The Austalian National University
School of Computing
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Tutorial 2

COMP/ENGN 4528/6528: Computer Vision

Question 1 — Colour Space Conversion

1. Given RGB value $(25, 30, 40)$, what would the pixel value be in the HSV space? (Please refer to the Szeliski Computer Vision textbook and the lecture slides)

   Solution.
   HSV space

$$(R', G', B') = (\frac{R}{255}, \frac{G}{255}, \frac{B}{255}) = (\frac{25}{255}, \frac{30}{255}, \frac{40}{255})$$

$$V = X_{max} = \max(R', G', B') = \frac{40}{255} = 0.157$$

$$X_{min} = \min(R', G', B') = \frac{25}{255}$$

$$C = X_{max} - X_{min} = \frac{15}{255}$$

$$\because V = B$$

$$\therefore H = 60^\circ \cdot (4 + \frac{R' - G'}{C}) = 60^\circ \times \frac{11}{3} = 220^\circ$$

$$\because V \neq 0$$

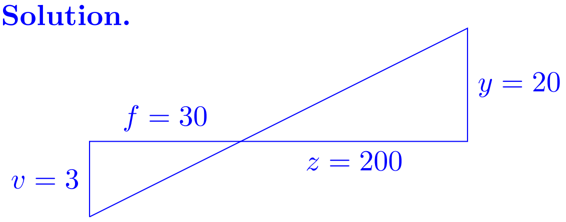
$$\therefore S = \frac{C}{V} = \frac{15}{40} = 0.375$$

$$\begin{bmatrix} H \\ S \\ V \end{bmatrix} = \begin{bmatrix} 220^\circ \\ 0.375 \\ 0.157 \end{bmatrix}$$

Question 2 — Camera Projections

1. Given a lens with a focal length of 30, and a world point at $(20, 20, 200)$, where the camera center sits at $(0, 0, 0)$, and the optical axis is in direction $(0, 0, 1)$, with the x and y axes aligned to the world axes, what pixel will this world point project to?

   Solution.



The plot demonstrates the calculation of projecting the y coordinate of a point from world to the image plane using the concept of similar triangles.
Note: the values in the above image are lengths and do not reflect a coordinate system. If the image plane is behind the origin, it is located at **-30** in world coordinates. This means that the image will appear inverted.

$$\frac{v}{f} = \frac{y}{z}$$

$$v = f\frac{y}{z} = -30 \times \frac{20}{200} = -3$$

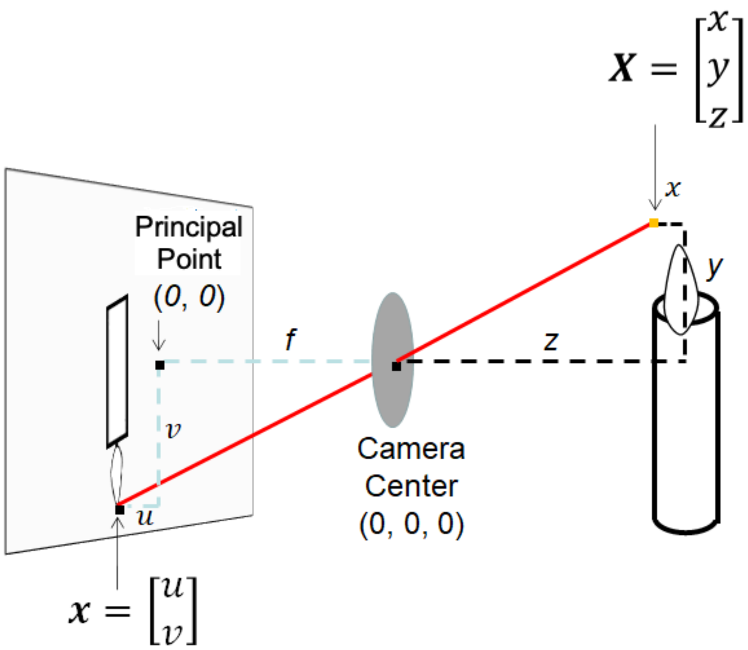


Figure 1: Q2.1 coordinate illustration

By similar triangles from 3D coordinates in Figure 1

$$(x, y, z) \rightarrow_{proj} (f\frac{x}{z}, f\frac{y}{z}, f\frac{z}{z}) = (f\frac{x}{z}, f\frac{y}{z}, f) = (-3, -3, -30)$$

The projection will be on the image plane. Drop the third coordinate (focal length),

$$(x, y, z) \rightarrow_{proj} (f\frac{x}{z}, f\frac{y}{z}) = (-3, -3)$$

Question 3 **(Homogeneous) Coordinates Transformations**

1. Image warping can be performed by pre-multiplying coordinate locations by a 2×2 matrix, and adding for translation. What would the effect be of the following transformation:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} d_x \\ d_y \end{bmatrix}$$

where $a_{11} = 2$, $a_{12} = 0$, $a_{21} = 0$, $a_{22} = 1$, $d_x = 1$ and $d_y = 2$

Solution.

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2x + 1 \\ y + 2 \end{bmatrix}$$

(a) Enlarge the image twice horizontally.

(b) Move the image 1 unit along the positive x-axis and 2 units along the positive y-axis.

2. Homogeneous coordinates simplify representation by allowing 2D image transformations to be represented by a single matrix operation (3×3). What are the effects of the following transformations:

(a) $\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$, where $t_x = 1$ and $t_y = 2$.

(b) $\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$, where $\theta = 45°$.

Solution.

(a)

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x + t_x \\ y + t_y \\ 1 \end{bmatrix} = \begin{bmatrix} x + 1 \\ y + 2 \\ 1 \end{bmatrix}$$

Move the image 1 unit along the positive x-axis and 2 units along the positive y-axis.

(b)

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos 45° & -\sin 45° & 0 \\ \sin 45° & \cos 45° & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Take the origin as the center of rotation, rotate the image 45° counterclockwise.

3. Suppose we have a set of transformations in the order of translation of $(1, 2)$, counterclockwise rotation of 25°, and scaling of $(2, 1)$. What is the resulting transformation matrix?

Solution.

$$\begin{aligned}
\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} &= \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix} \\
&= \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 25° & -\sin 25° & 0 \\ \sin 25° & \cos 25° & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix} \\
&= \begin{bmatrix} 2\cos 25° & -2\sin 25° & 2\cos 25° - 4\sin 25° \\ \sin 25° & \cos 25° & \sin 25° + 2\cos 25° \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix} \\
&= \begin{bmatrix} 1.8126 & -0.8452 & 0.1221 \\ 0.4226 & 0.9063 & 2.2352 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}
\end{aligned}$$