



## Final assessment

The final assessment questions are as follows. Submit all your solutions as a **single PDF file** via Turnitin. For typesetting, use LaTeX, MS Word, etc (do not use handwritten characters) whenever possible; you may handwrite formal proofs and insert them as pictures. The deadline is strict and cannot be extended, unless valid reasons with proper evidence are provided for special considerations \*at least three days in advance of the deadline\*; so please submit in time. Use of any generative AI such as ChatGPT and its relatives is not allowed in any question in this assessment (use of generative AI will be detected and lead to zero marks). The questions should not be too difficult if you have studied seriously. Good luck!

Q1 [11 marks]

Create a **sequent** and give a **sequent calculus proof** for it satisfying the following conditions:

- the sequent must include **implication, disjunction and universal quantifier**;
- the proof must include **both the left implication rule** and the **right universal quantifier rule**, and either **left or right disjunction rule**;
- there must be at **least one assumption** in the **sequent**;
- the assumptions must not **be contradictory**; the **conclusions** must **not be valid (always true)**;
- the **sequent/proof** must be **different** from those included in the course materials.

Q2 [3 marks]

The following ChatGPT-generated description on the **difference** between **classical and intuitionistic sequent calculi** is mathematically wrong. Concisely explain why it is wrong **within 60 words**.

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The difference in the **number of conclusions allowed** in sequent calculus for classical and intuitionistic logic has significant mathematical consequences, particularly in terms of **proof theory** and the **structure of logical systems**.

Classical Logic (Multiple Conclusions):

- In classical logic, sequent calculus allows for multiple conclusions in a single sequent. This means that a single step in a proof can yield several conclusions.
- The ability to have multiple conclusions often simplifies proofs, as it allows for a more direct expression of the consequences of assumptions.

Intuitionistic Logic (Single Conclusions):

- In contrast, intuitionistic logic restricts sequents to have only a single conclusion. This reflects the rejection of the **principle of explosion** and the law of the excluded middle.
- Intuitionistic logic emphasizes constructive reasoning, where a proof of a proposition asserts the existence of a witness or evidence for that proposition.
- The restriction to single conclusions enforces a more disciplined approach to proof construction. Each step in a proof **must lead directly to a single conclusion**, reflecting the constructive nature of intuitionistic reasoning.

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Q3 [9 marks]

Answer the following questions; the **sequents** you provide must **not be the same** as those included in the course materials.

1. Give a sequent that is valid in the **three-valued semantics** for **relevance logic** and **not valid** in the **three-valued semantics for fuzzy logic**, and briefly **explain why** that is the case within **85 words**. [3 marks]
2. Give a sequent that is valid in the three-valued semantics for **fuzzy logic** and not valid in the three-valued semantics for **intuitionistic logic**, and briefly explain why that is the case within 85 words. [3 marks]
3. Give a sequent that is valid in the three-valued semantics for intuitionistic logic and not valid in the three-valued semantics for relevance logic, and briefly explain why that is the case within 85 words. [3 marks]

Q4 [27 marks]

We have studied classical logic and three non-classical logics (i.e., **fuzzy, relevant and intuitionistic**). Choose your **favourite logic** among them and **argue** that it is a logical system which works better than each of the other three logics **within 990 words**. [9 marks for an argument for each of them, the length of which is expected to be **330 words or less**; each argument will be **evaluated with respect to originality and clarity, rigour and precision**, and **level of formal logical knowledge applied**]

Note that you can explore the literature to enrich your arguments but as usual you must **correctly and clearly cite all the sources you used** (if any source used is not cited, it will count as plagiarism and lead to zero marks). Note also that the specified word limit includes 10% leeway (and excludes the reference section if any). Use of generative AI will be detected and lead to zero marks (if there is suspicion, oral examination will be conducted where necessary; the same applies to the other questions as well).

Additional Notes (answering a question during lecture): if you wish you can choose intuitionistic fuzzy logic as a version of fuzzy logic, intuitionistic relevance logic as a version of relevance logic and Full Lambek Calculus as a version of fuzzy or relevance logic; if you choose Full Lambek, then please specify whether you regard it as an extension of fuzzy logic or an extension of relevance logic.

Deadline (strict):

31 May

Submission must be made via turnitin on wattle; no email or other form of submission will be accepted. Please ensure that your solutions are correctly submitted on wattle/turnitin; even if anything is wrong with your submitted solution file, we cannot replace it with another one and will mark the submitted one (we must be fair to everyone).

It is your responsibility to submit it in time; no later submission will be accepted. We provide you with two weeks to work on the questions, which take various uncertainties (such as mild illness) into account. Please note that we have to meet the deadline for final grade submission to the university and extension requests will rarely be accepted. Please work on the questions early enough and submit it strictly in time. Your cooperation will be greatly appreciated.