Final Assessment

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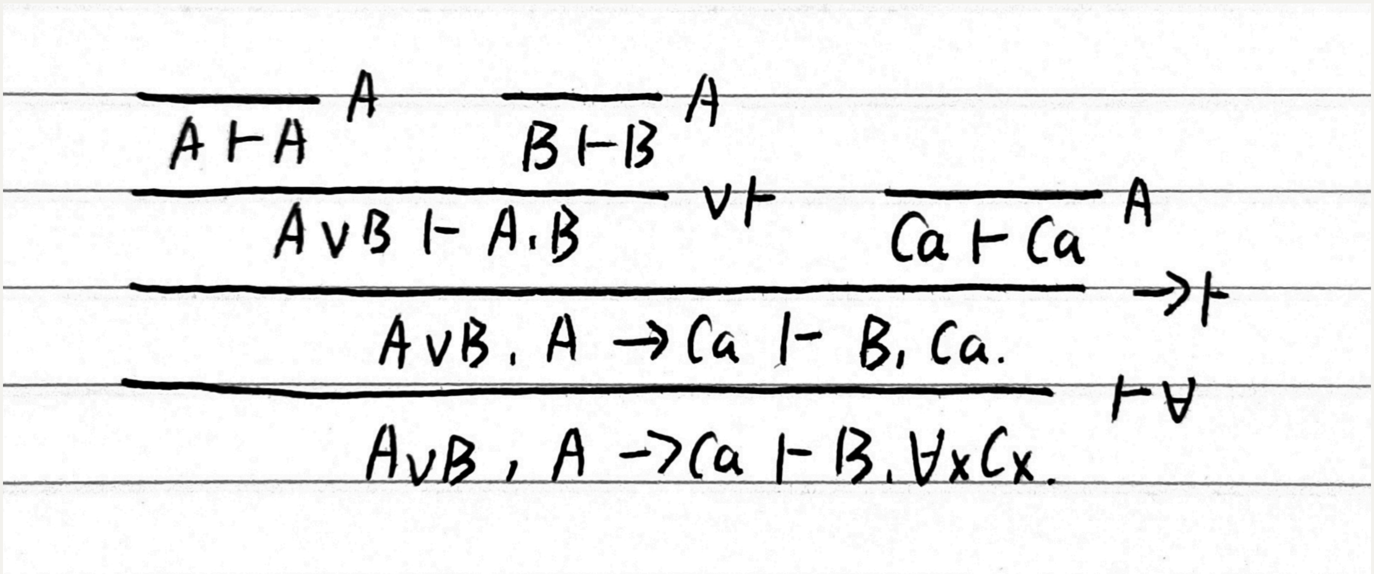
Q1

The sequent we want to prove is:

$$A \vee B, A \to Ca \vdash B, \forall x Cx \tag{1}$$

The left-hand side contains one implication and one disjunction, while the right-hand side includes a universal quantifier.

Now we show the proof:



Notice that the second line contains the left disjunction rule, the third line contains the left implication rule, and the last line contains the right universal quantifier rule.

Q2

For classical logic:

Having multiple conclusions can simplify proofs since it allows using non-constructive principles such as proof by contradiction.

For intuitionistic logic:

Intuitionistic logic does not reject the principle of explosion.

Each step does not necessarily lead to a single conclusion, but each step constructively establishes the result.

Q3

A sequent that is valid in relevance logic but not in fuzzy logic:

$$\neg A \to A \vdash A \tag{2}$$

- Validity in relevance logic: It is easy to prove that the right-hand side is always at least as good as the left-hand side.
- Invalidity in fuzzy logic: When A is i, then the left-hand side is T while the right-hand side is i.

A sequent that is valid in fuzzy logic but not in intuitionistic logic:

$$(A \wedge \neg A) \to B \vdash \neg B \to (A \vee \neg A) \tag{3}$$

- Validity in fuzzy logic: It is easy to check whenever the left-hand side is T, the right-hand side is also T. Notice that there is no way to set the right-hand side to F, since $A \vee \neg A$ is either T or i.
- Invalidity in intuition logic: When A is i and B is F, the left-hand is T, but the right-hand side if i.

A sequent that is valid in intuitionistic logic but not in relevance logic:

$$A \wedge B \vdash A \to B \tag{4}$$

- Validity in intuitionistic logic: Only when A and B are both T, $A \wedge B$ is T, then $A \to B$ is also T.

- Invalidity in relevance logic: When A is T and B is i, then $A \wedge B$ is i, but $A \to B$ is F.

Q4 - Fuzzy Logic vs Other Three

Fuzzy Logic vs Classic Logic

Classic logic, also known as binary or Boolean logic, operates on the principle of bivalence, which asserts that every proposition is either true or false [1]. This binary nature is the foundation of classical logical systems and is widely used in digital systems, mathematical proofs, and formal logical analysis [2]. However, its applicability becomes limited in real-world scenarios characterised by uncertainty and imprecision. For instance, consider the statement "Logic class is fun." This statement must be categorised as true or false in classic logic. Such rigid categorisation fails to capture the nuanced variations in temperature perceptions (e.g., not interesting at all, average fun, very intriguing).

Fuzzy logic, introduced by Lotfi Zadeh in 1965, addresses this limitation by allowing for degrees of truth [3]. Instead of a binary true or false evaluation, fuzzy logic assigns a truth value between 0 and 1 to represent the extent to which a proposition is true. This gradation is particularly useful in dealing with vague or imprecise concepts. For example, the feedback from the logic class can be represented as a fuzzy variable with values like 0.1 (not interesting), 0.5 (average interesting), or 0.9 (very interesting). This approach enables a more accurate and flexible representation of real-world phenomena, aligning closer to human reasoning and perception [4].

Fuzzy logic's ability to handle partial truth makes it more effective in real-world scenarios where binary distinctions are inadequate. Many systems, such as climate control, medical diagnostics, and consumer electronics, benefit from the nuanced approach of fuzzy logic [5]. For instance, a fuzzy logic-based HVAC system improves temperature control by considering the degree of deviation from the desired temperature and adjusting the heating or cooling output accordingly [6].

In summary, fuzzy logic expands on the formal reasoning and digital computation capabilities of classic logic by taking into account the ambiguity and complexity present in real-world scenarios. Because of this, fuzzy logic becomes a more useful and adaptable tool in situations where accuracy and subtlety are essential.

Fuzzy Logic vs Relevance Logic

Relevance logic, sometimes referred to as relevance logic, is a subset of non-classical logic that places emphasis on the requirement that premises and conclusions in logical inference be relevant. It seeks to stop the possibility of unimportant or trivial inferences in classical logic [7]. For instance, in classical logic, the inference from "Logic class is fun" to "I don't like coffee" is valid, but irrelevant. Relevant logic ensures that the premises have a meaningful connection to the conclusion. Relevant logic is especially helpful in formal systems, such as legal reasoning or certain computer applications, where the logical flow and contextual integrity of arguments are essential. This is because relevant logic focuses on relevance [8].

Fuzzy logic, on the other hand, deals with uncertainty and partial truth values. It does not explicitly address the relevance of premises to conclusions but excels in handling vague and imprecise information. Notice that combining fuzzy logic with relevance logic can harness the strengths of both systems. For example, in decision-making systems, one can ensure that the data inputs (premises) are contextually relevant to the decision (conclusion) using relevance logic [9], while fuzzy logic is utilized to interpret the varied degrees of truth in the inputs, enabling more adaptable and flexible decision-making based on the relative relevance and certainty of each input [10].

In alternative scenarios, real-time data on traffic density, speed, weather, and past traffic patterns can be gathered via sensors, while irrelevant data can then be filtered out using relevance logic. For instance, ignore data on pedestrian traffic if the primary goal is to manage vehicle flow during peak hours [11]. The pertinent data is evaluated using fuzzy logic. Instead of abruptly changing between preset timings, the traffic light duration could be progressively modified, for instance, if the traffic density is between light and moderate [11].

Fuzzy Logic vs Intuitionistic Logic

Intuitionistic logic, a form of constructive logic, emphasises the need for constructive proofs. Unlike classical logic, which accepts the law of excluded middle (a proposition is either true or false), intuitionistic logic requires explicit evidence of a proposition's truth. It does not allow proof by contradiction unless direct constructive proof is available [12]. This method is especially helpful in mathematical proofs that call for explicit constructs, as well as in fields like computer science where constructive processes and algorithms play a crucial role [13].

Combining fuzzy logic and intuitionistic logic, one can create systems that can handle the complexity and uncertainty of real-world data while ensuring that all actions and decisions are based on constructively proven and reliable information [14]. This integrated approach is especially helpful in areas where both evidence-based actions and complex data interpretation are essential, such as medical diagnosis, treatment planning [17] and smart building automation [15].

For example, smart building systems must be able to precisely control several environmental factors (such as lighting, temperature, and security) and make sure that decisions are made using accurate and useful data [15]. Intuitionistic logic ensures that actions taken by smart building systems are based on constructively proven algorithms and data, while fuzzy logic handles the continuous and uncertain data from sensors, such as temperature, light levels, and occupancy, allowing for nuanced control [16].

However, the focus of intuitionistic logic on constructive proofs has benefits in domains like as computer science and mathematics [19]. For instance, the process of theorem proving necessitates the provision of direct evidence in support of claims, encouraging rigorous thinking and improved theorem comprehension. Intuitionistic logic improves dependability and lowers errors in software verification by ensuring that programs are verified using explicit, constructive proofs [18]. This methodology is consistent with constructive mathematics, encouraging a more rigorous and verifiable approach to reasoning.

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