Corner Extraction

Week - 3

Some slides of this lecture are courtesy of profs. S. Lazebnik & K. Grauman

Image Feature Extraction

- Edge detection
- Interest Point Detection (Corner, SIFT)

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Three levels of Vision Processing

- **Low-level vision**:

  – image processing, denoise, filtering, image restoration.

- **Mid-level vision**:

  – image feature detection, image segmentation, edge, contour extraction, perceptual organization,

  – 3D vision reconstruction: Multiview Geometry: 2-1/2D representation, 3D information recovery.

- **High level vision**:

  – visual recognition, classification, object localisation, semantic understanding and labelling, action, activity, event detection.

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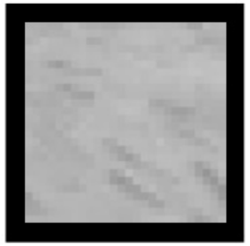
Content

- Harris Corner Detector
    - The most widely used corner point detector
- SIFT - Scale Invariant Feature Transform (next lecture)

Motivation

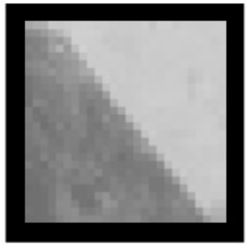
- Find “interesting” parts/pieces inside an image
  - e.g. corners, salient patches
  - Focus of attention, fixation.
  - Speed up computation.
  - Compress/extraction of information.
- Applications of interest points
  - Image Matching, Search
  - Object Detection, Object Recognition
  - Image Alignment & Stitching
  - Stereo
  - Tracking

Interest Points



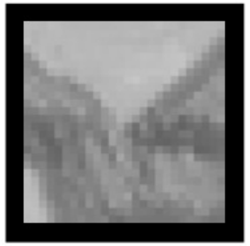
isotropic structure: flat region

➡ not interesting, 0D, not useful for matching



linear structure: edges, lines

➡ edge, can be localized in 1D, subject to the aperture problem

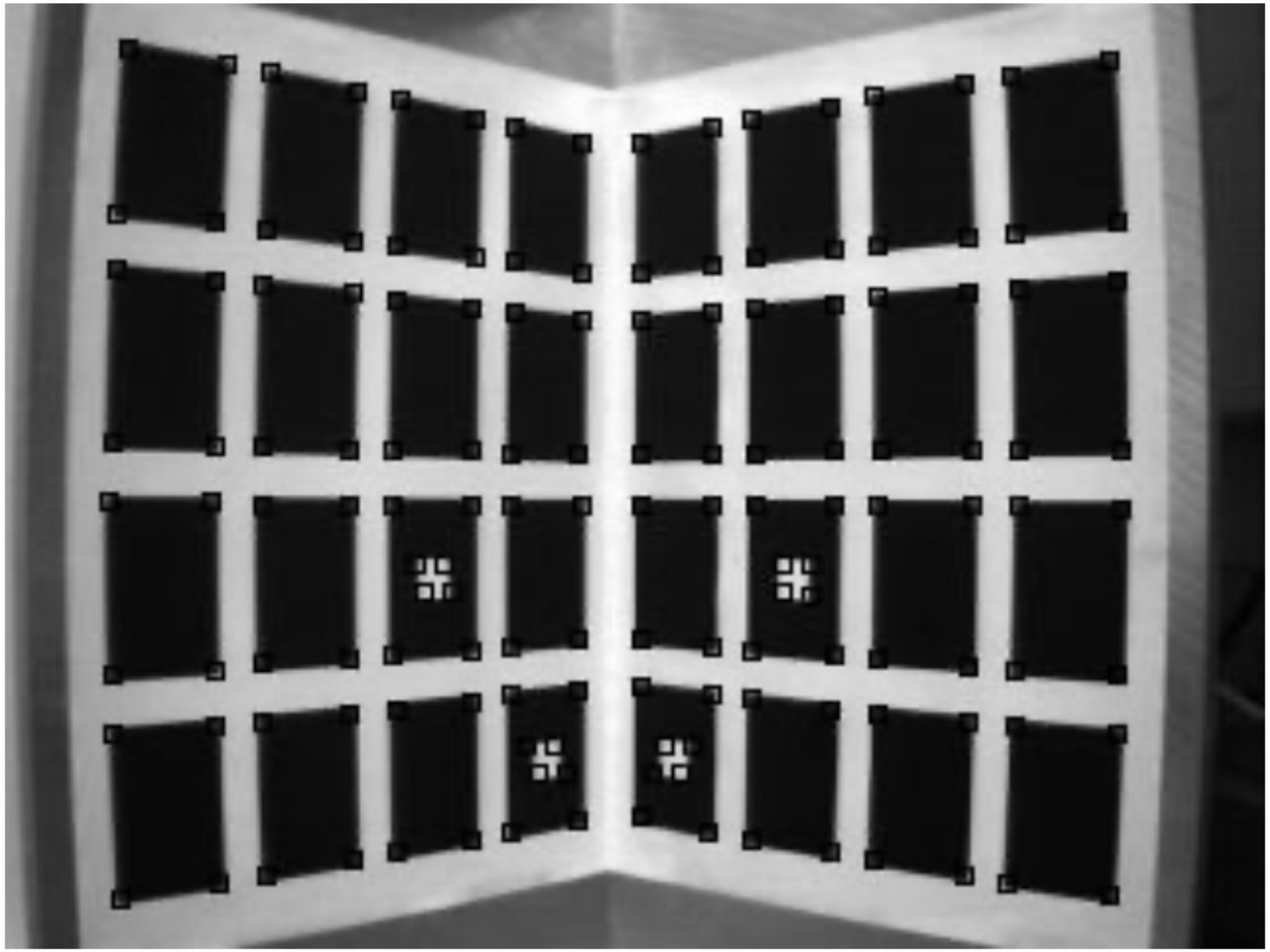


bi-directional structure: corners

➡ corner, or **interest point**, can be localised in 2D, good for matching

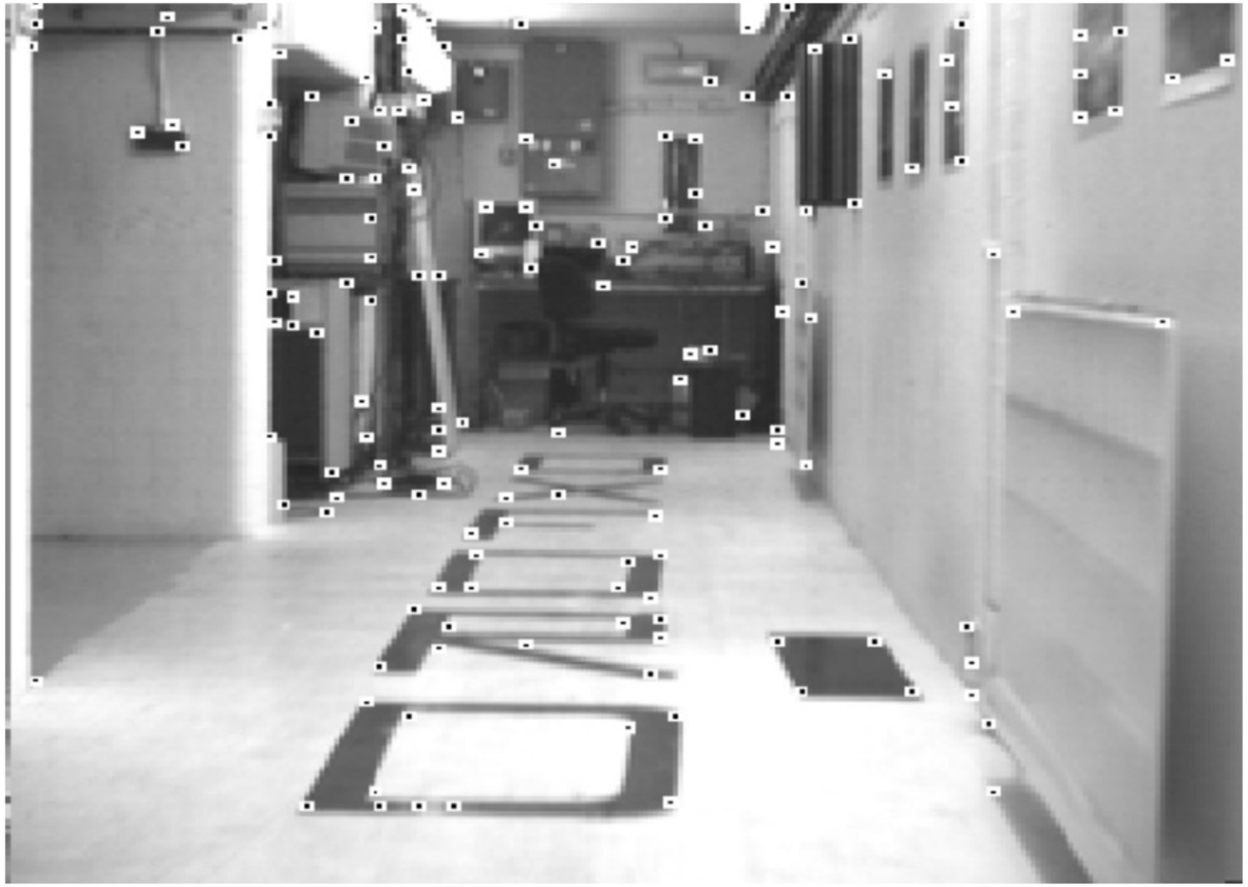
Interest Points have 2-directional structure.

Application: Corner Detection (for camera calibration)



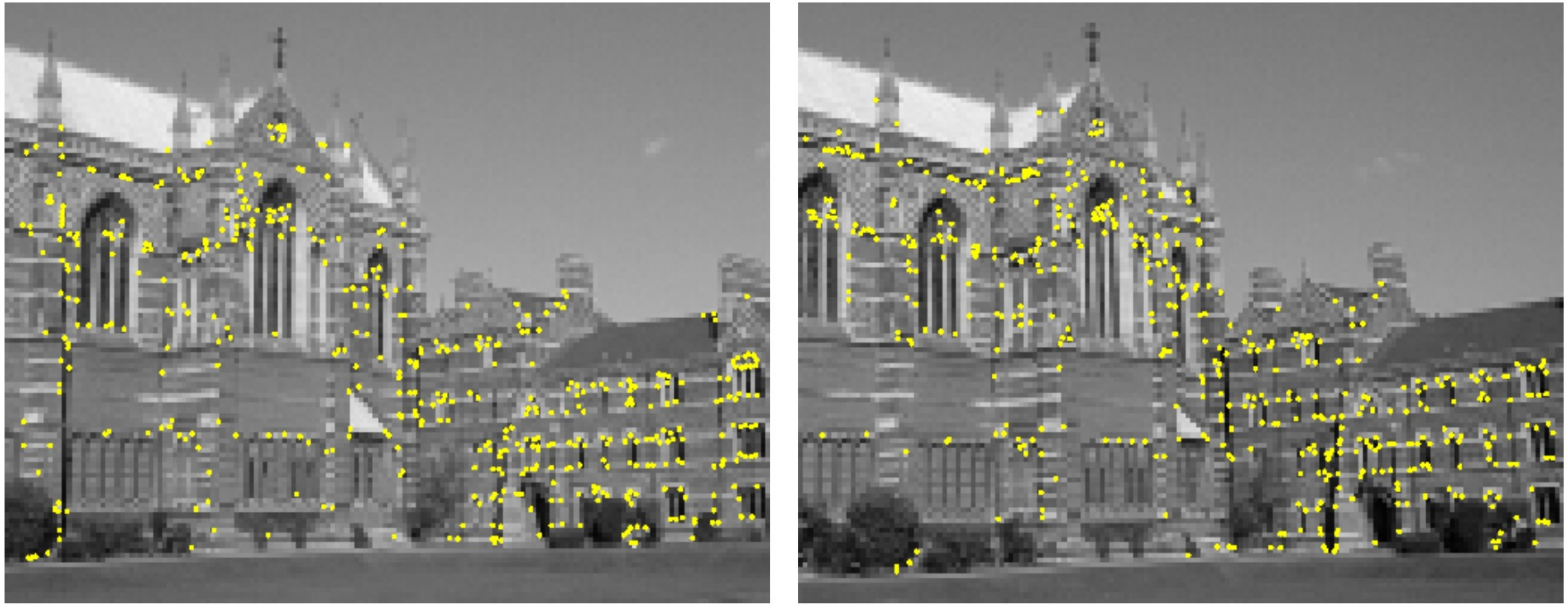
courtesy of B. Wilburn

Application: Robot navigation



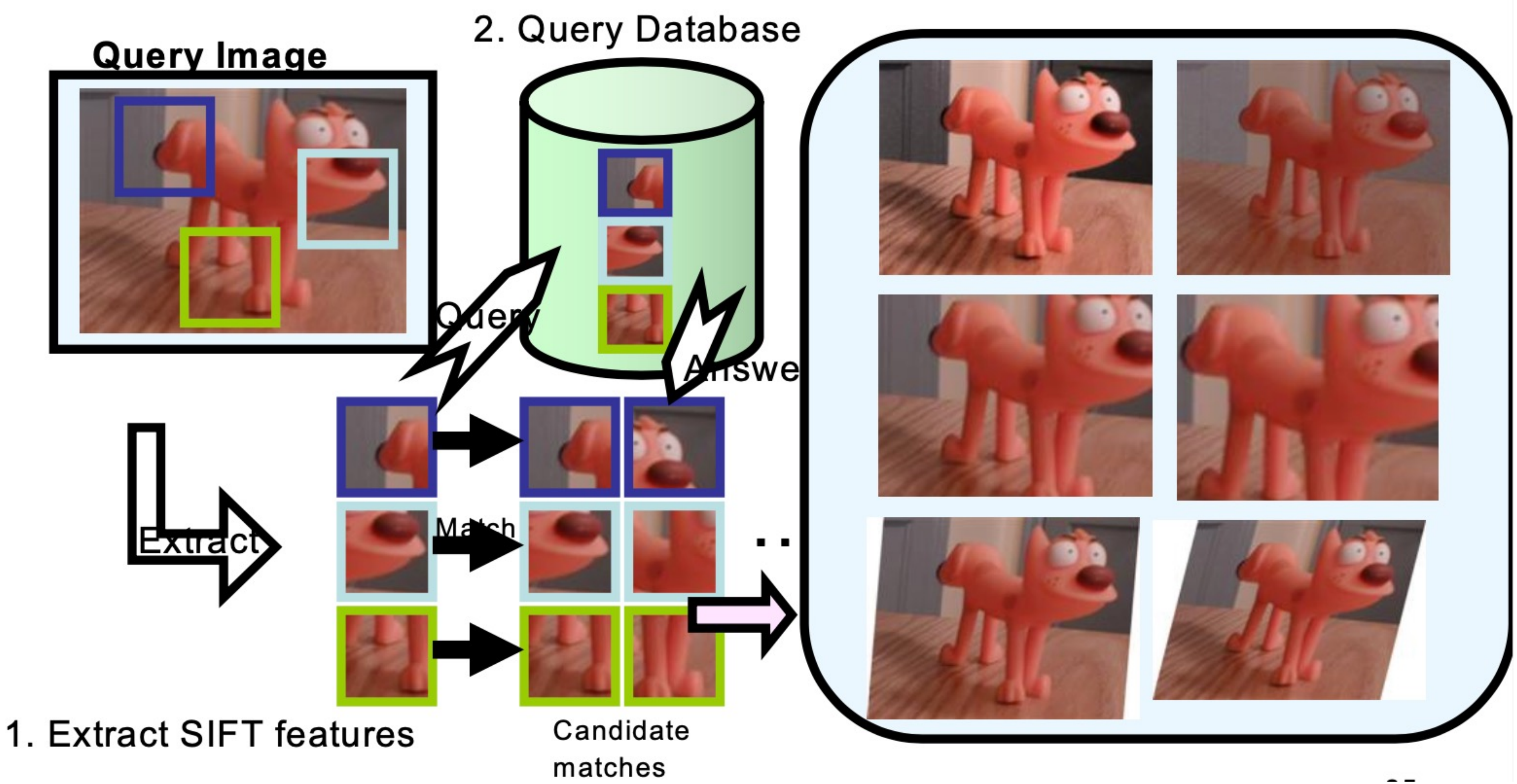
courtesy of S. Smith

Application: Matching between two images

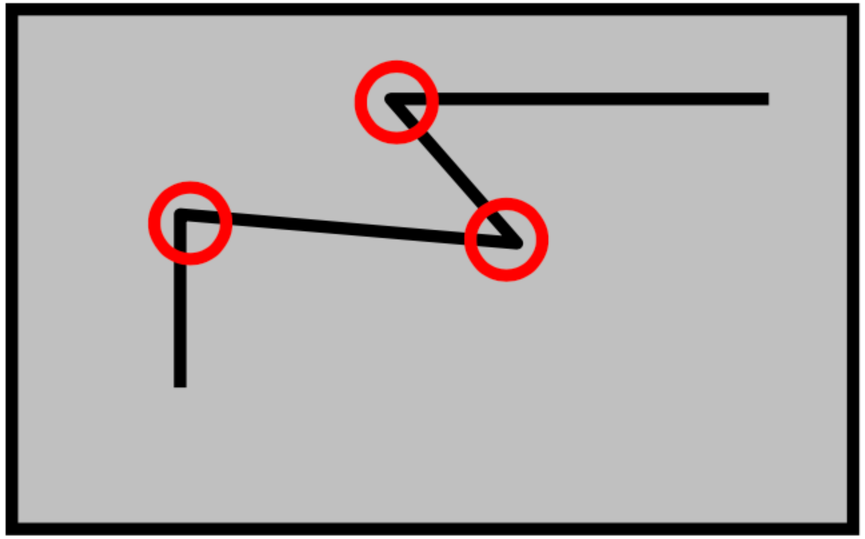


Interest points extracted with Harris (~ 500 points)

Content Based Image Retrieval (CBIR)



Harris Corner Detector



Reference

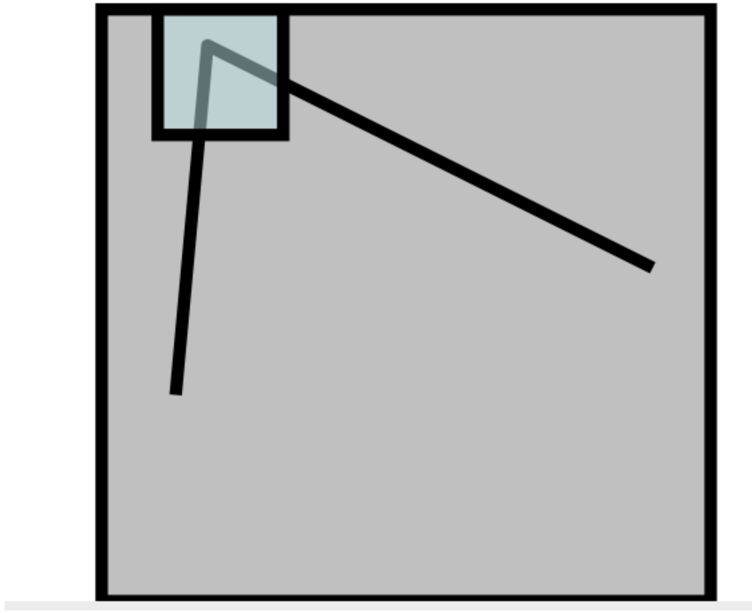
Chris Harris & Mike Stephens, CVIU, 1988

"A Combined Corner and Edge Detector"

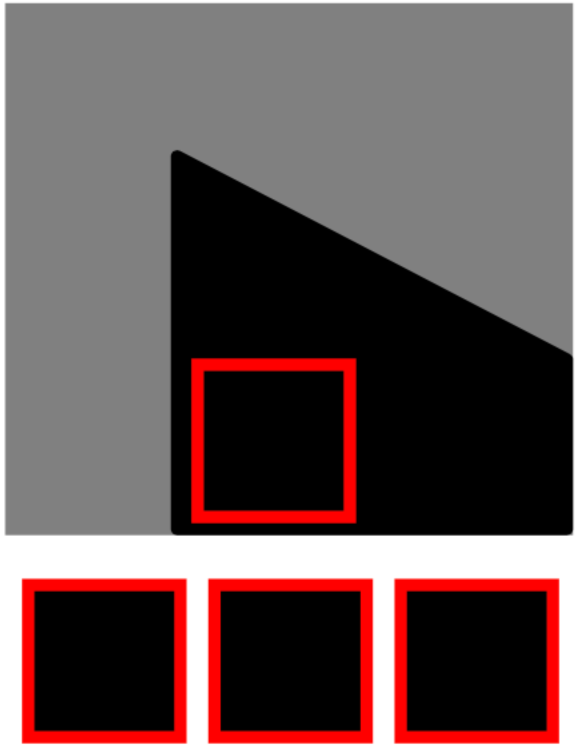
Harris Corner: Intuition

- We should easily recognize a corner point by looking through a small window
- Shifting a window in *any direction* should give *a large change* in intensity

shifting the window in any direction yield a large change in appearance

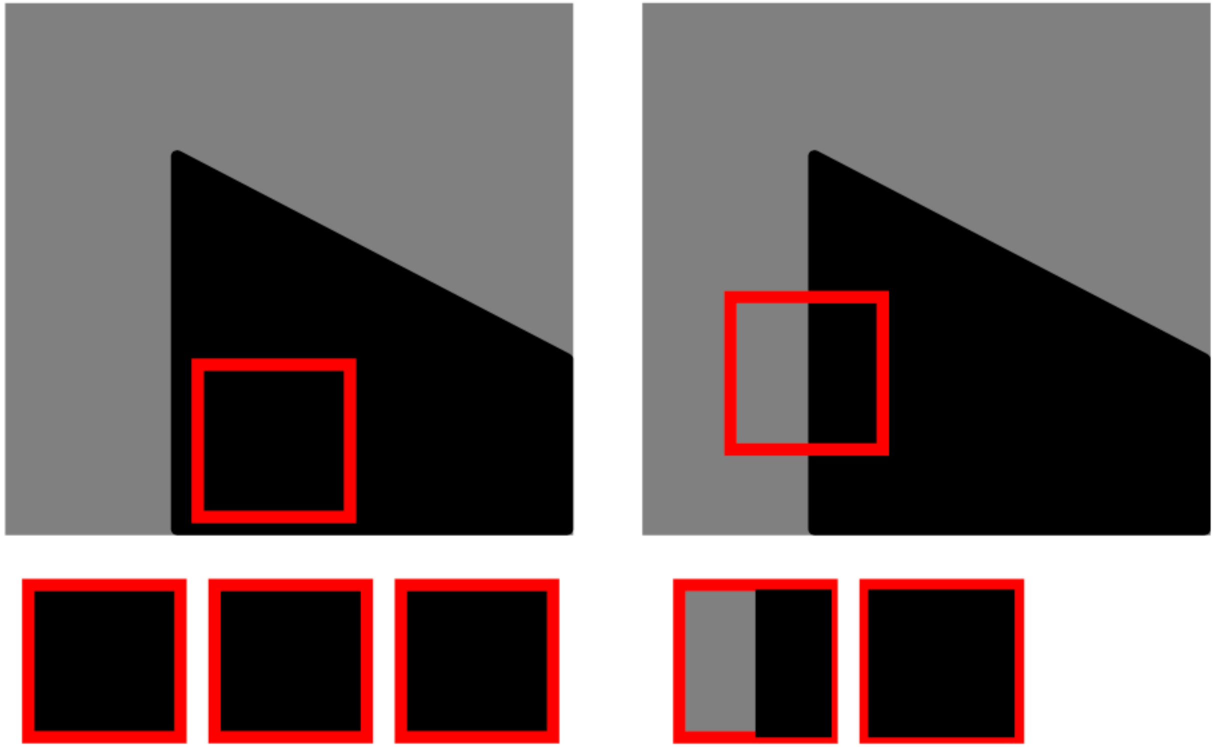


Corner detector



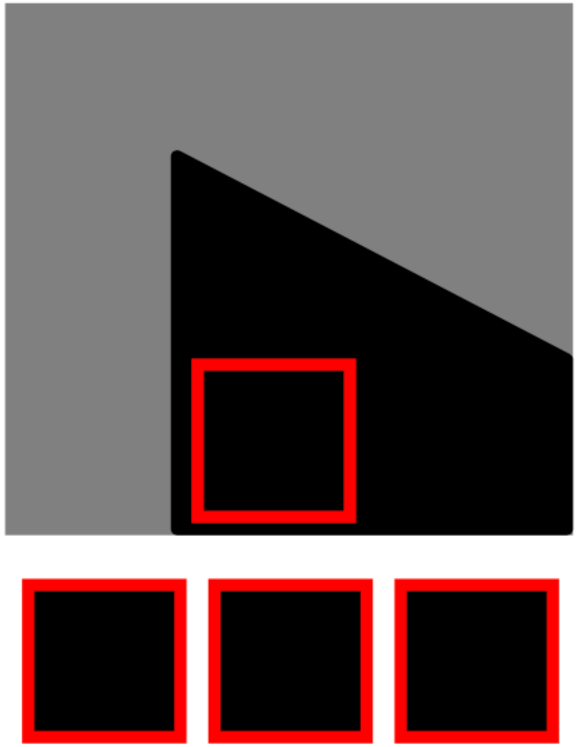
Flat

Corner detector

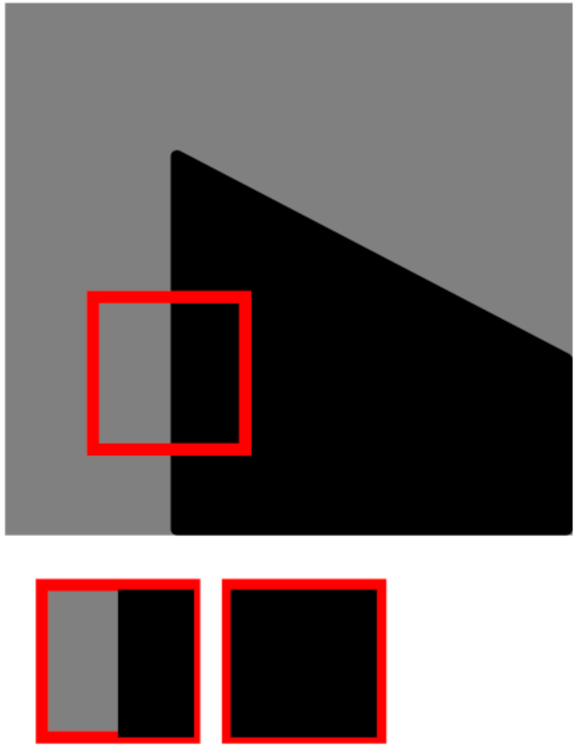


Flat Edge

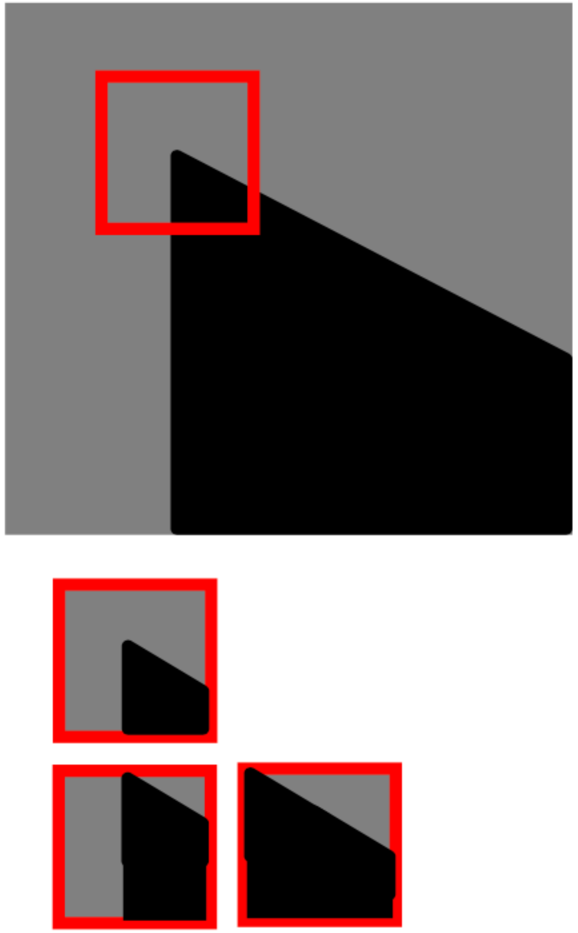
Corner detector



Flat



Edge



Corner

Harris Corner: Mathematics

Change of intensity for the shift $[u,v]$
(local auto-correlation analysis):

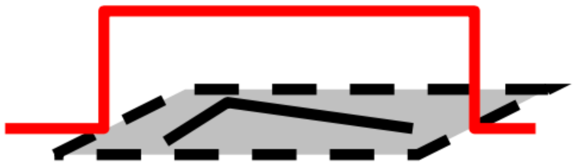
$$E(u,v) = \sum_{x,y} w(x,y)\left[I(x+u,y+v) - I(x,y)\right]^2$$

Window function

Shifted intensity

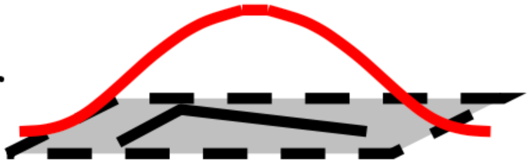
Intensity

Window function $w(x,y) =$



1 in window, 0 outside

or



Gaussian

Harris Corner: Mathematics

- For small shifts [u,v], we have first-order Tylor approximation for I(x+u,y+v)

$$I(x+u, y+v) \approx I(x,y) + uI_x + vI_y$$

Definition [edit]

The Taylor series of a real or complex-valued function $f(x)$, that is infinitely differentiable at a real or complex number a, is the power series

$$f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x -$$

Here, $n!$ denotes the factorial of n. The function $f^{(n)}(a)$ denotes the nth derivative of f evaluated at the point a. The derivative of order zero of f is defined to be f itself and $(x-a)^0$ and $0!$ are both defined to be 1. This series can be written by using sigma notation, as in the right side formula.[1] With $a = 0$, the Maclaurin series takes the form:[2]

$$f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \cdots = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!}x^n.$$

Harris Corner: Mathematics

$$
\begin{aligned}
E(u,v) &= \sum_{x,y} w(x,y)(I(x+u,y+v)-I(x,y))^2 \\
&= \sum_{x,y} w(x,y)(I(x,y)+uI_x+vI_y-I(x,y))^2 \\
&= \sum_{x,y} w(x,y)(uI_x+vI_y)^2 \\
&= \sum_{x,y} w(x,y)(u^2I_x^2+v^2I_y^2+2uvI_xI_y) \\
&= [u,v]\, M \begin{bmatrix} u \\ v \end{bmatrix}
\end{aligned}
$$

Harris Corner: Mathematics

- For small shift [u,v], we have 1st order Tylor approximation

$$E(u,v) \approx [u,v]\, M \begin{bmatrix} u \\ v \end{bmatrix}$$

- Where M is a 2x2 matrix computed from image derivatives:

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

M is called the second order moments (or auto correlation) matrix of gradients.

Harris Corner: Mathematics

Intensity change in shifting window: eigenvalue analysis

$$E(u,v) \cong [u,v] \quad M \quad \begin{bmatrix} u \\ v \end{bmatrix}$$

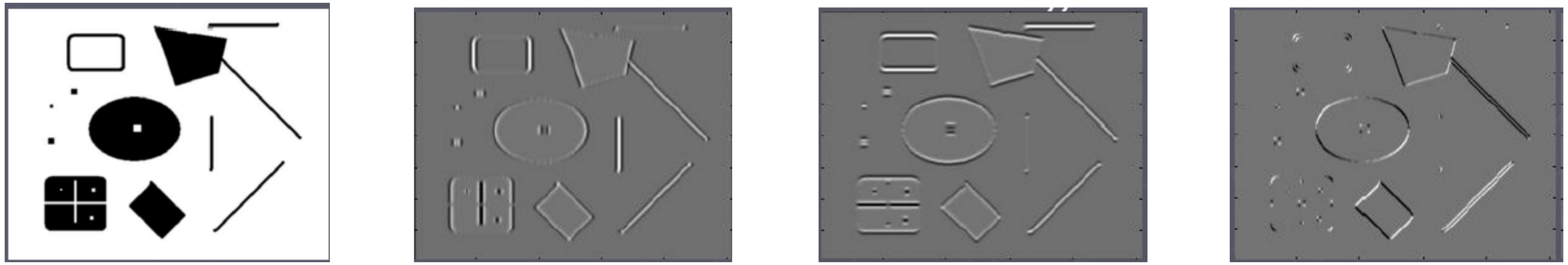
λ_1, λ_2 – eigenvalues of M

If we try every possible orientation $\mathbf{n}$,
the biggest change and smallest changes in intensity happen in λs.

Corners as distinctive interest points

$$M = \sum w(x,y) \begin{bmatrix} I_x I_x & I_x I_y \\ I_x I_y & I_y I_y \end{bmatrix}$$

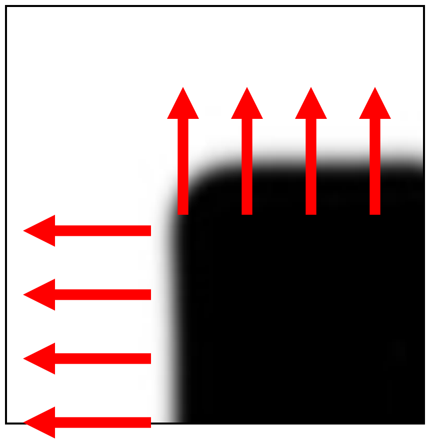
2 x 2 matrix of image derivatives (averaged in neighborhood of a point).



Notation: $I_x \Leftrightarrow \frac{\partial I}{\partial x}$ $\quad I_y \Leftrightarrow \frac{\partial I}{\partial y}$ $\quad I_x I_y \Leftrightarrow \frac{\partial I}{\partial x}\frac{\partial I}{\partial y}$

What does this matrix reveal?

First, consider an axis-aligned corner:



What does this matrix reveal?

First, consider an axis-aligned corner:

$$M = \sum \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

This means dominant gradient directions align with x or y axis

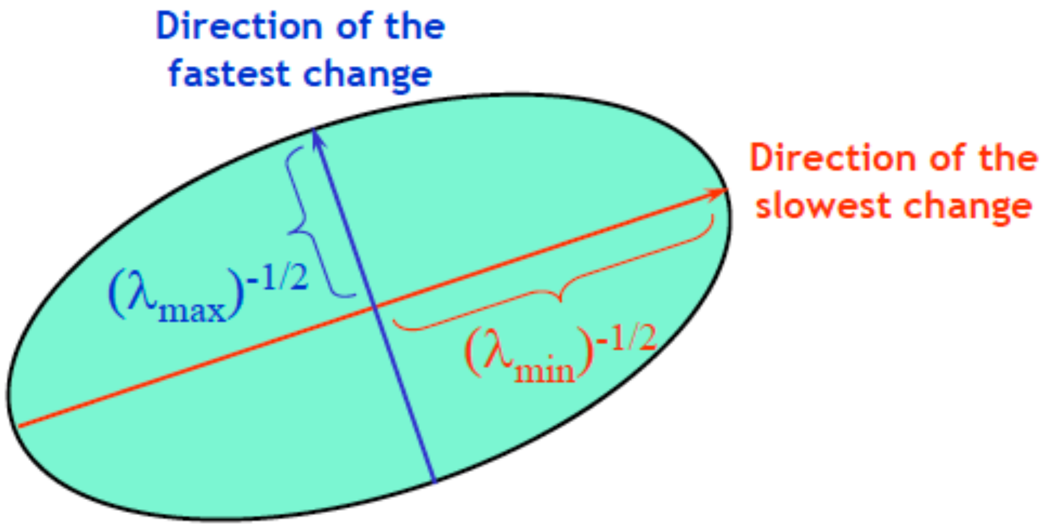
Look for locations where **both** λ's are large.

If either λ is close to 0, then this is **not** corner-like.

What if we have a corner that is not aligned with the image axes?

General Case

- Since M is symmetric, we have $M = X \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} X^T$
- (eigen value decomposition)
- We can visualize M as an eclipse with axis length determined by the eigenvalues and orientation determined by X



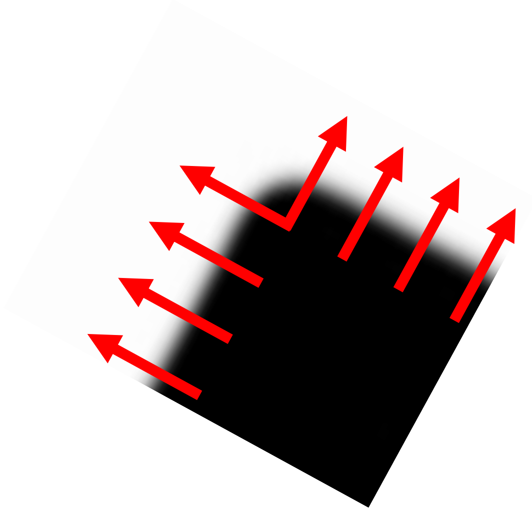
Explanation

- X = I (identity matrix)
-

$$\begin{pmatrix} u & v \end{pmatrix} \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} = \lambda_1 u^2 + \lambda_2 v^2 = 1$$

What does this matrix reveal?

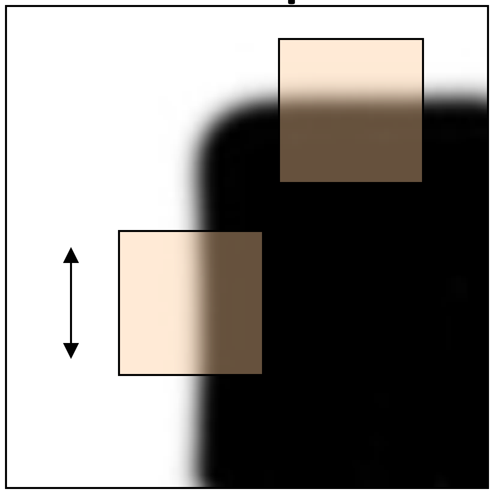
Since M is symmetric, we have $M = X \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} X^T$



$$Mx_i = \lambda_i x_i$$

The *eigenvalues of M* reveal the amount of intensity change in the two principal orthogonal gradient directions in the window.

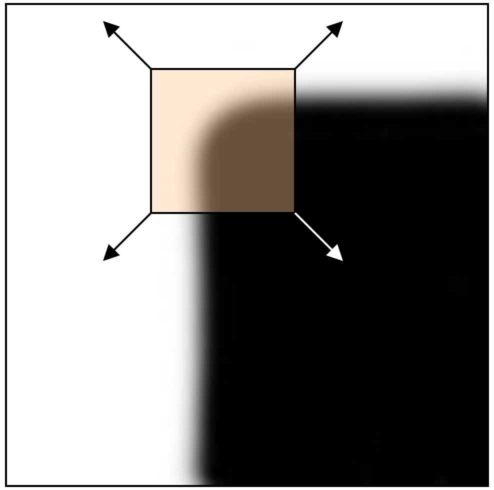
Corner response function



"edge":

$\lambda_1 >> \lambda_2$

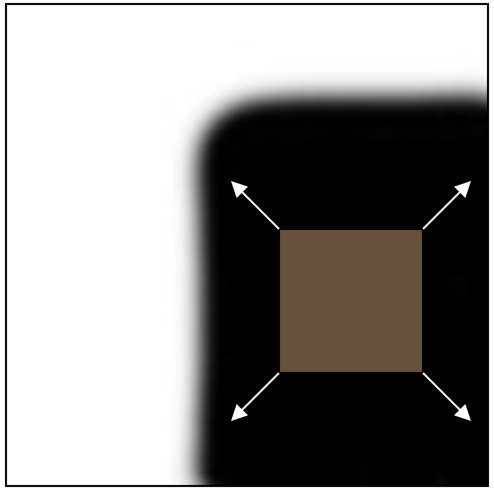
$\lambda_2 >> \lambda_1$



"corner":

λ_1 and λ_2 are large,

$\lambda_1 \sim \lambda_2$;



"flat" region

λ_1 and λ_2 are small;

Harris Corner Detector: Mathematics

Measure of corner response: (Cornerness)

$$R = \det M - k\left(\operatorname{trace} M\right)^2$$

$$\det M = \lambda_1 \lambda_2$$
$$\operatorname{trace} M = \lambda_1 + \lambda_2$$

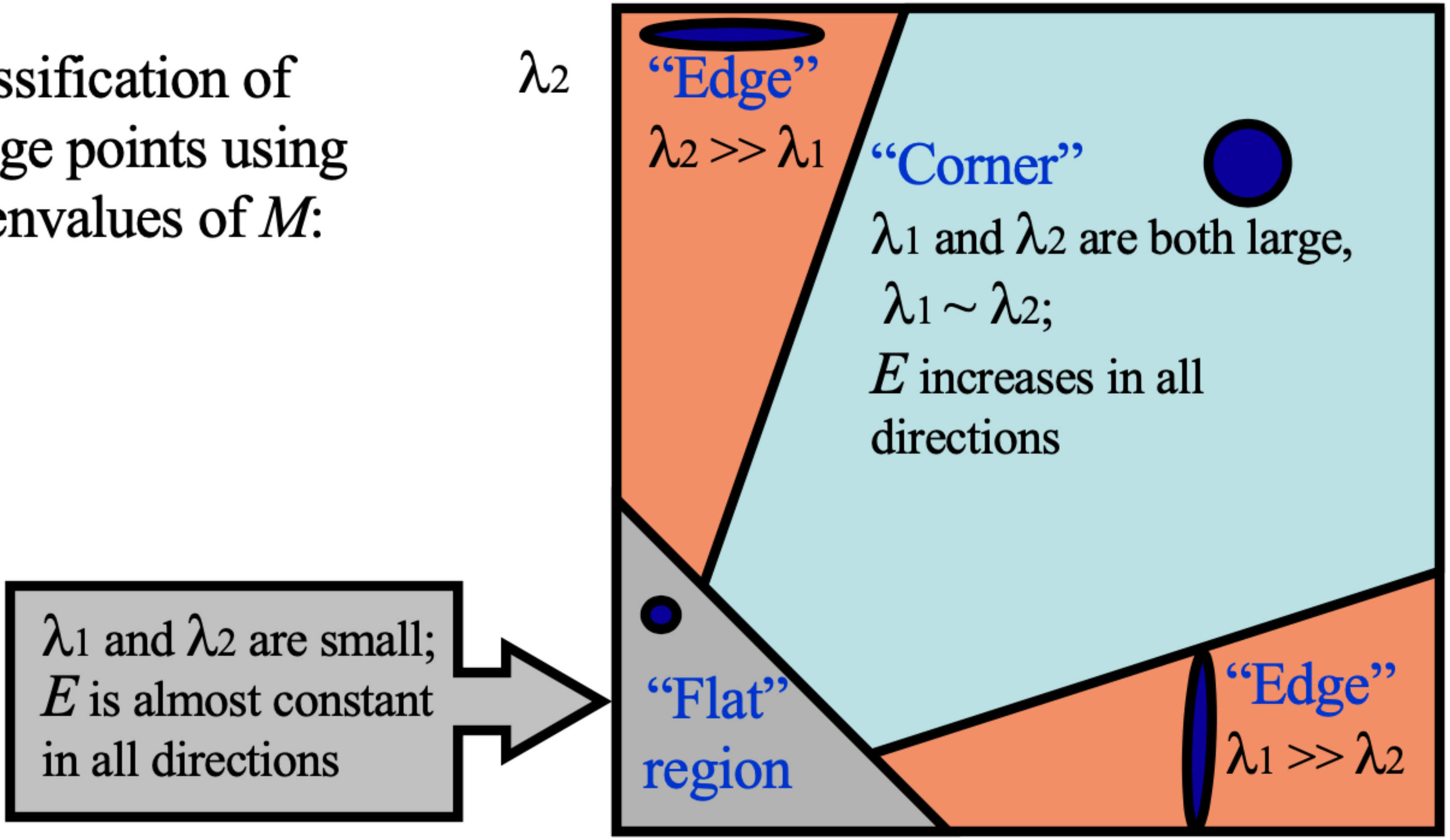
(k – empirical constant, $k = 0.01\text{-}0.1$)

Harris corner detector

1) Compute M matrix for each image window to get their *cornerness* scores.
2) Find points whose surrounding window gave large corner response (R> threshold)
3) Take the points of local maxima, i.e., perform non-maximum suppression

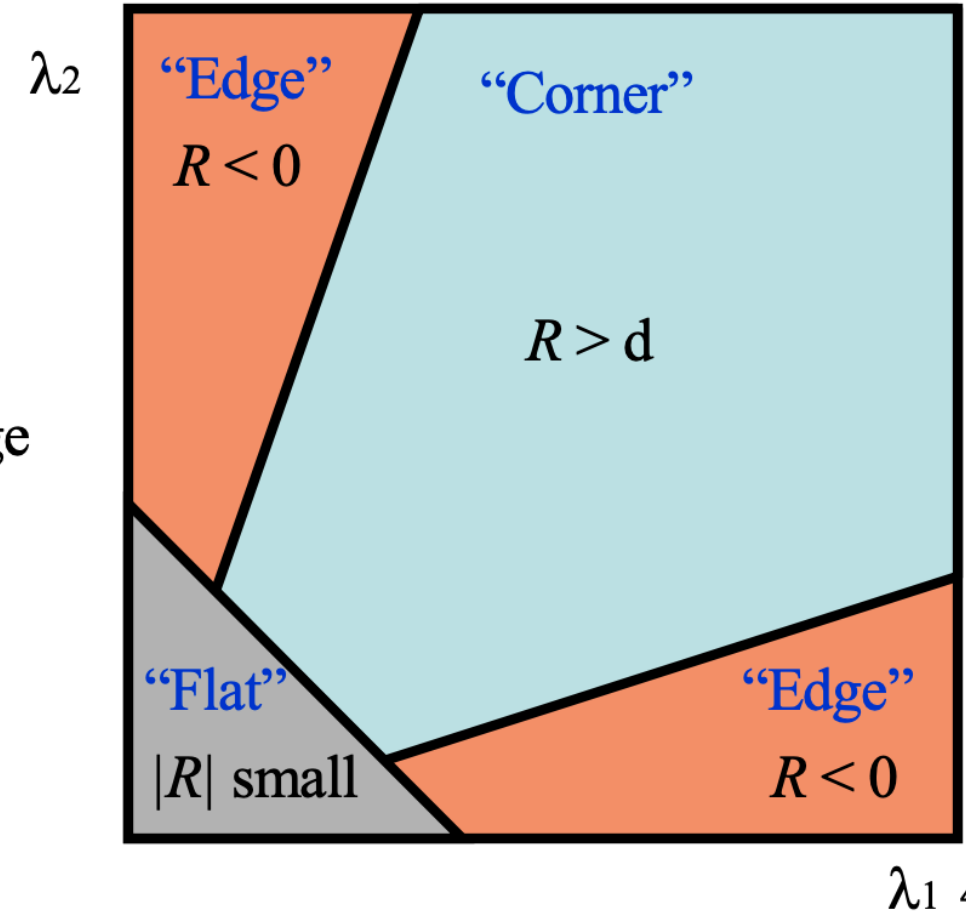
Harris Corner Detector

Classification of image points using eigenvalues of M:



Harris Corner Detector: Mathematics

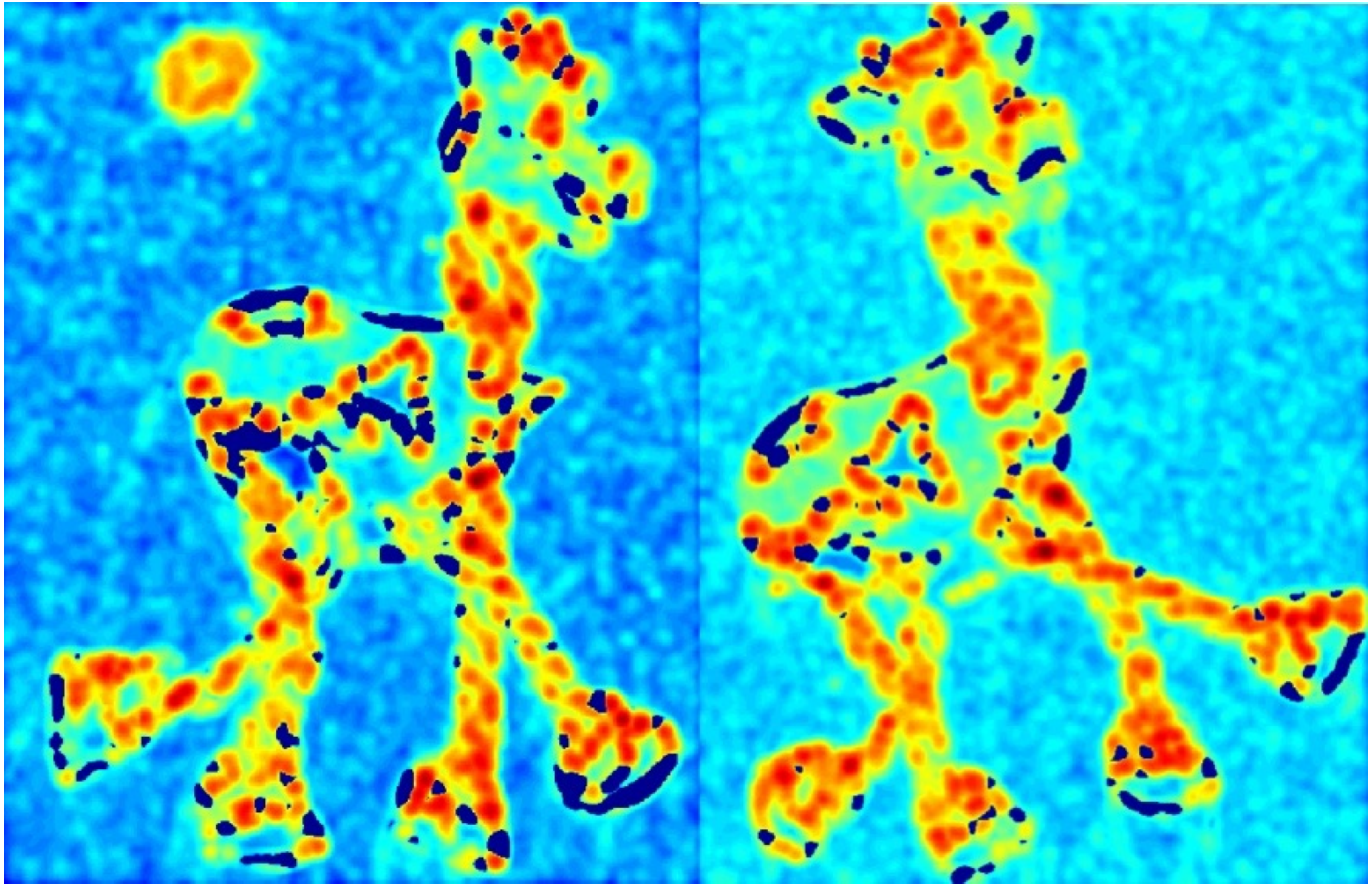
- R depends only on eigenvalues of M
- R is large for a corner
- R is negative with large magnitude for an edge
- $|R|$ is small for a flat region



Harris Detector: Steps

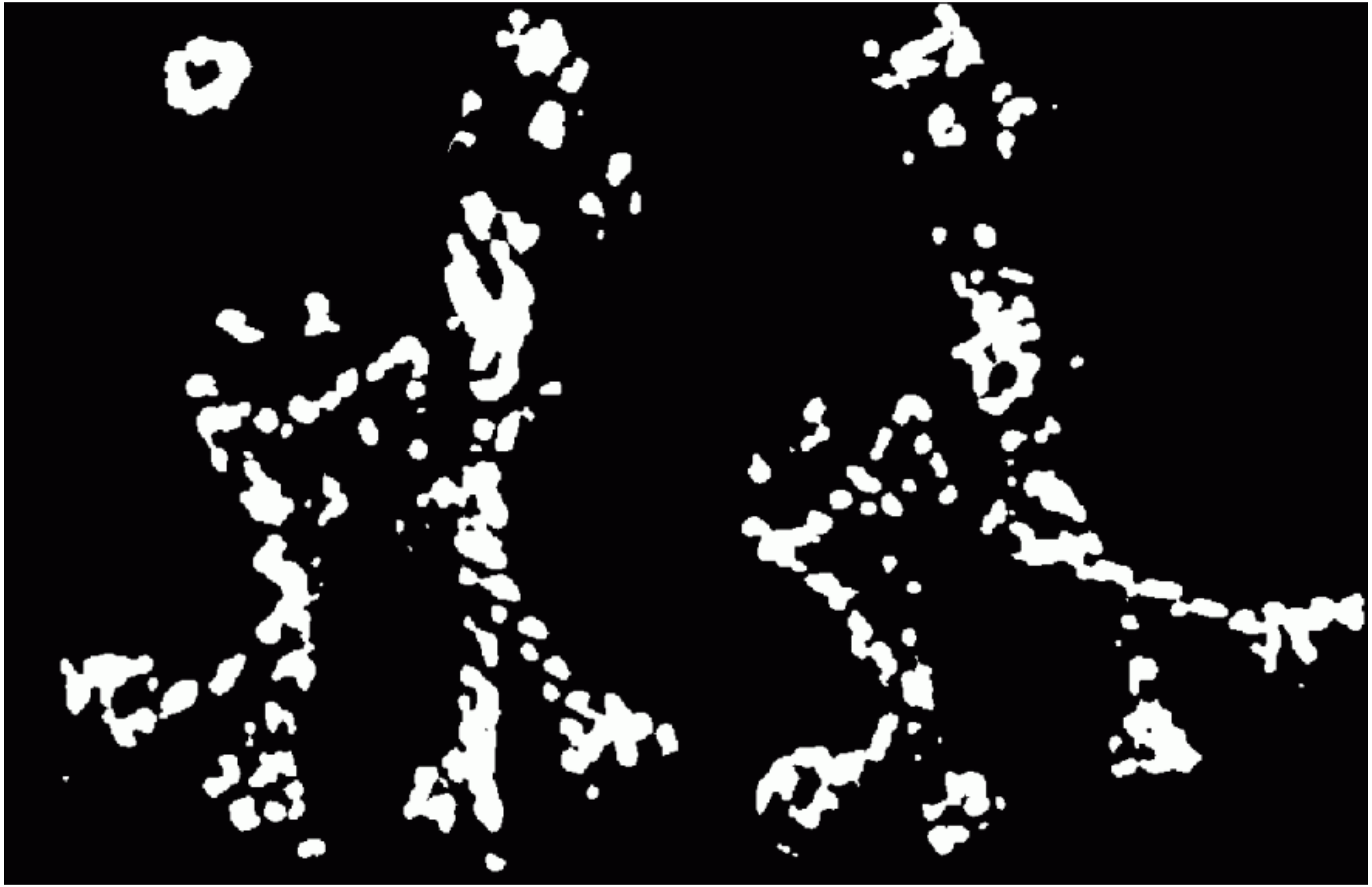
Harris Detector: Steps

Compute corner response R *(i.e. the "cornerness")*



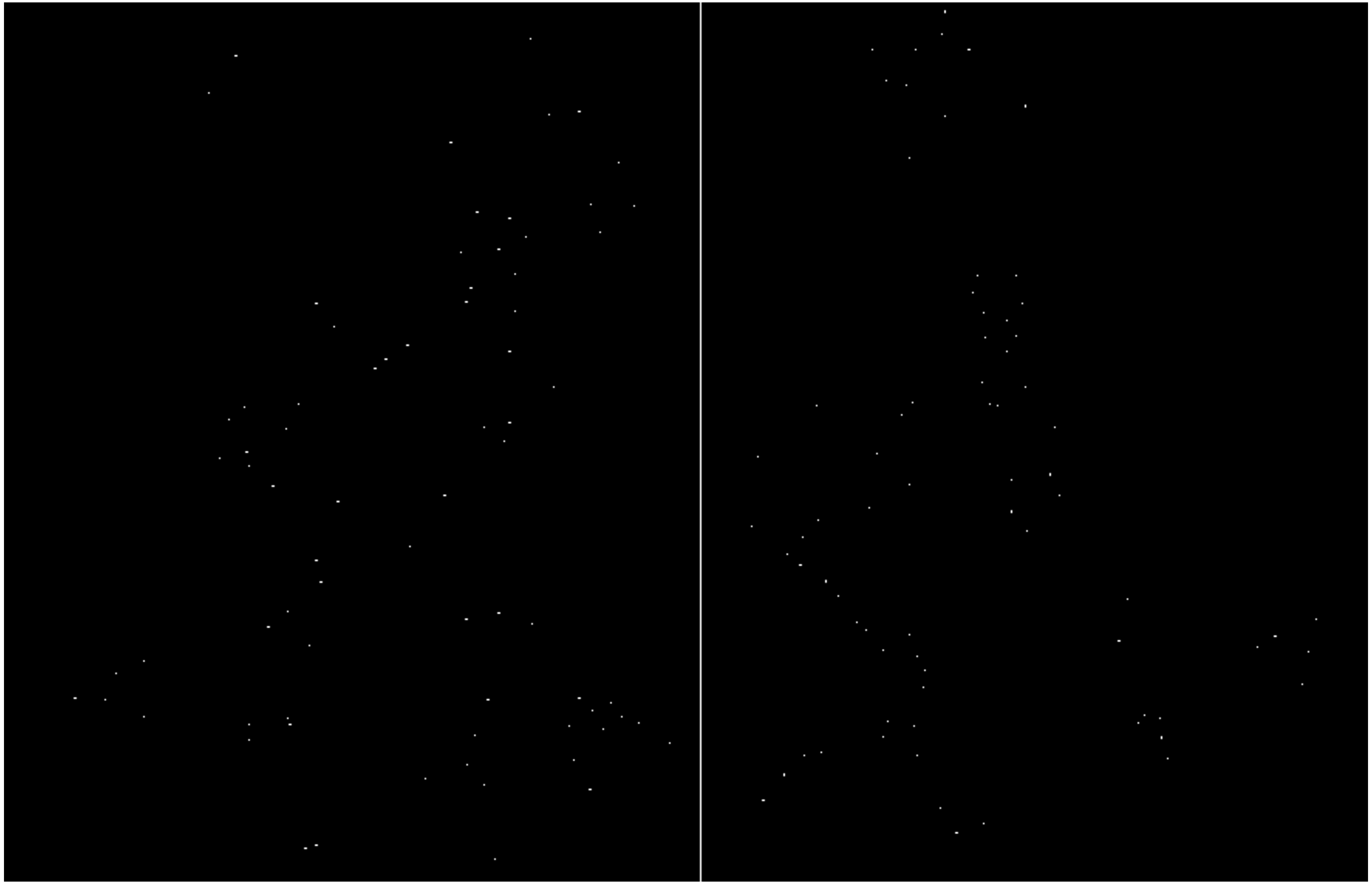
Harris Detector: Steps

Find points with large corner response: $R >$ threshold

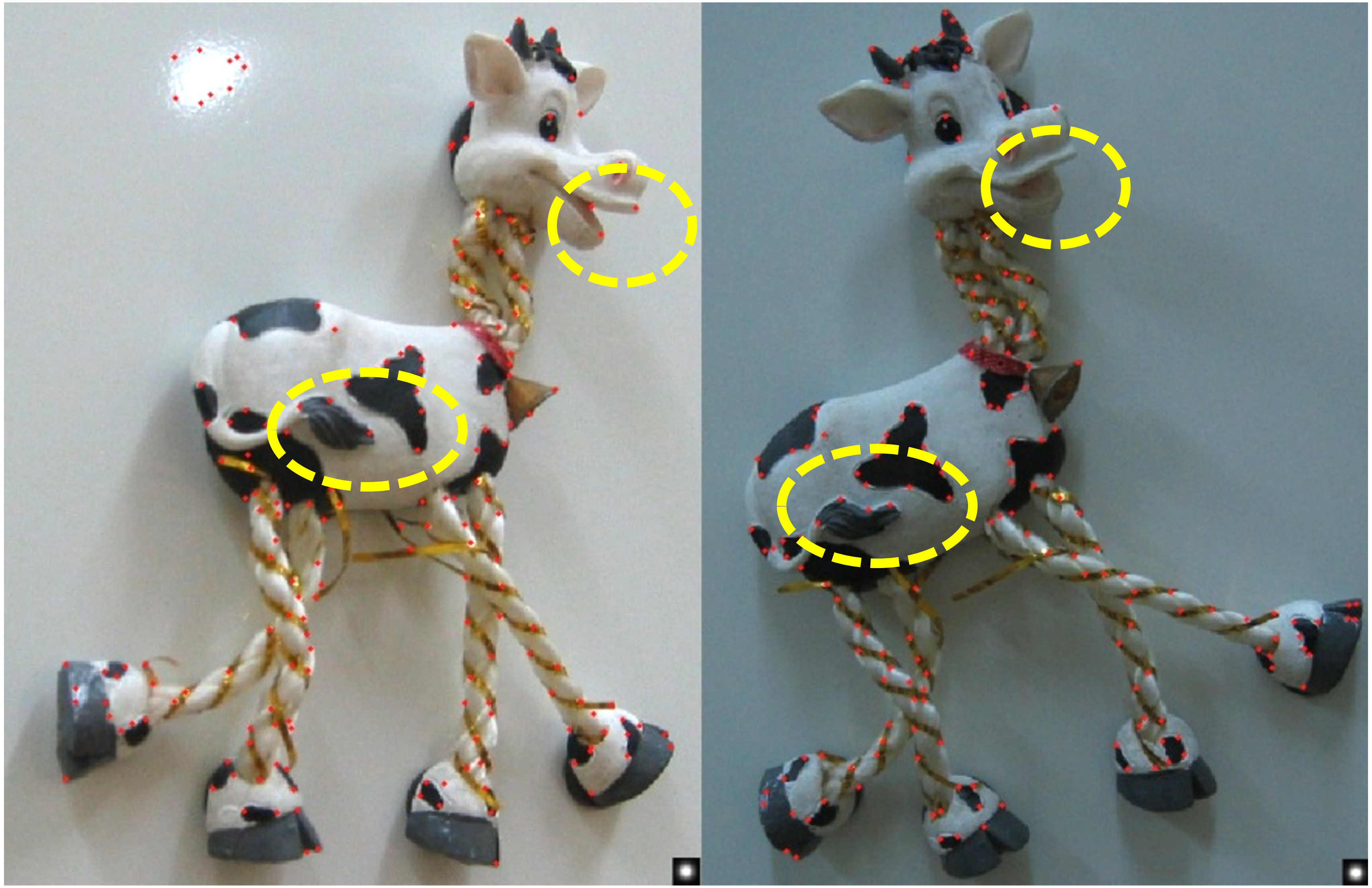


Harris Detector: Steps

Take only the points of local maxima of R



Harris Detector: Steps



Recap: Harris Corner

- Compute the moment (auto-correlation) matrix M
  - captures the structure of the local neighborhood
  - measure based on **eigenvalues** of M 2*2
    - 2 strong eigenvalues => interest point
    - 1 strong eigenvalue => edge or contour
    - 0 or very weak eigenvalues => flat region

- Interest point detection
  - threshold on the eigenvalues
  - local maximum for localization

Recap: Harris Corner

- Corner strength $R = \det(M) - k\, Tr(M)^2$
- Let α and β be the two eigenvalues. We don't have to calculate them! Instead, use trace and determinant:

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \qquad \begin{aligned} \det(\mathbf{A}) &= a_{11}a_{22} - a_{12}a_{21} \\ \mathrm{tr}(\mathbf{A}) &= a_{11} + a_{22} \end{aligned}$$

- R is positive for corners, negative for edges, and small for flat regions
- **Non-maximal suppression**: select corners that are 8-way local maxima

Expected Learning Outcome

- Understand why and where we need to use interest point detector.

- Learn how to implement Harris corner detector and how to use it.

Reference

- Section 7.1.1: Feature Detector (Computer Vision: Algorithms and Applications 2nd Edition)