Logic: Tutorial Week 7

Discussions

- What is an interpretation in First-Order Logic?

  See the course material of week 7 for formal definitions. (And compare the solution of the last exercise on this sheet.)

- How could an interpretation in Propositional Logic look like? (Keep in mind that Propositional Logic is a special case of FOL.)

  Here, an interpretation is simply a mapping from each propositional symbol to either true or false. So this is basically simply a line in a truth table. But as the question already states: This is already covered by interpretations of predicate logic since propositional symbols are simply predicate symbols with arity zero.

- Interpretations in FOL requires the set D, a so-called domain. Its elements (sometimes called objects) ... what is this? Are they constants? Are they the same? Where does D come from?

  It's important to understand the difference between syntax and semantics. Formulae – and constants are part of formulae – live in the world of syntax. The semantics on the other hand defined the concrete world that we are currently facing. So each "world" comes with its own interpretation. Thus, D specifies the objects of the real world, and is intependent from contants. However, contants may of course be interpreted with objects from the domain. If a formula is "correct" (i.e., we could state it as a theorem), then it's true for all possible interpretations. Otherwise we might just find some interpretations that make it true, or even none (if its unsatisfiable).

- Discuss the latter in the context of both valid and invalid sequents. Specifically, what does it mean for an interpretation (resp. all interpretations) if a FOL sequent is valid or invalid? Recall what we did in case a sequent is invalid!

  If a sequent is valid, it means that no matter which interpretation we come up with, it will always make the sequent valid! This is validity means: For all possible interpretations that make the left hand side true, its right hand side is also true. Now, what does it mean for a sequent to me invalid? It means that its *not* true that each interpretation that satisfies the left hand side also makes the right hand side true. Thus, it means that there must exist interpretations (at least one) that do satisfy the left hand side, but not the right hand side. This is what we can do with the semantic tableaux! It helps us finding exactly such an interpreation! Go back to week 6 after you understood that and look up what we did there! It should now be clearer what that meant...

Blocksworld Formalization

EXERCISE a:
Formalize the following statements:

- Any block that's somewhere above another one is also above it (not just the ones that are directly on top of another).
  $\forall x \forall y \forall z ((\mathit{ON}(x, y) \wedge \mathit{ABOVE}(y, z)) \Rightarrow \mathit{ABOVE}(x, z))$
- If one block is above another, then it's not the other way round.
  $\forall x \forall y (\mathit{ABOVE}(x, y) \Rightarrow \neg \mathit{ABOVE}(y, x))$
- Every block that's clear doesn't have any block sitting on it.
  $\forall x (\mathit{CLEAR}(x) \Leftrightarrow \neg \exists y\, \mathit{ON}(y, x))$
- Each block has at most one block sitting on top of it.
  Sorry, no sample solution. :)
- Each block is on the table or directly on another block.
  Sorry, no sample solution. :)
- If a block is above another block, then there is some block below the first that's directly on the table.
  Sorry, no sample solution. :)
- Each block that's clear has exactly two blocks sitting on it that are both on the table. (Recall: this statement is not correct, but we formalize it anyway.)
  Sorry, no sample solution. :)

EXERCISE b:
Can you think of other properties that you can formalize? Keep in mind that nobody prevents you from formalizing a sentence that doesn't hold in the world described! So you could come up with something fun and/or complicated to formalize in a group, and then try.

That's for you to find out...

EXERCISE c:
Provide a formal interpretation (i.e., in the correct mathematical syntax) that satisfies all (correct) statements from above (i.e., from the exercise) and that described the situation which is illustrated in the picture. Provide the complete interpretation, i.e., everything that's required according to the course. You do not need to formally prove that your interpretation is correct, but you should be able to defend, i.e., explain it in the class.

- $D = \{A, B, C, D, E\}$. Note that the *names* of these objects were arbitrary. We could have also named them 1 to 5, or used any other (funny, absurd, whatever) names. Doesn't matter! But using the same names as in the graphics clearly helps us understand what we mean.
- Now, the interpretation I must interpret all constants, functions (which are generalizations of constants), and predicate symbols. In this case, we only have predicate symbols to interpret! Interpretations of the predicates:
  - $I(ON) = \{(A, B), (B, C), (D, E)\}$,
  - $I(ABOVE) = \{(A, B), (A, C), (B, C), (D, E)\}$,
  - $I(CLEAR) = \{A, D\}$,
  - $I(ONTABLE) = \{C, E\}$

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A
B    D
C    E
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|   TABLE   |
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