

WHAT IS LOGIC ABOUT?

Yoshihiro Maruyama



WHAT IS IT ALL ABOUT?

Logic is about characterising the infinite truths in finitary terms; there are infinitely many semantically valid formulae, but they can be finitely generated by a finitary proof system.

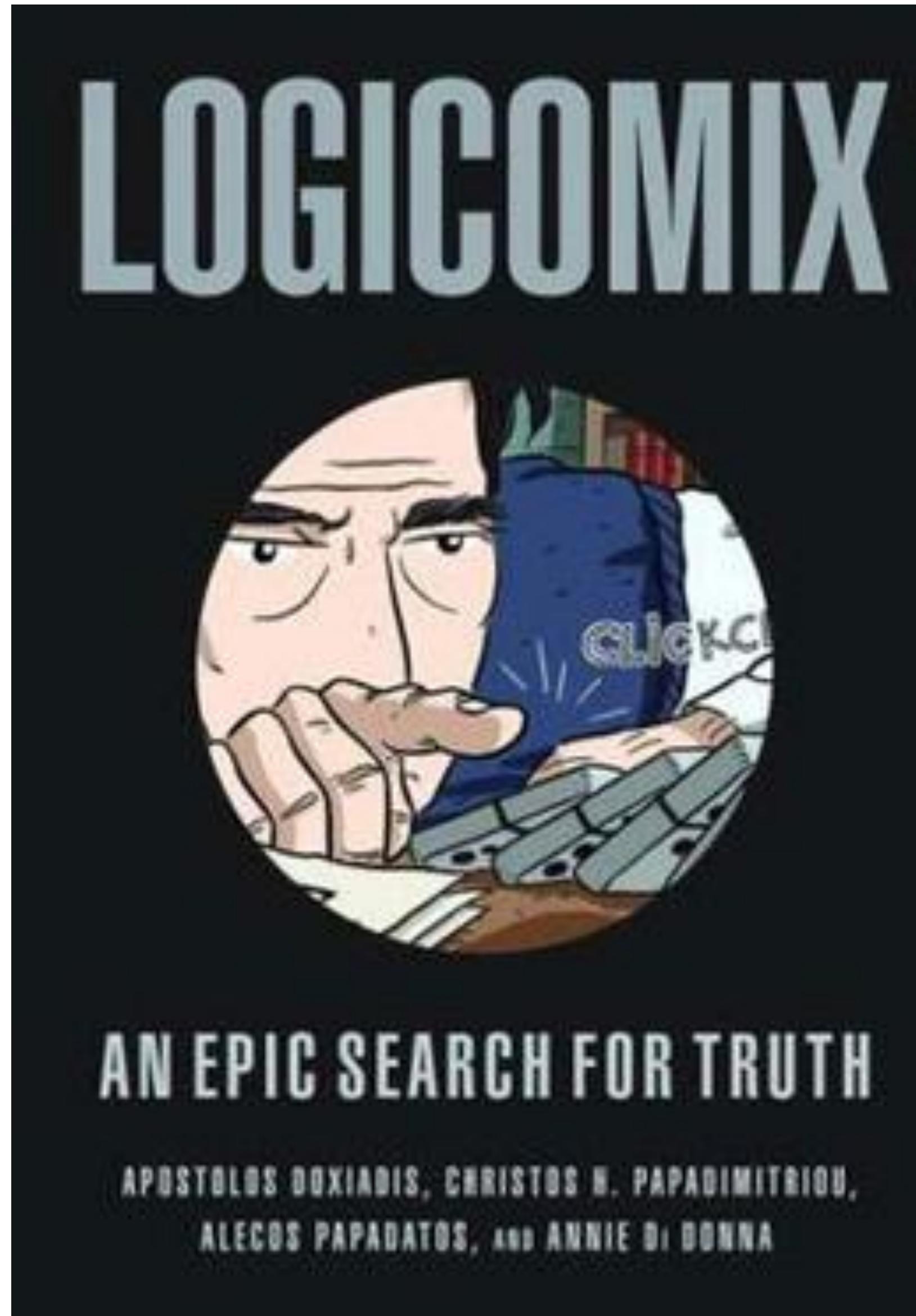
Logic is about truth, or
about finitary means to
characterise (generate)
infinitary truths.

NB. Wittgenstein is
one of the discoverers
of truth table semantics

Logicomix (2008, 2009)



LOGICOMIX (2018, 2019)



Highly enjoyable,
but not always precise

WHAT IS IT ALL ABOUT THEN?

- It is not obvious whether there is any finitary means to characterise (generate) all truths.
- **Completeness theorems** tell that is possible. It is possible to characterise the infinite set of all logical truths by finitary means, namely by a proof system. Natural deduction allows us to derive all logical truths by finitary proof procedures.
- **Incompleteness theorems** tell it is impossible; e.g., it is impossible to characterise the infinite set of all arithmetical truths by any finitary means, i.e., by any (computable) proof system.
- There is a mechanical procedure (program) to generate all logical truths, but no such procedure (program) to generate all arithmetical truths, let alone all mathematical truths. A small number of rules allow us to generate all truths, which is a highly non-trivial mathematical phenomenon.

GÖDEL ANNOUNCES INCOMPLETENESS THEOREMS



MORE HISTORY AND THE PRESENT STATE-OF-THE-ART

- A “very few logical concepts and axioms” allow us to generate all truths, which is amazing.
Note: the human being is a finitary entity, and only comprehends finitary things.
- Gödel (1944): “It was only [Russell's] Principia Mathematica that full use was made [...] for actually deriving large parts of mathematics from a very few logical concepts and axioms”
- Kleene (1952): “Leibniz (1666) first conceived of **logic as a science containing the ideas and principles underlying all other sciences.**” Can we generate all scientific truths via logic?
- It’s open, but we can, e.g., generate quantum-mechanical truths via logic (there is even an automated reasoning system, called Quantomatic, based on category-theoretical logic).

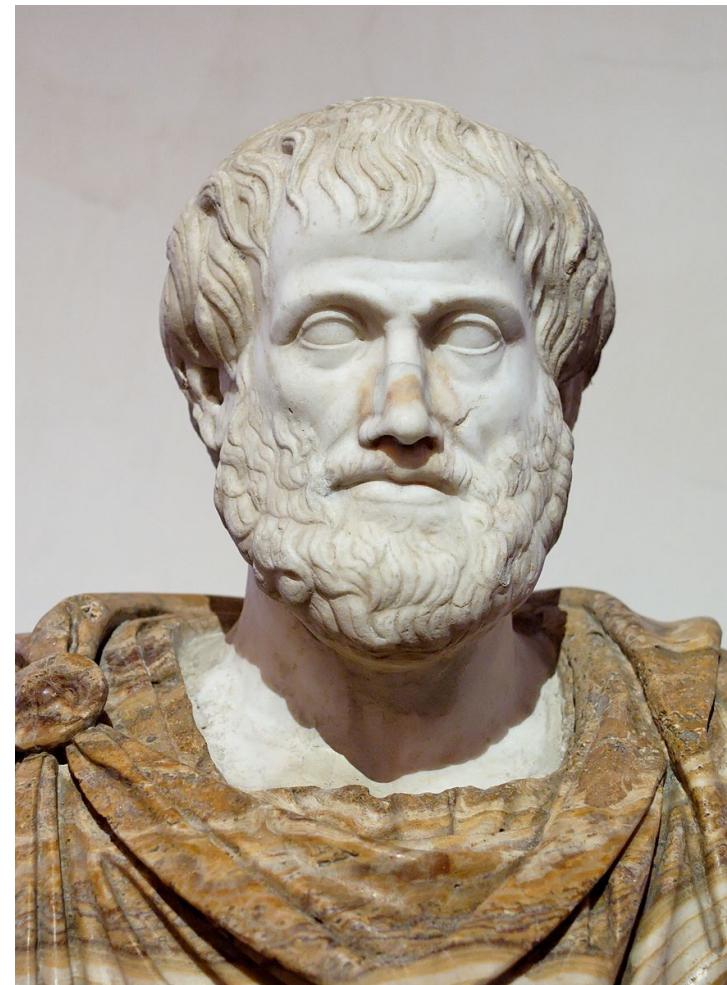
FACETS OF LOGIC

Yoshihiro Maruyama



TWO ORIGINS OF LOGIC, ANCIENT AND MODERN

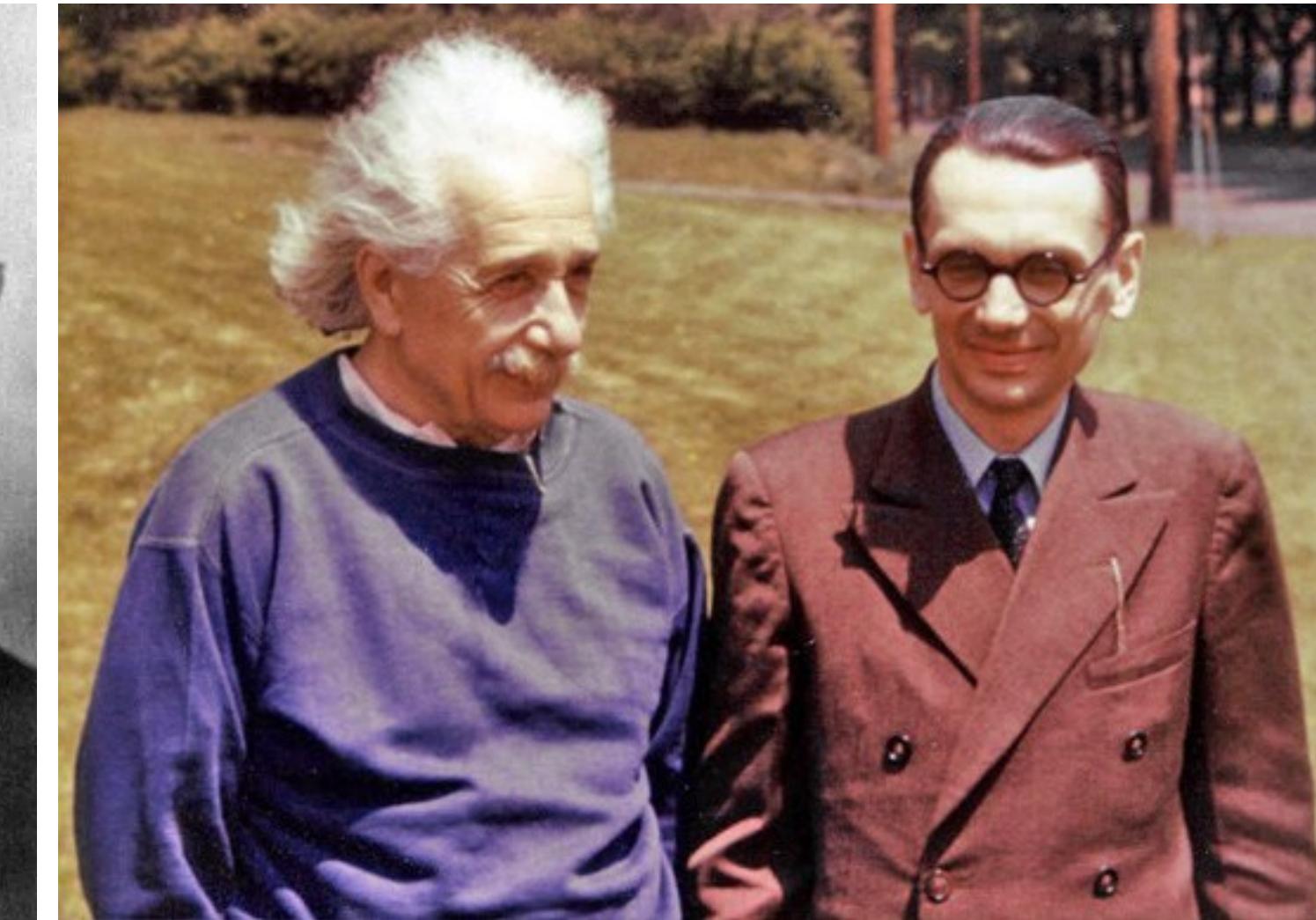
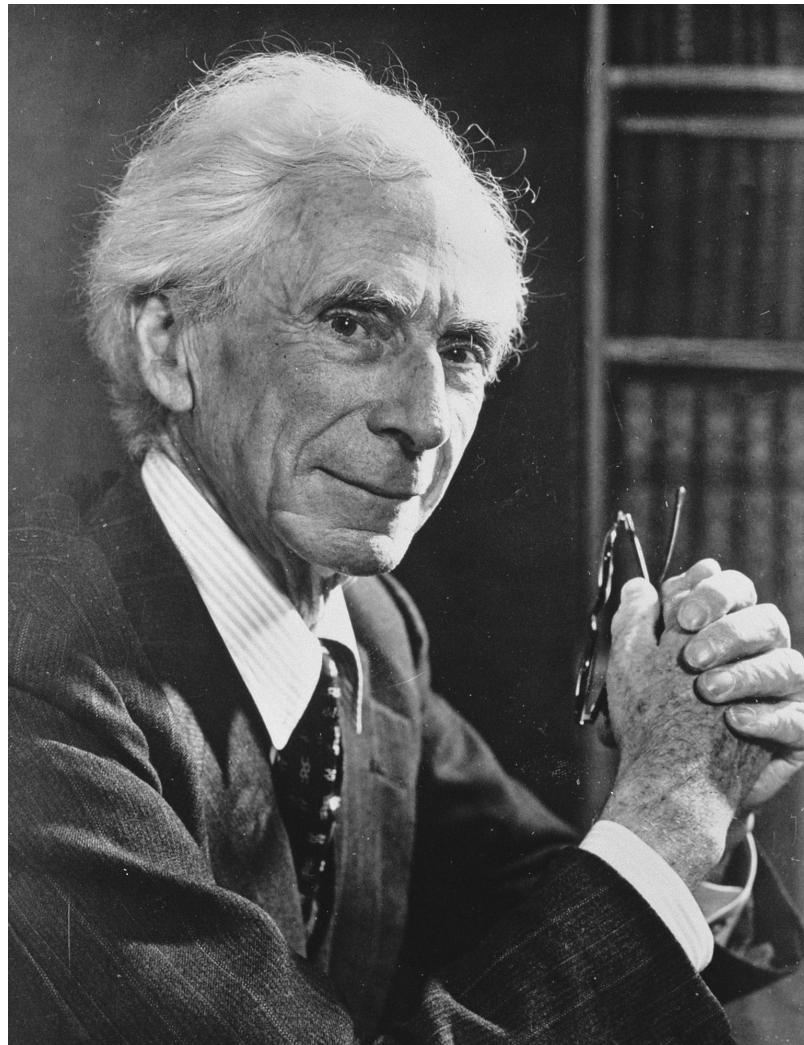
- Logic as **Foundations of Natural Language Reasoning**: you convince others via logos (as well as pathos and ethos; Aristotle's *Rhetoric*); an origin of Ancient Logic.



- Logic as **Foundations of Mathematics**: you use logical reasoning in math, but what does it mean to be logical / illogical? Where is the border? What reasoning exactly is allowed in math? Why is mathematical truth universal? Is math really consistent?

TWO ORIGINS OF LOGIC, ANCIENT AND MODERN

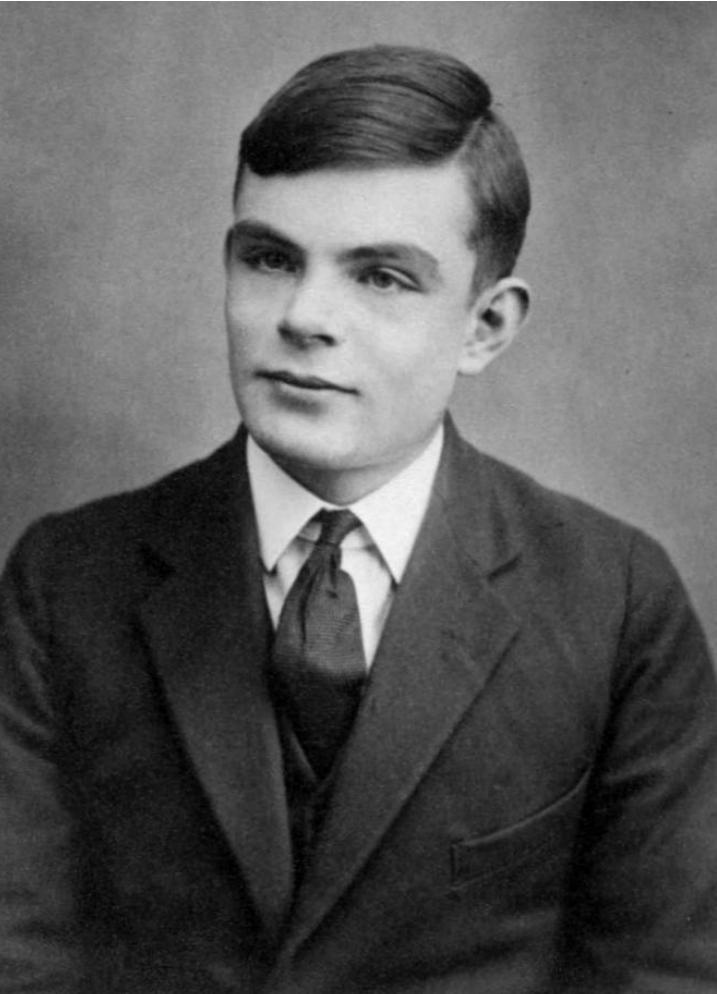
- The **Foundational Crisis of Mathematics**:
- Inconsistencies were discovered in math, and mathematicians and philosophers such as Russell and Hilbert worked hard to build solid foundations of mathematics



<https://www.brainpickings.org/2015/05/07/rebecca-goldstein-incompleteness-godel-einstein-time/>

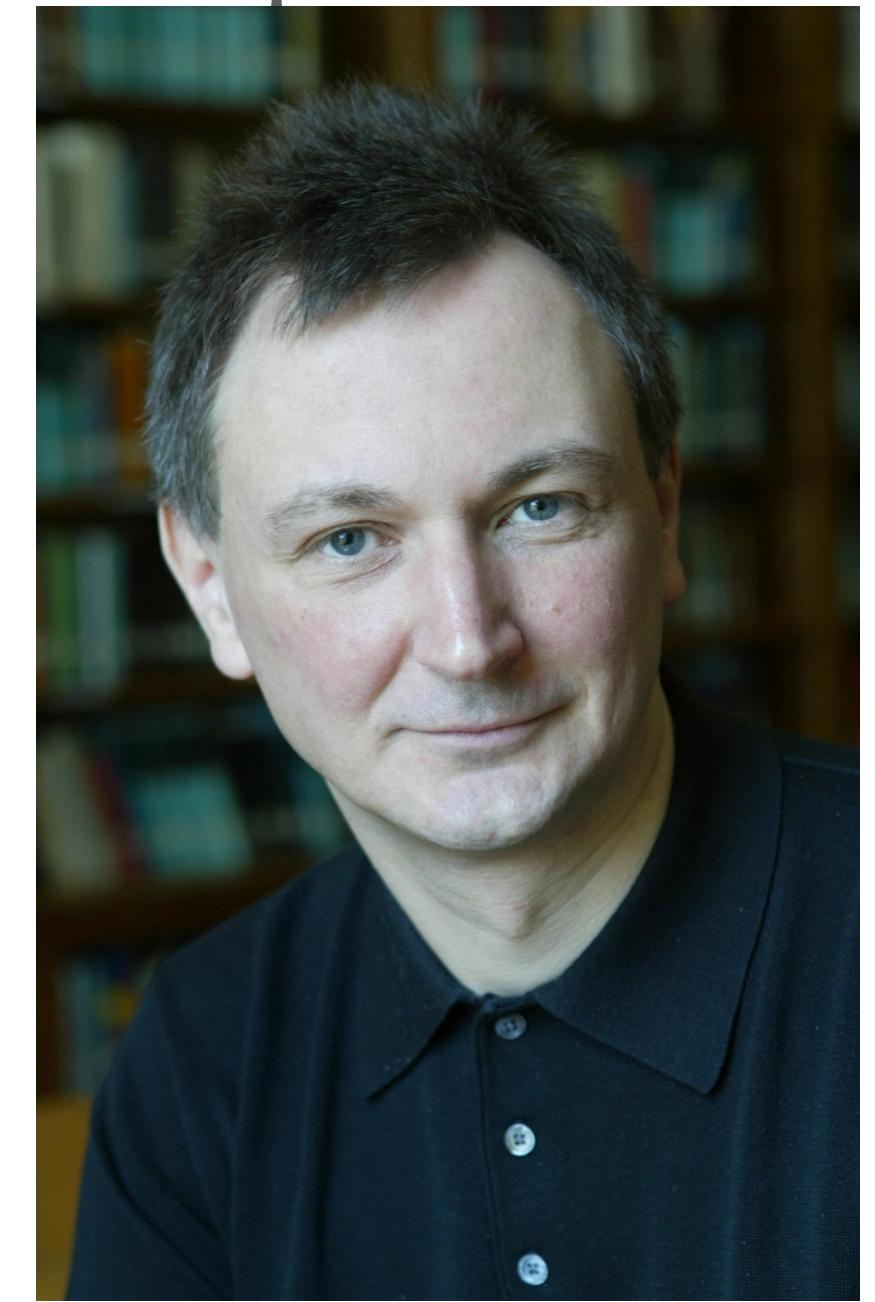
- And failed; Gödel proved it is impossible to prove the consistency of math (certain conditions, 1931). Yet the endeavour yielded modern logic as a scientific discipline. Before then: only mathematical philosophers and philosophical mathematicians.

APPLICATIONS OF LOGIC

- Logic as Foundations of Computer Science: the computer itself was born from logic (e.g., Turing, was a logician; no computer scientist at that time). 
- Logic as Foundations of Artificial Intelligence: the very idea of AI stemmed from Turing's thinking machines (older origins; Leibniz's alphabet of human thought).
- Logic as Foundations of Physics: the empirical nature of logic has been debated in foundations of quantum physics, the logic of which is essentially different from classical logic. Logical foundations of quantum physics and quantum computing. 
- Logic as Philosophical Methodology: analytic philosophy, arguably the strongest tradition of philosophy today, was born from Frege and Russell's logical studies.
- Logic as Foundations of Formal Linguistics, Microeconomics, Cognitive Sci., etc.

LOGIC AND THE CERTAINTY AND OBJECTIVITY OF MATHEMATICS

- Logic as Foundations of **Computer Verifications of Proofs**: Vladimir Voevodsky, a Fields Medalist in Mathematics (2002), found non-trivial mistakes in his published proofs (his theorems were used by other mathematicians as well).
- Voevodsky thus started to work on computer verifications of proofs (cf. **automated theorem proving**; **interactive proof assistant**).
- You can also verify the correctness of computer programs via logic. It is called **Hoare logic** (Hoare is a Turing award winner in 1980).
- Shinichi Mochizuki released a proof of the **ABC conjecture** in 2012, which was finally published in 2021 (after 10 years reviewing). But the proof is too complex and the mathematics community still has not agreed upon the correctness of the proof...



<https://www.nytimes.com/2017/10/06/obituaries/vladimir-voevodsky-revolutionary-mathematician-dies-at-51.html>

CONCLUSIONS

- **Conclusions:** Logic is foundations of everything (cf. a theory of everything; Leibniz's *characteristica universalis*), from mathematics and philosophy to physics and computer science / artificial intelligence (and thus logic is interesting).
- **Medieval Logic** I omitted (not because of dark ages), e.g. Ockham known for Razor:





THE LOGIC TRADITION IN AUSTRALASIA AND ANU

- Australasia has an interesting tradition in logic, especially substructural logic and paraconsistent logic, which is even related with Buddhist logic (an expert at ANU).
- The ANU School of Computing is especially well known for the tradition of substructural logic (Robert Meyer, John Slaney, Rajeev Goré, etc.); I work on it too.
- Even today, there are many logicians in the School of Computing, and in the School of Philosophy (esp., K. Tanaka working on paraconsistent logic and Buddhist logic).

Both A & Not-A Neither A Nor Not-A

Catuskoti

A

Not-A