"if...then..." and the paradoxes

The "implicational paradoxes" are treated by most contemporary logicians somewhat as follows.

> The two-valued propositional calculus sanctions as valid many of the obvious and satisfactoy principles which we recognise intuitively as valid... It consequently suggests itself as a candidate for a formal analysis of "if...then...." To be sure, there are certain odd theorems such as
>
> $$A \to (B \to A)$$
>
> and
>
> $$A \to (B \to B)$$
>
> which might offend the naive, and indeed these have been referred to in the literature as "paradoxes of implication". But this terminology reflects a misunderstanding. "If A then if B then A" really means no more that "Either not-A or not-B or A" and the latter is clearly a logical truth; hence so is the former. Properly understood there are no "paradoxes" of implication.

The position just outlined will be found stated in many places and by many people; we shall refer to it as the Official view. We agree with the Official view that there are no paradoxes of implication, but for reasons that are quite different from those ordinarily given. To be sure, there is a misunderstanding involved, but it does not consist in the fact that the material implication connective is an "odd kind" of implication, but rather in the view that material "implication" is a "kind" of implication at all.

Let us imagine a logician who offers the following formalization as an explication or reconstruction of implication in formal terms. In addition to the rule of *modus ponens* ($\to$E) he takes as primitive three axioms:

$$A \to A$$
$$(A \to B) \to ((B \to C) \to (A \to C))$$
$$(A \to B) \to (B \to A)$$

One might find those who would object that "if...then..." doesn't seem to be symmetrical, and that the third axiom is objectionable. But our logician has an answer to that.

> There is nothing paradoxical about the third axiom: it is just a matter of understanding the formulas properly. "If A then B" means simply "Either A and B are both true, or else they are both false," and if we understand the arrow in that way, then our rule will never allow us to infer a false proposition from a true one, and moreover all the axioms are evidently logical truths. The implication connective of this system may not *exactly* coincide with the intuitions of naive, untutored folk, but it is quite adequate for my needs. And it has the important property, common to all kinds of implication, of *never leading from truth to falsehood.*

There are of course some differences between the situation just sketched and the Official view, but in point of perversity, muddle-headedness and downright error, they seem to us entirely on a par. Of course proponents of the view that material "implication" has something to do with implication have frequently apologised by saying that the name "material implication" is "somewhat misleading," since it suggests a closer relation with implication than actually obtains. But we can think of lots of no more "misleading" names for the relation: "material conjunction," for example, or "material disjunction," or "immaterial negation." Material implicaion is *not* a "kind" of implication, or so we hold; it is no more a kind of implication than a blunderbuss is a kind of bus.

Alan Ross Anderson and Nuel D Belnap Jr, 1960.
(abridged)