Performance Evaluation Math Notes

1. Why z-transform?

    (a) $6g_n - 5g_{n-1} + g_{n-2} = 6(\frac{1}{5})^n, n = 2, 3, 4, ..., g_0 = 0, g_1 = \frac{6}{5}$

$$
\begin{aligned}
6\sum_{n=2}^{\infty} g_n z^n - 5\sum_{n=2}^{\infty} g_{n-1} z^n + \sum_{n=2}^{\infty} g_{n-2} z^n &= 6\sum_{n=2}^{\infty} (\frac{1}{5})^n z^n \\
6\sum_{n=2}^{\infty} g_n z^n - 5z\sum_{n=2}^{\infty} g_{n-1} z^{n-1} + z^2\sum_{n=2}^{\infty} g_{n-2} z^{n-2} &= 6\sum_{n=2}^{\infty} (\frac{z}{5})^n \\
6(G(z) - g_0 - g_1 z) - 5z(G(z) - g_0) + z^2 G(z) &= \frac{6z^2}{5(5-z)} \\
G(z)(z-2)(z-3) - \frac{36z}{5} &= \frac{6z^2}{5(5-z)}
\end{aligned}
$$

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\begin{aligned}
G(z) &= \frac{6z(6-z)}{(2-z)(3-z)(5-z)} \\
&= \frac{z(6-z)}{5(1-\frac{z}{2})(1-\frac{z}{3})(1-\frac{z}{5})} \\
&= \frac{8}{1-\frac{z}{2}} + \frac{-9}{1-\frac{z}{3}} + \frac{1}{1-\frac{z}{5}} \\
&= 8\sum_{n=0}^{\infty}(\frac{1}{2})^n z^n - 9\sum_{n=0}^{\infty}(\frac{1}{3})^n z^n + \sum_{n=0}^{\infty}(\frac{1}{5})^n z^n \\
g(z) &= 8(\frac{1}{2})^n - 9(\frac{1}{3})^n + (\frac{1}{5})^n
\end{aligned}
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    (b) $a_{n+1} = 2a_n + 3^n + 5n, n = 1, 2, 3, ..., a_0 = -3$

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\begin{aligned}
\sum_{n=0}^{\infty} a_{n+1} z^n &= 2\sum_{n=0}^{\infty} a_n z^n + \sum_{n=0}^{\infty} 3^n z^n + 5\sum_{n=0}^{\infty} n z^n \\
\frac{A(z) - a_0}{z} &= 2A(z) + \frac{1}{1-3z} + \frac{5z}{(1-z)^2} \\
A(z) + 3 &= 2zA(z) + \frac{z}{1-3z} + \frac{5z^2}{(1-z)^2} \\
A(z) &= \frac{-3}{1-2z} + \frac{z}{(1-2z)(1-3z)} + \frac{5z^2}{(1-2z)(1-z)^2} \\
&= \frac{-3}{1-2z} + \frac{1}{1-3z} - \frac{1}{1-2z} - \frac{5}{(1-z)^2} + \frac{5}{1-2z} \\
&= \frac{1}{1-2z} + \frac{1}{1-3z} - \frac{5}{(1-z)^2}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{1}{1-2z} + \frac{1}{1-3z} - \frac{5z}{(1-z)^2} - \frac{5}{1-z} \\
&= \sum_{n=0}^{\infty} 2^n z^n + \sum_{n=0}^{\infty} 3^n z^n - 5\sum_{n=0}^{\infty} n z^n - 5\sum_{n=0}^{\infty} z^n
\end{aligned}
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$$
a_n = 2^n + 3^n - 5n - 5
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