Deep Learning

Week - 5

Today's Lecture

- Review Machine Learning Basics
- Back-propagation
- Convolutional Neural Networks
- Short tutorial on building your own network.

Recipe for design a Machine Learning system

1. Given training data:

$$\{\boldsymbol{x}_i, \boldsymbol{y}_i\}_{i=1}^{N}$$

2. Choose each of these:
   - Decision function

$$\hat{\boldsymbol{y}} = f_{\boldsymbol{\theta}}(\boldsymbol{x}_i)$$

   - Loss function

$$\ell(\hat{\boldsymbol{y}}, \boldsymbol{y}_i) \in \mathbb{R}$$

3. Define Goal state:

$$\boldsymbol{\theta}^* = \arg\min_{\boldsymbol{\theta}} \sum_{i=1}^{N} \ell(f_{\boldsymbol{\theta}}(\boldsymbol{x}_i), \boldsymbol{y}_i)$$

4. Train with BP in SGD:

(take small steps opposite the gradient)

$$\boldsymbol{\theta}^{(t+1)} = \boldsymbol{\theta}^{(t)} - \eta_t \nabla \ell(f_{\boldsymbol{\theta}}(\boldsymbol{x}_i), \boldsymbol{y}_i)$$

Learning Algorithm

- Initialize $\boldsymbol{\theta}$ $\left(\boldsymbol{\theta} \equiv \{\mathbf{W}^{(1)}, \mathbf{b}^{(1)}, \ldots, \mathbf{W}^{(L+1)}, \mathbf{b}^{(L+1)}\} \right)$
- For N iterations:
  - For each training example $(\mathbf{x}^{(t)}, y^{(t)})$
  - $\Delta = -\nabla_{\boldsymbol{\theta}} l(f(\mathbf{x}^{(t)}; \boldsymbol{\theta}), y^{(t)}) - \lambda \nabla_{\boldsymbol{\theta}} \Omega(\boldsymbol{\theta})$
  - $\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} + \alpha \, \Delta$

Training epoch
=
Iteration over all
examples

- To apply this algorithm to neural network training, we need
  - The loss function $l(\mathbf{f}(\mathbf{x}^{(t)}; \boldsymbol{\theta}), y^{(t)})$
  - Procedure to compute parameter gradients $\nabla_{\boldsymbol{\theta}} l(\mathbf{f}(\mathbf{x}^{(t)}; \boldsymbol{\theta}), y^{(t)})$
  - The regularizer $\Omega(\boldsymbol{\theta})$ (and the gradient $\nabla_{\boldsymbol{\theta}} \Omega(\boldsymbol{\theta})$)
  - Initialization method

Back-propagation learning

Training deep neural networks

- Outlook:
  - Computing gradients is hard (many parameters)
    → Back-propagation!

- Network represents a chain of function calls (one per layer):

$$\boldsymbol{f}(\mathbf{x}) = \boldsymbol{f}_{L+1} \circ \boldsymbol{f}_L \circ \cdots \circ \boldsymbol{f}_1(\mathbf{x})$$

$$= \boldsymbol{f}_{L+1}(\boldsymbol{f}_L(\dots \boldsymbol{f}_1(\mathbf{x})))$$

- Gradients: Chain rule can be applied!

$$\frac{\partial \boldsymbol{f}(\mathbf{x})}{\partial \boldsymbol{\theta}} = \frac{\partial \boldsymbol{f}_{L+1}}{\partial \boldsymbol{f}_L} \cdot \frac{\partial \boldsymbol{f}_L}{\partial \boldsymbol{f}_{L-1}} \cdot \dots \cdot \frac{\partial \boldsymbol{f}_1}{\partial \boldsymbol{\theta}}$$

How to compute gradient?

Symbolic Differentiation

- Input formulae is a symbolic expression tree (computation graph).
- Implement differentiation rules, e.g., sum rule, product rule, chain rule

$$\frac{d(f+g)}{dx} = \frac{df}{dx} + \frac{dg}{dx} \qquad \frac{d(fg)}{dx} = \frac{df}{dx}g + f\frac{dg}{dx} \qquad \frac{d\big(h(x)\big)}{dx} = \frac{df\big(g(x)\big)}{dx} \cdot \frac{dg(x)}{x}$$

✗ For complicated functions, the resultant expression can be exponentially large.

✗ Wasteful to keep around intermediate symbolic expressions if we only need a numeric value of the gradient in the end

✗ Prone to error

How to compute gradient?

Numerical Differentiation

- We can approximate the gradient using

$$\frac{\partial f(\boldsymbol{x})}{\partial x_i} \approx \lim_{h \to 0} \frac{f(\boldsymbol{x} + h\boldsymbol{e_i}) - f(\boldsymbol{x})}{h}$$

$$f(W, x) = \quad W \quad \cdot \quad x$$

$$\begin{bmatrix} -0.8 & 0.3 \end{bmatrix} \cdot \begin{bmatrix} 0.5 \\ -0.2 \end{bmatrix}$$

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$$f(W, x) = \quad W \quad \cdot \quad x \qquad\qquad f(W, x) = \quad W \quad \cdot \quad x$$

$$[-0.8 \quad 0.3] \cdot \begin{bmatrix} 0.5 \\ -0.2 \end{bmatrix} \qquad\qquad [-0.8 + \varepsilon \quad 0.3] \cdot \begin{bmatrix} 0.5 \\ -0.2 \end{bmatrix}$$

How to compute gradient?

Numerical Differentiation

- We can approximate the gradient using

$$\frac{\partial f(\boldsymbol{x})}{\partial x_i} \approx \lim_{h \to 0} \frac{f(\boldsymbol{x} + h\boldsymbol{e}_i) - f(\boldsymbol{x})}{h}$$

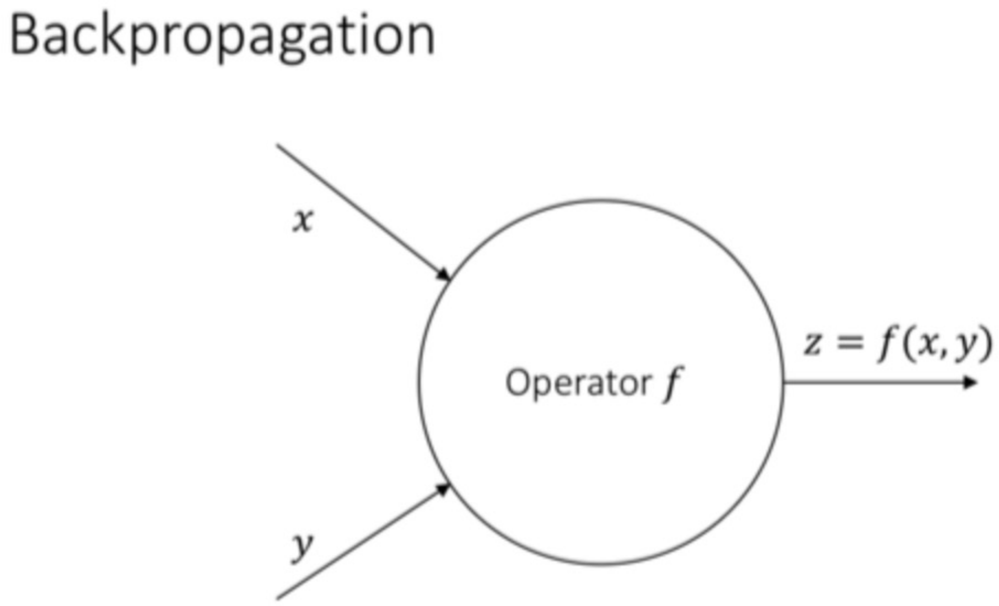
- Reduce the truncation error by using center difference

$$\frac{\partial f(\boldsymbol{x})}{\partial x_i} \approx \lim_{h \to 0} \frac{f(\boldsymbol{x} + h\boldsymbol{e}_i) - f(\boldsymbol{x} - h\boldsymbol{e}_i)}{2h}$$

✗ Bad: rounding error, and slow to compute

✓ A powerful tool to check the correctness of implementation, usually use h = 1e-6.

Back Propagation for computing differentiation



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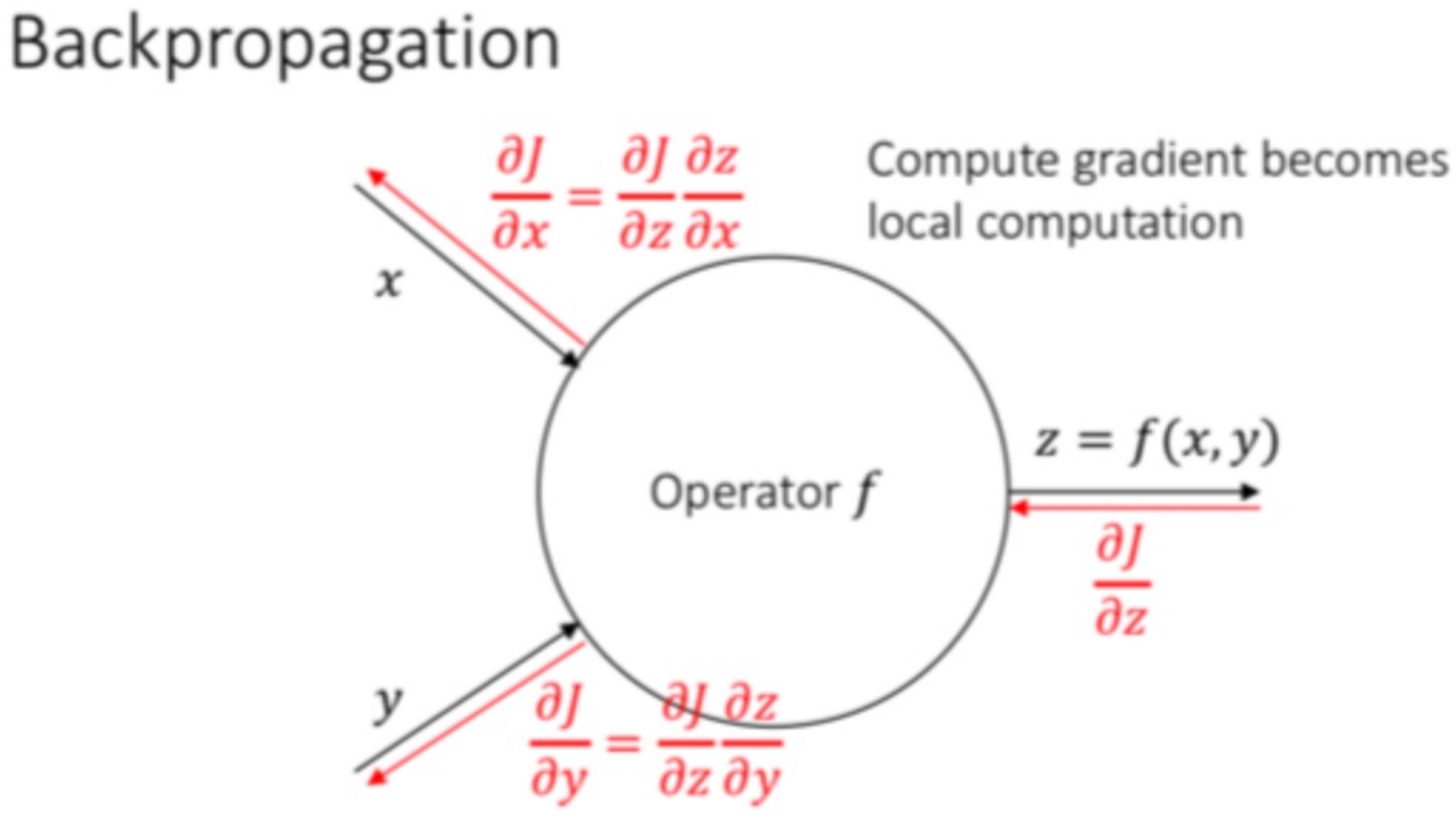
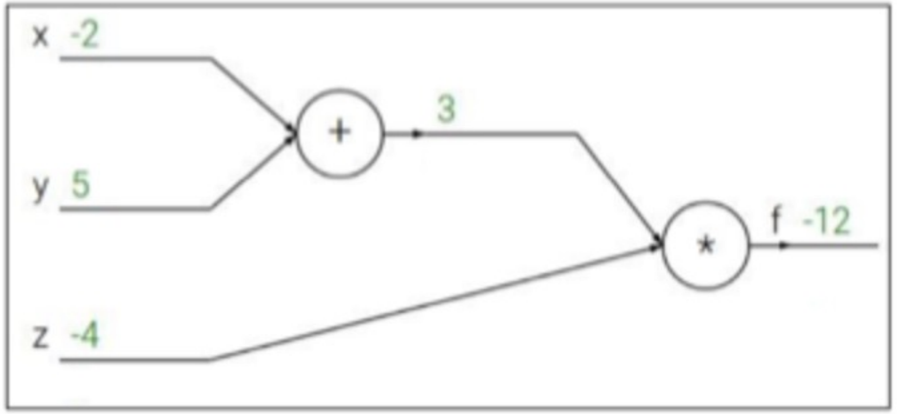


Example: Back-Prop

$$f(x, y, z) = (x + y)z$$

e.g. x = -2, y = 5, z = -4



Want: $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$

- slide source: Stanford CS231n lecture note.

Example: Back-Prop

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$$q = x + y \qquad \frac{\partial q}{\partial x} = 1, \frac{\partial q}{\partial y} = 1$$

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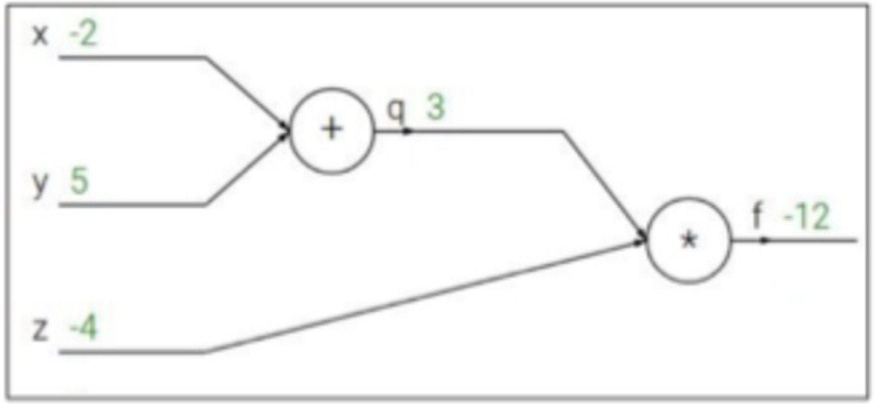

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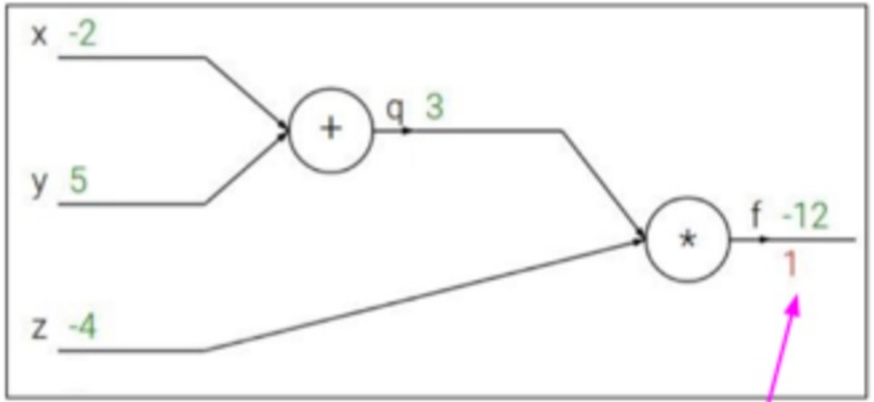

$$\boxed{\frac{\partial f}{\partial f}}$$

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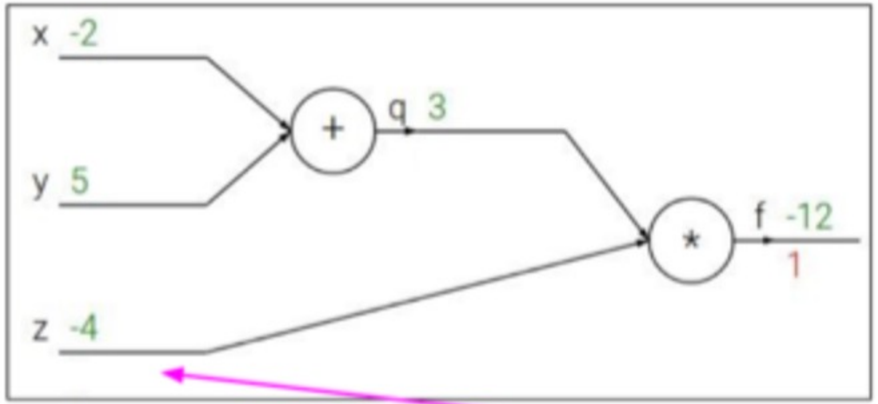

$$\frac{\partial f}{\partial z}$$

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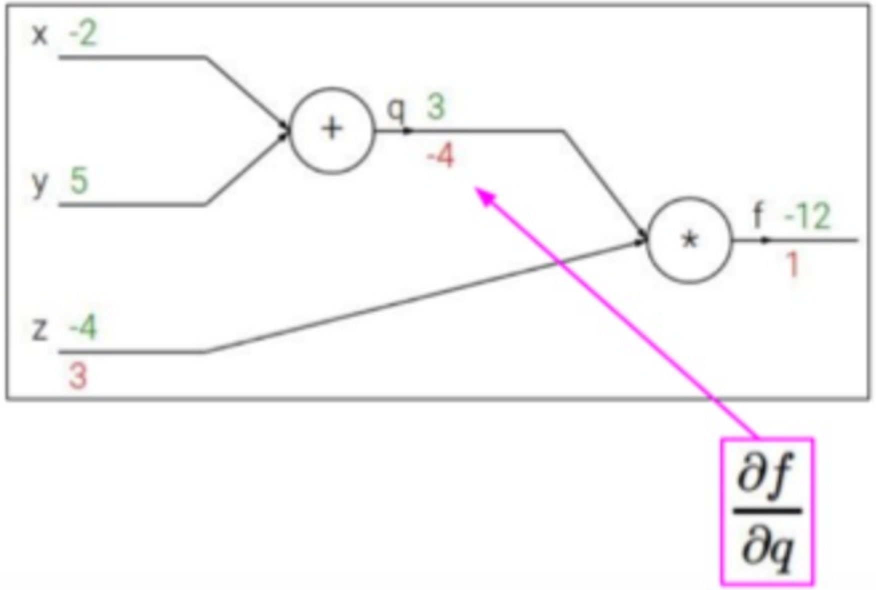

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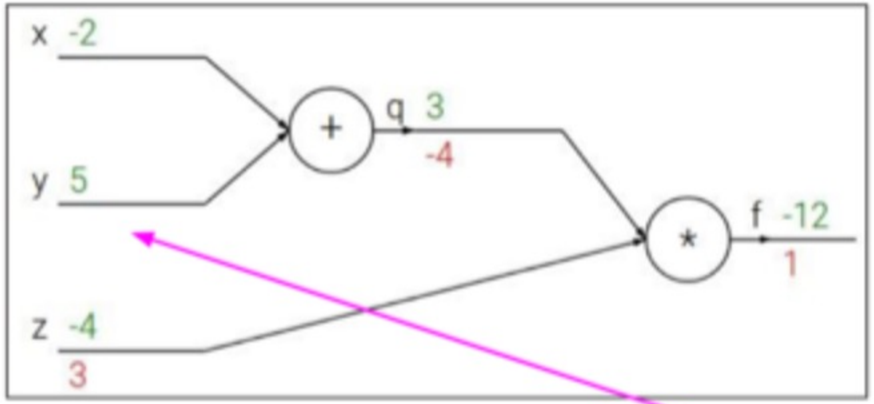

$\frac{\partial f}{\partial y}$

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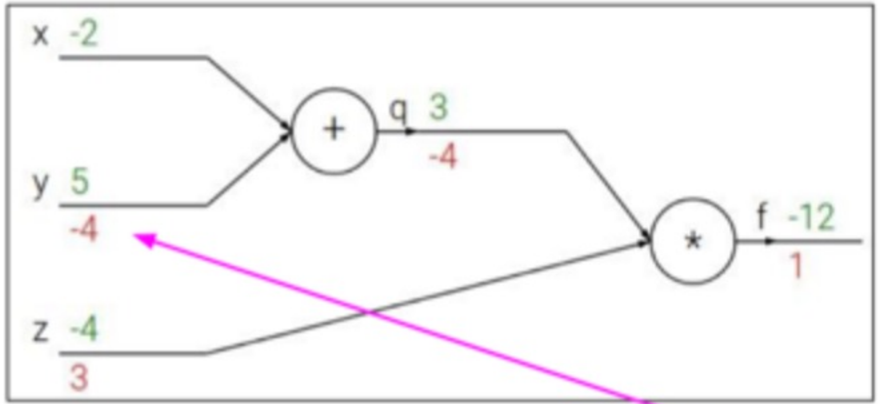

$$\frac{\partial f}{\partial y}$$

Chain rule:

$$\boxed{\frac{\partial f}{\partial y} = \frac{\partial f}{\partial q}\frac{\partial q}{\partial y}}$$

Upstream Gradient

Local Gradient

Example: Back-Prop

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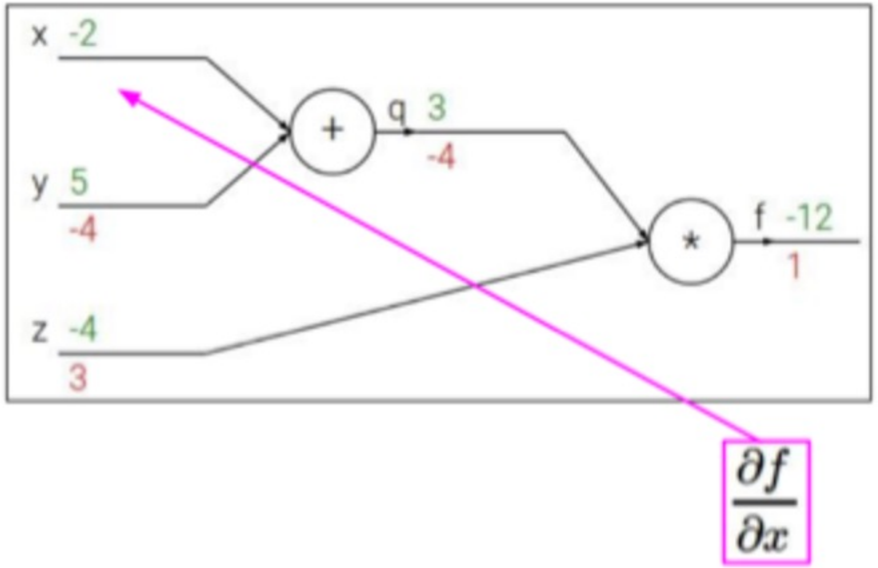

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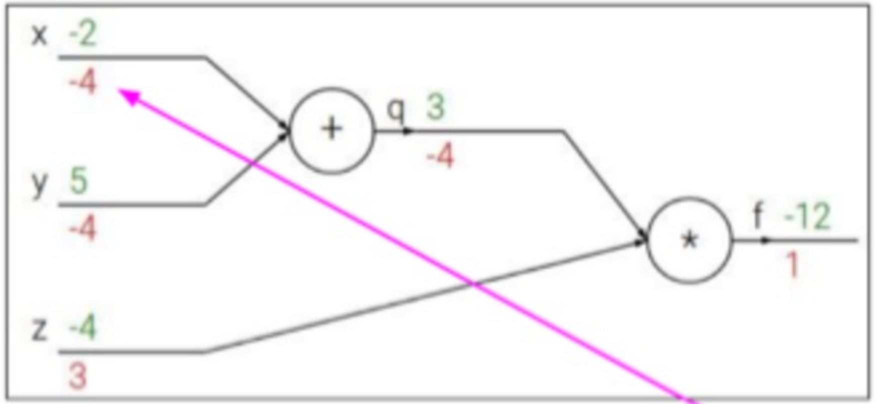

$$\frac{\partial f}{\partial x}$$

Chain rule:

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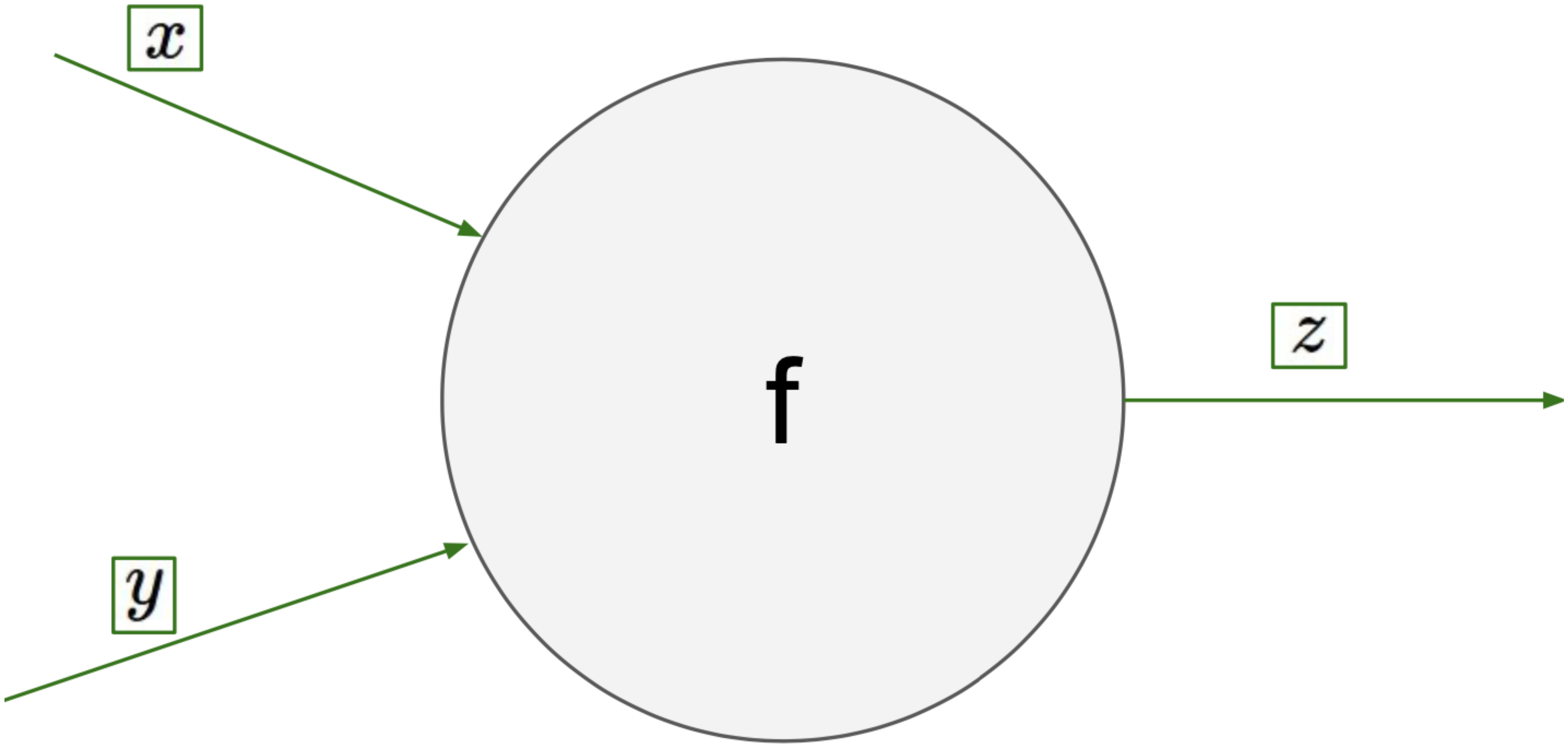
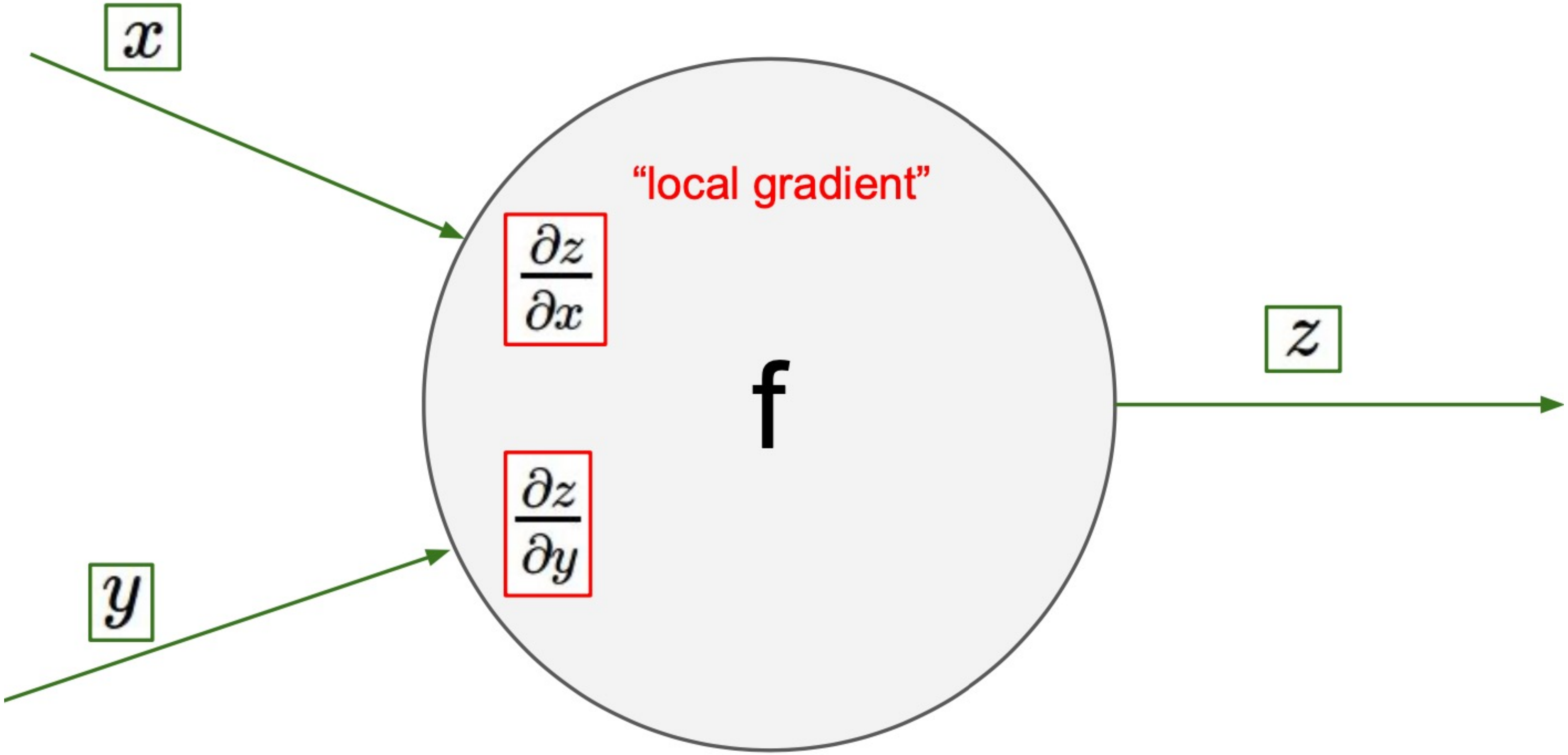
Upstream Gradient

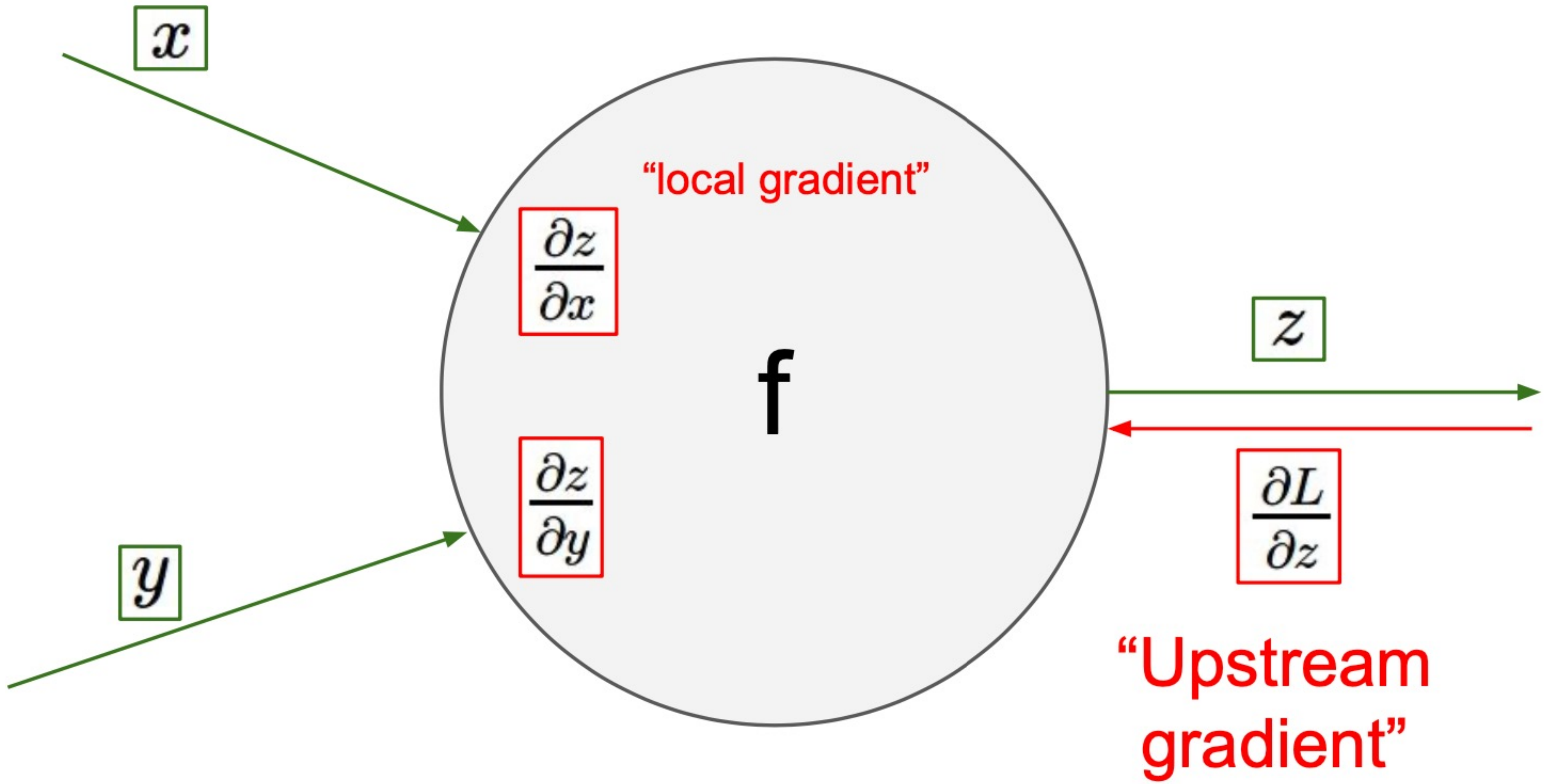
Local Gradient

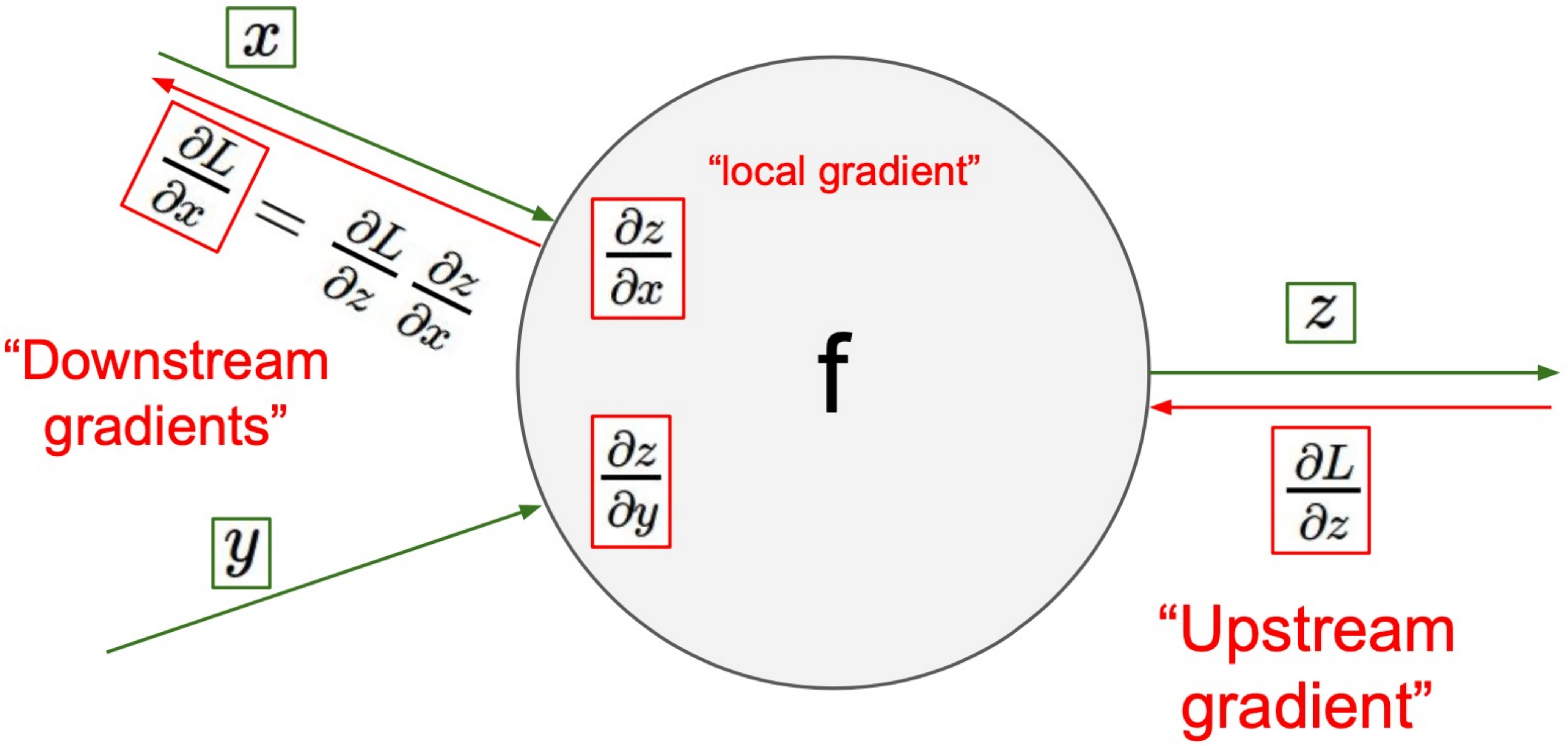


x
y
f
z

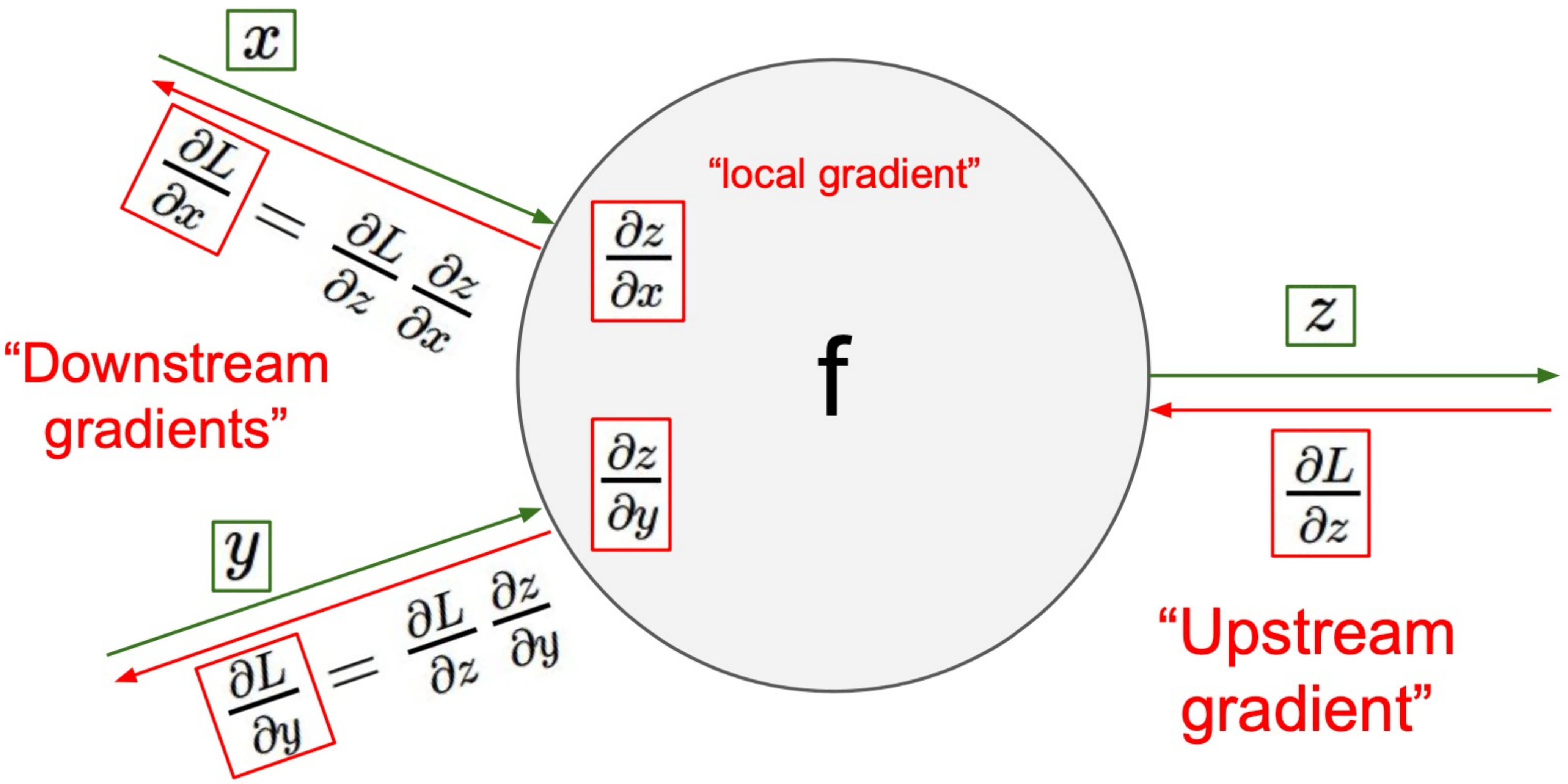


x
"local gradient"
$\frac{\partial z}{\partial x}$
f
z
$\frac{\partial z}{\partial y}$
y


x
y
"local gradient"
$\frac{\partial z}{\partial x}$
$\frac{\partial z}{\partial y}$
f
z
$\frac{\partial L}{\partial z}$
"Upstream gradient"



x
$\frac{\partial L}{\partial x} = \frac{\partial L}{\partial z} \frac{\partial z}{\partial x}$
"Downstream gradients"
y
"local gradient"
$\frac{\partial z}{\partial x}$
$\frac{\partial z}{\partial y}$
f
z
$\frac{\partial L}{\partial z}$
"Upstream gradient"



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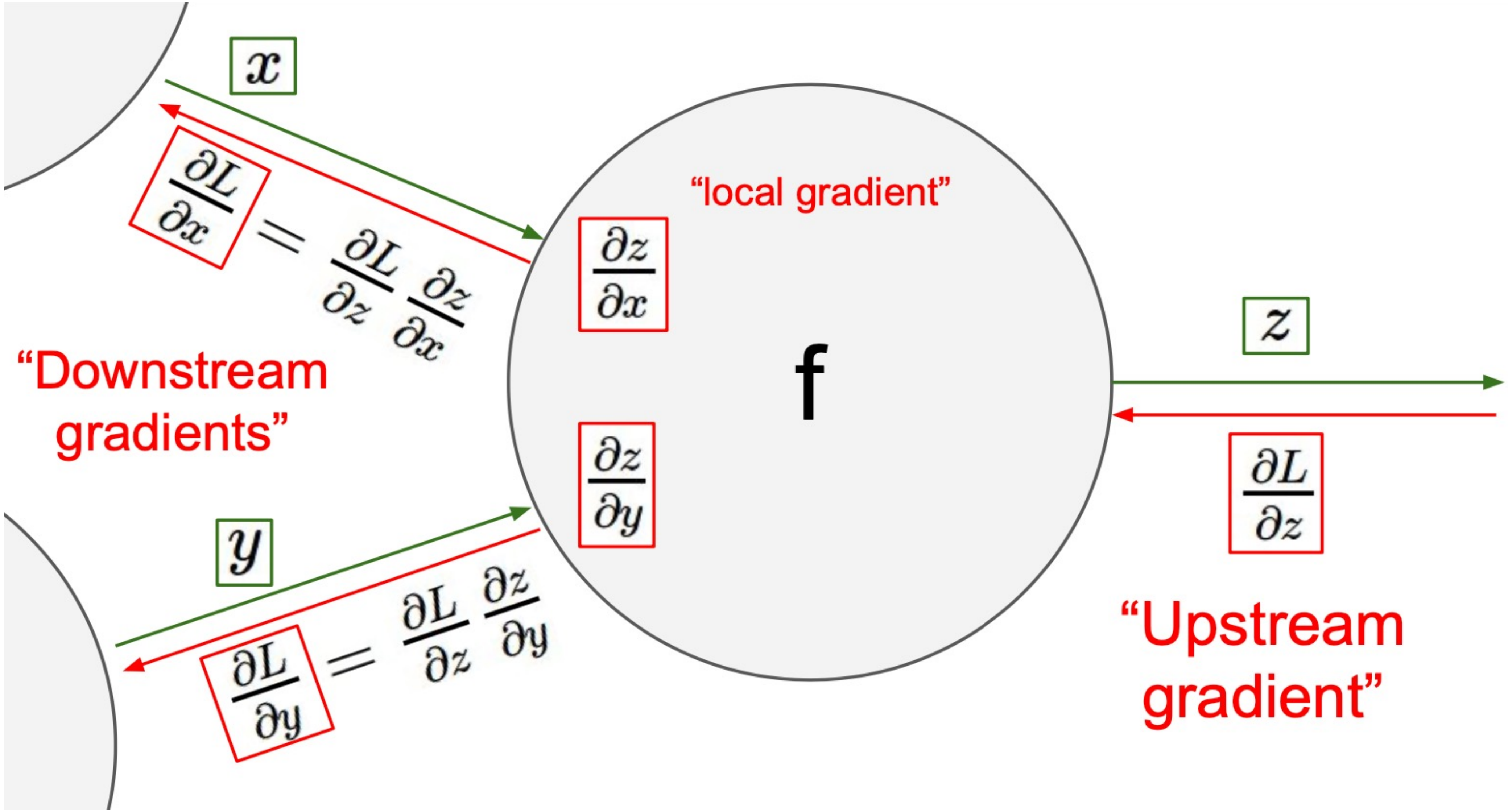

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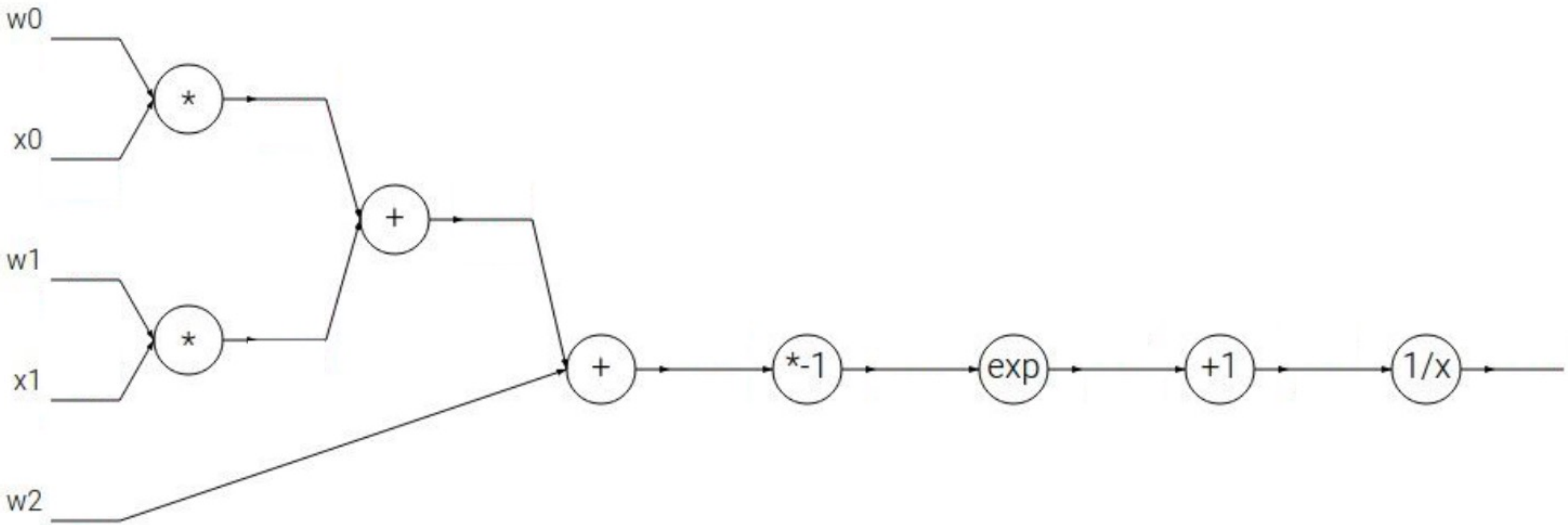
$\frac{\partial z}{\partial y}$
$\frac{\partial L}{\partial z}$
y
$\frac{\partial L}{\partial y} = \frac{\partial L}{\partial z}\frac{\partial z}{\partial y}$
"Upstream gradient"

Another example: $$f(w,x) = \frac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$$



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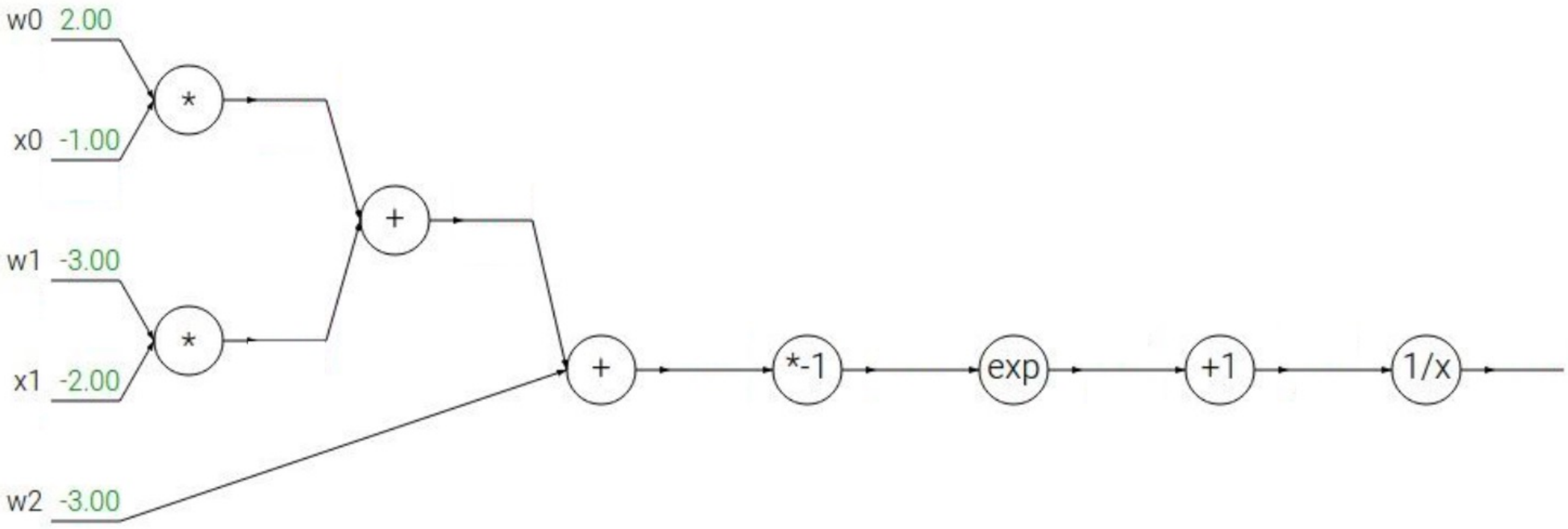

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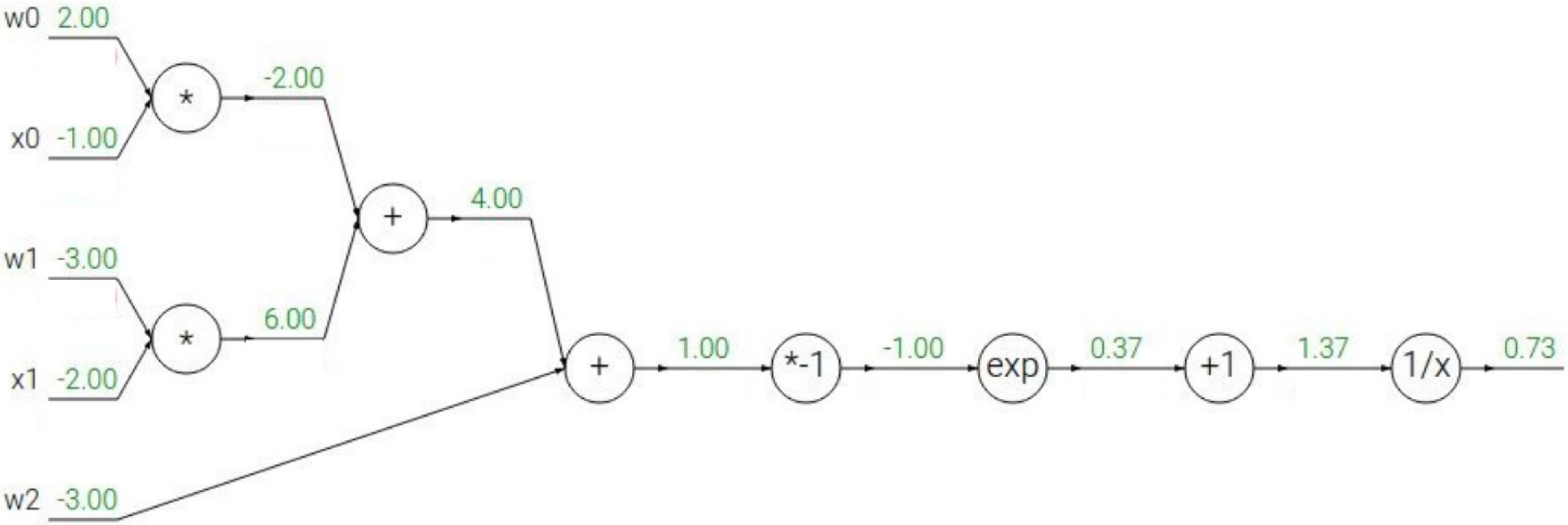
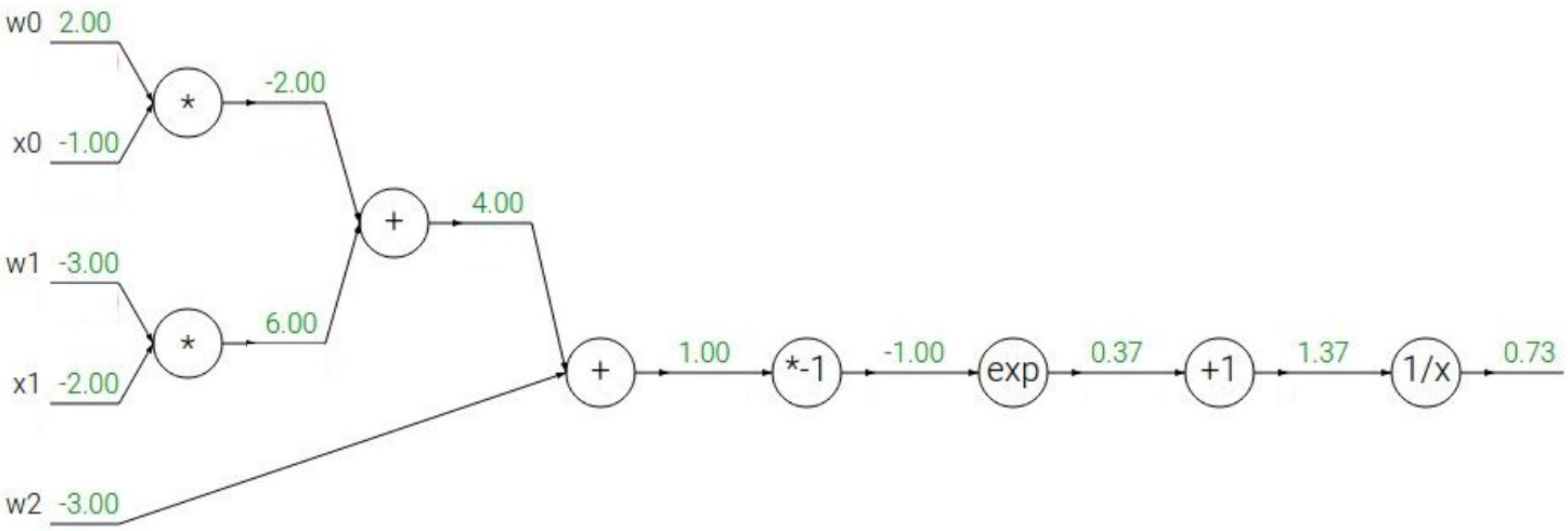


Another example: $f(w,x) = \frac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$



$f(x) = e^x \quad \rightarrow \quad \frac{df}{dx} = e^x$	$f(x) = \frac{1}{x} \quad \rightarrow \quad \frac{df}{dx} = -1/x^2$
$f_a(x) = ax \quad \rightarrow \quad \frac{df}{dx} = a$	$f_c(x) = c + x \quad \rightarrow \quad \frac{df}{dx} = 1$

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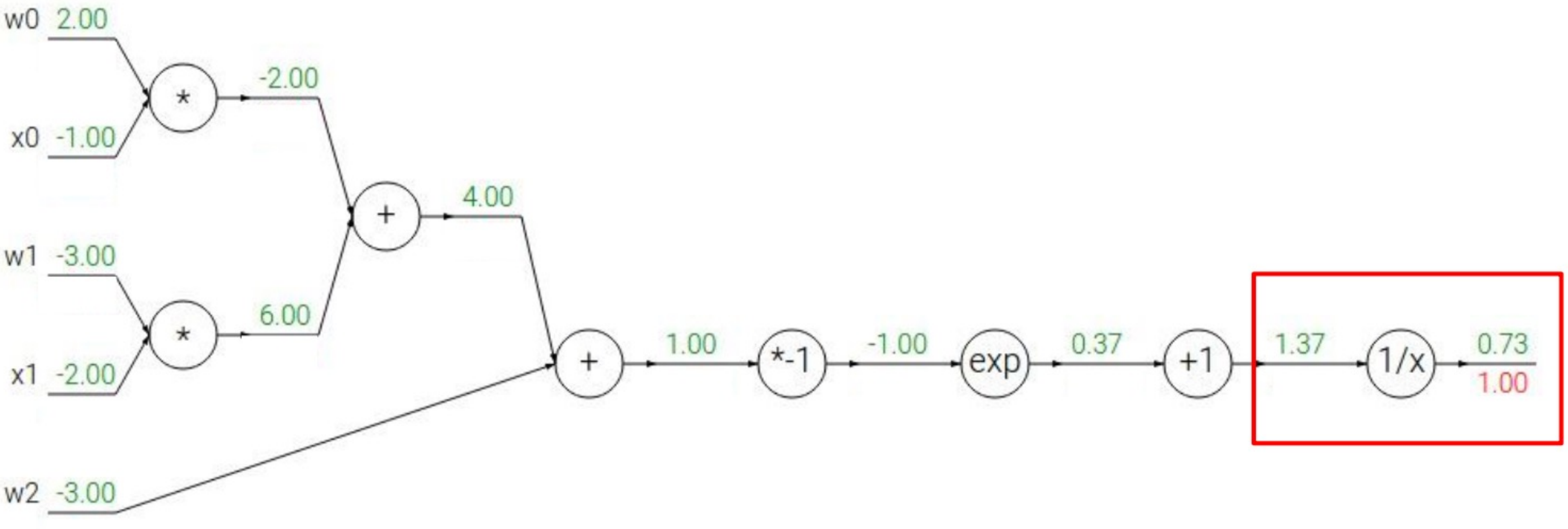

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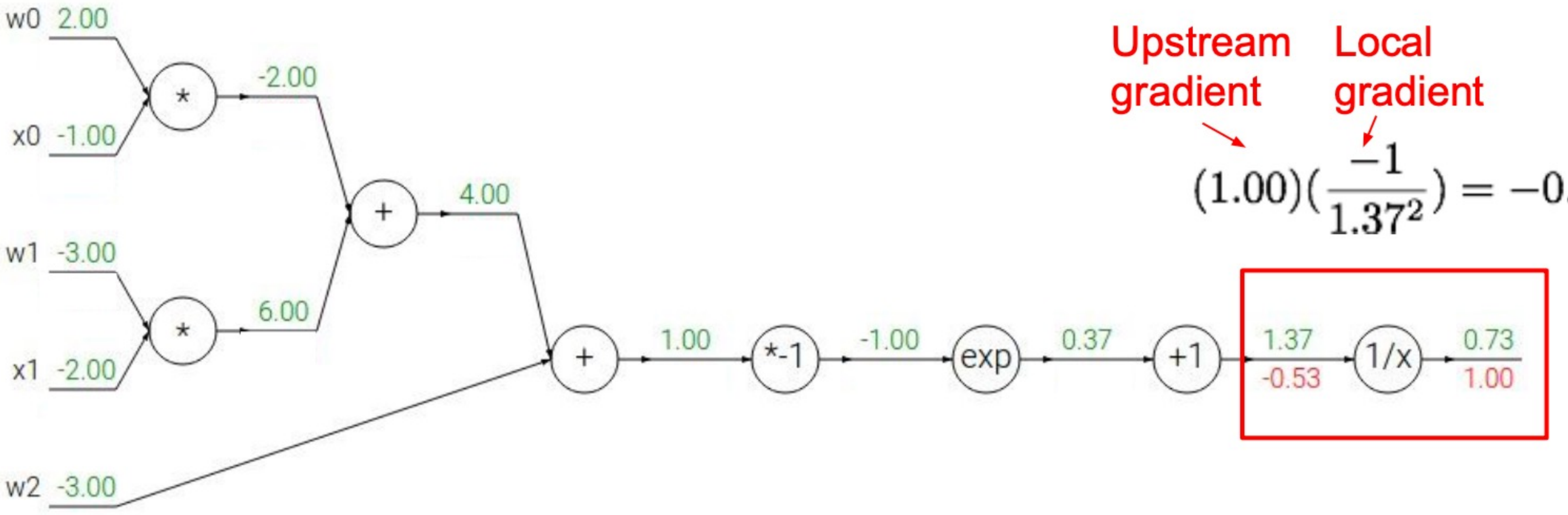

$$(1.00)\left(\frac{-1}{1.37^2}\right) = -0.53$$

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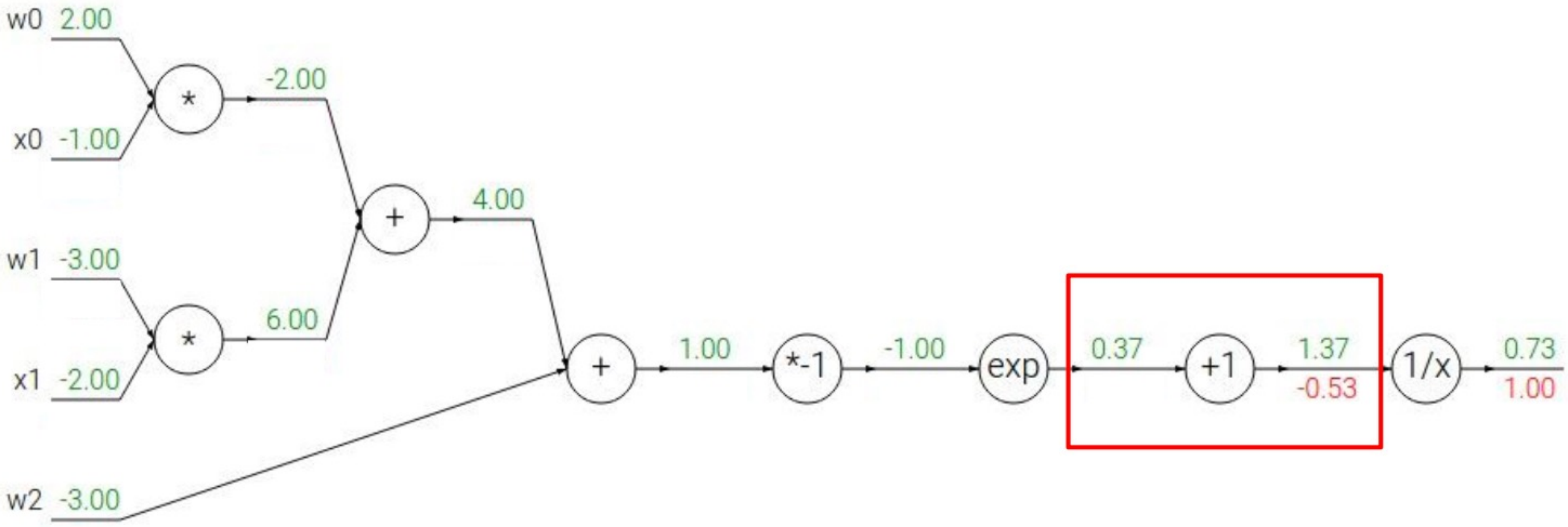

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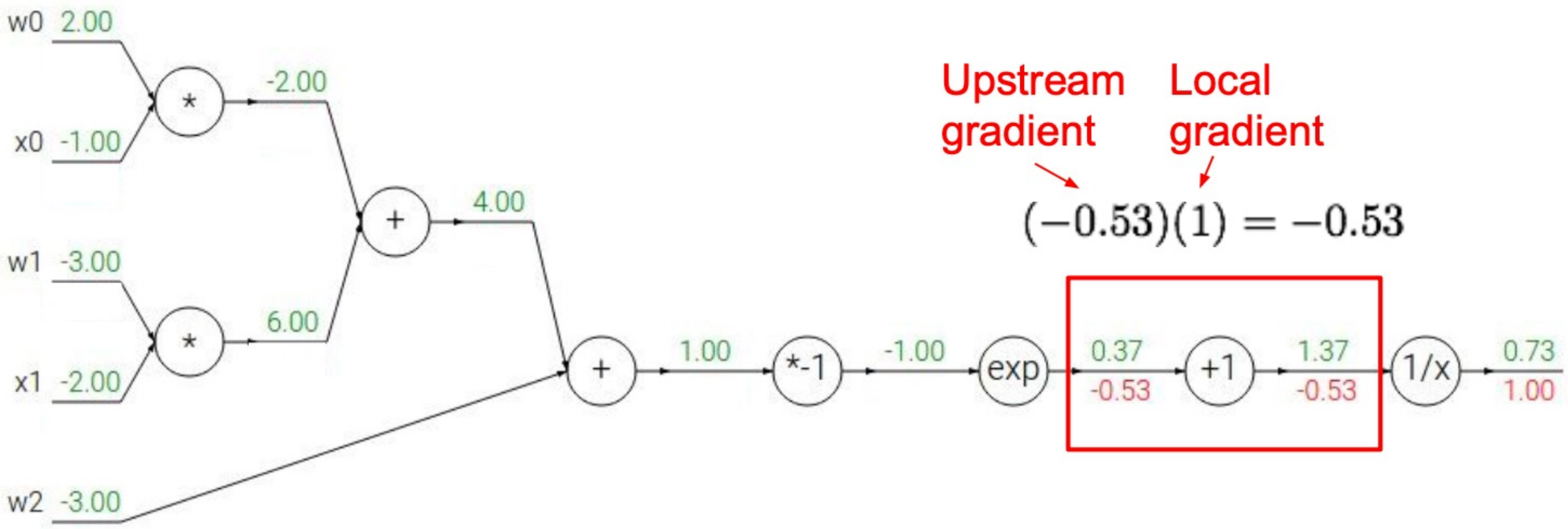

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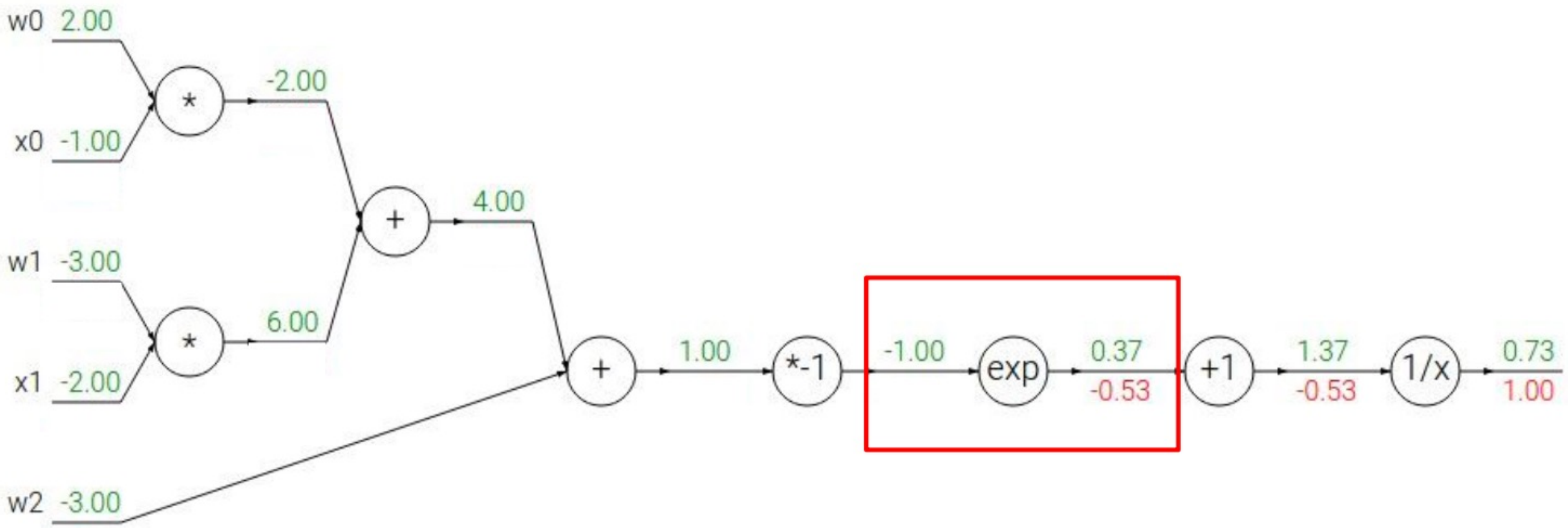

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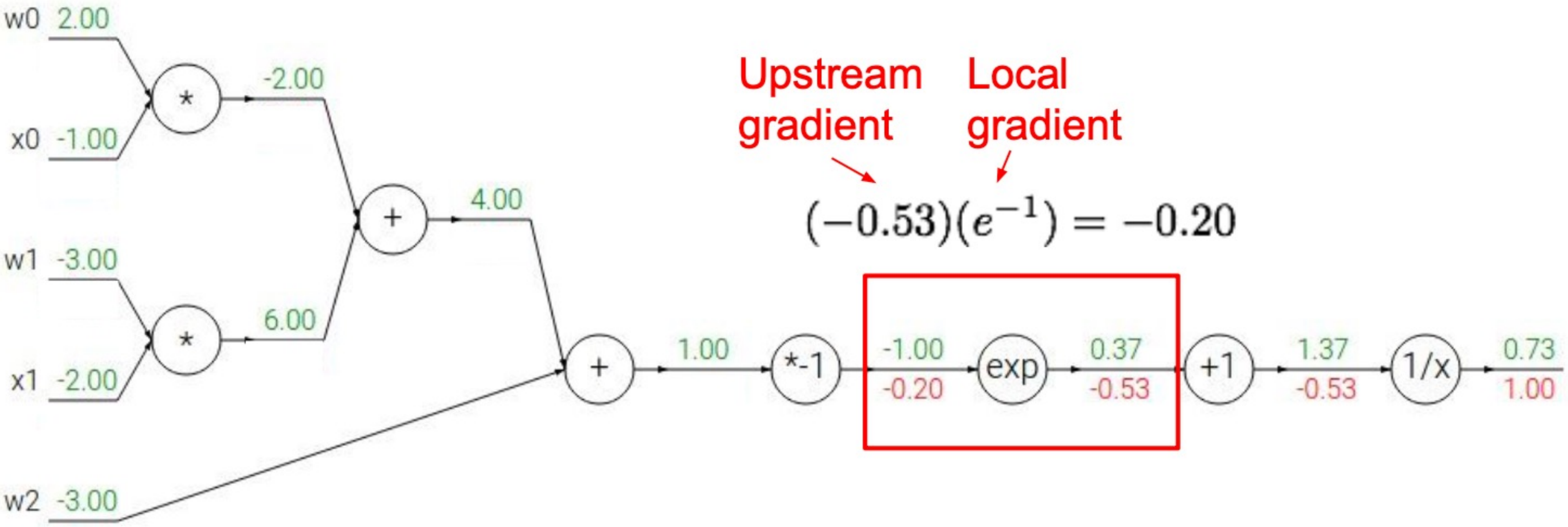

$$(-0.53)(e^{-1}) = -0.20$$

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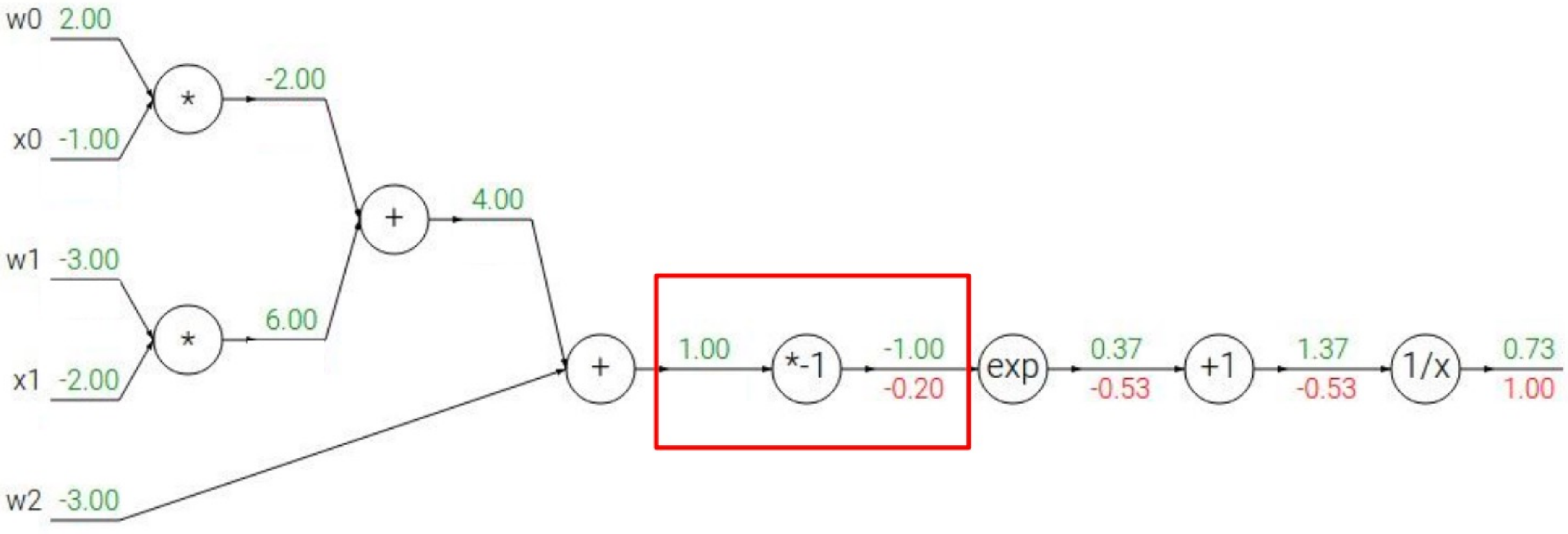

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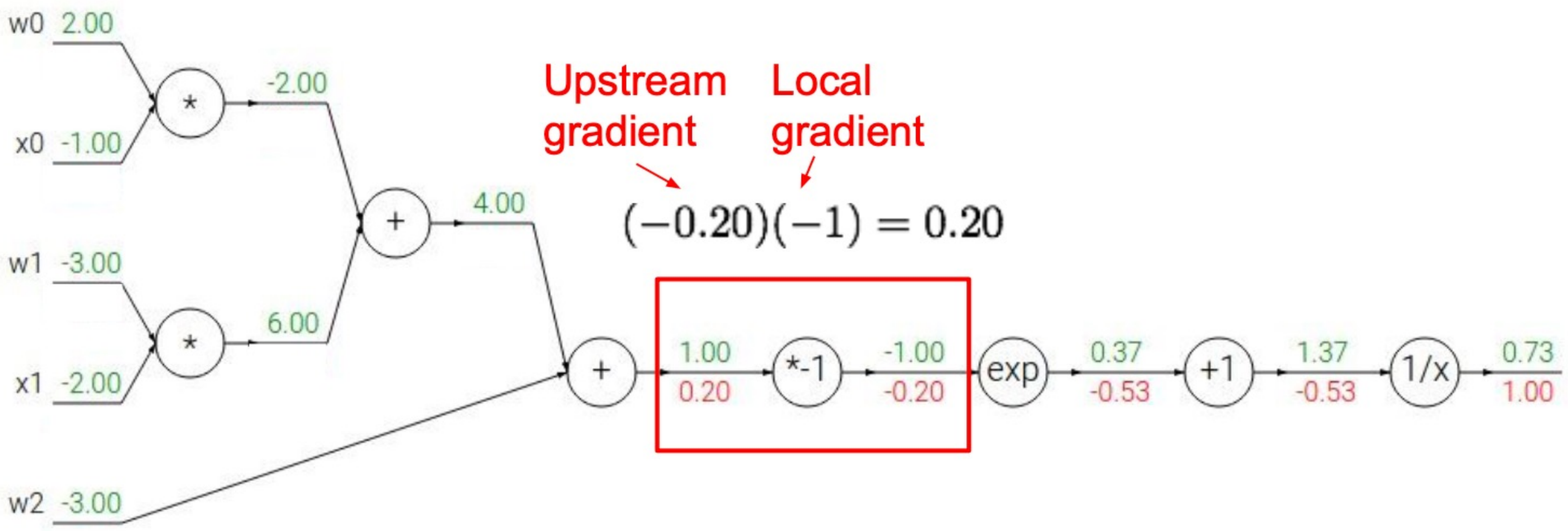

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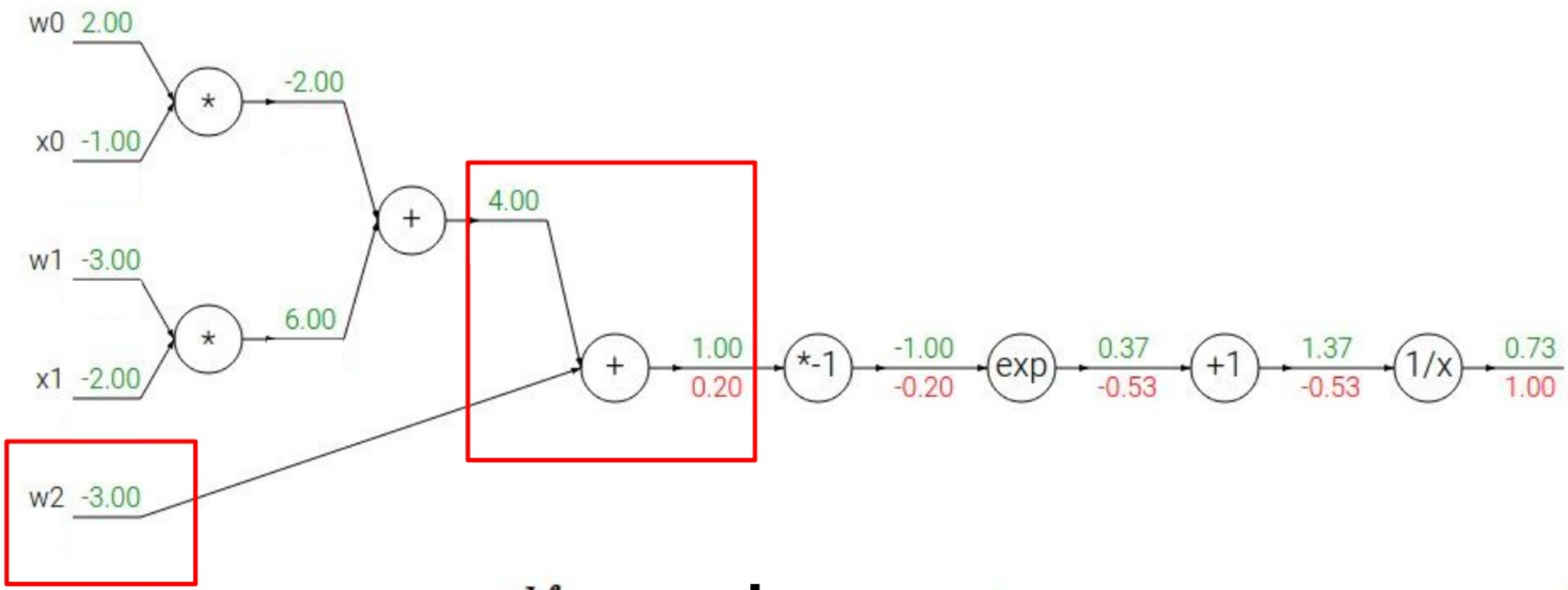


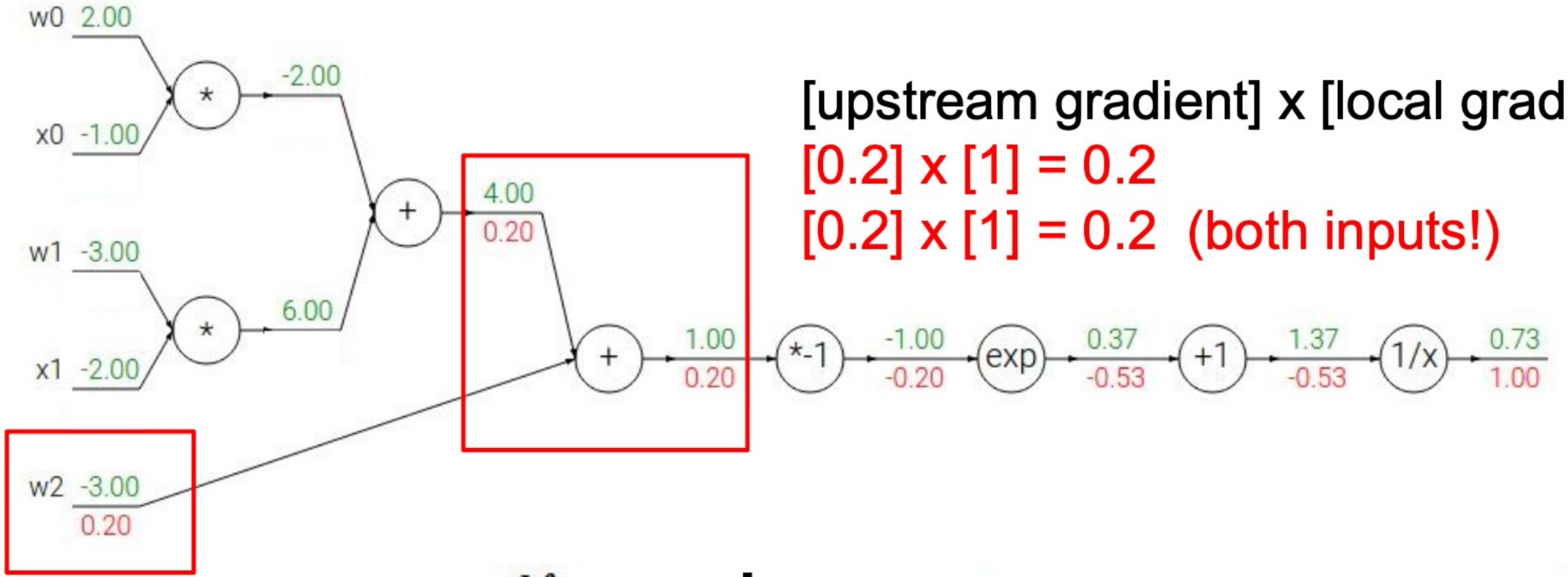
$$f(x) = e^x \quad \rightarrow \quad \frac{df}{dx} = e^x$$

$$f_a(x) = ax \quad \rightarrow \quad \frac{df}{dx} = a$$

$$f(x) = \frac{1}{x} \quad \rightarrow \quad \frac{df}{dx} = -1/x^2$$

$$f_c(x) = c + x \quad \rightarrow \quad \frac{df}{dx} = 1$$

Another example: $f(w,x) = \frac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$



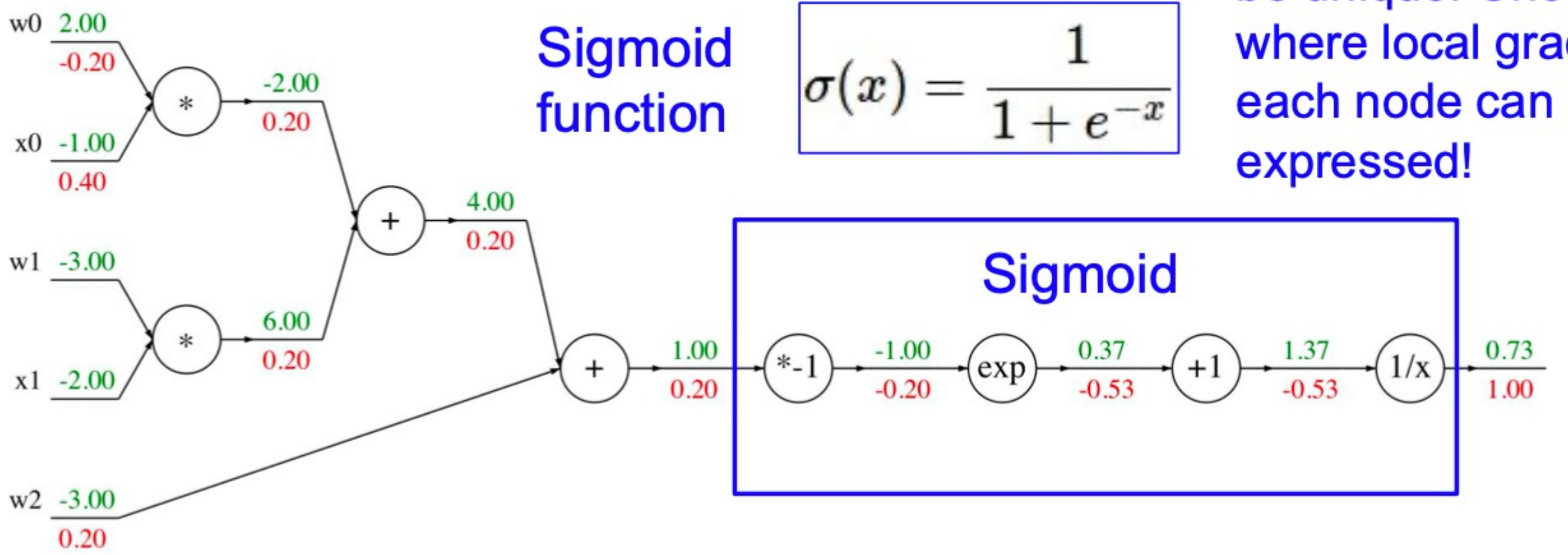
[upstream gradient] x [local gradient]
[0.2] x [1] = 0.2
[0.2] x [1] = 0.2 (both inputs!)

$f(x) = e^x \quad \rightarrow \quad \frac{df}{dx} = e^x$		$f(x) = \frac{1}{x} \quad \rightarrow \quad \frac{df}{dx} = -1/x^2$	
$f_a(x) = ax \quad \rightarrow \quad \frac{df}{dx} = a$		$f_c(x) = c + x \quad \rightarrow \quad \frac{df}{dx} = 1$	

Another example: $$f(w,x) = \frac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$$

Computational graph representation may not be unique. Choose one where local gradients at each node can be easily expressed!

Sigmoid function $$\boxed{\sigma(x) = \frac{1}{1+e^{-x}}}$$



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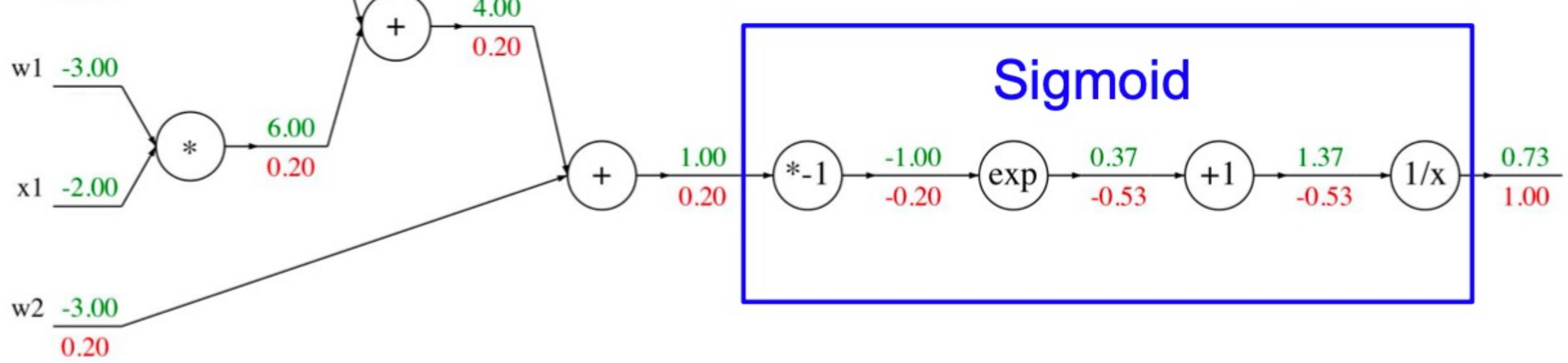


Sigmoid local gradient: $$\frac{d\sigma(x)}{dx} = \frac{e^{-x}}{(1+e^{-x})^2} = \left(\frac{1+e^{-x}-1}{1+e^{-x}}\right)\left(\frac{1}{1+e^{-x}}\right) = (1-\sigma(x))\,\sigma(x)$$

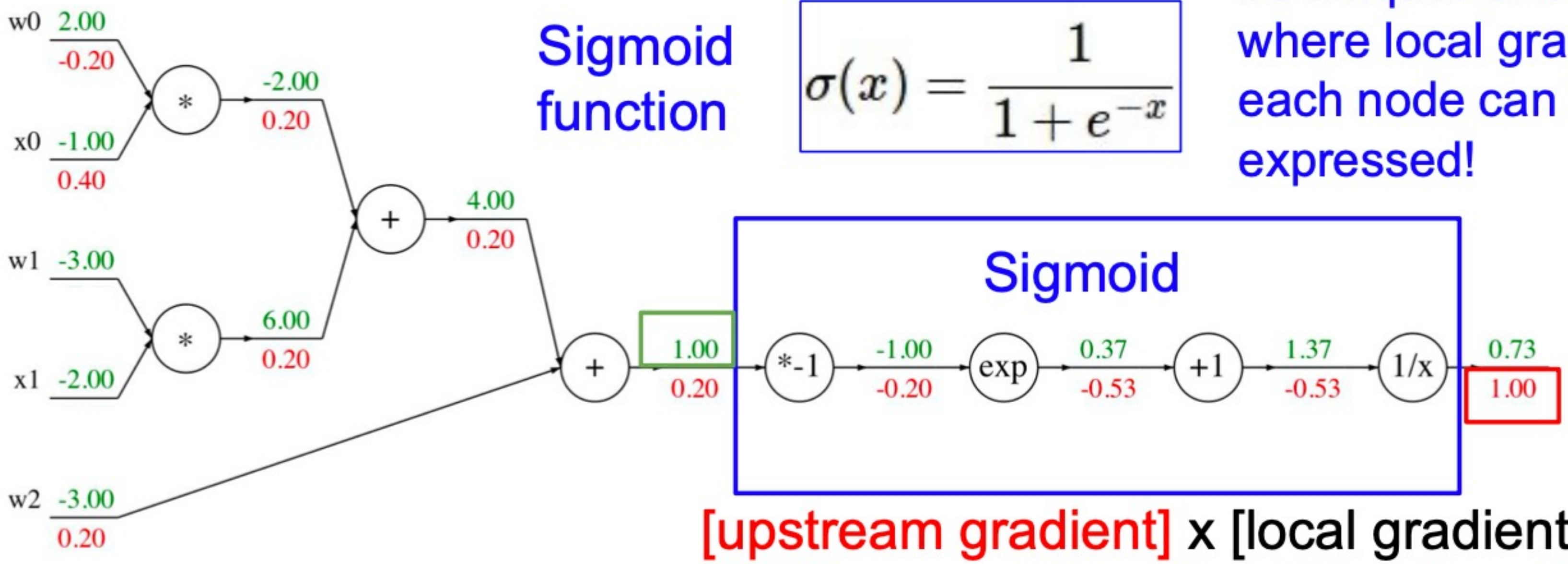
Another example:

$$f(w,x) = \frac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$$

Computational graph representation may not be unique. Choose one where local gradients at each node can be easily expressed!

Sigmoid function

$$\sigma(x) = \frac{1}{1+e^{-x}}$$



[upstream gradient] x [local gradient]

[1.00] x [(1 - 1/(1+e^1)) (1/(1+e^1))] = 0.2

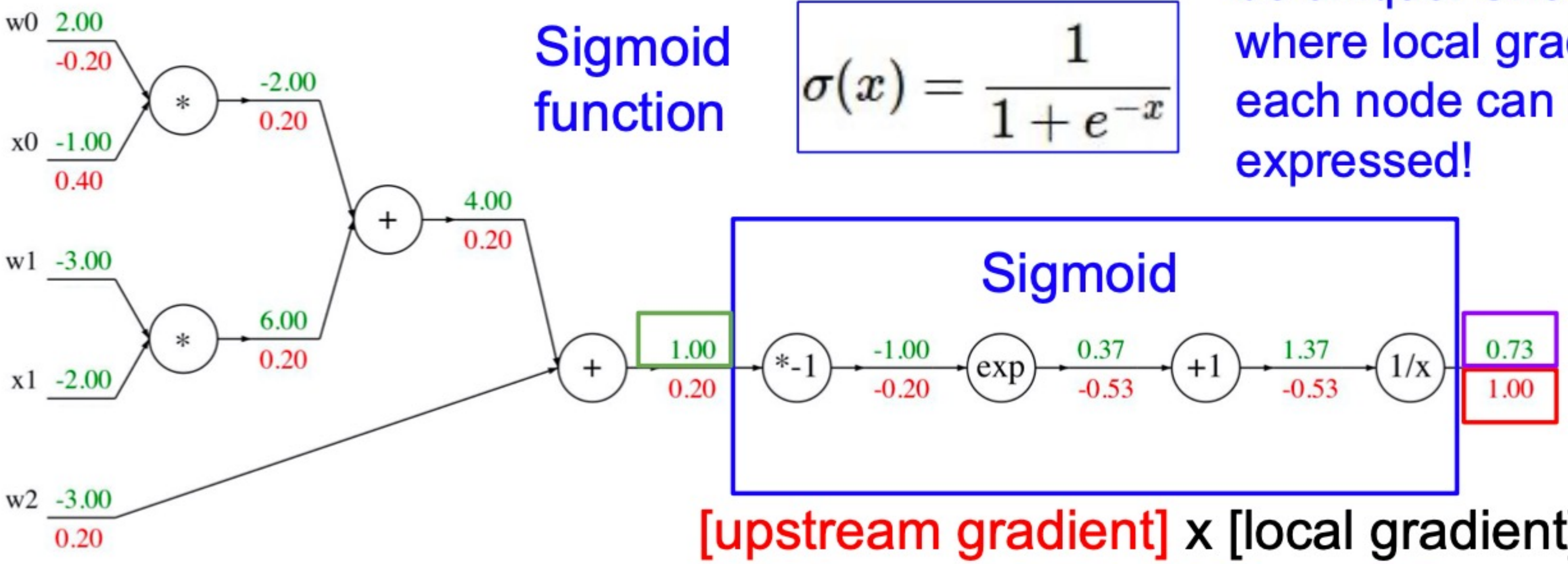
Sigmoid local gradient:

$$\frac{d\sigma(x)}{dx} = \frac{e^{-x}}{(1+e^{-x})^2} = \left(\frac{1+e^{-x}-1}{1+e^{-x}}\right)\left(\frac{1}{1+e^{-x}}\right) = (1-\sigma(x))\,\sigma(x)$$

Another example: $f(w,x) = \frac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$

Computational graph representation may not be unique. Choose one where local gradients at each node can be easily expressed!

Sigmoid function $\boxed{\sigma(x) = \frac{1}{1+e^{-x}}}$



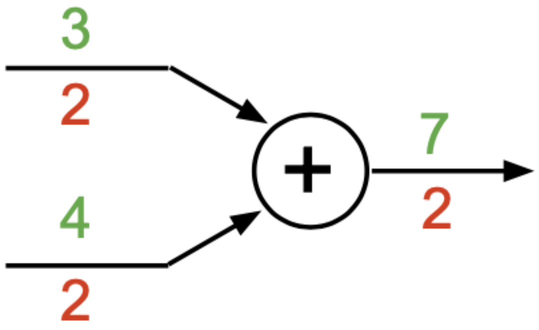
[upstream gradient] x [local gradient]
[1.00] x [(1 - 0.73) (0.73)] = 0.2

Sigmoid local gradient:

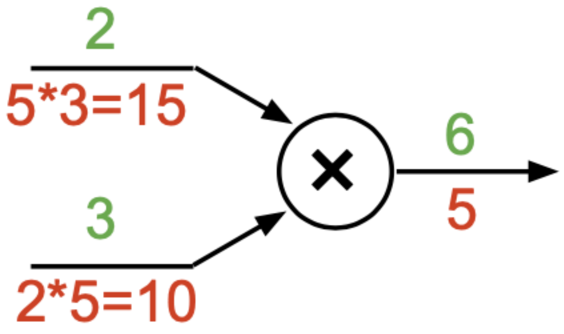
$$\frac{d\sigma(x)}{dx} = \frac{e^{-x}}{(1+e^{-x})^2} = \left(\frac{1+e^{-x}-1}{1+e^{-x}}\right)\left(\frac{1}{1+e^{-x}}\right) = (1-\sigma(x))\,\sigma(x)$$

Patterns in gradient flow

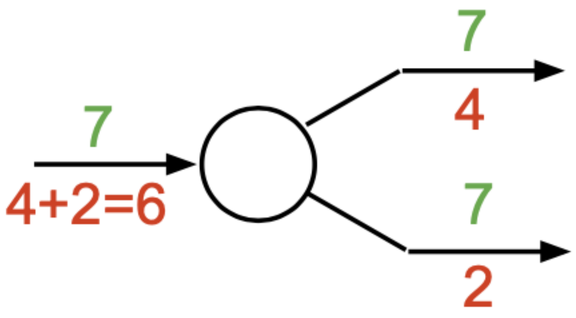
add gate: gradient distributor



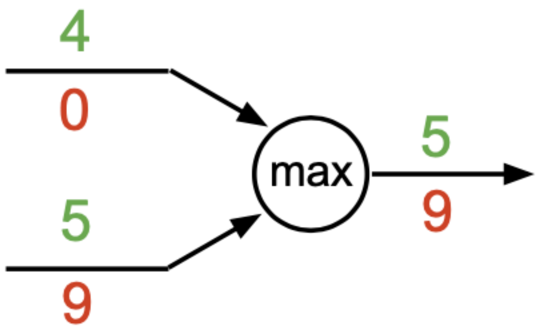
mul gate: “swap multiplier”



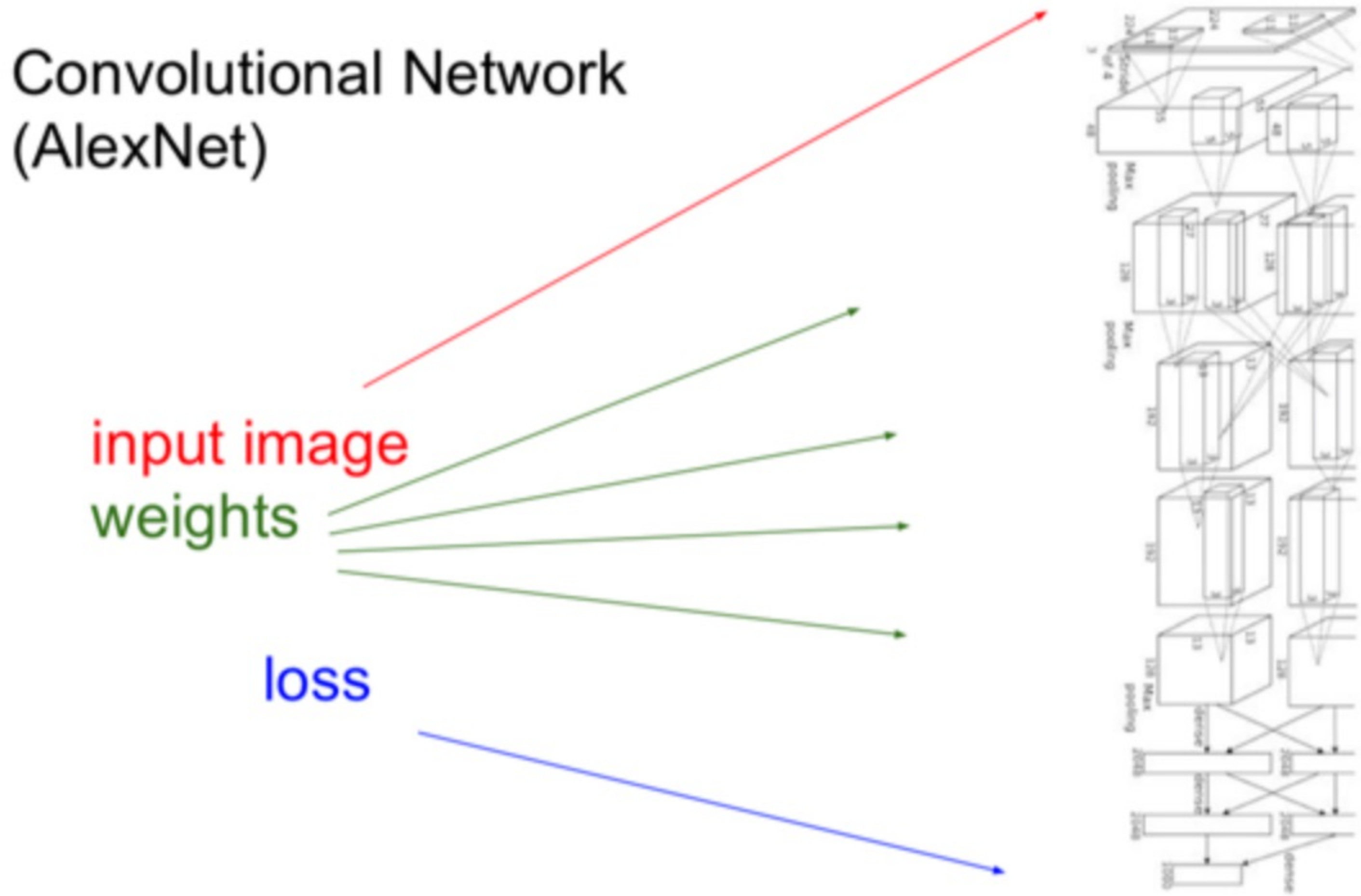
copy gate: gradient adder



max gate: gradient router



Can be applied to arbitrarily complex deep networks



Overall Steps of BP

After choosing the weights of the network randomly, the backpropagation algorithm is used to compute the necessary corrections. The algorithm can be decomposed in the following four steps:

i) Feed-forward computation

ii) Backpropagation to the output layer

iii) Backpropagation to the hidden layer

iv) Weight updates

The algorithm is stopped when the value of the error function has become sufficiently small.

Multilayer Neural Networks

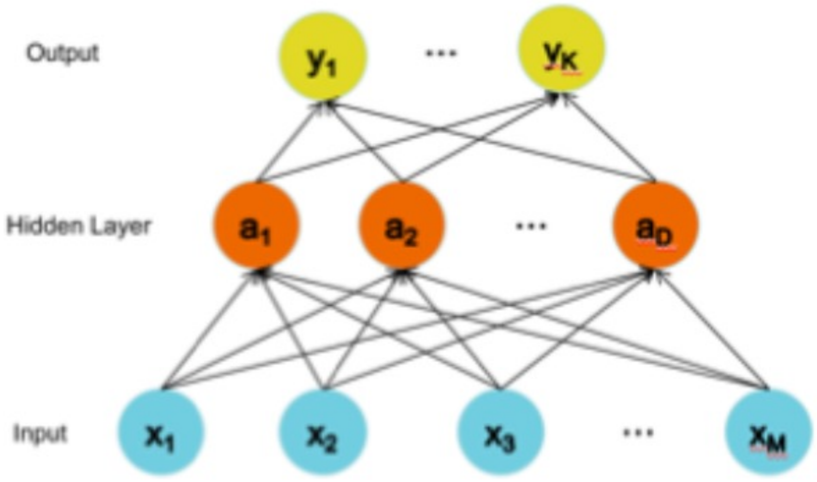
Network Design

Neural Network Architectures

- Even for a basic Multilayer Neural Network, there are many design decisions to make:

1) Number of hidden layers (depth of the network)
2) Number of neurons per hidden layer (width)
3) Type of activation functions
4) Form of objective functions

Example: Neural Network with sigmoid activation functions



(F) Loss
$J = \frac{1}{2}(y - y^*)^2$

(E) Output (sigmoid)
$y = \frac{1}{1+\exp(-b)}$

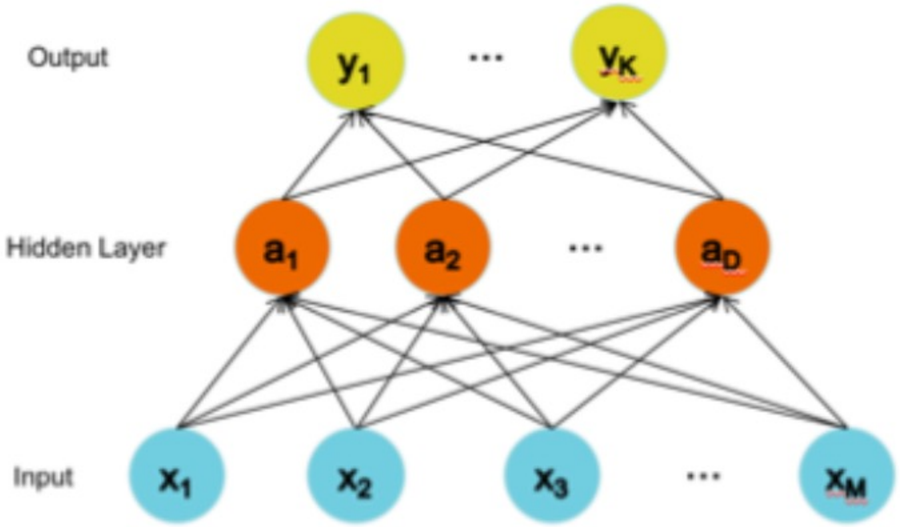
(D) Output (linear)
$b = \sum_{j=0}^{D} \beta_j z_j$

(C) Hidden (sigmoid)
$z_j = \frac{1}{1+\exp(-a_j)}, \ \forall j$

(B) Hidden (linear)
$a_j = \sum_{i=0}^{M} \alpha_{ji} x_i, \ \forall j$

(A) Input
Given $x_i, \ \forall i$

Neural Network with arbitrary nonlinear activation functions



(F) **Loss**
$J = \frac{1}{2}(y - y^*)^2$

(E) **Output (nonlinear)**
$y = \sigma(b)$

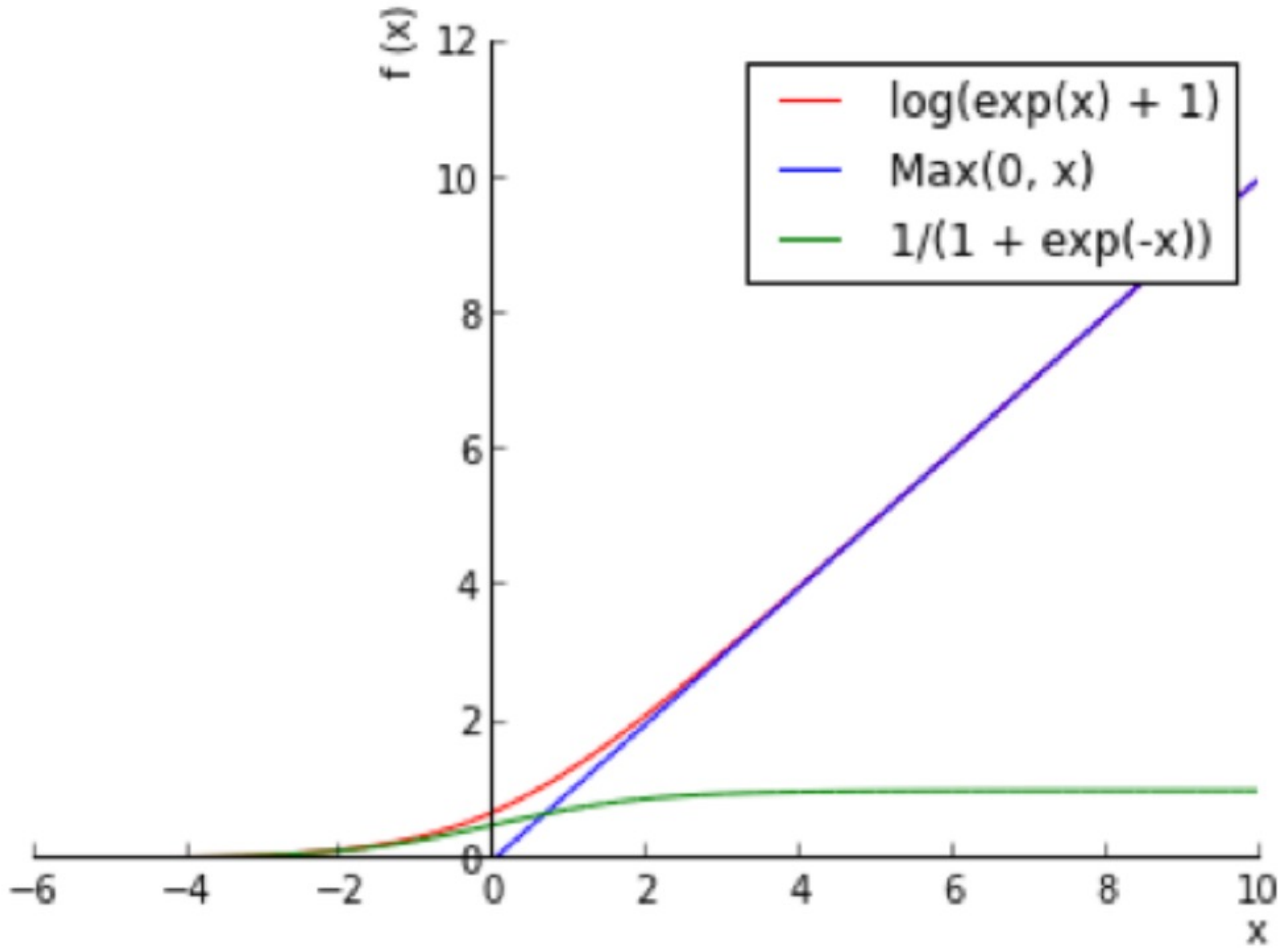
(D) **Output (linear)**
$b = \sum_{j=0}^{D} \beta_j z_j$

(C) **Hidden (nonlinear)**
$z_j = \sigma(a_j), \ \forall j$

(B) **Hidden (linear)**
$a_j = \sum_{i=0}^{M} \alpha_{ji} x_i, \ \forall j$

(A) **Input**
Given $x_i, \ \forall i$

ReLU often used in computer vision tasks



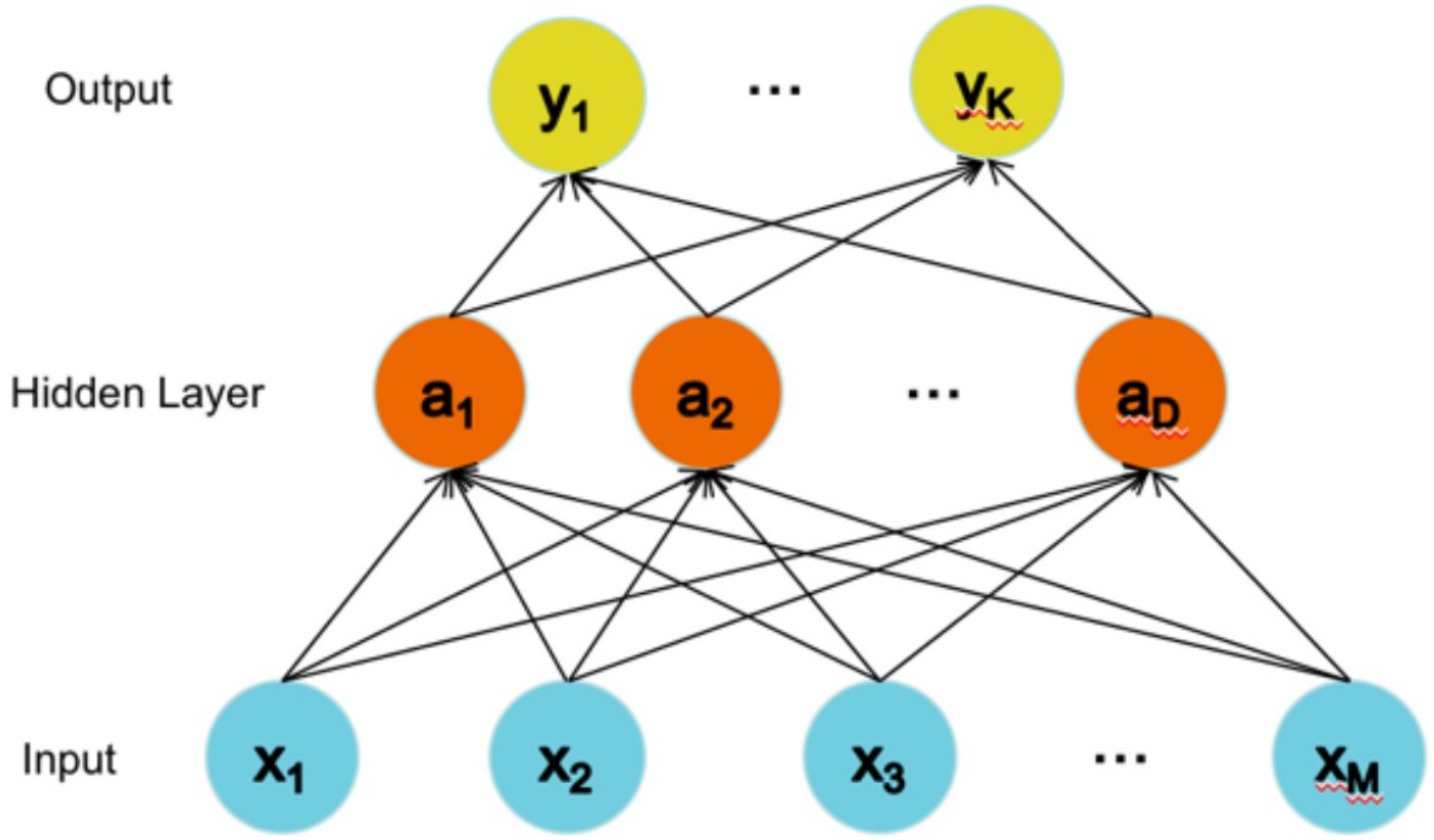
Loss Functions for NN

- **Regression:**
  - Use the same objective as Linear Regression
  - Quadratic loss (i.e. mean squared error)

- **Classification:**
  - Use the same objective as Logistic Regression
  - Cross-entropy (i.e. negative log likelihood)
  - This requires probabilities, so we add an additional "softmax" layer at the end of our network

	Forward	Backward
Quadratic	$J = \frac{1}{2}(y - y^*)^2$	$\frac{dJ}{dy} = y - y^*$
Cross Entropy	$J = y^* \log(y) + (1 - y^*) \log(1 - y)$	$\frac{dJ}{dy} = y^* \frac{1}{y} + (1 - y^*) \frac{1}{y - 1}$

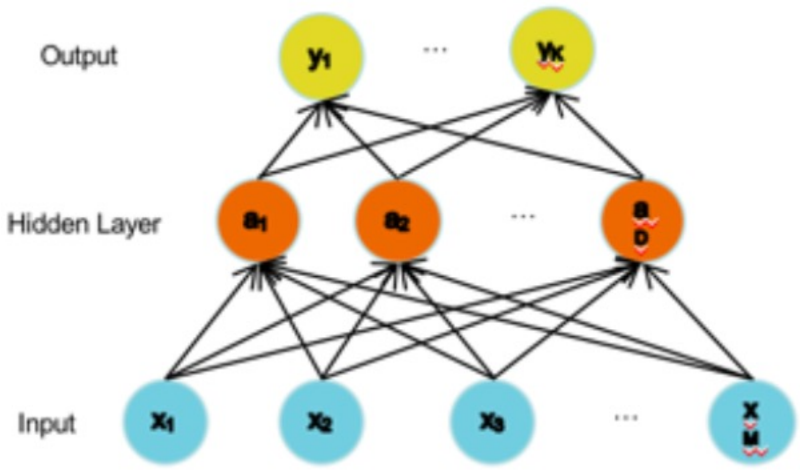
Multi-class Output

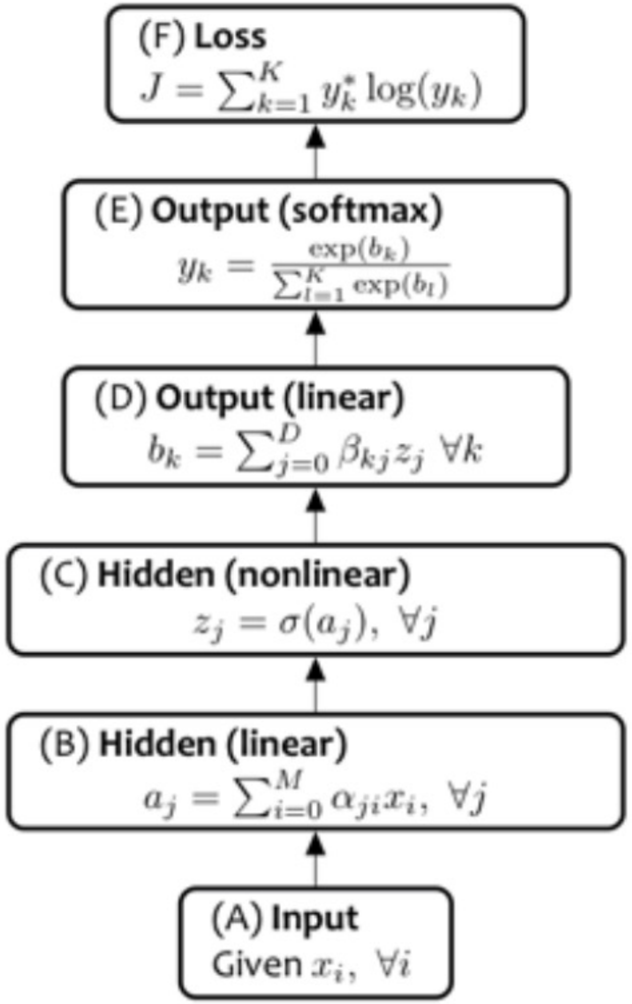


Multi-class Output

Softmax:

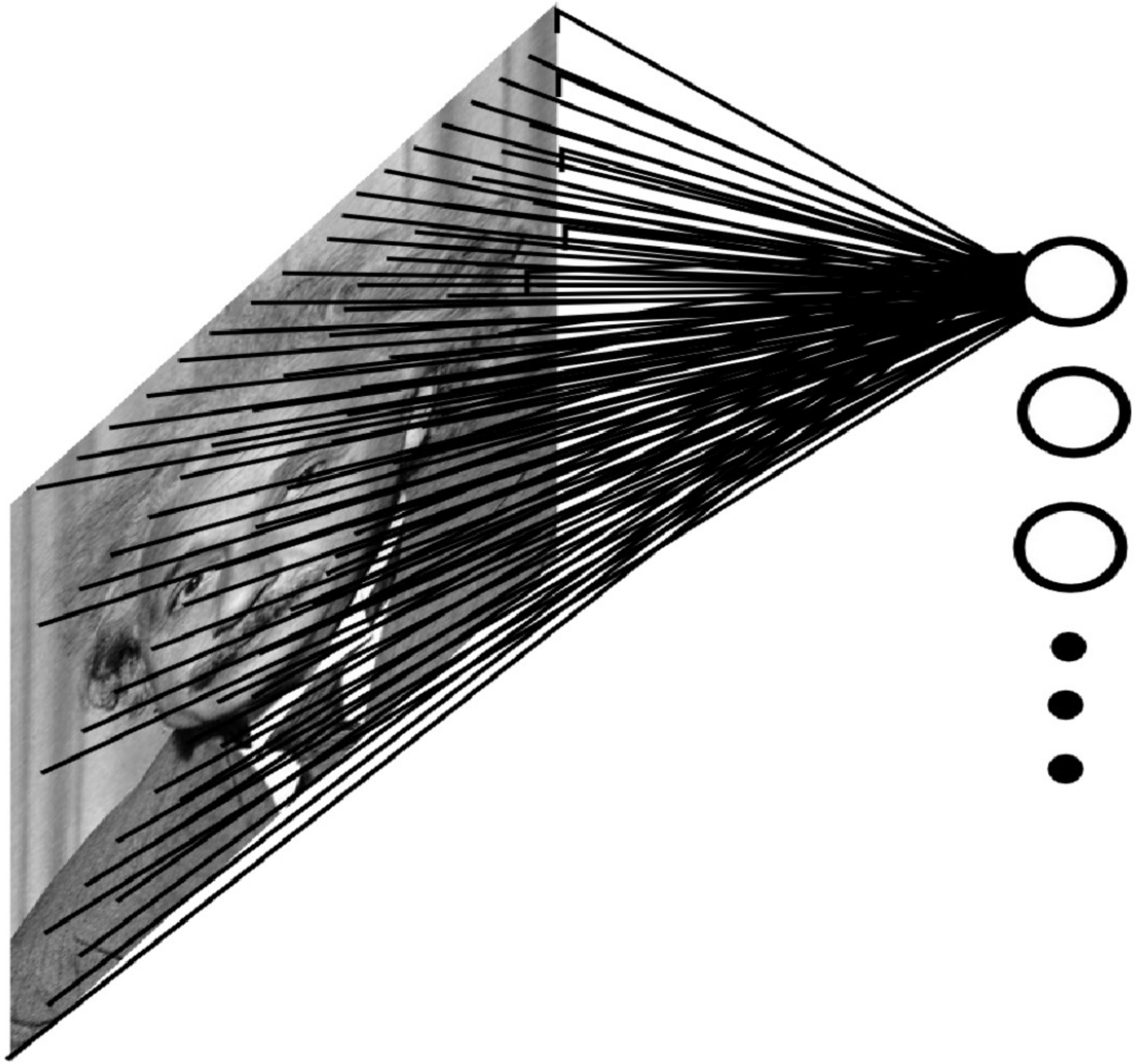
$$y_k = \frac{\exp(b_k)}{\sum_{l=1}^{K} \exp(b_l)}$$





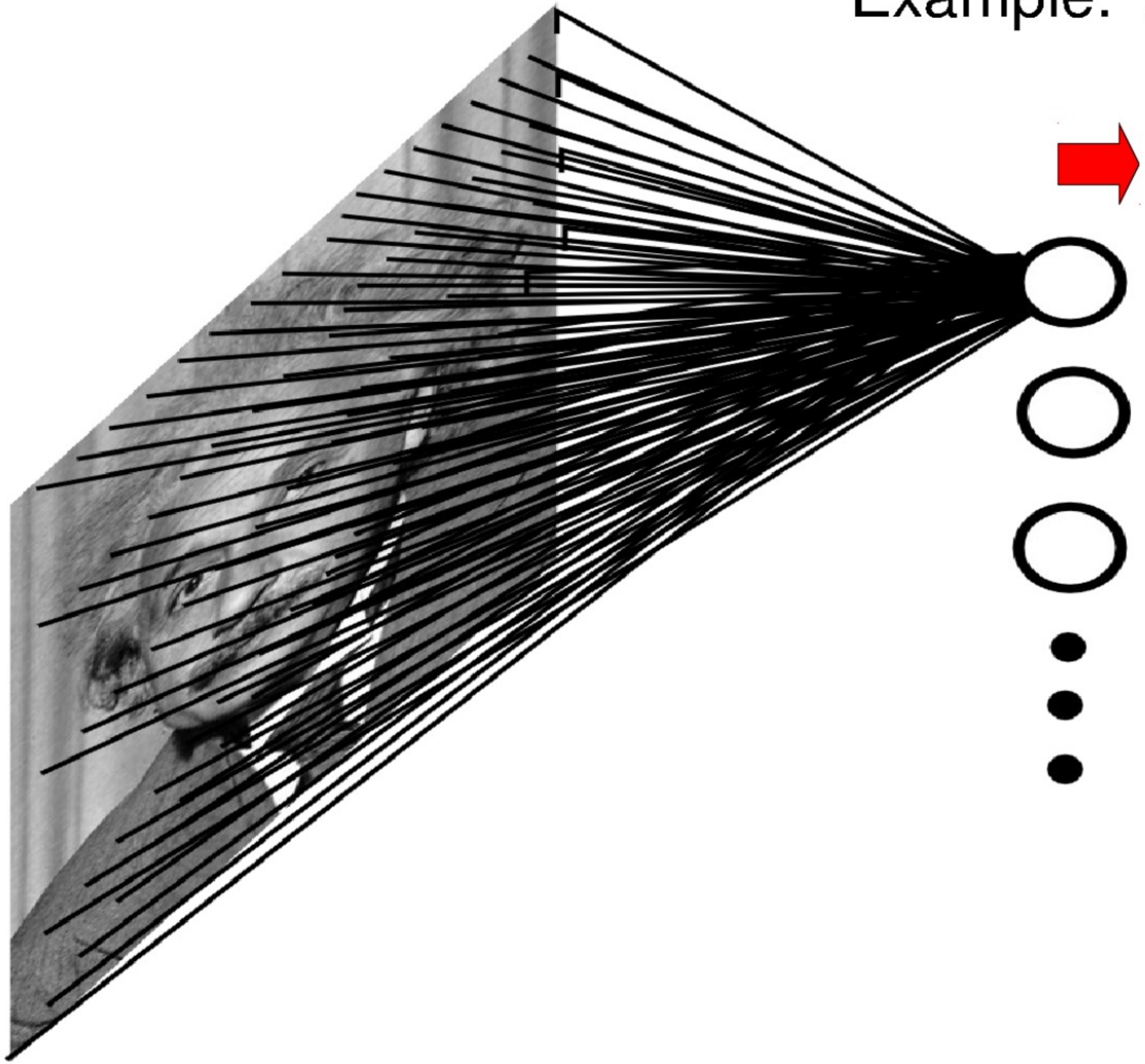
Convolutional Neural Network

Images as input to neural networks



Ranzato

Images as input to neural networks



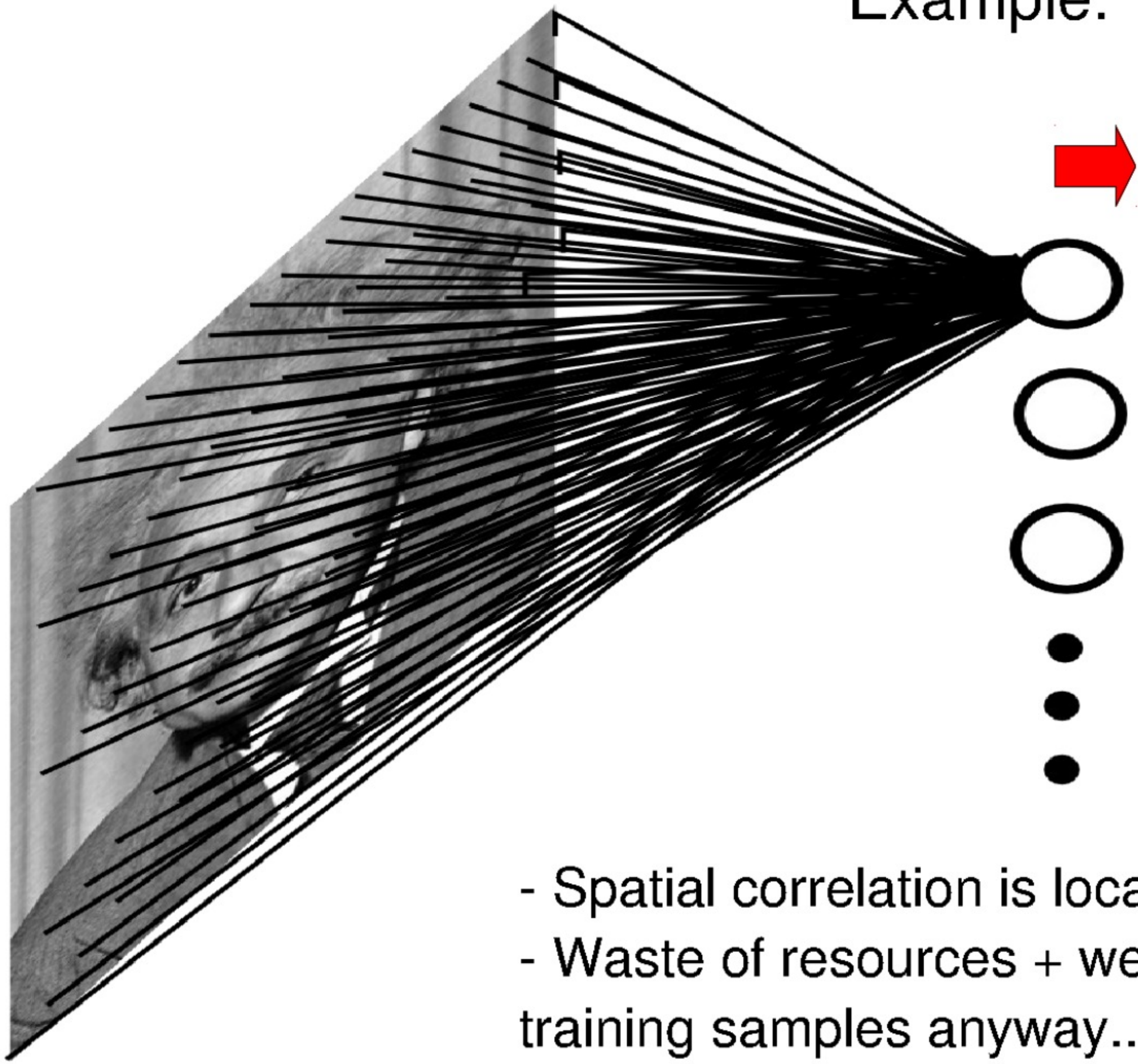
Example: 200x200 image

40K hidden units

~2B parameters!!!

Ranzato

Images as input to neural networks



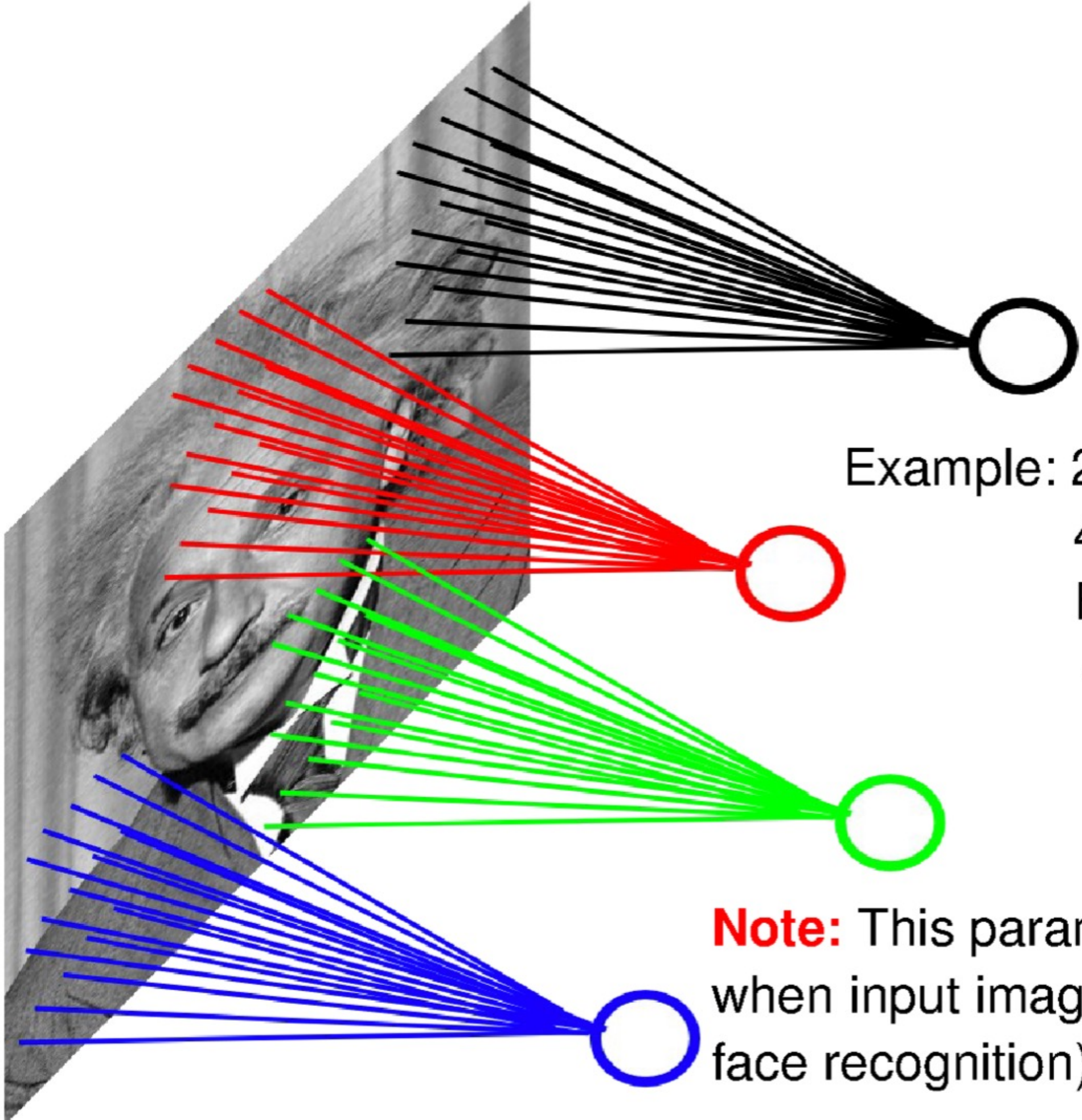
Example: 200x200 image
40K hidden units
~2B parameters!!!

- Spatial correlation is local
- Waste of resources + we have not enough training samples anyway..

Ranzato

Motivation

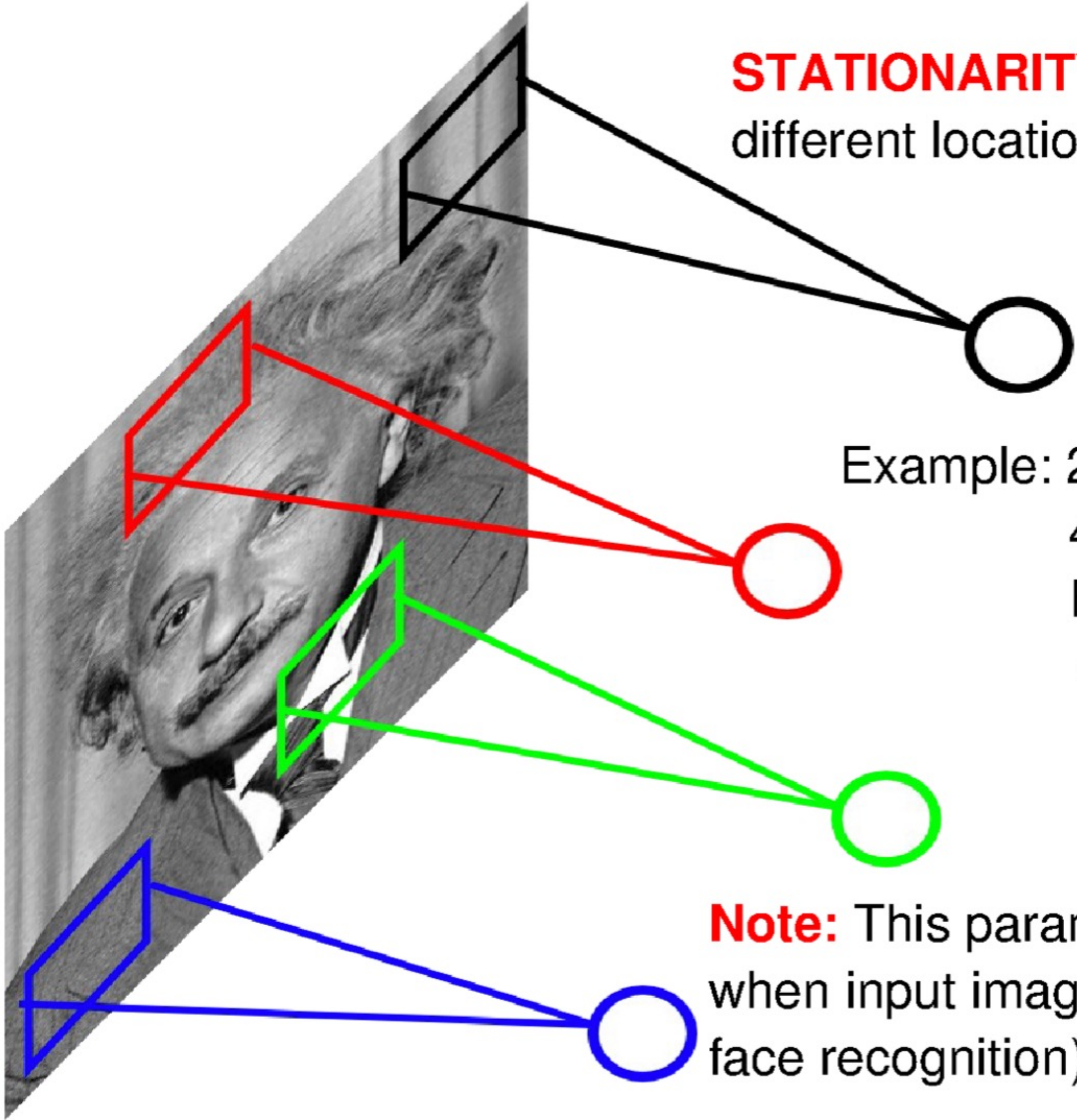
- Sparse interactions – *receptive fields*
  - Assume that in an image, we care about ‘local neighborhoods’ only for a given neural network layer.
  - Composition of layers will expand local -> global.



Example: 200x200 image
40K hidden units
Filter size: 10x10
4M parameters

Note: This parameterization is good when input image is registered (e.g., face recognition).

Ranzato



STATIONARITY? Statistics is similar at different locations

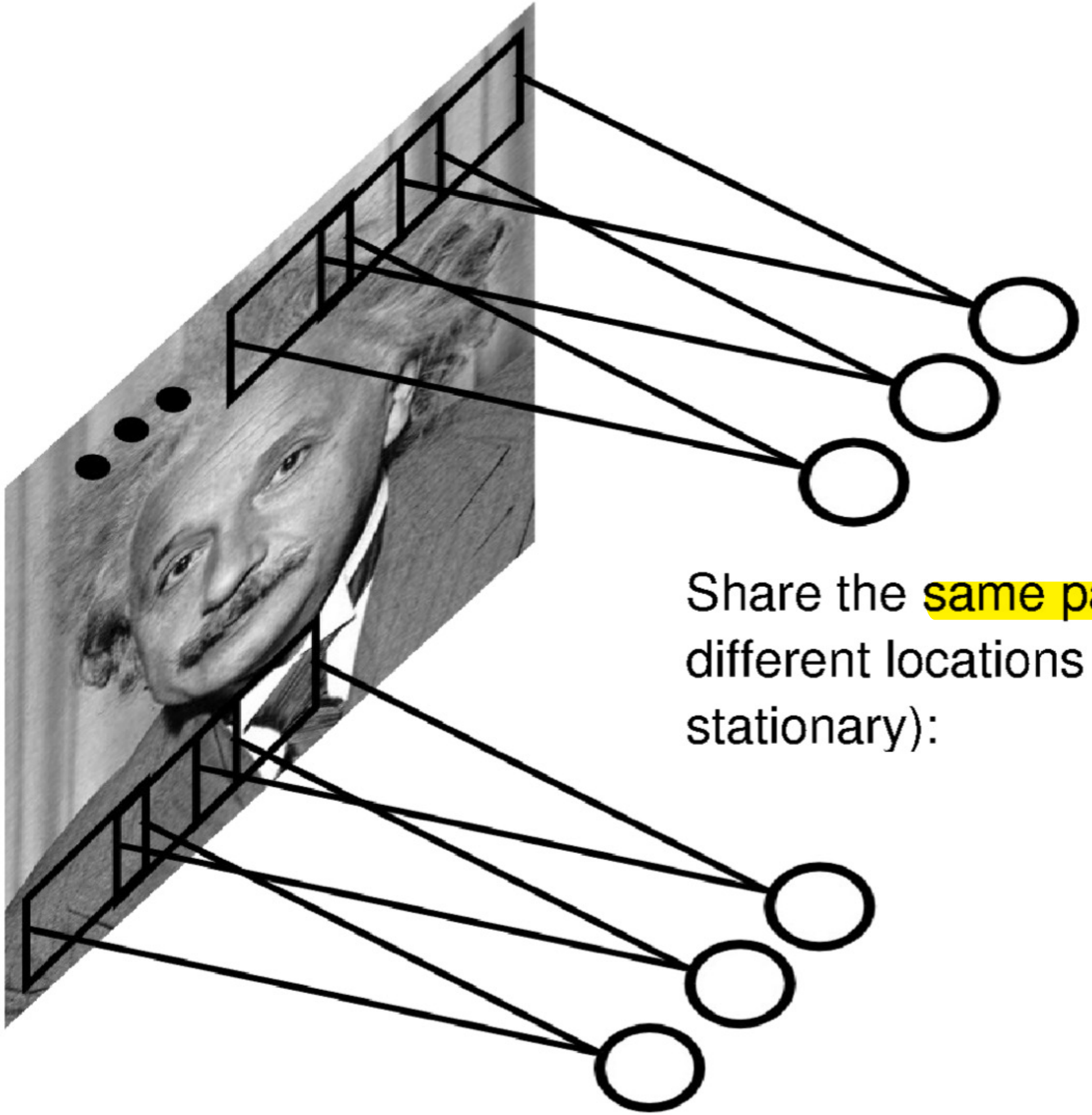
Example: 200x200 image
40K hidden units
Filter size: 10x10
4M parameters

Note: This parameterization is good when input image is registered (e.g., face recognition).

Motivation

- Sparse interactions – *receptive fields*
  - Assume that in an image, we care about 'local neighborhoods' only for a given neural network layer.
  - Composition of layers will expand local -> global.

- Parameter sharing
  - 'Tied weights' – use same weights for more than one perceptron in the neural network.
  - Leads to *equivariant representation*
    - If input changes (e.g., translates), then output changes similarly



Share the same parameters across different locations (assuming input is stationary):

Ranzato

Filtering reminder:
Correlation (rotated convolution)

$f[\cdot\,,\cdot\,]$ $\frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$

$I[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

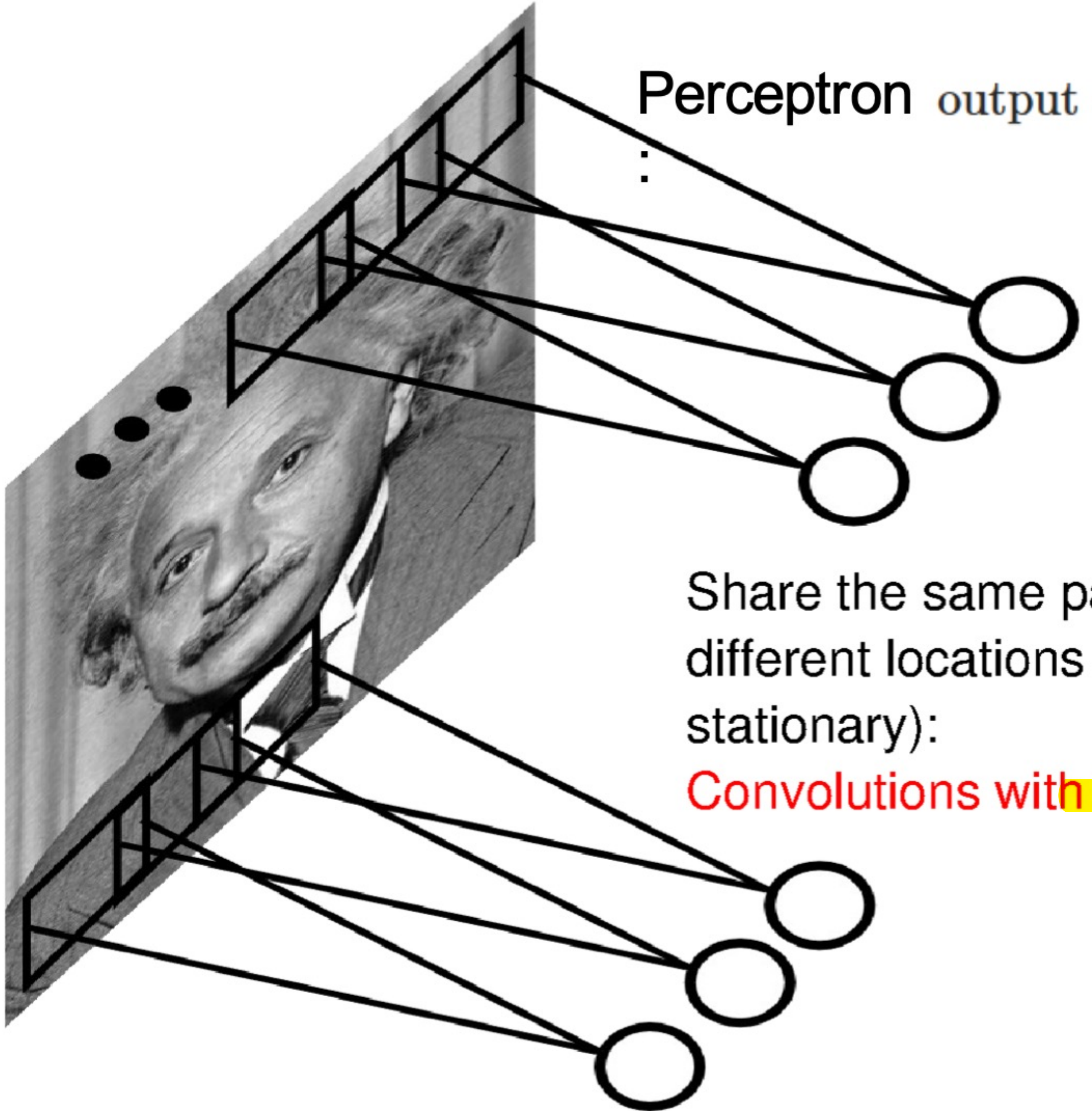
$h[\cdot,\cdot]$

	0	10	20	30	30	30	20	10	
	0	20	40	60	60	60	40	20	
	0	30	60	90	90	90	60	30	
	0	30	50	80	80	90	60	30	
	0	30	50	80	80	90	60	30	
	0	20	30	50	50	60	40	20	
	10	20	30	30	30	30	20	10	
	10	10	10	0	0	0	0	0	

$$h[m,n]=\sum_{k,l} f[k,l]\, I[m+k,n+l]$$

Credit: S. Seitz

Convolutional Layer



Perceptron:
$$\text{output} = \begin{cases} 0 & \text{if } w \cdot x + b \leq 0 \\ 1 & \text{if } w \cdot x + b > 0 \end{cases}$$

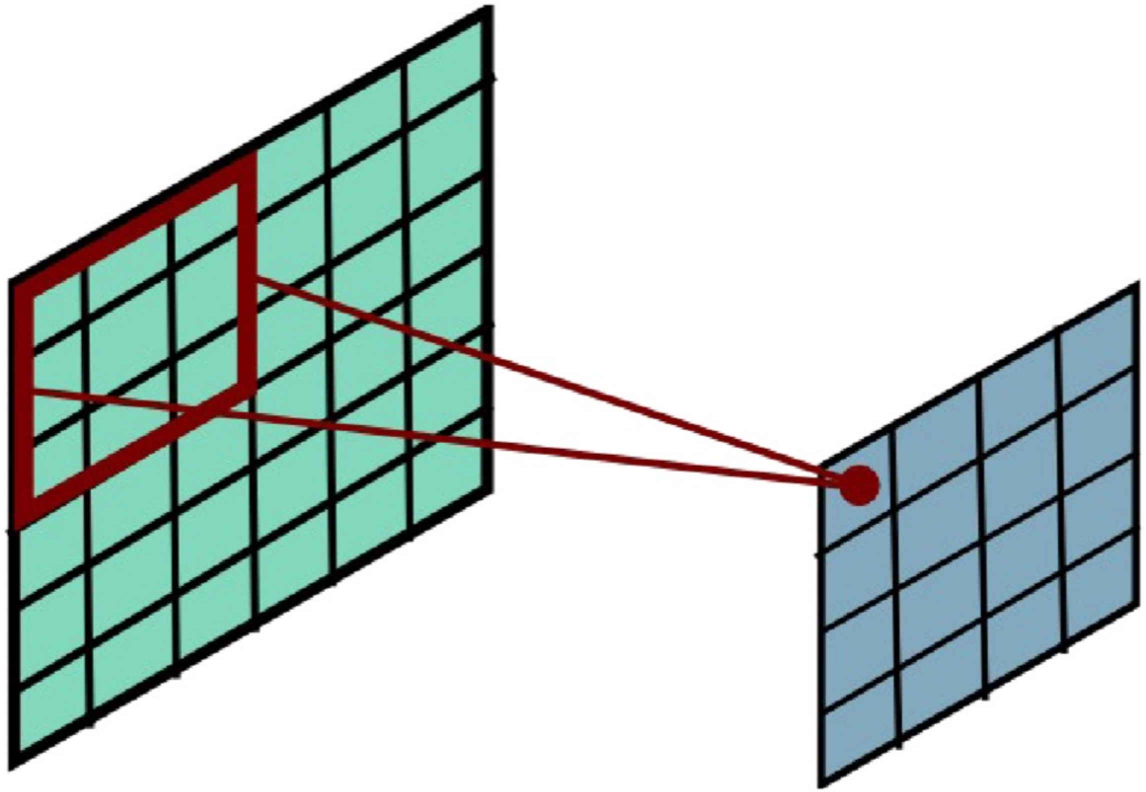
$$w \cdot x \equiv \sum_j w_j x_j,$$

This is convolution!

Share the same parameters across different locations (assuming input is stationary):
Convolutions with learned kernels

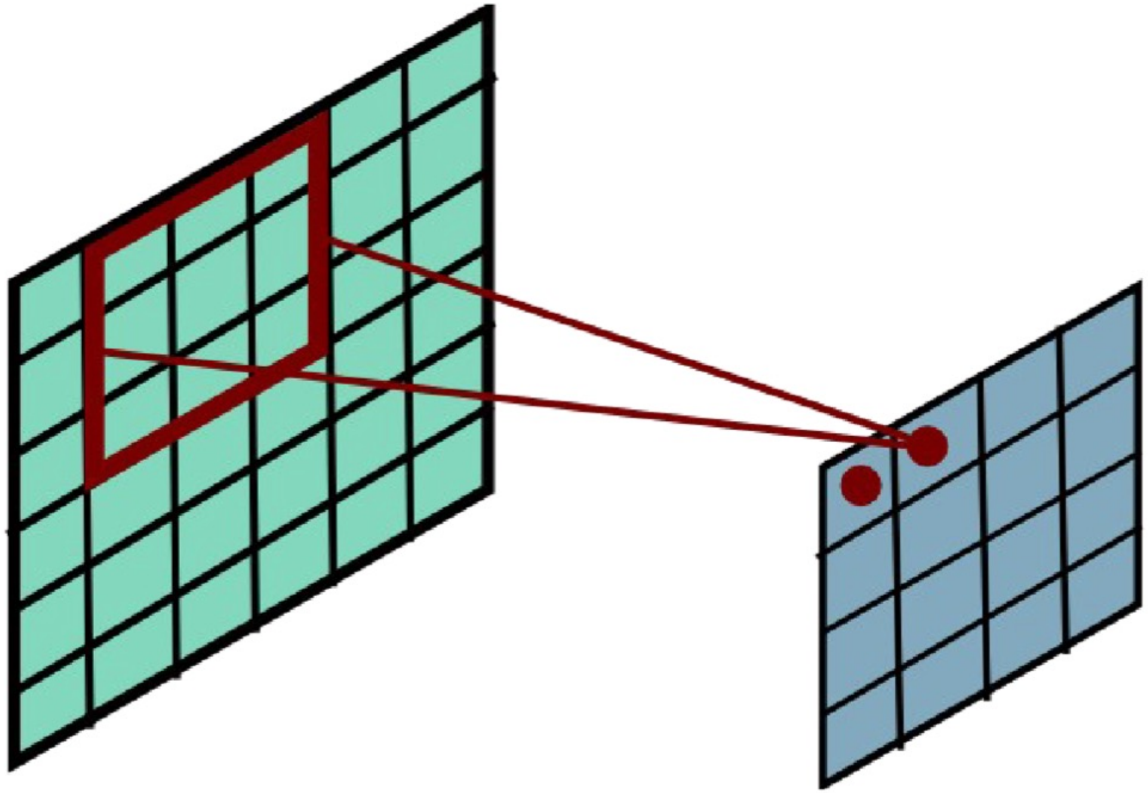
Ranzato

Convolutional Layer



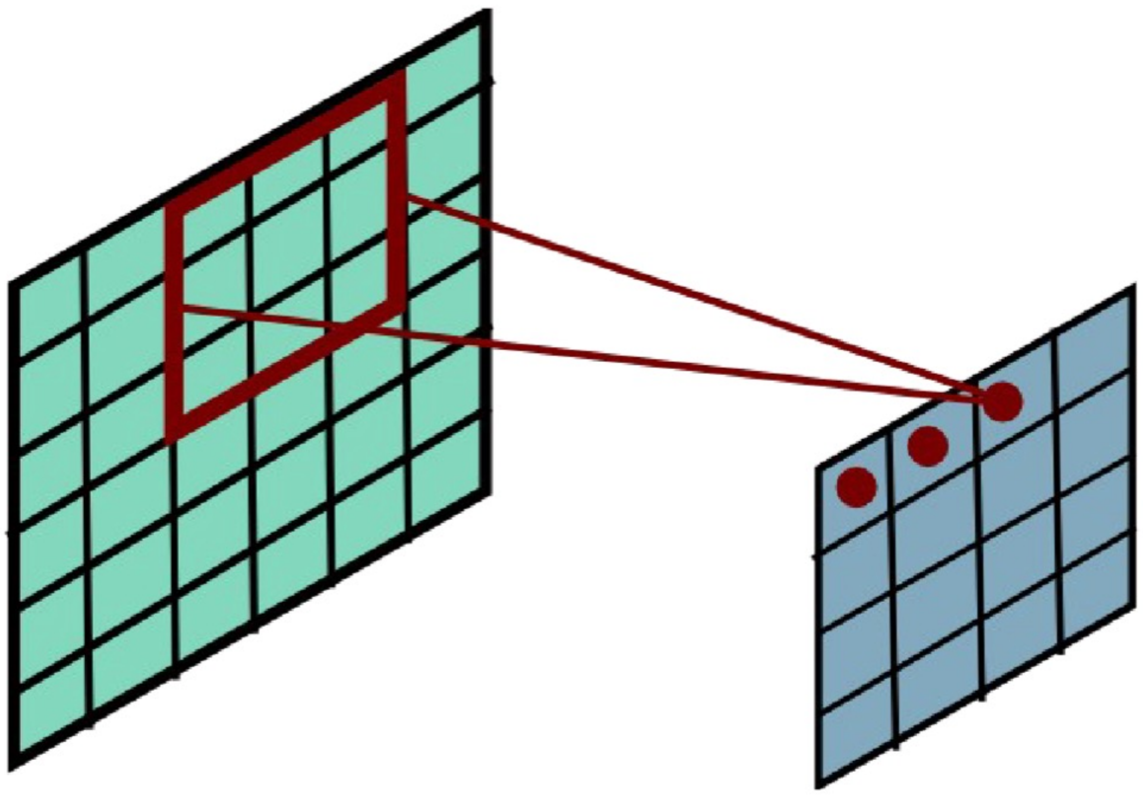
Ranzato

Convolutional Layer



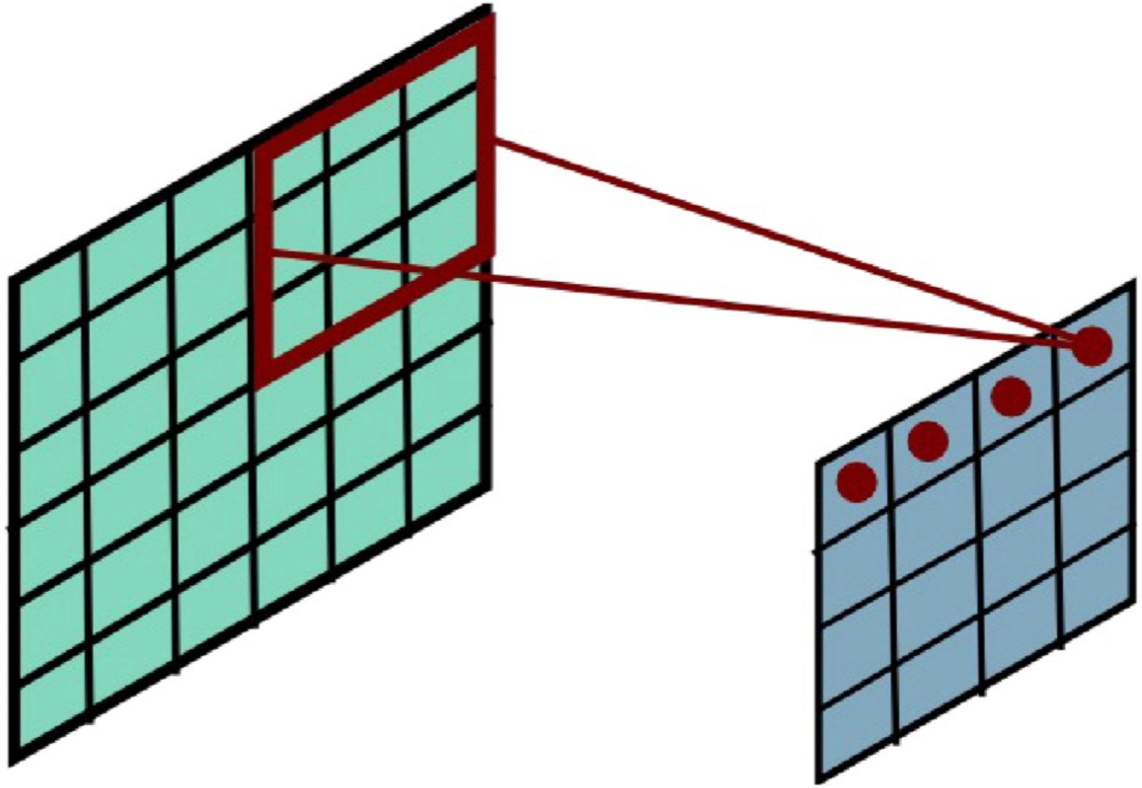
Ranzato

Convolutional Layer



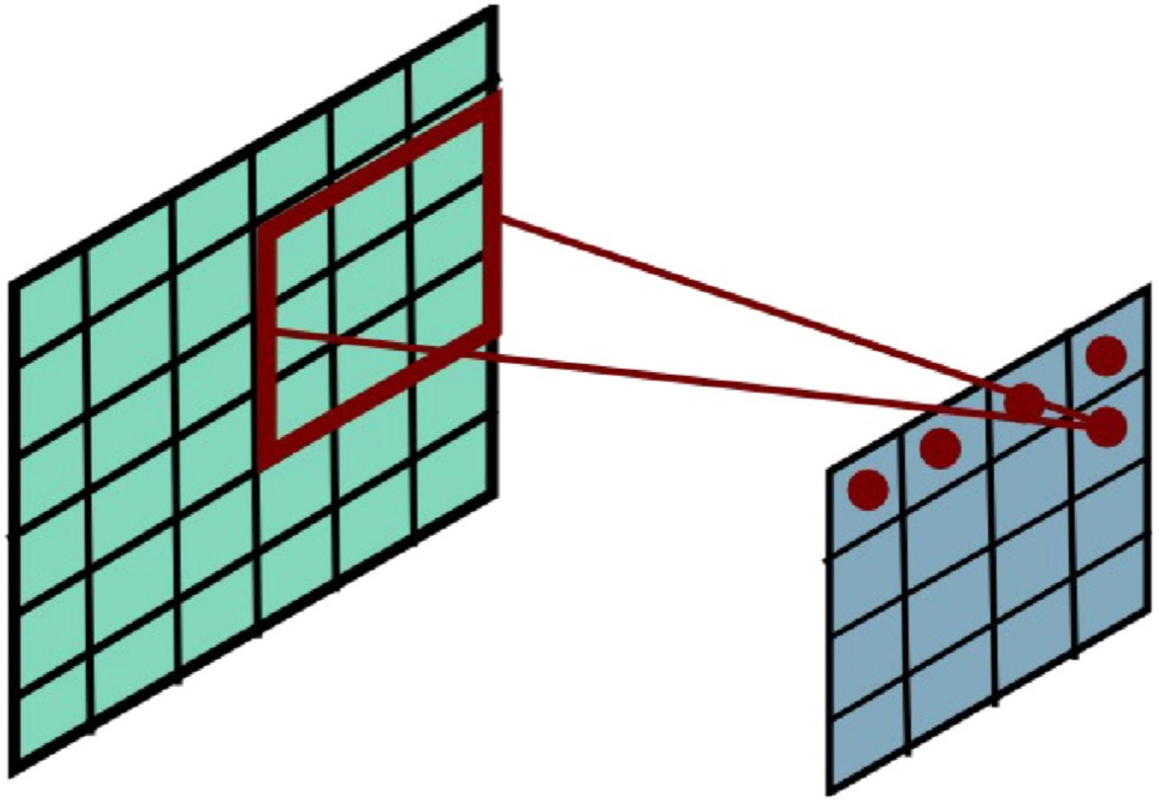
Ranzato

Convolutional Layer



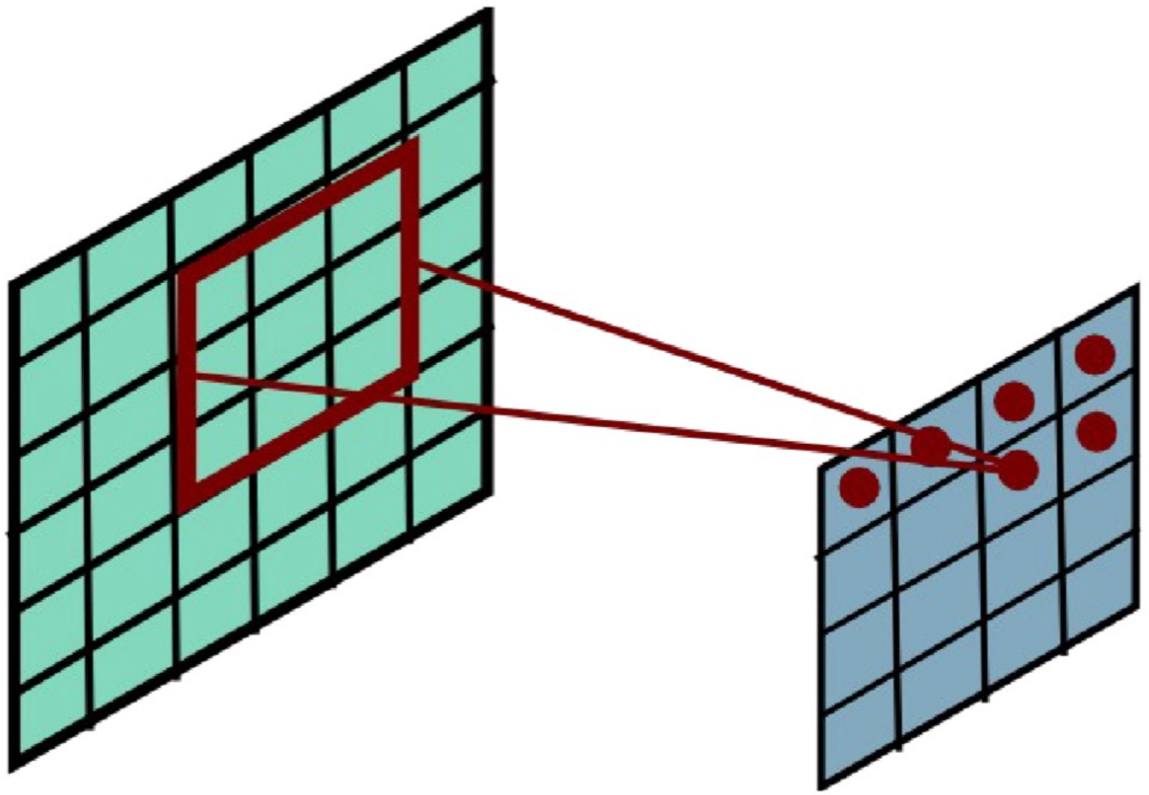
Ranzato

Convolutional Layer



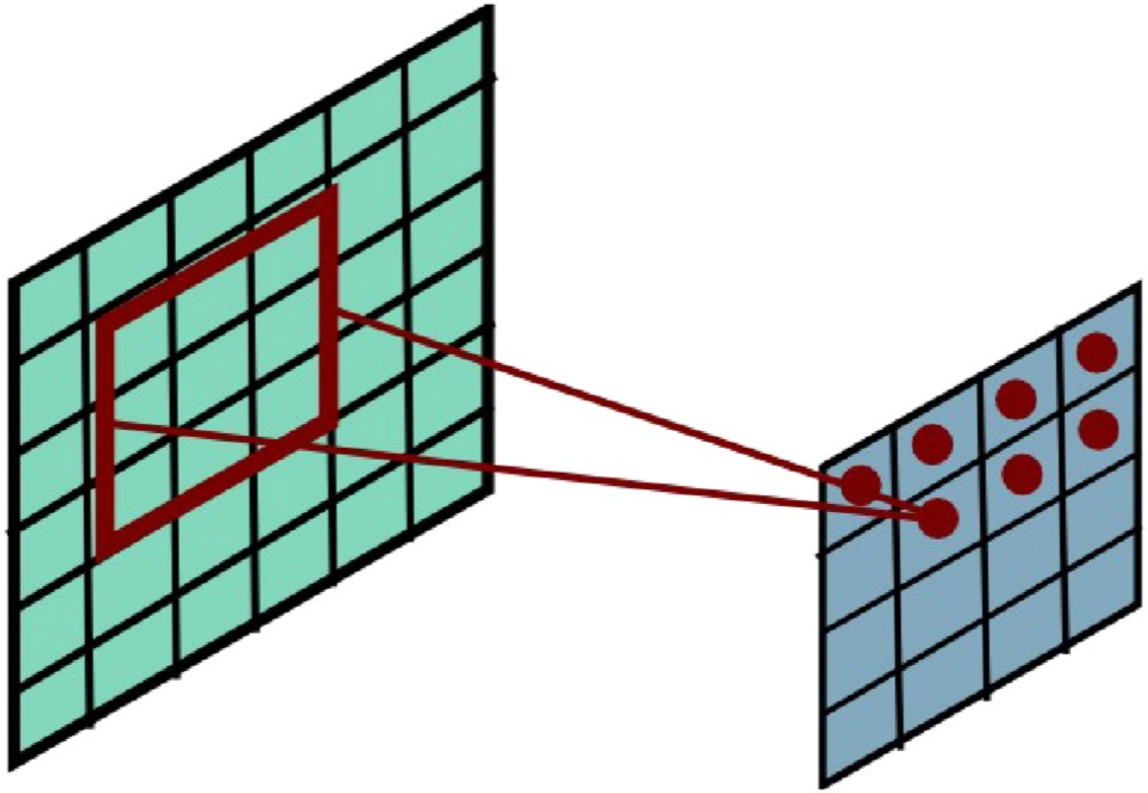
Ranzato

Convolutional Layer



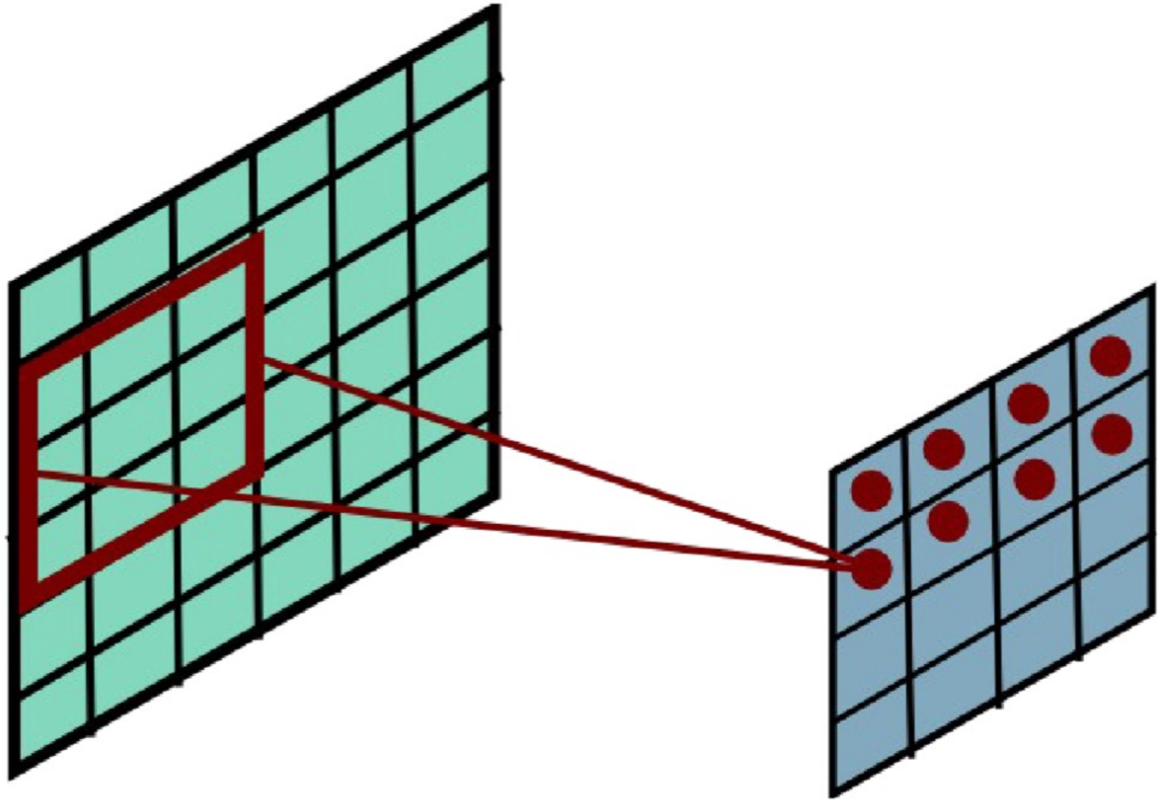
Ranzato

Convolutional Layer



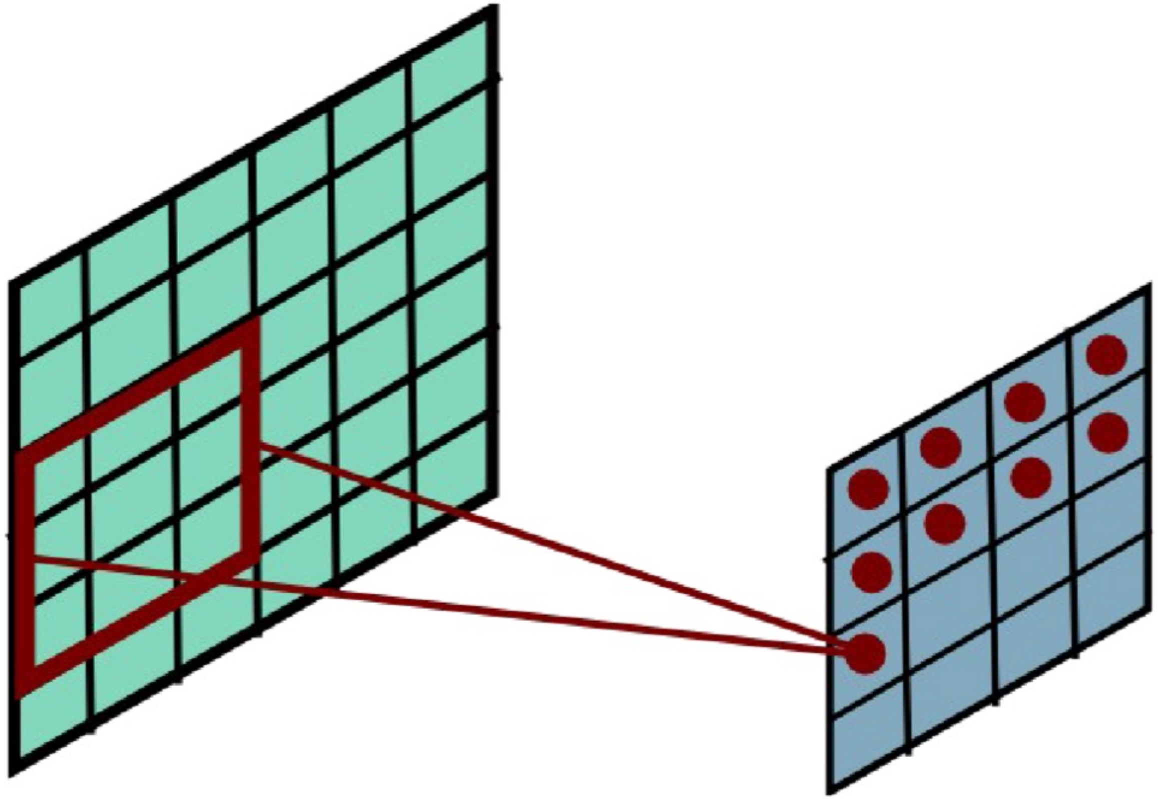
Ranzato

Convolutional Layer

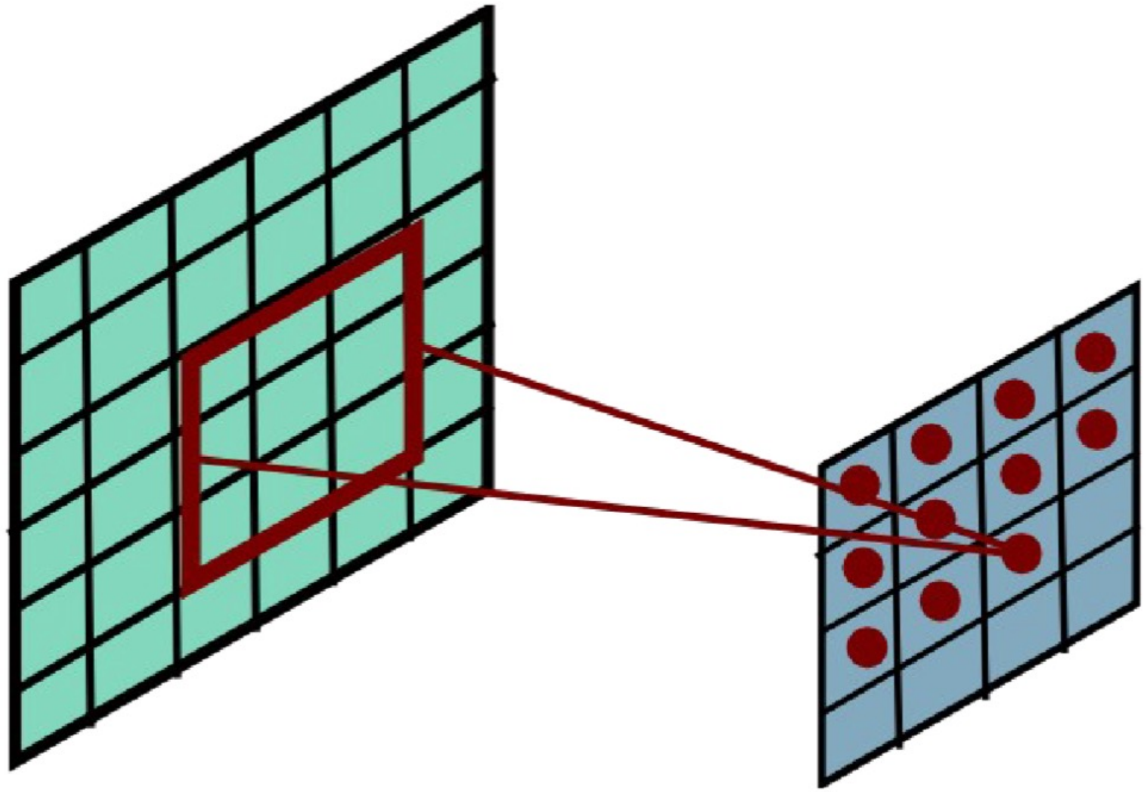


Ranzato

Convolutional Layer

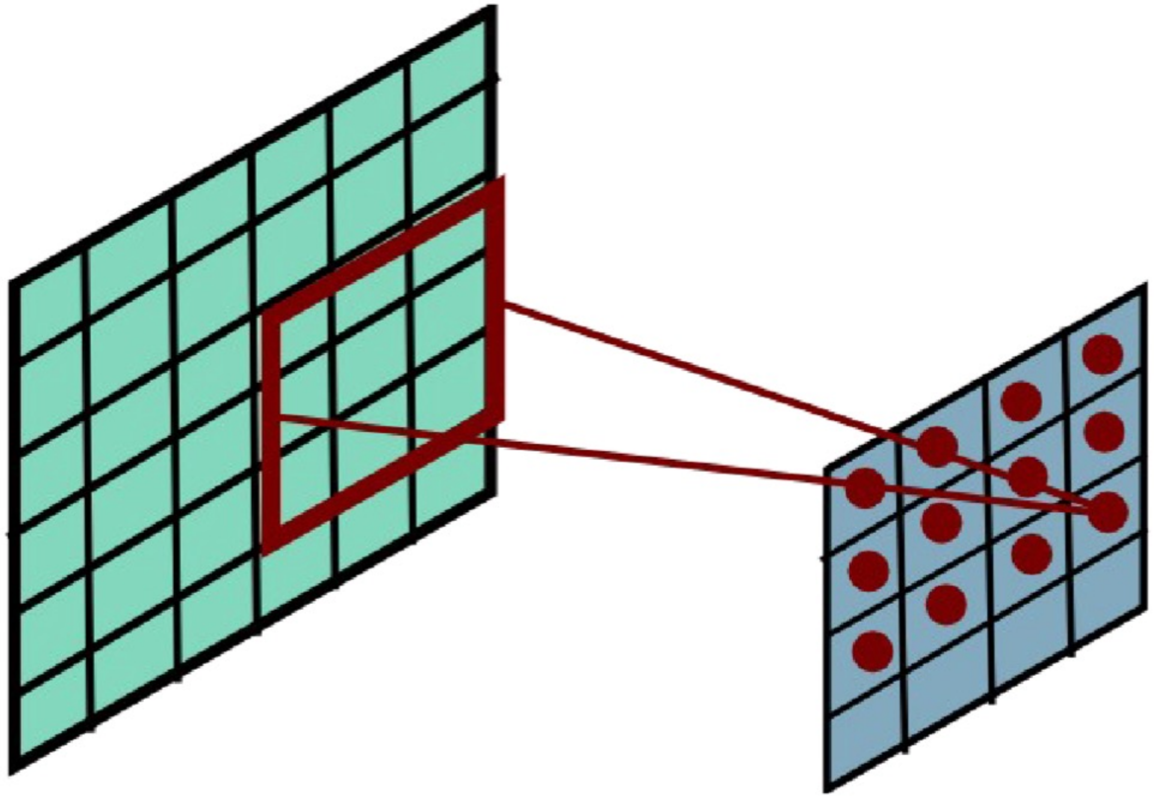


Convolutional Layer



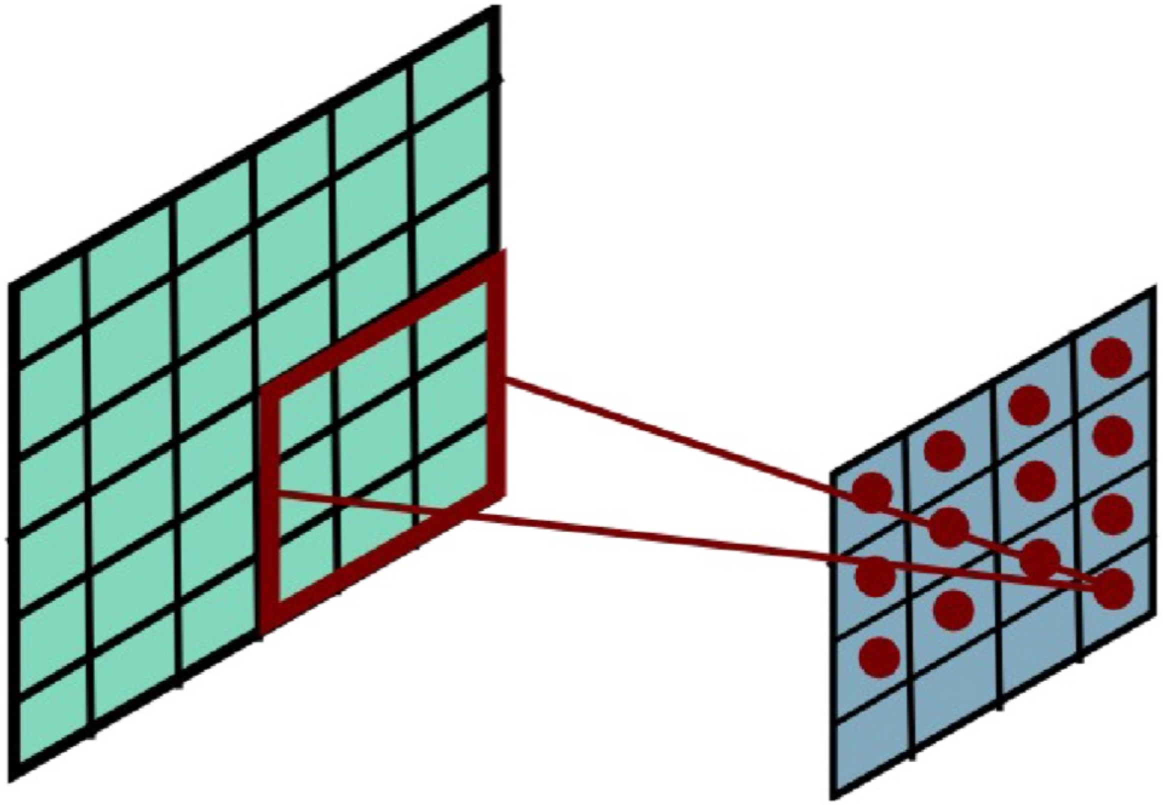
Ranzato

Convolutional Layer



Ranzato

Convolutional Layer

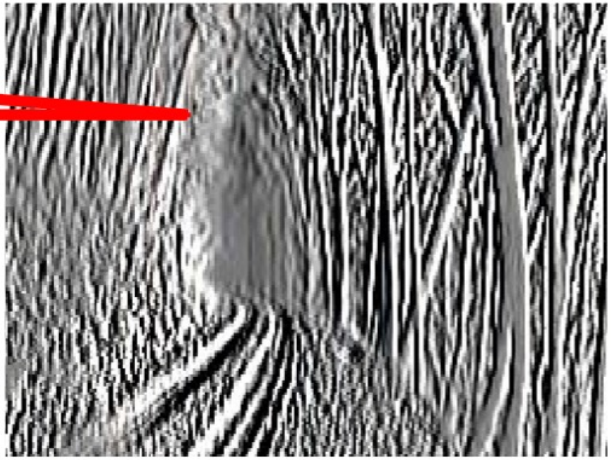


Ranzato

Convolutional Layer

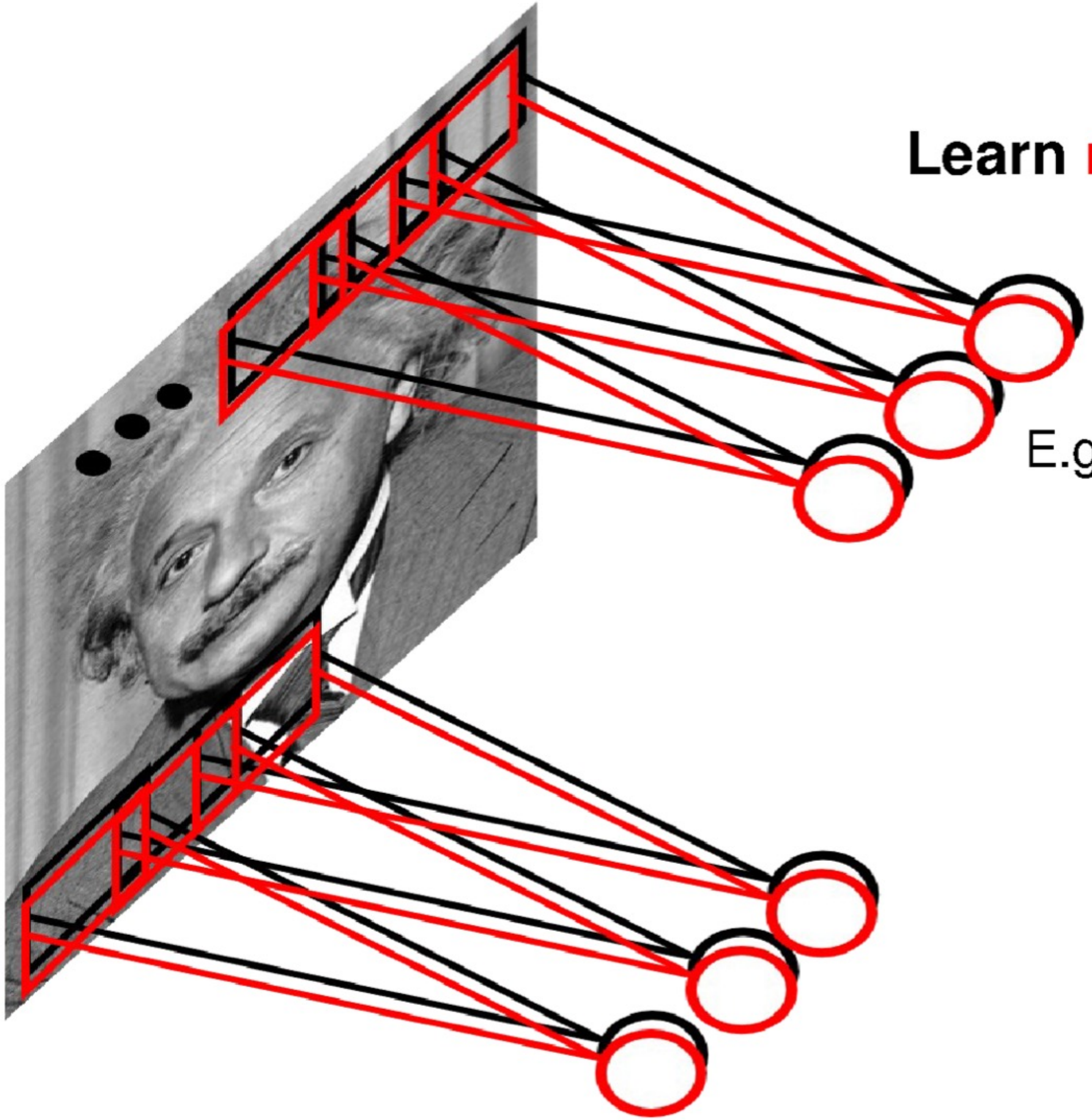
$$* \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} =$$

Shared weights



Ranzato

Convolutional Layer



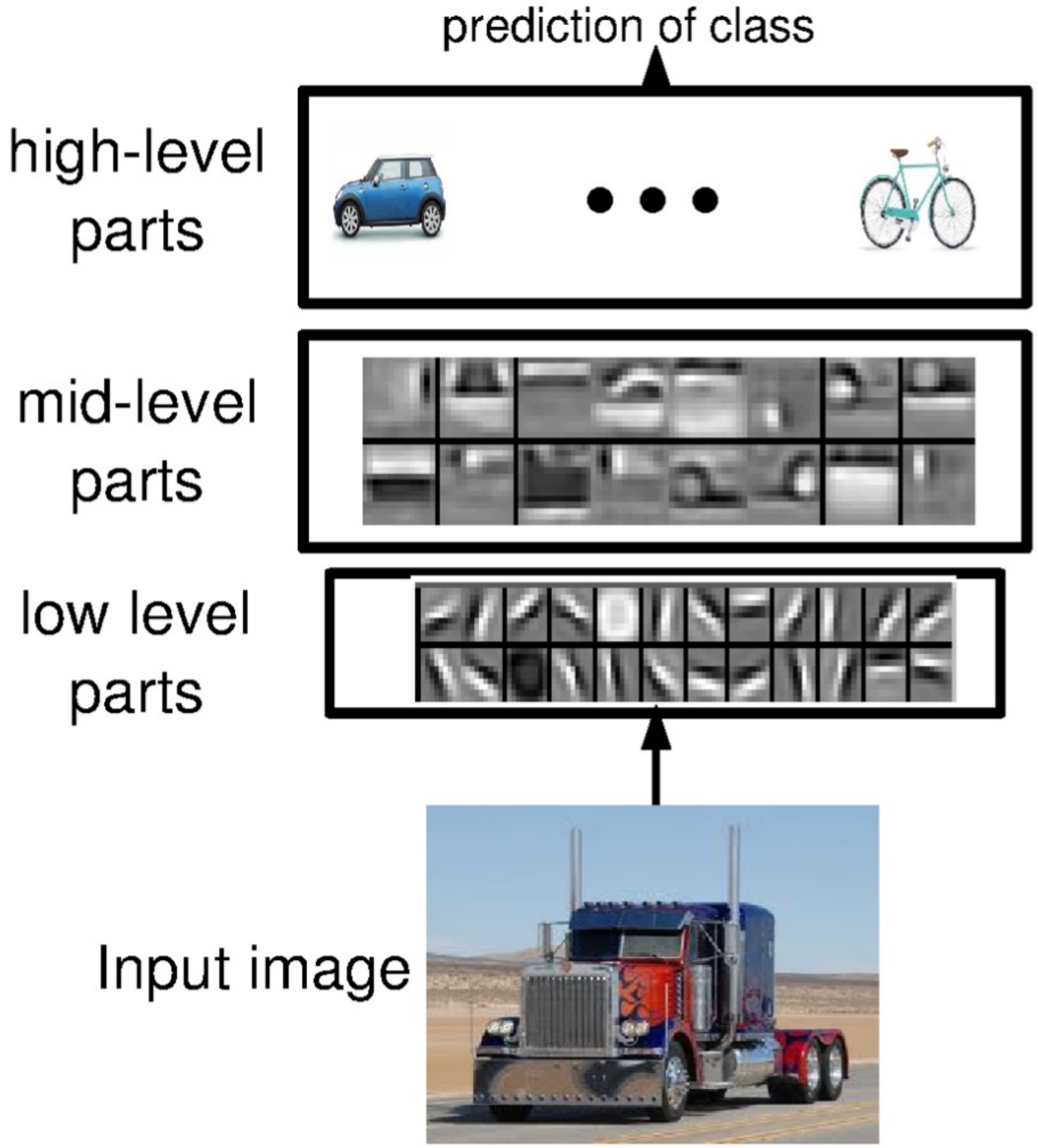
Learn multiple filters.

Filter = 'local' perceptron.
Also called *kernel*.

E.g.: 200x200 image
100 Filters
Filter size: 10x10
10K parameters

Ranzato

Interpretation



- distributed representations
- feature sharing
- compositionality

Lee et al. “Convolutional DBN's ...” ICML 2009

Convolutional Layer

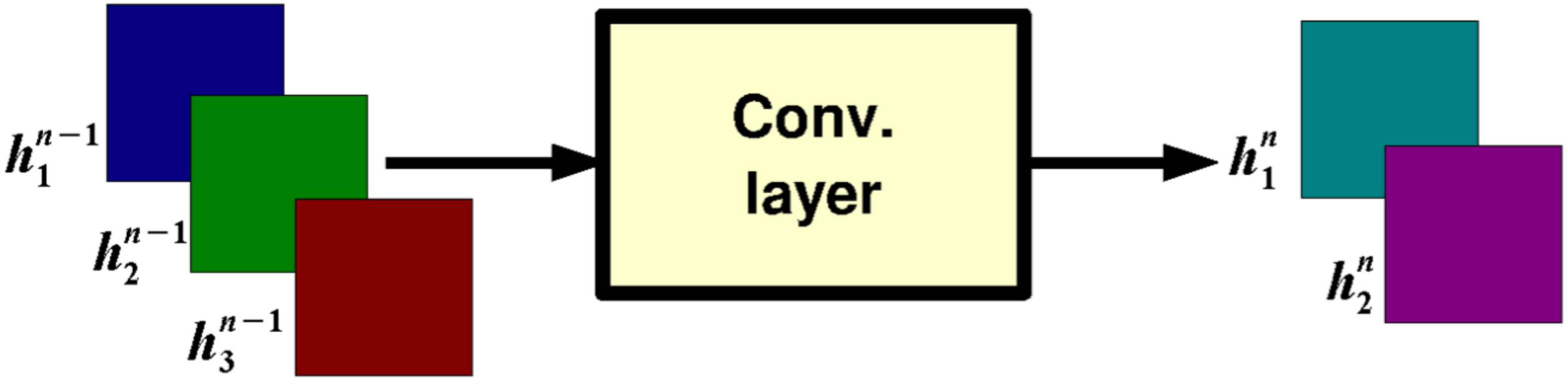
$$h_j^n = max\left(0, \sum_{k=1}^{K} h_k^{n-1} * w_{kj}^n\right)$$

output feature map

input feature map

kernel

n = layer number
K = kernel size
j = # channels (input)
or # filters (depth)



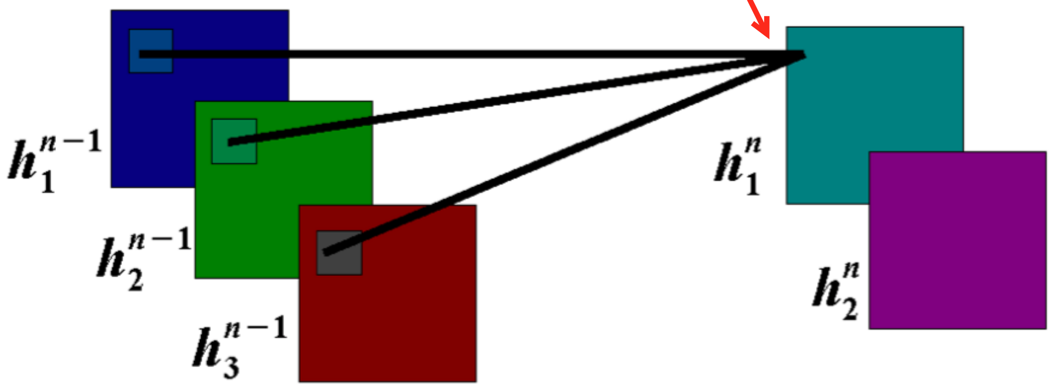
Convolutional Layer

$$h_j^n = max\left(0, \boxed{\sum_{k=1}^{K}} h_k^{n-1} * w_{kj}^n\right)$$

output feature map

input feature map

kernel



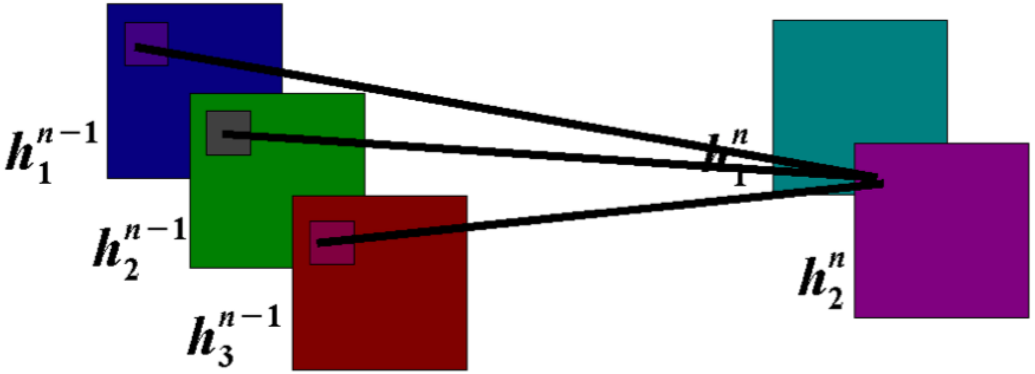
Convolutional Layer

$$h_j^n = max\,(0, \sum_{k=1}^{K} h_k^{n-1} * w_{kj}^n)$$

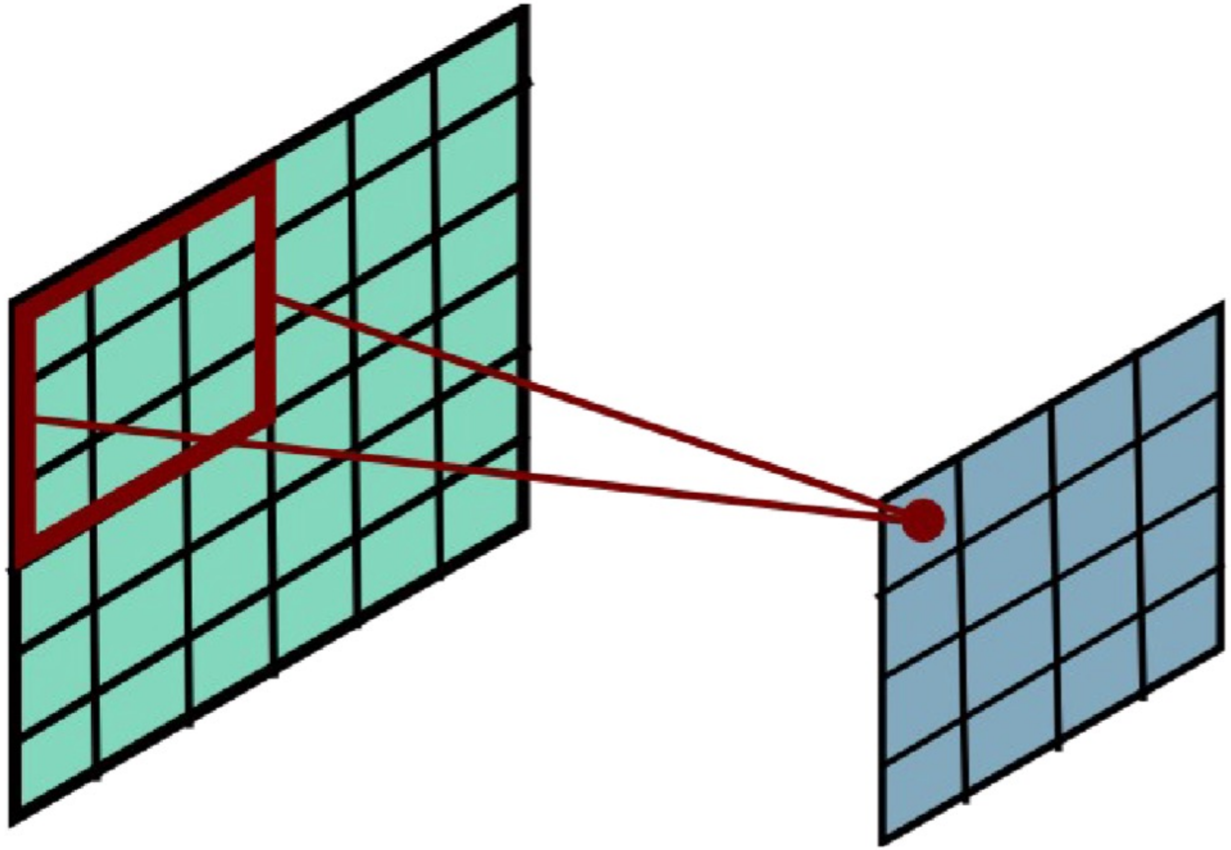
output feature map

input feature map

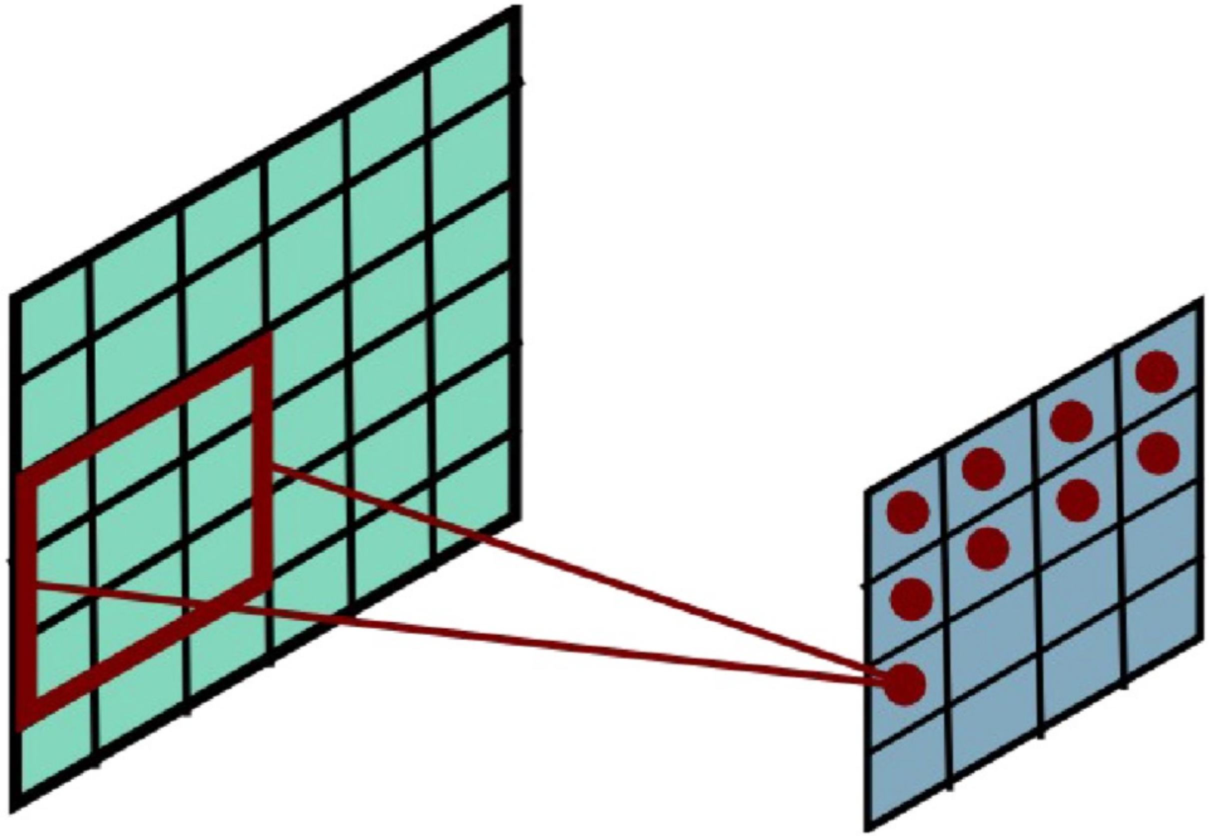
kernel



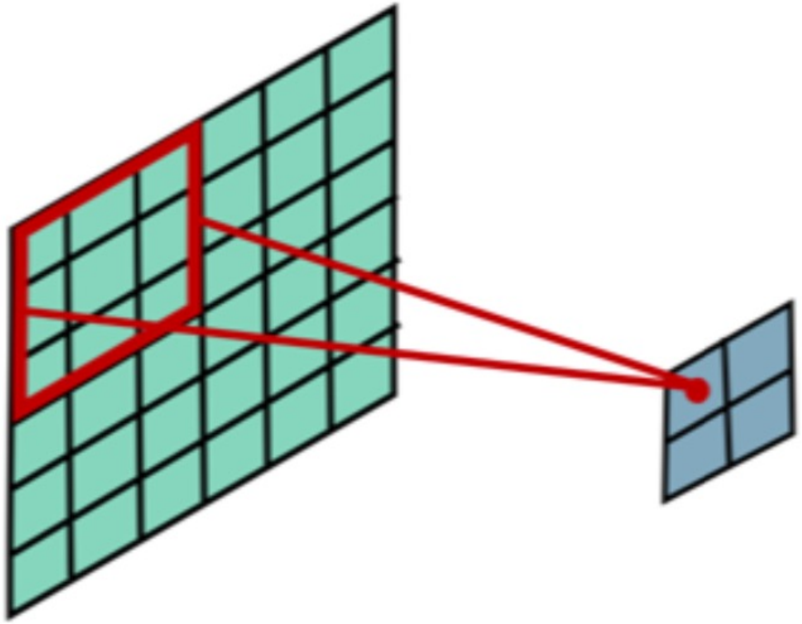
Stride = 1



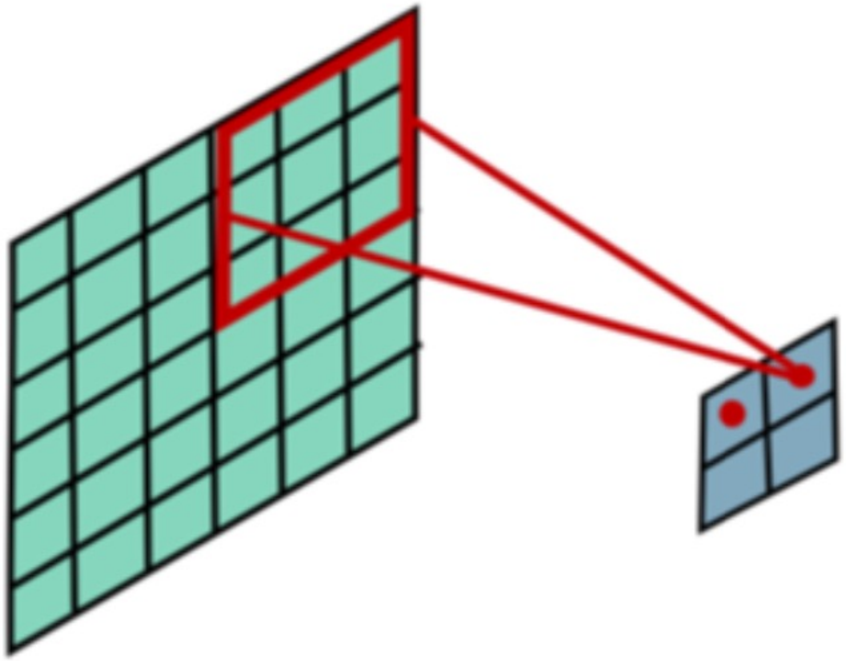
Stride = 1



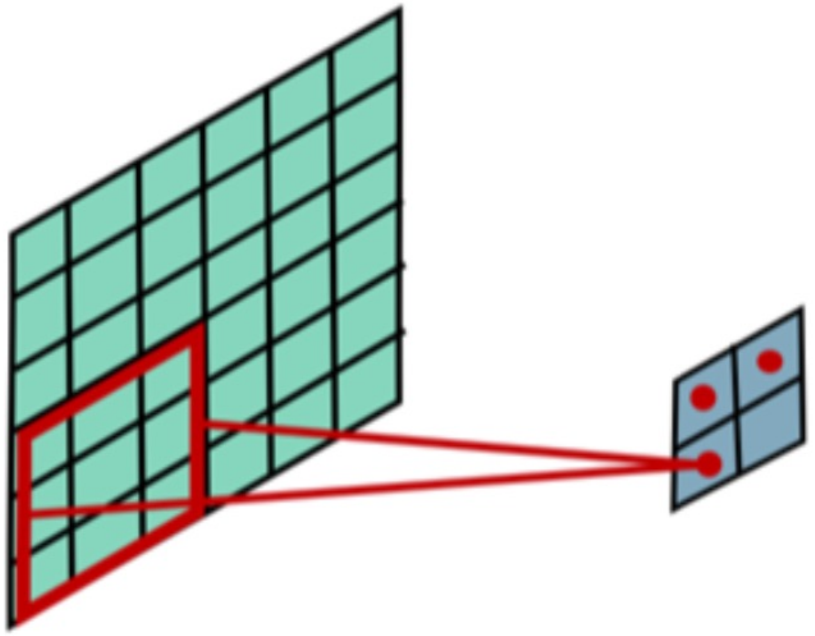
Stride = 3



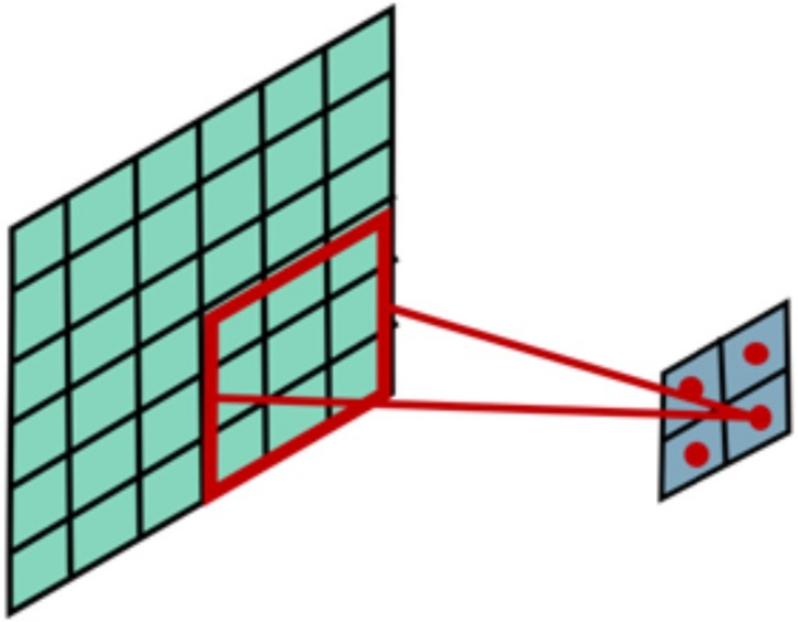
Stride = 3



Stride = 3



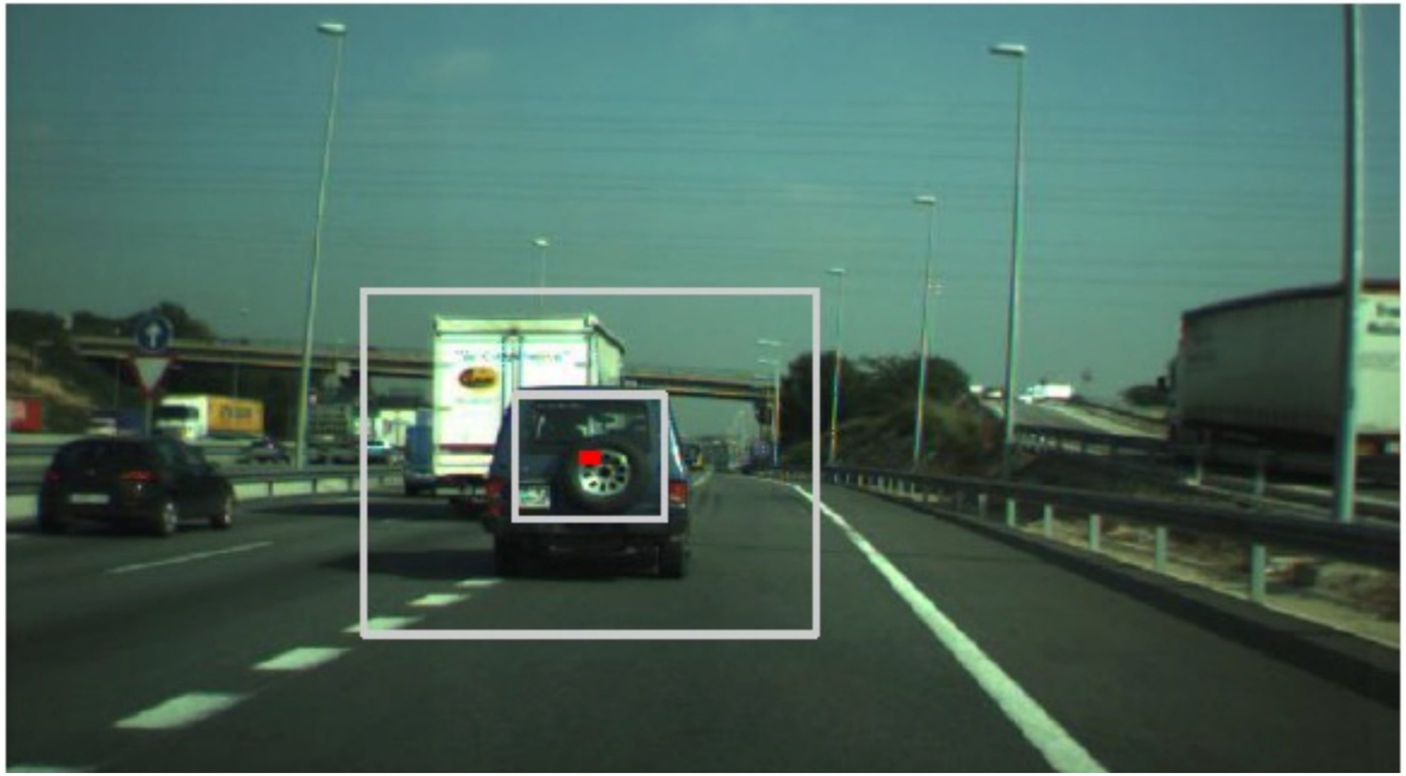
Stride = 3



Stride is the number of pixels shifted in both dimensions for convolution if Stride is applied to both.

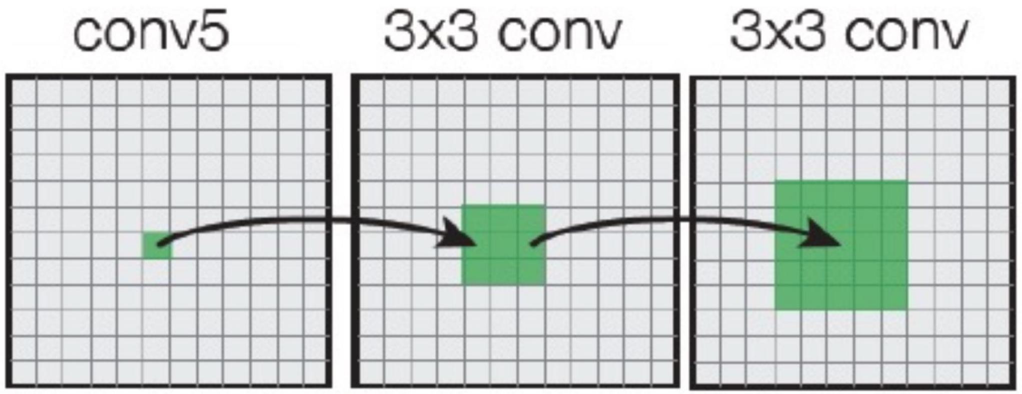
Receptive Field

- Is the spatial extent of the connectivity (equivalently this is the filter size).

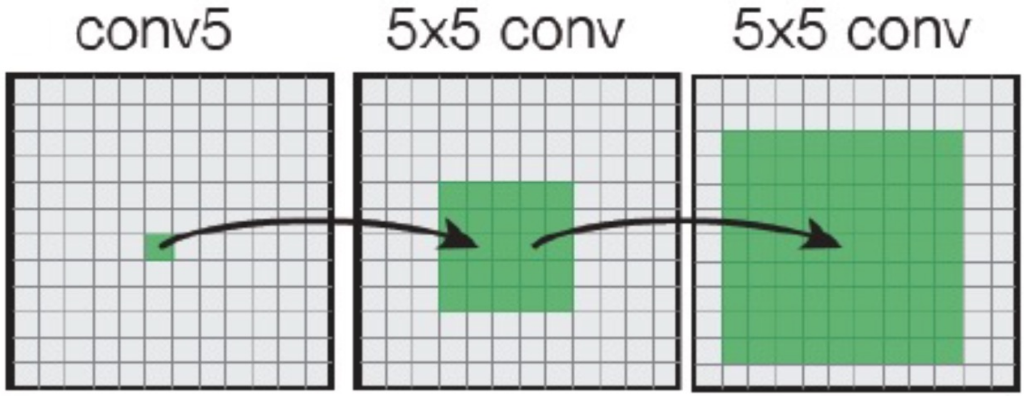


Receptive Field

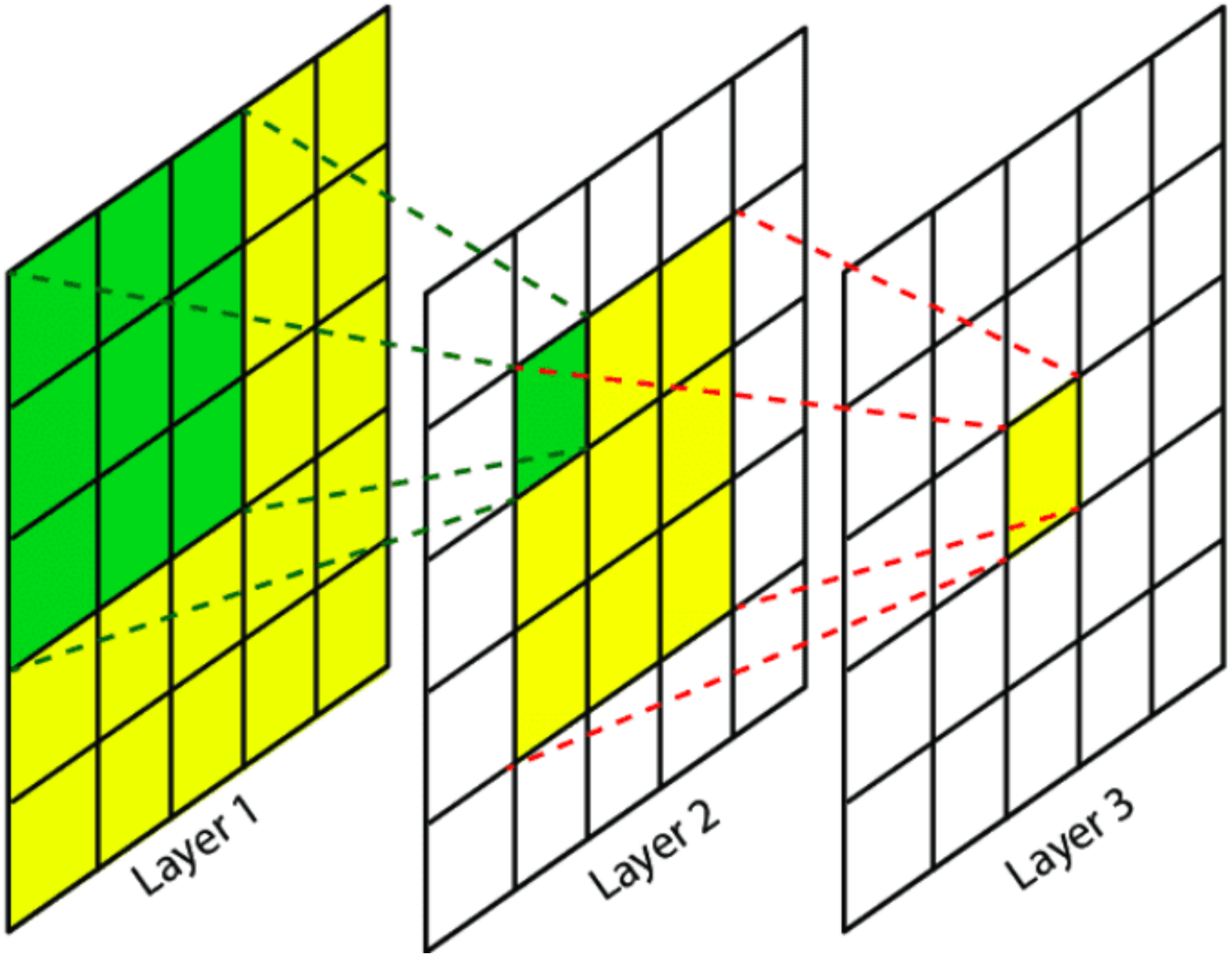
- Receptive Field



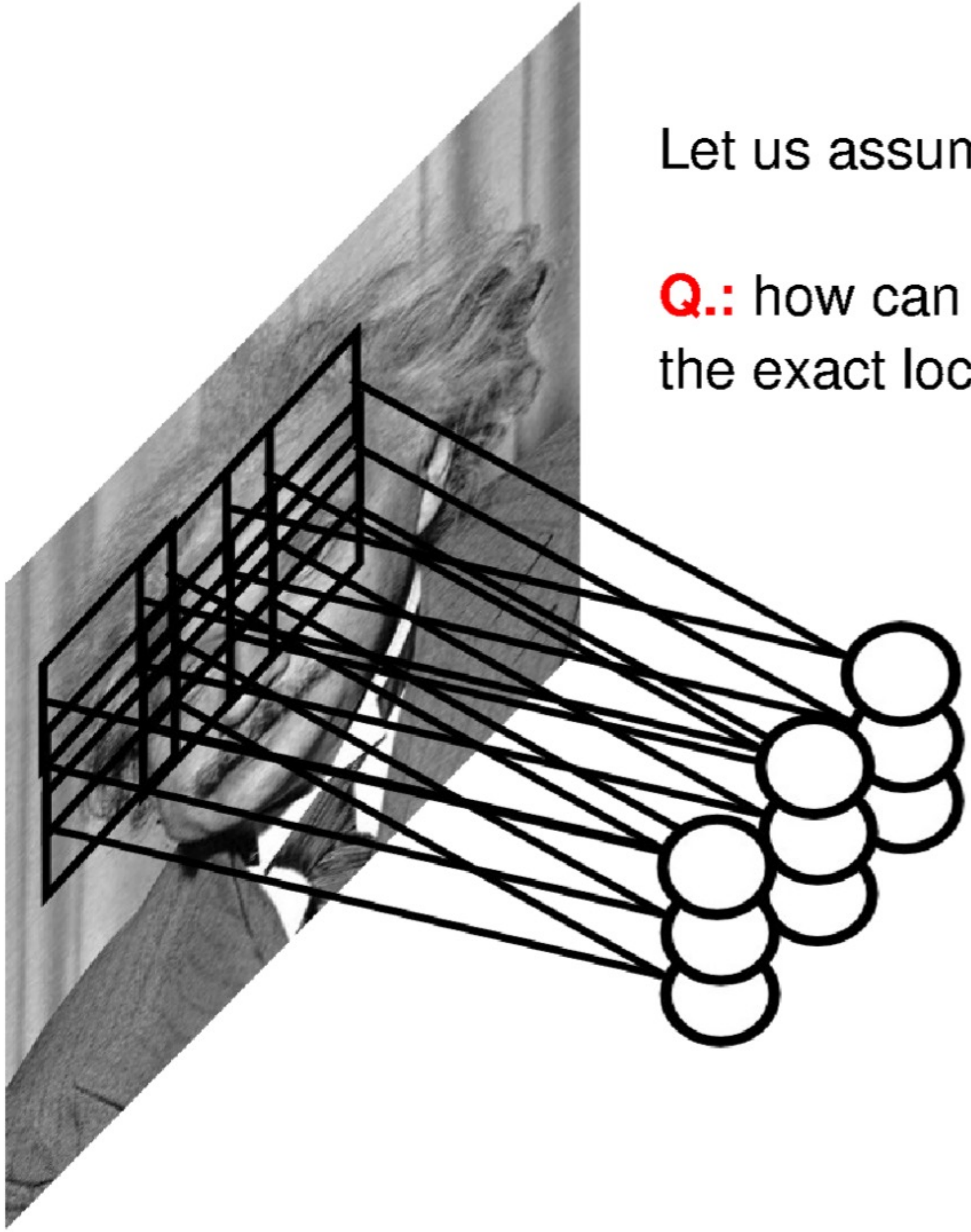
(a) two stacked 3x3 convolution layers



(b) two stacked 5x5 convolution layers


Layer 1
Layer 2
Layer 3

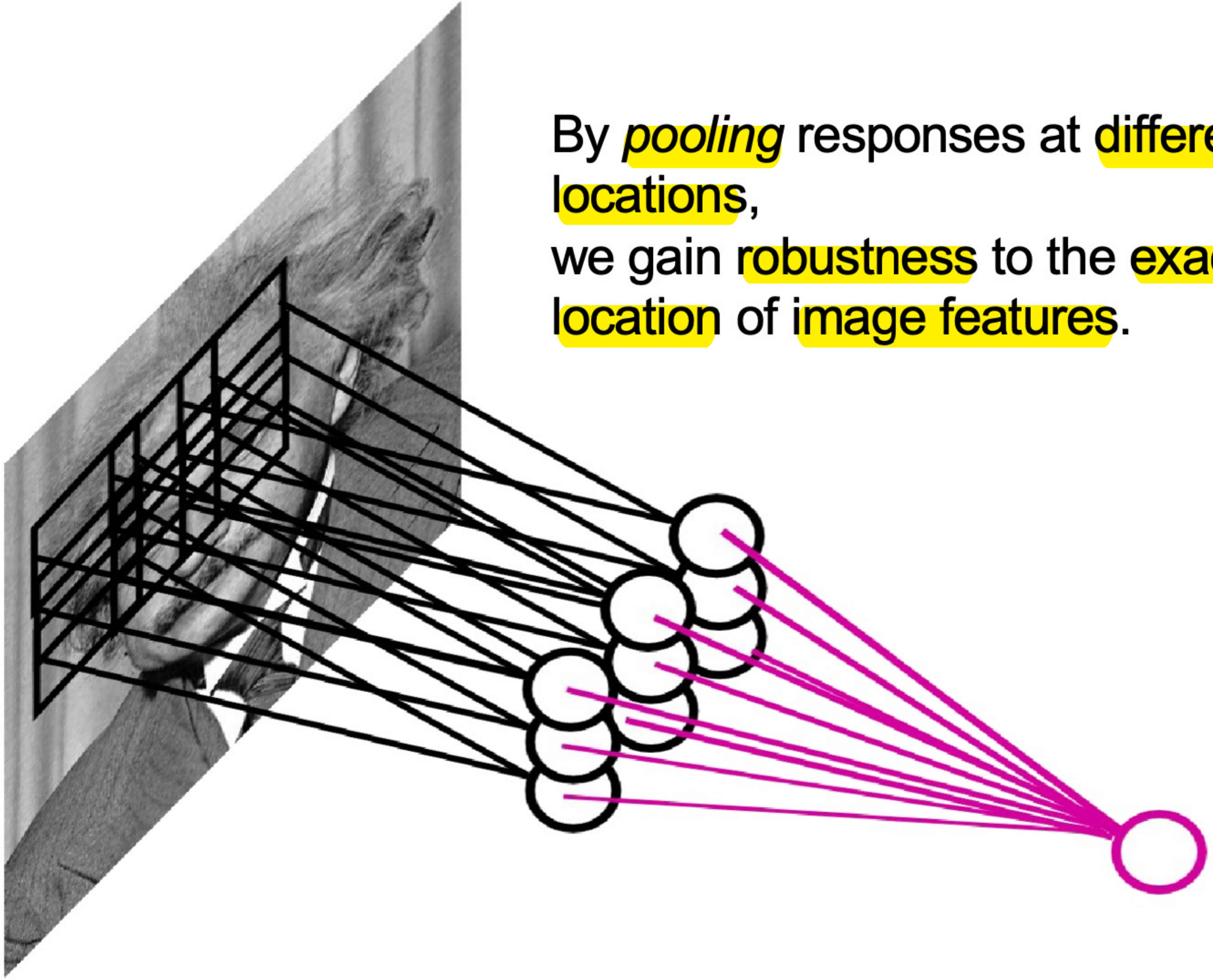
Pooling Layer



Let us assume filter is an “eye” detector.

Q.: how can we make the detection robust to the exact location of the eye?

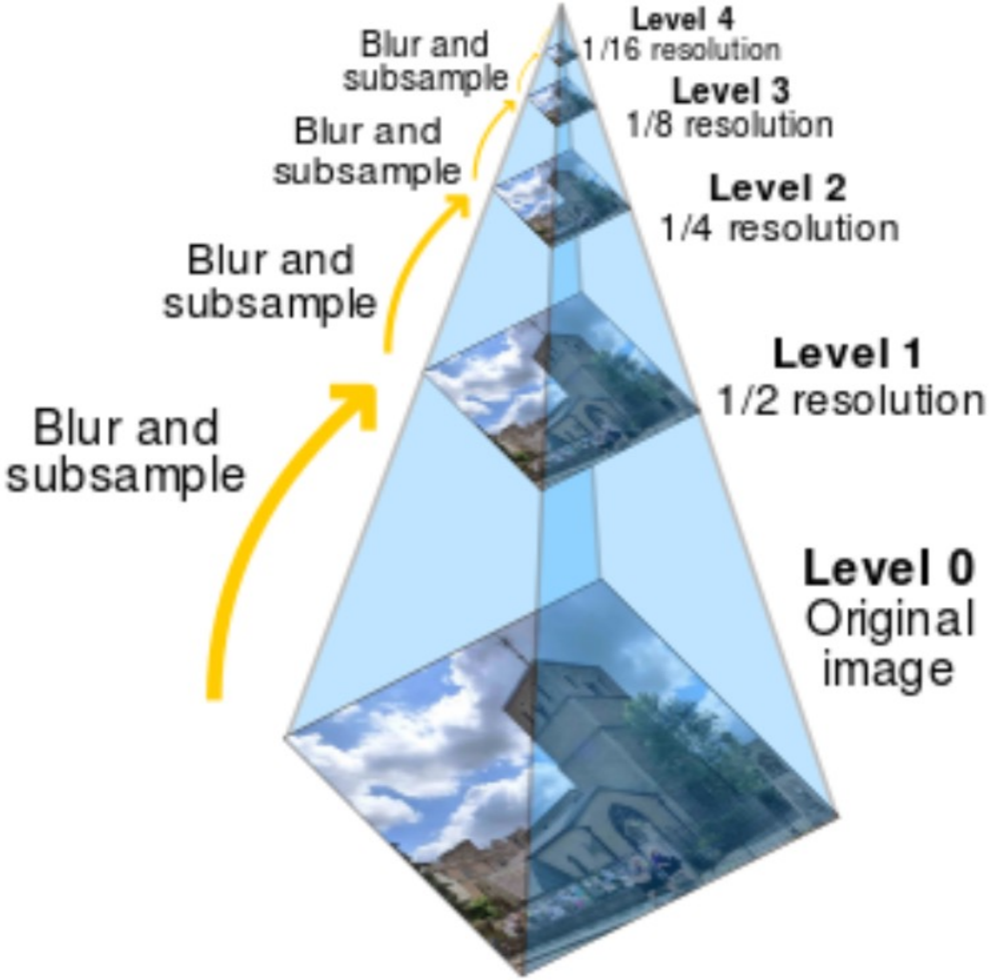
Pooling Layer



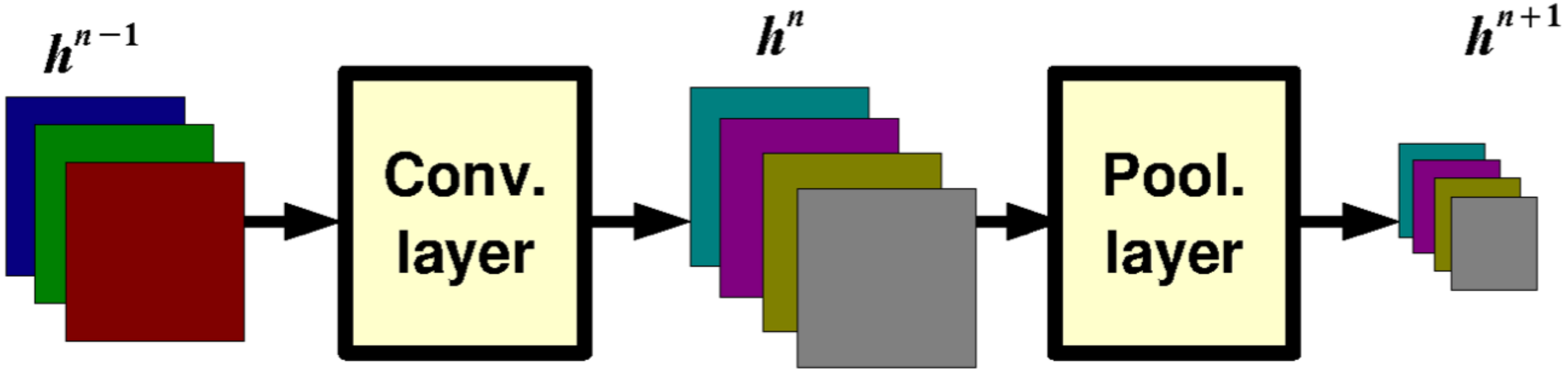
By *pooling* responses at different locations,
we gain robustness to the exact spatial location of image features.

Ranzato

Pooling is similar to pyrimid downsampling



Pooling Layer: Receptive Field Size



Pooling Layer: Examples

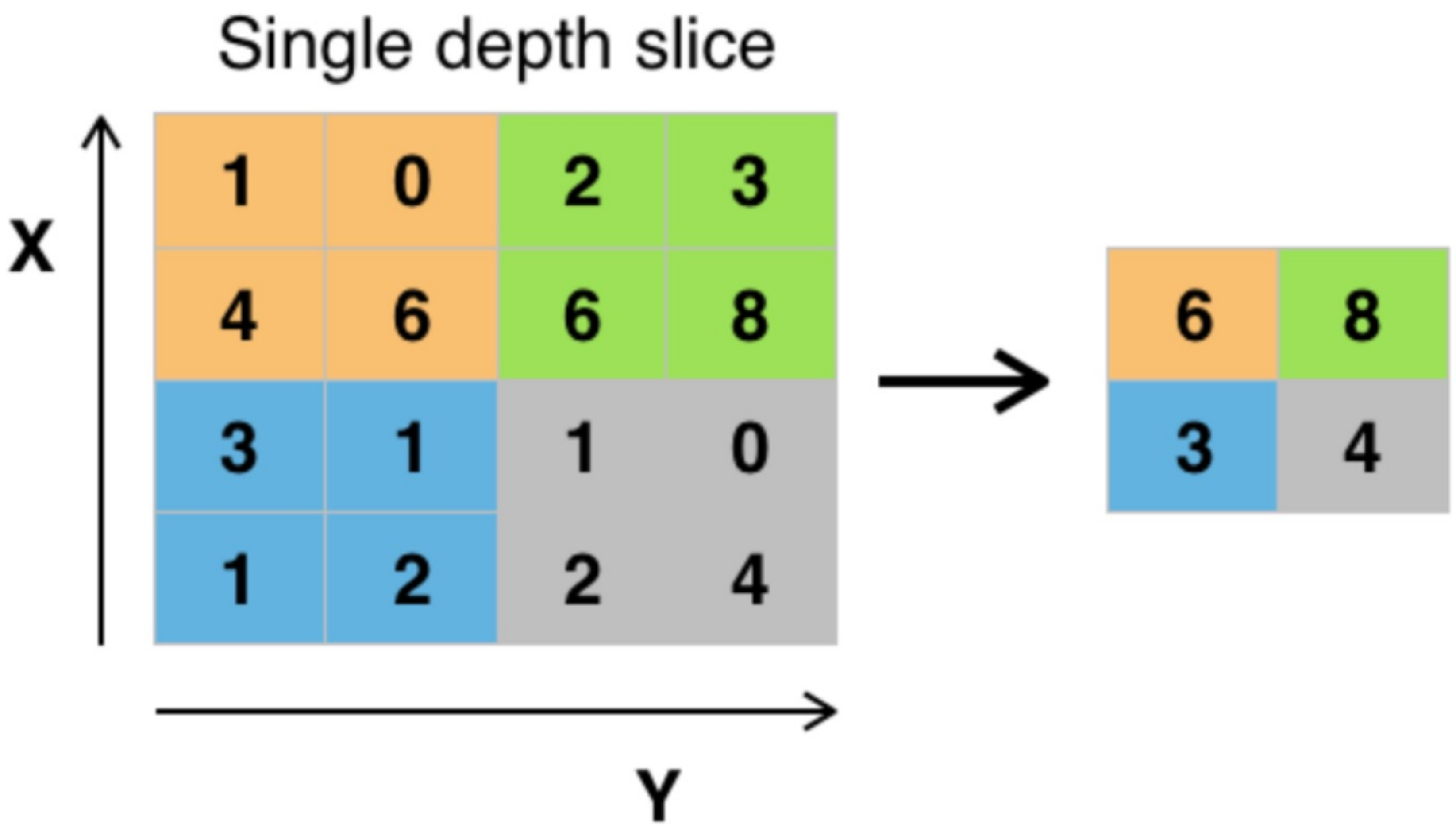
Max-pooling:

$$h_j^n(x, y) = max_{\bar{x} \in N(x), \bar{y} \in N(y)} h_j^{n-1}(\bar{x}, \bar{y})$$

Average-pooling:

$$h_j^n(x, y) = 1/K \sum_{\bar{x} \in N(x), \bar{y} \in N(y)} h_j^{n-1}(\bar{x}, \bar{y})$$

Max pooling



Wikipedia

Pooling Layer: Examples

Max-pooling:

$$h_j^n(x, y) = max_{\bar{x} \in N(x), \bar{y} \in N(y)} h_j^{n-1}(\bar{x}, \bar{y})$$

Average-pooling:

$$h_j^n(x, y) = 1/K \sum_{\bar{x} \in N(x), \bar{y} \in N(y)} h_j^{n-1}(\bar{x}, \bar{y})$$

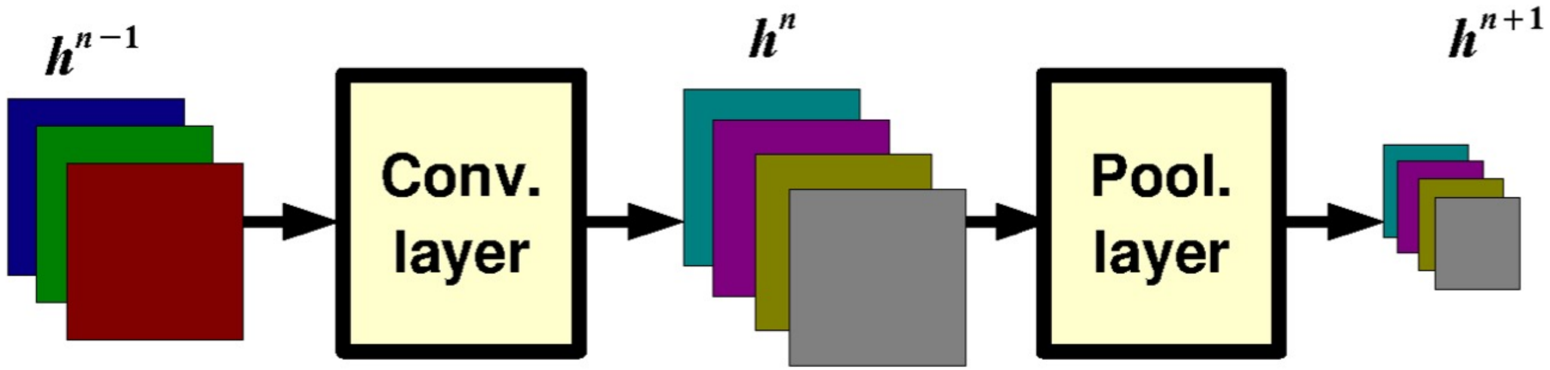
L2-pooling:

$$h_j^n(x, y) = \sqrt{\sum_{\bar{x} \in N(x), \bar{y} \in N(y)} h_j^{n-1}(\bar{x}, \bar{y})^2}$$

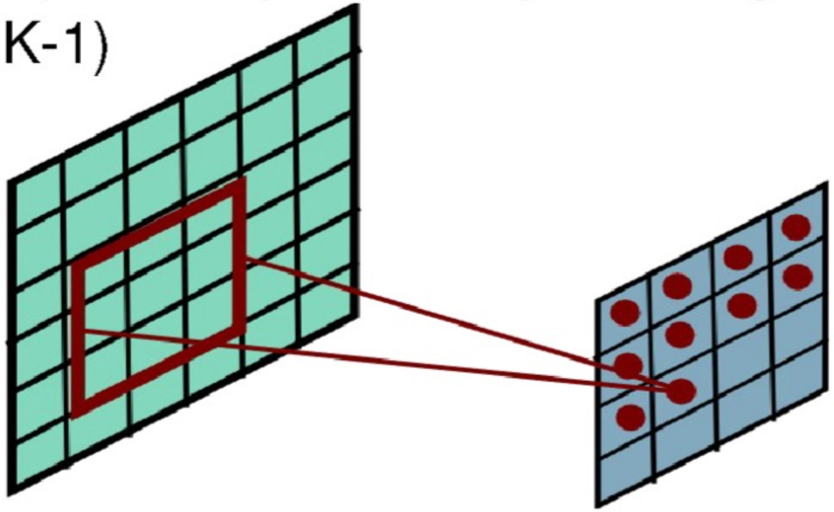
L2-pooling over features:

$$h_j^n(x, y) = \sqrt{\sum_{k \in N(j)} h_k^{n-1}(x, y)^2}$$

Pooling Layer: Receptive Field Size

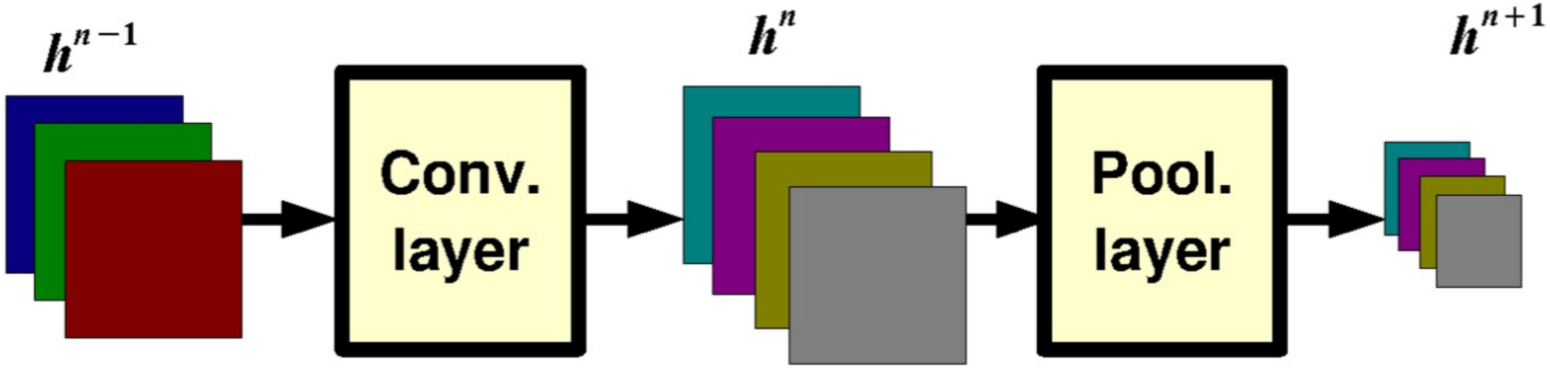


If convolutional filters have size KxK and stride 1, and pooling layer has pools of size PxP, then each unit in the pooling layer depends upon a patch (at the input of the preceding conv. layer) of size: (P+K-1)x(P+K-1)



Ranzato

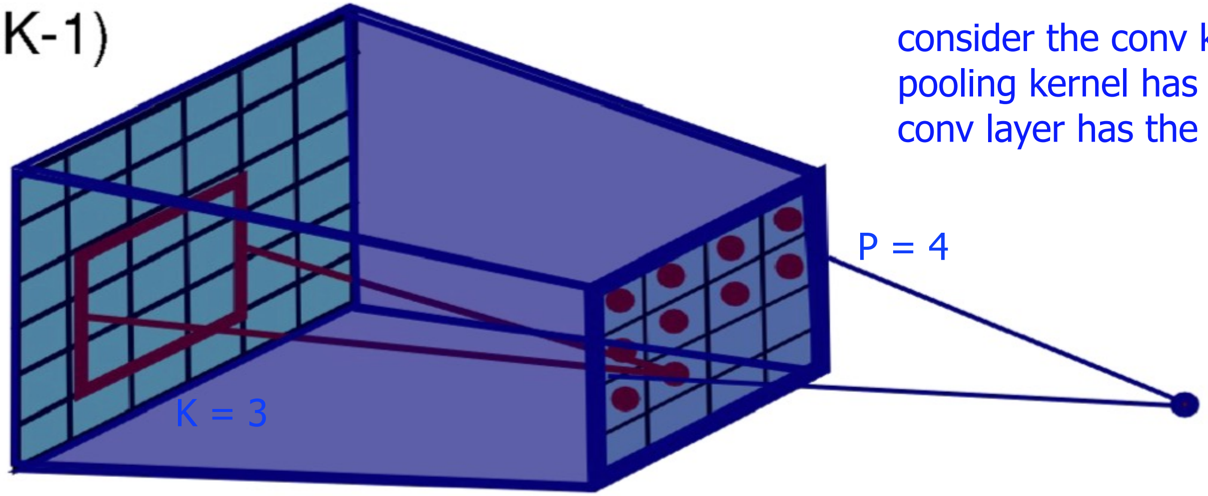
Pooling Layer: Receptive Field Size



If convolutional filters have size KxK and stride 1, and pooling layer has pools of size PxP, then each unit in the pooling layer depends upon a patch (at the input of the preceding conv. layer) of size: (P+K-1)x(P+K-1)

consider the conv kernel has the size of (K * K), the pooling kernel has the size of (P * P), then the input of conv layer has the size of (K + P - 1) * (K + P - 1).

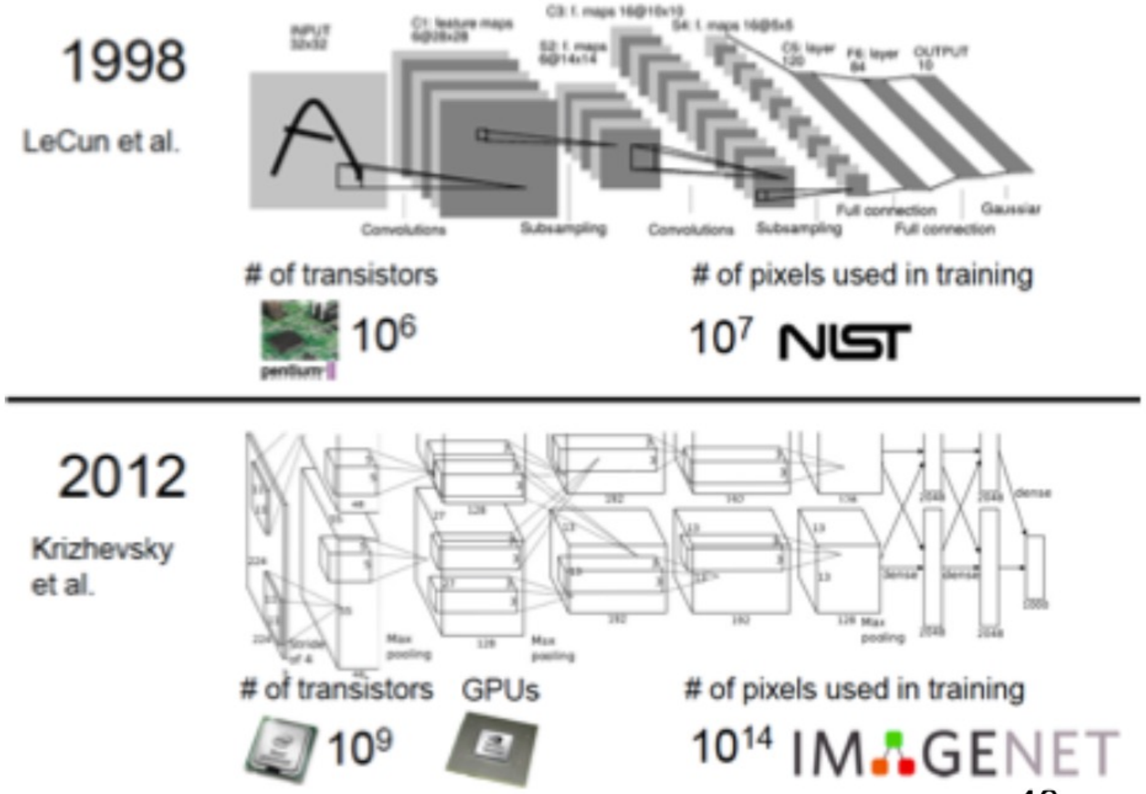
input size:
(K + P - 1) * (K + P- 1)



P = 4

Convolutional Neural Networks

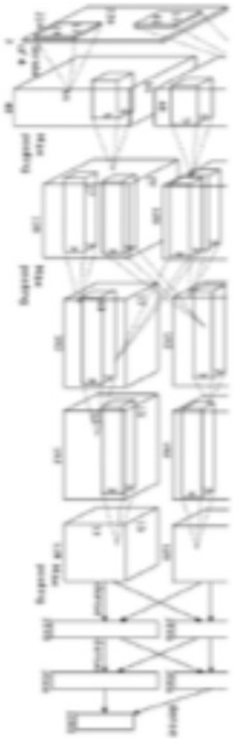
- Not invented over night
  - Back-propagation: ~1975
  - Early convolutional neural networks: ~1988 (Yann LeCun)



Other variants of CNN

Year 2012

SuperVision



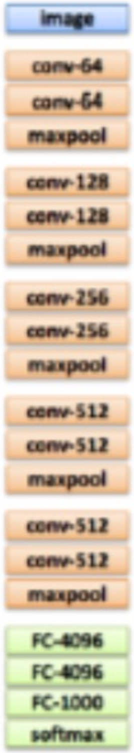
[Krizhevsky NIPS 2012]

Year 2014

GoogLeNet

[Szegedy arxiv 2014]

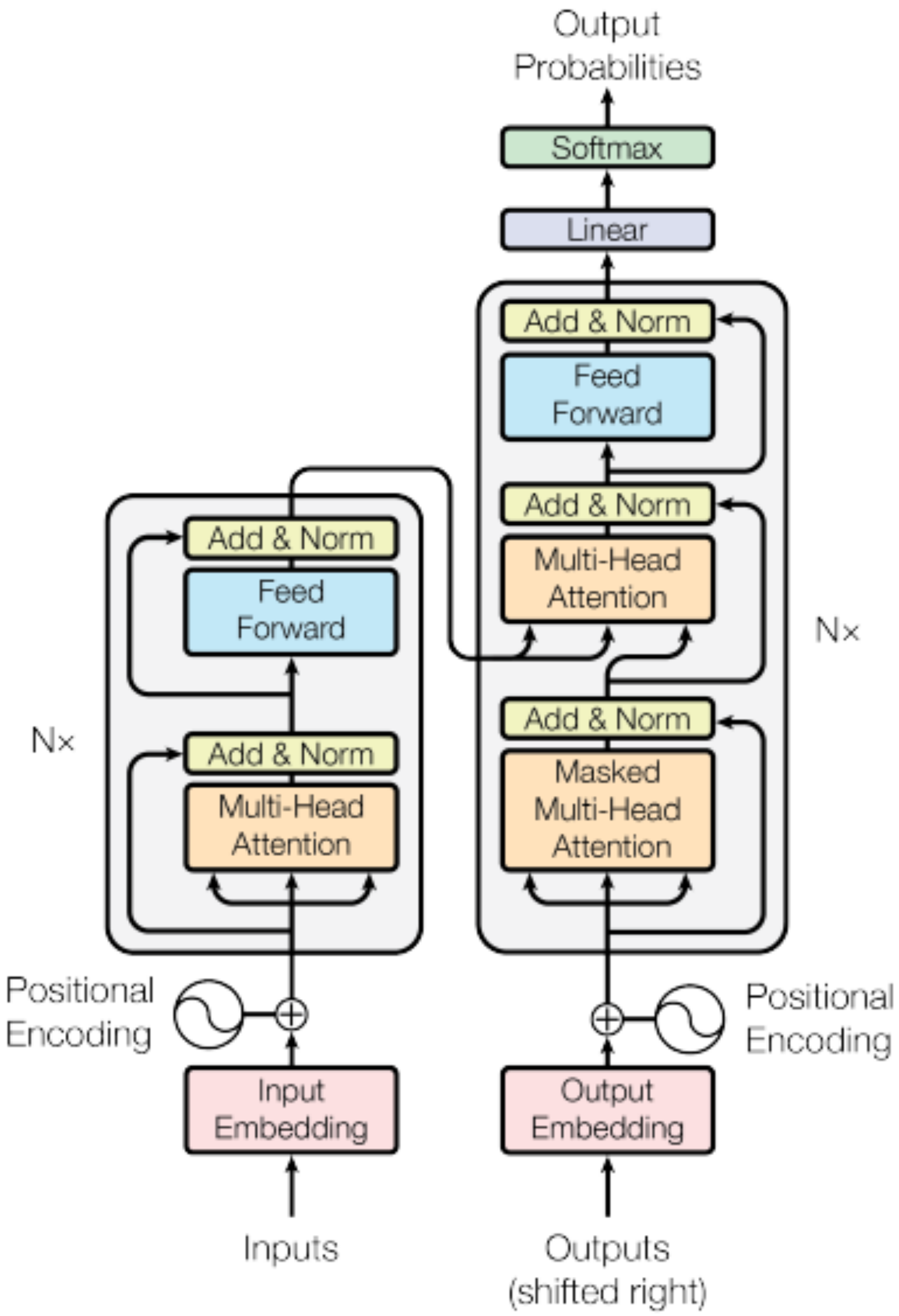
VGG



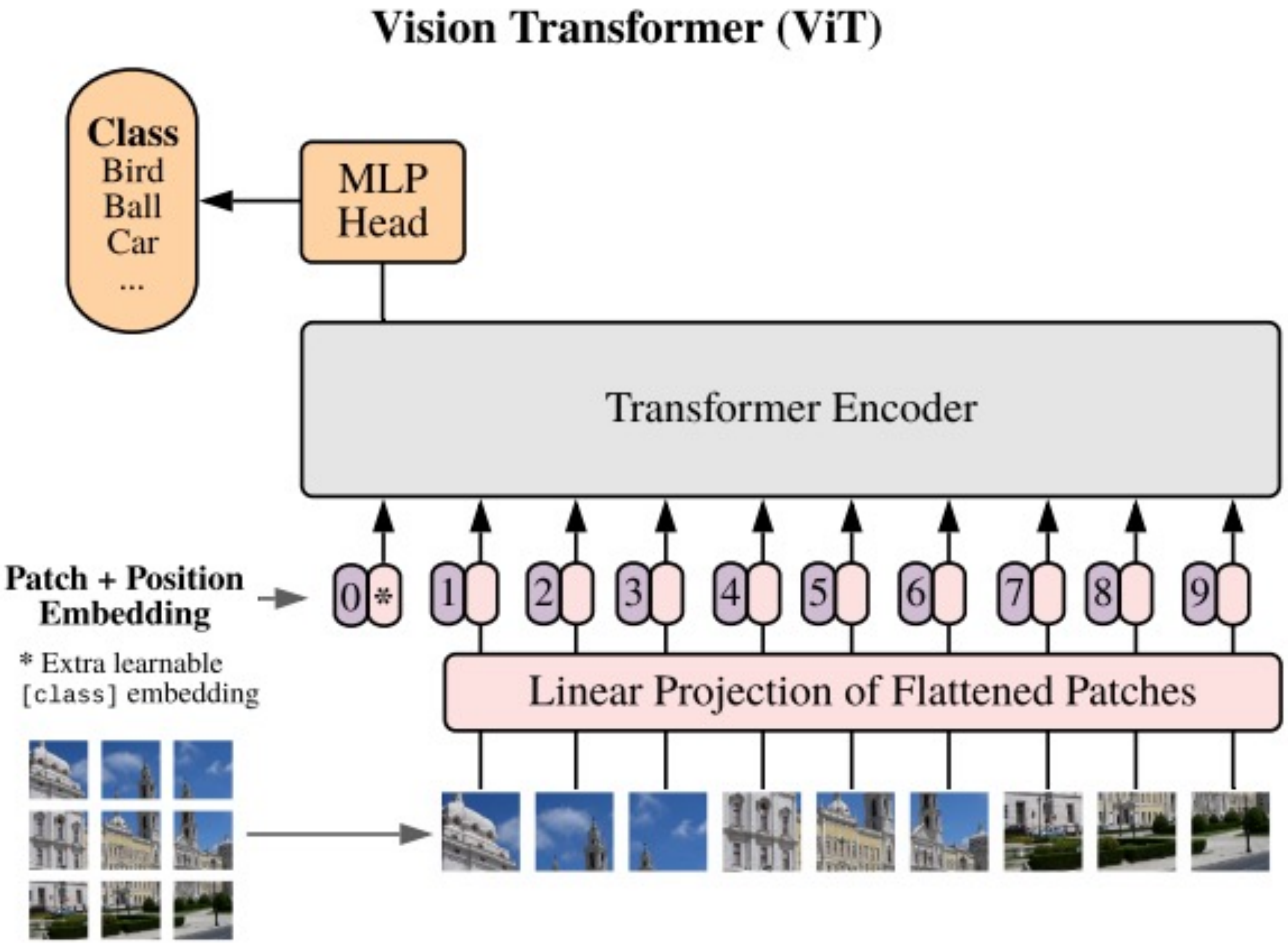
[Simonyan arxiv 2014]

Year 2015

MSRA



2017: Transformer, **Attention is all you need**



2020: VIT, transformer for computer vision tasks.

Reference

- Chapter 6 Deep feedforward networks, Deep Learning, https://www.deeplearningbook.org/