

Performance Evaluation HW1

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1. I think the statement is wrong. If it is true, then exactly the same situation (expectation value equals to $1.25Y$) will happen after we decide to swap to another door, this means that the action of swapping could last forever. A much more reasonable explanation is as follows. Assume we choose door A, then the expectation value of A is $\frac{1}{2} \cdot x + \frac{1}{2} \cdot 2x = 1.5x$. The expectation value of B, which is another door that may contains twice or half of money of A, is still $\frac{1}{2} \cdot x + \frac{1}{2} \cdot 2x = 1.5x$, which is equals to that of A. Therefore, it doesn't matter whether we swap the door or not. Reference: https://en.wikipedia.org/wiki/Two_envelopes_problem

2. Assume box A is the box that contains two red balls, box B is the box that contains two blue balls, and box C is the box that contains a red ball and a blue ball.

(a) $P(\text{The second ball is also blue.} \mid \text{The first ball is blue.})$

$$\begin{aligned} &= \frac{P(\text{Both the first and second balls are blue.})}{P(\text{The first ball is blue.})} \\ &= \frac{\frac{1}{3}}{\frac{1}{3} + \frac{1}{3} \cdot \frac{1}{2}} = \frac{2}{3} \end{aligned}$$

(b) $P(\text{The second ball is also blue.} \mid \text{There is one blue ball in the chosen box.})$

$$\begin{aligned} &= \frac{P(\text{Both balls are blue.})}{P(\text{There is one blue ball in the chosen box.})} \\ &= \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2} \end{aligned}$$

3. X_i are i.i.d. exponentially distributed $\Rightarrow f_{X_i}(x) = \lambda e^{-\lambda x}, \lambda \geq 0$.

$$\begin{aligned} F_{X_i}^*(s) &= \int_0^\infty \lambda e^{-\lambda x} e^{-sx} dx \\ &= \int_0^\infty \lambda e^{-(s+\lambda)x} dx \\ &= \frac{\lambda}{s+\lambda} [-e^{-(s+\lambda)x}]_0^\infty \\ &= \frac{\lambda}{s+\lambda} \end{aligned}$$

\tilde{N} is geometrically distributed $\Rightarrow P_{\tilde{N}(n=i)} = p(1-p)^{i-1}$.

$$\begin{aligned}\tilde{N}(z) &= \sum_{i=1}^{\infty} p(1-p)^{i-1} z^i \\ &= \frac{zp}{1 - (1-p)z}\end{aligned}$$

$$\begin{aligned}Y^*(s) &= \tilde{N}(F_{X_i}^*(s)) \\ &= p \cdot \frac{F_{X_i}^*(s)}{1 - (1-p)F_{X_i}^*(s)} \\ &= \frac{p \cdot \frac{\lambda}{s+\lambda}}{1 - (1-p)\frac{\lambda}{s+\lambda}} \\ &= \frac{\lambda p}{s + \lambda p}\end{aligned}$$

$$(a) \ E(Y) = \left[-\frac{dY^*(s)}{ds}\right]_{s=0} = \left[-\frac{-\lambda p}{(s+\lambda p)^2}\right]_{s=0} = \frac{1}{\lambda p}$$

$$(b) \ Var(Y) = \left[\frac{d^{(2)}Y^*(s)}{ds^2}\right]_{s=0} = \left[\frac{2\lambda p}{(s+\lambda p)^3}\right]_{s=0} = \frac{2}{(\lambda p)^2}$$