FBA/

- Wittgenstein (in his later philosophy): “Meaning is use”; “The words are not a translation of something else that was there before they were.” Autonomy of language.
- More on this: read Dummett (antirealism), Brandom (inferentialism), etc (or Derrida: “there is no outside-text”; language constructs, and is prior to, reality or the world).

COMPLETENESS THEOREMS ARE ALMOST EVERYWHERE (JUST TO MENTION)

- Logic is about characterising the infinite set of (logical/mathematical/scientific) truths by a finitary proof system.

- Turing machine is about characterising the infinite set of certain functions (recursive functions) by finitary mechanical procedures.

- Deep learning is about characterising the infinite set of certain functions (e.g., Bochner-Lebesgue p-integrable functions) by (functions generated from) finitary neural networks.

- Algebraic geometry is about characterising infinitely many geometric objects (and their geometric properties) by finitary algebraic equations (and their algebraic properties).