

# Modeling and Performance Analysis of BitTorrent-Like Peer-to-Peer Networks

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# Outline

- Introduction to P2P network.....
- Introduction to BitTorrent.....
- A simple fluid model.....
- The incentive mechanism.....
- Experiments.....
- Conclusions.....

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# Peer-to-peer network

- ❖ Client-Server network vs. Peer-to-peer (P2P) network
- ❖ Peers in a P2P network operate as both servers and clients.
- ❖ P2P traffic is starting to dominate the bandwidth of the Internet.
- ❖ Among P2P applications, file sharing is the most popular one.
- ❖ P2P file sharing system is scalable.
- ❖ BitTorrent, one of the most popular P2P applications, will be studied in this paper.

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# BitTorrent

- ❖ Some concepts of BitTorrent:
  - A server, called the **seed**, has the entire file of interest.
  - The file is separated into many pieces, where each piece is of 256 KB.
  - As peers arrive, they download random pieces of the file from the seed.
  - Now there are **many peers** in the system **with different or overlapping pieces** of the file.
  - The peers can act as servers even they only have parts of the file.
  - **Peers can then download from each other!**
- ❖ In a traditional client-server system (such as FTP), clients can only download from a single server.

# How do BitTorrent work?



Web Server



Tracker

Peer 1



Peer 2



Peer 3



Peer 4



- In this network, there are 4 peers downloading pieces of the file of interest.

# How do BitTorrent work?



Web Server



Tracker

Peer 1



Peer 5



Peer 4



Peer 2



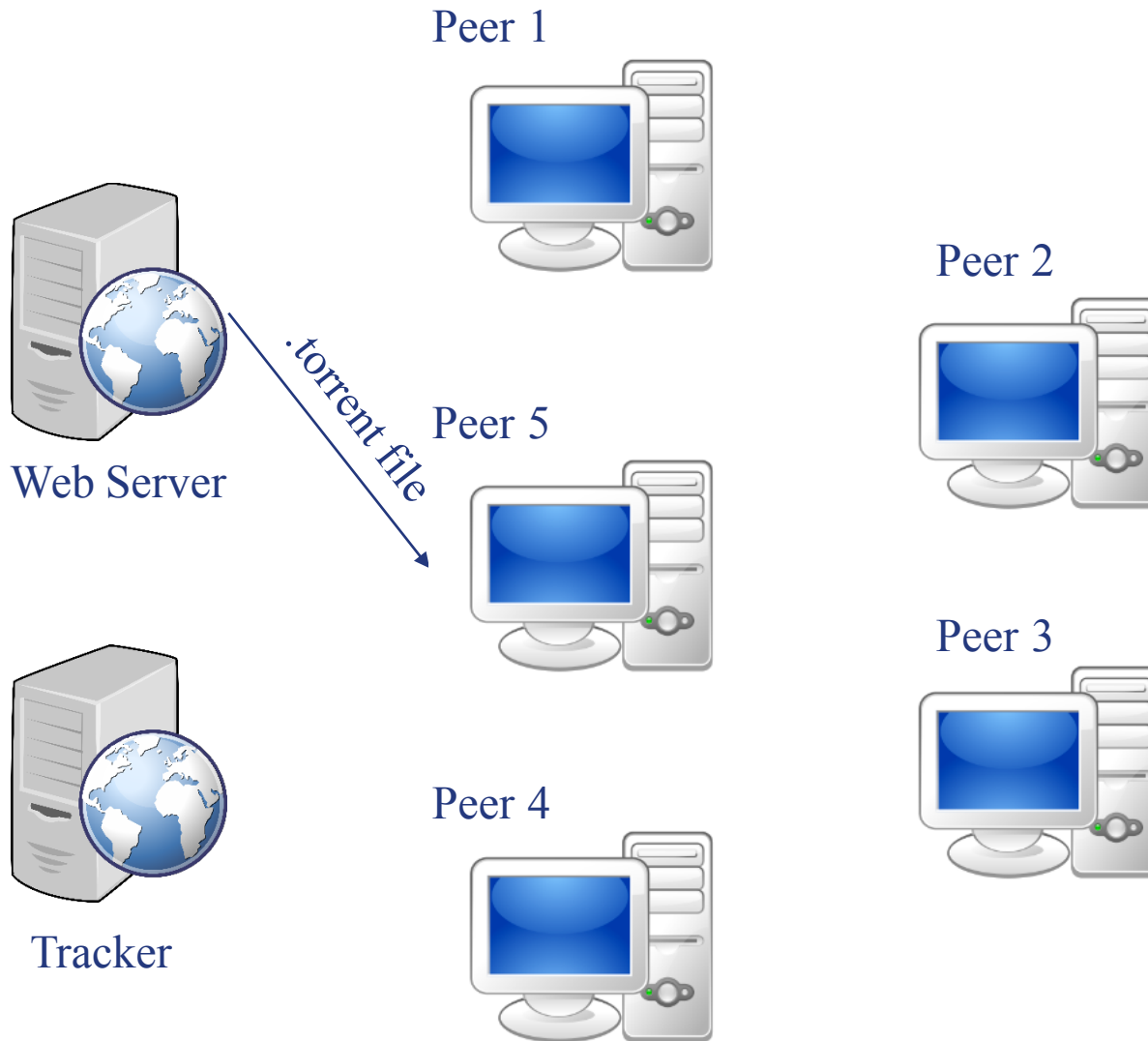
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- In this network, there are 4 peers downloading pieces of the file of interest.
- Peer 5 joins the network to download the file.

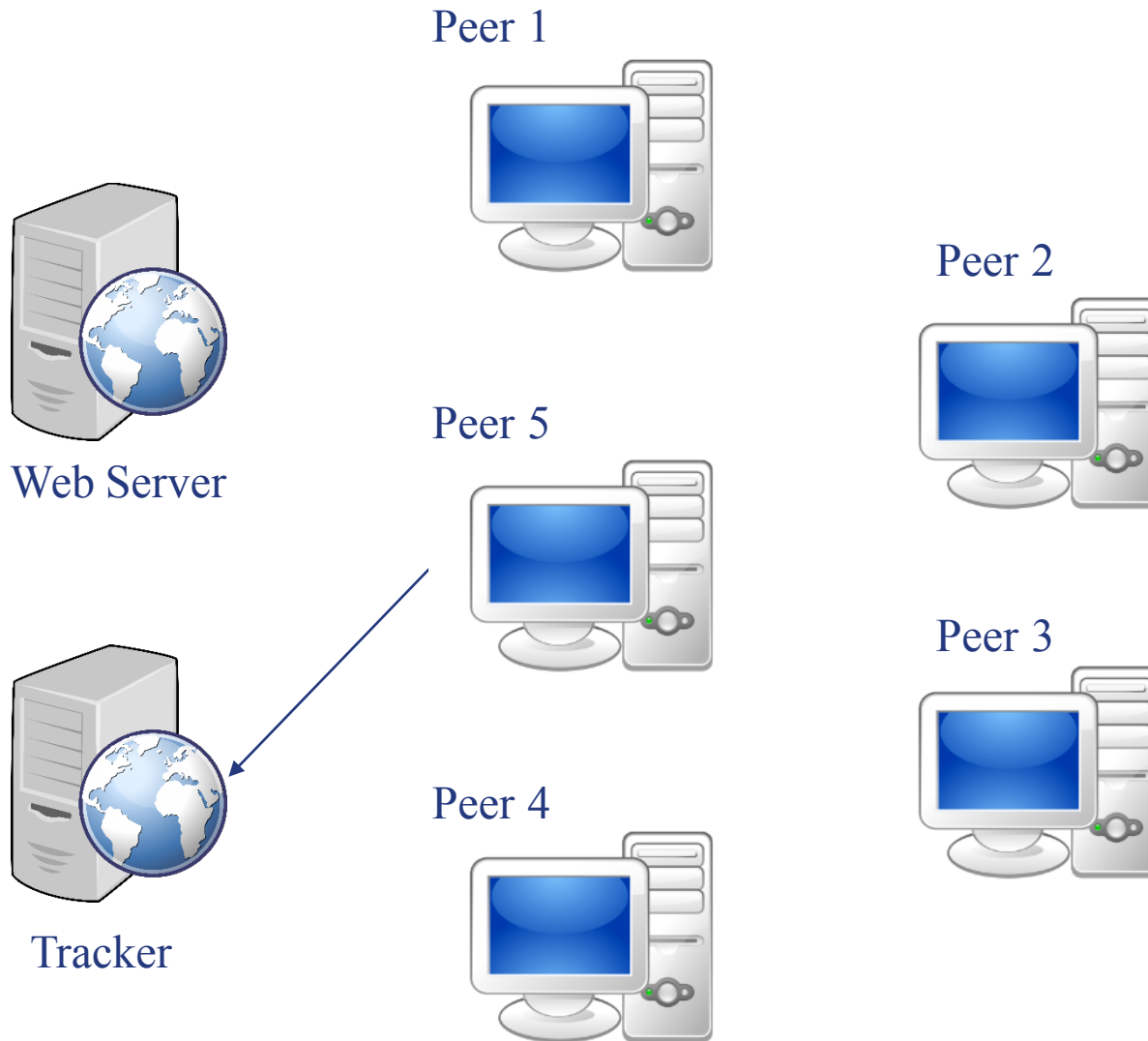


# How do BitTorrent work?



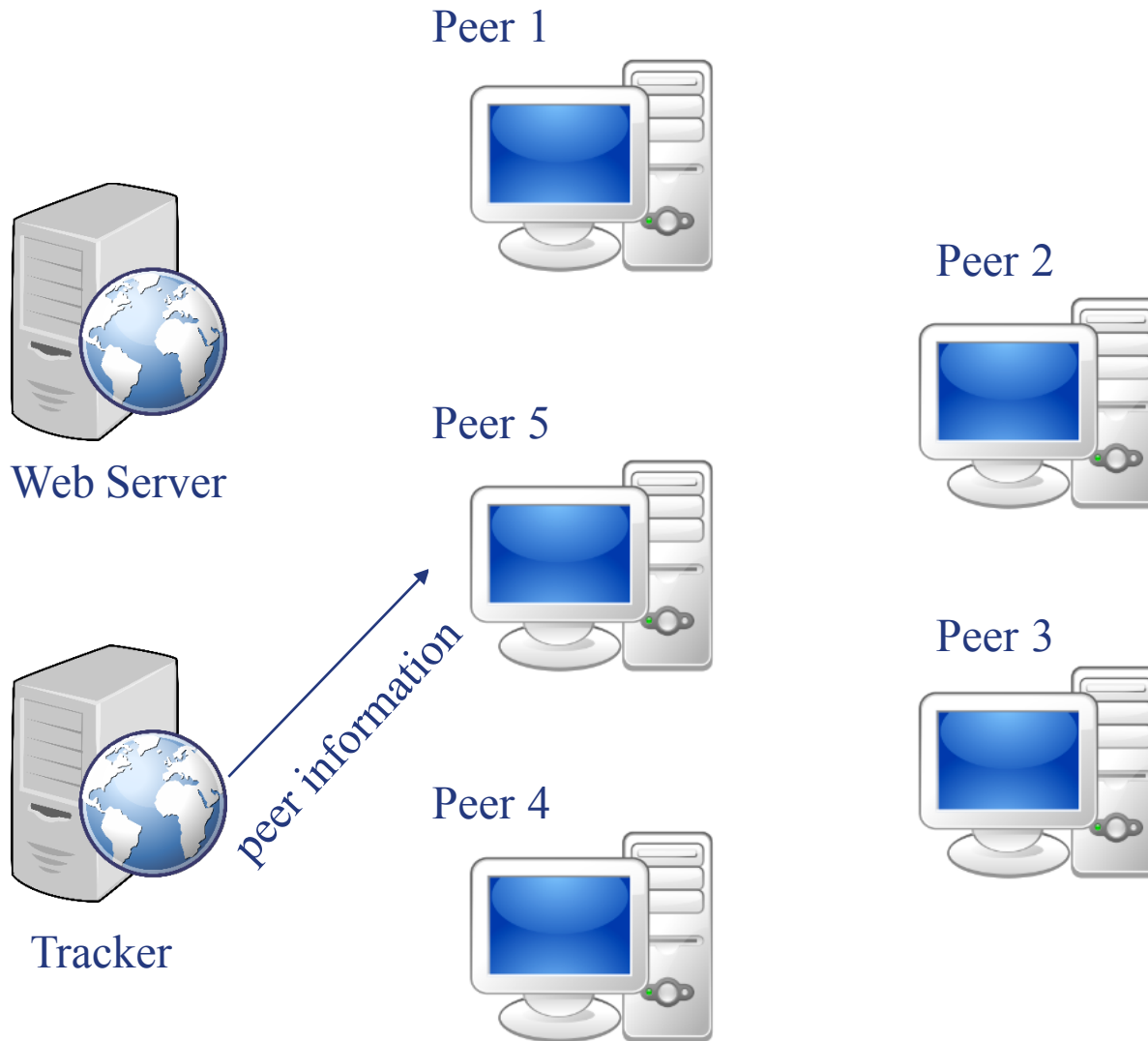
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- Peer 5 joins the network to download the file.
- It first downloads a .torrent file from the web server.

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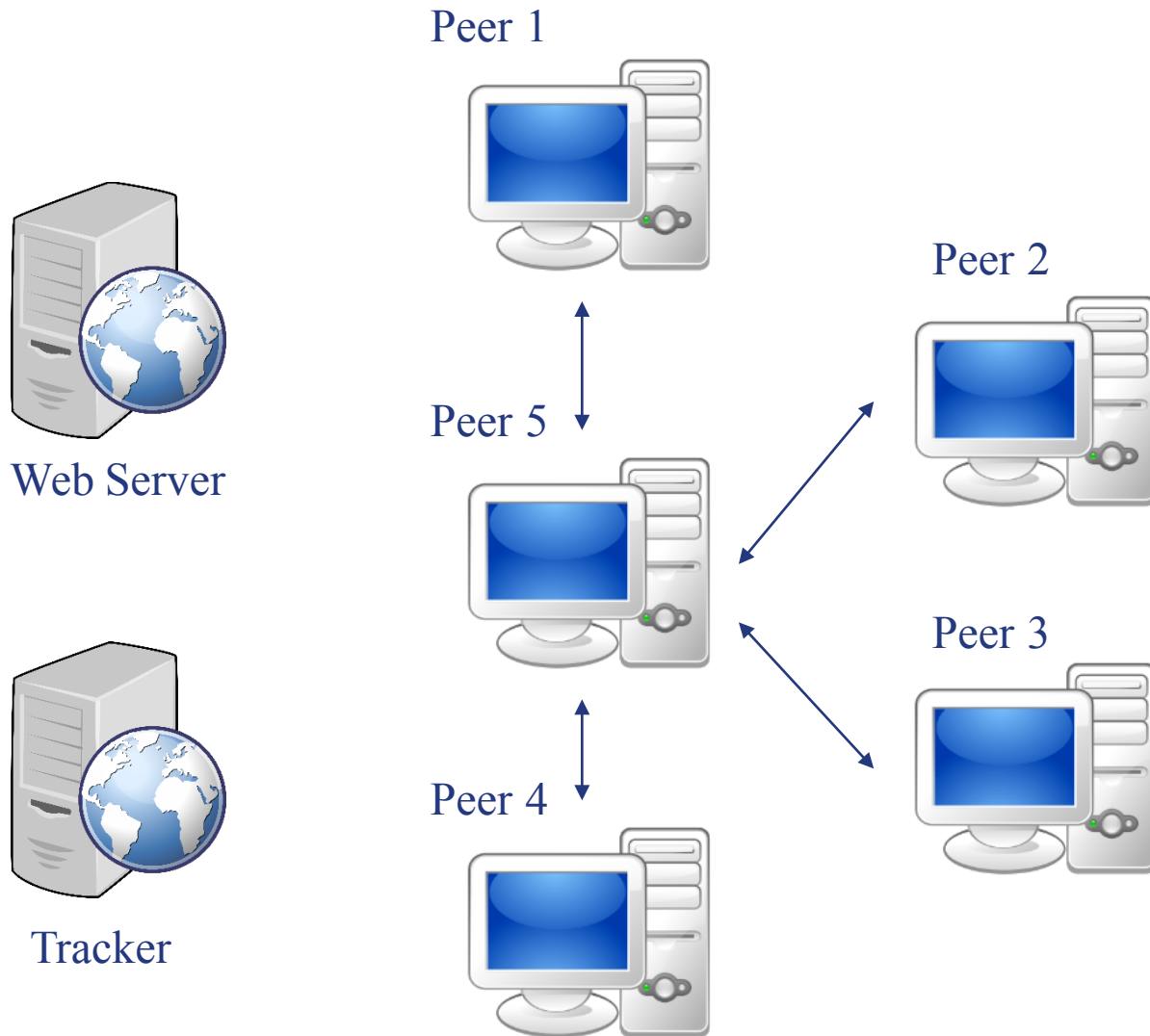
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- It first downloads a .torrent file from the web server.
- Next, it connects to the tracker.

# How do BitTorrent work?



- In this network, there are 4 peers downloading pieces of the file of interest.
- Peer 5 joins the network to download the file.
- It first downloads a .torrent file from the web server.
- Next, it connects to the tracker.
- The tracker returns the peer information.

# How do BitTorrent work?



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- Peer 5 joins the network to download the file.
- It first downloads a .torrent file from the web server.
- Next, it connects to the tracker.
- The tracker returns the peer information.
- Peer 5 connects to 4 other peers and begins downloading and exchanging.

# Some notes behind the operation of BitTorrent

- ❖ **Downloaders** are peers who only have a part (or none) of the file while **seeds** are peers who have all the pieces (the complete file) but stay in the system to allow other peers to download from them.
- ❖ Each peer is allowed to upload only to a fixed number (by default 4) of other peers at a given time.
- ❖ What is and why **unchocking**? To prevent **free-riding**, the most important concern in BitTorrent.
- ❖ The mechanism of **Optimistic Unchoking** → to search for potential peer with high uploading/downloading speed.

# How do Optimistic Unchocking work

Peer 1



Peer 2



Peer 5



Peer 3

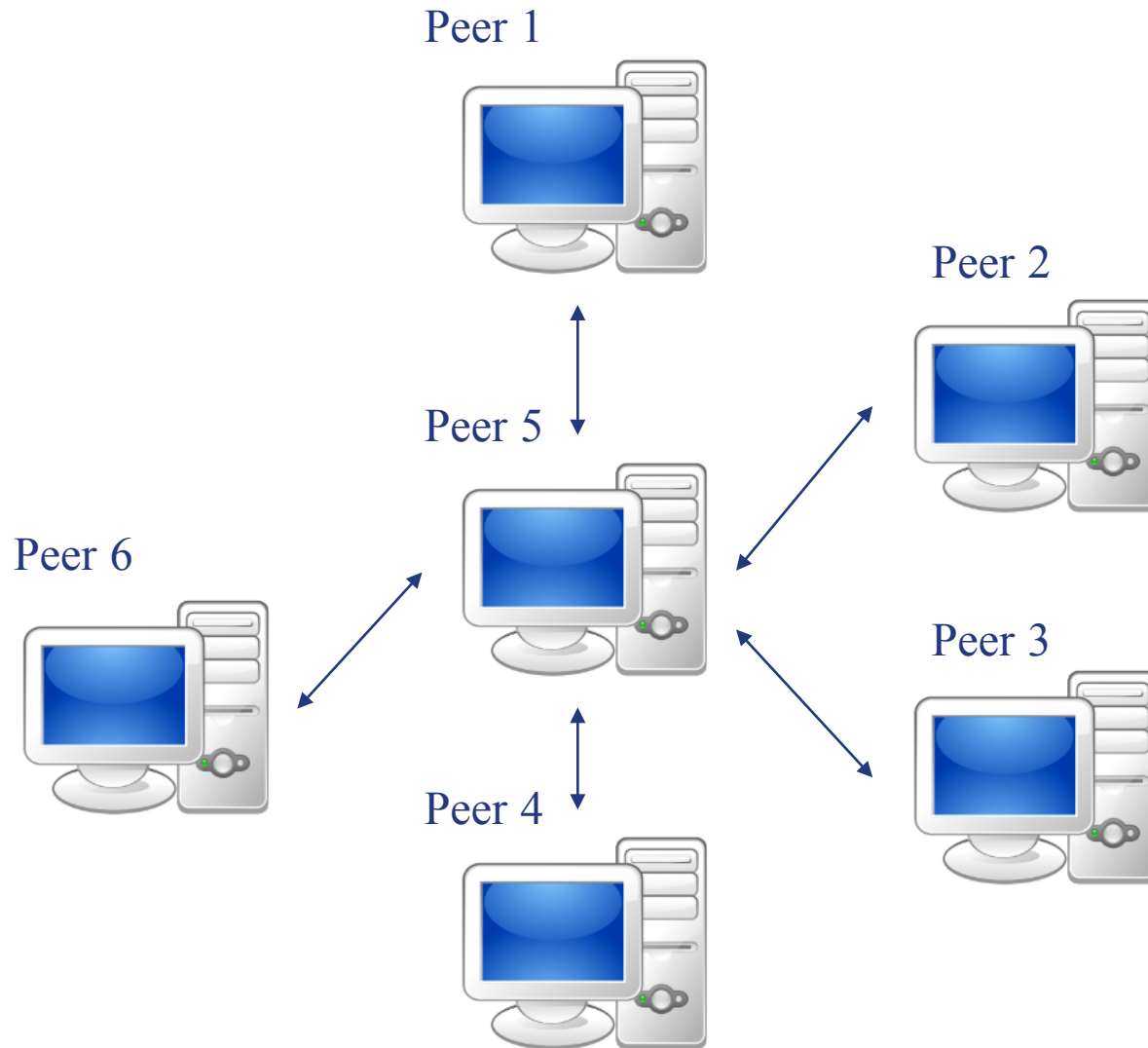


Peer 4



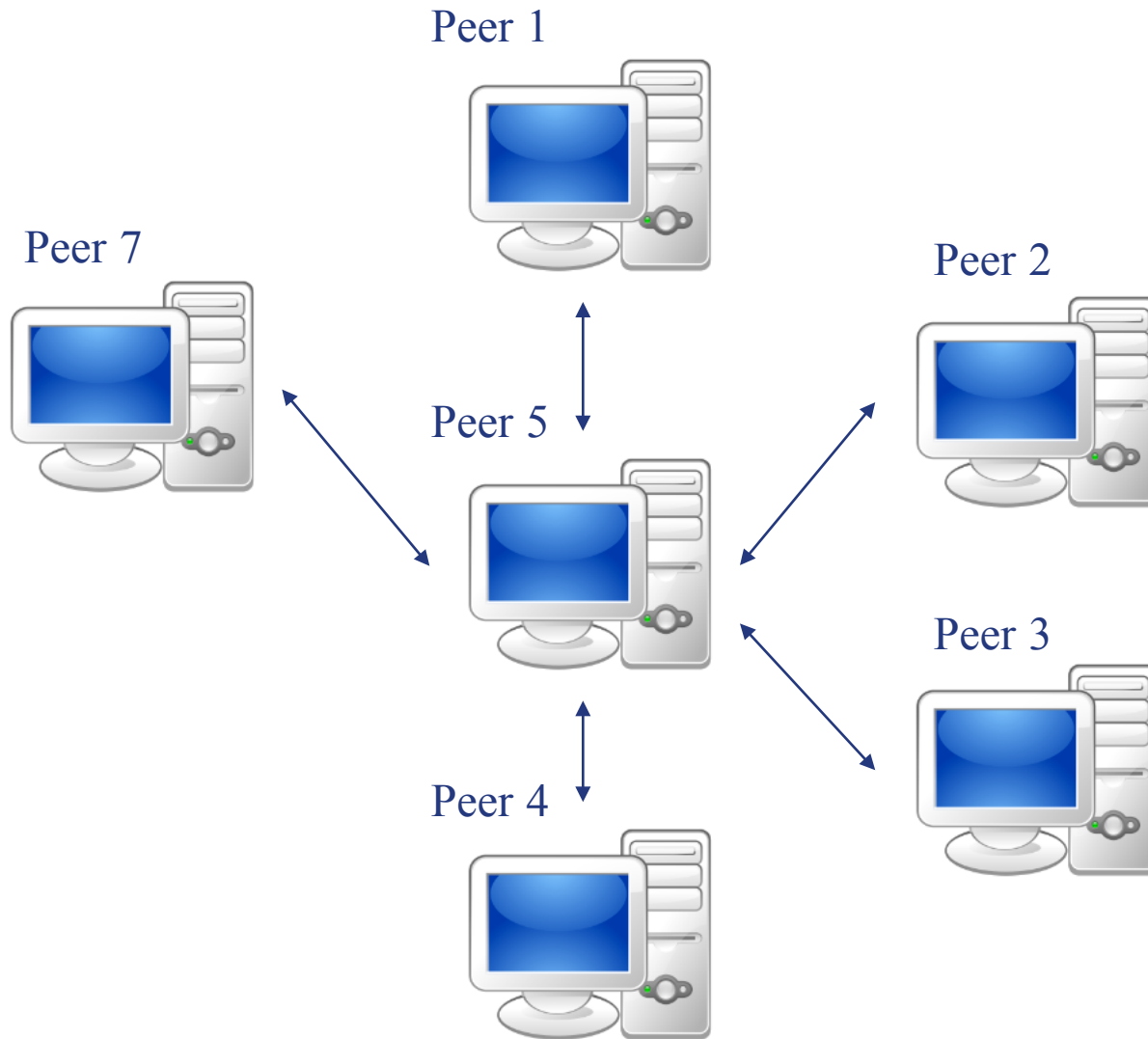
- Peer 5 is now connecting with peer 1 ~ 4 for downloading / uploading the file of interest.

# How do Optimistic Unchocking work



- Peer 5 is now connecting with peer 1 ~ 4 for downloading / uploading the file of interest.
- After 30 seconds, peer 5 randomly connects to another peer, say peer 6, and finds that the transmission speed of peer 6 does not exceed any of the peers that peer 5 has already connected to.

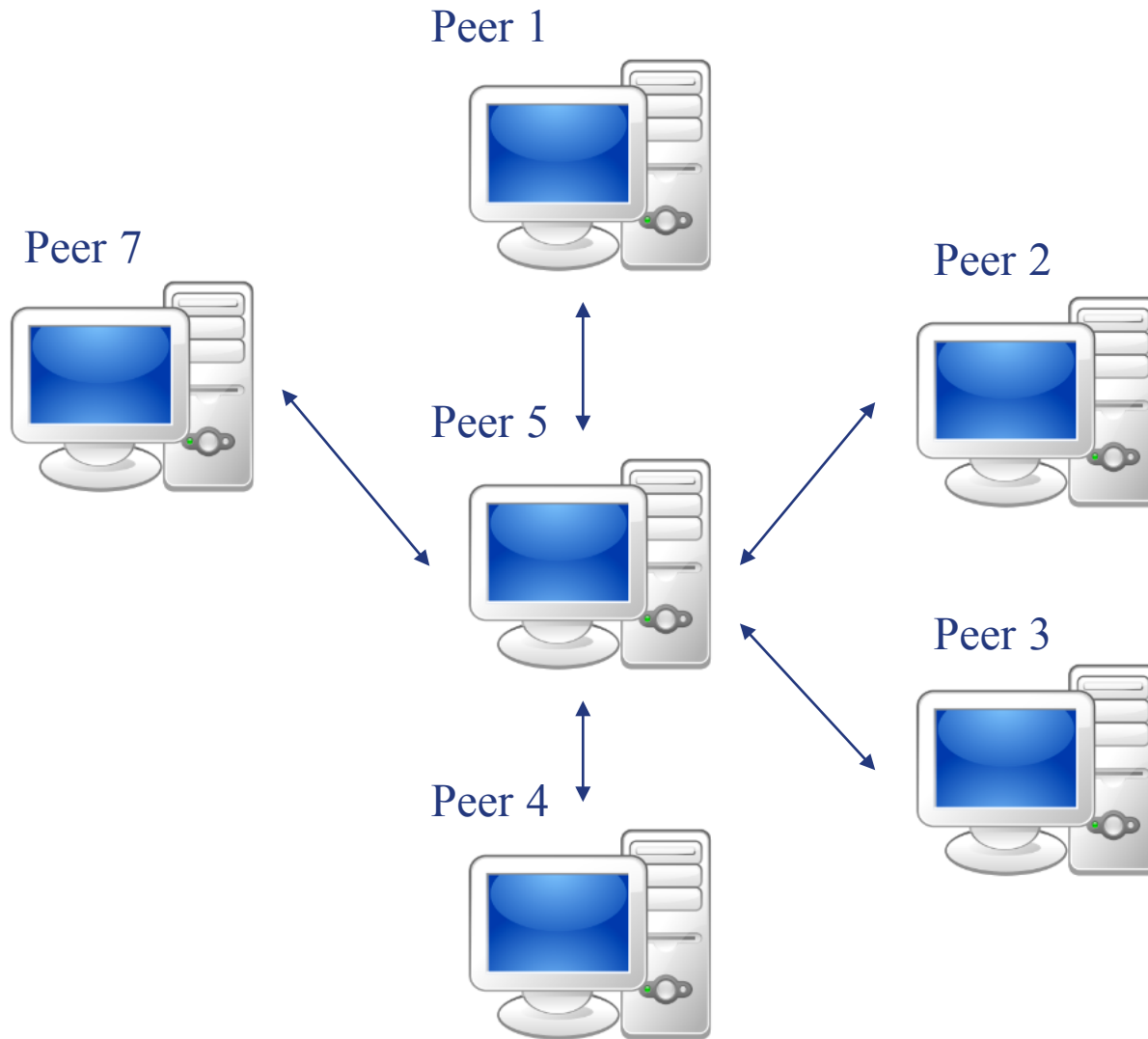
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- Peer 5 is now connecting with peer 1 ~ 4 for downloading / uploading the file of interest.
- After 30 seconds, peer 5 randomly connects to another peer, say peer 6, and finds that the transmission speed of peer 6 does not exceed any of the peers that peer 5 has already connected to.
- Therefore, after another 30 seconds, peer 5 disconnects with peer 6, and connects with another peer, say peer 7.

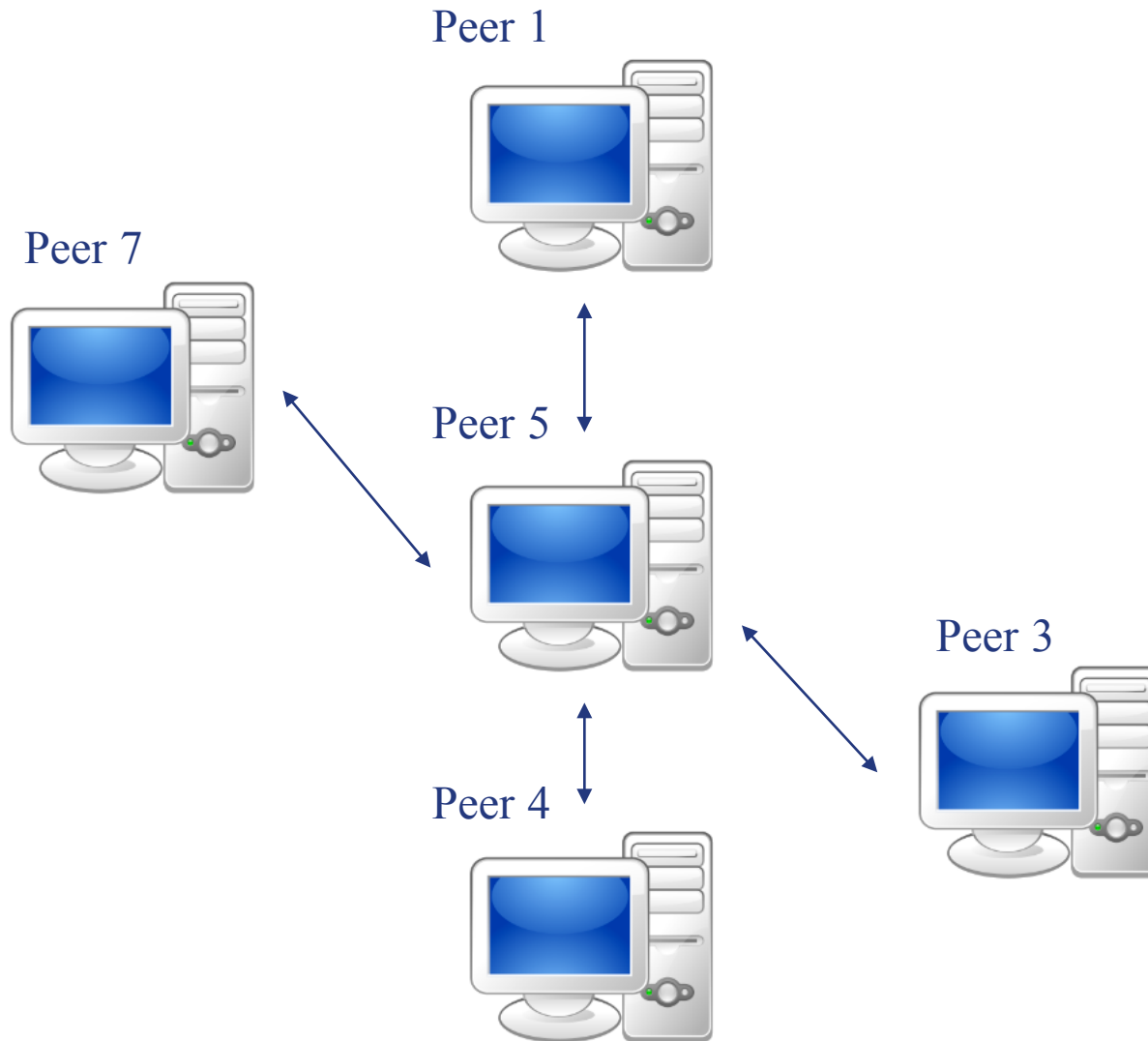


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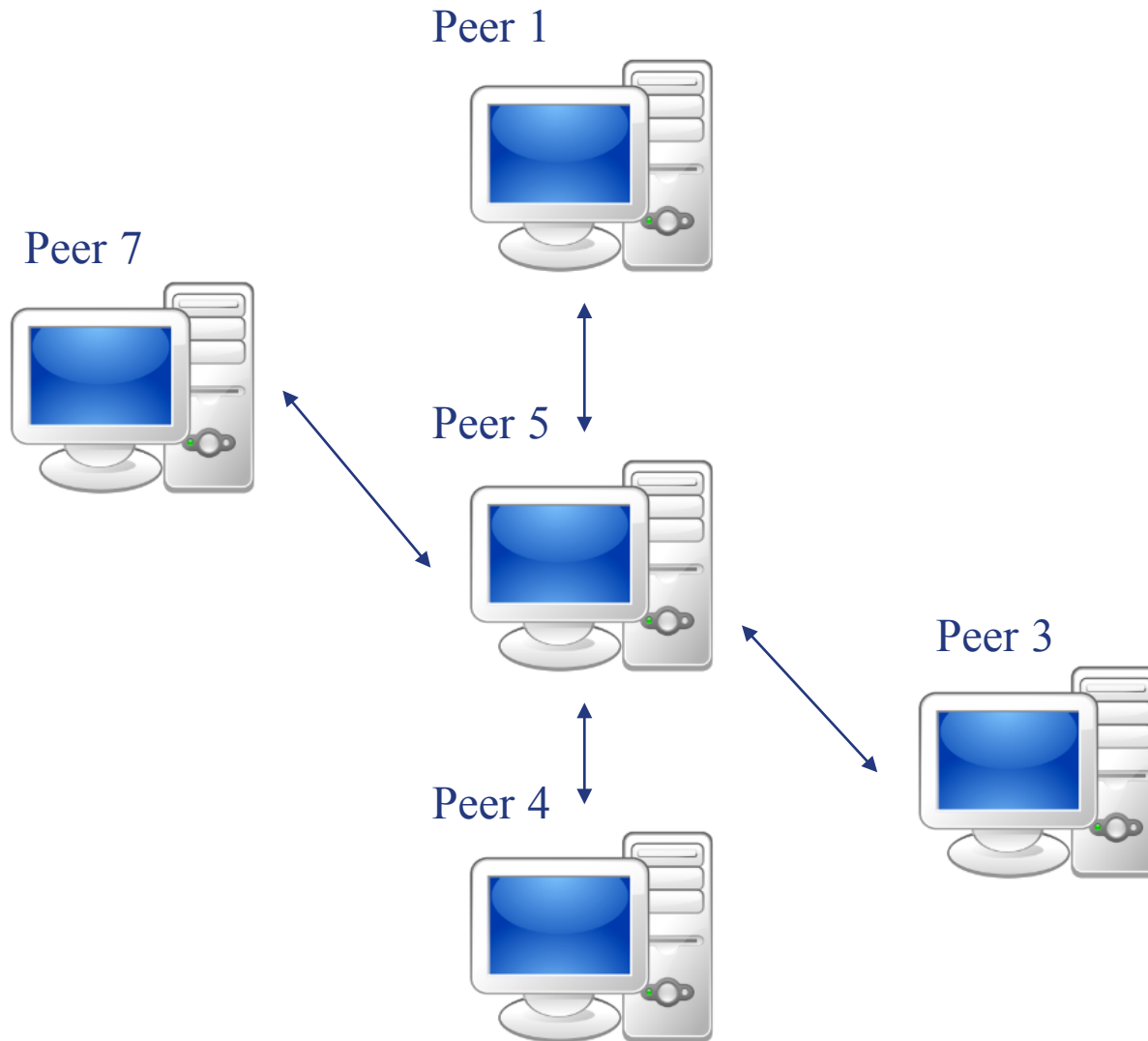
- Peer 5 finds that after it connects to peer 7, peer 2 becomes the one that has the slowest transmission speed.

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- Therefore, peer 5 disconnects with peer 2 after another 30 seconds.

# How do Optimistic Unchoking work



- Peer 5 finds that after it connects to peer 7, peer 2 becomes the one that has the slowest transmission speed.
- Therefore, peer 5 disconnects with peer 2 after another 30 seconds.
- The same process is repeated again and again ...

# Some Issues of BitTorrent

- ❖ **Peer Evolution:** how the number of peers evolves as a function of the request arrival rate, the peer departure rate, the uploading/downloading bandwidth of each peer, etc.
- ❖ **Scalability:** as the size of the network (number of peers) grows, the network performance (average file downloading time) should preferably to actually improve.
- ❖ **File Sharing Efficiency:** as peers often have different uploading/downloading bandwidths, it is important to design a good file-sharing protocol such that the uploading/downloading bandwidths are fully utilized.
- ❖ **Incentives to prevent free-riding:** need to a mechanism to deter peers from free-riding (download from other peers while not uploading to others).

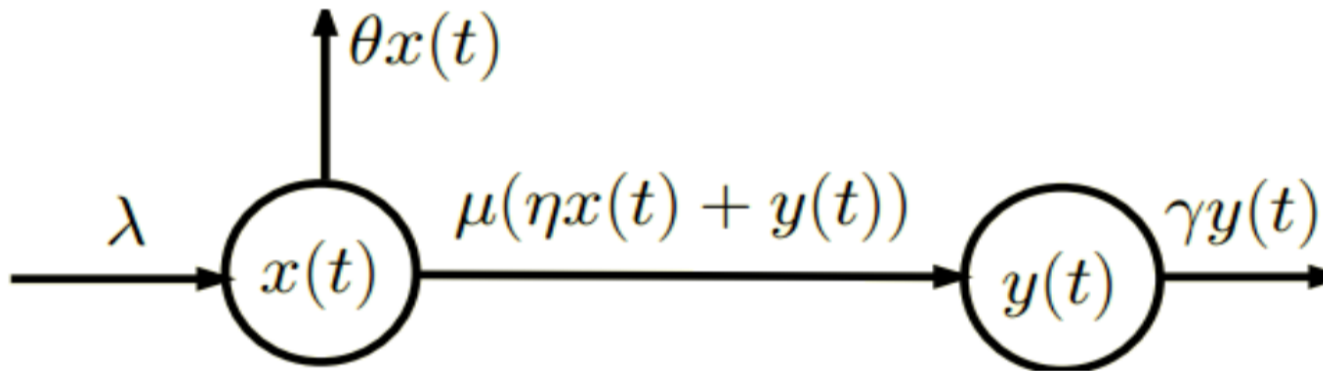
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# Fluid Model

- ❖  $x(t)$  - number of downloaders in the system at time  $t$ .
- ❖  $y(t)$  - number of seeds in the system at time  $t$ .
- ❖  $\lambda$  - the arrival rate of new requests (Poisson process)
- ❖  $\mu$  - the uploading bandwidth of a given peer (normalized by file size)
- ❖  $C$  - the downloading bandwidth of a given peer (normalized by file size).
- ❖  $\theta$  - the rate at which downloaders abort the download (Poisson process)
- ❖  $\gamma$  - the rate at which seeds leave the system (Poisson process)
- ❖  $\eta$  - indicates the effectiveness of the file sharing , takes values in  $[0, 1]$ .

# Fluid Model



$$\begin{aligned}\frac{dx}{dt} &= \lambda - \theta x(t) - \min\{cx(t), \mu(\eta x(t) + y(t))\}, \\ \frac{dy}{dt} &= \min\{cx(t), \mu(\eta x(t) + y(t))\} - \gamma y(t),\end{aligned}$$

# Steady-State Performance

- ❖ To study the system in steady-state, we let

$$\begin{aligned}\frac{dx(t)}{dt} = \frac{dy(t)}{dt} = 0 \quad & 0 = \lambda - \theta\bar{x} - \min\{c\bar{x}, \mu(\eta\bar{x} + \bar{y})\}, \\ & 0 = \min\{c\bar{x}, \mu(\eta\bar{x} + \bar{y})\} - \gamma\bar{y},\end{aligned}$$

- ❖  $\bar{X}$ ,  $\bar{Y}$  are equilibrium values

$$\begin{aligned}\bar{x} &= \frac{\lambda}{\beta(1 + \frac{\theta}{\beta})} \\ \bar{y} &= \frac{\lambda}{\gamma(1 + \frac{\theta}{\beta})}.\end{aligned}$$

- ❖  $\beta$  is determined by the bottleneck between download rate and upload rate

$$\frac{1}{\beta} = \max\left\{\frac{1}{c}, \frac{1}{\eta}\left(\frac{1}{\mu} - \frac{1}{\gamma}\right)\right\}$$



# Steady-State Performance(cont.)

- ❖ Little's law: Average download time

$$\frac{\lambda - \theta \bar{x}}{\lambda} \bar{x} = (\lambda - \theta \bar{x})T, \quad T = \frac{1}{\theta + \beta}.$$

$$\frac{1}{\beta} = \max\left\{\frac{1}{c}, \frac{1}{\eta}\left(\frac{1}{\mu} - \frac{1}{\gamma}\right)\right\}$$

- ❖ Scalability: T is not a function of  $\lambda$ , the request arrival rate
- ❖ Need for incentives: When the seed departure rate  $\gamma$  increases, T increases
- ❖ Initially when the downloading rate  $c$  increases, T decreases. However, beyond a point the uploading rate becomes the bottleneck

# Effectiveness of File Sharing

- ❖ For a given downloader  $i$ , we assume that it is connected to  $k$  other downloaders.
  - $\eta = 1 - P(\text{downloader } i \text{ has no piece that the connected peers need})$ .
- ❖ We assume that the piece distributions between different peers are independent and identical. Then
  - $\eta = 1 - P(\text{downloader } j \text{ needs no piece from downloader } i)^k$ ,
  - where  $j$  is a downloader connected to  $i$ .
- ❖ For each downloader, we assume that the number of pieces it has is uniformly distributed in  $\{0, \dots, N - 1\}$ , where  $N$  is the number of pieces of the served file.

# Effectiveness of File Sharing(cont.)

❖ Let  $n_i$  be number of pieces at downloader  $i$ .

$$\begin{aligned} & \mathbf{P} \left\{ \begin{array}{l} \text{downloader } j \text{ needs no} \\ \text{piece from downloader } i \end{array} \right\} \\ &= \mathbf{P}\{j \text{ has all pieces of downloader } i\} \\ &= \sum_{n_j=1}^{N-1} \sum_{n_i=0}^{n_j} \frac{1}{N^2} \mathbf{P}\{j \text{ has all pieces of } i | n_i, n_j\} \\ &= \sum_{n_j=1}^{N-1} \sum_{n_i=0}^{n_j} \frac{1}{N^2} \frac{\binom{N-n_i}{n_j-n_i}}{\binom{N}{n_j}} \\ &= \frac{N+1}{N^2} \sum_{m=2}^N \frac{1}{m} \approx \frac{\log N}{N} \end{aligned}$$

# Effectiveness of File Sharing(cont.)

$$\eta \approx 1 - \left( \frac{\log N}{N} \right)^k .$$

- ❖ In BitTorrent, each piece is typically 256KB. For a file that is a few hundreds of megabytes in size,  $N$  is of the order of several hundreds.
- ❖ Even if  $k = 1$ ,  $\eta$  is very close to one.
- ❖ When  $k$  increases,  $\eta$  also increases but very slowly and the network performance increases slowly. Hence, when  $\lambda$  increases, the network performance increases but very slowly.
- ❖ When  $k = 0$ ,  $\eta = 0$ .

# Local Stability

- ❖ eigenvalues of  $A_1$  and  $A_2$  have negative real parts – system is stable.

$$\frac{dx(t)}{dt} = \lambda - \theta x(t) - \mu(\eta x(t) + y(t))$$

$$\frac{dy(t)}{dt} = \mu(\eta x(t) + y(t)) - \gamma y(t).$$

$$\mathbf{A}_1 = \begin{bmatrix} -(\mu\eta + \theta) & -\mu \\ \mu\eta & -(\gamma - \mu) \end{bmatrix}.$$

$$\frac{dx(t)}{dt} = \lambda - \theta x(t) - cx(t)$$

$$\frac{dy(t)}{dt} = cx(t) - \gamma y(t).$$

$$\mathbf{A}_2 = \begin{bmatrix} -(\theta + c) & 0 \\ c & -\gamma \end{bmatrix}.$$

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# Peer Selection Algorithm

- ❖ Peer  $i$  selects the  $n_u$  (default 4) peers that give it the best upload to download to
- ❖ Assumptions : Global information , No optimistic unchoking , No download limit
- ❖ Sort the peers according to their uploading bandwidth (physical or determined), first peer has the highest uploading bandwidth.
- ❖ peer  $i$  choosing peers to upload at step  $i$ .
- ❖  $N$  total number of peers
- ❖  $u_i$ : the uploading bandwidth of peer  $i$ .

# Peer Selection Algorithm(cont.)

1. If peer  $i$  is selected by peer  $j$  ( $j < i$ ), then  $i$  selects  $j$ . For any peer  $k$  ( $k \geq i$ ), let  $n_k^i$  be the number of peers that have selected peer  $k$  prior to step  $i$ .
2. If  $n_i^i < n_u$  and  $n_u - n_i^i \leq N - i$ , peer  $i$  selects  $n_u - n_i^i$  peers from the set  $\{k | k > i\}$  using the following set of rules to prioritize a peer, say  $k1$ , over another peer  $k2$  :
  - (a) If  $\mu_{k1} > \mu_{k2}$ , select  $k1$ .
  - (b) If  $\mu_{k1} = \mu_{k2}$  and  $n_{k1}^i < n_{k2}^i$ , select  $k1$ .
  - (c) If  $\mu_{k1} = \mu_{k2}$ ,  $n_{k1}^i = n_{k2}^i$ , and  $k1 < k2$ , select  $k1$ .
3. If  $n_i^i < n_u$  and  $n_u - n_i^i > N - i$ , peer  $i$  selects all peers in  $\{k | k > i\}$  and also randomly selects  $(n_u - n_i^i) - (N - i)$  peers from the peers that  $i$  has not selected yet.



# Peer Selection Algorithm(cont.)

- ❖ Lemma 1. With the peer selection algorithm, when peer  $i$  selects uploading peers,  $n_i^i \leq n_u$  and for any  $k_2 > k_1 \geq I$ ,  $n_{k_2}^i \leq n_{k_1}^i \leq n_u$
- ❖ Lemma 2. Suppose that peers  $i, i+1, \dots, j$  have the same uploading bandwidth  $\mu$ . If  $j - i + 1 > n_u \geq 2$ , then for any  $k > j$ , we have
  1.  $d(i) \geq d(i+1) \geq \dots \geq d(j) \geq d(k)$ ,
  2.  $d(i) > d(k)$ ,
  3.  $d(\mu) > d(k)$

where

$$d_i = \frac{1}{n_u} \sum_{k \in D_i} \mu_k. \quad d(\mu) = \frac{1}{j - i + 1} \sum_{k=i}^j d_k.$$

# Peer Strategy

- ❖ First maximizing download bandwidth , then minimizing upload bandwidth
- ❖ Then peer  $i$  choose  $u_i$  s.t.

$$\mu_i = \min\{\tilde{\mu}_i | d_i(\tilde{\mu}_i, \mu_{-i}) = d_i(p_i, \mu_{-i})\}.$$

$P_i$  denote the physical upload rate

- ❖ Or more realistic

$$\mu_i = \min\{\inf\{\tilde{\mu}_i | d_i(\tilde{\mu}_i, \mu_{-i}) = d_i(p_i, \mu_{-i})\} + \varepsilon, p_i\}$$

- ❖ Where  $\varepsilon$  is the difference between two rates that a peer can differentiate.

# Peer Strategy(ex.)

- ❖ consider a small BitTorrent network with 6 peers
- ❖ The number of uploads  $n_u = 4$  for all peers
- ❖ peers have different physical uploading bandwidth
- ❖ If the minimum uploading bandwidth  $\min\{p_i\} > 2\varepsilon$ , there is no Nash equilibrium point for the system

# Nash Equilibrium Point

- ❖ Divide the network to sub-groups . In each group  $j$ , all peers have the same physical uploading bandwidth  $p_j$ .

*PROPOSITION 1. If  $n_u \geq 2$  and the number of peers in a group  $\|g_j\| > n_u + 1$  for all groups, there exists a Nash equilibrium point for the system, in which  $\bar{\mu}_i = p_j$  if peer  $i \in g_j$ . Moreover, with any initial setting of  $\{\mu_i^0\}$ , the system converges to the Nash equilibrium point  $\{\bar{\mu}_i\}$ .*

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# Optimistic Unchoking Recap

- ❖ Each peer uploads to other  $n_u$  (by default 4) peers which provide it with the best downloading rate.
- ❖ Under optimistic unchoking, each peer randomly selects a fifth peer to download from exploring other download rates.
- ❖ Then peer with the least downloading rate is dropped.
- ❖ Optimistic unchoking happens once every 30 seconds.
- ❖ However, such mechanism gives chances to free-riders!

# Free-riding Recap

- ❖ Free-riding means that a peer does not contribute anything to the system, but attempts to obtain service (or downloading) from other peers.
- ❖ Lack of global knowledge (full downloading rates information) requires the use of optimistic unchoking.
- ❖ This allows for peers with no uploading bandwidth to get download bandwidth.

# Free-riding Formulation

- ❖ Consider a group of peers that have the same uploading bandwidth  $\mu$  and number of peers in the group  $N$ .
- ❖ Assume that each peer has  $n_u$  uploads and 1 optimistic unchoking upload.
- ❖ By optimistic unchoking, peer  $j$  which is a free-rider get selected  $1/(N - n_u)$  of the time by any other peer.
- ❖ So, total average downloading rate of peer  $j$  will be

$$N \frac{1}{N - n_u} \frac{\mu}{n_u + 1} \approx \frac{\mu}{n_u + 1}$$

when  $N$  is large.

- ❖ The free-riders problem is not yet solved in BitTorrent.  
(in BitTorrent,  $n_u = 4$  so free-rider gets 20% of the possible maximum downloading rate.)



# Notations Recap

- ❖  $x(t)$  : number of downloaders in the system at time  $t$ .
- ❖  $y(t)$  : number of seeds in the system at time  $t$ .
- ❖  $\lambda$  : the arrival rate of new request.
- ❖  $\mu$  : the uploading bandwidth of a given peer.
- ❖  $c$  : the downloading bandwidth of a given peer.
- ❖  $\theta$  : the rate at which downloaders abort the system.
- ❖  $\gamma$  : the rate at which seeds leaves the system.
- ❖  $\eta$  : the effectiveness of the file sharing.  $\eta$  takes values in  $[0,1]$ .

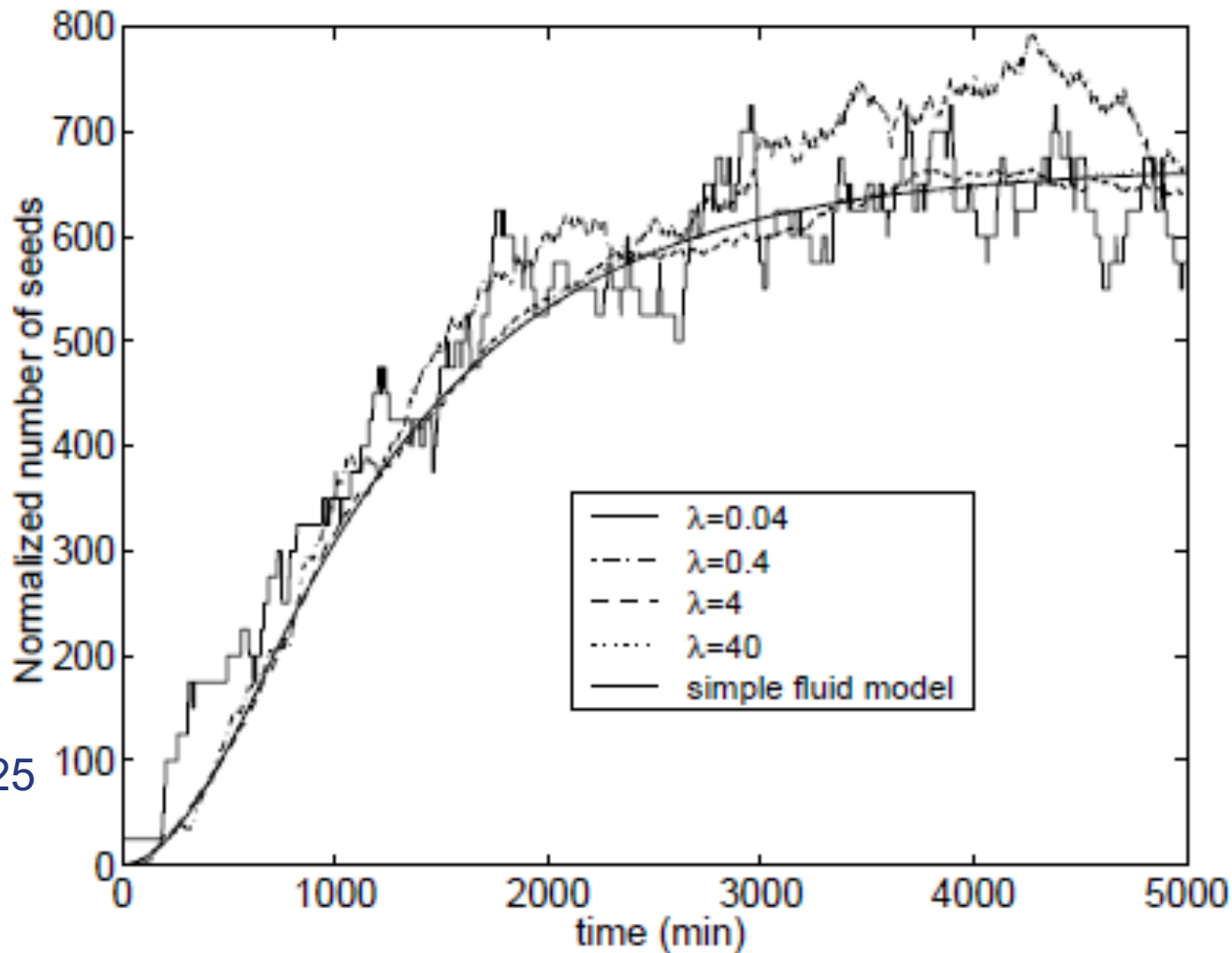
# Experiment Setup

❖ Compare fluid model to:

- (1) Simulated BitTorrent network that is download-limited ( $\gamma < \mu$ )
  - Discrete-event simulation based on the Markov model
  - The arrival rate  $\lambda \in \{0.04, 0.4, 4, 40\}$ .
  - Normalize the number of seeds/downloaders by dividing by  $\lambda$
- (2) Simulated BitTorrent network that is upload-limited (same setup with (1), but  $\gamma > \mu$ )
- (3) Real-world BitTorrent network
  - collect the log files of the BitTorrent tracker for a time period of around three days.

Set  $\eta = 1$  for all cases.

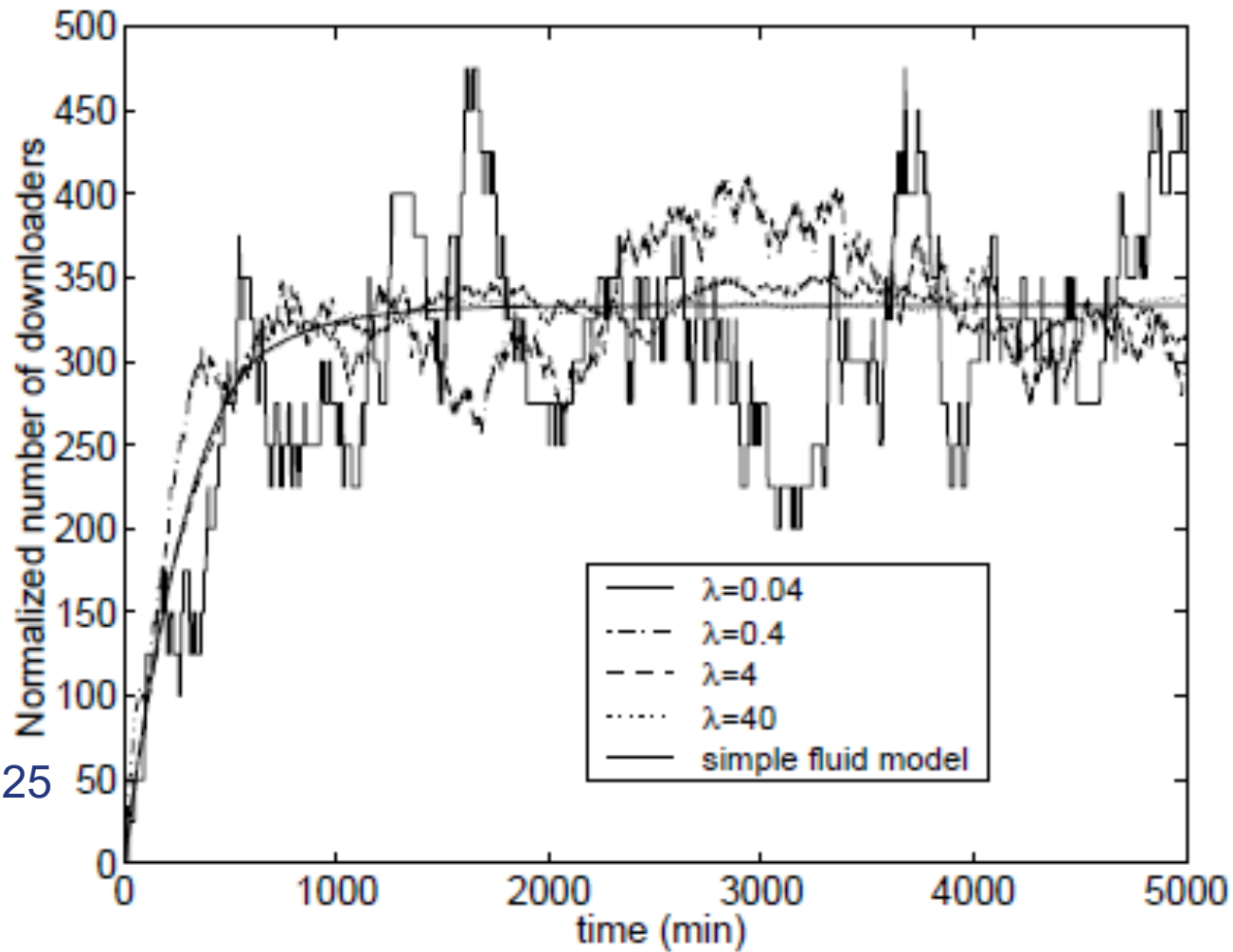
# Experiment 1



$\mu = 0.00125$   
 $\theta = 0.001$   
 $c = 0.002$   
 $\gamma = 0.001$

$\lambda$  : arrival rate of new request

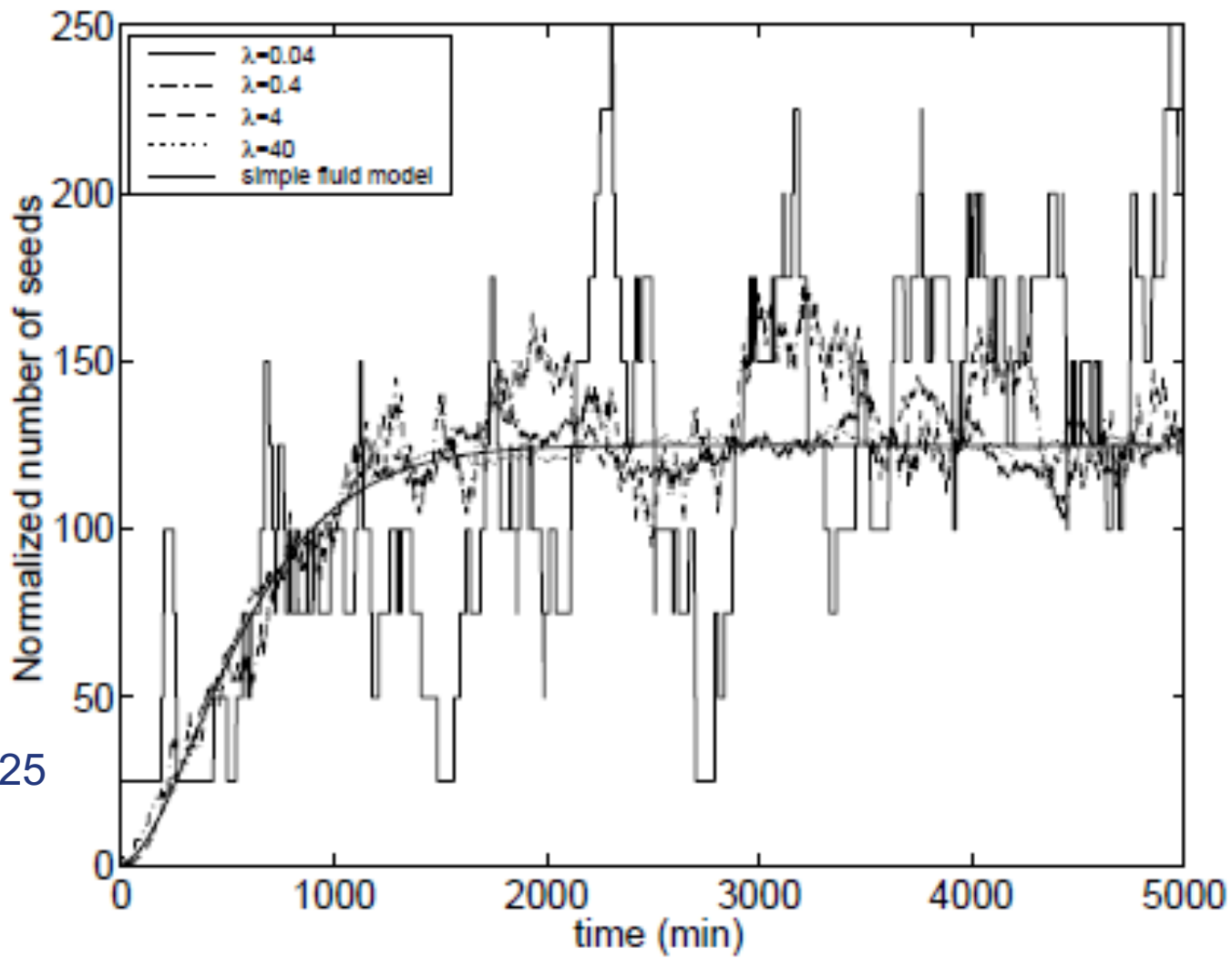
# Experiment 1



$\mu = 0.00125$   
 $\theta = 0.001$   
 $c = 0.002$   
 $\gamma = 0.001$

$\lambda$  : arrival rate of new request

# Experiment 2



$\mu = 0.00125$

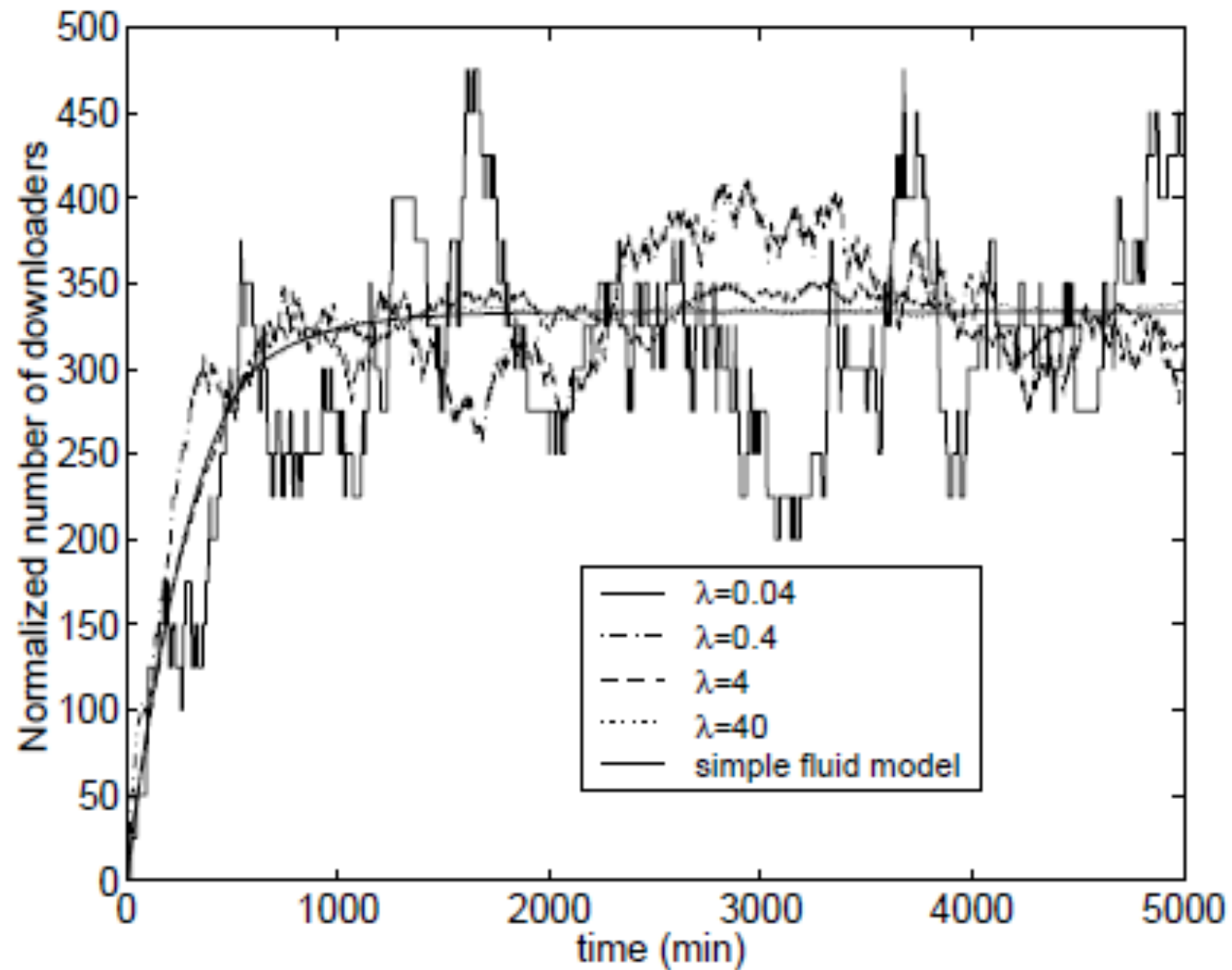
$\theta = 0.001$

$c = 0.002$

$\gamma = 0.005$

$\lambda$  : arrival rate of new request

# Experiment 2



$\mu = 0.00125$

$\theta = 0.001$

$c = 0.002$

$\gamma = 0.005$

$\lambda$  : arrival rate of new request

# Analysis of (1) & (2)

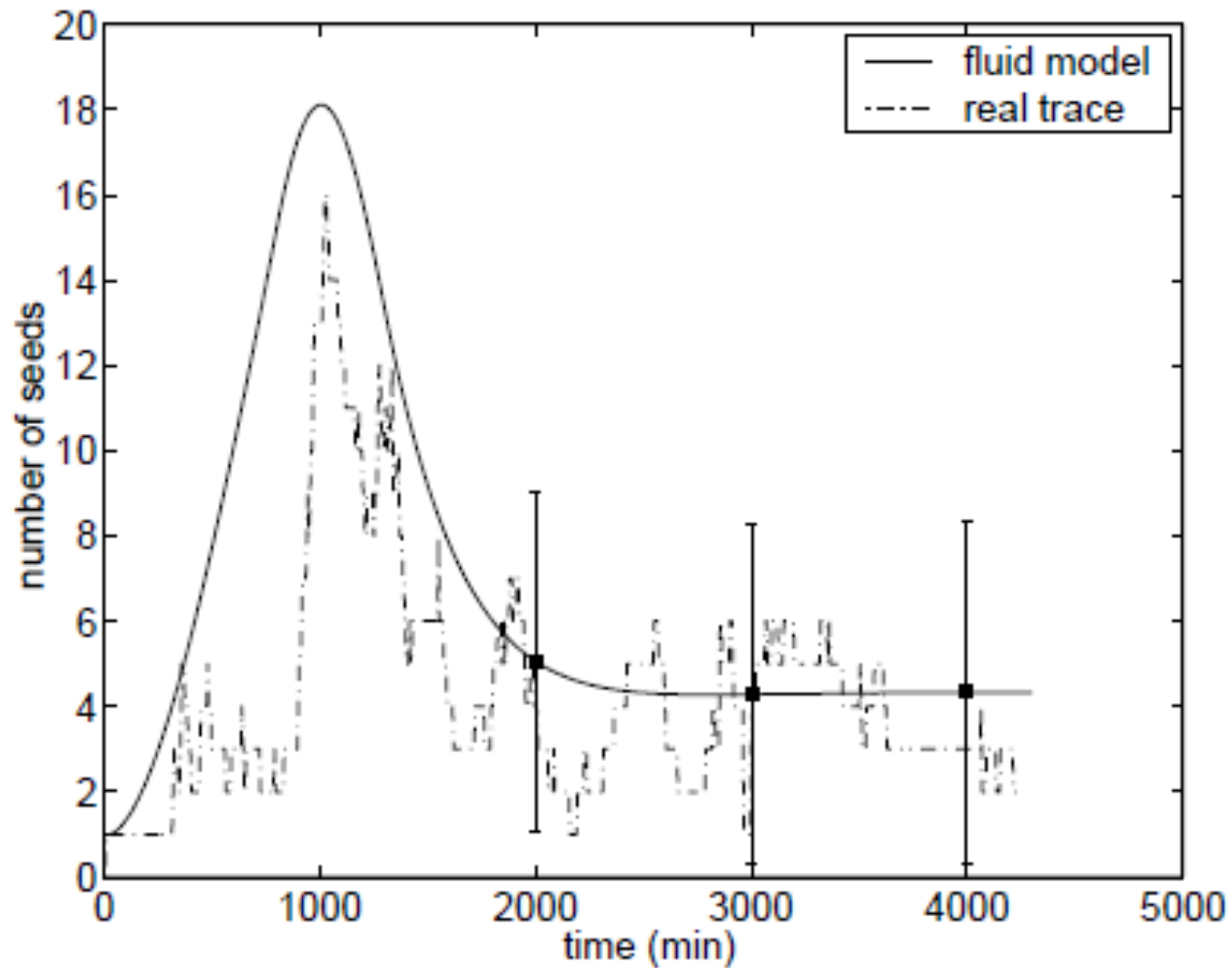
- ❖ The fluid model is a good approximation when the arrival rate  $\lambda$  is large regardless of the relationship between  $\mu$  and  $\gamma$ .
- ❖ The approximation is also good for small  $\lambda$ .

# Experiment 3 – Real-world Data

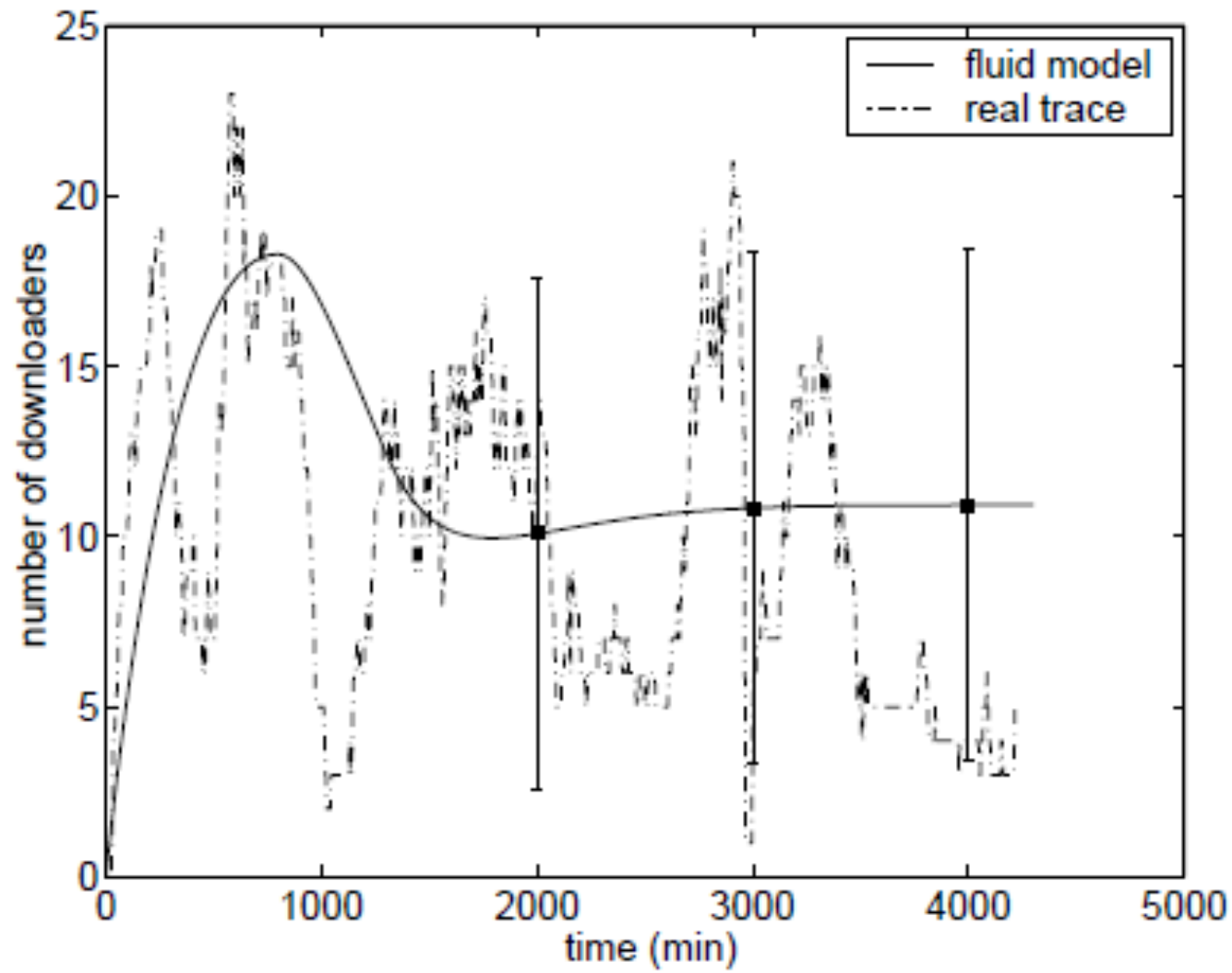
- ❖ Assume the system is upload-limited ( $\gamma > \mu$ ).
- ❖ Measured  $\lambda$  and  $\gamma$  are time-dependent
  - for  $t < 800$  min,  $\lambda = 0.06$   $\gamma = 0.001$
  - for  $t > 1300$  min,  $\lambda = 0.03$   $\gamma = 0.0044$
  - in between, they vary linearly
- ❖ Also numerically calculate the expected variation from the model as a 95% confidence interval.



# Experiment 3



# Experiment 3



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# Conclusions

- ❖ Presented a simple fluid model for BitTorrent-like networks.
- ❖ Studied the steady-state network performance and stability.
- ❖ Obtained insight into the effect of different parameters on network performance.
- ❖ The effect of optimistic unchoking on free-riding.
- ❖ The experiments show that the simple fluid model is able to capture the behavior of the system even when the arrival rate is small.